A Multi-Layer Market for Vehicle-to-Grid Energy Trading in the Smart Grid

Albert Y.S. Lam, Longbo Huang, Alonso Silva
University of California, Berkeley
Email: {ayslam, huang, asilva}@eecs.berkeley.edu

Walid Saad
ECE Department, University of Miami
Email: walid@miami.edu

***Abstract*—In this paper, we propose a novel multi-layer market for analyzing the energy exchange process between electric vehicles and the smart grid. The proposed market consists essentially of two layers: a macro layer and a micro layer. At the macro layer, we propose a double auction mechanism using which the aggregators, acting as sellers, and the smart grid elements, acting as buyers, interact so as to trade energy. We show that this double auction mechanism is strategy-proof and converges asymptotically. At the micro layer, the aggregators, which are the sellers in the macro layer, are given monetary incentives so as to sell the energy of associated plug-in hybrid electric vehicles (PHEVs) and to maximize their revenues. We analyze the interaction between the macro and micro layers and study some representative cases. Depending on the elasticity of the supply and demand, the utility functions are analyzed under different scenarios. Simulation results show that the proposed approach can significantly increase the utility of PHEVs, compared to a classical greedy approach.**

I. Introduction

The rising oil prices combined with the ongoing trend for developing environmental-friendly technological solutions, implies that electrically-operated vehicles will lie at the heart of future transportation systems. In particular, it is envisioned that plug-in hybrid electric vehicles (PHEVs), which are essentially electric vehicles equipped with storage devices, will constitute one key component towards realizing the vision of green, environment-friendly transportation networks. For instance, it is forecast that up to 2.7 million electric vehicles will be put on the road in the United States, by 2020 [1].

The presence of energy storage devices implies that PHEVs can not only serve as a green means of transportation, but also, if properly configured, they can function as a “moving” energy reservoir that can store and, possibly, supply power back to the power grid. While current PHEV deployment are mostly concerned with grid-to-vehicle interactions, enabling two-way vehicle-to-grid (V2G) interactions between the grid and PHEVs, has recently started to receive considerable attention both in research and standardization agencies [2], [3], [4], [5], [6] and is expected to lie at the heart of the emerging smart grid system.

Enabling V2G interactions has several advantages such as providing a backup power source during outages or supplying ancillary services back to the grid for regulation services. However, in order to fully reap the benefits of V2G systems, several key challenges must be addressed at different level such as control, communications, implementation, and market mechanisms. In [7], the authors propose a scheme that uses PHEV batteries to absorb the randomness in intermittent wind power generation. The authors in [8] study the use of game theory for providing frequency regulation through V2G operation. Using the PHEVs as storage units is studied and analyzed in [9] while communication architectures suitable for V2G systems are discussed in [10]. Further, the authors in [11] considers the problem of optimally providing energy and ancillary services using electric vehicles.

Clearly, most existing work are focused on implementation, communication, and energy transfer in V2G systems. However, the need for energy transfer and exchange from PHEVs to the grid has also an economic aspect that must be addressed. In this respect, the work in [12] sheds a light on this aspect by investigating the price and quantities exchanged if the PHEVs and the grid elements form an energy trade market. Beyond [12], little work seems to have been focused on the economics of V2G exchanges which are essential for a better understanding on the potential of using V2G in future power grid systems.

The main contribution of this paper is to propose a general framework and algorithm for studying the economics of the market emerging between PHEVs, aggregators, and the smart grid elements. To address this problem, we propose a multi-layered market mechanisms in which the agreggators, the PHEVs, and the grid elements can decide on the quantity and prices at which they wish to trade energy while optimizing the tradeoff between the benefits (e.g., revenues) and costs from this energy exchange. In this proposed market, first, the aggregators and the smart grid elements (e.g., substations) submit their reservation prices and bids so as to agree on a price and energy trading mechanism. These interactions are modeled using a double auction whose result is then fed back into the second layer, which deals with the management of PHEV resources at each aggregator. In this layer, within each PHEV group, the aggregator negotiates with the PHEVs to settle for an agreement on the resource usage. In particular, the aggregator will announce its energy buying price to the PHEVs, and each PHEV determines how much energy it is willing to supply to the aggregator. The outcome of this layer is directly linked to the previous market due to the fact that the aggregator has to carefully balance its earnings from the energy market and its payments to the PHEVs within its group. Hence, unlike existing work such as in [12], the proposed scheme depends, not only on pricing issues, but also on modeling the PHEVs-to-aggregator interactions (from an economical perspective) as well as on providing incentives for the PHEVs to participate in the foreseen market and, hence, it can leverage V2G, for improving the overall smart grid performance.

