WATER RESOURCES RESEARCH, VOL. 47, W08520, doi:10.1029/2011WR010728, 2011

A new model for predicting relative nonwetting phase permeability from soil water retention curves

Xingxing Kuang[1] and Jiu Jimmy Jiao[1]

Received 30 March 2011; revised 18 June 2011; accepted 29 June 2011; published 19 August 2011.

[1] Relative permeability of the nonwetting phase in a multiphase flow in porous media is a function of phase saturation. Specific expressions of this function are commonly determined by combining soil water retention curves with relative nonwetting phase permeability models. Experimental evidence suggests that the relative permeability of the nonwetting phase can be significantly overestimated by the existing relative permeability models. A new model for the prediction of relative nonwetting phase permeability from soil water retention curves is proposed in this paper. A closed form expression can be obtained in combination with soil water retention curves. The model is mathematically simple and can easily and efficiently be implemented in numerical models of multiphase flow processes in porous media. The predicting capability of the proposed model is contrasted with well-supported models by comparing the measured and predicted relative air permeability data for 11 soils, representing a wide range of soil textures, from sand to silty clay loam. In most of the cases the proposed model improves the agreement between the predicted relative air permeability and the measured data.

Citation: Kuang, X., and J. J. Jiao (2011), A new model for predicting relative nonwetting phase permeability from soil water retention curves, *Water Resour. Res., 47*, W08520, doi:10.1029/2011WR010728.

1. Introduction

[2] Relative nonwetting phase permeability is an important parameter to many science and engineering fields, such as soil and agriculture sciences, petroleum engineering, hydrology, and environmental engineering [*Springer et al.*, 1995; *Dury et al.*, 1999]. It is indispensable for subsurface multiphase flow numerical modeling [*Touma and Vauclin*, 1986; *Kueper and Frind*, 1991; *Celia and Binning*, 1992]. In many cases, the nonwetting phase is air and the wetting phase is water.

[3] The relative permeability of air is a function of air saturation, or equivalently, a function of water saturation. However, experimental work on the determination of this function is limited [e.g., *Collis-George*, 1953; *Brooks and Corey*, 1964; *Stonestrom and Rubin*, 1989; *Detty*, 1992; *Stylianou and DeVantier*, 1995; *Dury et al.*, 1998; *Springer et al.*, 1998; *Tuli and Hopmans*, 2004]. *Springer et al.* [1995] presented a comprehensive review on laboratory measurement of air permeability.

[4] There are basically two categories of models to describe the relative air permeability–saturation relationships, i.e., the empirical and the statistical model. The empirical model expresses the relative air permeability as a power function of water saturation [*Corey*, 1954; *Pirson*, 1958; *Wyllie*, 1962; *Falta et al.*, 1989]. In the statistical model, specific expressions of relative air permeability are derived from soil water retention curves (also called the capillary pressure–saturation relationship). A large number of functional forms of the soil water retention curves exist in the literature [e.g., *Gardner*, 1958; *Brooks and Corey*, 1964; *Farrel and Larson*, 1972; *van Genuchten*, 1980; *Fredlund and Xing*, 1994; *Kosugi*, 1994; *Assouline et al.*, 1998]. These soil water retention curves can be used in combination with relative permeability models [e.g., *Burdine*, 1953; *Mualem*, 1976] to derive specific expressions for the relative air permeability-saturation relationship.

[5] Using *Burdine*'s [1953] theory, *Brooks and Corey* [1964] derived the Brooks and Corey-Burdine (BCB) model for relative nonwetting phase permeability. *Parker et al.* [1987] derived an expression for relative air permeability on the basis of the *van Genuchten* [1980] capillary pressure–saturation relationship and *Mualem*'s [1976] relative hydraulic conductivity model, namely, the van Genuchten-Mualem (VGM) model. *Demond and Roberts* [1993] derived the expression of the van Genuchten-Burdine (VGB) model [*Burdine* 1953; *van Genuchten*, 1980] for relative nonwetting phase permeability. The Brooks and Corey-Mualem (BCM) model is presented by *Dury et al.* [1999]. Based on the *Gardner* [1958] relative wetting phase permeability model, *Russo* [1988] derived the *Gardner* [1958] capillary pressure–saturation relationship, after which the Gardner-Mualem (GM) relative nonwetting phase permeability model can be obtained [*Chen et al.*, 1999]. *Brutsaert* [1967] derived the closed form Brutsaert-Burdine (BRB) model. The lognormal distribution-Mualem (LNM) model [*Kosugi*, 1994, 1996; *Mualem*, 1976] is given by *Chen et al.* [1999].

