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Integral identities based on symmetric and skew-symmetric weight functions for a semi-infinite interfacial crack in anisotropic magnetoelectroelastic bimaterials

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Abstract

In this paper, we address a semi-infinite interfacial crack problem in an anisotropic magnetoelectroelastic (MEE) bimaterial system subjected to a magnetoelectromechanical asymmetric load on the crack surface. First, the symmetric and skew-symmetric weight functions are derived for a two-dimensional (2-D) deformation problem. Using these weight functions and extending the Betti formula to MEE materials, the integral identities are further obtained and the present crack problem is formulated in terms of singular integral equations, which establish the relationship between the applied external load and the generalized displacement jump across the crack faces. The illustrative examples in relation to Mode III, and Mode I and Mode II problems show that the method developed in this study avoids the use of Green's function and is very convenient for the fracture analysis of MEE solids, in which a multi-field coupled effect is observed.

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1. Introduction

Weight functions were defined by Bueckner (1970, 1987, 1989) as the singular non-trivial solutions of the homogeneous traction-free problem, and were fundamental in the determination of stress intensity factors for asymptotic representations near cracks. The theory of weight functions has also been employed to solve interfacial crack problems in elastic bimaterials (Gao, 1991; Gao, 1992; Ma and Chen, 2004), where the symmetric opening loads were applied on the crack faces. However, for the in-plane deformation of interfacial cracks problem, skew-symmetric loads with forces of the same magnitude and opposite directions produce non-zero stress intensity factors, and for the Mode III problem there is also a non-vanishing skew-symmetric component of the weight functions. Moreover, in most situations, the loads are not symmetric; therefore, an investigation of the influence of skew-symmetric loads on fracture analysis is necessary. With these considerations, skew-symmetric weight functions were introduced by Piccolroaz et al. (2009) to perform a fracture analysis on the interfacial crack problem in isotropic elastic bimaterials (Piccolroaz et al., 2010; Piccolroaz and Mishuris, 2013; Vellender et al., 2013) and anisotropic elastic bimaterials (Morini et al., 2012; Morini et al., 2013). The weight function method has also been extended by Pryce et al. (2013) to analyze dynamic steady-state propagation of interfacial problems under arbitrary loading. In the past couple of decades, more and more attention has been paid to magnetoelectroelastic (MEE) materials, which exhibit the coupled effect within magnetic, electric and elastic fields, and many achievements have been made to the interface crack problems of MEE materials (Gao and Noda, 2004; Li and Kardomateas, 2007; Zhao et al., 2008; Herrmann et al., 2010; Zhu et al., 2010; Feng et al., 2009; Feng et al., 2012; Ma et al., 2015a). Although many achievements regarding weight functions have been made in relation to elastic interfacial cracks, to our best knowledge, the symmetric and skew-symmetric weight functions for an anisotropic MEE bimaterial system have not been reported for the plane strain case, nor even for the Mode III deformation.

On the other hand, the method of singular integral equations is often employed to investigate fracture problems, and the generalized displacement and stress field defined by singular integral formulations is derived based on Green's function in many circumstances (Zhu and Qin, 2007a; Zhu and Qin, 2007b; Zhao et al., 2007b; Li et al., 2011; Zhao et al., 2013; Zhang and Wang, 2014). Although Green's functions for various two-dimensional (2-D) and three-dimensional (3-D) problems have been obtained (Liu et al., 2001; Pan, 2002; Wang and Shen, 2002; Jiang and Pan, 2004; Ding et al., 2005; Hou et al., 2005; Hou et al., 2009; Jiang et al., 2007; Rojas-Diaz et al., 2008; Ma and Lee, 2009; Xiong et al., 2010; Buroni, and Saez, 2010; Zhao et al., 2007a; Zhao et al., 2015), their application in the determination of displacement and stress on the crack surface usually implies challenging numerical integral calculations, and the convergence of these integrals should be carefully identified, especially in the case of anisotropic MEE materials with multi-field coupled effects. Additionally, the singular integral formulations based on Green's functions can only be used to analyze problems relating to symmetric loads on the crack surface, and are not applicable to cases involving asymmetric loads.