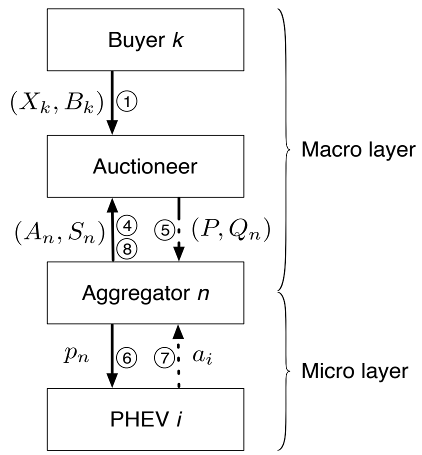


Fig. 1. The two-layer market model. The numbers in the figure correspond to the step numbers in Algorithm 1.

We characterize the equilibria resulting from the proposed multi-layered market and we show their existence. Further, using linear approximation techniques we provide a large-system analysis on the economics of V2G energy trading. Using simulations, we assess the properties and performance of the energy trading mechanisms resulting from proposed scheme and we show that our approach can significantly increase the utility of PHEVs.

The rest of this paper is organized as follows: In Section II, we present the proposed system model. In Section III, we analyze the system for different PHEV energy supply costs. Simulation results are discussed and analyzed in Section IV while conclusions are drawn in Section V.

II. System Model and Proposed Market Mechanism

Consider a smart grid system consisting of K grid elements (e.g., substations) with $\mathcal{K} = \{1, \ldots, K\}$ denoting the set of all such elements. In this grid, N electric vehicles aggregators are deployed. We let $\mathcal{N} = \{1, \ldots, N\}$ denote the set of all aggregators. Each aggregator $n \in \mathcal{N}$ manages a group of PHEVs denoted by $\mathcal{I}_n = \{1, \ldots, I_n\}$. In this model, we are particularly interested in smart grid elements that are unable to meet their demand and, hence, need to buy energy from alternative sources such as the PHEV aggregators. Hence, hereinafter, all grid elements are referred to as buyers while the aggregators are referred to as sellers.

The energy exchange process between the aggregators and the grid elements is modeled using a two-layered market model as shown in Fig. 1 with the utility company's control center acting as a middleman that handles the prospective energy trading mechanisms. On the one hand, at the first layer, referred to as the *macro layer*, the aggregators and the grid elements interact so as to trade energy. At this layer, the buyers wish to optimize their performance and meet their demand by buying energy from the PHEVs through the aggregators while the aggregators wish to strategically choose their price and quantity to trade so as to optimize their revenues. On the other hand, at the second layer, referred to as the *micro layer*, the aggregators must interact with the PHEVs so as to optimize the energy resources and provide incentives for the PHEVs to actually participate in the trade. Clearly, the outcomes of these two layers are coupled and, thus, any market solution must take into account this inter-layer dependence. Below, we discuss and analyze, in details, the market operation at each layer.

A. Macro Layer

Auction theory is essentially an analytical framework used to study the interactions between a number of sellers, each of which has some commodity or good to sell (in this example, the commodity is power), and a number of buyers interested in obtaining the good so as to optimize their objective functions [13]. The outcome of an auction is the price at which the trade takes place as well as the amount of good sold to each buyer. The use of game-theoretic techniques has recently emerged as a suitable approach for analyzing and predicting the outcomes of an auction. At the macro layer of our model, we propose a double auction model [14], [15], [12] using which the aggregators and the smart grid elements can exchange energy. For instance, at this layer, each potential seller or aggregator $n \in \mathcal{N}$ sends the quantity of energy A_n that it intends to supply and its reservation price S_n to the auctioneer. The reservation price sent by the potential sellers corresponds to the minimum price at which the seller is willing to sell its offered amount of energy. Each buyer $k \in \mathcal{K}$ proposes a bid B_k and the quantity it requests, denoted by X_k, to the auctioneer. Here, we are mainly focused on the interactions between buyers and sellers in a certain window of time during which the bids and reservation prices do not vary. This can correspond to an energy trading market in which decisions are based on medium or long-term energy needs such as in a day-ahead market. In each round, each aggregator n decides its own A_n with the fixed S_n. After receiving all A_n's, the auctioneer determines the price $P(\mathbf{A})$ of the energy, where $\mathbf{A} = (A_n, 1 \leq n \leq N)$, and $Q_n(\mathbf{A})$, which corresponds to the total quantity sold by aggregator n, $\forall n \in \mathcal{N}$, by a double auction.

Here, we propose a double auction mechanism based on [14], [12], [16], [15] and which proceeds as follows:

- The sellers are ordered in an increasing order of their reservation price. W.l.o.g. we consider
$$S_1 < S_2 < \ldots < S_N. \quad (1)$$
- The buyers are ordered in a decreasing order of their reservation bids. W.l.o.g. we consider
$$B_1 > B_2 > \ldots > B_K. \quad (2)$$
- If two sellers (respectively, buyers) have equal reservation prices (bids), they are aggregated into one single "virtual" seller (or buyer).
- We generate the supply curve (selling reservation price S_n versus the amount of energy put out for sale A_n)
- We generate the demand curve (offered bids B_k versus quantity needed X_k).
- We find an intersection point.