[6] The aim of this paper is to derive a new model which improves the prediction of relative nonwetting phase permeability from soil water retention curves in a two-phase flow through porous media. Model performance is subsequently

[1]Department of Earth Sciences, University of Hong Kong, Hong Kong.

0043-1397/11/2011WR010728

evaluated by comparing results with the measured data and two well-supported models.

2. Theory

2.1. General Model

[7] Relative nonwetting phase permeability is usually defined as the ratio of the effective nonwetting phase permeability to the intrinsic permeability of the porous medium [*Brooks and Corey*, 1964; *Demond and Roberts*, 1993],

$$k_{rn}(S) = \frac{k_n(S)}{k}, \tag{1}$$

where k_{rn} is the relative permeability of the nonwetting phase, k_n is the effective permeability of the nonwetting phase as a function of phase saturation S (volumetric phase content $\theta = \phi S$, where ϕ is the porosity of the porous medium), and k is the intrinsic permeability of the porous medium. *Hoffmann-Riem et al.* [1999] proposed a general model for relative wetting phase permeability. *Brooks and Corey* [1964] and *Parker et al.* [1987] show that the relative permeability of the nonwetting phase can be obtained by changing the integration interval of the relative wetting phase permeability model. After *Hoffmann-Riem et al.* [1999], the general expression for relative nonwetting phase permeability can be written as

$$k_{rn}(S_{ew}) = (1 - S_{ew})^{\mu} \left\{ \frac{\int_{S_{ew}}^{1} [h(x)]^{-\beta} dx}{\int_0^1 [h(x)]^{-\beta} dx} \right\}^{\eta}, \tag{2}$$

where S_{ew} is the effective wetting phase saturation given by

$$S_{ew}(h) = \frac{\theta_w(h) - \theta_{rw}}{\theta_{sw} - \theta_{rw}}, \tag{3}$$

in which θ_{rw} and θ_{sw} are the residual and saturated volumetric wetting phase content, respectively, x is a dummy variable for integration representing S_{ew} in the inverted function $h(S_{ew})$ of the capillary pressure–saturation relationship. The values of the parameters μ, β, and η can be varied to derive more specific expressions. For the Burdine model [*Burdine*, 1953], $\mu = 2$, $\beta = 2$, and $\eta = 1$. For the Mualem model [*Mualem*, 1976], $\mu = 1/2$, $\beta = 1$, and $\eta = 2$. *Assouline* [2001] derived a model that lumps all the unknown powers into one value η with $\beta = 1$. On the basis of the Mualem model [*Mualem*, 1976], *Luckner et al.* [1989] assumed $\mu = 1/3$, $\beta = 1$, and $\eta = 2$. However, experimental evidence shows that large discrepancies can be observed between predicted and measured data [*Collis-George*, 1953; *Demond and Roberts*, 1993; *Dury et al.*, 1999; *Fagerlund et al.*, 2006].

2.2. Proposed Model

[8] On the basis of *Mualem*'s [1976] work for relative wetting phase permeability, the expression for relative nonwetting phase permeability can be written as

$$k_{rn}(\theta_w) = T(R, r, \rho) G(R, r, \rho) \left[\frac{\int_R^{R_{\max}} r f(r) dr}{\int_{R_{\min}}^{R_{\max}} r f(r) dr} \right]^2, \tag{4}$$

where r and ρ are pore radii in a homogeneous porous medium, $R_{\min}$ and $R_{\max}$ are the minimum and maximum pore radii, respectively, $f(r)$ is the function describing pore water distribution, $T(R, r, \rho)$ is the tortuosity factor (a correction factor to account for flow path eccentricity), and $G(R, r, \rho)$ is a partial correlation factor (a correction factor to account for partial correlation between the pores r and ρ at a given water content θ_w) [*Mualem*, 1976]. *Assouline* [2001] derived an expression for the tortuosity factor $T(R, r, \rho)$ as a power function. According to *Carman* [1937] and *Porter et al.* [1960], a power of 2 was further assumed herein:

$$T(R, r, \rho) = \left[\frac{\int_R^{R_{\max}} r f(r) dr}{\int_{R_{\min}}^{R_{\max}} r f(r) dr} \right]^2. \tag{5}$$

