# FULL-RANGE ANALYSIS OF MULTI-SPAN PRESTRESSED CONCRETE SEGMENTAL BRIDGES

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**ABSTRACT**

The in-situ concrete stitches of prestressed concrete segmental bridge are locations of potential weakness for the entire bridge deck but relatively little work has been carried out in this area. The effects of the performance of in-situ stitches on the global behaviour of bridge deck are not well understood. As most existing techniques cannot cope with such full-range analyses, a numerical technique has been developed for conducting full-range analyses of continuous prestressed concrete bridges under incremental loads or displacements. The bridge is modelled as a series of beam elements each of which is governed by the corresponding moment-curvature relationship of a representative section within it. While most of the existing techniques are only capable of analysing the behaviour of continuous prestressed concrete beams up to the peak load-carrying capacity, the present technique can extend well into the post-peak range, which is crucial to the investigation of ductility or deformability. The development and verification of the technique are presented in this paper.

**Keywords:** Full-range analysis, nonlinear analysis, post-peak behaviour, prestressed concrete, segmental bridges.

## 1. INTRODUCTION

The balanced cantilever method for the construction of precast concrete segmental bridges involves sequentially extending precast segments outwards from each pier in a roughly balanced manner. A gap of 100 to 200 mm in width is usually provided around the mid-span location between the last two approaching segments to facilitate erection. In-situ concrete is then cast to 'stitch' the segments together, thus making the bridge deck continuous. The in-situ stitch is usually designed to be capable of sustaining considerable sagging moment but only nominal hogging moment. If rupturing of the in-situ stitch occurred when the hogging moment became high under exceptional circumstances, it could potentially trigger a progressive collapse mechanism. The in-situ concrete stitches are therefore locations of potential weakness for the entire bridge deck, but relatively few studies have been conducted in this area. The effects of the performance of the in-situ stitches on

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the global behaviour of the entire bridge deck, namely moment redistribution and collapse mechanism, are not well understood.

The amount of moment redistribution that a bridge deck can undergo is strongly dependent upon the post-peak inelastic deformability of the structure at the plastic hinge locations. Moreover, whether the bridge deck will suffer localized damage only or experience progressive collapse in an extreme event actually depends on how the internal moments redistribute themselves. In other words, the post-peak behaviour of a structural element does have marked influence on its ability to redistribute moments and, more importantly, it does affect the robustness of a structure.

However, most of the existing numerical methods have been developed for analysing prestressed concrete elements up to the peak load-carrying capacity only. To study the effects of the performance of the in-situ stitches on moment redistribution and robustness, an appropriate method with the necessary numerical stability is necessary for the full-range nonlinear analysis of prestressed concrete structures. This has led to the development of a tailor-made numerical technique for the present study. The technique has been verified by comparing the calculated load-deflection response of various prestressed concrete beams against those obtained experimentally by previous researchers. The background development of the technique and the verification are presented in this paper.

## 2. METHOD OF ANALYSIS

### 2.1. General scheme

The continuous segmental bridge deck is idealized as a series of beam elements whose constitutive behaviour is governed by the corresponding moment-curvature relationship of a representative section within the element. Incremental load or displacement is then applied on the structure, upon which a series of iterations are performed to obtain the admissible nodal displacements that satisfy the constitutive behaviour of each element. In summary, the technique essentially involves three steps, namely (i) discretising the bridge deck with a series of beam elements; (ii) performing section analysis on a representative section within each element to obtain the moment-curvature relationship; and (iii) performing iterations to obtain the admissible nodal forces and displacements for each imposed load or displacement increment. They will be further elaborated below.

The present technique is to certain extent similar to that suggested by Warner and Yeo (1986) in that the prestressed concrete beam is idealized as a series of beam elements with pre-generated moment-curvature curves governing the flexural behaviour of elements. The technique of Warner and Yeo (1986) has been adopted later by Campbell and Kodur (1990) in conducting nonlinear analyses of prestressed concrete continuous beams. However, there are essentially two major differences between the present technique and the one developed by Warner and Yeo (1986), namely (i) the iteration scheme adopted; and (ii) the constitutive modelling of the unloading sections as the structure enters post-peak range.