This intersection is at the level of a certain seller L and buyer M, such that $B_M \geq S_L$ and $B_{M+1} < S_{L+1}$. Having found seller L and buyer M, the proposed double auction dictates that the first $L-1$ and $M-1$ buyers will participate in the energy trading. Seller L and buyer M do not participate in this trade which allows to match the total supply and demand while maintaining a strategy-proof mechanism [15].

Thus, all sellers with index $n < L$ and all buyers with index $k < M$ become the participants in the double auction at the macro layer. Subsequently, the trading prices for the sellers and the buyers can be chosen within any point in the range $[S_L, B_M]$ [16]. For any action choice $\mathbf{A}$ (or $\mathbf{A}_n, \forall n \in \mathcal{N}$) by the sellers (i.e. aggregators), given seller L and buyer M at the intersection, we consider that all sellers $i < L$ and buyers $k < M$ trade at a price $P(\mathbf{A})$, given by

$$P(\mathbf{A}) = \frac{S_L(\boldsymbol{A}) + B_M(\boldsymbol{A})}{2}, \tag{3}$$

where the dependence on $\mathbf{A}$ is due to the fact that each $\mathbf{A}$ can give different interaction points with the demand and supply curves, and thus we can have different L and M.

At the end of the auction, numerous criteria can be used for determining the amount of traded energy between each one of the $L-1$ sellers and $M-1$ buyers. We adopt the approach of [15] in which the volume is divided so as to ensure a strategy-proof auction. Using this approach, the total quantity $Q_n(\mathbf{A})$ sold by any PHEV group n, for a given choice $\mathbf{A}$ is:

$$Q_n(\mathbf{A}) = \begin{cases} A_n & \text{if } \sum_{k=1}^{M-1} X_k \geq \sum_{j=1}^{n} A_j, \\ (A_n - \Psi)^+ & \text{if } n = L-1, \\ 0 & \text{if } n > L-1, \end{cases} \tag{4}$$

where $(x)^+ = \max\{0, x\}$ and Ψ is the oversupply, i.e., $\Psi = \sum_{j=1}^{L-1} A_j - \sum_{k=1}^{M-1} X_k$.

B. Micro Layer

The micro layer deals with the interactions between the aggregators and the PHEVs. At this layer, each aggregator announces a price p_n to its PHEVs by

$$p_n = \gamma_n P(\mathbf{A}), \forall n \in \mathcal{N}, \tag{5}$$

where $0 < \gamma_n < 1$ is the commission rate. The price difference, $P(\mathbf{A}) - p_n$, is the commission (i.e., cost of management) earned by aggregator n from its managed PHEVs. This commission provides a profitable monetary incentive for each aggregator to help the PHEVs in their market participation. and maximize their profits. Essentially, each PHEV $i \in \mathcal{I}_n$ determines its available supply $a_i \in [0, a_i^{\max}]$, with $a_i^{\max}$ being the amount of energy the owner of PHEV i can afford to sell, i.e., after reserving enough for its own use, so as to maximize a well-defined utility function (defined later in this section). For the same type of vehicle, a PHEV which needs more reserve for its proper operation has a lower $a_i^{\max}$. In fact, we have

$$A_n = \sum_{i=1}^{I_n} a_i, \forall n \in \mathcal{N}. \tag{6}$$

Let $\mathbf{a}_n = (a_i, 1 \leq i \leq I_n)$ be the action of the PHEVs in group i. Aggregator n determines the actual quantity $q_i(\mathbf{a}_n, Q_n(\mathbf{A}))$ allocated to PHEV i in group n proportional to a_i by

$$q_i(\mathbf{a}_n, Q_n(\mathbf{A})) = \frac{a_i}{\sum_{j \in \mathcal{I}_n} a_j} \times Q_n(\mathbf{A}), \tag{7}$$

and thus

$$Q_n(\mathbf{A}) = \sum_{i=1}^{I_n} q_i(\mathbf{a}_n, Q_n(\mathbf{A})), \forall n \in \mathcal{N}. \tag{8}$$

Consider that each PHEV $i \in \mathcal{I}_n$ imposes a cost of discharging its battery to supply energy to its aggregator. We denote this cost by $c_i(a)$, where a is the amount of energy *sold* by PHEV i. We assume that during each transaction, each PHEV will try to maximize its utility (representing the profit) by choosing the amount of energy a_i as follows:

$$a_i^* = \arg\max_{a_i} u_i(a_i) = \arg\max_{a_i} \{p_n(\mathbf{a}_n) - c_i(a_i)\}, \tag{9}$$

where $u_i(\cdot)$ and $c_i(\cdot)$ accounts for the utility function and the inconvenience cost (e.g., for not having enough energy to operate normally, for discharging the battery, etc.) of PHEV i, respectively.

