On the Theory and Design of a Class of PR Uniform and Recombination Nonuniform Causal-Stable IIR Cosine Modulated Filter Banks

S. S. Yin, S. C. Chan, K. M. Tsui, and X. M. Xie

***Abstract*—This paper studies the theory and design of a class of perfect reconstruction (PR) uniform causal-stable infinite-impulse response (IIR) cosine modulated filter banks (CMFBs). The design approach is also applicable to the design of PR recombination nonuniform (RN) IIR CMFBs. The polyphase components of the prototype filters of these IIR CMFBs are assumed to have the same denominator so as to simplify the PR condition. In designing the proposed IIR CMFB, a PR FIR CMFB with similar specifications is first designed. The finite-impulse response prototype filter is then converted to a nearly PR (NPR) IIR CMFB using a modified model reduction technique. The NPR IIR CMFB so obtained has a reasonably low reconstruction error. Its denominator is designed to be a polynomial in z^M, where M is the number of channels, to simplify the PR condition. Finally, it is employed as the initial guess to constrained nonlinear optimization software for the design of the PR IIR CMFB. Design results show that both NPR and PR IIR CMFBs with good frequency characteristics and different system delays can be obtained by the proposed method. By using these IIR CMFBs in the RN CMFBs, new RN NPR and PR IIR CMFBs can be obtained similarly.**

***Index Terms*—Causal-stable infinite-impulse response (IIR) filters, cosine-modulated filter banks, perfect reconstruction (PR), recombination nonuniform filter banks.**

I. Introduction

PERFECT RECONSTRUCTION (PR) filter banks (FBs) have important applications in speech, audio, image and array processing. A class of very efficient FB is cosine-modulated FB (CMFB) [1], [2], which has very low design and implementation complexities and excellent frequency selectivity. Infinite-impulse response (IIR) CMFBs, in particular, have received considerable attention for their potential advantages of low system delay, sharp cutoff and high stopband attenuation.

Due to the absence of the stability constraint, the design of PR finite-impulse response (FIR) CMFBs is considerably simpler than PR IIR CMFBs. Although the latter also involves nonlinear constrained optimization, satisfactory results are usually obtained without much difficulty. Furthermore, a number of efficient design methods are now available for improving the design efficiency and performance of nearly PR (NPR) and PR FIR CMFBs [7]–[9]. However, the design of IIR CMFBs is complicated by highly nonlinear objective function, PR and stability constraints. In [3], allpass-based PR IIR CMFBs was considered. Because of the difficulties in designing such FBs, only a two-channel design example was given. Recently, Mao *et al.*. [4] studied the design of IIR CMFBs and proposed to simplify the PR constraints by employing prototype filters having the same denominator in their polyphase components. However, since the initial guess is only obtained from a PR FIR CMFB, it is difficult for the optimization procedure to converge to a good solution, especially for high order IIR filter.

In this paper, a new method for designing NPR and PR IIR CMFBs is studied based on the simplified PR condition in [4]. The denominators of the prototype filters of such IIR CMFBs are chosen as polynomials in z^M, where M is the number of channel. In particular, we propose to employ the modified reduction technique in [6] to convert a PR FIR CMFB to an IIR CMFB, because this approach is capable of designing the prototype filters with the above special form. Other advantages of this approach are that the resulting IIR filter is guaranteed to be stable and to closely approximate the frequency response of its FIR counterpart. Therefore, if this modified model reduction technique is applied to the prototype filter of a PR FIR CMFB, an IIR CMFB with similar characteristics can be obtained. Design results show that the IIR CMFB so obtained is NPR with a reasonably low reconstruction error (about 1e-3). Furthermore, since this NPR IIR CMFB also has an identical denominator in its polyphase components, the PR constraints are simplified as in [4] and it can be used as initial guess to constrained nonlinear optimizers for the design of PR IIR CMFBs. Significantly better convergence speed and reliability over the direct nonlinear optimization approach can be achieved as the pole locations can be approximately located.

