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INVESTIGATION ON SEMI-ACTIVE CONTROL OF VEHICLE SUSPENSION USING ADAPTIVE INERTER

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The analysis of passive control with inerter in suspension system has been well studied in previous work by employing different configurations and optimizing the spring stiffness, damping coefficient and inertance simultaneously. In this paper, we study the suspension performance with semi-active control under the assumption that the inertance may be adjusted in real-time. The suspension system is designed to attenuate the vertical acceleration of the sprung mass. A quarter-car model is considered, and the inerter is installed parallel to the spring and damper. First, an analysis is provided on the influence of a fixed inerter to a given suspension system. Then, a state-feedback H_2 controller for active suspension system is designed. The active force is approximated by an inerter with adaptive inertance. Simulation results show that comparing with the passive suspension with a fixed inerter, the designed H_2 controller realized by adaptive inerter can achieve good improvement of ride comfort at the sprung mass natural frequency at the expense of a relatively small deterioration at the unsprung mass natural frequency.

1. Introduction

Vehicle suspension is designed to provide ride comfort, road holding, good handling and support the vehicle static weight [1]. There are three types of vehicle suspensions, namely, passive, active and semi-active ones. Passive system is simple and reliable, but it is only effective in a certain frequency band. Active suspension, which requires large energy supply, has been extensively studied [2–5] because it can improve the performance over a wide frequency range and meet complicated requirements. Semi-active suspension systems have drawn much attention in recent years [6–11] as it can achieve desirable performances while relatively a small amount of energy is required. MR and ER dampers are commonly adopted to adjust the damping coefficients.

Inerter is a new device which is firstly proposed by Smith [12]. It is a mechanical two-terminal element and the applied force F is proportional to the relative acceleration $\ddot{z}_1 - \ddot{z}_2$ between these two terminals, that is, $F = b(\ddot{z}_1 - \ddot{z}_2)$, with b as the constant of proportionality called the inertance, which has units of kilograms. An inerter broadens the class of mechanical realizations of complex impedances which can only be operated with springs and dampers in the past. It can be mechanically realized in various ways. A rack-pinion inerter is proposed in [12], which uses a flywheel and gears. A ball-screw device can achieve the functionality of an inerter as well [13]. More description about inerters can be found in [14] and its references. The first successful application of inerter in vehicle suspension can be found in [15]. It proposes several layouts of the suspension system with springs, dampers and inerters. By optimizing the damping and inertance, when comparing with those systems which only optimize damping, about 10% improvement can be achieved in terms of ride comfort, suspension deflection and tyre deflection for both quarter-car and full-car models. Some installation configurations are more suitable for a particular performance index than others. The analytical solutions of ride comfort and tyre deflection in six different configurations are derived in [16]. A method formulated using linear matrix inequalities (LMIs) to optimize the transfer function with fixed order is proposed in [17]. In [13], the nonlinearities of a ball-screw ineter are studied, which include friction, backlash and elastic effect, and a nonlinear theoretical model is obtained. Multiple performance requirements of suspension system with inerter are studied in [18]. Almost all research on passive system with inerters are based on the different layouts of springs, dampers and inerters. To simplify the complexity, a general class of suspension admittances which can be realized by one damper, one inerter and arbitrary number of springs is reported in [19]. Semi-active control of suspension with inerter is studied in [20] based on different configurations. However, in this study, the variable component is the damper, the control method is proposed to adjust the damping coefficient. Inerter is a fixed-value device belonging to the passive system.