For these aforementioned considerations, in this study we firstly derive general expressions for the symmetric and skew-symmetric weight functions of a 2-D interfacial crack problem in an anisotropic MEE bimaterial system. Then, referring to Morini et al. (2013) and using these weight functions together with Betti's reciprocal theorem, we further develop an alternative method for deriving integral identities for MEE bimaterials, which describe the relationship between the applied magnetoelectromechanical load and the generalized displacement jump across crack faces. For the 2-D problem, the obtained identities are composed of Cauchy type singular operators and algebraic terms, with the latter ceasing to exist for homogeneous MEE materials. The approach proposed in this paper avoids the use of Green's function and provides a powerful tool with which to solve the interfacial crack problems of MEE bimaterials.

2. Symmetric and skew-symmetric weight functions for anisotropic MEE bimaterials

In this section, we will derive the general expressions for symmetric and skew-symmetric weight functions in relation to the present semi-infinite interfacial crack under generalized traction-free conditions. As shown in Fig. 1a, a semi-infinite plane crack is situated at the interface of two dissimilar anisotropic MEE materials and the origin of the coordinate system is at the crack tip. In this paper, we consider the 2-D deformation, in which both the generalized displacement vector $\mathbf{u} = \{u_1,\ u_2,\ u_3,\ \varphi,\ \phi\}^{\mathrm{T}}$ and the generalized traction vector $\mathbf{t} = \{t_1,\ t_2,\ t_3,\ t_4,\ t_5\}^{\mathrm{T}} = \{\sigma_{31},\ \sigma_{32},\ \sigma_{33},\ D_3,\ B_3\}^{\mathrm{T}}$ are only dependent on variables x_1 and x_3, where $u_i (i = 1, 2, 3)$, ϕ and φ are elastic displacements, electric and magnetic potentials, respectively, and σ_{3i}, and D_3 and B_3 are mechanical stresses, electric displacement and magnetic induction, respectively. In addition, the interfacial crack is assumed to be magnetoelectrically impermeable. Therefore, the boundary conditions at the interface can be written as

$$[\mathbf{u}](x_1) = \mathbf{u}^{(1)}(x_1, 0) - \mathbf{u}^{(2)}(x_1, 0) = 0,\ \mathbf{t}^{(1)}(x_1, 0) = \mathbf{t}^{(2)}(x_1, 0), \quad x_1 > 0\,, \tag{1}$$

$$\mathbf{t}^{(1)}(x_1, 0) = \mathbf{t}^{(2)}(x_1, 0) = 0, \quad x_1 < 0\,, \tag{2}$$

where the superscripts "(1)" and "(2)" refer to Materials 1 and 2, respectively. In the following sections, for simplicity, we will use $\mathbf{t}(x_1)$ instead of $\mathbf{t}(x_1, 0)$ to denote the generalized traction at the material interface.

In the present problem, the asymmetric loads are imposed on the crack faces, which are assumed to be self-balanced and vanish at infinity (Morini et al., 2012). The applied magnetoelectromechanical load can be defined by generalized traction acting on the crack faces, and is expressed as

$$\begin{aligned}\mathbf{q}^{\pm}(x_1) &= \left\{q_1^{\pm}(x_1),\ q_2^{\pm}(x_1),\ q_3^{\pm}(x_1),\ q_4^{\pm}(x_1),\ q_5^{\pm}(x_1)\right\}^{\mathrm{T}} \\ &= \left\{\sigma_{31}\left(x_1,0^{\pm}\right),\ \sigma_{32}\left(x_1,0^{\pm}\right),\ \sigma_{33}\left(x_1,0^{\pm}\right),\ D_3\left(x_1,0^{\pm}\right),\ B_3\left(x_1,0^{\pm}\right)\right\}^{\mathrm{T}}, \quad x_1<0,\end{aligned} \tag{3}$$

where $q_j^{\pm}(x_1)$ are the prescribed load functions, and the superscripts "+" and "-" stand for the upper and lower crack faces, respectively. In the absence of body forces, electric charges and electric currents, the symmetric and skew-symmetric components of the magnetoelectromechanical load are written as

$$\langle\mathbf{q}\rangle(x_1) = \frac{1}{2}\left\{\mathbf{q}^{+}(x_1)+\mathbf{q}^{-}(x_1)\right\}, \quad [\mathbf{q}](x_1) = \mathbf{q}^{+}(x_1)-\mathbf{q}^{-}(x_1). \tag{4}$$

Obviously, the arbitrary load boundary condition can be defined with the help of Eq. (4) since all the applied load combinations can be decomposed into the superposition of the symmetric and skew-symmetric parts.