The iteration scheme employed by Warner and Yeo (1986) is primarily based on updating of the secant stiffness of each element at each load increment to yield a set of displacements and internal forces that satisfy equilibrium. However, this method often leads to difficulty in achieving convergence for sections that have a steep post-peak branch in the moment curvature curve, as well as for unloading sections of which the unloading path intersects with the secant at an extremely small angle. In the proposed technique that ensures better convergence, the initial stiffness method is used. Instead of iterating based on the secant stiffnesses of elements, iterations will be carried out based on the residual curvatures of elements that vary, and the initial stiffnesses of elements that remain unchanged throughout the entire load range.

### 2.2. Finite element modelling and formulation

The segmental bridge deck is modelled as a continuous girder using a series of beam elements. In-plane translational and rotational degrees of freedom are associated with each of the two nodes of each element. The moment-curvature relationship of each element is governed by

$$M = EI(\phi - \phi_r) \tag{1}$$

where $M$ is the moment applied on a section, $EI$ is the flexural rigidity expressed in terms of the initial Young's modulus $E$ and second moment of area $I$, and $\phi$ and $\phi_r$ are the section curvature and section residual curvature respectively. The constitutive model is illustrated graphically in Figure 1 where the flexural rigidity $EI$ of the elastic region is used to define other parameters.

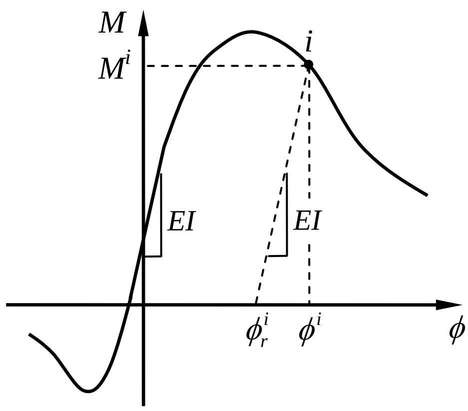


**Figure 1: Constitutive model of the beam element used.**

Subsequent derivation using the potential energy approach based on the constitutive model shown in Equation (1) yields the force-displacement relationship for each element

$$\mathbf{f} = \mathbf{K}\,\boldsymbol{\delta} - \int \mathbf{B}^{\mathrm{T}}(EI)\phi_r\,dx \tag{2}$$

where the stiffness matrix **K** and strain matrix **B** are given by

$$\mathbf{K} = \int \mathbf{B}^{\mathrm{T}} (EI) \mathbf{B} \, dx \tag{3}$$

$$\mathbf{B} = \begin{bmatrix} -\frac{6}{L^2} + \frac{12x}{L^3} & -\frac{4}{L} + \frac{6x}{L^2} & \frac{6}{L^2} - \frac{12x}{L^3} & -\frac{2}{L} + \frac{6x}{L^2} \end{bmatrix} \tag{4}$$

$x$ is the coordinate in the axial direction of element, and $L$ the length of the element. In Equation (2), the residual curvature $\phi_r$ is taken as that at the middle of element.

The finite element mesh of the structural model should be sufficiently fine so that the flexural rigidity can be assumed constant over the length of each element. Therefore, the length of element is chosen to be not exceeding the overall depth of the section. Elements located within the potential plastic hinges are identified and grouped during discretisation. All the elements that lie within a potential plastic hinge are assumed to have the same curvature as the element that first reaches the peak moment capacity in the corresponding group. The potential plastic hinge is centred at the point where there is a local peak in the bending moment diagram. The plastic hinge length is estimated approximately using the formula by Mattock (1967), which has been adopted by researchers such as Du et al. (2008) in analyzing the ductility of prestressed concrete beams with unbonded tendons. The formula for estimating the plastic hinge length $l_p$ is

$$l_p = 0.5d_p + 0.05Z \tag{5}$$

where $d_p$ is the depth to the centroid of the prestressing tendon; $Z$ is the shear span or the distance between the point of maximum moment and point of contra-flexure. It is believed that the length of the plastic hinge can have marked influence on the results. Therefore, the formulae for plastic hinge length as proposed by various researchers will also be examined. Parametric study will be conducted in the near future to investigate the effects of the plastic hinge length on the moment redistribution of bridge deck.