Passive suspension with inerter can obtain better performance than that without it, but it calls for complex suspension layout, which is not suitable for general application. Although the conditions of realization with one damper and one inerter and several springs have been proposed, the requirements are not easily met. In view of this, in this paper, we study the performance with semi-active control method by employing an adaptive inerter. Such an inerter is being studied in the authors' research team. A quarter-car model with the simplest suspension layout, that is, involving a spring, a damper and an inerter installed in parallel, is considered. The influence of an inerter to the suspension system is analyzed. An H_2 controller aiming at attenuating the sprung mass vibration is designed, the desired control force is approximated by adaptively varying the inertance. The performance of the proposed scheme is demonstrated through simulation that the designed semi-active vehicle suspension can achieve good improvement at the sprung mass natural frequency while not deteriorating the behaviour too significantly at the unsprung mass natural frequency.

The rest of this paper is organized as follows. A quarter-car model is given in Section 2. Section 3 gives an analysis of the effects of an inerter on a given suspension system. The design of a semi-active H_2 control scheme employing adaptive inerter is developed in Section 4. Section 5 presents the simulation results and discussions of the proposed method. Conclusions are given in Section 6.

2. Quarter-car model

In this study, a quarter-car model is shown in Figure 1, in which m_s represents the sprung mass; m_u represents the unsprung mass; k_s, c_s are stiffness and damping of the suspension, respectively; k_t is tyre stiffness; z_s, z_u are the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; u stands for the external input force which can be operated by means of an actuator for active control or by means of an inerter for semi-active control. This quarter-car model combines two modes, that is, the sprung mass mode and unsprung mass mode.

The dynamic equations for the sprung and unsprung masses of a quarter-car model are given by

$$\begin{aligned} m_s\ddot{z}_s + k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) &= u, \\ m_u\ddot{z}_u + k_t(z_u - z_r) - k_s(z_s - z_u) - c_s(\dot{z}_s - \dot{z}_u) &= -u, \end{aligned} \tag{1}$$

Choose the set of state variables as

$$x_1 = z_s - z_u, \quad x_2 = z_u - z_r, \quad x_3 = \dot{z}_s, \quad x_4 = \dot{z}_u, \tag{2}$$

where x_1 is the suspension deflection, x_2 is the tyre deflection, x_3, x_4 are the velocities of sprung and unsprung masses, respectively.

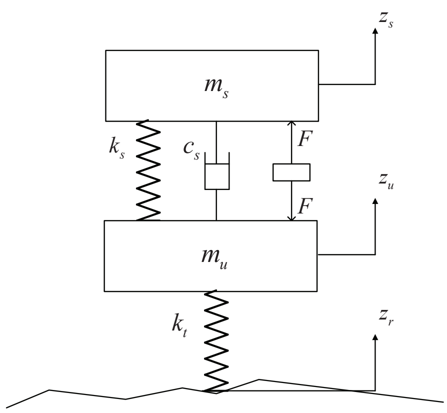

Figure 1: Quarter-car model

By defining $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}$, $w = \dot{z}_r$, the dynamic equations can be written in the following state-space form:

$$\dot{x} = Ax + Bu + B_w w, \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_u} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s}{m_u} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}. \tag{4}$$

Define the output $z(t)$ as $z = [\ddot{z}_s, \alpha(z_s - z_u), \beta(z_u - z_r)]^T$, then z can be rewritten as

$$z = Cx + Du, \tag{5}$$

where

$$C = \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} \frac{1}{m_s} \\ 0 \\ 0 \end{bmatrix}. \tag{6}$$

where $\alpha > 0, \beta > 0$ are weighting coefficients for suspension deflection and tyre deflection, respectively. These coefficients are used to control the trade-off between different performance indexes.

Parameters of the suspension chosen for analysis are listed in Table 1 and we assume the inertance resides in $[0, 500]$ kg.