C. Market Mechanism

For analyzing the interaction between the macro and micro layers, we propose the algorithm shown in Algorithm 1, which is used to find the market equilibrium P^*, A_n^*. This proposed algorithm terminates when a pre-determined number of iterations $t^{\max}$ has been reached or the percentage change of the market price is less than a certain threshold ξ.

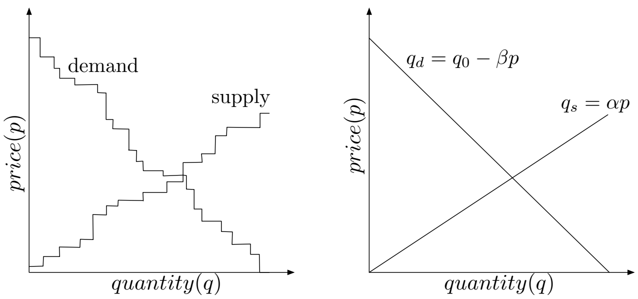
Algorithm 1 Market Mechanism

For all k, buyer k submits its bid B_k and its requested amount X_k to the auctioneer
$t \leftarrow 1$
repeat
  For all n, aggregator n submits its reservation price S_n and its proposed supply A_n, by (6), to the auctioneer
  The auctioneer implements a double auction and determines the market price $P(t)$, by (3), and the allocated quantity Q_n, by (4), for aggregator n for all n
  For each aggregator n, the selling price p_n, by (5), is announced to its PHEVs
  For each PHEV i, it determines its proposed selling amount a_i by maximizing its utility according to p_n, by (9), and returns a_i back to its aggregator
  Each aggregator n sums up all a_i's from its PHEVs by (6)
  $t \leftarrow t + 1$
until $t > t^{\max}$ or $|\frac{P^{(t)} - P^{(t-1)}}{P^{(t)}}| < \xi$

As we can see in simulation later, Algorithm 1 performs very efficiently and converges within a few step to the market equilibrium.

III. Linear Approximation for Large systems

In a practical smart grid, both the number of PHEVs and smart grid elements are expected to be large. Hence, it is of interest to analyze the behavior of the proposed market mechanism in large-scale systems. Hereinafter, for simplicity, we restrict our attention to case in which N and M are sufficiently large such that the corresponding supply and demand curves can be approximated by a linear function, depending on the distribution of aggregators' reservation price (buyers' bids). Thus, we assume that the reservation prices S_n from each aggregator can be ordered into a line with each point being one unit apart. Fig. 2 shows one example of the supply and demand curves, each of which is approximated by one single linear function. Since the



(a) Real situation (b) Linear approximation

Fig. 2. Supply and demand curves

focus of this work is on the supply side, we will study different types of supply curves while maintaining a fixed demand curve.

A. Linear Cost Function with Homogeneous PHEVs

Here, we consider that the cost function of PHEV i is a linear function with respect to the amount of energy it provides, i.e., $c_i(a) = \eta_i a$, within a certain interval $[0, a_i^{\max}]$. If the amount of energy to be sold exceeds $a_i^{\max}$, the cost will become infinite. Note that this case also takes into account the inconvenience cost. Then, PHEV i's utility function is given by $u_i(a) = (\gamma_n P - \eta_i)a$. The problem PHEV i needs to address is the following:

$$u_i^* = \underset{a_i \in [0, a_i^{\max}]}{\text{maximize}}\ u_i(a_i). \tag{10}$$

We note that, a PHEV can always decide not to participate in the market in which case it maximum utility will be $u_i^* = 0$. As a result, the maximum utility in (10) is always nonnegative. In this case, the solution a_i^* of (10) is

$$a_i^* = \begin{cases} a_i^{\max} & \text{if } \eta_i \geq p_n, \\ 0 & \text{else.} \end{cases} \tag{11}$$

We can then subdivide the set of PHEVs $\mathcal{I}_n$ in two disjoint sets $\mathcal{I}_n^{(1)}$ and $\mathcal{I}_n^{(2)}$, such that $\mathcal{I}_n^{(1)} \cup \mathcal{I}_n^{(2)} = \mathcal{I}_n$, where $a_i^* = 0, \forall i \in \mathcal{I}_n^{(1)}$, and $a_i^* = a_i^{\max}, \forall i \in \mathcal{I}_n^{(2)}$. Thus, we obtain

$$A_n = \sum_{i \in \mathcal{I}_n} a_i = \sum_{i \in \mathcal{I}_n^{(1)}} a_i + \sum_{i \in \mathcal{I}_n^{(2)}} a_i. \tag{12}$$

Since for all $i \in \mathcal{I}_n^{(1)}$, $a_i^* = 0$, then $\sum_{i \in \mathcal{I}_n^{(1)}} a_i = 0$, and in consequence

$$A_n = \sum_{i \in \mathcal{I}_n^{(2)}} a_i = \sum_{i \in \mathcal{I}_n^{(2)}} a_i^{\max}. \tag{13}$$