### 2.3. Section analysis

Each element in the continuous girder model is assigned a pre-generated moment-curvature curve that governs its flexural behaviour. To obtain the moment-curvature curve as illustrated in Figure 1, section analysis is performed for a representative section chosen at the middle of the element. Such analysis is done numerically by a computer programme developed based on the approach of Ho et al. (2003), which is intended for analysing reinforced concrete sections. Modifications have been made such that fully and partially prestressed sections can be analysed.

In the computer programme for section analyses, both the non-prestressing steel and prestressing steel are assumed to be perfectly bonded to the concrete. An iterative process with the prescribed curvature applied incrementally is adopted. At each iteration step, the strain variation is determined assuming that plane sections remain plane after bending, and the stresses in the concrete and steel are evaluated from their respective constitutive models. Axial equilibrium is used to determine the position of neutral axis after which the resisting moment is calculated. This iterative process is repeated until sufficient length of the full-range moment-curvature curve has been obtained. The moment-curvature curves obtained from the section analyses are then input into the computer programme for global structural analyses for assignment to each element.

### 2.4. Iteration process

The analysis starts by forming the element and global stiffness matrices $\mathbf{K_e}$ and $\mathbf{K_g}$ respectively. As mentioned above, the iteration scheme of the present technique adopts the initial stiffnesses, and hence the global stiffness matrix remains unchanged throughout the entire analysis. The value of flexural rigidity *EI* required for computing the element stiffness matrix $\mathbf{K_e}$ is taken as the slope of the elastic region of the moment-curvature curve corresponding to each element, as shown in Figure 1. The cumulative incremental load or displacement is applied at the specified location, upon which iterations are performed and the residual curvature of each element is updated until a set of admissible displacements and forces at all nodes is obtained. The procedure of the iteration process at any load step *i* is explained as follows.

Step 1. A set of nodal displacements and forces is determined by solving Equation (2). The curvature $\phi_i^n$ at the representative section of each element can be calculated from the nodal displacement vector $\boldsymbol{\delta}$ of that element by

$$\phi_i^n = \mathbf{B}\,\boldsymbol{\delta}^n \qquad (6)$$

where the subscripts *i* and *n* refer to the $i^{th}$ load step and $n^{th}$ iteration step respectively. The moment $m_i^n$ corresponding to curvature $\phi_i^n$ is then calculated using Equation (1). The residual curvature $\phi_r$ in Equation (1) is taken as the residual curvature determined from the previous load step or iteration step.

Step 2. For each element, the maximum permissible moment $M_i^n$ corresponding to the calculated curvature $\phi_i^n$ is obtained from the moment-curvature curve. The moment-curvature curve is treated effectively as an envelope in the sense that the calculated moment cannot exceed the moment given by the curve at a certain curvature. In other words, $M_i^n$ is the moment on the moment-curvature curve at the calculated curvature $\phi_i^n$.

Step 3. The calculated moment $m_i^n$ is checked against the maximum permissible moment $M_i^n$. If $m_i^n$ is greater than $M_i^n$ by a certain tolerance, the maximum moment $M_i^n$ corresponding to the calculated curvature is adopted and the residual curvature is updated accordingly, namely

$$(\phi_r)_i^{n+1} = \phi_i^n - \frac{M_i^n}{EI} \qquad (7)$$

which is obtained from the re-arrangement of Equation (1) and $(\phi_r)_i^{n+1}$ is the updated residual curvature to be used in the next iteration step. On the contrary, if $m_i^n$ is less than $M_i^n$, then $m_i^n$ will be taken as the moment that the section is subject to and the residual curvature will not be updated.