The partial correlation factor $G(R, r, \rho)$ is generally assumed to be a power function of S_{ew} for relative wetting phase permeability [*Burdine*, 1953; *Millington and Quirk*, 1961; *Mualem*, 1976]. For nonwetting phase, $G(R, r, \rho)$ can be expressed as

$$G(R, r, \rho) = (1 - S_{ew})^{\mu}. \tag{6}$$

Substituting (5) and (6) into (4) leads to

$$k_{rn}(\theta_w) = (1 - S_{ew})^{\mu} \left[\frac{\int_R^{R_{\max}} r f(r) dr}{\int_{R_{\min}}^{R_{\max}} r f(r) dr} \right]^4. \tag{7}$$

Applying the capillary law $r = C/h$ ($C = 2\gamma$, where γ is the surface tension of the wetting phase) and the relationship $d\theta_w = f(r) dr$ [*Mualem*, 1976] in (7) leads to

$$k_{rn}(\theta_w) = (1 - S_{ew})^{\mu} \left[\frac{\int_{\theta_w}^{\theta_{sw}} \frac{1}{h} d\theta_w}{\int_0^{\theta_{sw}} \frac{1}{h} d\theta_w} \right]^4. \tag{8}$$

For an analytical capillary pressure–saturation relationship $h(\theta_w)$, a specific expression can be derived for $k_{rn}(\theta_w)$. On the basis of the analysis of 45 soils representing a wide range of texture, *Mualem* [1976] pointed out that $\mu = 1/2$ is the best value for the partial correlation factor. Hence, the proposed model for $k_{rn}(S_{ew})$ becomes

$$k_{rn}(S_{ew}) = (1 - S_{ew})^{1/2} \left[\frac{\int_{S_{ew}}^{1} \frac{1}{h} dS_{ew}}{\int_0^1 \frac{1}{h} dS_{ew}} \right]^4. \tag{9}$$

Comparing (9) with (2) leads to $\mu = 1/2$, $\beta = 1$, and $\eta = 4$.

[9] To solve (9), an expression relating the effective wetting phase saturation S_{ew} to the capillary pressure head h is required. The following expression is presented by *van Genuchten* [1980] for the h–S_{ew} relationship

$$S_{ew}(h) = \frac{1}{[1 + (\alpha h)^n]^m}, \tag{10}$$

where α (>0) is related to the inverse of the air entry pressure, and n (>1) is a measure of the pore size distribution.

Solving (10) for $h = h(S_{ew})$ and then substituting the resulting expression into (9) leads to

$$k_{rn}(S_{ew}) = (1 - S_{ew})^{1/2}(1 - S_{ew}^{1/m})^{4m}, \quad m = 1 - 1/n. \tag{11}$$

Equation (11) is the expression of the relative nonwetting phase permeability function when the van Genuchten capillary pressure–saturation relationship is combined with the proposed model (equation (9)).

[10] Applying the Mualem model to (10), the resulting expression is [*Parker et al.*, 1987]

$$k_{rn}(S_{ew}) = (1 - S_{ew})^{1/2}(1 - S_{ew}^{1/m})^{2m}, \quad m = 1 - 1/n. \tag{12}$$

Equation (12) is referred to as the VGM model. This model is widely used in the literature for multiphase-flow investigations [e.g., *Finsterle and Pruess*, 1995; *Jacobs and Gelhar*, 2005; *Papafotiou et al.*, 2008; *Amaziane et al.*, 2010]. The VGM model is used herein as a reference model.

[11] As a representative of the empirical models, the Corey model [*Corey*, 1954] is expressed as

$$k_{rn}(S_{ew}) = (1 - S_{ew})^2(1 - S_{ew}^2). \tag{13}$$

Corey's model is also widely applied in the investigation of multiphase flow problems in porous media [e.g., *Demond and Roberts*, 1993; *Pruess et al.*, 1999; *Vasco*, 2004]. The Corey model is used as another reference model to compare with the proposed model.

3. Results and Discussion

3.1. Testing Data Sets

[12] Experimental data sets are selected from the literature to evaluate the predicting capability of the proposed model. These data sets of soils were selected because both measured soil water retention curve and relative air permeability data are available. These soils represent a wide range of soil structure and texture from sand to silty clay loam (Table 1).

[13] The van Genuchten soil water retention function (10) is fitted to the measured data. For each soil the parameters θ_{rw}, θ_{sw}, α, and n were determined, using an iterative nonlinear regression procedure on the basis of the Marquardt-Levenberg algorithm. The values are shown in Table 1.

3.2. Illustrative Examples

[14] Comparisons between predicted and measured relative air permeability curves for four soils are given in this section (Figure 1). The referenced relative air permeability models (VGM model and Corey model) are computed in each case.