1) Linear Approximation: Consider the linear case as in Fig. 2(b). The PHEVs are price takers and the supply and demand curves are given by

$$\text{Supply}(P, Q) = \alpha P, \tag{14}$$

$$\text{Demand}(P, Q) = Q_0 - \beta P. \tag{15}$$

Here Q_0 is the total demand from all the buyers and P is the price determined from the double auction in the macro layer. Also, α will depend on $\mathcal{I}_n^{(2)}$ for all aggregator n. In that case, the equilibrium price P^* and the quantity sold Q^* can be determined when the supply meets the demand, i.e.,

$$\text{Supply}(P^*, Q^*) = \text{Demand}(P^*, Q^*) \Rightarrow \alpha P^* = Q_0 - \beta P^*,$$

or equivalently, when

$$P^* = \frac{Q_0}{\alpha + \beta}. \tag{16}$$

The utility for each PHEV i in group n will be thus given by

1) If $\eta_i \geq \frac{\gamma_n Q_0}{(\alpha+\beta)}$, then $a_i^* = 0$ and $u_i^* = 0$.
2) If $\eta_i < \frac{\gamma_n Q_0}{(\alpha+\beta)}$, then $a_i^* = a_i^{\max}$ and

$$u_i^* = (\gamma_n P^* - \eta_i)\, a_i^* = \left(\frac{\gamma_n Q_0}{(\alpha + \beta)} - \eta_i \right) a_i^{\max}. \tag{17}$$

The total utility U_n of aggregator n is given by

$$U_n(\alpha, \beta, Q_0) = \sum_{i \in \mathcal{I}_n^{(2)}} \left(\frac{\gamma_n Q_0}{(\alpha + \beta)} - \eta_i \right) a_i^{\max}. \tag{18}$$

From this simple example, *ceteris paribus*, we can deduce the following properties:

1) If the supply slope of a market 1 is higher than that of a market 2, i.e., $\alpha_1 > \alpha_2$, then the utility gained in market 1 is smaller than that in market 2.
2) If the demand slope of a market 1 is higher than that of a market 2, i.e., $\beta_1 > \beta_2$, then the utility gained in market 1 is smaller than that in market 2.
3) If the total possible demand of a market 1 is higher than that of a market 2, i.e., $Q_0^1 > Q_0^2$, then the utility gained in market 1 is greater than that in market 2.

B. Quadratic Cost Function

Each PHEV i will supply the following amount of energy:

$$a_i^* = \left[\frac{p_n(\mathbf{a}_n) - \eta_i}{2\upsilon_i} \right]_0^{a_i^{\max}}, \tag{19}$$

where $[x]_0^y = \min\left[\max[x, 0], y\right]$. For ease of analysis, we assume that $a_i^* \leq a_i^{\max}$ for all i. Then, we have:

$$a_i^* = \frac{p_n(\mathbf{a}_n) - \eta_i}{2\upsilon_i}, \quad \text{and} \quad A_n^* = C_n p_n(\mathbf{a}_n) - d_n, \tag{20}$$

where

$$C_n = \sum_i \frac{1}{2\upsilon_i} \quad \text{and} \quad d_n = \sum_i \frac{\eta_i}{2\upsilon_i}. \tag{21}$$

In this case, A_n^* is the supply from one aggregator and A_n^* is the slope of the supply curve, i.e., $\alpha = A_n^*$. Thus, we have, by a derivation similar to that of (16),

$$P^* = \frac{Q_0}{A_n^* + \beta}. \tag{22}$$

Hence, we can use the following iterative procedure to find the market price:

- In iteration t, with $A_n(t)$ computed, we obtain the optimal price $P^*(t) = \frac{Q_0}{A_n(t)+\beta}$.
- In iteration $t+1$, we compute the new supply quantity by (20), i.e., $A_n(t+1) = C_n \gamma_n P^*(t) - d_n$.

Based on the above observation, we have the following simple lemma characterizing the necessary conditions under which there exists market equilibria, i.e., (20) and (22) both hold.

***Lemma** 1:* The following conditions are necessary for the above iterative process to converge to a market equilibrium (P^*, A_n^*):

$$\beta d_n - C_n \gamma_n Q_0 \geq 0. \tag{23}$$

Proof: In equilibrium, this price must result in a supply that is exactly equal to the resulting A_n, i.e.,

$$A_n = \frac{C_n Q_0 \gamma_n}{A_n + \beta} - d_n. \tag{24}$$

This gives rise to the following condition on A_n:

$$A_n^2 + (\beta + d_n)A_n + \beta d_n - C_n \gamma_n Q_0 = 0. \tag{25}$$

Note that the above argument also shows an interesting fact that there is an implicit iteration for the market price P as follows:

$$P^* \quad = \quad \frac{Q_0}{C_n\gamma_n P^* - d_n + \beta}. \tag{26}$$

This similarly implies that the fixed point should satisfy:

$$C_n\gamma_n P^{*2} + (\beta - d_n)P^* - Q_0 = 0. \tag{27}$$

It is not difficult to see that (27) will always have a nonnegative solution since $d_n - \beta + \sqrt{(\beta - d_n)^2 + 4C_n\gamma_n Q_0} \geq 0$. Now for (25) to have a nonnegative solution, we only need $\beta d_n - C_n\gamma_n Q_0 \geq 0$, because then we have:

$$\frac{-\beta - d_n + \sqrt{(\beta + d_n)^2 - 4(\beta d_n - C_n\gamma_n Q_0)}}{2} \geq 0. \tag{28}$$