Once the residual curvatures of all elements have been determined, Steps 1 to 3 are repeated until the calculated moments and curvatures of all elements are sufficiently close to the moment-curvature curve of the corresponding element. The iteration process can also be demonstrated graphically using Figure 2. Figure 2(a) shows the moment and curvature of a typical element increase from Point I to Point J. An enlarged view of the moment-curvature curve between Point I and Point J is shown in Figure 2(b) to illustrate the iteration process. Suppose that the initial moment and curvature of the element at the beginning of load step *i* lie on Point I, and the element has residual curvature $(\phi_r)_i$. Referring to Figure 2(b), the calculated moment and curvature after the first iteration step are $m_i^1$ and $\phi_i^1$ respectively, giving Point 1 with moment exceeding the maximum permissible moment $M_{\mathrm{i}}^1$ as denoted by Point 2. The residual curvature is thus updated with a new value $(\phi_r)_i^1$. The computer programme then proceeds to the second iteration step, which gives the moment and curvature corresponding to Point 3. As the moment of Point 3 does not go beyond the moment-curvature curve, the residual curvature is kept unchanged. Subsequent iterations yield Point 4 and so forth. The iteration cycles will be terminated as the values of moment and curvature converge to those at Point J.

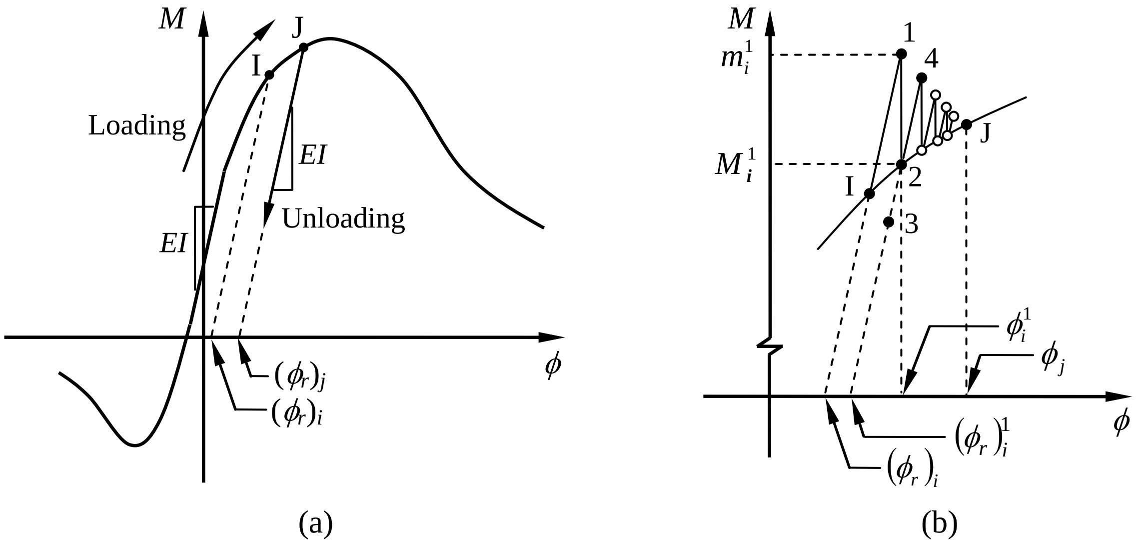


**Figure 2: The iteration process. (a) Moment and curvature of a typical element increase from Point I to Point J; (b) iterations of moment and curvature from Point I to Point J.**

Suppose that upon reaching Point J, the section in this element undergoes unloading (Figure 2(a)) because some other elements of the structure have reached their peak moments and gone to the post-peak range. The moment and curvature of the unloading element will follow the unloading path of the moment-curvature curve from Point J, which is assumed to be parallel to the elastic slope. Neither Warner and Yeo (1986) nor Campbell and Kodur (1990) have clearly explained their treatment of the moment-curvature relationship as a section undergoes unloading.

