Inflation Dynamics: The Role of Public Debt and Policy Regimes

Saroj Bhattarai
Penn State

Jae Won Lee
Rutgers University

Woong Yong Park*
University of Hong Kong

Abstract

We investigate the impact of a time-varying inflation target and changing monetary and fiscal policy stances on the dynamics of inflation in a DSGE model. Under an active monetary and passive fiscal policy regime, inflation closely follows the path of the inflation target and a stronger reaction of monetary policy to inflation decreases the equilibrium response of inflation to non-policy shocks. In sharp contrast, under an active fiscal and passive monetary policy regime, inflation moves in an opposite direction from the inflation target and a stronger reaction of monetary policy to inflation increases the equilibrium response of inflation to non-policy shocks. Moreover, a weaker response of fiscal policy to debt decreases the response of inflation to non-policy shocks. These results are due to variation in the value of public debt that lead to wealth effects on households. Finally, under a passive monetary and passive fiscal policy regime, because of equilibrium indeterminacy, theory provides no clear answer on the behavior of inflation. We characterize these results analytically in a simple model and numerically in a richer quantitative model.

JEL Classification: E31, E52, E63

Keywords: Time-varying inflation target; Inflation response; Monetary and fiscal policy regimes; Public debt; Monetary and fiscal policy stances; DSGE model

---

*Bhattarai: 615 Kern Building, University Park, PA 16802, sub31@psu.edu. Lee: 75 Hamilton Street, NJ Hall, New Brunswick, NJ 08901, jwlee@econ.rutgers.edu. Park: School of Economics and Finance, KK Leung Bldg, Pokfulam Road, Hong Kong, wypark@hku.hk. We are grateful to seminar participants at University of Hong Kong for comments. This version: June 2012.

1 Introduction

Using a micro-founded model, we address two classic questions in monetary economics and policy in this paper. First, can a time-varying inflation target decisively influence the path of actual inflation? In other words, does monetary policy properly control the dynamics and path of inflation? Second, what are the effects of changes in monetary and fiscal policy stances on the equilibrium response of inflation to various shocks impinging on the economy? For example, what happens to the equilibrium behavior of inflation when the monetary policy stance changes to a more aggressive response to inflation? Does the fiscal policy stance with respect to public debt matter for inflation dynamics? If yes, then how does a variation in the fiscal policy stance affect inflation?

These issues, while long of great interest in monetary economics, have received a renewed interest recently in the literature.[1] A prominent illustration is provided by the research that aims to provide an explanation for the low frequency movement of inflation in the U.S., especially, the rise of inflation in the 1970s and the subsequent fall in the 1980s. Proposed explanations typically rely on changes in the dynamics of the inflation target and/or changes in policy stances.[2]

For example, Ireland (2007) and Cogley, Primiceri, and Sargent (2010) propose a rise in a persistent time-varying inflation target as an explanation for the rise of inflation in the 1970s. Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), and Bhattarai, Lee, and Park (2012) argue that a weak monetary policy stance with respect to inflation, or a passive monetary policy regime, in the pre-Volcker period implied indeterminacy of equilibria, which in turn, led to a rise of inflation due to self-fulfilling beliefs.[3] These papers provide evidence that post-Volcker, inflation stabilization was successful because of an aggressive monetary policy stance with respect to inflation, that is, an active monetary policy regime. Finally, Sims (2011) and Bianchi and Ilut (2012) argue that a weak response of taxes to debt, or an active fiscal policy regime, led to an increase in inflation in the 1970s as a response to increases in government spending. These authors argue that after the 1970s, the fiscal policy stance changed to one that implied a passive policy regime, that is, taxes responded strongly

[1] For a recent survey of the literature on monetary and fiscal policy interactions, see Canzoneri, Cumby, and Diba (2011).

[2] There are also some well-known papers that provide a learning-based explanation for the rise and fall of U.S. inflation. See for example, Primiceri (2006) and Sargent, Williams, and Zha (2006). Moreover, some papers have attributed the rise and fall of U.S. inflation mostly to time-varying volatility of shocks. See for example, Sims and Zha (2006).

[3] We use the language of Leeper (1991) in characterizing policies as active and passive. Under an active monetary policy regime, nominal interest rates react strongly to inflation while under an active fiscal policy regime, taxes respond weakly to debt outstanding. What exactly constitutes active and passive monetary and fiscal policy is model-specific. Later, we precisely state the bounds on policy parameters that lead to a particular policy regime combination in our model.

to debt. This choked off the possibility of rising inflation in response to fiscal shocks.

Motivated by these theoretical and empirical considerations, we provide a complete and analytical characterization, to the best of our knowledge for the first time in the literature, of these questions in a standard dynamic stochastic general equilibrium (DSGE) model. At first, we use a relatively simple model that enables us to derive sharp and clear closed-form results. The baseline model that we solve in closed-form is a standard sticky price model that features simple monetary and fiscal policy rules, lump-sum taxes, and one-period nominal government bonds. We focus on the correlation between actual inflation and the inflation target and conduct comparative static exercises related to the impact on inflation of changing monetary and fiscal policy stances. We find that the results of these experiments depend critically on the prevailing monetary and fiscal policy regimes.

In particular, we analyze three different policy regimes. First, an active monetary and passive fiscal policy regime, where a high response of interest rates to inflation is coupled with a high response of taxes to outstanding public debt. This is the most common policy regime considered in the literature where a unique bounded equilibrium exists. In this regime, inflation closely follows the path of the inflation target. In fact, stronger the systematic reaction of monetary policy to inflation, more closely will actual inflation follow the inflation target. Moreover, a stronger reaction of monetary policy to inflation decreases the response of inflation to the various non-policy shocks impinging on the economy. Finally, as is well-known, in this case, fiscal policy plays no role in price level determination.[4]

These results are standard since in this regime, monetary policy controls inflation dynamics. An unanticipated decrease in the inflation target, which is equivalent to an unanticipated increase in the nominal interest rate, decreases expected inflation in this regime, since the systematic response of interest rate to inflation is more than one-for-one. Then, due to the increase in the ex-ante real interest rate, the output gap and thereby, actual inflation, decrease. Moreover, stronger the systematic response of interest rates to inflation, greater is the effect on the ex-ante real interest rate, which decreases the effect on inflation when non-policy shocks hit the economy.

Second, we analyze an active fiscal and passive monetary policy regime, where a low response of interest rates to inflation is coupled with a low response of taxes to outstanding public debt.[5] A unique bounded equilibrium exists with this combination of monetary and fiscal policies as well. In this regime, in sharp contrast to the previous regime, inflation moves in an opposite direction from the inflation target on impact.[6] In fact, stronger the

[4] We focus on a model with lump-sum taxes and transfers only.

[5] For early treatments of this policy regime in simple models, see Leeper (1991), Sims (1994), and Woodford (1995).

[6] We analytically characterize the impact responses while computing the entire path of responses numeri-

systematic reaction of monetary policy to inflation, greater will be the divergence between the inflation target and actual inflation. In addition, and again in sharp contrast to the active monetary and passive fiscal regime, in this regime, a stronger reaction of monetary policy to inflation increases the response of inflation to the various non-policy shocks impinging on the economy.[7] Moreover, now, the fiscal policy stance matters for the dynamics of inflation. We show that a weaker response of taxes to debt leads to a weaker response of inflation to non-policy shocks.

These results arise because of the wealth effect on households of interest rate and tax changes.[8] Under the previous regime, because the systematic response of interest rates to inflation was greater than one, expected inflation decreases in response to an unanticipated increase in interest rates. In this regime, however, interest rate increases increase the value of outstanding government debt. Since tax response to government debt is low, this increase in interest rate leads to a positive wealth effect on households, the government bond holders. The positive wealth effect then leads to increased spending and thereby, higher inflation. Moreover, greater the systematic response of interest rates to inflation, as long as this response is less than one-for-one, it only serves to make this positive wealth effect stronger. Then, the equilibrium response of inflation will be even higher. Finally, given the crucial role of government debt dynamics on equilibrium determination, it is natural that fiscal policy stance now matters for the dynamics of inflation. In particular, a weaker response of taxes to debt implies that the wealth effect due to tax changes is lower. Thus, inflation responds by less when non-policy shocks hit the economy.

Third, we explore a passive monetary and passive fiscal policy regime, where a low response of interest rates to inflation is coupled with a high response of taxes to outstanding government debt. In this regime, there is equilibrium indeterminacy and theory provides no clear answer on the behavior of inflation. Generally, in this regime, both fundamental and sunspot shocks play a role in price level determination. Moreover, the potential for self-fulfilling beliefs implies that the effects of monetary and fiscal policy on inflation can be very different from the cases where there is equilibrium determinacy.

While at first we provide closed-form solutions for a simple model, in the second part of the paper, we also conduct a quantitative experiment with a richer DSGE model that includes a variety of shocks and frictions. In particular, we use a medium-scale DSGE model along the lines of Smets and Wouters (2007), Del Negro, Schorfheide, Smets, and Wouters (2007),

cally.

[7]This result is somewhat similar to that of Loyo (1999), who considered only a flexible price economy and showed that a strong response of interest rates to inflation can lead to a hyperinflationary spiral under a passive monetary and active fiscal policy regime. Here, we work with a determinate equilibrium and a sticky-price model.

[8]Changes in lump-sum taxes thus affect consumption in this economy due to the wealth effect.

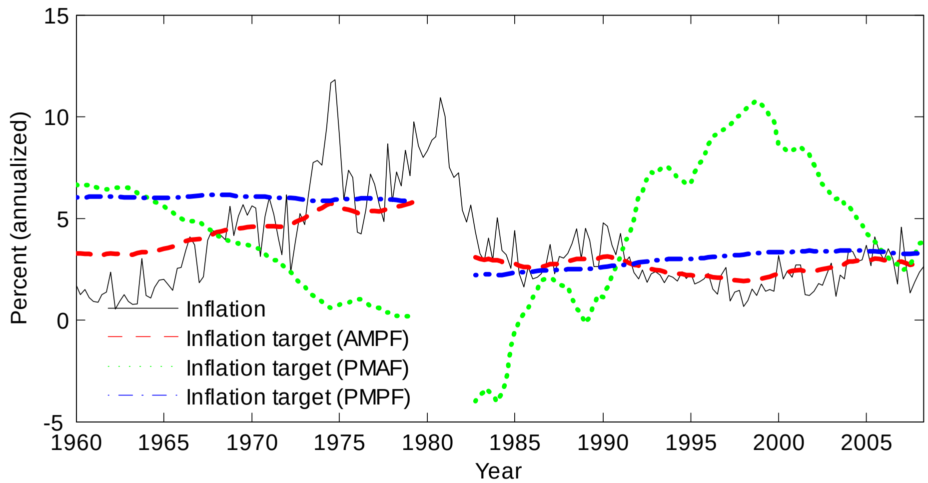

Figure 1: Inflation vs. smoothed inflation target under different policy regimes in an estimated model using U.S. data. Source: Bhattarai, Lee, and Park (2012).

and Justiniano, Primiceri, and Tambalotti (2010). The model features stochastic growth, sticky prices and wages with partial dynamic indexation, habit formation, endogenous capital accumulation, and variable capacity utilization. Moreover, the economy is subject to a wide range of shocks, such as neutral and investment specific technology shocks, preference shock, government spending shock, price and wage markup shocks, and policy shocks.[9] We show that for a wide range of realistic parameter values, our analytical results continue to apply in such a model.

Our results have implications for both the empirical and theoretical literature in monetary economics. First, consider the recent practice, in papers that estimate monetary DSGE models, of using a time-varying inflation target process to explain the low frequency movement in actual inflation. For example, in a recent comprehensive study of various monetary policy reaction functions, Curdia, Ferrero, Ng, and Tambalotti (2011) show that using a time-varying and slow-moving inflation target improves the fit of the model since it helps capture the low frequency variation in inflation. Our results show that this strategy works only if one imposes an active monetary and passive fiscal policy regime while estimating the model.[10] Indeed, using an estimated DSGE model and a pre-Volcker and a post-Volcker subsample analysis, in Bhattarai, Lee, and Park (2012), we show that the correlation between inflation and the smoothed inflation target shock backed out after the estimation varies significantly

[9] We keep the fiscal block of the model relatively simple even in this case by considering only lump-sum taxes and one-period nominal government bonds.

[10] Curdia, Ferrero, Ng, and Tambalotti (2011) use U.S. data from $1987:III$ to $2009:III$, a period during which an active monetary and passive fiscal policy regime is certainly a reasonable description of policy.

depending on which policy regime one imposes during estimation.[11] Figure 1, reproduced from that paper, makes the point especially clear. It shows for example that while under an active monetary and passive fiscal policy regime, the long-run correlation between the inflation target and actual inflation is high and positive, under a passive monetary and active fiscal policy regime, it is strongly negative. Moreover, under a passive monetary and passive fiscal policy regime, while theoretically the correlation between the inflation target and actual inflation is not pinned down by theory, the figure shows that empirically, the correlation is close to zero.

Second, our theoretical results show that the effects of an aggressive monetary policy stance, or a "hawkish" central bank, on inflation depends critically on the joint behavior of monetary and fiscal policy. In particular, we show that in a passive monetary and active fiscal policy regime, an aggressive reaction to inflation by the central bank actually ends up increasing the response and volatility of inflation to non-policy shocks. Thus, any prescription for monetary policy behavior has to take into account the prevailing fiscal policy regime.[12]

2 Simple Model

We use a standard DSGE model with nominal rigidities that can be solved analytically. We lay out the basic model features below while providing a complete description in the appendix. The main actors and their decision problems are as follows.

2.1 Description

2.1.1 Households

Households, a continuum in the unit interval, face an infinite horizon problem and maximize expected discounted utility over consumption and leisure. The utility function is additively separable over consumption and labor effort.

2.1.2 Firms

Firms, a continuum in the unit interval, produce differentiated goods using labor as input. Firms have some monopoly power over setting prices, which are sticky in nominal terms.

[11] The estimated model in Bhattarai, Lee, and Park (2012), while richer than the analytical model we work with in this paper, is relatively small-scale. For example, it does not feature sticky wages and endogenous capital accumulation, features that are present in the quantitative model we use in this paper.

[12] Loyo (1999) uses a similar result from a flexible price model to interpret the experience of Brazil in the 1970s and 1980s. Relatedly, Sims (2004) shows in a very different set-up, also a flexible price model, that a central bank might lose control of inflation if it is not adequately backed up by the treasury.

Price stickiness is modelled using the Calvo formulation where every period, firms face a constant probability of not adjusting prices.

2.1.3 Government

The government is subject to a flow budget constraint and conducts monetary and fiscal policies using endogenous feedback rules. For simplicity, we assume that the government issues only one-period nominal debt and levies lump-sum taxes. The government controls the one-period nominal interest rate R_t. Monetary policy is modelled using an interest rate rule that features a systematic response of the nominal interest rate to the deviation of inflation π_t from a time-varying target π_t^*. The feedback parameter on inflation deviation is given by ϕ. Fiscal policy is modelled using a tax rule that features a systematic response of the tax revenues τ_t to the level of outstanding government debt b_{t-1}. The feedback parameter on debt is given by ψ.

2.2 Approximate Model

We first solve the problem of households and firms given the monetary and fiscal policy rules and derive the equilibrium conditions. We then use approximation methods to solve the model: we obtain a first-order approximation to the equilibrium conditions around the non-stochastic steady state.[13] We provide the detailed derivations in the appendix. The resulting model can be summarized by the following linearized equations:

$$\begin{aligned}
\tilde{Y}_t &= E_t\tilde{Y}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \hat{r}_t^* && (1)\\
\hat{\pi}_t &= \kappa\tilde{Y}_t + \beta E_t\hat{\pi}_{t+1} && (2)\\
\hat{R}_t &= \phi\left(\hat{\pi}_t - \hat{\pi}_t^*\right) && (3)\\
\hat{\tau}_t &= \psi\hat{b}_{t-1} && (4)\\
\hat{b}_t &= \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\hat{\pi}_t - \beta^{-1}\hat{\tau}_t + \bar{b}\hat{R}_t && (5)\\
\hat{r}_t^* &= \rho_r\hat{r}_{t-1}^* + \varepsilon_{r,t} && (6)\\
\hat{\pi}_t^* &= \rho_\pi\hat{\pi}_{t-1}^* + \varepsilon_{\pi,t}. && (7)
\end{aligned}$$

Here, $\tilde{Y}_t \equiv \hat{Y}_t - \hat{Y}_t^n$ is the output gap. That is, it is the difference between actual output

[13] In the equations below, we use $\hat{X}_t$ to denote the log deviation of a variable X_t from its steady state $\bar{X}$ ($\hat{X}_t = \ln X_t - \ln\bar{X}$), except for two fiscal variables, $\hat{b}_t$ and $\hat{\tau}_t$. Following Woodford (2003), we let them represent respectively the deviation of the maturity value of government debt and of government tax revenues (net of transfers) from their steady-state levels, measured as a precentage of steady-state output: $\hat{b}_t = \frac{b_t - \bar{b}}{\bar{Y}}$ and $\hat{\tau}_t = \frac{\tau_t - \bar{\tau}}{\bar{Y}}$.

$\hat{Y}_t$ and the natural level of output $\hat{Y}_t^n$, the output that would prevail under flexible prices. Moreover, $\hat{r}_t^*$ is a composite shock that is a linear combination of the structural shocks in the model such as technology and preference shocks. It is often referred to as the natural rate of interest because it is the real interest rate that would prevail under flexible prices. Equation (6) shows that we assume that it follows an exogenous AR (1) process with $0 < \rho_r < 1$. The innovation $\varepsilon_{r,t}$ has zero mean and variance σ_r^2. Equation (7) shows that we assume that the inflation target follows an exogenous AR(1) process with $0 < \rho_\pi < 1$. The innovation $\varepsilon_{\pi,t}$ has zero mean and variance σ_π^2. Ireland (2007) models that the Federal Reserve adjusts the inflation target in response to the economy's supply shocks but finds that the response is not statistically significant. In light of this result, we make the exogeneity assumption on $\hat{\pi}_t^*$. Cogley and Sbordone (2008) and Cogley, Primiceri and Sargent (2010) also use an exogenous AR process to model the inflation target.[14]

Equation (1), the dynamic "IS" equation, expresses how the output gap today is determined by the expected output gap tomorrow and the ex ante real interest rate. Equation (2), the dynamic "AS" equation, describes how inflation today is determined as a function of discounted expected inflation tomorrow and the output gap today. Here, β is the discount factor of the household and κ, which determines the slope of the AS equation, is a composite parameter of the structural parameters. Equation (3) is the monetary policy rule which governs the response of the nominal interest rate to the deviation of inflation from the inflation target while Equation (4) is the fiscal policy rule which governs the response of taxes to the real maturity value of the outstanding debt. Finally, Equation (5) is the flow budget constraint of the government.

As is well-known, in the approximate model, the existence and uniqueness of equilibrium depends crucially on the prevailing monetary and fiscal policy regime. The equilibrium of the economy will be determinate either if monetary policy is active while fiscal policy is passive (the AMPF regime) or if monetary policy is passive while fiscal policy is active (the PMAF regime). The equilibrium is indeterminate and multiple equilibria exist if both monetary and fiscal policies are passive (the PMPF regime). In our model, monetary policy is active if $\phi > 1$ and fiscal policy is active if $\psi < 1 - \beta$. Table 1 summarizes these policy regime

[14] The assumption that $\hat{\pi}_t^*$ is stationary implies that the monetary authority does not permanently keep the inflation target at the same level but in the long-run drives the inflation target back to the non-stochastic steady state level. We introduce this assumption for two reasons. First, the stationarity assumption allows us to work with a standard framework. To assume that the inflation target has a unit root results in time-varying coefficients of the Phillips curve as in Cogley and Sbordone (2008) or leads to a non-standard monetary policy rule for with which the Taylor principle should be modified. Therefore, to present our point clearly within a familiar framework, we assume that the inflation target is very persistent but stationary. Second, we will generally restrict ρ_π to values close to 1, thereby effectively ensuring that $\hat{\pi}_t^*$ captures the persistent behavior of the inflation target set by the central bank. Cogley, Primiceri, and Sargent (2010) use the same assumption.