Using the IIR CMFBs obtained, it is possible to develop a new class of PR nonuniform IIR CMFBs. PR FBs with nonuniform frequency spacing have the potential advantage of offering more flexibility in time-frequency partitioning, but they are more difficult to design. There has been considerable interest in designing nonuniform FBs [10]–[15]. The proposed nonuniform IIR CMFBs are based on the indirect method proposed in [14], [15], where certain channels of an original uniform FB are merged using the synthesis bank of a recombination FB or transmultiplexer (TMUX). An advantage of this indirect method or recombination nonuniform FBs (RNFBs) is that the PR property is structurally imposed as long as the original uniform and recombination FBs are PR. Furthermore, dynamic recombination of consecutive channels in the original uniform FB by pre-designed TMUXs is possible.

To obtain FIR RNFBs with good frequency characteristics, a matching condition between the original uniform FB and the

Manuscript received January 27, 2006; revised March 26, 2007, and January 3, 2008. First published July 16, 2008; last published August 13, 2008 (projected). This work was presented in part at the International Symposium on Circuits and Systems, Kobe, Japan, May 23–26, 2005. This paper was recommended by Associate Editor T. Davidson.

The authors are with the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong (e-mail: scchan@eee.hku.hk; ssyin@eee.hku.hk; kmtsui@eee.hku.hk; xmxie@eee.hku.hk).

Digital Object Identifier 10.1109/TCSII.2008.922435

recombination FB is imposed [14], [15]. Unlike the FIR RN CMFBs in [15], we propose to employ PR IIR CMFBs as the original uniform FB and the recombination TMUXs. Similar to the design of PR uniform IIR CMFBs mentioned earlier, the proposed method is applied to a PR RN FIR CMFB satisfying the matching condition and the given specifications. It yields an NPR RN IIR CMFB which can be for further optimized to obtain the desired PR RN IIR CMFB.

The paper is organized as follows. The theories of PR uniform and RN IIR CMFBs are respectively introduced in Sections II and III. The modified model reduction technique is discussed in Section IV. Design examples and comparisons are given in Section V. Finally conclusion is drawn in Section VI.

II. PR Uniform IIR CMFBs

Recall that the analysis and synthesis filters of a type-IV M-channel CMFB [1] are obtained by modulating a prototype filter $H(z) = \Sigma_{n=0}^{L-1} h(n)z^{-n}$

$$h_k(n) = 2h(n)\cos\left[\frac{\pi}{M}\left(k+\frac{1}{2}\right)\left(n-\frac{D}{2}\right)+(-1)^k\frac{\pi}{4}\right]$$
$$f_k(n) = 2h(n)\cos\left[\frac{\pi}{M}\left(k+\frac{1}{2}\right)\left(n-\frac{D}{2}\right)-(-1)^k\frac{\pi}{4}\right] \quad (1)$$

for $k = 0,\ldots,M-1, n = 0,\ldots,L-1$, where $\{h(n)\}$ and L are the impulse response and filter length of the prototype filter, respectively. D is the system delay. For the uniform IIR CMFB, the prototype filter is an IIR filter of the form: $\hat{H}(z) = (\Sigma_{n=0}^{L_p-1} p(n)z^{-n})/(\Sigma_{n=0}^{L_q-1} q(n)z^{-n})$, where L_p and L_q are respectively the lengths of the numerator and denominator. Let $E_k(z)$ be the type-I polyphase component of $\hat{H}(z)$ such that $\hat{H}(z) = \Sigma_{k=0}^{2M-1} E_k(z^{2M})z^{-k}$. If $E_k(z)'s$ have the same denominator, that is $E_k(z) = N_k(z)/D(z)$ for $k = 0,\ldots,2M-1$, then the PR condition of the IIR CMFB is simplified to [4]

$$N_k(z)N_{2M-k-1}(z) + N_{M+k}(z)N_{M-k-1}(z) = \beta\cdot z^{-n_k}D^2(z) \quad (2)$$

for $k = 0,\ldots,(M/2)-1$, where β is a constant and n_k is an integer. All the roots of $D(z^{2M})$ should remain inside the unit circle to ensure that the analysis filters and synthesis filters are stable. A possible objective function is given as

$$\mathbf{\Phi}(\mathbf{x}) = \int_{\omega_s}^{\pi} |\hat{H}(e^{j\omega})|^2 d\omega + \lambda \cdot \int_0^{\omega_{p\_1}} (|H_0(e^{j\omega})|-1)^2 d\omega \quad (3)$$