## 3. VERIFICATION

A series of prestressed concrete beams were tested by Mitchell et al. (1993) to study the effects of concrete strength on the transfer length of pretensioning strands. The simply supported beam selected for verification has a span of 3730 mm with a point load acting at the mid-span. The beam section was rectangular with a depth of 250 mm and a breadth of 200 mm. The beam had a straight tendon with cross sectional area of 146.4 $mm^2$ located at 75 mm below the centroidal axis stressed to 1286 MPa. The cylinder strength of concrete was 31.0 MPa, while the tendon had an ultimate strength of 1793 MPa. The load-displacement response at the loaded point as shown in Figure 4 indicates good agreement between the numerical and experimental results. Note that the present technique can generate the load-displacement relationship well into the post-peak branch, while most existing numerical techniques stop after predicting the peak load-carrying capacity.

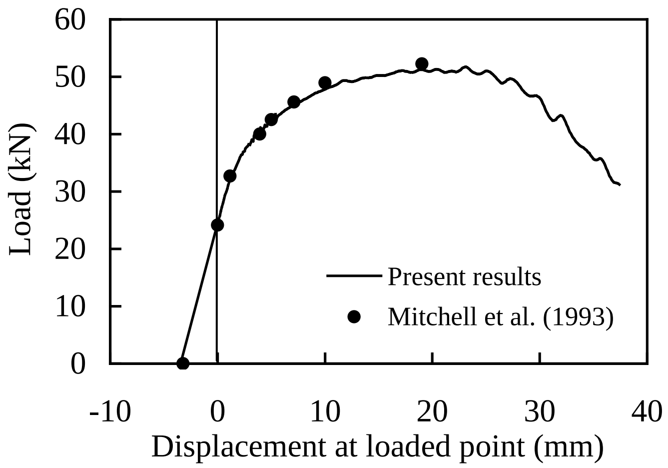


**Figure 4: Load-displacement response of a simply supported prestressed concrete beam.**

Priestley and Park (1972) conducted experimental study on the moment redistribution in continuous prestressed concrete beams. The load-displacement relationship of one of the beams tested was analysed numerically by the present technique. The beam had two equal spans of 3015 mm each and was symmetrically loaded with a point load on each span at 1489 mm from the end. The beam section is rectangular having a depth of 203 mm and a breadth of 99 mm. A single layer of deflected tendons comprising two 7 mm diameter wires stressed to 750 MPa was provided, where the eccentricity was zero from each end to the nearer loaded point and the tendons were 63.5 mm above centroidal axis at the central support. The concrete cube strength was 48.0 MPa, while the ultimate strength of tendon was taken to be 1860 MPa. The load-displacement response at the loaded point is shown in Figure 5, which indicates good agreement between the numerical and experimental results.

## 4. CONCLUSIONS

The development of a numerical technique for full-range nonlinear analysis of prestressed concrete segmental bridge has been presented. The technique idealizes the bridge deck as a series of beam elements whose behaviour is governed by the moment-curvature relationship of a representative section within each element. Iterations are then carried out using the initial stiffness method to obtain a set of admissible nodal displacements and internal forces that satisfy the constitutive behaviour of each element. Experimental results of two prestressed concrete beams previously tested by various researchers were used for verifying the present technique. The load-displacement responses obtained by the present technique are compared with available experimental results, and good agreement is observed.

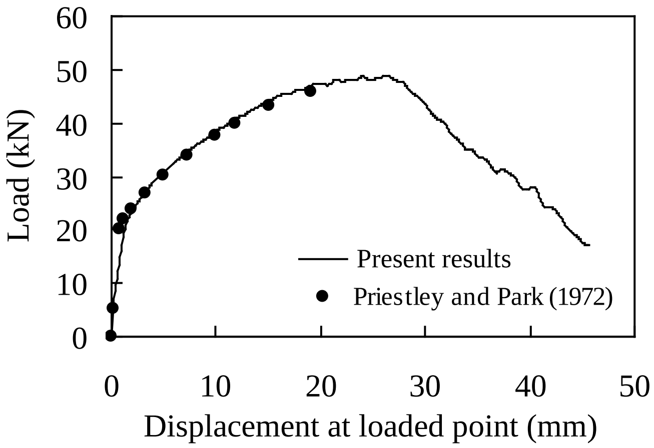


**Figure 5: Load-displacement response of a 2-span continuous prestressed concrete beam.**

## 5. ACKNOWLEDGEMENTS


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