Table 1: Monetary/Fiscal Policy Regimes and Equilibrium Properties

	Active Money ($\phi > 1$)	Passive Money ($\phi < 1$)
Active Fiscal ($\psi < 1 - \beta$)	No Equilibrium	Unique Equilibrium
Passive Fiscal ($\psi > 1 - \beta$)	Unique Equilibrium	Multiple Equilibria

combinations and the associated equilibrium outcomes.

2.3 Results

We analytically characterize the solution of the model either when a determinate equilibrium exists or when there are multiple equilibria. We then derive several results regarding the dynamics of inflation. Specifically, we study how the path of inflation depends on the path of the inflation target and how the response of inflation changes when monetary and fiscal policy stances change within a policy regime combination. All the details of the derivations and the proofs of the various propositions are in the appendix.

2.3.1 Active Monetary and Passive Fiscal Policy

Under an active monetary and passive fiscal policy regime, we can express the solution for inflation as:

$$\hat{\pi}_t = \Phi(\phi)\,\hat{\pi}_t^* + \Gamma(\phi)\,\hat{r}_t^*, \tag{8}$$

where $\Phi(\phi)$ and $\Gamma(\phi)$ are functions of the monetary policy response parameter ϕ.[15] Note that in this case, as Equation (8) makes clear, the dynamics of inflation do not depend on the dynamics of government debt and fiscal policy. Therefore, debt $\hat{b}_{t-1}$ is not a state variable that determines inflation and the fiscal policy response parameter ψ does not affect inflation. Moreover, this implies that inflation is solely a function of the two exogenous processes, $\hat{\pi}_t^*$ and $\hat{r}_t^*$, since other than the budget constraint, the rest of the model is completely forward-looking.

We next characterize several properties of the solution. We first start with the response of inflation to changes in the inflation target.

[15] Obviously, $\Phi(\phi)$ and $\Gamma(\phi)$ in Equation (8) are a function of other structural parameters as well. Here on after, we write the coefficients in a solution for inflation as a function of policy parameters only so as to highlight their role in determining inflation dynamics.

Proposition 1 (Direction of inflation response) *When monetary policy is active and fiscal policy is passive (AMPF), inflation moves in the same direction as the inflation target – that is,*

$$\Phi(\phi) \geq 0.$$

The equality holds when prices are completely sticky ($\kappa = 0$). Moreover, inflation responds more (or less) than one-for-one to changes in the inflation target if prices are sufficiently flexible (or sticky):

$$\begin{aligned} 1 &< \Phi(\phi), & \text{for } \kappa > \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}, \\ 0 &\leq \Phi(\phi) \leq 1, & \text{for } 0 \leq \kappa \leq \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}. \end{aligned}$$

In this regime, since inflation moves in the same direction as the inflation target, we see clearly that monetary policy controls the dynamics of inflation. Consider a positive shock to $\hat{\pi}_t^*$. From Equation (3), this leads to a decrease in the nominal interest rate $\hat{R}_t$. Since in this regime the central bank systematically changes nominal interest rates more than one-for-one to changes in inflation ($\phi > 1$), a decrease in $\hat{R}_t$ brings expected inflation up. This implies that the ex-ante real interest rate $\hat{R}_t - E_t\hat{\pi}_{t+1}$ goes down. From, Equation (1), this leads to an increase in the output gap $\tilde{Y}_t$ and then from Equation (2), it leads to an increase in actual inflation $\hat{\pi}_t$. Thus, $\hat{\pi}_t$ and $\hat{\pi}_t^*$ are positively correlated.

Moreover, as is natural, the response of inflation to the inflation target shock depends on the extent of price stickiness in the economy: greater the degree of price stickiness, smaller is the response of inflation. As a limiting result, the proposition also shows that inflation does not respond to the inflation target at all when prices are completely sticky because the price level would not respond at all to any shocks. In contrast, when prices are sufficiently flexible, inflation responds more than one-for-one to changes in the inflation target. This is possible because due to two factors. First, from Equation (3), we see that on impact, a unit increase in $\hat{\pi}_t^*$ decreases $\hat{R}_t$ by more than a unit (since $\phi > 1$).[16] At a given level of $\hat{\pi}_t$, from Equation (1), $\tilde{Y}_t$ increases and in equilibrium, if κ is large enough, which implies that inflation is quite sensitive to changes in the output gap, then Equation (2) shows that the increase in $\hat{\pi}_t$ can be by more than a unit.

Now let us consider a comparative static exercise with respect to the monetary policy parameter ϕ, which is a measure of the monetary policy stance.

Proposition 2 (Magnitude of inflation response and monetary policy stance) *When*

[16] To emphasize, this is in a partial equilibrium sense.

monetary policy is active and fiscal policy is passive (AMPF), the response of inflation to changes in the inflation target is decreasing (or increasing) in ϕ if prices are sufficiently flexible (or sticky):

$$\begin{aligned}\frac{\partial \Phi(\phi)}{\partial \phi} &< 0, \qquad \text{for } \kappa > \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}, \\ \frac{\partial \Phi(\phi)}{\partial \phi} &> 0, \qquad \text{for } 0 \le \kappa \le \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}.\end{aligned}$$

In combination with Proposition 1, we now have the intuitive result that $\hat{\pi}_t$ will move *more closely* with $\hat{\pi}_t^*$ as ϕ increases – for all values of κ. Again, in this sense, monetary policy controls inflation successfully in this regime. Thus, if the central bank's objective is to stabilize the "inflation gap," $\hat{\pi}_t - \hat{\pi}_t^*$, it needs to have a large value for ϕ, or respond strongly to the inflation gap. As ϕ is higher, in this active monetary regime, expected inflation increases by less for a given increase in $\hat{\pi}_t^*$, thereby dampening down the response of inflation.

To make the results even more transparent, we show in Figure 2 the impulse response of inflation to an exogenous change in the inflation target, varying the degree of monetary policy stance.[17] Figure 2 clearly shows that inflation dynamics closely mimic those of the inflation target. The completely forward-looking nature of the model with the fiscal variables being redundant makes the inflation dynamics particularly simple. Note that if the inflation target moves persistently, so does inflation. In addition, we can see that inflation responds more than one-for-one to changes in the inflation target because $\Phi(\phi) > 1$ at our benchmark parameterization.[18] Not surprisingly, however, inflation is closer to the target rate as the monetary authority responds more strongly to the inflation gap.

We next analyze the response of inflation to the non-policy shock $\hat{r}_t^*$.

Proposition 3 (Direction of inflation response) *When monetary policy is active and fiscal policy is passive (AMPF), inflation moves in the same directions in response to the non-policy shock, $\hat{r}_t^*$ – that is,*

$$\Gamma(\phi) > 0.$$

The proposition thus establishes that when monetary policy is active, inflation moves in the same direction as the natural rate of interest $\hat{r}_t^*$. In particular, we show in the appendix that demand-type shocks such as preference shocks increase $\hat{r}_t^*$, while supply-type shocks such as technology shocks lower $\hat{r}_t^*$. Hence, the proposition tells us that inflation increases in

[17] For this and the other figures in the analytical model section, we assign some standard values to model parameters: $\beta = 0.99$; $\alpha = 0.75$; $\bar{b} = 0.4$; $\rho_r = 0.9$; and $\rho_\pi = 0.995$.

[18] The lower bound of κ for $\Phi(\phi) > 1$, found in Proposition 1, is very small when ρ_π has a vlaue close to one. At the benchmark parameterization, the lower bound is less than 0.0001, which is not restrictive at all.

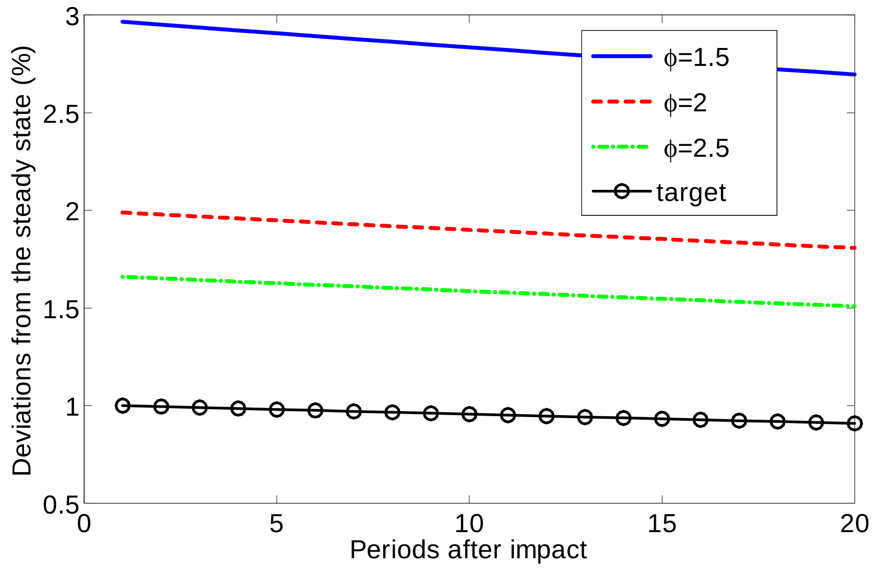

Figure 2: The response of inflation to a one percentage point increase in the inflation target under the AMPF regime.

response to favorable demand shocks and decreases in response to favorable supply shocks. This is a conventional result under the AMPF regime. Equation (1) implies that a positive $\hat{r}_t^*$ increases output gap for a given level of expected output gap and the real interest rate. Then from Equation (2) it is clear that this in turn increases inflation and expected inflation in equilibrium.

We next assess how this response depends on the monetary policy stance.

Proposition 4 (Magnitude of inflation response and monetary policy stance) *When monetary policy is active and fiscal policy is passive (AMPF), inflation responds less to non-policy shocks as the monetary authority becomes more aggressive—that is, Γ decreases as ϕ increases:*

$$\frac{\partial \Gamma(\phi)}{\partial \phi} < 0.$$

We therefore show that greater the systematic response of monetary policy to inflation, lower will be the response of inflation to non-policy shocks. As ϕ increases, under active monetary policy, expected inflation gets damped down more. Since this decreases the response of the ex ante real interest rate, inflation will increase by a lower amount in equilibrium.

Finally, because of the forward-looking nature of the model variables, it is straightforward to establish a result regarding the variance of inflation in response to non-policy shocks.

Proposition 5 (Unconditional variance) *When monetary policy is active and fiscal policy is passive (AMPF), the unconditional variance of inflation decreases in ϕ:*

$$\frac{\partial VAR_{NP}(\hat{\pi}_t)}{\partial \phi} < 0,$$

where $VAR_{NP}(\cdot)$ denotes the unconditional (long-run) variance associated with non-policy shocks only (i.e. the influence of policy shifts by the government is shut down: $\varepsilon_{\pi,t} = 0$).

The proposition thus establishes that the long-run variance of inflation in this policy regime decreases when the monetary authority reacts systematically strongly to inflation. The two previous propositions have essentially the same implication: As the central bank responds more strongly to the inflation gap, the volatility of inflation due to non-policy shocks decreases.

Again, we illustrate these results in Figure 3. It shows the response of inflation to non-policy shocks under three different values of ϕ and clearly illustrates our analytical findings. In addition, we can see that the response of inflation decays at a much faster rate here than in Figure 2. The reason for this is straightforward. As should be clear from Equation (8), the response of inflation to a shock to each of the exogenous processes – aside from the size of the response – is entirely dictated by the response of the respective exogenous process itself. Therefore, to the extent that the inflation target is more persistent than other exogenous variables ($\rho_\pi > \rho_r$), inflation should return to its steady state level more slowly in response to inflation target shocks.

Our results raise an interesting point. Empirical studies in the recent DSGE literature such as Cogley, Primiceri and Sargent (2010) have found that the low-frequency components of the inflation rate are explained almost entirely by a time-varying inflation target. Our analytical analysis however suggests that fixing ρ_π to a large value is necessary for this well-established finding. The finding is not obtained without such a tight restriction in modeling and estimation. Figure 4 compares the response of inflation to the inflation target shock with the response to the non-policy shock. For a more direct comparison, we have normalized the initial responses to one. We can clearly see that the inflation target – relative to other shocks – dominates inflation dynamics, especially in the long run. As discussed above, however, this result is entirely due to the fact that we fix ρ_π at a higher value than ρ_r. If we instead treated all shocks symmetrically (i.e. $\rho_\pi = \rho_r$), the model would not distinguish between the two shocks with respect to inflation dynamics, as can be seen from Figure 4.

We now move on to analyzing another policy regime combination that leads to a determinate equilibrium. As we will see, the results related to the correlation between the inflation target and the comparative statics on monetary policy response parameters will be in stark

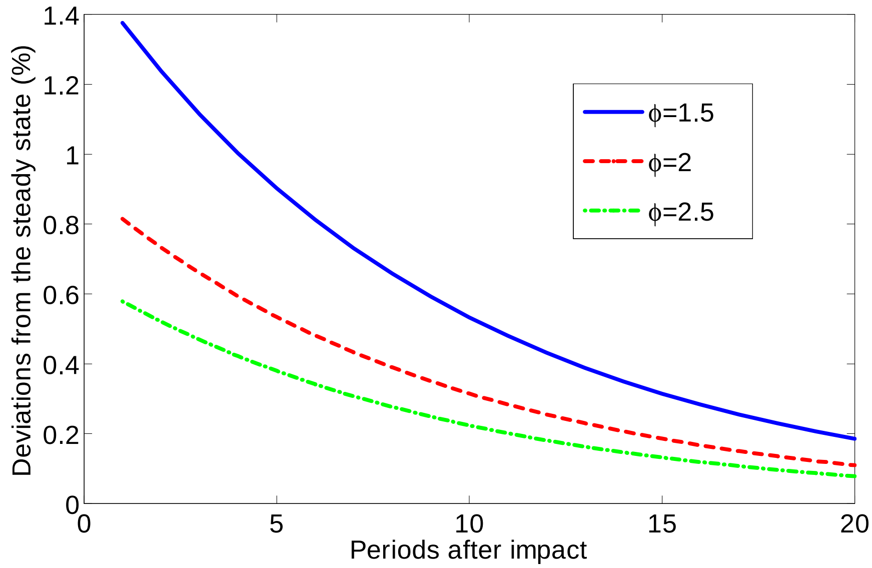

Figure 3: The response of inflation to a one percentage point increase in the non-policy shock under the AMPF regime.

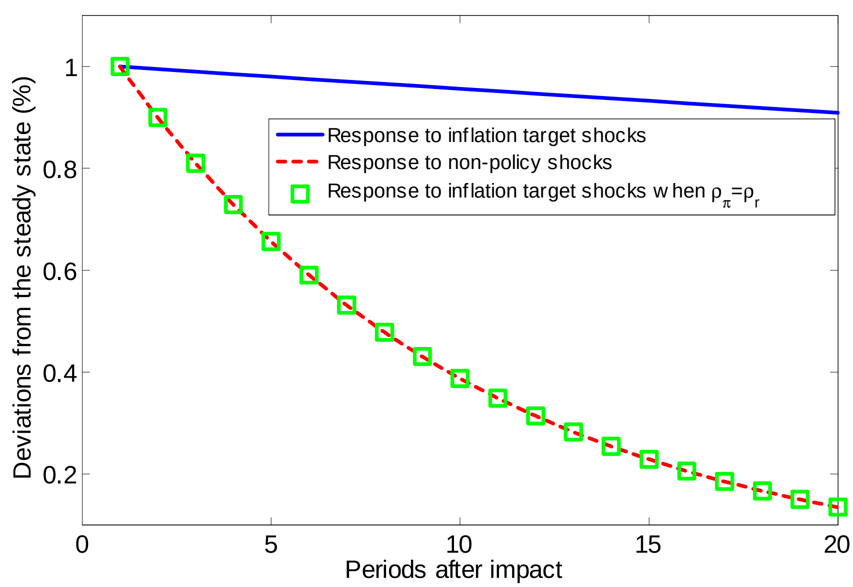

Figure 4: The response of inflation to a one percentage point increase in the inflation target and the non-policy shock under the AMPF regime.

contrast in this case compared to the active monetary and passive fiscal policy regime.

2.3.2 Passive Monetary and Active Fiscal Policy

Under a passive monetary and active fiscal policy regime, we can express the solution for inflation as:

$$\hat{\pi}_t = \Omega(\phi,\psi)\,\hat{b}_{t-1} - \Phi(\phi,\psi)\,\hat{\pi}_t^* + \Gamma(\phi,\psi)\,\hat{r}_t^*, \tag{9}$$

where $\Omega(\phi,\psi)$, $\Phi(\phi,\psi)$, and $\Gamma(\phi,\psi)$ are functions of both the monetary policy response parameter ϕ and the fiscal policy response parameter ψ.[19] Note that in this policy regime, as Equation (9) makes clear, the dynamics of inflation depend on public debt outstanding $\hat{b}_{t-1}$ as well. This implies that there is an endogenous state variable in this case, which in turn imparts endogenous dynamics to the model. These are extremely important differences from the case we analyzed in the previous section where monetary policy was active and fiscal policy was passive.

We next characterize several properties of the solution.

Proposition 6 (Direction of inflation response) *When fiscal policy is "sufficiently" active and monetary policy is passive (PMAF), inflation moves in the opposite direction in response to a change in the inflation target – that is,*

$$\begin{aligned}\Phi(\phi,\psi) \geq 0 \quad & \textit{for} \\ & -\infty < \psi < \bar{\psi}^{**} \textit{ and } 0 \leq \phi < 1,\end{aligned} \tag{10}$$

where $0 < \bar{\psi}^{**} \leq 1-\beta$ *is a reduced-form parameter. The equality holds when* $\phi = 0$.

This result, which is in stark contrast to Proposition 1 under the AMPF regime, arises because now changes in the value of government debt influences inflation dynamics. Consider a negative shock to the inflation target. From Equation (3), this increases the nominal interest rate on impact. An increase in the nominal interest rate results in an increase of the outstanding value of government debt. In this active fiscal policy regime, since taxes do not adjust by enough, the increase in the value of government debt leads to a positive wealth effect on households who hold government debt. This positive wealth effect then leads to higher spending, which pushes up inflation. Proposition 6 is therefore, the key result behind the negative relationship between inflation and the inflation target under the PMAF regime shown in Figure 1. The wealth effect on households due to changes in the value of government

[19] The complete solution, including the solution for debt, is provided in the appendix.

debt and taxes, which in turn affects households' spending, is the main mechanism behind our results in this section.

Now let us consider a comparative static exercise with respect to the monetary policy parameter ϕ.

Proposition 7 (Magnitude of inflation response and monetary policy stance) *When fiscal policy is "sufficiently" active and monetary policy is passive (PMAF), inflation deviates even further from the inflation target when the monetary authority is more aggressive – that is,* Φ *increases in* ϕ *in the domain of* $[0, 1)$*:*

$$\frac{\partial \Phi(\phi, \psi)}{\partial \phi} > 0 \qquad \text{for}$$
$$-\infty < \psi < \bar{\psi}^{***} \text{ and } 0 \leq \phi < 1,$$

where $0 < \bar{\psi}^{***} \leq 1 - \beta$ *is a reduced-form parameter.*

In sharp contrast to our result under the AMPF policy regime, here, as the reaction of monetary policy to inflation increases, so does the equilibrium impact on inflation of the inflation target shock. The mechanism is as follows. When the reaction of monetary policy to inflation increases, then for a given decrease in the inflation target, the interest rate increase will be higher. This means that the outstanding value of government debt increases by more, which in turn, increases the size of the wealth effect discussed above. This then implies a greater effect on spending, and thereby, on inflation. Thus, unless the monetary authority decides to respond to inflation by enough such that monetary policy moves from a passive regime to an active regime and at the same time fiscal policy moves from an active regime to a passive regime, a stronger response of monetary policy to inflation ends up stabilizing inflation by less.

In this regime, since fiscal policy also matters for inflation dynamics, we next establish a result related to the fiscal policy stance.