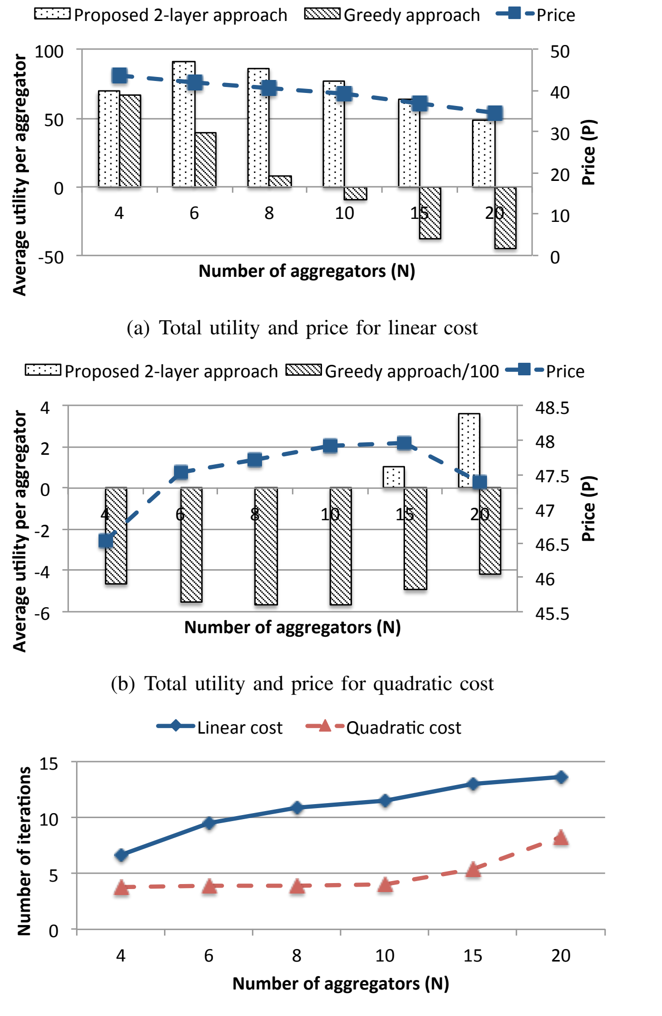
This completes the proof of the lemma. ■

IV. Simulation Results

For simulations, we consider a smart grid in which a number of aggregators sell their energy surplus to smart grid elements (buyers) through a utility company. The simulation setting is as follows. Each aggregator manages a certain number of PHEVs, randomly generated in the range of $[500, 1000]$. Each PHEV has a maximum battery capacity of 250 miles with power consumption 22kWh per 100 miles [17], [18] out of which an arbitrary amount of energy, between 30 and 100 miles, is reserved for the PHEV's private use. The reservation prices of the aggregators are uniformly selected in $[10, 50]$ dollars/MWh while the buyers' bids are randomly chosen from $[15, 60]$ dollars/MWh. Each buyer requests energy demand with the amount chosen in $[20, 60]$ MWh. The commission rate is set to $\gamma_n = 0.91, \forall n \in \mathcal{N}$. Each PHEV i has random cost function parameters $\eta_i \in [10, 50]$ and $\upsilon_i \in [1000, 2000]$ for quadratic cost function and $\beta_i = 0$ for the linear cost function. The algorithm always starts by setting $a_i = a_i^{\max}, \forall i \in \mathcal{I}_n, \forall n \in \mathcal{N}$.

A. Small Numbers of Buyers and Aggregators

First, we simulate cases with small numbers of buyers and aggregators. We consider 5 buyers ($K = 5$) in each case and we compare our two-layer approach with a greedy approach, in which each PHEV always proposes to sell $a_i^{\max}$. Fig. 3 shows the average results of 1000 independent simulation runs for each case. Figs. 3(a) and 3(b) present the average utility per aggregator corresponding to the linear and quadratic cost functions, respectively. We can see that our approach always yields a higher average utility than the greedy scheme, as shown in Figs. 3(a) and 3(b). For linear cost functions, we can see that the average utility starts by increasing with N but then, it starts to decrease when the number of aggregators reaches that of buyers ($N = 6$). This result is due to the fact that, for small N, an increase in the number of participating aggregators leads to a larger amount of energy sold which, subsequently, improves the average utility. However, when $N \geq 6$, an increase in the number of aggregators N will yield a decrease in the settled price which leads to a decrease in the utility. For quadratic cost functions, the average utility increases with N since more aggregators can participate in the market which results in a larger amount of total energy sold. Hence, in general, the equilibrium trading price decreases with N since an increase in the number of aggregators leads to an increased competition which subsequently imposes