[15] The experimental data sets for Oakley sand [*Stonestrom*, 1987; *Stonestrom and Rubin*, 1989] were taken from *Dury et al.* [1999]. This soil has a rather narrow pore size distribution, which is indicated by the relatively high n value. Figure 1 shows that the calculated soil water retention curve is in very good agreement with the measured data. Furthermore, the relative air permeability predicted by the proposed model is also in very good agreement with the measured data. However, the reference models significantly overestimate the relative air permeability over the entire range of water saturation.

[16] The experimental data sets for mixed sand [*Dury*, 1997; *Dury et al.*, 1998]; were also taken from *Dury et al.* [1999]. As can be seen from Figure 1, somewhat similar results are obtained. The proposed model predicts the relative air permeability fairly well and slightly overestimates the experimental data only when the water saturation is larger than 0.4. However, the VGM model overestimates the data over almost the entire range of water saturation, and the Corey model overestimates the data when water saturation is greater than 0.25.

[17] For the Amarillo silty clay loam [*Brooks and Corey*, 1964], the proposed model predicts the relative air permeability very satisfactorily over the entire range of water saturation. The Corey model also presents a reasonable fit. However, the VGM model still significantly overestimates the experimental data.

[18] For the Grenoble sand [*Touma and Vauclin*, 1986], Figure 1 shows that in this case the Corey model performs the best. Both the VGM and the proposed model overestimate the relative air permeability when water saturation is less than 0.7.

3.3. Statistical Analysis

[19] Further comparison of the proposed model and the two existing models with more measured data sets are carried out by means of the root-mean-square error (RMSE), which is an indicator of the magnitude of the differences

Table 1. Fitted Parameters for the Van Genuchten Soil Water Retention Function

Soil Name	ϕ	θ_{sw}	θ_{rw}	α (cm^{-1})	n	Reference
Mixed sand	0.360	0.285	0.0216	0.031	5.65	*Dury* [1997]
Oakley sand	0.365	0.314	0.102	0.023	5.62	*Stonestrom* [1987]
Grenoble sand	0.370	0.312	0.0265	0.044	2.22	*Touma et al.* [1984]
Silty sand	0.431	0.431	0.0138	0.040	1.52	*Springer et al.* [1998]
Cambridge sand	0.380	0.380	0.0327	0.069	8.00	*Collis-George* [1953]
Fine sand	0.377	0.377	0.066	0.021	6.20	*Brooks and Corey* [1964]
Poudre river sand	0.364	0.364	0.0455	0.059	6.20	*Brooks and Corey* [1964]
Volcanic sand	0.351	0.351	0.055	0.045	4.20	*Brooks and Corey* [1964]
Glass beads	0.370	0.370	0.036	0.031	11.5	*Brooks and Corey* [1964]
Berea sandstone	0.206	0.206	0.0616	0.020	5.80	*Brooks and Corey* [1964]
Amarillo silty clay loam	0.455	0.455	0.114	0.020	4.50	*Brooks and Corey* [1964]