Table 1: Model parameters

m_s	m_u	k_s	c_s	k_t
320 kg	40 kg	18 KN/m	1000 Ns/m	200 KN/m

3. Analysis of the inerter effect on a given suspension

In this section, analysis of the effect of an inerter on the given suspension system is carried out. Figure 2 shows the frequency responses of the sprung mass vertical acceleration with respect to different fixed inertance. The response comparison of the passive suspension without inerter and with three different inerters is shown in Figure 3. From these two figures, it can be seen that adding a fixed inerter to a passive suspension system will lead to the decrease of natural frequency values and change in resonant peak values. To be more specific, both the sprung mass and unsprung mass natural frequencies decrease, the reduction of the unsprung mass natural frequency is more apparent, a larger inertance will result in more reduction of natural frequencies. The first resonant peak value is reduced with the presence of the inerter, and increasing the inertance can eliminate the peak further. However, for the second resonant peak, the peak value will dramatically increase with increasing inertance. The system dynamic characteristics have been changed due to the change in the system inertia by the inerter. The change is quantified in Table 1, f_1, f_2 correspond to the sprung mass and unsprung mass frequencies. The table shows that comparing with the passive suspension system without inerter, there are $38\%, 54\%, 58\%$ improvement for $b = 100, 300, 500$ at the sprung mass natural frequency, respectively. However, at the unsprung mass natural frequency, the acceleration is $8, 34, 60$ times larger than the passive system, which is not acceptable. It indicates that adding a fixed inerter to a given passive suspension system can improve the performance at the sprung mass natural frequency but will dramatically aggravate the vibration at the unsprung mass natural frequency.

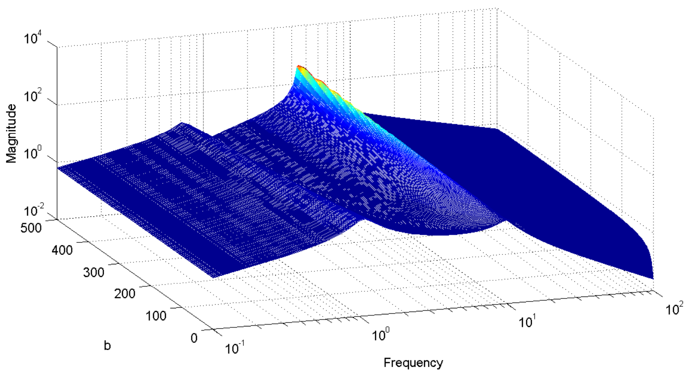

Figure 2: Frequency response of vertical acceleration

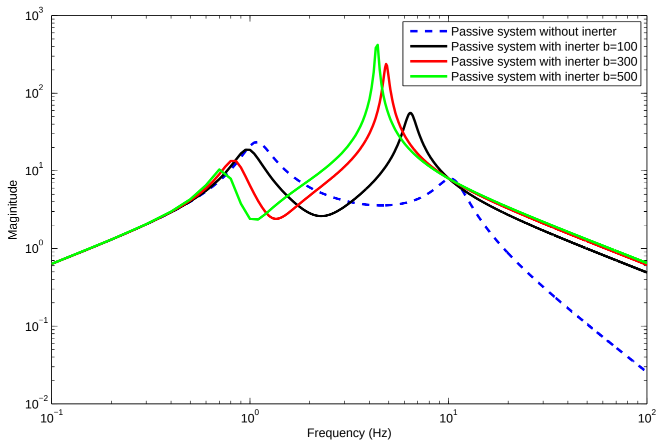

Figure 3: Frequency response comparison of vertical acceleration

4. Semi-active Inerter Control of Suspension System

Motivated by the design of semi-active suspension control with variable damper, an active H_2 controller is designed for the suspension system initially, than an inerter is used to apply force to the suspension system by tracking the ideal control force.