(a) Total utility and price for linear cost

(b) Total utility and price for quadratic cost

(c) Iterations required for convergence

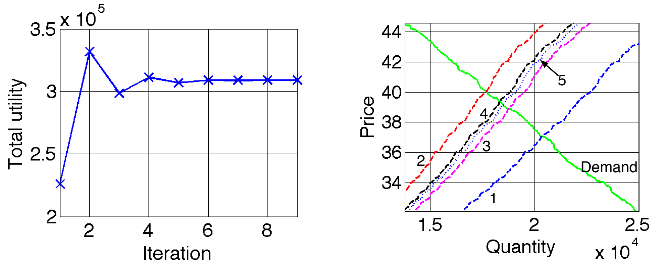
Fig. 3. Basic simulations for small numbers of buyers and aggregators.

a lower price. Note that it is possible to have an situation with zero total utility (i.e., no energy being sold). Thus our mechanism weakly dominates.

Fig. 3(c) shows the average number of iterations required to reach an equilibrium. Clearly, as more aggregators participate in the market, the number of iterations till convergence increases. Moreover, Fig. 3(c) shows that the convergence time is faster in the case with quadratic cost. With a linear cost function, each PHEV i takes either 0 or $a_i^{\max}$ in each iteration, and thus, the algorithm will oscillate more around the equilibrium point before convergence. With a quadratic cost function, due to the concavity of the utility, the algorithm moves toward the equilibrium in a smoother manner which is further corroborated in the subsequent simulations.

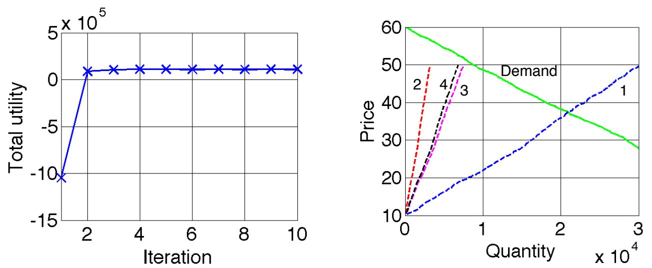
B. Large Numbers of Buyers and Aggregators

Here, we simulate cases in which a large number of buyers ($K = 1000$) and aggregators ($N = 1000$) are deployed so as to verify the analytical results induced from the linear approximation studied in Section III-A. As previously mentioned, when N (K) increases, the supply (demand) curve approaches a linear



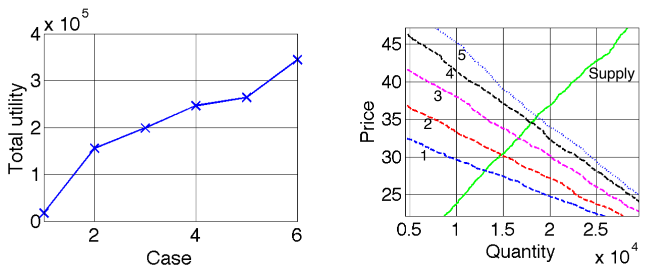
(a) Total utility in each iteration (b) Supply and demand curves

Fig. 4. Linear cost function with 1000 buyers (K) and 1000 aggregators (N)



(a) Total utility in each iteration (b) Supply and demand curves

Fig. 5. Quadratic cost function with 1000 buyers (K) and 1000 aggregators (N)



(a) Total Utility in each case (b) Supply and demand curves

Fig. 6. Fixed Q_0 with 1000 buyers (K) and 1000 aggregators (N)

function as all random numbers are generated uniformly. We first study the results with a fixed demand curve (i.e. with β and Q_0 fixed) and they correspond to cases in which the value evolves in a particular simulation. Fig. 4[1] shows the results for linear PHEV cost functions when the algorithm iterates. Fig. 4(a) gives the total utility computed from the double auction for each iteration while having the corresponding supply and demand curves shown in Fig. 4(b). [2] In Fig. 4, we can also see that the total utility decreases with the supply slope (α). For example, consider iterations 1 and 2. For these two iterations, we can see that α_1 is larger than α_2 while the utility gained in iteration 1 is smaller than that in iteration 2. We also study the quadratic PHEV cost functions and the results are shown in Fig. 5.[1] Clearly, with the quadratic cost function, the algorithm converges faster and smoother. Next, we study the results for six different demand curves with a fixed supply curve (i.e. α fixed). Fig. 6[1,2] shows the six cases with a fixed Q_0. These results are also aligned with those of Section III-A: the larger the β, the smaller the utility.