where $\mathbf{x}$ is the variable vector containing the coefficients of $\{p(n)\}$ and $\{q(n)\}, \omega_s$ is the stopband cutoff frequency of the prototype filter, ω_{p_1} is the passband cutoff frequency of the first analysis filter $H_0(z)$ and λ equals to a positive weighting factor. The design problem can be formulated as a constrained optimization problem as

$$\min \mathbf{\Phi}(\mathbf{x})$$
$$\text{subject to}: \ \{\max(|r_i|) < 1\} \text{ and the PR conditions} \quad (4)$$

where $r_i^{'}s$ are the roots of $D(z^{2M})$. Note the PR conditions are imposed by enforcing (2) in time domain. For FIR CMFBs, $D(z) = 1$ and the design problem is considerably simplified.

III. PR RN IIR CMFBs

Fig. 1 shows the structure of the RNFB considered in this paper. In the analysis bank of the proposed RNFB, consecutive channels of the analysis bank of a uniform analysis FB, called the original uniform FB, are combined using the synthesis bank of another FB, called the recombination FB, which has a fewer number of channels, say m_k as shown in the figure. The sampling rate after recombination is reduced by a factor of m_k/M. For simplicity, only the merging of m_k channels of the original M-channel analysis bank is shown. Further merging of consecutive channels can be performed similarly. Let the merged outputs be indexed by an integer $k, k = 0,\ldots,K-1$, and m_k be the number of channels merged at the k-th output. For critical sampling, we have $\Sigma_{k=0}^{K-1}(m_k/M) = 1$. In the synthesis bank of the RNFB, each merged output will pass through the analysis filters of the recombination FBs and they will be fed to the synthesis bank of the uniform FB for reconstruction. Each synthesis–analysis structure, involving an m_k-channel FB, is called a TMUX. To ensure that the whole system is PR, the TMUXs and the original uniform FB should be PR. Furthermore, in order to suppress the spurious response of a RN FIR CMFB due to the mismatch of the magnitude and phase responses in the transition bands of its prototype filters, the following matching conditions should be satisfied [15]:

$$(L_{m_k}/L_M) = (m_k/M)$$
$$(\omega_{p\_m_k}/\omega_{p\_M}) = (\omega_{s\_m_k}/\omega_{s\_M}) = (M/m_k) \quad (5)$$

where $L_{m_k}, \omega_{p\_m_k}$, and $\omega_{s\_m_k}(L_M, \omega_{p\_M}$ and $\omega_{s\_M})$ are respectively the length, passband and stopband cutoff frequencies of the prototype filters for the m_k -channel TMUX (M-channel FB). By using the model reduction approach, this requirement on the frequency response of the IIR prototype filters is approximately satisfied. It should be noted that if m_k and M are coprime to each other, then the equivalent analysis filter at the k-th output is linear time-invariant (LTI), and is given by $\hat{H}_k(z) = \Sigma_{i=0}^{m_k-1} H_{l_k+i}(z^{m_k})G_i(z^M)$, where l_k is the position of the first sub-channel to be merged for the k-th output. If l_k is odd, it is necessary to multiply the sequence $(-1)^n$ to the merged output to avoid the problem of spectral inversion. On the other hand, the analysis filters are linear periodic time varying (LPTV) for noncoprime case. Fortunately, the spurious response resulting from the merging of the original uniform FB and recombination TMUXs can still be suppressed by imposing (5) to the prototype filters.

IV. Modified Model Reduction Method

Since it is much simpler to design a PR FIR CMFB than a PR IIR CMFB, we propose to obtain an initial guess to the IIR CMFB design problem mentioned above by a model reduction approach. More precisely, the model reduction method proposed in [5] is modified so that the denominator of the IIR filter can be directly expressed as a polynomial in the integer power of z.