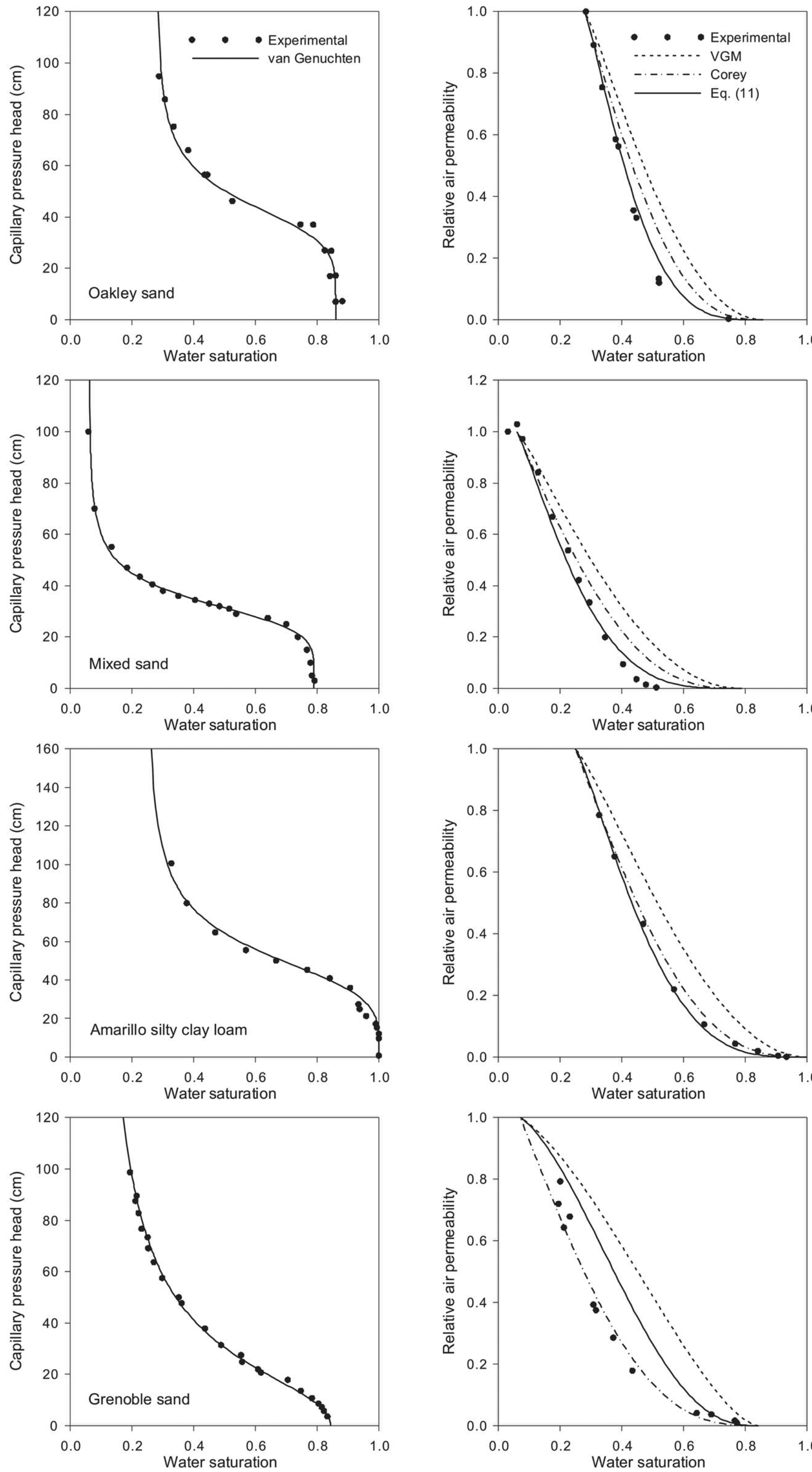

Figure 1. A comparison of measured data for four soils with predicted results. (left) Soil water retention curve and (right) relative air permeability.

Table 2. RMSE Values Obtained With the VGM Model, the Corey Model, and Equation (11) for the Relative Air Permeability

Soil Name	RMSE		
	VGM	Corey	Equation (11)
Mixed sand	0.146	0.076	**0.037**
Oakley sand	0.173	0.100	**0.037**
Grenoble sand	0.224	**0.049**	0.149
Silty sand	0.161	0.328	**0.106**
Cambridge sand	0.090	**0.045**	0.053
Fine sand	0.111	0.065	**0.022**
Poudre river sand	0.105	0.057	**0.020**
Volcanic sand	0.095	0.023	**0.021**
Glass beads	0.128	0.113	**0.051**
Berea sandstone	0.271	0.192	**0.115**
Amarillo silty clay loam	0.109	0.023	**0.012**
Mean RMSE	0.147	0.097	**0.057**

between the predicted and measured data. The RMSE is computed as

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[k_{rn,i} - k_{rn}(S_{ewi})\right]^2}, \quad (14)$$

where N is the number of measurements in the data set, and $k_{rn,i}$ and $k_{rn}(S_{ewi})$ are the measured and predicted relative air permeability, respectively. Table 2 summarizes the RMSE of the proposed model and the other two models for 11 data sets, including four sets elaborated in section 3.2. For each soil, the lowest values of RMSE are highlighted in bold. Table 2 shows that the proposed model (equation (11)) is the best model for nine out of 11 testing data sets. The averaged value of RMSE for (11) is 0.057, which is 2.6 times smaller than that of the VGM model and 1.7 times smaller than that of the Corey model. As shown, the proposed model improves the agreement between the predicted and measured data. It should be noted that the Corey model has one fitting parameter less than the VGM and the proposed model. However, Table 2 shows that it is the best model in two cases.

[20] In (9), the parameter $\mu = 1/2$ was assumed. In order to investigate the impact of μ on the predicted results, seven different values of μ were used: $\mu = -1 + 0.5i$, $i = 0, 1, \ldots, 6$. For each soil and each value of i, the RMSE was computed. For each value i the average RMSE was computed for the 11 soils. Figure 2 presents the variation of the average RMSE with μ, which shows that $\mu = 1/2$ may indeed be considered as the optimal value.