Proposition 8 (Magnitude of inflation response and fiscal policy stance) *When monetary policy is passive and fiscal policy is active (PMAF), inflation deviates even further from the inflation target as the fiscal authority becomes more active – that is,* Φ *increases as* ψ *decreases in the domain of* $(-\infty, 1 - \beta)$*:*

$$\frac{\partial \Phi(\phi, \psi)}{\partial \psi} < 0 \qquad \text{for}$$
$$-\infty < \psi < 1 - \beta \text{ and } 0 \leq \phi < 1.$$

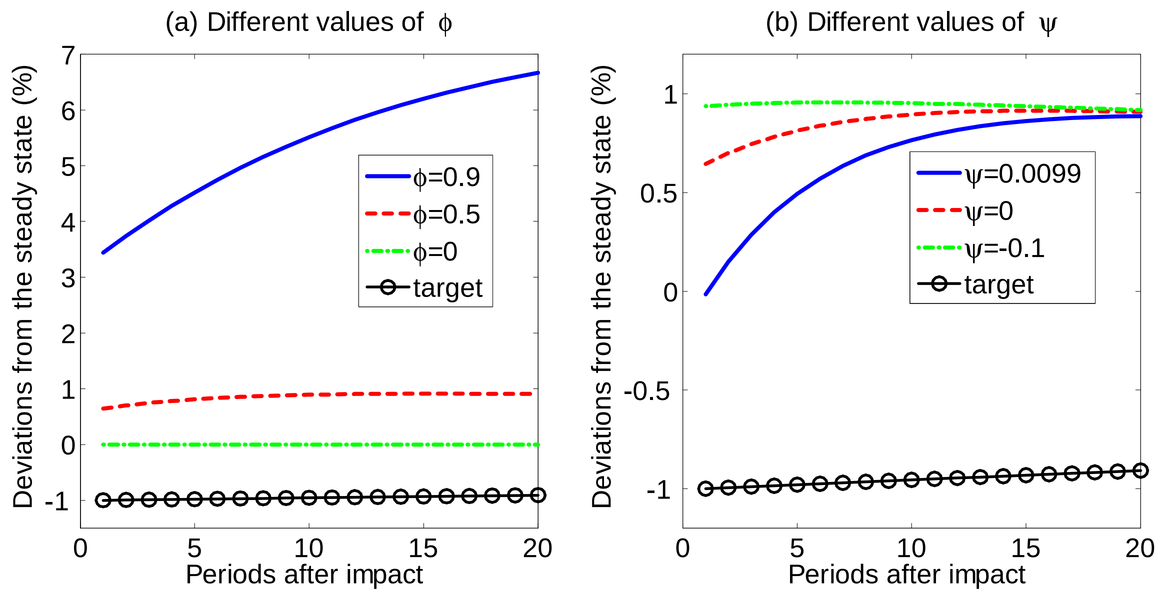

Figure 5: The response of inflation to a one percentage decrease in the inflation target under the PMAF regime.

Here, as fiscal policy becomes more active, we see that inflation will respond more strongly, and in the opposite direction, to changes in the inflation target. This result arises because as ψ decreases, taxes respond less strongly to debt as given by Equation (4). Then the wealth effect due to interest rate changes described above becomes amplified. This increased wealth effect in turn leads to greater spending and thereby a stronger response of inflation.

As an illustration, we show in Figure 5 the responses of inflation to a one percent increase in the inflation target shock under varying degrees of monetary and fiscal policy stances to inflation and debt. The figure highlights our theoretical results above. In addition, it shows that although our theoretical findings are focused on the impact response of inflation, the same economic intuition can be extended to longer horizons. Indeed, it clearly shows that the deviation of inflation from the target continues to be greater in periods following the shock, as monetary and fiscal policies become more active. The reason is that when ϕ is higher (and/or ψ is lower), the interest rate will be persistently higher after a negative shock to the inflation target to the extent that the inflation target is persistent. This in turn leads to persistently higher value of public debt. This then leads to a persistently positive wealth effect, which in turn, leads to a persistently higher inflation. What is more, when monetary and/or fiscal policy is more active, inflation depends more strongly on government indebtedness – that is, as shown below in a proposition, $\Omega(\phi, \psi)$ is increasing in ϕ and decreasing in ψ. This property obviously magnifies the mechanism through which higher debt influences the dynamics of inflation.

We now move on to analyzing the response of inflation to the non-policy shock.

Proposition 9 (Direction of inflation response) *When monetary policy is passive and fiscal policy is active (PMAF), inflation moves in the same directions in response to the non-policy shock,* $\hat{r}_t^*$ *– that is,*

$$\Gamma(\phi, \psi) \geq 0 \quad \textit{for} \\ -\infty < \psi < 1 - \beta \textit{ and } 0 \leq \phi < 1. \tag{11}$$

The equality holds when $\kappa = 0$.

This proposition under the PMAF policy regime is the same as Proposition 3 under the AMPF policy regime. That is, inflation moves in the same direction as the natural rate of interest. Thus inflation increases in response to a positive demand shock while it decreases in response to a positive supply shock. This is because even under the PMAF policy regime, the effect of the non-policy shock on the economy is still to increase the output gap given the expectations as implied by Equation (1), and in turn, inflation as implied by Equation (2). The comparative statics of inflation responses with respect to the monetary policy stance however, is dramatically different in the PMAF regime compared to the AMPF regime, as we establish next.

Proposition 10 (Magnitude of inflation response and monetary policy stance) *When monetary policy is passive and fiscal policy is active (PMAF), inflation responds more to non-policy shocks as the monetary authority becomes more aggressive—that is,* Γ *increases as* ϕ *increases in the domain of* $[0, 1)$*:*

$$\frac{\partial \Gamma(\phi, \psi)}{\partial \phi} > 0 \quad \textit{for} \\ -\infty < \psi < 1 - \beta \textit{ and } 0 \leq \phi < 1.$$

Thus, the stronger the systematic response of monetary policy to inflation, the greater will be the response of inflation to the non-policy shocks in equilibrium. Why is this the case? When a positive $\hat{r}_t^*$ shock hits the economy, it raises inflation. Now with a higher ϕ, interest rates will rise by more in response to this increase in inflation, as given by Equation (3). Under the AMPF policy regime, this increase in interest rates would bring expected inflation down. In this PMAF regime, however, the greater increase in interest rates raises the outstanding value of government debt by a greater amount. As we have explained before, this leads to a greater wealth effect on the households, which increases inflation by a larger amount.

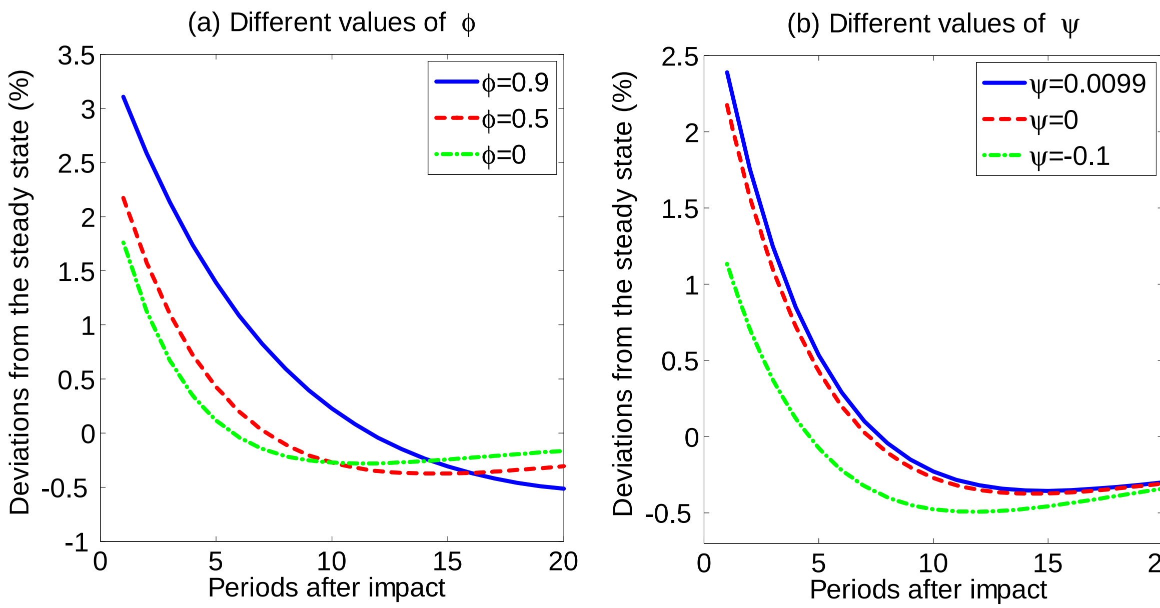

Figure 6: The response of inflation to a one percentage increase in the non-policy shock under the PMAF regime.

We next conduct a similar comparative static exercise with respect to the stance of fiscal policy.

Proposition 11 (Magnitude of inflation response and fiscal policy stance) *When monetary policy is passive and fiscal policy is active (PMAF), inflation responds less in response to non-policy shocks as the fiscal authority becomes more active—that is,* Γ *decreases as* ψ *decreases in the domain of* $(-\infty, 1-\beta)$*:*

$$\frac{\partial \Gamma(\phi, \psi)}{\partial \psi} > 0 \qquad \textit{for}$$

$$-\infty < \psi < 1-\beta \textit{ and } 0 \leq \phi < 1.$$

This proposition shows that the weaker is the response of taxes to debt, the lower is the response of inflation to the non-policy shock. When a positive $\hat{r}_t^*$ hits the economy, as we discussed above, it leads to higher inflation. This lowers the outstanding value of government debt. From the fiscal policy rule (4) this implies that taxes will decrease. Now the lower is ψ, the smaller is the decrease in taxes. Even though taxes are lump-sum in our model, when the regime is PMAF, tax changes lead to a wealth effect on households. With a smaller decrease in taxes, the wealth effect is smaller, which in turn leads to a smaller change in spending and thereby inflation.

Figure 6 illustrates our results on inflation response to non-policy shocks under varying degrees of monetary and fiscal policy reactions to inflation and debt respectively. The figure highlights our analytical results above on the impact response of inflation. Higher ϕ or a higher ψ leads to a greater initial impact of inflation. In addition to the initial impact, the dynamic responses of inflation reveal an interesting pattern that is different from the case under AMPF. While the initial response of inflation is positive in response to the shock $\hat{r}_t^*$ and it remains positive for a number of periods, after some time, inflation goes below steady state. The intuition for this result is again related to the dynamics of government debt. Initially, the increase in inflation lowers the outstanding value of government debt. In this regime, this decrease in the value of government debt leads to a negative wealth effect on households. This negative wealth effect leads to a decrease in spending by households, which in turn, eventually leads to inflation decreasing and going below steady state. Moreover, note that while analyzing the dynamic response of inflation under different values of ϕ, one sees that the paths intersect after a certain number of periods. This feature arises because when inflation goes below steady state, it leads to a decrease in nominal interest rates, as given by Equation (3). This decrease in interest rate leads to a negative wealth effect. Higher the value of ϕ, greater is this negative wealth effect. Thus, once inflation goes below steady state, due to the negative wealth effect that depresses spending, there is a tendency for inflation to continue below steady for a while. This effect is more pronounced when ϕ is higher, which in turn, implies that the paths for different levels of ϕ will cross.

Figure 7 shows results for the inflation response to non-policy shocks under varying degrees of monetary policy reaction to inflation and for different levels of persistence of the non-policy shocks. As is to be expected, the greater the persistence of the shock, the more persistent will be the response of inflation. Moreover, the pattern of inflation initially remaining above steady state and then eventually going below steady state is robust to various levels of persistence of the shock.

The results on the dynamic responses of inflation have an important implication for the relationship between the monetary policy stance and the volatility of inflation – a primary policy objective of central banks. Due to endogenous dynamics under this policy regime, we are unable to provide closed form expressions for the variance of inflation. We thus resort to numerical illustrations. As Figure 6 illustrates, under PMAF, while the response of inflation deviation is not greater for every time period when ϕ is higher, it is certainly the case for most periods – especially the initial period. To the extent that initial responses of inflation to shocks dominate in the second moment of inflation dynamics, the volatility of inflation will be larger when monetary policy reaction is stronger.[20] Figure 8 illustrates this result. Under

[20]Initial responses are disproportionately important for the variance of the inflation rate because the *squared* size of the initial response to a shock is substantially bigger than those of the responses in the following periods as can be seen clear in Figure 7. This argument is reminiscent of the difference in outcomes

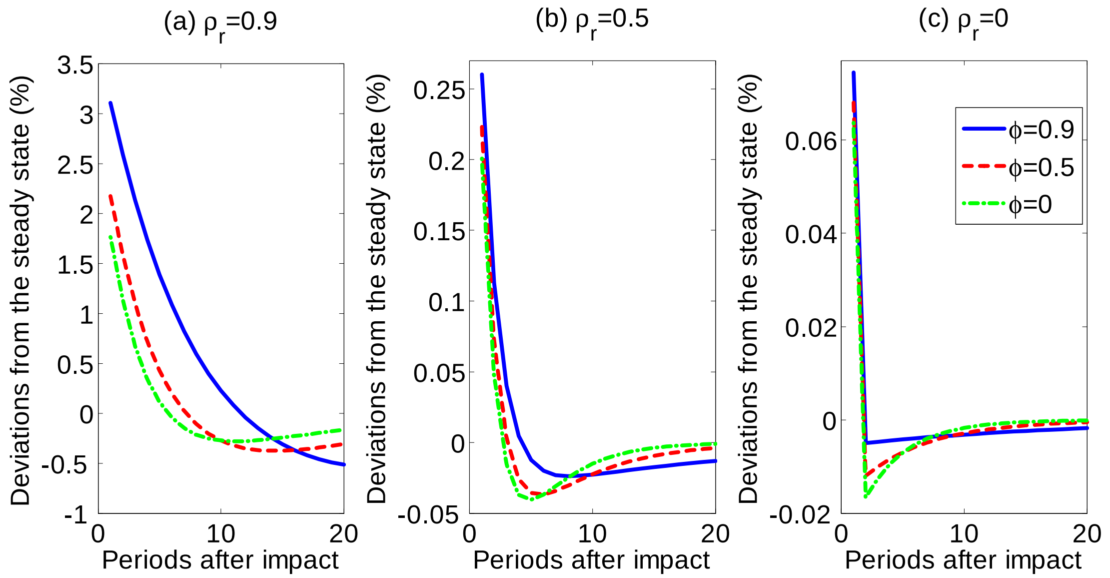

Figure 7: The response of inflation to a one percentage point increase in the non-policy shock with different degrees of persistence under the PMAF regime.

AMPF, a more hawkish monetary policy leads to a smaller standard deviation of inflation as proved earlier. Under PMAF, however, a stronger monetary policy reaction to inflation instead leads to a higher volatility of inflation.

We next provide some properties of the solution related to the response of inflation to public debt outstanding. Note again that this feature of the solution is unique to the passive monetary and active fiscal policy regime (PMAF).

Proposition 12 (Direction of inflation response) *When fiscal policy is active and monetary policy is passive (PMAF), inflation moves in the same direction in response to a change in public debt outstanding – that is,*

$$\Omega(\phi, \psi) > 0 \qquad \textit{for}$$
$$-\infty < \psi < 1 - \beta \ \textit{and}\ 0 \leq \phi < 1.$$

Thus, inflation is affected positively by changes in public debt outstanding in this regime. This result is again a direct derivative of the wealth effect on households that is a crucial mechanism under active fiscal policy. A higher level of public debt outstanding will increase the extent of wealth effect of changes in the value of public debt. This then changes spending

when monetary policy is analyzed under commitment and under discretion, also known as the *stabilization bias.*

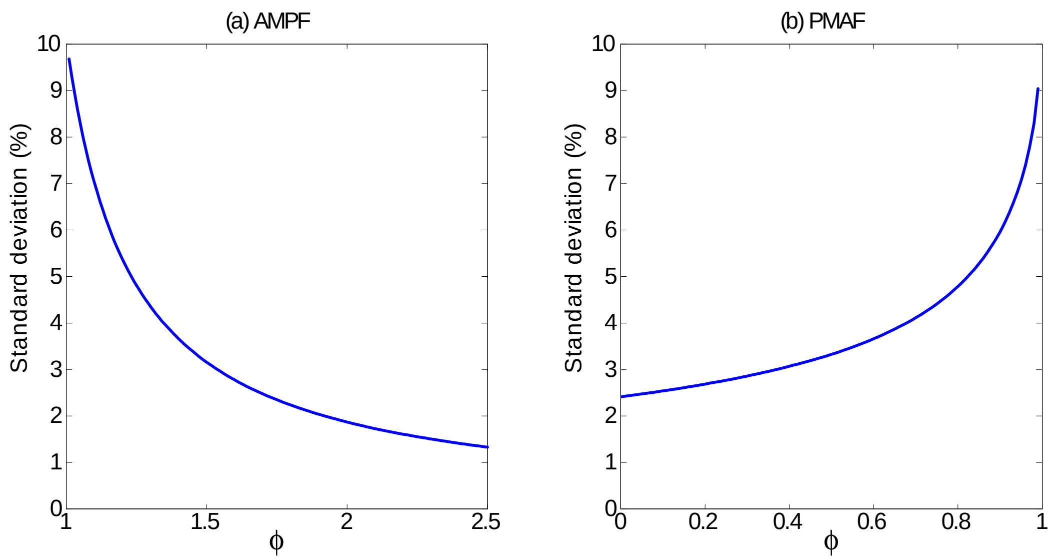

Figure 8: Standard deviation of inflation (in percent) across different values of ϕ conditional on the non-policy shocks under the PMAF regime. The standard deviation of the non-policy shock is normalized to one percent.

and thereby, inflation by a greater extent. We now conduct comparative static exercises with respect to the monetary policy and fiscal policy parameters.

Proposition 13 (Magnitude of inflation response and monetary policy stance) *When fiscal policy is "sufficiently" active and monetary policy is passive (PMAF), inflation responds more to a change in public debt outstanding when the monetary authority is more aggressive—that is, Ω increases as ϕ increases in the domain of $[0, 1)$:*

$$\frac{\partial \Omega(\phi, \psi)}{\partial \phi} > 0 \qquad \text{for}$$
$$-\infty < \psi < \bar{\psi}^* \text{ and } 0 \leq \phi < 1,$$

where $\bar{\psi}^*$ is a positive reduced-form parameter that lies between 0 and $1 - \beta$ (i.e. $0 < \bar{\psi}^* \leq 1 - \beta$).

Proposition 14 (Magnitude of inflation response and fiscal policy stance) *When fiscal policy is active and monetary policy is passive (PMAF), inflation responds more to a change in public debt outstanding when the fiscal authority is more active—that is, Ω de-*

creases as ψ decreases in the domain of $(-\infty, 1-\beta)$:

$$\frac{\partial \Omega(\phi,\psi)}{\partial \psi} < 0 \qquad \textit{for}$$
$$-\infty < \psi < 1-\beta \textit{ and } 0 \leq \phi < 1.$$

The first of the propositions above thus shows that a greater systematic response of interest rates to inflation lead to a greater response of inflation to public debt outstanding. Again, this result arises because with a stronger response of interest rates to inflation, the wealth effect on households of changes in the value of outstanding government debt gets amplified. The result of the second proposition arises also because the wealth effect gets magnified when taxes respond less to public debt outstanding.

Finally, we consider the case where there is equilibrium indeterminacy as both monetary and fiscal policies are passive.

2.3.3 Passive Monetary and Passive Fiscal Policy

Under a passive monetary and passive fiscal policy regime (PMPF), multiple equilibria exist. We can express the solution for inflation as:

$$\hat{\pi}_t = \underbrace{\left[e_2\hat{\pi}_{t-1} + \phi\left(\Omega\bar{b} - \frac{\kappa\beta^{-1}}{e_1-\rho_\pi}\right)\hat{\pi}^*_{t-1} - \left(\frac{\kappa\beta^{-1}}{e_1-\rho_r}\right)\hat{r}^*_{t-1} + \Omega\left(\hat{b}_{t-1} - e_2\hat{b}_{t-2}\right)\right]}_{E_{t-1}\hat{\pi}_t}$$
$$+\begin{bmatrix} m_\pi & m_r \end{bmatrix}\begin{bmatrix} \varepsilon_{\pi,t} \\ \varepsilon_{r,t} \end{bmatrix} + \zeta_t,$$

where ζ_t is a sunspot shock.[21] By decomposing the expectational error $\hat{\pi}_t - E_{t-1}\hat{\pi}_t$ into the innovations to the fundamental shocks, $\varepsilon_{\pi,t}$ and $\varepsilon_{r,t}$, and the sunspot shock, ζ_t, we maintain that the sunspot shock is independent of the fundamental shocks to the economy. Note that parameters m_π and m_r are not uniquely determined in this solution.