Table 2: System dynamics characteristic comparison with different inertance of passive system

	without inerter	$b = 100$	$b = 300$	$b = 500$
f_1 (Hz)	1.11	1.02	0.78	0.71
f_2 (Hz)	10.45	6.45	4.85	4.40
Peak value (f_1)	23.39	18.63	13.40	10.42
Peak value (f_2)	7.87	55.58	237.70	419.30

4.1 Formulation of H_2 controller

Generally speaking, ride comfort, suspension deflection and road holding are the main performance indexes in suspension design. Ride comfort is related to the vertical acceleration of the sprung mass, it amounts to keep the transfer function from road disturbance w to sprung mass acceleration $\ddot{z}_s$ small. Suspension deflection calls for hard constraint on the suspension deflection $z_s - z_u$ in order to avoid structural damage and deterioration on the ride comfort when hitting the deflection limit. Road holding is required to minimize the transfer function from disturbance w to tyre deflection $z_u - z_r$ to ensure a firm contact of the wheels to road. According to the above requirements, an H_2 controller is designed to deal with the problem. Weighting coefficients α, β are use to shape and comprise different performance objectives. In this work, our objective focuses on the improvement of the ride comfort. State-feedback control $u = Kx$ is considered here where K is the control gain matrix which is to be designed such that the resulting closed-loop system is asymptotically stable, and the H_2 norm of the closed-loop transfer function from disturbance w to output z, defined as $\|T\|_2$, is bounded by a constant $\gamma > 0$. The following lemma is about the H_2 performance [21].

Lemma 1 *The suspension closed-loop system with state-feedback is asymptotically stable with* $\|T\|_2 \leq \gamma$ *if there exit matrices* $P > 0$ *and* W *satisfying*

$$\begin{aligned} \mathbf{tr}(W) < \gamma^2, \\ \begin{bmatrix} PA^T + AP + K^TB^T + BK & B_w \\ * & -P \end{bmatrix} < 0, \\ \begin{bmatrix} -W & CP + DW \\ * & -P \end{bmatrix} < 0. \end{aligned} \tag{7}$$

4.2 Semi-active control law with inerter

The desired active control force F_{des} is obtained according to the above designed active controller. Since the inerter cannot actively supply energy, when F_{des} and the relative suspension acceleration $\ddot{z}_s - \ddot{z}_u$ has the same sign, that is, the required control force is in the same direction as the relative velocity, such an F_{des} cannot be provided by the inerter. In this case, the value of the inertance b is set to be zero. When the active control force F_{des} and the relative acceleration $\ddot{z}_s - \ddot{z}_u$ have the opposite sign, then the inerter can provide the desired force. In this case, the value of the semi-active inertance b is chosen to be $b = -\frac{F_{des}}{\ddot{z}_s - \ddot{z}_u}$.

In the presence of the constraints $0 < b < b_{max}$, the optimal control inertance b can be obtained as

$$b = \begin{cases} 0, & \text{if } -F_{des}(\ddot{z}_s - \ddot{z}_u) \leq 0, \\ -\frac{F_{des}}{\ddot{z}_s - \ddot{z}_u}, & \text{if } 0 < -\frac{F_{des}}{\ddot{z}_s - \ddot{z}_u} < b_{max}, \\ b_{max}, & \text{if } -\frac{F_{des}}{\ddot{z}_s - \ddot{z}_u} \geq b_{max}, \end{cases} \tag{8}$$

then the semi-active control force are obtained as $F_{semi} = -b \times (\ddot{z}_s - \ddot{z}_u)$.

5. Simulation Results and Discussions

In this section, the simulation result concerning the proposed semi-active inerter control method is studied. The road velocity input is given as $\dot{z}_r = 0.1\sin(2\pi ft)$. By setting $\alpha = 5, \beta = 10$ to focus on the ride comfort performance index, the obtained gain matrix is $K = 10^4 \times [1.8795, -0.1091, -0.0037, -0.0928]$. Figure 4 compares the frequency responses of open-loop and closed-loop system with the H_2 controller, it can be seen that the designed controller can considerably reduce the sprung mass acceleration over a broad frequency range except at the unsprung mass natural frequency. This phenomenon is well-known and also can be found from previous works, see [1].