[21] Figure 3 shows scatter charts of measured versus predicted relative air permeability values. It can be seen from Figure 3 that the VGM model tends to generally overestimate relative air permeability over the entire range of water saturation. The Corey model overestimates the measured data to a lesser extent but can underestimate the measured data significantly for relatively fine textured soils. In contrast, the values predicted by the proposed model are close to the measured values, which is shown by a much better linearship.

4. Conclusions

[22] A new model is proposed to predict the relative nonwetting phase permeability from soil water retention curves. The performances of the proposed model are tested on 11 data sets and compared with two other well-supported models. In most of the cases, the relative air permeability predicted by the proposed model is in better agreement with the measured data. The VGM model generally overestimates the measured data for all of the cases.

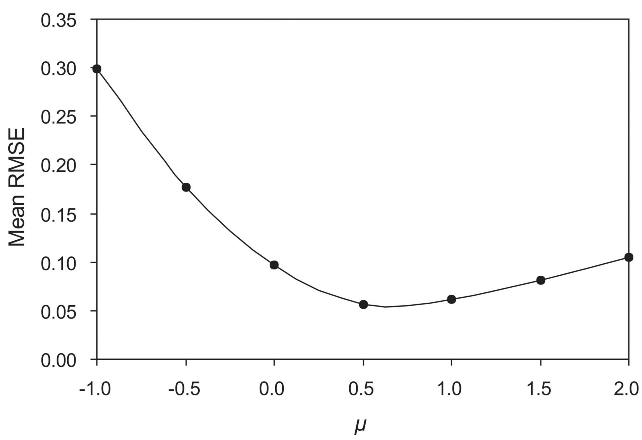

Figure 2. Variations of the mean RMSE with μ.

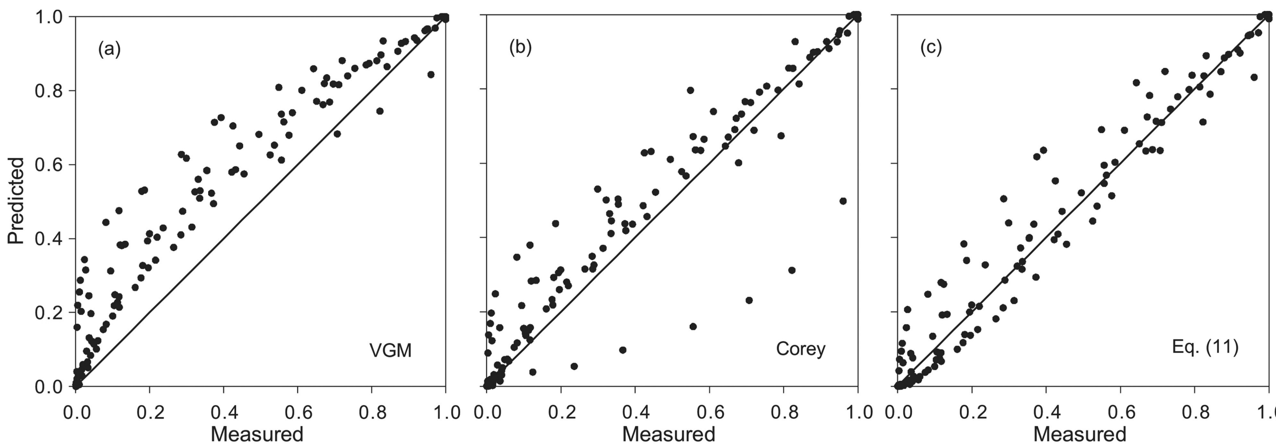

Figure 3. Scatter charts of measured versus predicted relative air permeability values by (a) the VGM model, (b) the Corey model, and (c) equation (11).

The Corey model overestimates the measured data to a lesser extent but can significantly underestimate the measured data in some cases. The proposed model is mathematically simple and can easily be integrated into existing numerical models of multiphase flow phenomena in porous media.

[23] **Acknowledgments.** The authors thank the reviewers for their insightful comments. This research was supported by the Research Grants Council of the Hong Kong Special Administrative Region, China (HKU 701908P).

References

Amaziane, B., M. Jurak, and A. Ž. Keko (2010), Modeling and numerical simulations of immiscible compressible two-phase flow in porous media by the concept of global pressure, *Transp. Porous Media*, *84*, 133–152, doi:10.1007/s11242-009-9489-8.

Assouline, S. (2001), A model for soil relative hydraulic conductivity based on the water retention characteristic curve, *Water Resour. Res.*, *37*(2), 265–271. (Correction, *Water Resour. Res.*, *40*, W02901, doi:10.1029/2004WR003025.)