It is easy to see that the relationship between $\hat{\pi}_t$ and $\hat{\pi}^*_t$ and the effect of $\hat{r}^*_t$ on $\hat{\pi}_t$ is ambiguous. For example, $\hat{\pi}_t$ and $\hat{\pi}^*_t$ may be positively or negatively related. If agents form self-fulfilling expectations that a shock to $\hat{\pi}^*_t$ decreases inflation significantly ($m_\pi << 0$), then the self-fulfilling expectations can dominate and inflation can respond negatively to a shock to $\hat{\pi}^*_t$. But, in general, the question of how $\hat{\pi}_t$ responds to $\hat{\pi}^*_t$ can be answered only empirically. The answer will depend the numerical values of non-structural as well as

[21] For the complete description of the notation, see the appendix.

structural parameters. In their estimated model, Bhattarai, Lee, and Park (2012) find that the inflation target is not a significant driving force of inflation dynamics under the PMPF regime, as depicted in Figure 1.

3 Quantitative Model

In this section, we assess whether the results found analytically with the simple model in the previous sections also hold in a quantitative model. We use a standard medium-scale DSGE model that features a rich set of frictions and shocks along the lines of Smets and Wouters (2007), Del Negro, Schorfheide, Smets, and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2010). We lay out the basic model features below while providing a complete description in the appendix. The main actors and their decision problems are as follows.

3.1 Description

3.1.1 Households

Households, a continuum in the unit interval, face an infinite horizon problem and maximize expected discounted utility over consumption and leisure. The utility function is additively separable over consumption and labor effort. There is time-varying external habit formation in consumption and a discount factor shock. Households own capital that they rent to firms. The model features variable capital utilization rate, which is chosen optimally by households. Households also make capital accumulation decision, and in doing so, take into account capital adjustment costs. There is a investment shock in the capital accumulation equation that leads to a variation in the efficiency with which the consumption good is converted into capital.

Each household is a monopolistic supplier of differentiated labor. The elasticity of substitution over the differentiated labor varieties is time-varying. A large number of competitive employment agencies combine the differentiated labor services into a homogeneous labor input that is sold to firms. Each household enjoys some monopoly power over setting wages, which are sticky in nominal terms. Wage stickiness is modelled following Calvo (1983). There is a constant probability of not adjusting wages every period, with wages that do not adjust partially indexed to past inflation.

3.1.2 Firms

Firms, a continuum in the unit interval, produce differentiated goods using the homogenous labor input and capital. The elasticity of substitution over the differentiated goods varieties is time-varying. There is a fixed cost in production, which ensures zero profits in steady-state. The production function, which takes a Cobb-Douglas form, is subject to an aggregate neutral technology shock. Each firms enjoys some monopoly power over setting prices, which are sticky in nominal terms. Price stickiness is modelled following Calvo (1983). There is a constant probability of not adjusting prices every period, with prices that do not adjust partially indexed to past inflation.

3.1.3 Government

The government is subject to a flow budget constraint and conducts monetary and fiscal policies using endogenous feedback rules. For simplicity, we assume that the government issues only one-period nominal debt and levies lump-sum taxes. The government controls the one-period nominal interest rate. Monetary policy is modeled using an interest rate rule that features interest rate smoothing and a systematic response of the nominal interest rate to the deviation of inflation from a time-varying target and the deviation of output from the natural level of output.[22] Monetary policy shock is the non-systematic component of this policy rule. Fiscal policy is modelled using a tax rule that features a systematic response of the tax revenues to the level of outstanding government debt. Government spending-to-output ratio evolves exogenously as a time-varying fraction of output.

3.2 Approximate Model

We first solve the problem of households and firms given the monetary and fiscal policy rules and derive the equilibrium conditions. We then use approximation methods to solve the model. First, the model features a stochastic balanced growth path since the neutral technology shock contains a unit root. Therefore, we de-trend variables on the balanced growth path by the level of the technology shock and write down all the equilibrium conditions of the transformed model. Second, we compute the non-stochastic steady state of this transformed model. Third, we obtain a first-order approximation of the equilibrium conditions around this steady state. We then solve the approximated model using standard methods. The approximated equations are provided in the appendix.

As in the simple model, the existence and uniqueness of equilibrium depends crucially on the prevailing monetary and fiscal policy regime. The equilibrium of the economy will be determinate either if monetary policy is active while fiscal policy is passive (the AMPF regime) or if monetary policy is passive while fiscal policy is active (the PMAF regime).

[22] The natural level of output is the output that would prevail under flexible wages and prices and in the absence of time-variation in the elasticity of substitution over the different varieties of labor and goods.

An equilibrium is indeterminate and multiple equilibria exist if both monetary and fiscal policies are passive (the PMPF regime). In this richer model, we are unable to analytically characterize the exact parameter boundaries that lead to active and passive policies. We therefore determine the boundaries numerically.

3.3 Results

The results from this quantitative model are consistent with our analytical results, as we discuss below in detail.[23] We will present results with respect to the inflation target shock and six non-policy shocks: neutral technology shock, government spending, investment specific technology shock, price markup shock, wage markup shock, and a preference shock.

3.3.1 Parameter Values

Our model, other than a slightly different specification of monetary policy rule and an inclusion of a fiscal block, is the same as in Del Negro, Schorfheide, Smets, and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2010).[24] For our numerical exercises, we use the posterior median estimates of Justiniano, Primiceri, and Tambalotti (2010) for all the parameters related to preferences and technology. For all three policy regimes, we also use the same value as their posterior median estimates for the monetary policy feedback parameter on output gap. For the tax smoothing parameter in the fiscal policy rule we use the posterior estimate of Bhattarai, Lee, and Park (2012) while for the steady state level of the maturity value of debt-to-output, we use the sample average from U.S. data. We then conduct several comparative static exercises with respect to the policy feedback parameters on inflation and debt outstanding to show that the numerical results from this model are consistent with our analytical results from the simple model. All the parameter values that we use are provided in the appendix.

3.3.2 Inflation Target Shock

Panels (a)-(d) of Figure 9 show for the three policy regimes the impulse response of inflation to an exogenous change in the inflation target, varying the degree of monetary policy stance. They clearly illustrate one of the main results of our paper: under AMPF, actual inflation moves in the same direction as the inflation target and that higher the systematic response of monetary policy to inflation, lower is the gap between inflation and the inflation target,

[23] There are a two cases of differences with respect to the intial impact on inflation, for which we provide detailed explanations.

[24] The main differences in the monetary policy rule specification is that we include a time-varying inflation target while excluding the growth rate of the output gap.

while under PMAF, in sharp contrast, actual inflation moves in an opposite direction from the inflation target and higher is the systematic response of monetary policy to inflation, higher is the gap between inflation and the inflation target. Moreover, under PMPF, depending on the value of the parameter M_{π}^{*}, which governs how self-fulfilling beliefs are formed under indeterminacy, inflation could either move in the same direction as the inflation target or in an opposite direction. Finally, under PMAF, panel (e) of Figure 9 shows that lower is the response of taxes to debt, greater is the gap between inflation and the inflation target.

3.3.3 Non-policy Shocks

Figure 10 shows under AMPF the impulse response of inflation to six non-policy shocks, varying the degree of monetary policy stance. It is clear that inflation responds less on impact to a non-policy shock when the systematic response of monetary policy to inflation is higher for all cases, except for the investment specific technology shock. Even for this shock however, after 5 periods or so, the response is lower for a greater ϕ_{π}. In our simple model, where we abstract from investment, this shock is not present. In this quantitative model, the initial response of inflation is higher for a greater ϕ_{π} because this shock directly and significantly affects the capital rental cost for firms. Thus, when ϕ_{π} is greater, it can be the case that the rise in marginal cost due to a positive investment specific shock outweighs the usual inflation stabilization effect, thereby leading to a greater response of inflation. This result however, depends on all the other parameters of the model, in particular, the extent of wage stickiness in the economy. This is because wage stickiness determines the dynamics of wages, an important component of marginal cost. In fact in the appendix, in an alternate parameterization, we show a case where inflation responds less on impact to this shock with a greater ϕ_{π}, which makes it completely consistent with our analytical results. In this alternate parameterization, we decrease the extent of wage stickiness compared to the baseline case presented here, which magnifies the inflation stabilization effect of monetary policy on wage costs and, thereby, damps down the increase in inflation following an investment specific shock.

Figure 11 shows under PMAF the impulse response of inflation to six non-policy shocks, varying the degree of monetary policy stance. It is clear that for all cases and in sharp contrast to AMPF, inflation responds more on impact to a non-policy shock when the systematic response of monetary policy to inflation is higher.

Figure 12 shows under PMAF the impulse response of inflation to six non-policy shocks, varying the degree of fiscal policy stance It is clear that inflation responds less on impact to a non-policy shock when the systematic response of fiscal policy to debt is lower for all cases, except for the neutral technology shock. This result is different from our analytical results in

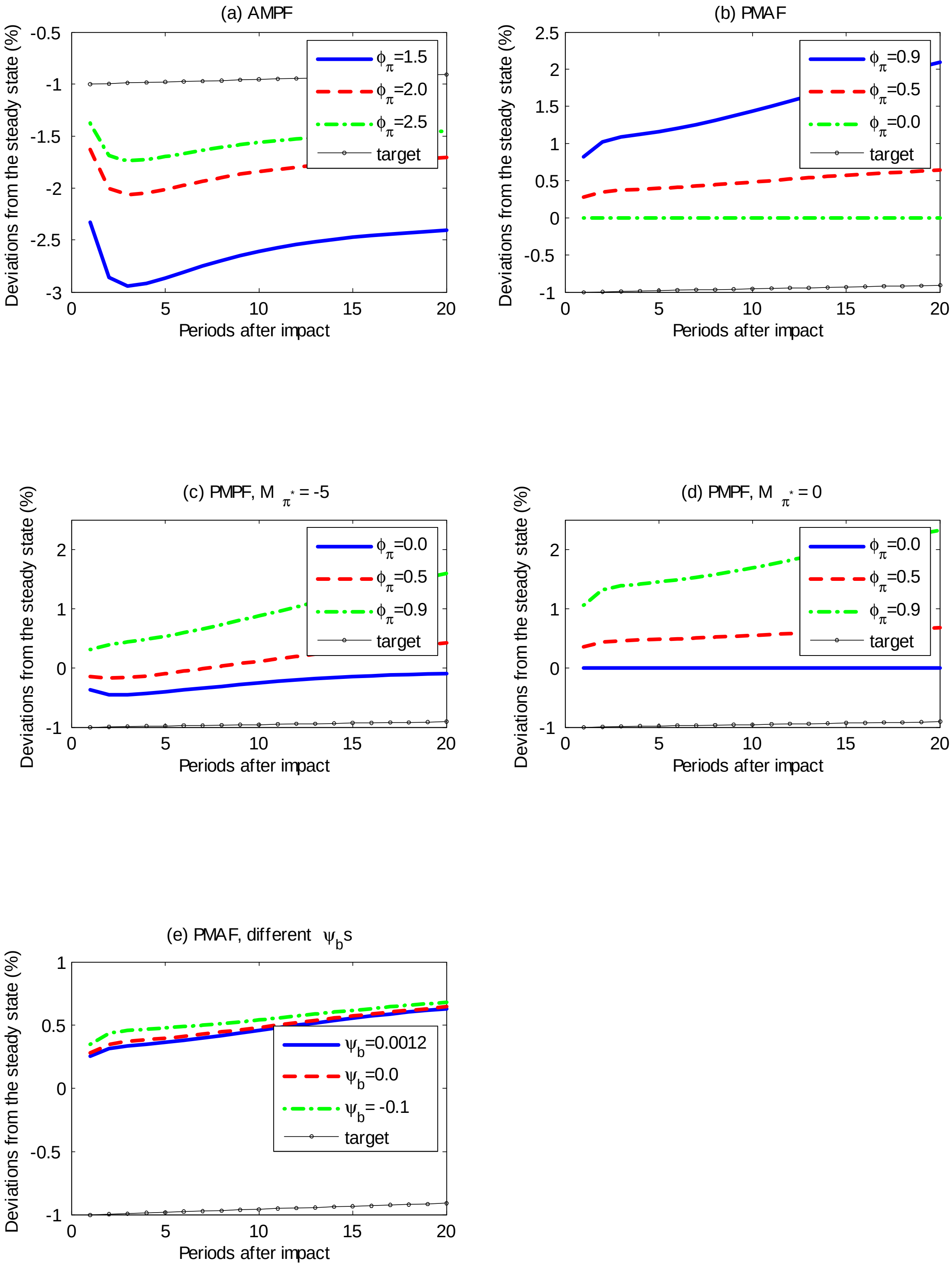

Figure 9: The response of inflation to a one percentage decrease in the inflation target.

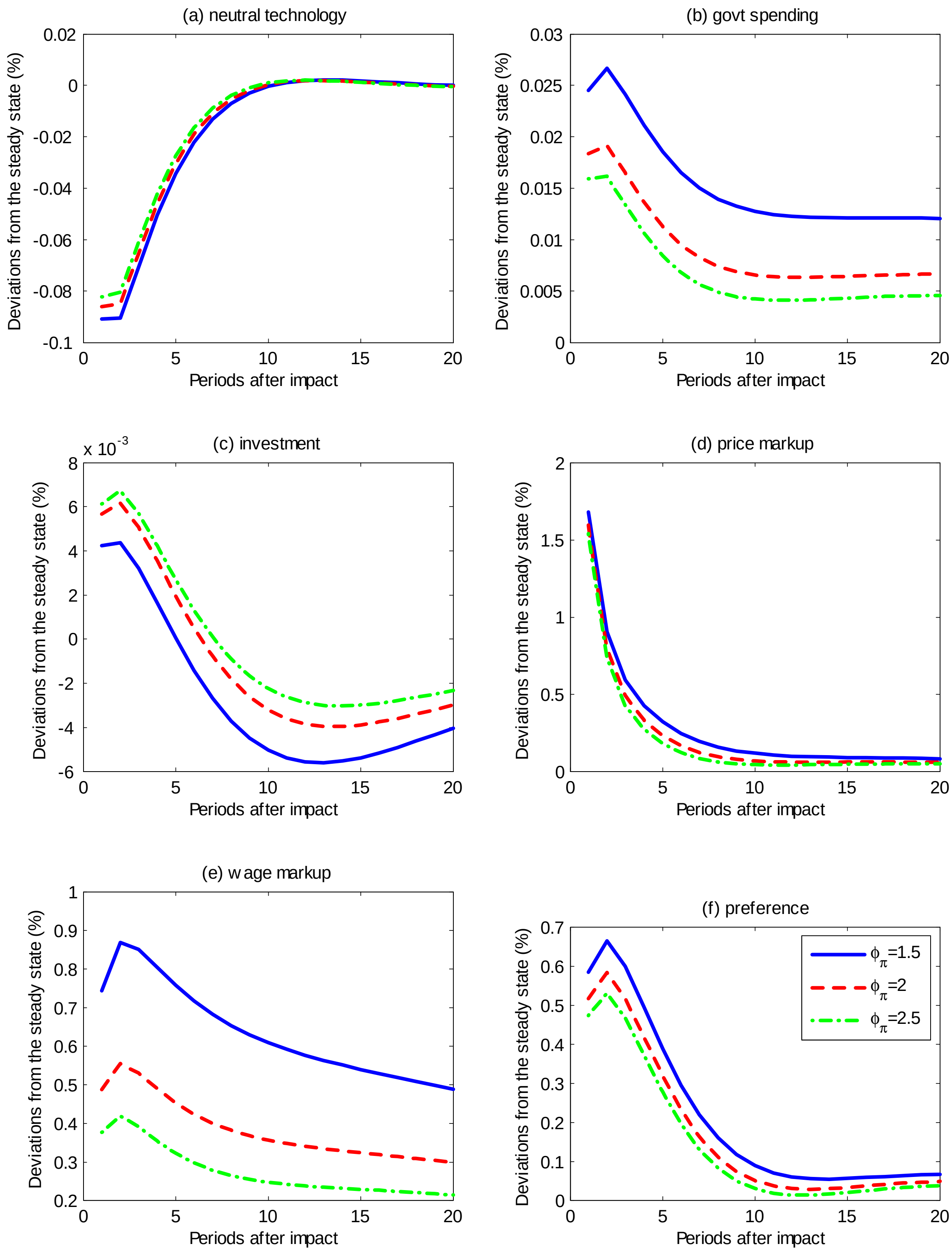

Figure 10: The response of inflation to a one standard deviation increase in the non-policy shock under AMPF.

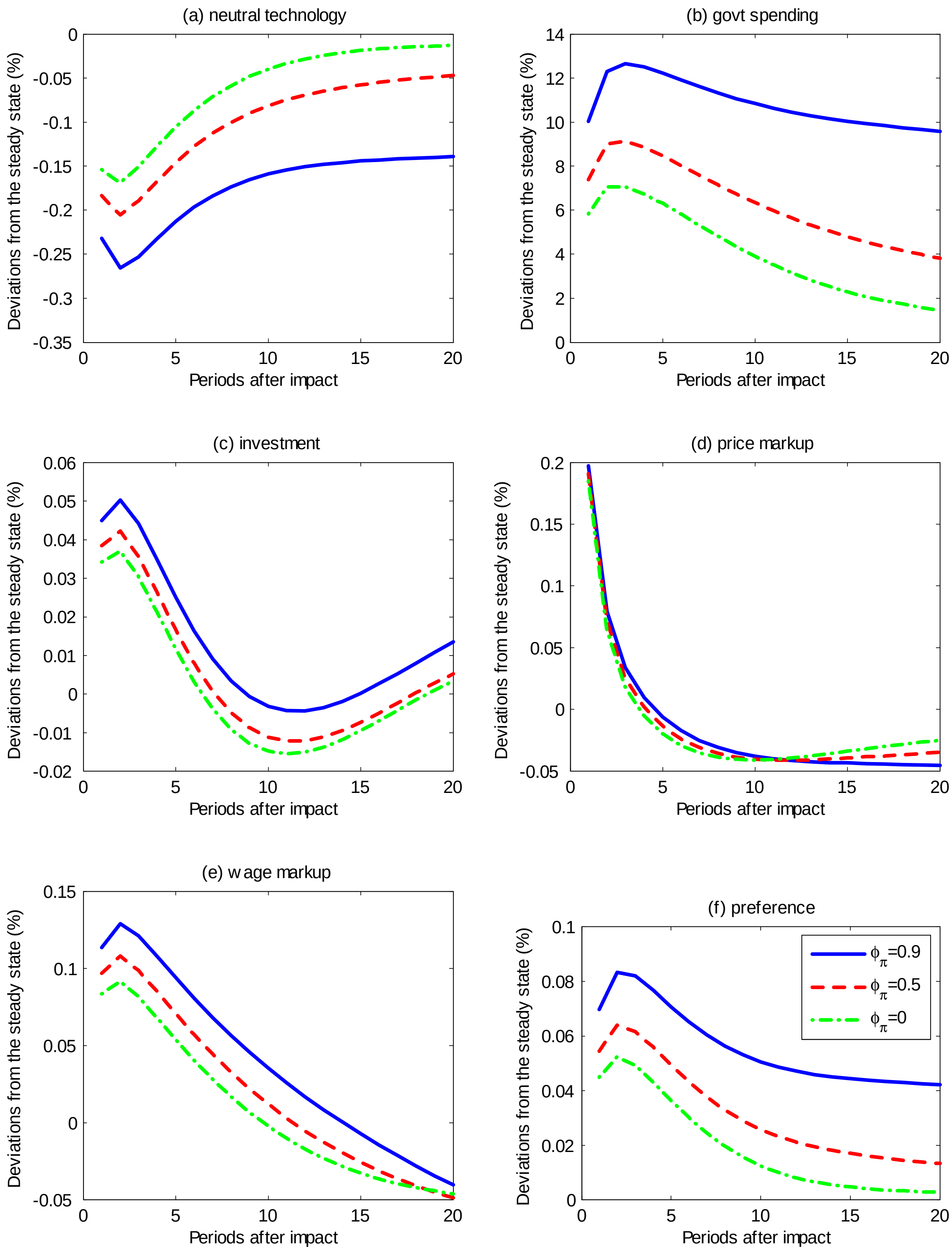

Figure 11: The response of inflation to a one standard deviation increase in the non-policy shock under PMAF.

the simple model. The reason is that this quantitative model features stochastic growth and the technology shock is therefore a shock to the growth rate as opposed to a shock to the level of technology, which was the case in the simple model. Thus, due to this, the shock can significantly affect the dynamics of inflation as it plays a prominent role in the government budget constraint.