It was found in [5] that the denominator $\hat{D}(z)$ of an IIR filter $N(z)/\hat{D}(z)$ for approximating an FIR filter $H(z)$ can be determined using the following iterative procedure. Let

$$\hat{D}^{(k)}(z) = 1 + \Sigma_{n=1}^{L_q-1} q^{(k)}(n)z^{-n}, \hat{D}^{(0)}(z) = 1 \quad (6)$$

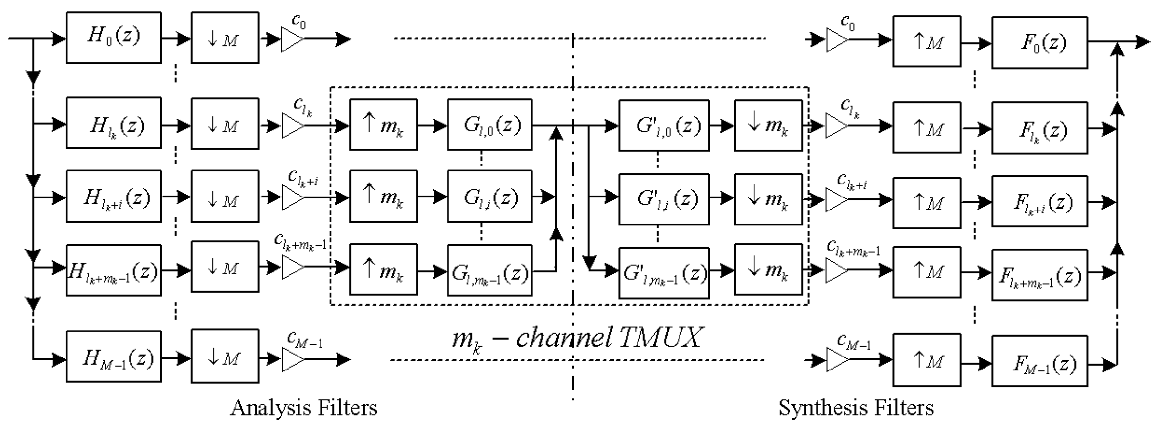

Fig. 1. Structure of recombination nonuniform filter bank.

be the polynomial which approximates $\hat{D}(z)$ at the k-th iteration. From the sequence

$$\begin{aligned} X^{(k)}(z) &= z^{-(L-1)}H(z^{-1})/\hat{D}^{(k-1)}(z) \\ &= \Sigma_{n=0}^{\infty} x^{(k)}(n) z^{-n} \end{aligned} \tag{7}$$

we calculate $q^{(k)}(n)$ by minimizing the following objective function:

$$F^{(k)} = (\boldsymbol{B}^{(k)}\boldsymbol{q}^{(k)} - \boldsymbol{d}^{(k)})^T \cdot (\boldsymbol{B}^{(k)}\boldsymbol{q}^{(k)} - \boldsymbol{d}^{(k)}), \tag{8}$$

where

$$\boldsymbol{B}^{(k)} = \begin{bmatrix} x^{(k)}(0) & 0 & \cdots & 0 \\ x^{(k)}(1) & x^{(k)}(0) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ x^{(k)}(L_q-2) & x^{(k)}(L_q-3) & \cdots & x^{(k)}(0) \\ \vdots & \vdots & & \vdots \\ x^{(k)}(L-2) & x^{(k)}(L-3) & \cdots & x^{(k)}(L-L_q) \end{bmatrix}$$

$\boldsymbol{q}^{(k)} = \left[q^{(k)}(L_q-1), \ldots, q^{(k)}(1)\right]^T$ and

$\boldsymbol{d}^{(k)} = -\left[0, \ldots, 0, x^{(k)}(0), \ldots, x^{(k)}(L-L_q-1)\right]^T.$

The required denominator is the one that minimizes $F^{(k)}$ for a sufficiently large value of k ($k = 300$ in this paper). The numerator $N(z)$ can be determined separately using the least squares criterion [16]. More importantly, the IIR filter so obtained is always stable. As mentioned earlier, the denominator of the IIR prototype filter should have the form $D(z^{2M})$. If direct model reduction is used, the numerator and denominators have to be multiplied by certain factors and the filter length is thus unnecessarily increased. As suggested in [6], the above procedure can be modified to obtain a filter with the following form:

$$\hat{H}(z) = \frac{N(z)}{D(z^{2M})} = \frac{\Sigma_{n=0}^{L_p-1} p(n) z^{-n}}{1 + \Sigma_{n=1}^{d} q(n) z^{-2nM}}, \tag{9}$$