Referring to Suo (1990) and Suo et al. (1992), the present crack problem can be reduced to a Riemann-Hilbert problem and the corresponding details are presented in Appendix A. According to the Plemelj formula, the solution of the Riemann-Hilbert problem Eq. (A.12) $\mathbf{h}(z)$ can be expressed as

$$\mathbf{h}(z) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{+\infty}\frac{\mathbf{t}(\eta)}{\eta - z}\mathrm{d}\eta\,. \tag{5}$$

Introducing the following Fourier transform

$$\hat{\rho}(\xi) = \mathcal{F}_{\xi}\left\{\rho(x_1)\right\} = \int_{-\infty}^{+\infty}\rho(x_1)\mathrm{e}^{\mathrm{i}\xi x_1}\mathrm{d}x_1,\ \rho(x_1) = \mathcal{F}_{x_1}^{-1}\left\{\rho(\xi)\right\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{\rho}(\xi)\mathrm{e}^{-\mathrm{i}\xi x_1}\mathrm{d}x_1\,, \tag{6}$$

and applying the Fourier transform to Eq. (5) as $x_3 = 0^{\pm}$, one can get (Morini et al., 2012)

$$\hat{\mathbf{h}}\left(\xi,0^{\pm}\right) = \mathbf{h}^{\pm}(\xi) = \pm\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}\xi x_1}H(\pm\xi)\,\mathbf{t}(\eta)\mathrm{d}\eta = \pm H(\mp\xi)\,\hat{\mathbf{t}}^{(+)}(\xi), \quad \xi\in\mathrm{R}\,, \tag{7}$$

where H is the Heaviside function and $\hat{\mathbf{t}}^{(+)}(\xi)$ is the transform of the generalized traction $\mathbf{t}^{(+)}(\xi)$ at the material interface, namely

$$\hat{\mathbf{t}}^{(+)}(\xi) = \mathscr{F}_\xi\left\{\mathbf{t}^{(+)}(x_1)\right\} = \int_0^\infty \mathbf{t}^{(+)}(x_1)\mathrm{e}^{\mathrm{i}\xi x_1}\mathrm{d}x_1, \tag{8}$$

where $\mathbf{t}^{(+)}(x_1)$ with the superscript "$(+)$", different from $\mathbf{t}(x_1)$, denotes the generalized traction on the positive semi-axis $x_1 > 0$. In what follows, the function with the superscripts "$(+)$" and "$(-)$" means that the support of this function is restricted to the positive semi-axis and negative semi-axis, respectively. In deriving Eq. (7), the following relation is used (Morini et al., 2012)

$$\int_{-\infty}^{+\infty} \frac{\mathrm{e}^{\mathrm{i}\xi(x_1 \pm \mathrm{i}0)}}{\eta - (x_1 \pm \mathrm{i}0)}\mathrm{d}x_1 = \pm 2\pi \mathrm{i}\mathrm{e}^{\mathrm{i}\xi\eta} H(\mp\xi). \tag{9}$$

The Fourier transforms of the functions $\mathbf{g}(z)$ and $\overline{\mathbf{g}}(\overline{z})$ in Eq. (A.6) at the interface can be written as

$$\hat{\mathbf{g}}(\xi) = \mathbf{B}^{-1}\hat{\mathbf{h}}^{+}(\xi) = H(-\xi)\mathbf{B}^{-1}\hat{\mathbf{t}}^{(+)}(\xi), \quad \xi \in \mathrm{R}, \tag{10}$$

$$\hat{\overline{\mathbf{g}}}(\xi) = -\overline{\mathbf{B}}^{-1}\hat{\mathbf{h}}^{-}(\xi) = H(\xi)\overline{\mathbf{B}}^{-1}\hat{\mathbf{t}}^{(+)}(\xi), \quad \xi \in \mathrm{R}. \tag{11}$$