To preserve space, we do not present impulses responses under PMPF as the results clearly depend on the calibration of M_π^*. We now move on to presenting results on the volatility of inflation. This is especially pertinent because arguably, focusing on the volatility of inflation is a more sensible metric for inflation dynamics in this quantitative model which features various adjustment costs and internal propagation mechanisms. Figure 13 shows under the three policy regimes the standard deviation of inflation, varying the degree of monetary policy stance. Here we present the standard deviation of inflation when all six non-policy shocks hit the economy. Panels (a) and (b) clearly depict one of the main results of our paper: in response to non-policy shocks under AMPF, inflation volatility decreases as monetary policy responds strongly to inflation, while in sharp contrast, under PMAF, inflation volatility increases.[25] In this particular parameterization, panel (c) shows that under PMPF, inflation volatility increases when monetary policy responds strongly to inflation.

4 Conclusion

In this paper we characterize the dynamics of inflation under different monetary and fiscal regime combinations in a standard DSGE model. First, using a simple set-up that allows for closed-form solutions, we show that answers to some classic questions on inflation dynamics depend crucially on the prevailing policy regime. Second, we show that our insights continue to hold in a richer quantitative model.

Our results show that under an active monetary and passive fiscal policy regime, inflation closely follows the path of the inflation target and a stronger reaction of monetary policy to inflation decreases the response of inflation to shocks. This is the usual case studied in the literature and the results are standard since in this regime, monetary policy has control over inflation. In sharp contrast, under an active fiscal and passive monetary policy regime, inflation moves in an opposite direction from the inflation target and a stronger reaction of

[25] Note that inflation is extremely volatile under PMAF. This is mostly because of the effect of government spending shocks in this regime. Government spending shocks have a direct impact on the government budget constraint and thereby, require extremely volatile movements in inflation for debt stabilization. Moreover, note that since we use parameter values from a model estimated under AMPF, inflation volatility under PMAF is perhaps unreasonably high. Therefore, the main point of this exercise is simply to show how the volatility of inflation depends on the monetary policy reaction function parameter within a policy regime, rather than a direct comparison of the volatility of inflation across policy regimes.

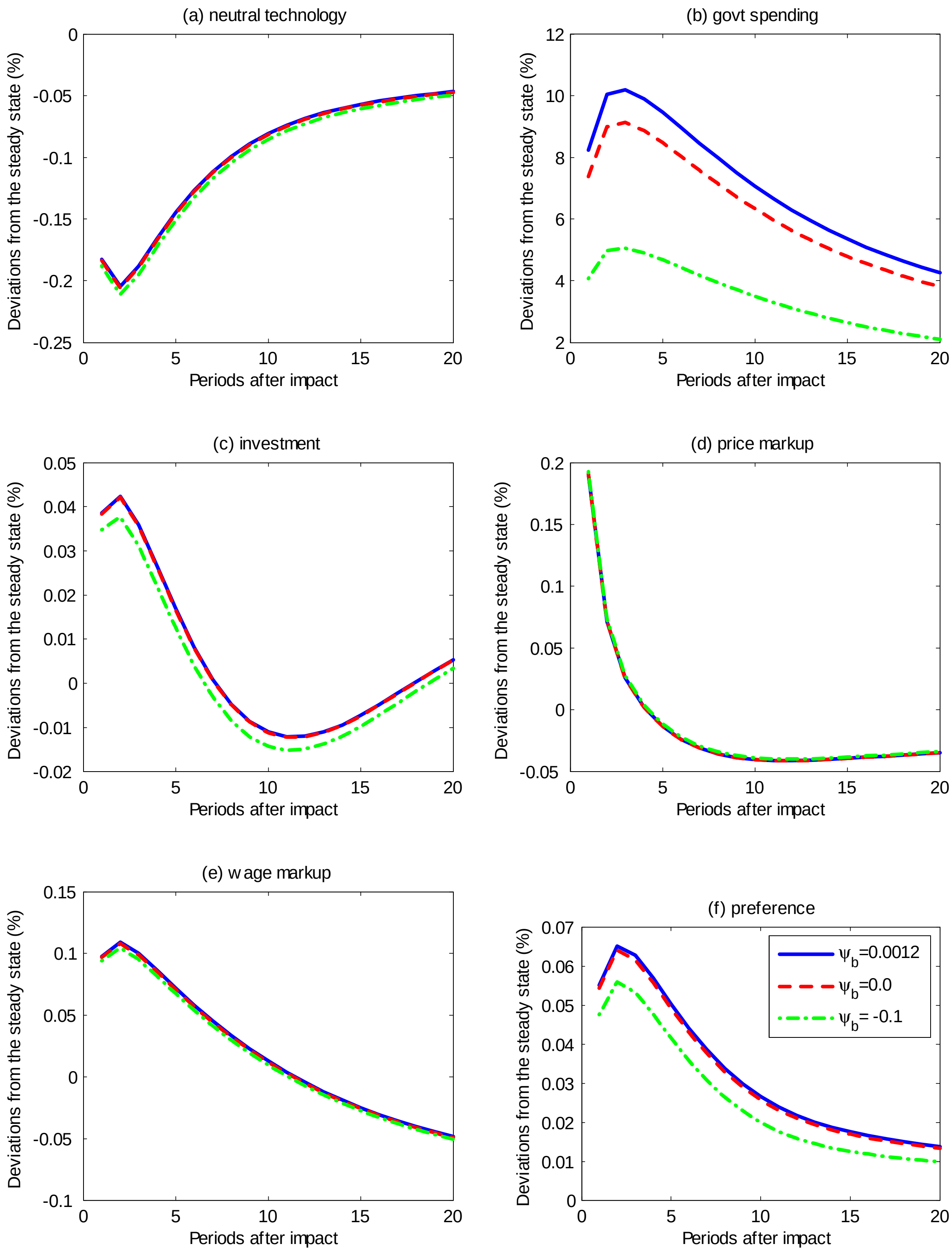

Figure 12: The response of inflation to a one standard deviation increase in the non-policy shock under PMAF.

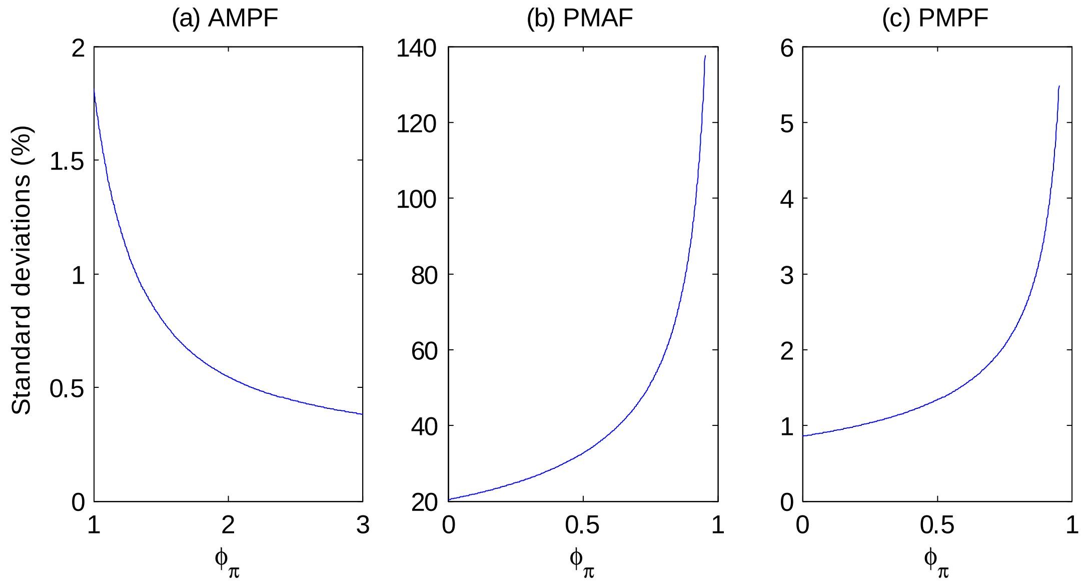

Figure 13: Standard deviation of inflation across different values of ϕ_π conditional on the non-policy shocks.

monetary policy to inflation increases the response of inflation to shocks. These effects arise crucially because of the prevalence of a wealth effect in response to interest rate movements that change the value of government debt. In particular, an increase in interest rate, because of a positive wealth effect, increases spending, and thereby inflation. Moreover, in this case, a weaker response of fiscal policy to debt decreases the response of inflation to shocks. Finally, under a passive monetary and passive fiscal policy regime, because of equilibrium indeterminacy, theory provides no clear answer on the behavior of inflation, which can only be ascertained by estimating the model.

References

[1] Bhattarai, Saroj, Jae W. Lee, and Woong Y. Park. 2012. "Policy Regimes, Policy Shifts, and U.S. Business Cycles." Unpublished.

[2] Bianchi, Francesco and Cosmin Ilut. 2012. "Monetary/Fiscal Policy Mix and Agents' Beliefs." Unpublished.

[3] Canzoneri, Matthew, Robert Cumby, and Behzad Diba. 2011. "The Interaction Between Monetary and Fiscal Policy." In *Handbook of Monetary Economics*, Vol. 3B, ed. Benjamin M. Friedman and Michael Woodford, 935-99. Amsterdam: Elsevier Science, North-Holland.

[4] Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules And Macroeconomic Stability: Evidence And Some Theory." *Quarterly Journal of Economics,* 115(1): 147-180.

[5] Cogley, Timothy, Giorgio E. Primiceri, and Thomas J. Sargent. 2010. "Inflation-Gap Persistence in the U.S." *American Economic Journal: Macroeconomics*, 2(1): 43–69.

[6] Cogley, Timothy and Argia M. Sbordone. 2008. "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve." *American Economic Review*, 98(5): 2101-26.

[7] Curdia, Vasco, Andrea Ferrero, Ging Cee Ng, and Andrea Tambalotti. 2011. "Evaluating Interest Rate Rules in an Estimated DSGE Model." Unpublished.

[8] Del Negro, Marco, Frank Schorfheide, Frank Smets, and Raf Wouters. 2007. "On the Fit and Forecasting Performance of New Keynesian Models." *Journal of Business and Economic Statistics, 25(2): 123-162.*

[9] Dixit, Avinash K. and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review,* 67: 297-308.

[10] Ireland, Peter N. 2007. "Changes in the Federal Reserve's Inflation Target: Causes and Consequences." *Journal of Money, Credit and Banking,* 39(8): 1851-1882.

[11] Justiniano, Alejandro, Giorgio E. Primiceri, and AndreaTambalotti. 2010. "Investment Shocks and Business Cycles." *Journal of Monetary Economics, 57(2): 132-145.*

[12] Kim, Soyoung. 2003. "Structural Shocks and the Fiscal Theory of the Price Level in the Sticky Price Model." *Macroeconomic Dynamics, 7: 759-782.*

[13] Leeper, Eric M. 1991. "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics*, 27(1): 129-47.

[14] Loyo, Eduardo. 1999. "Tight Money Paradox on the Loose: A Fiscalist Hyperinflation." Unpublished.

[15] Lubik, Thomas A. and Frank Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94(1): 190-217.

[16] Primiceri, Giorgio E. 2006. "Why Inflation Rose and Fell: Policy-makers Beliefs and U.S. Postwar Stabilization Policy." *Quarterly Journal of Economics,* 121(3): 867-901.

[17] Sargent, Thomas J., Noah Williams, and Tao Zha. 2006. "Shocks and Government Beliefs: The Rise and Fall of American Inflation." *American Economic Review,* 96(4), 1193-1224.

[18] Sims, Christopher A. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory*, 4(3): 381-99.

[19] Sims, Christopher A. 2004. "Limits to Inflation Targeting." In *The Inflation Targeting Debate*, ed. Ben S. Bernanke and Michael Woodford, 283-309. Chicago: University of Chicago Press.

[20] Sims, Christopher A. and Tao Zha**.** 2006. "Were There Regime Switches in U.S. Monetary Policy?." *American Economic Review*, 96(1): 54-81.

[21] Sims, Christopher A. 2011. "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the1970s." *European Economic Review*, 55: 48-56.

[22] Smets, Frank and Raf Wouters. 2007. "Shocks and Frictions in U. S. Business Cycles: A Bayesian Approach." *American Economic Review,* 97(3): 586-606.

[23] Woodford, Michael. 1995. "Price-level Determinacy Without Control of a Monetary Aggregate." *Carnegie-Rochester Conference Series on Public Policy*, 43(1): 1-46.

[24] Woodford, Michael. 2003. *Interest and Prices.* Princeton, NJ: Princeton University Press.

Appendix

A Simple Model

A.1 Households

Identical households choose sequences of $\{C_t, B_t, N_t, D_{t+1}\}$ to solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t d_t \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$P_t C_t + B_t + E_t \left[Q_{t,t+1} D_{t+1} \right] = R_{t-1} B_{t-1} + D_t + W_t N_t + \Pi_t - P_t \tau_t,$$

where C_t is consumption, N_t is labor hours, P_t is the price level, B_t is the amount of one-period risk-less nominal government bond, R_t is the gross nominal interest rate, W_t is the nominal wage rate, Π_t is profits of intermediate firms, and τ_t is government taxes net of transfers. The parameter, $\varphi \geq 0$, denotes the inverse of the Frisch elasticity of labor supply, while d_t represents an intertemporal preference shock. In addition to the government bond, households trade at time t one-period state-contingent nominal securities D_{t+1}at price $Q_{t,t+1}$.

A.2 Firms

Perfectly competitive firms produce the final good, Y_t, by assembling intermediate goods, $Y_t(i)$, through a Dixit and Stiglitz (1977) technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ denotes the elasticity of substitution between intermediate goods. The corresponding price index for the final consumption good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of the intermediate good i. The optimal demand for $Y_t(i)$ is given by $Y_t(i) = \left(P_t(i)/P_t \right)^{-\theta} Y_t$.

Monopolistically competitive firms produce intermediate goods using the production function, $Y_t(i) = a_t N_t(i)$, where $N_t(i)$ denotes the labor hours employed by firm i and a_t represents exogenous economy-wide productivity. Prices are sticky as in Calvo. A firm adjusts its price, $P_t(i)$, with probability $1-\alpha$ each period, to maximize the present discounted value of future profits:

$$E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left[P_t(i) - \frac{W_{t+k}}{A_{t+k}} \right] Y_{t+k}(i).$$

A.3 Government

Each period, the government collects lump-sum tax revenues τ_t and issues one-period nominal bonds B_t to finance its consumption G_t, and interest payments. Accordingly, the flow budget constraint is given by:

$$\frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_t} + G_t - \tau_t.$$

For simplicity, we assume $G_t = 0$, which is inconsequential for our theoretical results. The flow budget constraint can be rewritten as:

$$R_t^{-1} b_t = b_{t-1}\frac{1}{\pi_t} - \tau_t,$$

where $b_t \equiv R_t \frac{B_t}{P_t}$ denotes the real maturity value of government debt.

The monetary and fiscal policies are described by simple rules. The monetary authority responds to deviations of the inflation rate from its time-varying target rate, π_t^*, by setting the nominal interest rate according to:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\pi_t^*}\right)^{\phi},$$

where $\bar{R}$ is the steady-state value of R_t. Similarly, the fiscal authority sets the tax revenues according to:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\psi},$$

where $\bar{\tau}$ and $\bar{b}$ are respectively the steady state value of τ_t and b_t.

A.4 Approximate Model

We log-linearize the equilibrium conditions around non-stochastic steady state values: $\{\bar{\pi},\ \bar{Y},\ \bar{R},\ \bar{b},\ \bar{\tau}\}$. Since the log-linearized model is completely standard, we omit a detailed derivation. The approximate model is characterized by the following equations:

$$\begin{aligned}
\hat{Y}_t &= E_t\hat{Y}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) - E_t[\Delta\hat{d}_{t+1}], \\
\hat{\pi}_t &= \kappa\left(\hat{Y}_t - \hat{Y}_t^n\right) + \beta E_t\hat{\pi}_{t+1}, \\
\hat{R}_t &= \phi\left(\hat{\pi}_t - \hat{\pi}_t^*\right), \\
\hat{\tau}_t &= \psi\hat{b}_{t-1}, \\
\hat{b}_t &= \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\hat{\pi}_t - \beta^{-1}\hat{\tau}_t + \bar{b}\hat{R}_t.
\end{aligned}$$

In the equations above, we use $\hat{X}_t$ to denote the log deviation of a variable X_t from its steady state $\bar{X}$ ($\hat{X}_t = \ln X_t - \ln \bar{X}$), except for two fiscal variables, $\hat{b}_t$ and $\hat{\tau}_t$. Following Woodford (2003), we let them represent respectively the deviation of the maturity value of government debt and of government tax revenues (net of transfers) from their steady-state levels, measured as a percentage of steady-state output: $\hat{b}_t = \frac{b_t - \bar{b}}{\bar{Y}}$ and $\hat{\tau}_t = \frac{\tau_t - \bar{\tau}}{\bar{Y}}$. In our simple model, (the log-deviation of) the natural level of output and the slope of the Phillips curve are respectively given as $\hat{Y}_t^n = \hat{a}_t$ and $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$.

The model can be reduced to a dynamic system of $\left\{\hat{\pi}_t, \hat{b}_t, \tilde{Y}_t\right\}$:

$$\begin{aligned} \tilde{Y}_t &= E_t \tilde{Y}_{t+1} - \phi\left(\hat{\pi}_t - \hat{\pi}_t^*\right) + E_t \hat{\pi}_{t+1} + \hat{r}_t^*, \\ \hat{\pi}_t &= \kappa \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1}, \\ \hat{b}_t &= \beta^{-1}(1-\psi)\hat{b}_{t-1} - \bar{b}\left(\beta^{-1} - \phi\right)\hat{\pi}_t - \bar{b}\phi\hat{\pi}_t^*, \end{aligned} \tag{12}$$

where $\tilde{Y}_t \equiv \hat{Y}_t - \hat{Y}_t^n$ represents the output gap and $\hat{r}_t^*$ is a linear combination of all non-policy shocks (of both supply and demand types). It is often referred to as the natural rate of interest because it is the real interest rate that would prevail under flexible prices. In our simple model, it is specifically given as:

$$\hat{r}_t^* = E_t\left[\Delta \hat{a}_{t+1}\right] - E_t[\Delta \hat{d}_{t+1}].$$

Note that demand-type shocks raise $\hat{r}_t^*$, while supply-type shocks lower $\hat{r}_t^*$.