Assouline, S., D. Tessier, and A. Bruand (1998), A conceptual model of the soil water retention curve, *Water Resour. Res.*, *34*(2), 223–231. (Correction, *Water Resour. Res.*, *36*, 3769, doi:10.1029/97WR03039.)

Brooks, R. H., and A. T. Corey (1964), Hydraulic properties of porous media, *Hydrol. Pap. 3*, Civ. Eng. Dep., Colo. State Univ., Fort Collins, Colo.

Brutsaert, W. (1967), Some methods of calculating unsaturated permeability, *Trans. ASAE*, *10*(3), 400–404.

Burdine, N. T. (1953), Relative permeability calculations from pore size distribution data, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *198*, 71–78.

Carman, P. C. (1937), Fluid flow through granular beds, *Trans. Inst. Chem. Eng.*, *15*, 150–166.

Celia, M. A., and P. Binning (1992), A mass conservative numerical solution for two-phase flow in porous media with application to unsaturated flow, *Water Resour. Res.*, *28*(10), 2819–2828, doi:10.1029/92WR01488.

Chen, J., J. W. Hopmans, and M. E. Grismer (1999), Parameter estimation of two-fluid capillary pressure-saturation and permeability functions, *Adv. Water Resour.*, *22*(5), 479–493.

Collis-George, N. (1953), Relationship between air and water permeabilities in porous media, *Soil Sci.*, *76*(4), 239–250.

Corey, A. T. (1954), The interrelation between gas and oil relative permeabilities, *Prod. Mon.*, *19*, 32–41.

Demond, A. H., and P. V. Roberts (1993), Estimation of two-phase relative permeability relationships for organic liquid contaminants, *Water Resour. Res.*, *29*(4), 1081–1090, doi:10.1029/92WR02987.

Detty, T. E. (1992), Determination of air and water relative permeability relationships for selected unconsolidated porous materials, M. S. thesis, Univ. of Ariz., Tucson, Ariz.

Dury, O. (1997), Organic pollutants in unsaturated soils: Effect of butanol as a model contaminant on phase saturation and flow characteristics of a quartz sand packing, Ph.D. thesis, Swiss Fed. Inst. of Technol., Zürich, Switzerland.

Dury, O., U. Fischer, and R. Schulin (1998), Dependence of hydraulic and pneumatic characteristics of soils on a dissolved organic compound, *J. Contam. Hydrol.*, *33*, 39–57.

Dury, O., U. Fischer, and R. Schulin (1999), A comparison of relative nonwetting-phase permeability models, *Water Resour. Res.*, *35*(5), 1481–1493, doi:10.1029/1999WR900019.

Fagerlund, F. F., A. Niemi, and M. Odén (2006), Comparison of relative permeability-fluid saturation-capillary pressure relations in the modelling of non-aqueous phase liquid infiltration in variably saturated, layered media, *Adv. Water Resour.*, *29*, 1705–1730.

Falta, R. W., I. Javandel, K. Pruess, and P. A. Witherspoon (1989), Density-driven flow of gas in the unsaturated zone due to the evaporation of volatile organic compounds, *Water Resour. Res.*, *25*(10), 2159–2169, doi:10.1029/WR025i010p02159.

Farrell, D. A., and W. E. Larson (1972), Modeling the pore structure of porous media, *Water Resour. Res.*, *8*(3), 699–706, doi:10.1029/WR008i003p00699.

Finsterle, S., and K. Pruess (1995), Solving the estimation-identification problem in two-phase flow modeling, *Water Resour. Res.*, *31*(4), 913–924, doi:10.1016/S0301-9322(97)88605-3.

Fredlund, D. G., and A. Xing (1994), Equations for the soil-water characteristic curve, *Can. Geotech. J.*, *31*, 521–532.

Gardner, W. R. (1958), Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table, *Soil Sci.*, *85*(4), 228–232.

Hoffmann-Riem, H., M. Th. van Genuchten, and H. Flühler (1999), A general model of the hydraulic conductivity of unsaturated soils, in *Proc. Int. Workshop on Characterization and Measurement of the Hydraulic Properties of Unsaturated Porous Media*, edited by M. Th. van Genuchten, F. J. Leij, and L. Wu, pp. 31–42, University of California, Riverside, Calif.

Jacobs, B., and L. W. Gelhar (2005), Effective properties of two-phase flow in heterogeneous aquifers, *Water Resour. Res.*, *41*, W01018, doi:10.1029/2004WR003232.

Kosugi, K. (1994), Three-parameter lognormal distribution model for soil water retention, *Water Resour. Res.*, *30*(4), 891–901, doi:10.1029/93WR02931.

