Comment on "Density and Spin Response of a Strongly Interacting Fermi Gas in the Attractive and Quasirepulsive Regime"

In Ref. [1], the authors summarize a linear response theory for Fermi gases undergoing Bardeen-Cooper-Schrieffer (BCS) to Bose-Einstein condensation (BEC) crossover. Invoking Popov theory, they include a rather complex set of diagrams on the basis of avoiding a divergence in the density response associated with noninteracting bosons. We wish to point out that repairing this divergence as they do above T_c leads to violations of Ward identities or conservation principles. The essence of the Ward identity in question is the verification that the diamagnetic and paramagnetic contributions cancel to avoid an unphysical normal-state Meissner effect. Indeed, their previous work [2] implied that consistency was not yet established in their Popov-based approach; the authors indicated that in the last step of the program they would need to "modify the number equation to be consistent with the above approximation" for their self-energy diagrams. Although they did not specify the precise form of the self-energy Σ_P, the Supplemental Material in Ref. [1] suggested that there were no incompatibilities between the linear response vertex Λ and Σ_P. Here we prove that this is not the case.

We confine our attention in this Comment to the attractive interaction regime which was considered by us in earlier work [3–7] to address a systematic theory of spin and charge linear response above and below T_c. Our work, not cited by the authors, was shown to be demonstrably consistent with a number of sum rules, reflecting gauge invariance. In Appendix D of Ref. [3], we presented the appropriate set of diagrams for the vertex Λ which is compatible with the self-energy Σ associated with the Nozieres–Schmitt-Rink approach above T_c. Here the t-matrix contains only noninteracting Green's functions. Adding additional diagrams in any transport theory has to be done with considerable care to maintain the Ward identity between Λ and Σ.

The diagrams of Ref. [1] involve the Maki-Thompson (MT) and Aslamazov-Larkin (AL) diagrams, which as we showed earlier [3] can be made compatible with a Ward identity; however, in addition to this subset, the authors of Ref. [1] introduce an infinite set of diagrams of the AL type. The fermionic self-energy (Σ_P, to which they refer in the Supplemental Material) is not available for explicit checking of the Ward identity: $Q \cdot \Lambda(K, K^+) = G^{-1}(K) - G^{-1}(K^+)$ with $K^+ = K + Q$. Nevertheless it is possible to prove that, by reconstructing additional self-energy contributions from the new AL diagrams, there must be additional diagrams in the response vertex as well. In particular, we find a violation of gauge invariance in the form

$$Q \cdot [\lambda(K, K^+) + \mathrm{MT}(K, K^+) + \mathrm{AL}(K, K^+) + \sum_{n=2}^{\infty} \mathrm{AL}_n(K, K^+)] \neq G^{-1}(K) - G^{-1}(K^+). \quad (1)$$

The reason that this Ward identity cannot be respected is because the repeated AL diagrams will introduce factors of 2^n in the response vertex due to the two possible ways of connecting the two fermion propagators inside. In order to find consistency, it is essential to include diagrams of the MT or mixed MT and AL forms and thereby cancel these factors of 2^n which do not appear naturally in self-energy diagrams.

The consistency with a Ward identity is a crucial check of the gauge invariance of a theory. The reliability of any theory of transport in a many-body system needs to be built on this level of consistency. When there is such a violation, f sum rules will not be satisfied. Moreover, an unphysical normal-state Meissner effect ensues, which implies a finite superfluid density above T_c. This Comment is intended to emphasize the key role played by sum rules and other conservation constraints in transport and scattering theories of Fermi gases, as we have repeatedly stressed in our papers [3–7]. As pointed out in textbooks [8], it is difficult using approximate methods to obtain the same answer for the compressibility from the static correlation function and the thermodynamic derivative. Although quantitative agreement with experiments is reported in Ref. [1], the theoretical inconsistencies addressed in this Comment should not be overlooked.

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