B Solution of the Simple Model

In this section, we solve for the equilibrium time paths of $\left\{\hat{\pi}_t, \hat{b}_t, \tilde{Y}_t\right\}$ given exogenous variables summarized by the policy and non-policy shocks, $\{\hat{\pi}_t^*, \hat{r}_t^*\}$. To this end, we assume the exogenous random variables follow AR(1) processes:

$$\begin{aligned} \hat{\pi}_t^* &= \rho_\pi \hat{\pi}_{t-1}^* + \varepsilon_{\pi,t}, \\ \hat{r}_t^* &= \rho_r \hat{r}_{t-1}^* + \varepsilon_{r,t}. \end{aligned}$$

We first write (12) in state space form:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} E_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} 1 & \phi & 0 \\ -\kappa & 1 & 0 \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) & \beta^{-1}(1-\psi) \end{pmatrix} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}^*_t \\ r^*_t \end{pmatrix} \tag{13}$$

We then pre-multiply $\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$ to both sides of the equation (13):

$$E_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \underbrace{\begin{pmatrix} \kappa\beta^{-1}+1 & \phi-\beta^{-1} & 0 \\ -\kappa\beta^{-1} & \beta^{-1} & 0 \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) & \beta^{-1}(1-\psi) \end{pmatrix}}_{=M} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}^*_t \\ \hat{r}^*_t \end{pmatrix}$$

The coefficient matrix, M, can be decomposed as $M = VDV^{-1}$, where D is a diagonal matrix whose elements are the eigenvalues of M. The system then can be written as:

$$E_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = V \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} V^{-1} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}^*_t \\ \hat{r}^*_t \end{pmatrix},$$

where

$$\begin{aligned} e_1 &= \frac{1}{2\beta}\left(\beta+\kappa+1+\sqrt{(\beta+\kappa+1)^2-4\beta\left(1+\kappa\phi\right)}\right) \\ e_2 &= \beta^{-1}(1-\psi) \\ e_3 &= \frac{1}{2\beta}\left(\beta+\kappa+1-\sqrt{(\beta+\kappa+1)^2-4\beta\left(1+\kappa\phi\right)}\right) \end{aligned}$$

$$V = \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad V^{-1} = \begin{pmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix}.$$

The elements of V and V^{-1} are nonlinear functions of the model parameters. For later use, we note that:

$$v_{23} = \frac{2(1-\psi) - \left(\beta + \kappa + 1 - \sqrt{(\beta+\kappa+1)^2 - 4\beta(1+\kappa\phi_\pi)}\right)}{2\bar{b}(1-\beta\phi)}.$$

Finally, letting $X_t \equiv \begin{pmatrix} x_{1,t} & x_{2,t} & x_{3,t} \end{pmatrix}^T \equiv V^{-1} \begin{pmatrix} \tilde{Y}_t & \hat{\pi}_t & \hat{b}_{t-1} \end{pmatrix}^T$, we rewrite the system as:

$$E_t X_{t+1} = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} X_t + \begin{pmatrix} -\phi q_{11} & -q_{11} \\ -\phi\left(q_{21} + \bar{b}\right) & -q_{21} \\ -\phi q_{31} & -q_{31} \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix}. \tag{14}$$

Each element of X_t is given by:

$$\begin{aligned} x_{1,t} &= q_{11}\tilde{Y}_t + q_{12}\hat{\pi}_t, \\ x_{2,t} &= q_{21}\tilde{Y}_t + q_{22}\hat{\pi}_t + \hat{b}_{t-1}, \\ x_{3,t} &= q_{31}\tilde{Y}_t + q_{32}\hat{\pi}_t. \end{aligned}$$

B.1 Active Monetary and Passive Fiscal Policy

Under AMPF, e_1 and e_3 are outside the unit circle, while e_2 is inside the circle. We thus use the first and third rows of the system (14) to draw linear restrictions between model variables. Substituting out the future values of $x_{1,t}$ and $x_{3,t}$ recursively, we obtain:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1}\right)^k E_t z_{1,t+k}^*, \tag{15}$$

$$x_{3,t} = \frac{1}{e_3} \sum_{k=0}^{\infty} \left(\frac{1}{e_3}\right)^k E_t z_{3,t+k}^*, \tag{16}$$

where

$$\begin{aligned} z_{1,t}^* &= \phi q_{11}\hat{\pi}_t^* + q_{11}\hat{r}_t^*, \\ z_{3,t}^* &= \phi q_{31}\hat{\pi}_t^* + q_{31}\hat{r}_t^*. \end{aligned}$$

These equations imply:

$$\begin{aligned} E_t z_{1,t+k}^* &= \phi q_{11}\rho_\pi^k \hat{\pi}_t^* + q_{11}\rho_r^k \hat{r}_t^* \\ E_t z_{3,t+k}^* &= \phi q_{31}\rho_\pi^k \hat{\pi}_t^* + q_{31}\rho_r^k \hat{r}_t^* \end{aligned}$$

Plugging these equations into (15) and (16), we obtain:

$$
\begin{aligned}
x_{1,t} &= \frac{1}{e_1}\sum_{k=0}^{\infty}\left(\frac{1}{e_1}\right)^k E_t z^*_{1,t+k} = \phi q_{11}\frac{1}{e_1-\rho_\pi}\hat{\pi}^*_t + q_{11}\frac{1}{e_1-\rho_r}\hat{r}^*_t, \\
x_{3,t} &= \frac{1}{e_3}\sum_{k=0}^{\infty}\left(\frac{1}{e_3}\right)^k E_t z^*_{3,t+k} = \phi q_{31}\frac{1}{e_3-\rho_\pi}\hat{\pi}^*_t + q_{31}\frac{1}{e_3-\rho_r}\hat{r}^*_t,
\end{aligned}
$$

which leads to:

$$
\begin{aligned}
\tilde{Y}_t &= -\frac{q_{12}}{q_{11}}\hat{\pi}_t + \phi\frac{1}{e_1-\rho_\pi}\hat{\pi}^*_t + \frac{1}{e_1-\rho_r}\hat{r}^*_t \\
\tilde{Y}_t &= -\frac{q_{32}}{q_{31}}\hat{\pi}_t + \phi\frac{1}{e_3-\rho_\pi}\hat{\pi}^*_t + \frac{1}{e_3-\rho_r}\hat{r}^*_t
\end{aligned}
$$

Since the above system has two equations and two endogenous variables, we can easily solve for $\hat{\pi}_t$ and $\tilde{Y}_t$. Moreover, $\hat{\pi}_t$ and $\tilde{Y}_t$ do not depend on $\hat{b}_{t-1}$. We use the method of undetermined coefficients and obtain:

$$
\begin{aligned}
\hat{\pi}_t &= \Phi\left(\phi\right)\hat{\pi}^*_t + \Gamma\left(\phi\right)\hat{r}^*_t \\
\tilde{Y}_t &= \Phi^Y\left(\phi\right)\hat{\pi}^*_t + \Gamma^Y\left(\phi\right)\hat{r}^*_t
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi\left(\phi\right) &\equiv \frac{\kappa\phi}{\kappa\left(\phi-\rho_\pi\right)+\left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)} \\
\Gamma\left(\phi\right) &\equiv \frac{\kappa}{\kappa\left(\phi-\rho_r\right)+\left(1-\rho_r\right)\left(1-\beta\rho_r\right)}
\end{aligned}
$$

$$
\begin{aligned}
\Phi^Y\left(\phi\right) &\equiv \frac{\phi\left(1-\beta\rho_\pi\right)}{\kappa\left(\phi-\rho_\pi\right)+\left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)} \\
\Gamma^Y\left(\phi\right) &\equiv \frac{\left(1-\beta\rho_r\right)}{\kappa\left(\phi-\rho_r\right)+\left(1-\rho_r\right)\left(1-\beta\rho_r\right)}.
\end{aligned}
$$

B.2 Passive Monetary and Active Fiscal Policy

We consider the case in which $\phi_\pi \in [0,1)$ and $\psi \in \left(-\infty,\bar{\psi}\right)$ where $\bar{\psi} \equiv 1-\beta$ is the upper bound for active fiscal policy. We then can show that $e_1 > 1$, $e_2 > 1$ and $e_3 \in (0,1)$ in that parameter space. Consequently the first two rows in (14) provide linear restrictions. From the rows, we obtain:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1} \right)^k E_t z^*_{1,t+k}, \tag{17}$$

$$x_{2,t} = \frac{1}{e_2} \sum_{k=0}^{\infty} \left(\frac{1}{e_2} \right)^k E_t z^*_{2,t+k}, \tag{18}$$

where

$$\begin{aligned} z^*_{1,t} &= \phi q_{11} \hat{\pi}^*_t + q_{11} \hat{r}^*_t, \\ z^*_{2,t} &= \phi \left(q_{21} + \bar{b} \right) \hat{\pi}^*_t + q_{21} \hat{r}^*_t. \end{aligned}$$

The equations above imply:

$$\begin{aligned} E_t z^*_{1,t+k} &= \phi q_{11} \rho^k_\pi \hat{\pi}^*_t + q_{11} \rho^k_r \hat{r}^*_t \\ E_t z^*_{2,t+k} &= \phi \left(q_{21} + \bar{b} \right) \rho^k_\pi \hat{\pi}^*_t + q_{21} \rho^k_r \hat{r}^*_t \end{aligned}$$

Plugging these equations into (15) and (16), we obtain:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1} \right)^k E_t z^*_{1,t+k} = \phi q_{11} \frac{1}{e_1 - \rho_\pi} \hat{\pi}^*_t + q_{11} \frac{1}{e_1 - \rho_r} \hat{r}^*_t \tag{19}$$

$$x_{2,t} = \frac{1}{e_2} \sum_{k=0}^{\infty} \left(\frac{1}{e_2} \right)^k E_t z^*_{2,t+k} = \phi \left(q_{21} + \bar{b} \right) \frac{1}{e_2 - \rho_\pi} \hat{\pi}^*_t + q_{21} \frac{1}{e_2 - \rho_r} \hat{r}^*_t \tag{20}$$

Equation (19) implies:

$$\tilde{Y}_t = -\frac{q_{12}}{q_{11}} \hat{\pi}_t + \phi \frac{1}{e_1 - \rho_\pi} \hat{\pi}^*_t + \frac{1}{e_1 - \rho_r} \hat{r}^*_t \tag{21}$$

We plug (21) into (20) to get:

$$q_{21} \left[-\frac{q_{12}}{q_{11}} \hat{\pi}_t + \phi \frac{1}{e_1 - \rho_\pi} \hat{\pi}^*_t + \frac{1}{e_1 - \rho_r} r^*_t \right] + q_{22} \hat{\pi}_t + \hat{b}_{t-1} = \phi \left(q_{21} + \bar{b} \right) \frac{1}{e_2 - \rho_\pi} \hat{\pi}^*_t + q_{21} \frac{1}{e_2 - \rho_r} \hat{r}^*_t$$

Solving for $\hat{\pi}_t$, we obtain $\hat{\pi}_t$ as a function of state variables, $\left\{ \hat{b}_{t-1}, \hat{\pi}^*_t, r^*_t \right\}$:

$$\hat{\pi}_t = \Omega \hat{b}_{t-1} - \Omega \phi \left[\left(q_{21} + \bar{b} \right) \frac{1}{e_2 - \rho_\pi} - q_{21} \frac{1}{e_1 - \rho_\pi} \right] \hat{\pi}^*_t + \Omega q_{21} \left[\frac{1}{e_1 - \rho_r} - \frac{1}{e_2 - \rho_r} \right] r^*_t \tag{22}$$

where

$$
\begin{aligned}
\Omega &= \frac{q_{11}}{q_{12}q_{21} - q_{11}q_{22}} \\
q_{21} &= \frac{\bar{b}\kappa\left(1 - \beta\phi\right)}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}.
\end{aligned}
$$

For further analysis, it is useful to express the coefficients on the state variables in terms of model parameters. To this end, we use the results in the following lemmas.

Lemma 1: $\Omega = \frac{\beta(e_2 - e_3)}{\bar{b}(1-\beta\phi)} > 0$.

Proof of Lemma 1: Note that

$$
\begin{aligned}
\frac{q_{13}q_{21} - q_{11}q_{23}}{\det\left(V^{-1}\right)} &= v_{23}, \\
\frac{q_{11}q_{22} - q_{12}q_{21}}{\det\left(V^{-1}\right)} &= v_{33}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
0 \times q_{21} - q_{11} = -q_{11} &= \det\left(V^{-1}\right) \times v_{23}, \\
q_{11}q_{22} - q_{12}q_{21} &= \det\left(V^{-1}\right) \times v_{33} = \det\left(V^{-1}\right) \times 1.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\Omega &= \frac{q_{11}}{q_{12}q_{21} - q_{11}q_{22}} = v_{23} \\
&= \frac{2\left(1 - \psi\right) - \left(\beta + \kappa + 1 - \sqrt{\left(\beta + \kappa + 1\right)^2 - 4\beta\left(1 + \kappa\phi\right)}\right)}{2\bar{b}\left(1 - \beta\phi\right)} \\
&= \frac{2\left(1 - \psi\right) - 2\beta e_3}{2\bar{b}\left(1 - \beta\phi\right)} = \frac{\beta\left(e_2 - e_3\right)}{\bar{b}\left(1 - \beta\phi\right)} > 0.
\end{aligned}
$$

Lemma 2: $\Omega q_{21}\left(e_1 - e_2\right) = -\kappa\beta^{-1}$.

Proof of Lemma 2: We have

$$
\begin{aligned}
\Omega q_{21}\left(e_1 - e_2\right) &= \frac{\beta\left(e_2 - e_3\right)}{\bar{b}\left(1 - \beta\phi\right)} \frac{\bar{b}\kappa\left(1 - \beta\phi\right)}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}\left(e_1 - e_2\right) \\
&= \kappa\beta \frac{\left(e_2 - e_3\right)\left(e_1 - e_2\right)}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} = \kappa\beta \frac{e_1e_2 - e_2^2 - e_1e_3 + e_2e_3}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} \\
&= \kappa\beta \frac{e_2\left(e_1 + e_3 - e_2\right) - e_1e_3}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} = \kappa\beta \frac{\frac{(1-\psi)(\beta+\kappa+\psi)}{\beta^2} - e_1e_3}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \kappa\beta \frac{\frac{(1-\psi)(\beta+\kappa+\psi)}{\beta^2} - \frac{1+\kappa\phi}{\beta}}{\psi^2 + (\beta+\kappa-1)\psi - \kappa(1-\beta\phi)} = \kappa\beta \frac{\frac{1}{\beta^2}\left[(1-\psi)(\beta+\kappa+\psi) - \beta - \kappa\beta\phi\right]}{\psi^2 + (\beta+\kappa-1)\psi - \kappa(1-\beta\phi)} \\
&= \kappa\beta^{-1} \frac{-\left[\psi^2 + (\beta+\kappa-1)\psi - \kappa(1-\beta\phi)\right]}{\psi^2 + (\beta+\kappa-1)\psi - \kappa(1-\beta\phi)} = -\kappa\beta^{-1}.
\end{aligned}
$$

Using the results from these two lemmas, we can simplify (22) as:

$$
\hat{\pi}_t = \Omega(\phi,\psi)\,\hat{b}_{t-1} - \Phi(\phi,\psi)\,\hat{\pi}_t^* + \Gamma(\phi,\psi)\,\hat{r}_t^* \tag{23}
$$

where

$$
\begin{aligned}
\Omega(\phi,\psi) &\equiv \frac{\beta(e_2 - e_3)}{\bar{b}(1-\beta\phi)}, \\
\Phi(\phi,\psi) &\equiv \phi \times \Theta(\phi,\psi), \qquad \text{where } \; \Theta(\phi,\psi) \equiv \frac{\Omega\bar{b}(e_1-\rho_\pi) - \kappa\beta^{-1}}{(e_1-\rho_\pi)(e_2-\rho_\pi)}, \\
\Gamma(\phi,\psi) &\equiv \frac{\kappa\beta^{-1}}{(e_1-\rho_r)(e_2-\rho_r)}.
\end{aligned}
$$

It then follows that the low of motion for $\hat{b}_t$ is given as:

$$
\hat{b}_t = e_3\hat{b}_{t-1} - \bar{b}\phi\left[1 - \left(\beta^{-1} - \phi\right)\Theta\right]\hat{\pi}_t^* - \bar{b}\left(\beta^{-1} - \phi\right)\Gamma\hat{r}_t^*.
$$

B.3 Passive Monetary and Passive Fiscal Policy

Finally, we consider the case in which $\phi_\pi \in [0,1)$ and $\psi \in \left(\bar{\psi}, \infty\right)$. Then, only one root ($e_1$) is explosive and there will exist multiple solutions to the model.

Solution for $x_{1,t}$ is the same as in AMPF or PMAF

$$
\tilde{Y}_t = -\frac{q_{12}}{q_{11}}\hat{\pi}_t + \phi\frac{1}{e_1-\rho_\pi}\hat{\pi}_t^* + \frac{1}{e_1-\rho_r}\hat{r}_t^*
$$

Plug the equation above into the equation for $x_{2,t}$:

$$
x_{2,t} = -\frac{1}{\Omega}\hat{\pi}_t + \phi\frac{q_{21}}{e_1-\rho_\pi}\hat{\pi}_t^* + \frac{q_{21}}{e_1-\rho_r}\hat{r}_t^* + \hat{b}_{t-1}
$$

Note that

$$
\begin{aligned}
E_t x_{2,t+1} &= -\frac{1}{\Omega}E_t\hat{\pi}_{t+1} + \phi\frac{q_{21}}{e_1-\rho_\pi}E_t\hat{\pi}_{t+1}^* + \frac{q_{21}}{e_1-\rho_r}E_t\hat{r}_{t+1}^* + \hat{b}_t \\
&= -\frac{1}{\Omega}E_t\hat{\pi}_{t+1} + \phi\frac{q_{21}}{e_1-\rho_\pi}\rho_\pi\hat{\pi}_t^* + \frac{q_{21}}{e_1-\rho_r}\rho_r\hat{r}_t^* + \hat{b}_t,
\end{aligned}
$$

and therefore we can plug this into the difference equation for $x_{2,t}$ as

$$E_t\hat{\pi}_{t+1} = e_2\hat{\pi}_t + \phi\left(\Omega\bar{b} - \frac{\kappa\beta^{-1}}{e_1-\rho_\pi}\right)\hat{\pi}_t^* - \left(\frac{\kappa\beta^{-1}}{e_1-\rho_r}\right)\hat{r}_t^* + \Omega\left(\hat{b}_t - e_2\hat{b}_{t-1}\right).$$

Now a solution to this equation can be characterized as

$$\begin{aligned}\hat{\pi}_{t+1} = & \underbrace{\left[e_2\hat{\pi}_t + \phi\left(\Omega\bar{b} - \frac{\kappa\beta^{-1}}{e_1-\rho_\pi}\right)\hat{\pi}_t^* - \left(\frac{\kappa\beta^{-1}}{e_1-\rho_r}\right)\hat{r}_t^* + \Omega\left(\hat{b}_t - e_2\hat{b}_{t-1}\right)\right]}_{E_t\hat{\pi}_{t+1}} \\ & + \begin{bmatrix} m_\pi & m_r \end{bmatrix}\begin{bmatrix} \varepsilon_{\pi,t+1} \\ \varepsilon_{r,t+1} \end{bmatrix} + \zeta_{t+1},\end{aligned}$$

where ε_{t+1} is a vector of fundamental shocks and ζ_{t+1} is a sunspot shock.

C Proofs

Proof of Proposition 1 Showing $\Phi(\phi) \geq 0$ is straight forward since $\kappa \geq 0$; $\phi > \rho_\pi$; $(1-\rho_\pi) > 0$; and $(1-\beta\rho_\pi) > 0$. In addition, it is straight forward to show:

$$\Phi(\phi) > 1, \qquad \text{for } \kappa > \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}.$$

Proof of Proposition 2 Take the partial derivative:

$$\frac{\partial\Phi(\phi)}{\partial\phi} = \frac{-\kappa\left[\kappa\rho_\pi - (1-\rho_\pi)(1-\beta\rho_\pi)\right]}{\left[\kappa(\phi-\rho_\pi) + (1-\rho_\pi)(1-\beta\rho_\pi)\right]^2}.$$

The denominator is always positive. Therefore, the sign of the numerator will determine the sign of the derivative. We thus have:

$$\begin{aligned}\frac{\partial\Phi(\phi)}{\partial\phi} &> 0, \qquad \text{for } 0 \leq \kappa < \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi} \\ \frac{\partial\Phi(\phi)}{\partial\phi} &< 0, \qquad \text{for } \kappa > \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}.\end{aligned}$$

Proof of Proposition 3 Showing $\Gamma(\phi) \geq 0$ is straight forward since $\kappa \geq 0$; $\phi > \rho_r$; $(1-\rho_r) > 0$; and $(1-\beta\rho_r) > 0$.

Proof of Proposition 4 It is trivial.

Proof of Proposition 5 This result follows directly from Propositions 3 and 4.

