A Vector Potential Integral Equation Method For Electromagnetic Scattering

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***Abstract*—In this paper, we implement a vector potential integral equation method for electromagnetic scattering problems. The proposed equations are formulated by imposing both the tangential and normal components along the surface of a scatterer. They can be solved by a subspace projection method such as the Galerkin's or moment methods by considering the induced surface currents and the normal component $\hat{n}\cdot\mathbf{A}$ as unknowns. Numerical results validate the effectiveness, stability and accuracy of the proposed formulation.**

I. Introduction

Today, quantum theories are showing their importance in solving interdisciplinary electromagnetic problems with the rapid development of quantum optics applications. To bridge the quantum and classical physics regimes, a potential-based solution (vector potential $\mathbf{A}$ and scalar potential Φ) is always preferred, while the field quantities $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$ and $\mathbf{B}$ are popularly introduced to describe the classical electromagnetic physics. Maxwell's equations, the foundation of electromagnetics theory, are also formulated with those classical physical quantities [1], which have been well solved by differential equation (DE) and integral equation (IE) methods. However, in quantum mechanics, the potential-based formulations frequently appear [2]: the fields $\mathbf{E}$, $\mathbf{H}$ might become zero while the potentials are present for some situations like the Aharonv-Bohm effect. Since the vector and scalar potential equations are still valid from quantum regime to classical regime, it is a possible medium to link the two regimes.

The potential-based formulations are mainly derived from the differential equations [3–7]. For the surface integral equation (SIE) methods, the formulation are mainly based on the field quantities. By considering both the vector and scalar potentials as unknowns [8], the IE-based formulation was obtained to calculate the scattering from dielectric objects at middle frequencies. Recently, a vector potential integral equation was also presented for solving scattering problems [9], where the decoupled potential integral equation was constructed by solving the boundary value problems and the system should be well-conditioned at low frequencies.

By following the generalized Green's theorem and equivalent principle, a vector potential integral equation was recently proposed in [10]. By involving both the induced surface currents and the normal component $\hat{n}\cdot\mathbf{A}$ as unknowns, the equation system can be established along tangential and normal directions and solved in a general manner of moments method. Similar matrix solution method was used in [11] where a overdetermined system has to be solved. By introducing $\hat{n}\cdot\mathbf{A}$ as addition unknowns, the resultant matrix system becomes more symmetrical, thus avoiding the saddle point problem in the augmented electric field integral equation in [12]. In this paper, the proposed potential-based integral formulation is first-time developed and verified for the analysis of electromagnetic scattering of perfect electrically conducting (PEC) objects.

II. Formulation and Discretization

The vector potential integral formulation for a PEC problem can be written as [10]

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}_{\text{inc}}(\mathbf{r}) + \int_S dS' \left\{\mu g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}') + \Sigma(\mathbf{r}')\nabla' g(\mathbf{r},\mathbf{r}')\right\} \quad \mathbf{r}\in S^+ \quad (1)$$

where μ is the permeability. Here, $\mathbf{J}(\mathbf{r}')$ and $\Sigma(\mathbf{r}') = \hat{n}'\cdot\mathbf{A}(\mathbf{r}')$ are the unknowns and $\mathbf{A}_{\text{inc}}$ is the known incident vector potential, $\hat{n}'$ denotes the unit vector pointing out of the surface at the source point, and $g(\mathbf{r},\mathbf{r}')$ is the scalar Green's function in free space, where $\mathbf{r}$ and $\mathbf{r}'$ denote the field and source points, respectively.

By taking the tangential and normal components of (1), the dimension of the matrix system can match the number of unknowns. Here, the surface current unknowns are discretized with RWG basis functions based on edge, while the normal component $\Sigma(\mathbf{r}')$ as charge unknowns are discretized with pulse basis function based on patches. The second equation for the normal component is given by

$$\Sigma(\mathbf{r}) = \Sigma_{\text{inc}}(\mathbf{r}) + \hat{n}\cdot\int_S dS' \left\{\mu g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}') + \Sigma(\mathbf{r}')\nabla' g(\mathbf{r},\mathbf{r}')\right\} \quad \mathbf{r}\in S^+ \quad (2)$$

Denote the basis function as Λ_m and the testing function as $\mathbf{T}_n$. By using the integration by parts, the second term under the integral in (1) can be written as

$$\langle \nabla\cdot\mathbf{T}_m(\mathbf{r}), g(\mathbf{r},\mathbf{r}'), \Lambda(\mathbf{r}')\rangle \quad (3)$$

$g(\mathbf{r},\mathbf{r}')$ is continuous across the PEC surface. Thus, by using the extinction theorem, the left hand side of (1) can be chosen to be zero on the surface. The first equation of the system is a first kind integral equation. Since the normal component of $\nabla' g(\mathbf{r}, \mathbf{r}')$ is not continuous across the surface, the left hand side of (2) cannot be chosen to be zero. Hence, the equation becomes a second-kind equation. The singularities in the second term under the integral should be carefully treated. Also it should be noted that the normal unit vector is located at the field point, thus the first term under the integral should equal to zero when the field patch meets with the source RWG edge.

III. Numerical Results

Numerical examples are shown here to verify the accuracy of the proposed formulation. Consider a unit PEC sphere at 300 MHz with the discretization of 1,568 triangle patches and 2,352 unknowns. As shown in Fig. 1, the far field results match well with the analytical solution.

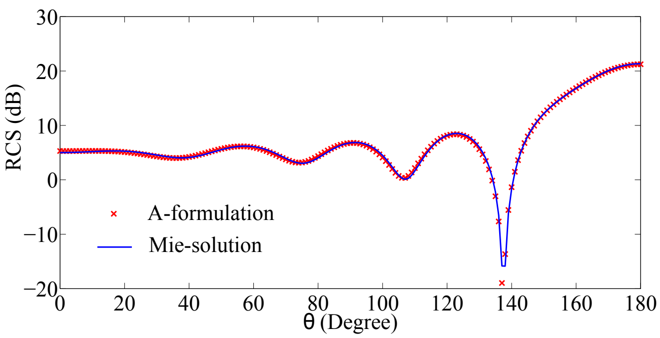

Fig. 1. RCS of the unit PEC sphere at 300 MHz.

Then a PEC cube with the side length of 1 m is excited with a x-polarized plane wave from z-direction at 100 MHz. The model is discretized with 2,760 patches and 4,140 edges. Fig. 2 shows the current distribution, where the currents are reasonably concentrated along the edges.

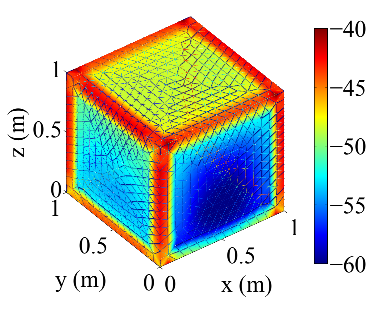

Fig. 2. Current distribution the PEC cube scattering problem at 100 MHz.

IV. Conclusion and Discussion

A vector potential integral equation method has been proposed and implemented in this work. The results show good agreement with the analytical solutions. However, to capture the electroquasistatic solutions well, as shall be shown, the surface integral equations for vector potential, has to be solved in tandem with the surface integral equation for the scalar potential, in order to capture both low-frequency physics well.

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