Kosugi, K. (1996), Lognormal distribution model for unsaturated soil hydraulic properties, *Water Resour. Res.*, *32*(9), 2697–2703, doi:10.1029/96WR01776.

Kueper, B. H., and E. O. Frind (1991), Two-phase flow in heterogeneous porous media 1. Model development, *Water Resour. Res.*, *27*(6), 1049–1057, doi:10.1029/91WR00266.

Luckner, L., M. Th. van Genuchten, and D. R. Nielsen (1989), A consistent set of parametric models for the two-phase flow of immiscible fluids in the subsurface, *Water Resour. Res.*, *25*(10), 2187–2193, doi:10.1029/WR025i010p02187.

Millington, R. J., and J. P. Quirk (1961), Permeability of porous solids, *Trans. Faraday Soc.*, *57*, 1200–1207.

Mualem, Y. (1976), A new model for predicting the hydraulic conductivity of unsaturated porous media, *Water Resour. Res.*, *12*(3), 513–522, doi:10.1029/WR012i003p00513.

Papafotiou, A., et al. (2008), From the pore scale to the lab scale: 3-D lab experiment and numerical simulation of drainage in heterogeneous porous media, *Adv. Water Resour.*, *31*, 1253–1268.

Parker, J. C., R. J. Lenhard, and T. Kuppusamy (1987), A parametric model for constitutive properties governing multiphase flow in porous media, *Water Resour. Res.*, *23*(4), 618–624, doi:10.1029/WR023i004p00618.

Pirson, S. J. (1958), *Oil Reservoir Engineering*, 326 pp., McGraw-Hill, New York.

Porter, L. K., W. D. Kemper, R. D. Jackson, and B. A. Stewart (1960), Chloride diffusion in soils as influenced by moisture content, *Soil Sci. Soc. Am. Proc.*, *24*, 460–463.

Pruess, K., C. Oldenburg, and G. Morisdis (1999), TOUGH2 user's guide, version 2.0, *Rep. LBNL-43134*, Lawrence Berkeley Natl. Lab, Berkeley, Calif.

Russo, D. (1988), Determining soil hydraulic properties by parameter estimation: On the selection of a model for the hydraulic properties, *Water Resour. Res.*, *24*(3), 453–459, doi:10.1029/WR024i003p00453.

Springer, D. S., S. J. Cullen, and L. G. Everett (1995), Laboratory studies on air permeability, in *Handbook of Vadose Zone Characterization and Monitoring*, pp. 217–247, edited by L. G. Wilson, L. G. Everett, and S. J. Cullen, Lewis, Boca Raton, Fla.

Springer, D. S., H. A. Loaiciga, S. J. Cullen, and L. G. Everett (1998), Air permeability of porous materials under controlled laboratory conditions, *Ground Water*, *36*(4), 558–565.

Stonestrom, D. A. (1987), Co-determination and comparison of hysteresis-affected, parametric functions of unsaturated soils: Water content dependence of matric pressure, air trapping, and fluid permeabilities in a non swelling soil, Ph.D. thesis, Stanford Univ., Stanford, Calif.

Stonestrom, D. A., and J. Rubin (1989), Air permeability and trapped-air content in two soils, *Water Resour. Res.*, *25*(9), 1959–1969, doi:10.1029/WR025i009p01959.

Stylianou, C., and B. A. DeVantier (1995), Relative air permeability as function of saturation in soil venting, *J. Environ. Eng.*, *121*(4), 337–347.

Touma, J., and M. Vauclin (1986), Experimental and numerical analysis of two-phase infiltration in a partially saturated soil, *Transp. Porous Media*, *1*, 27–55.

Tuli, A., and J. W. Hopmans (2004), Effect of degree of fluid saturation on transport coefficients in disturbed soils, *Eur. J. Soil. Sci.*, *55*(1), 147–164, doi:10.1046/j.1365-2389.2003.00551.x.

van Genuchten, M. Th. (1980), A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am. J.*, *44*, 892–898.

Vasco, D. W. (2004), An asymptotic solution for two-phase flow in the presence of capillary forces, *Water Resour. Res.*, *40*, W12407, doi:10.1029/2003WR002587.

Wyllie, M. R. J. (1962), Relative permeability, in *Petroleum Production Handbook*, vol. 2, *Reservoir Engineering*, pp. 25.1–25.14, Soc. of Pet. Eng., Richardson, Tex.

J. J. Jiao and X. Kuang, Department of Earth Sciences, University of Hong Kong, Pokfulam Road, Hong Kong. (jjiao@hku.hk)