Proof of Proposition 6 Since $\phi \geq 0$, it suffices to show $\Theta(\phi, \psi)$ is positive. Let us rewrite $\Theta(\phi, \psi)$ by substituting out Ω:

$$\Theta = \frac{\beta(e_2 - e_3)(e_1 - \rho_\pi) - \kappa\beta^{-1}(1 - \beta\phi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{\beta(e_2 - e_3)(e_1 - e_2 + e_2 - \rho_\pi) - \kappa\beta^{-1}(1 - \beta\phi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{\beta(e_2 - e_3)(e_1 - e_2) - \kappa\beta^{-1}(1 - \beta\phi) + \beta(e_2 - e_3)(e_2 - \rho_\pi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{-\frac{1}{\beta}\left[\psi^2 + (\beta + \kappa - 1)\psi - \kappa(1 - \beta\phi)\right] - \frac{\kappa}{\beta}(1 - \beta\phi) + \beta(e_2 - e_3)(e_2 - \rho_\pi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{-\frac{1}{\beta}\left[\psi^2 + (\beta + \kappa - 1)\psi\right] + \beta(e_2 - e_3)(e_2 - \rho_\pi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

Use $\psi = 1 - \beta e_2$:

$$\Theta = \frac{-\frac{1}{\beta}\left[\psi^2 + (\beta + \kappa - 1)\psi\right] + \beta(e_2 - e_3)(e_2 - \rho_\pi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)} \tag{24}$$

$$= \frac{-\frac{1}{\beta}\left[(1 - \beta e_2)^2 + (\beta + \kappa - 1)(1 - \beta e_2)\right] + \beta(e_2 - e_3)(e_2 - \rho_\pi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{\beta^2(e_2 - e_3)(e_2 - \rho_\pi) - (1 - \beta e_2)^2 - (\beta + \kappa - 1)(1 - \beta e_2)}{\beta(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{\beta^2(e_2^2 - (e_3 + \rho_\pi)e_2 + \rho_\pi e_3) - \left(\beta^2 e_2^2 - 2\beta e_2 + 1\right) - (\beta + \kappa - 1) + (\beta + \kappa - 1)\beta e_2}{\beta(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{\beta^2(\rho_\pi e_3 - (e_3 + \rho_\pi)e_2) + 2\beta e_2 - (\beta + \kappa) + (\beta + \kappa - 1)\beta e_2}{\beta(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$$

$$= \frac{\left[\beta^2(1 - e_3) + \beta(1 - \beta\rho_\pi) + \beta\kappa\right] e_2 + \beta^2\rho_\pi e_3 - (\beta + \kappa)}{\beta(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}.$$

The denominator is unambiguously positive for all parameter values under PMAF. Thus Θ will be positive if and only if the numerator is also positive. Note that the numerator is a linear and increasing function of e_2 because the slope is positive. This implies that $\Theta > 0$ for sufficiently large e_2 – or sufficiently small ψ. It is straightforward to show that $\Theta > 0$ if and only if $-\infty < \psi < \bar{\psi}^{**}$. $\bar{\psi}^{**}$ is a reduced-form parameter that crucially depends on the slope of the Phillips curve and satisfies

$$\begin{aligned}
\bar{\psi}^{**} &\equiv 1-\frac{(\beta+\kappa)-\beta^2\rho_\pi e_3}{\beta(1-e_3)+(1-\beta\rho_\pi)+\kappa}=\frac{\beta+\kappa-\beta^2\rho_\pi e_3}{\beta(e_1-\rho_\pi)} \\
0 &< \bar{\psi}^{**} \leq 1-\beta \\
\lim_{\kappa\to\infty}\bar{\psi}^{**} &= 0 \\
\lim_{\kappa\to 0}\bar{\psi}^{**} &= 1-\beta.
\end{aligned}$$

Therefore, the interval of ψ that makes Φ positive – i.e. that makes the coefficient on π_t^* negative – *always* contains zero (i.e. always includes the benchmark FTPL case). In addition, the interval covers the AF region ($-\infty<\psi<\bar{\psi}$) *almost* entirely. The interval coincides exactly with the entire AF region only when prices are fully sticky (i.e. when $\kappa=0$). In intermediate cases, the upper bound for positive Φ, $\bar{\psi}^{**}$, is positive, but is slightly below the upper bound for AF, $\bar{\psi}$.

Proof of Proposition 7 Let us first consider $\frac{\partial\Theta(\phi,\psi)}{\partial\phi}$:

$$\begin{aligned}
\frac{\partial\Theta(\phi,\psi)}{\partial\phi} &= \frac{\left\{\begin{array}{c}\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1-\rho_\pi)^2(e_2-\rho_\pi)+\frac{\partial e_1}{\partial\phi}\Omega\bar{b}(e_1-\rho_\pi)(e_2-\rho_\pi)\\ -\frac{\partial e_1}{\partial\phi}(e_2-\rho_\pi)\left[\Omega\bar{b}(e_1-\rho_\pi)-\kappa\beta^{-1}\right]\end{array}\right\}}{(e_1-\rho_\pi)^2(e_2-\rho_\pi)^2} \\
&= \frac{\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1-\rho_\pi)^2(e_2-\rho_\pi)-\frac{\partial e_3}{\partial\phi}(e_2-\rho_\pi)\kappa\beta^{-1}}{(e_1-\rho_\pi)^2(e_2-\rho_\pi)^2}=\frac{\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1-\rho_\pi)^2-\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1}}{(e_1-\rho_\pi)^2(e_2-\rho_\pi)}
\end{aligned}$$

Now let us take the partial derivative of $\Phi(\phi,\psi)$ with respect to ϕ:

$$\begin{aligned}
\frac{\partial\Phi(\phi,\psi)}{\partial\phi} &\equiv \Theta(\phi,\psi)+\phi\frac{\partial\Theta(\phi,\psi)}{\partial\phi} \\
&= \frac{\Omega\bar{b}(e_1-\rho_\pi)-\kappa\beta^{-1}}{(e_1-\rho_\pi)(e_2-\rho_\pi)}+\phi\frac{\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1-\rho_\pi)^2-\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1}}{(e_1-\rho_\pi)^2(e_2-\rho_\pi)} \\
&= \frac{\frac{\beta(e_2-e_3)}{(1-\beta\phi)}(e_1-\rho_\pi)^2-(e_1-\rho_\pi)\kappa\beta^{-1}+\phi\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1-\rho_\pi)^2-\phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1}}{(e_1-\rho_\pi)^2(e_2-\rho_\pi)}
\end{aligned}$$

Since the denominator is positive, $\frac{\partial\Phi(\phi,\psi)}{\partial\phi} > 0$ if and only if the numerator is positive – that is,

$$
\begin{aligned}
&0 < \frac{\partial\Phi(\phi,\psi)}{\partial\phi} \iff \\
&0 < \frac{\beta(e_2 - e_3)}{(1-\beta\phi)}(e_1-\rho_\pi)^2 - (e_1-\rho_\pi)\kappa\beta^{-1} + \phi\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1-\rho_\pi)^2 - \phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1} \\
&\iff \frac{\beta(e_2-e_3)}{(1-\beta\phi)} + \phi\frac{\partial\Omega}{\partial\phi}\bar{b} \geq \frac{\phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1} + (e_1-\rho_\pi)\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} \\
&\iff \frac{\beta(e_2-e_3)}{(1-\beta\phi)} + \phi\beta\frac{\beta e_2 - \beta e_3 - (1-\beta\phi)\frac{\partial e_3}{\partial\phi}}{(1-\beta\phi)^2} > \frac{\phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1} + (e_1-\rho_\pi)\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} \\
&\iff \beta(e_2-e_3)\frac{1}{(1-\beta\phi)^2} > \frac{\phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1} + (e_1-\rho_\pi)\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} + \frac{\beta\phi}{(1-\beta\phi)}\frac{\partial e_3}{\partial\phi} \\
&\iff \beta e_2 > \beta e_3 + (1-\beta\phi)^2\left\{\frac{\phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1} + (e_1-\rho_\pi)\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} + \frac{\beta\phi}{(1-\beta\phi)}\frac{\partial e_3}{\partial\phi}\right\} \\
&\iff \psi < 1 - \left[\beta e_3 + (1-\beta\phi)^2\left\{\frac{\phi\frac{\partial e_3}{\partial\phi}\kappa\beta^{-1} + (e_1-\rho_\pi)\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} + \frac{\beta\phi}{(1-\beta\phi)}\frac{\partial e_3}{\partial\phi}\right\}\right] \\
&\iff \psi < 1 - \left[\beta e_3 + (1-\beta\phi)^2\left\{\frac{\kappa\beta^{-1}}{(e_1-\rho_\pi)} + \frac{\phi\kappa\beta^{-1}}{(e_1-e_3)}\left[\frac{\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} + \frac{\beta}{(1-\beta\phi)}\right]\right\}\right]
\end{aligned}
$$

In sum,

$$
\frac{\partial\Phi(\phi,\psi)}{\partial\phi} > 0 \iff \psi < \bar{\psi}^{***}
$$

where

$$
\begin{aligned}
\bar{\psi}^{***} &\equiv 1 - \left[\beta e_3 + (1-\beta\phi)^2\left\{\frac{\kappa\beta^{-1}}{(e_1-\rho_\pi)} + \frac{\phi\kappa\beta^{-1}}{(e_1-e_3)}\left[\frac{\kappa\beta^{-1}}{(e_1-\rho_\pi)^2} + \frac{\beta}{(1-\beta\phi)}\right]\right\}\right] \\
\lim_{\kappa\to\infty}\bar{\psi}^{***} &= 0 \\
\lim_{\kappa\to 0}\bar{\psi}^{***} &= 1-\beta.
\end{aligned}
$$

It is straightforward to show that $\bar{\psi}^{***}$ is positive. First, suppose $\bar{\psi}^{***} \geq \bar{\psi}^{*}$ or $\bar{\psi}^{***} \geq \bar{\psi}^{**}$. We have already shown $\bar{\psi}^{*} > 0$ and $\bar{\psi}^{**} > 0$ above. Thus, it must be that $\bar{\psi}^{***} > 0$. Suppose instead $\bar{\psi}^{***} \leq \bar{\psi}^{*}$ and $\bar{\psi}^{***} \leq \bar{\psi}^{**}$. In this case, if $\psi < \bar{\psi}^{***}$, then $\Theta(\phi,\psi) > 0$ and $\frac{\partial\Omega(\phi,\psi)}{\partial\phi} > 0$. But, we can show from (24) that $\frac{\partial\Theta(\phi,\psi)}{\partial\phi} > 0$ at $\psi = 0$ if $\frac{\partial\Omega(\phi,\psi)}{\partial\phi} > 0$, which implies $\frac{\partial\Phi(\phi,\psi)}{\partial\phi} > 0$ at $\psi = 0$. Since $\psi < \bar{\psi}^{***}$ is the sufficient and necessary condition, it should always contain zero. Therefore, $\bar{\psi}^{***} > 0$.

Proof of Proposition 8 Take the partial derivative of $\Theta = \frac{\beta(e_2-e_3)(e_1-\rho_\pi)-\kappa\beta^{-1}(1-\beta\phi)}{(e_1-\rho_\pi)(e_2-\rho_\pi)(1-\beta\phi)}$ with respect to e_2:

$$\frac{\partial\Theta}{\partial e_2} = \frac{\beta\left(e_1-\rho_\pi\right)^2\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)-\left(e_1-\rho_\pi\right)\left(1-\beta\phi\right)\left[\beta\left(e_2-e_3\right)\left(e_1-\rho_\pi\right)-\kappa\beta^{-1}\left(1-\beta\phi\right)\right]}{\left[\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)\right]^2}$$

$$= \frac{\left(e_1-\rho_\pi\right)\left(1-\beta\phi\right)\left[\beta\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)-\beta\left(e_2-e_3\right)\left(e_1-\rho_\pi\right)+\kappa\beta^{-1}\left(1-\beta\phi\right)\right]}{\left[\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)\right]^2}$$

$$= \frac{\left(e_1-\rho_\pi\right)\left(1-\beta\phi\right)}{\left[\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)\right]^2}g$$

where

$$g = \beta\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)-\beta\left(e_2-e_3\right)\left(e_1-\rho_\pi\right)+\kappa\beta^{-1}\left(1-\beta\phi\right).$$

Since $\frac{(e_1-\rho_\pi)(1-\beta\phi)}{[(e_1-\rho_\pi)(e_2-\rho_\pi)(1-\beta\phi)]^2} > 0$, we focus on g, which can be written as:

$$g = \beta\left[\rho_\pi^2-\left(e_1+e_3\right)\rho_\pi+e_1e_3\right]+\kappa\beta^{-1}\left(1-\beta\phi\right).$$

Note that

$$e_1+e_3 = \frac{\beta+\kappa+1}{\beta}$$
$$e_1e_3 = \frac{\kappa\phi+1}{\beta}.$$

Using these, rewrite g and regard g as a function of $\rho_\pi \in (0,1)$ given other parameters:

$$g\left(\rho_\pi\right) = \beta\rho_\pi^2-\left(\beta+\kappa+1\right)\rho_\pi+1+\kappa\beta^{-1}.$$

Note that $g\left(\rho_\pi\right)$ is a convex and quadratic function of ρ_π, and

$$g\left(0\right) = 1+\kappa\beta^{-1} > g(1) = \kappa\left(\beta^{-1}-1\right) > 0.$$

Moreover,

$$g'\left(0\right) < 0 \text{ and } g'\left(1\right) < 0.$$

Therefore, it must be that $g > 0$ for $\rho_\pi \in (0,1)$, $\beta \in (0,1)$ and $\kappa \in [0,\infty)$. Hence,

$$\frac{\partial\Theta}{\partial e_2} = \frac{\left(e_1-\rho_\pi\right)\left(1-\beta\phi\right)}{\left[\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)\right]^2}g > 0.$$

This implies that

$$\frac{\partial \Theta}{\partial \psi} < 0 \text{ and } \frac{\partial \Phi}{\partial \psi} < 0,$$

because e_2 is decreasing in ψ.

Proof of Proposition 9 It is straight forward.

Proof of Proposition 10 $e_2 > 1$ and $\frac{\partial e_1}{\partial \phi} < 0$.

Proof of Proposition 11 $e_1 > 1$ and $\frac{\partial e_2}{\partial \psi} < 0$.

Proof of Proposition 12 $\Omega(\phi, \psi) \equiv \frac{\beta(e_2 - e_3)}{\bar{b}(1-\beta\phi)} > 0$ because $e_2 > e_3$ and $1 > \beta\phi$.

Proof of Proposition 13 Take the partial derivative of $\Omega(\phi, \psi)$ with respect to ϕ:

$$\frac{\partial \Omega}{\partial \phi} = \frac{\beta}{\bar{b}} \frac{\beta e_2 - \beta e_3 - (1-\beta\phi)\frac{\partial e_3}{\partial \phi}}{(1-\beta\phi)^2} = \frac{\beta}{\bar{b}} \frac{h(\phi)}{(1-\beta\phi)^2},$$

where

$$h(\phi) \equiv \beta e_2 - \beta e_3 - (1-\beta\phi)\frac{\partial e_3}{\partial \phi}.$$

Since $\frac{\beta}{\bar{b}}\frac{1}{(1-\beta\phi)^2}$ is clearly positive, $\frac{\partial \Omega}{\partial \phi}$ and of $h(\phi)$ must have the same sign. Note that

$$h(\phi) > 0 \iff \beta e_2 > \beta e_3 + (1-\beta\phi)\frac{\partial e_3}{\partial \phi}$$

$$\iff \beta e_2 > \beta e_3 + (1-\beta\phi)\frac{\partial e_3}{\partial \phi}$$

$$\iff \beta e_2 > \beta e_3 + (1-\beta\phi)\frac{\kappa}{\beta + \kappa + 1 - 2\beta e_3} \quad \left(\because \frac{\partial e_3}{\partial \phi} = \frac{\kappa}{\beta + \kappa + 1 - 2\beta e_3} > 0\right)$$

$$\iff \beta e_2 > \beta e_3 + (1-\beta\phi)\frac{\kappa}{\beta(e_1 - e_3)}$$

$$\iff \beta e_2 > \frac{\beta^2 e_1 e_3 - \beta^2 e_3^2 + (1-\beta\phi)\kappa}{\beta(e_1 - e_3)}$$

$$\iff \beta e_2 > \frac{\beta + \kappa - \beta^2 e_3^2}{\beta(e_1 - e_3)}$$

$$\iff \psi < \underbrace{1 - \frac{\beta + \kappa - \beta^2 e_3^2}{\beta(e_1 - e_3)}}_{\equiv \bar{\psi}^*} = \frac{\beta(e_1 - e_3) - \left(\beta + \kappa - \beta^2 e_3^2\right)}{\beta(e_1 - e_3)}$$

It remains to show that $\bar{\psi}^*$ is positive. Consider the numerator of $\bar{\psi}^*$, $g(\phi) \equiv \beta(e_1 - e_3) - \left(\beta + \kappa - \beta^2 e_3^2\right)$. Given other parameters, $g(\phi)$ has the smallest value at $\phi = 1$ because $g'(\phi) < 0$. Then

$$g(1) = \beta(e_1 - e_3) - \left(\beta + \kappa - \beta^2 e_3^2\right) = \beta\left(\frac{\kappa+1}{\beta} - 1\right) - \left(\beta + \kappa - \beta^2\right)$$
$$= (1-\beta)^2 > 0.$$

In addition, we can show that

$$\lim_{\kappa \to \infty} \bar{\psi}^* = 0$$
$$\lim_{\kappa \to 0} \bar{\psi}^* = 1 - \beta.$$

Proof of Proposition 14 It is straight forward as $\frac{\partial e_2}{\partial \psi} < 0$.

Discussion on $\bar{\psi}^*$, $\bar{\psi}^{**}$, $\bar{\psi}^{***}$ and $\bar{\psi} \equiv 1 - \beta$. Although the upper bounds, $\bar{\psi}^*$, $\bar{\psi}^{**}$ and $\bar{\psi}^{***}$, generally have different values, they converge to the same numbers as the slope of NKPC goes to infinity and zero. In particular, all of them equal $\bar{\psi} \equiv 1 - \beta$ – the "true" upper bound for AF – when $\kappa = 0$. However, $\bar{\psi}^*$, $\bar{\psi}^{**}$, $\bar{\psi}^{***}$ and $\bar{\psi}$ would be indistinguishable in practice because $\bar{\psi}^*, \bar{\psi}^{**}, \bar{\psi}^{***} \in (0, \bar{\psi}]$ and $\bar{\psi}$ has a tiny value.

D Quantitative Model

D.1 Households

There is a continuum of households in the unit interval. Each household specializes in the supply of a particular type of labor. A household that supplies labor of type-j maximizes the utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \delta_t \left[\log \left(C_t^j - \eta C_{t-1} \right) - \varpi \frac{\left(H_t^j\right)^{1+\varphi}}{1+\varphi} \right] \right\},$$

where H_t^j denotes the hours of type-j labor services, C_t is aggregate consumption, and C_t^j is consumption of household j. The parameters β, φ, and η are, respectively, the discount factor, the inverse of the (Frisch) elasticity of labor supply, and the degree of external habit formation, while δ_t represents an intertemporal preference shock that follows:

$$\delta_t = \delta_{t-1}^{\rho_\delta} \exp(\varepsilon_{\delta,t}),$$

where $\varepsilon_{\delta,t} \sim$ i.i.d. $N\left(0, \sigma_\delta^2\right)$.

Household j's flow budget constraint is:

$$P_t C_t^j + P_t I_t^j + B_t^j + E_t \left[Q_{t,t+1} V_{t+1}^j \right] = W_t(j) H_t^j + V_t^j + R_{t-1} B_{t-1}^j + R_t^k u_t \bar{K}_{t-1}^j - P_t a(u_t) \bar{K}_{t-1}^j + \Pi_t - T_t,$$

where P_t is the price level, B_t^j is the amount of one-period risk-less nominal government bond held by household j, R_t is the interest rate on the bond, $W_t(j)$ is the nominal wage rate for type-j labor, Π_t denotes profits of intermediate firms, and T_t denotes government taxes.[26] In addition to the government bond, households trade at time t one-period state-contingent nominal securities V_{t+1}^jat price $Q_{t,t+1}$, and hence fully insure against idiosyncratic risk.

Moreover, I_t^j is investment, R_t^k is the rental rate of effective capital $u_t \bar{K}_{t-1}^j$ where u_t is the variable capacity utilization rate, and $a(u_t)$ is the cost of capital utilization. In steady-state, $u = 1$ and $a(1) = 0$. Moreover, in the first-order approximation of the model, the only parameter that matters for the dynamic solution of the model is the curvature $\chi \equiv \frac{a''(1)}{a'(1)}$. The capital accumulation equation is then given by:

$$\bar{K}_t^j = (1-d)\, \bar{K}_{t-1}^j + \mu_t \left(1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) \right) I_t^j,$$

[26] The budget constraint reflects our assumptions that each household owns an equal share of all intermediate firms and receives the same amount of net lump-sum transfers from the government.

where d is the depreciation rate and $S(.)$ is the adjustment cost function. In steady-state, $S = S' = 0$ and $S'' > 0$. μ_t represents an investment shock that follows:

$$\mu_t = \mu_{t-1}^{\rho_\mu} \exp(\varepsilon_{\mu,t}),$$

where $\varepsilon_{\mu,t} \sim$ i.i.d. $N\left(0, \sigma_\mu^2\right)$.