where $L_p \geq 2dM, d$ is the number of nonzero coefficients of $D(z^{2M})$. The vector $\boldsymbol{q}^{(k)}$ in (8) is then modified as $\boldsymbol{q}^{(k)} = [q^{(k)}(2dM), q^{(k)}(2(d-1)M), \ldots, q^{(k)}(2M)]^T$, which can be solved by modifying the corresponding rows of $\boldsymbol{B}^{(k)}$ and $\boldsymbol{d}^{(k)}$. According to [5], this modification does not violate the stability theorem, which holds for arbitrarily given $X^{(k)}(z)$. Hence, the filter in (9) is still stable, provided that the index k with the smallest value of $F^{(k)}$ is chosen.

V. Design Procedure and Examples

Due to page limitation, we only summarize the design procedure of the RN IIR CMFB because the design of the uniform IIR CMFBs is similar. Given decimation ratios (m_k/M) for $k = 0, \ldots, K-1$ with $\Sigma_{k=0}^{K-1}(m_k/M) = 1$, the following are true.

1) Design the M -channel and m_k -channel PR uniform FIR CMFBs, which satisfy the matching condition in (5), by solving the constrained nonlinear optimization problem in (4).
2) Determine the corresponding NPR RN IIR CMFB by applying the proposed model reduction method in Section IV to the prototype filters obtained in step 1.
3) The NPR IIR CMFBs are used as initial guesses to the constrained nonlinear optimization problem in (4) to obtain the original and then the recombination PR IIR CMFBs.

In this paper, a local solution to the constrained nonlinear optimization problem in steps 1 and 3 is obtained using the function fmincon in MATLAB. The termination tolerance on the constraint value and the maximum number of iterations are set to 1e-15 and 1000, respectively.

Example 1: NPR and PR Uniform IIR CMFBs: In this example, low-delay (LD) NPR and PR IIR CMFBs are considered. An 16-channel PR LD FIR CMFB with a filter length of 224 is firstly designed and the model reduction method described in Section IV is then applied to convert the prototype filter to an IIR filter in the form of (9). The running time of the model reduction method is 0.281 s using MATLAB version 7.0 in a Pentium(R) 4 CPU 3-GHz PC. The lengths of the numerator and denominator of the IIR filter are chosen as $L_p = 192$ and $L_q = 97$ (i.e., $d = 3$), respectively. Table I summarizes the design parameters. D is the system delay. (ω_p, ω_s) are the passband and stopband cutoff frequencies of the prototype filter.

To obtain a PR IIR CMFB, the above NPR IIR CMFB is used as initial guess to fmincon, which converged to the solution after 315 iterations. The reconstruction error is 6.97e-14. For the sake of presentation, the proposed NPR and PR IIR CMFBs are repectively denoted by NPR IIR CMFB-1 and PR IIR CMFB-1

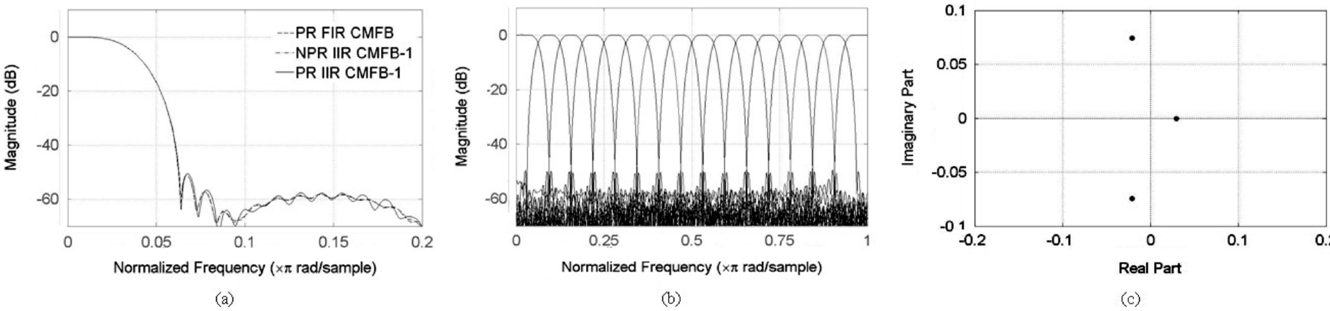

Fig. 2. (a) Frequency responses of prototype filters of 16-channel NPR/PR FIR/IIR CMFBs. (b) Frequency responses of the analysis filters of the proposed PR IIR CMFB-1. (c) Pole locations of the polyphase components of NPR IIR CMFB-1 (marked with stars) and PR IIR CMFB-1 (marked with circles).