[1]The numbers in Subfigure (b) correspond to iteration/case numbers in Subfigure (a). For clearer demonstration, In Subfigure (b), only the curves corresponding to the first few iterations/cases are given.

[2]For clarification, Fig. 4(b) shows the part of the whole graph near the intersection points only.

V. Conclusion

In this paper, we have proposed a multi-layer game-theoretic framework for modeling the market that enables aggregators, PHEVs, and the smart grid elements to exchange energy. The proposed framework consists of two layers: a macro layer and a micro layer. At the macro layer, the smart grid elements, which act as energy buyers buyers and the aggregators, which act as energy sellers, engage in a double auction so to determine the amount of energy that will be traded along with the associated trading price. At the micro layer, each aggregator assists its managed PHEVs in maximizing their utilities which captures the profits from the energy exchange. A novel mechanism is proposed to coordinate the interaction between the macro and micro layers. We have analyzed the system performance resulting from the proposed approach for several cases, including large-scale deployments. Simulation results have shown that our mechanism is always better than the greedy approach and verify some of our analytical results.

References

[1] T. A. Becker and I. Sidhu, "Electric vehicles in the united states: A new model with forecasts to 2030," Center for Entrepreneurship & Technology, University of California, Berkeley, Tech. Rep. 2009.1.v.2.0, Aug. 2009.

[2] A. N. Brooks and S. H. Thesen, "PG&E and Tesla motors: Vehicle to grid demonstration and evaluation program," in *Proc. 23rd International Electric Vehicles Symposium and Exposition*, Anaheim, CA, USA, Dec. 2007.

[3] Electric Power Research Institute (EPRI), "PAP11- electric vehicle roaming scenarios," *EPRI Research Program, Use Case*.

[4] National Institute for Standards and Technology (NIST), "SGIP CoS: SAE J2836 use cases for communication between plug-in vehicles and the utility grid," *NIST Standards*.

[5] C. Guille and G. Gross, "A conceptual framework for the vehicle-to-grid (V2G) implementation," *Energy Policy*, vol. 37, no. 11, pp. 4379–4390, Nov. 2009.

[6] B. K. Sovacool and R. F. Hirsh, "Beyond batteries: An examination of the benefits and barriers to plug-in hybrid electric vehicles (PHEVs) and a vehicle-to-grid (V2G) transition," *Energy Policy*, vol. 37, no. 3, pp. 1095–1103, Mar. 2009.

[7] J. Pillai and B. Bak-Jensen, "Vehicle-to-grid for islanded power system operation in bornholm," in *Power and Energy Society General Meeting, 2010 IEEE*, july 2010, pp. 1 –8.

[8] C. Wu, H. Mohsenian-rad, and J. Huang, "Vehicle-to-aggregator interaction game," *IEEE Trans. Smart Grid*, to appear 2011.

[9] M. Erol-Kantarci and H. Mouftah, "Management of phev batteries in the smart grid: Towards a cyber-physical power infrastructure," in *Wireless Communications and Mobile Computing Conference (IWCMC), 2011 7th International*, july 2011, pp. 795 –800.

[10] N. Matta, R. Rahim-Amoud, L. Merghem-Boulahia, and A. Jrad, "A cooperative aggregation-based architecture for vehicle-to-grid communications," in *Global Information Infrastructure Symposium (GIIS), 2011*, aug. 2011, pp. 1 –6.

[11] E. Sortomme and M. A. El-Sharkawi, "Optimal scheduling of vehicle-to-grid energy and ancillary services," *Smart Grid, IEEE Transactions on*, vol. PP, no. 99, p. 1, 2011.

[12] W. Saad, Z. Han, H. V. Poor, and T. Basar, "A noncooperative game for double auction-based energy trading between phevs and distribution grids," in *Proc. IEEE International Conference on Smart Grid Communications (SmartGridComm)*, Brussels, Belgium, Oct. 2011.

[13] V. Krishna, *Auction Theory*, A. Press, Ed., 2002.

[14] P. Huang, A. Scheller-Wolf, and K. Sycara, "A strategy-proof multiunit double auction mechanism," in *Proceedings of the first international joint conference on Autonomous agents and multiagent systems: part 1*, ser. AAMAS '02, 2002.

[15] ——, "Design of a multi-unit double auction e-market," *Computational Intelligence*, vol. 18, no. 4, pp. 596–617, Feb. 2002.

[16] D. Friedman and J. Rust, *The Double Auction Market: Institutions, Theories, and Evidence*. Boulder, CO, USA: Westview Press, 1993.

[17] C. Silva, M. Ross, and T. Farias, "Evaluation of energy consumption, emissions and cost of plug-in hybrid vehicles," *Elsevier Energy Conversion and Management*, vol. 50, no. 7, pp. 1635–1643, Jul. 2009.

[18] T. Motors, "Roadster innovations: Motor," Feb. 2011.