Each household monopolistically provides differentiated labor. There are competitive employment agencies that assemble these differentiated labor into a homogenous labor input that is sold to intermediate goods firms. The assembling technology is a Dixit and Stiglitz (1977) production technology $H_t = \left(\int_0^1 H_t^{j\frac{\theta_{l,t}-1}{\theta_{l,t}}} dj\right)^{\frac{\theta_{l,t}}{\theta_{l,t}-1}}$, where $\theta_{l,t}$ denotes the time-varying elasticity of substitution between differentiated labor. The corresponding wage index for the homogenous labor input is $W_t = \left(\int_0^1 W_t(j)^{1-\theta_{l,t}} dj\right)^{\frac{1}{1-\theta_{l,t}}}$ and the optimal demand for H_t^j is given by $H_t^j = \left(W_t(j)/W_t\right)^{-\theta_{l,t}} H_t$. The elasticity of substitution $\theta_{l,t}$ follows:

$$\left(\frac{\theta_{l,t}}{\theta_{l,t}-1}\right) = \left(\frac{\bar{\theta}_l}{\bar{\theta}_l - 1}\right)^{1-\rho_l} \left(\frac{\theta_{l,t-1}}{\theta_{l,t}-1}\right)^{\rho_l} \exp(\varepsilon_{l,t} - \upsilon_l \varepsilon_{l,t-1})$$

where $\varepsilon_{l,t} \sim$ i.i.d. $N\left(0, \sigma_l^2\right)$.

As in Calvo (1983), each household resets its nominal wage optimally with probability $1 - \alpha_w$ every period. Households that do not optimize adjust their wages according to the simple partial dynamic indexation rule:

$$W_t(j) = W_{t-1}(j) \left[\pi_{t-1} a_{t-1}\right]^{\gamma_w} \left[\bar{\pi}\bar{a}\right]^{1-\gamma_w},$$

where γ_w measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing households choose a common wage W_t^* to maximize the present discounted value of future utility:

$$E_t \sum_{k=0}^{\infty} \alpha_w^k \beta^k \left[-\delta_{t+k} \frac{\left(H_{t+k}^j\right)^{1+\varphi}}{1+\varphi} + \Lambda_{t+k} W_t^* H_{t+k}^j\right]$$

where Λ_{t+k} is the marginal utility of nominal income.

D.2 Firms

The final good Y_t, which is consumed by the government and households as well as used to invest, is produced by perfectly competitive firms assembling intermediate goods, $Y_t(i)$, with a Dixit and Stiglitz (1977) production technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta_{p,t}-1}{\theta_{p,t}}} di \right)^{\frac{\theta_{p,t}}{\theta_{p,t}-1}}$, where $\theta_{p,t}$ denotes the elasticity of substitution between intermediate goods. The corresponding price index for the final consumption good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta_{p,t}} di \right)^{\frac{1}{1-\theta_{p,t}}}$, where $P_t(i)$ is the price of the intermediate good i. The optimal demand for $Y_t(i)$ is given by $Y_t(i) = (P_t(i)/P_t)^{-\theta_{p,t}} Y_t$. The elasticity of substitution $\theta_{p,t}$ follows:

$$\left(\frac{\theta_{p,t}}{\theta_{p,t}-1} \right) = \left(\frac{\bar{\theta}_p}{\bar{\theta}_p - 1} \right)^{1-\rho_p} \left(\frac{\theta_{p,t-1}}{\theta_{p,t}-1} \right)^{\rho_p} \exp(\varepsilon_{p,t} - \upsilon_p \varepsilon_{p,t-1})$$

where $\varepsilon_{p,t} \sim$ i.i.d. $N\left(0, \sigma_p^2\right)$.

Monopolistically competitive firms produce intermediate goods using the production function:

$$Y_t(i) = \max\{(A_t H_t(i))^{1-\lambda} K_t(i)^{\lambda} - A_t F; 0\},$$

where $H_t(i)$ and $K_t(i)$ denote the homogenous labor and capital employed by firm i and A_t represents exogenous economy-wide technological progress. The gross growth rate of technology $a_t \equiv A_t/A_{t-1}$ follows:

$$a_t = \bar{a}^{1-\rho_a} a_{t-1}^{\rho_a} \exp(\varepsilon_{a,t}),$$

where $\bar{a}$ is the steady-state value of a_t and $\varepsilon_{a,t} \sim$ i.i.d. $N(0, \sigma_a^2)$. F is a fixed cost of production that ensure that profits are zero in steady state.

As in Calvo (1983), a firm resets its price optimally with probability $1 - \alpha_p$ every period. Firms that do not optimize adjust their price according to the simple partial dynamic indexation rule:

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^{\gamma_p} \bar{\pi}^{1-\gamma_p},$$

where γ_p measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing firms choose a common price P_t^* to maximize the present discounted value of future profits:

$$E_t \sum_{k=0}^{\infty} \alpha_p^k Q_{t,t+k} \left[P_t^* X_{t,k} Y_{t+k}(i) - W_{t+k} H_{t+k}(i) - R_{t+k}^k K_{t+k}(i) \right],$$

where

$$X_{t,k} \equiv \begin{cases} (\pi_t \pi_{t+1} \cdots \pi_{t+k-1})^{\gamma} \bar{\pi}^{(1-\gamma)k}, & k \geq 1 \\ 1, & k = 0 \end{cases}.$$

D.3 Government

D.3.1 Budget Constraint

Each period, the government collects lump-sum tax revenues T_t and issues one-period nominal bonds B_t to finance its consumption G_t, and interest payments. Accordingly, the flow budget constraint is given by:

$$\frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_t} + G_t - T_t.$$

The flow budget constraint can be rewritten as:

$$R_t^{-1} b_t = b_{t-1}\frac{1}{\pi_t}\frac{y_{t-1}}{y_y}\frac{A_{t-1}}{A_t} + \tilde{G}_t - \tau_t,$$

where $b_t \equiv R_t \frac{B_t}{P_t Y_t}$ denotes the real maturity value of government debt relative to output, $\tilde{G}_t \equiv \frac{G_t}{Y_t}$, and $\tau_t \equiv \frac{T_t}{Y_t}$.

D.3.2 Monetary Policy

The central bank sets the nominal interest rate according to a Taylor-type rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_t^*}\right)^{\phi_\pi}\left(\frac{X_t}{X_t^*}\right)^{\phi_Y}\right]^{1-\rho_R}\exp\left(\varepsilon_{R,t}\right),$$

which features interest rate smoothing and systematic responses to deviation of GDP from its natural level X_t^* and deviation of inflation from a time-varying target π_t^*.[27] $\bar{R}$ is the steady-state value of R_t and the non-systematic monetary policy shock $\varepsilon_{R,t}$ is assumed to follow i.i.d. $N\left(0, \sigma_R^2\right)$. The inflation target evolves exogenously as:

$$\pi_t^* = \bar{\pi}^{1-\rho_\pi}\left(\pi_{t-1}^*\right)^{\rho_\pi}\exp(\varepsilon_{\pi,t}),$$

where $\varepsilon_{\pi,t} \sim$ i.i.d. $N\left(0, \sigma_\pi^2\right)$.

D.3.3 Fiscal Policy

The fiscal authority sets the tax revenues according to:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{\tau_{t-1}}{\bar{\tau}}\right)^{\rho_\tau}\left[\left(\frac{b_{t-1}}{\bar{b}}\right)^{\tilde{\psi}}\right]^{1-\rho_\tau},$$

[27] The natural level of output is the output that would prevail under flexible wages and prices and in the absence of time-variation in the elasticity of subsitution over differentiated labor and goods varieties.

which features tax smoothing and a systematic response to lagged debt. $\bar{\tau}$ is the steady-state value of τ_t while $\bar{b}$ is the steady-state value of b_t. Government spending follows an exogenous process given by:

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t$$

where the government spending shock follows:

$$g_t = \bar{g}^{1-\rho_g} g_{t-1}^{\rho_g} \exp(\varepsilon_{g,t}),$$

where $\varepsilon_{g,t} \sim$ i.i.d. $N\left(0, \sigma_g^2\right)$.

D.4 Equilibrium

Equilibrium is characterized by the prices and quantities that satisfy the households' and firms' optimality conditions, the government budget constraint, monetary and fiscal policy rules, and the clearing conditions for the product, labor, and asset markets:

$$\begin{aligned}
\int_0^1 C_t^j dj + G_t + \int_0^1 I_t^j dj + a(u_t) \int_0^1 \bar{K}_{t-1}^j dj &= Y_t, \\
\int_0^1 H_t(i) di &= H_t \\
\int_0^1 V_t^j dj &= 0, \\
\int_0^1 B_t^j dj &= B_t.
\end{aligned}$$

Note that $C_t^j = C_t$, $I_t^j = I_t$, and $\bar{K}_{t-1}^j = \bar{K}_{t-1}$ due to the complete market assumption and the separability between consumption and leisure. The capital accumulation equation in the aggregate is then given by:

$$\bar{K}_t = (1-d)\bar{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

and the aggregate resource constraint and the definition of GDP then take the form:

$$C_t + I_t + G_t + a(u_t)\bar{K}_{t-1} = Y_t$$

$$X_t = C_t + I_t + G_t.$$

D.5 Approximate Model

We first solve the problem of households and firms given the monetary and fiscal policy rules and derive the equilibrium conditions. We then use approximation methods to solve the model. First, the model features a stochastic balanced growth path since the neutral technology shock contains a unit root. Therefore, we de-trend variables on the balanced growth path by the level of the technology shock and write down all the equilibrium conditions of the transformed model. Second, we compute the non-stochastic steady state of this transformed model. Third, we obtain a first-order approximation of the equilibrium conditions around this steady state. We then solve the approximated model using standard methods.

For a variable $X_t,$ let $x_t = \frac{X_t}{A_t}$. We denote by $\hat{x}_t$ the log-deviation from steady state of $x_t,$ except for fiscal variables, which are in terms of deviation from steady state.

We also, define some new variables: $\lambda_t = \Lambda_t P_t A_t,$ and ϕ_t,which is the lagrange multiplier on the capital accumulation equation for the household's optimization problem (that is, it is the shadow value of installed capital), $\varrho_{p,t} = \frac{\theta_{p,t}}{\theta_{p,t-1}},$ and $\varrho_{l,t} = \frac{\theta_{l,t}}{\theta_{l,t-1}}$. We omit a detailed derivation and the equations characterizing the approximate equilibrium, after some manipulations, are given by:

$$\hat{y}_t = \frac{\bar{y}+F}{\bar{y}}\left[\lambda\hat{k}_t + (1-\lambda)\hat{H}_t\right]$$

$$\chi\hat{u}_t = \hat{w}_t + \hat{L}_t - \hat{k}_t$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p}E_t\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} + \kappa_p\left[\lambda\left(\chi\hat{u}_t\right) + (1-\lambda)\hat{w}_t\right] + \kappa_p\hat{\varrho}_{p,t}$$

$$\hat{\lambda}_t = \hat{R}_t + E_t\left(\hat{\lambda}_{t+1} - \hat{a}_{t+1} - \hat{\pi}_{t+1}\right)$$

$$\begin{aligned}\hat{\lambda}_t \quad = \quad & \frac{\eta\beta\bar{a}}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}E_t\hat{c}_{t+1} - \frac{\bar{a}^2+\eta^2\beta}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}\hat{c}_t + \frac{\eta\bar{a}}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}\hat{c}_{t-1} \\ & + \frac{\eta\beta\bar{a}\rho_a - \eta\bar{a}}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}\hat{a}_t + \frac{(\bar{a}-\eta\beta\rho_\delta)}{(\bar{a}-\eta\beta)}\hat{\delta}_t\end{aligned}$$

$$\hat{\phi}_t = (1-d)\,\beta\bar{a}^{-1}E_t\left(\hat{\phi}_{t+1} - \hat{a}_{t+1}\right) + \left(1-(1-d)\,\beta\bar{a}^{-1}\right)E_t\left[\hat{\lambda}_{t+1} - \hat{a}_{t+1} + \chi\hat{u}_{t+1}\right]$$

$$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t - \bar{a}^2 S'' \left(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{a}_t\right) + \beta \bar{a}^2 S'' E_t \left[\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{a}_{t+1}\right]$$

$$\hat{k}_t = \hat{u}_t + \widehat{\bar{k}}_{t-1} - \hat{a}_t$$

$$\widehat{\bar{k}}_t = (1-d)\,\bar{a}^{-1}\left(\widehat{\bar{k}}_{t-1} - \hat{a}_t\right) + \left(1 - (1-d)\,\bar{a}^{-1}\right)\left(\hat{\mu}_t + \hat{u}_t\right)$$

$$\begin{aligned}
\hat{w}_t \;=\; & \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - \left(\varphi L_t + \hat{\delta}_t - \hat{\lambda}_t\right)\right] \\
& + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\gamma_p}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} \\
& + \frac{\gamma_w}{1+\beta}\hat{a}_{t-1} - \frac{1+\beta\gamma_w - \beta\rho_a}{1+\beta}\hat{a}_t + \kappa_w\hat{\varrho}_{l,t}
\end{aligned}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_y\left(x_t - x_t^*\right)\right] + \varepsilon_{R,t}$$

$$\hat{x}_t = \hat{y}_t - \frac{\rho\bar{k}}{\bar{y}}\hat{u}_t$$

$$\frac{1}{\bar{g}}\hat{y}_t = \frac{1}{\bar{g}}\hat{g}_t + \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{\imath}}{\bar{y}}\hat{\imath}_t + \frac{\rho\bar{k}}{\bar{y}}\hat{u}_t$$

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\left[\hat{\pi}_t + \hat{y}_t - \hat{y}_{t-1} + \hat{a}_t\right] - \bar{R}\hat{\tau}_t + \bar{b}\hat{R}_t + \bar{R}\frac{1}{\bar{g}}\hat{g}_t.$$

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\,\psi\hat{b}_{t-1}$$

where $\rho = \frac{\bar{a}}{\beta} - (1-d)\,,\ \kappa_p = \frac{(1-\beta\alpha_p)(1-\alpha_p)}{\alpha_p\left(1+\gamma_p\beta\right)},\ \ \kappa_w = \frac{(1-\beta\alpha_w)(1-\alpha_w)}{\alpha_w(1+\beta)(1+\varphi\theta_l)},$ and $\psi \equiv \frac{\bar{\tau}}{\bar{b}}\tilde{\psi}.$

D.6 Parameterization

Table 2: Benchmark Parameterization

Parameter	Value	Description
λ	0.17	Capital share
d	0.025	Depreciation rate of capital
γ_p	0.24	Price indexation
γ_w	0.11	Wage indexation
$100 \log \bar{a}$	0.48	SS technology growth rate
η	0.78	Consumption habit
$1/\left(1-\bar{\theta}_p\right)$	0.23	SS price markup
$1/\left(1-\bar{\theta}_l\right)$	0.15	SS wage markup
$\log \bar{H}$	0.38	SS log-hours
$100(\bar{\pi}-1)$	0.71	SS quarterly inflation
$100(\beta^{-1}-1)$	0.13	Discount factor
φ	3.79	Inverse Frisch elasticity
α_p	0.84	Calvo prices
α_w	0.70	Calvo wages
χ	5.30	Elasticity capital utilization costs
$S^{''}$	2.85	Investment adjustment costs
ρ_R	0.82	Monetary rule smoothing
ρ_τ	0.80	Fiscal rule smoothing
ρ_a	0.23	Neutral technology growth
ρ_g	0.99	Government spending
ρ_μ	0.72	Investment
ρ_p	0.94	Price markup
ρ_l	0.97	Wage markup
ρ_δ	0.67	Intertemporal preference
υ_p	0.77	Price markup MA
υ_l	0.91	Wage markup MA
$100\sigma_R$	0.22	Monetary policy
$100\sigma_a$	0.88	Neutral technology growth
$100\sigma_g$	0.35	Government spending
$100\sigma_\mu$	6.03	Investment
$100\sigma_p$	0.14	Price markup
$100\sigma_w$	0.20	Wage markup
$100\sigma_\delta$	0.04	Intertemporal preference
$\bar{b}$	0.40	SS public debt
$\bar{g}$	0.22	SS government spending
ϕ_y	0.07	Monetary rule output
ϕ_π	$[0,3]$	Monetary rule inflation
ψ	$[-0.1,0.1]$	Fiscal rule public debt

Note: The benchmark values of the parameters other than ρ_τ, $\overline{b}$, ϕ_π and ψ are the estimated median of Justiniano, Primiceri and Tambalotti (2010). The value of ρ_τ is taken from Bhattarai, Lee and Park (2012) and $\overline{b}$ is the sample average ratio of public debt to output in the U.S. For policy parameters ϕ_π and ψ, we try different values for comparative statics.

D.7 Dynamics under alternative parameterization

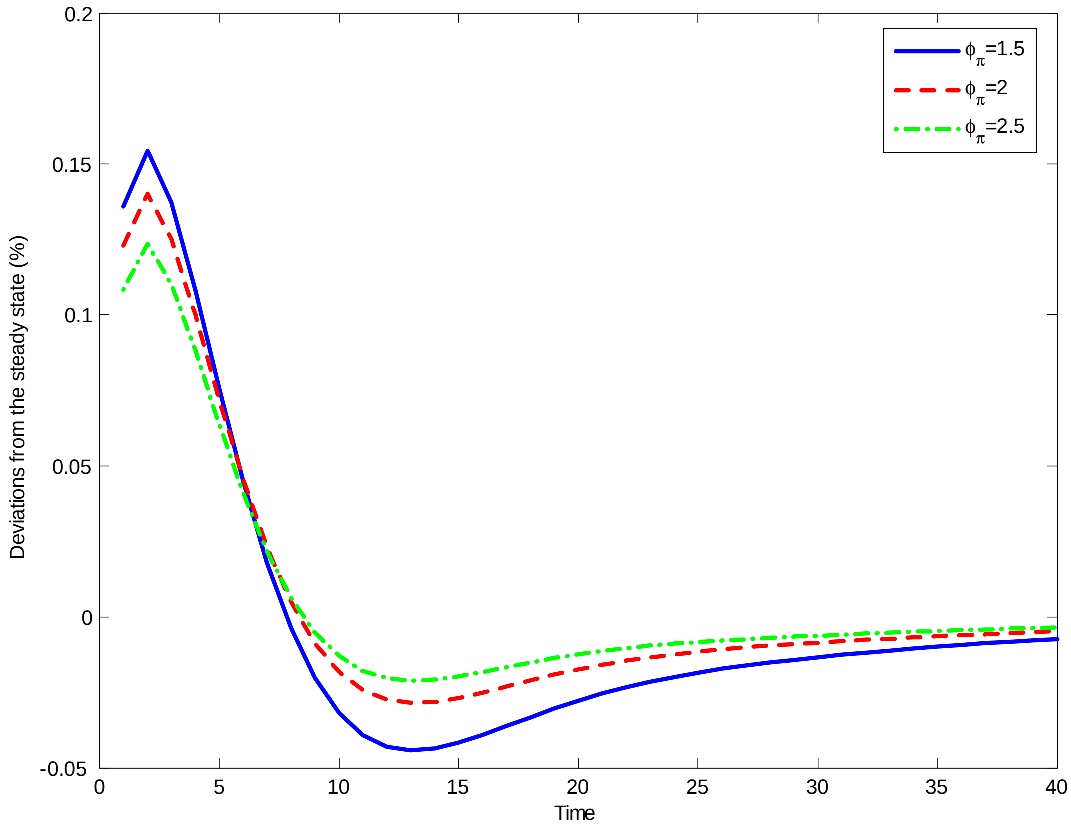

Figure 14: The response of inflation to a one standard deviation increase in the investment specific shock under an alternate parameterization ($\alpha_w = 0.3$).