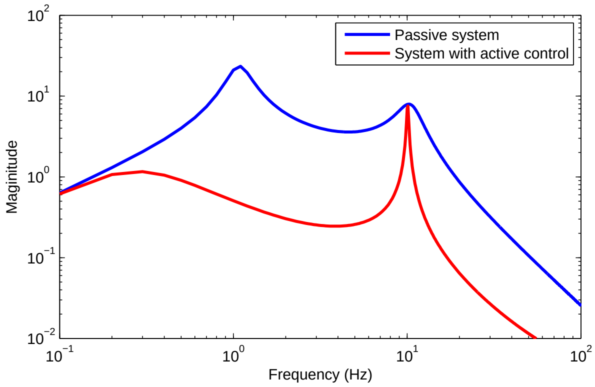

Figure 4: Frequency response of suspension system with and without active control

Figure 5 shows the RMS value comparison of vertical accelerations of suspension system with and without adaptive inerter under sinusoidal disturbance. Three adaptive inertance ranges are considered here, that is, $b \in [0, 100]$ and $b \in [0, 300]$ and $b \in [0, 500]$ kg. Table 3 lists the natural frequencies and corresponding RMS values under different conditions. From the figure and the table, it can be seen that the variation of natural frequencies are not so prominent as that in passive system with fixed inertance. Semi-active control with inerter can considerably reduce the acceleration at the sprung mass natural frequency, $24\%, 42\%, 50\%$ improvements can be obtained for different inertance range when comparing with the passive system without inerter. At the same time, only $0.7, 2.2, 3.2$ times amplifications are observed at the second resonant peak, which are much smaller than the passive suspension with a fixed inertance. It indicates that the adaptive inerter can achieve better performance than the fixed inerter for the same passive suspension system. On the other hand, this phenomenon implies that the designer might need to make a compromise between the two natural frequencies, more reduction at the first resonant peak is at the expense of a larger increase at the second resonant peak within an acceptable range.

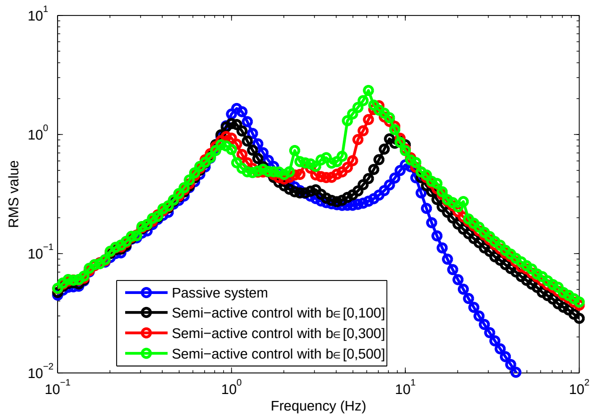

Figure 5: Comparison of vertical acceleration RMS value

Table 3: Natural frequencies and RMS accelerations for different adjustable inertance range

	Passive system	$b \in [0, 100]$	$b \in [0, 300]$	$b \in [0, 500]$
f_1 (Hz)	1.11	1.01	0.93	0.87
f_2 (Hz)	10.45	8.10	7.06	6.14
Peak value (f_1)	1.65	1.24	0.96	0.82
Peak value (f_2)	0.54	0.92	1.75	2.34

6. Conclusion

A suspension with parallel layout of a spring, a damper and an inerter is considered in this work. An analysis of the influence of a fixed inerter on the passive suspension system indicates that adding an inerter may attenuate the acceleration at the sprung mass natural frequency, but the acceleration at the unsprung mass natural frequency will be dramatically amplified, which may not be acceptable in practice. A semi-active control of suspension with adjustable inertance is proposed to deal with this problem. In the semi-active control, the inerter tracks the desired control force of a suitably designed state-feedback H_2 controller. The performance of this scheme, as validated by simulations, has shown that good reduction of vibration at the sprung mass frequency at the expense of a relatively small deterioration at the unsprung mass frequency.

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