TABLE I
DESIGN PARAMETERS OF FIR AND IIR CMFBS

Channel number	L	(L_p, L_q, d)	D	(ω_p, ω_s)
16	224	(192,97,3)	159	$(0.15\pi/16, \pi/16)$
8	112	(96,49,3)	79	$(0.15\pi/8, \pi/8)$
7	98	(84,43,3)	69	$(0.15\pi/7, \pi/7)$

TABLE II
RESULTS OF 16-CHANNEL NPR/PR FIR AND IIR CMFBS

FBs	*Filter order*	A_s (dB)	E_{pp}	Iters
NPR FIR CMFB	223	-86.15	2.44e-02	N/A
PR FIR CMFB	223	-51.03	1.70e-14	335
NPR IIR CMFB-1	191 ($d=3$)	-51.10	2.73e-04	N/A
PR IIR CMFB-1	191 ($d=3$)	-50.49	7.57e-14	235
NPR IIR CMFB-2	191 ($d=3$)	-64.45	2.28e-02	N/A
NPR IIR CMFB-2'	191 ($d=3$)	-50.60	1.32e-08	1000
NPR IIR CMFB [4]	191 ($d=3$)	-51.02	5.71e-10	1000

in Table II. The performances of the original PR FIR CMFB and its initial guess (i.e., the NPR FIR CMFB) are also listed in Table II. A_s (dB) is the highest stopband attenuation of the prototype filter and E_{pp} is the maximum peak-to-peak ripple of the transfer function $T(z) = \Sigma_{k=0}^{M-1} H_k(z)F_k(z)$. Iters denotes the number of iterations needed for fmincon to converge to a solution. Fig. 2(a) shows the frequency responses of the prototype filters for the PR FIR CMFB, NPR IIR CMFB-1 and PR IIR CMFB-1. The stopband attenuation of the proposed PR IIR CMFB-1 is comparable to its FIR counterpart, Moreover, its implementation complexity is relatively lower because there are fewer nonzero coefficients. Fig. 2(b) shows the frequency responses of the analysis filters of PR IIR CMFB-1. The pole locations of the polyphase components of the prototype filters of NPR IIR CMFB-1 (marked with stars) and PR IIR CMFB-1 (marked with circles) are almost overlapped as illustrated in Fig. 2(c).

Next, we shall examine how different initial guesses affect the convergence speed of the nonlinear optimizer. First of all, we consider an initial guess, which is obtained by applying the model reduction technique to the NPR FIR CMFB in Table I, instead of the PR FIR CMFB we proposed. Compared to the proposed NPR IIR CMFB-I, the NPR IIR CMFB so obtained (denoted by NPR IIR CMFB-2 in Table II) offer a higher stopband attenuation (64.45 dB), but a lower reconstruction error (2.28e-2). However, fmincon is unable to find a PR solution when the maximum number of iteration is reached (i.e., 1000 iterations), although the reconstruction error of the resulting IIR CMFB is reduced to 1.32e-8. For convenience, this NPR solution is denoted by NPR IIR CMFB-2' in Table II.

Another initial guess we consider is a 191-order FIR prototype filter of an 16-channel PR FIR CMFB with the denominator being set to one. It is designed according to the method in [4]. As shown in Table II, only a NPR solution with reconstruction error of 5.71e-10 is obtained after 1000 iterations. The above comparisons suggest that the proposed initial guess helps the optimizer to achieve a faster speed of convergence since the pole positions are approximately located.

Example 2: NPR and PR RN IIR CMFBs: In this example, LD NPR and PR RN IIR CMFBs with decimation factors $\{1/2, 1/16, 7/16\}$ are considered. To realize these decimation factors, a 16-channel PR uniform CMFB is first designed (Example 1). Then, the first eight channels and the last seven channels of this 16-channel CMFB are respectively merged by the 8- and 7-channel PR CMFB-based TMUXs. Therefore, we have $M = 16, m_0 = 8$ and $m_1 = 7$. The 16-, 8- and 7-channel PR uniform FIR CMFBs with respective system delays $D^{'}s$ of 159, 79, and 69 samples are designed according to the matching condition in (5). Then they are converted to an NPR RN IIR CMFB by the model reduction method. The parameters (length L, passband cutoff frequency ω_p, stopband cutoff frequency ω_s) of the prototype filters for the 16-, 8- and 7-channel PR uniform FIR CMFBs are, respectively, chosen as $(224, 0.15\pi/16, \pi/16), (112, 0.15\pi/8, \pi/8)$ and $(98, 0.15\pi/7, \pi/7)$. Whereas the parameters (length of numerator L_p, length of denominator L_q, number of nonzero coefficients of the denominator d) of the prototype filters for the 16-, 8- and 7-channel uniform IIR CMFBs are respectively chosen as (192, 97, 3),(96, 49, 3) and (84, 43, 3). The above design parameters are summarized in Table I. The reconstruction errors of the IIR CMFBs so designed are of order 1e-3. These NPR IIR CMFBs are used as initial guesses for further optimization to obtain the PR IIR CMFBs.

Note that since the rate-$1/2$ does not satisfy the coprime condition, the first analysis filter is LPTV. The bi-frequency response of the first analysis filter of the proposed PR RN IIR CMFB is shown in Fig. 3a, where the amplitudes and phases of all frequency components are generated by inputting a sinusoidal signal to the system at a certain frequency with unit amplitude and zero phase. One of the aliasing terms generated in

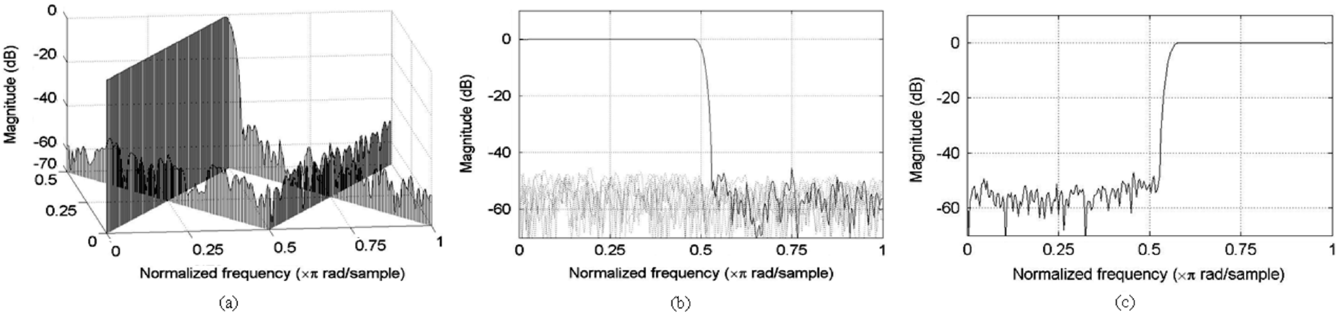

Fig. 3. Frequency responses of the PR RN IIR CMFB $(\text{decimation ratios} = \{1/2, 1/16, 7/16\})$. (a) Bi-frequency response of first LPTV analysis filter and one of aliasing terms. (b) Projection of first LPTV analysis filter (solid line) and all aliasing terms (dot lines) on the digital frequency axis. (c) Frequency response of third LTI analysis filter.

the recombination process is also illustrated. Fig. 3(b) shows the corresponding response of the first analysis filter (in solid line) and all aliasing terms (in dot lines) projected onto the frequency axis. The last analysis filter is LTI because the ratio is $7/16$. The frequency response of the last analysis filter is given in Fig. 3(c). This example shows that RN IIR CMFBs with good equivalent frequency characteristics can be obtained, in both coprime and noncoprime cases, by the proposed method. Compared with the original PR RN FIR CMFB, the proposed PR RN IIR CMFB has a comparable performance and lower implementation complexity.

VI. Conclusion

The theory and design of a class of PR uniform and RN CMFBs with causal-stable IIR filters are presented. The polyphase components of the prototype filters of these IIR CMFBs have the same denominator so that the PR condition can be simplified. The design procedure starts with the design of a PR uniform or RN FIR CMFBs of similar specifications. The prototype filters of the CMFBs are then converted to an NPR uniform or RN IIR CMFBs by a modified model reduction technique. These NPR uniform or RN IIR CMFBs are further optimized to obtain PR uniform or RN IIR CMFBs.

References

[1] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[2] R. D. Koilpillai and P. P. Vaidyanathan, “A spectral factorization approach to pseudo-QMF design,” *IEEE Trans. Signal Process.*, vol. 41, no. 1, pp. 82–92, Jan. 1993.

[3] T. Q. Nguyen, T. I. Laakso, and T. E. Tuncer, “On perfect-reconstruction allpass-based cosine-modulated IIR filter banks,” in *Proc. IEEE ISCAS*, May 1994, vol. 2, pp. 33–36.

[4] J. S. Mao, S. C. Chan, and K. L. Ho, “Theory and design of a class of M-channel IIR cosine-modulated filter banks,” *IEEE Signal Process. Lett.*, vol. 7, no. 2, pp. 38–40, Feb. 2000.

[5] H. Brandenstein and R. Unbehauen, “Least-squares approximation of FIR by IIR digital filters,” *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 21–30, Jan. 1998.

[6] K. M. Tsui and S. C. Chan, “Multiplet two-channel FIR and causal stable IIR perfect reconstruction filter banks,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 53, no. 12, pp. 2804–2817, Dec. 2006.

[7] C. D. Creusere and S. K. Mitra, “A simple method for designing high-quality prototype filters for M-band pseudo-QMF banks,” *IEEE Trans. Signal Process.*, vol. 43, no. 4, pp. 1005–1007, Apr. 1995.

[8] T. Q. Nguyen, “Near-perfect-reconstruction pseudo-QMF banks,” *IEEE Trans. Signal Process.*, vol. 42, no. 1, pp. 65–76, Jan. 1994.

[9] S. C. Chan and S. S. Yin, “On the design of low delay nearly-PR and PR FIR cosine modulated filter banks having approximate cosine-rolloff transition band,” in *Pro. EUSIPCO*, Sep. 2004, pp. 1253–1256.

[10] S. Akkarakaran and P. P. Vaidyanathan, “New results and open problems on nonuniform filter-banks,” in *Proc. IEEE ICASSP’*, Mar. 1999, vol. 3, pp. 1501–1504.

[11] T. Blu, “A new design algorithm for two-band orthonormal rational filter banks and orthonormal rational wavelets,” *IEEE Trans. Signal Process.*, vol. 46, no. 6, pp. 1494–1504, Jun. 1998.

[12] F. Argenti and E. Del Re, “Rational sampling filter banks based on IIR filters,” *IEEE Trans. Signal Process.*, vol. 46, no. 12, pp. 3403–3408, Dec. 1998.

[13] J. Kovacevic and M. Vetterli, “Perfect reconstruction filter banks with rational sampling factors,” *IEEE Trans. Signal Process.*, vol. 41, no. 6, pp. 2047–2066, Jun. 1993.

[14] R. V. Cox, “The design of uniformly and nonuniformly spaced pseudo-quadrature mirror filters,” *IEEE Trans. Acoust. Speech Signal Process.*, vol. 34, no. 5, pp. 1090–1096, Oct. 1986.

[15] X. M. Xie, S. C. Chan, and T. I. Yuk, “Design of perfect-reconstruction nonuniform recombination filter-banks with flexible rational sampling factors,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 9, pp. 1965–1981, Sep. 2005.

[16] S. C. Chan, K. M. Tsui, and K. W. Tse, “Design of constrained causal stable IIR filters using a new second-order-cone-programming-based model-reduction technique,” *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 54, no. 2, pp. 107–111, Feb. 2007.

[17] S. C. Chan and S. S. Yin, “On the design of a class of PR causal-stable IIR nonuniform recombination cosine modulated filter banks,” in *Proc. IEEE ISCAS*, May 2005, vol. 2, pp. 1094–1097.