Taking the Fourier transforms to Eq. (A.6), we have

$$-\mathrm{i}\xi\hat{\mathbf{u}}(\xi, 0^{\pm}) = \mathbf{A}\hat{\mathbf{f}}(\xi) + \overline{\mathbf{A}}\hat{\overline{\mathbf{f}}}(\xi), \quad \xi \in \mathrm{R}. \tag{12}$$

From Eq. (12), the Fourier transform of the generalized displacement on the boundary of the upper half-plane can be found as (Morini et al., 2012)

$$\hat{\mathbf{u}}(\xi, 0^{+}) = \frac{1}{\xi}\left\{H(-\xi)\mathbf{Y}^{(1)} - H(\xi)\overline{\mathbf{Y}}^{(1)}\right\}\hat{\mathbf{t}}^{(+)}(\xi), \quad \xi \in \mathrm{R}, \tag{13}$$

where $\mathbf{Y} = \mathrm{i}\mathbf{A}\mathbf{B}^{-1}$ is a Hermitian matrix.

Rewriting the Heaviside function in the following form

$$H\left(\pm\xi\right)=\frac{1}{2}\left\{1\pm\mathrm{sign}\left(\xi\right)\right\}, \tag{14}$$

one gets

$$\hat{\mathbf{u}}\left(\xi,0^{+}\right)=\left\{\frac{1}{2\xi}\left(\mathbf{Y}^{(1)}-\bar{\mathbf{Y}}^{(1)}\right)-\frac{1}{2|\xi|}\left(\mathbf{Y}^{(1)}+\bar{\mathbf{Y}}^{(1)}\right)\right\}\hat{\mathbf{t}}^{(+)}\left(\xi\right),\quad \xi\in\mathrm{R}. \tag{15}$$

Similarly, the Fourier transform of the generalized displacement on the boundary of the lower half-plane can be given as

$$\hat{\mathbf{u}}\left(\xi,0^{-}\right)=\left\{\frac{1}{2\xi}\left(\mathbf{Y}^{(2)}-\bar{\mathbf{Y}}^{(2)}\right)+\frac{1}{2|\xi|}\left(\mathbf{Y}^{(2)}+\bar{\mathbf{Y}}^{(2)}\right)\right\}\hat{\mathbf{t}}^{(+)}\left(\xi\right),\quad \xi\in\mathrm{R}. \tag{16}$$

For elasticity, the weight functions are defined as a non-trivial singular solution of the homogeneous traction free problem (Bueckner, 1987). By extending this concept to the MEE case, the weight functions related to MEE materials can be similarly obtained. Following the theory proposed by Wills and Movchan (1995), herein we introduce $\mathbf{U}=\left\{U_1,\ U_2,\ U_3,\ U_4,\ U_5\right\}^{\mathrm{T}}$ as the weight function in a different domain with respect to the physically generalized displacement, where the crack is located on the positive semi-axis $x_1>0$, as shown in Fig. 1b. Therefore, the symmetric and skew-symmetric weight functions are, respectively, presented as (Piccolroaz et al., 2013)

$$\left[\mathbf{U}\right]\left(x_1\right)=\mathbf{U}\left(x_1,0^{+}\right)-\mathbf{U}\left(x_1,0^{-}\right), \tag{17}$$

$$\left\langle\mathbf{U}\right\rangle\left(x_1\right)=\frac{1}{2}\left\{\mathbf{U}\left(x_1,0^{+}\right)+\mathbf{U}\left(x_1,0^{-}\right)\right\}. \tag{18}$$

Corresponding to the singular solution $\mathbf{U}$, the generalized traction vector $\mathbf{\Sigma}=\left\{\Sigma_1,\ \Sigma_2,\ \Sigma_3,\ \Sigma_4,\ \Sigma_5\right\}^{\mathrm{T}}$, where $\mathbf{\Sigma}_i\,(i=1,2,3)$ correspond to mechanical components whereas $\mathbf{\Sigma}_4$ and $\mathbf{\Sigma}_5$ correspond to electric and magnetic components, respectively, is also introduced with the following boundary conditions

$$\mathbf{\Sigma}\left(x_1,0\right)=0,\quad x_1>0\,, \tag{19}$$

$$\mathbf{\Sigma}^{(1)}\left(x_1,0\right)=\mathbf{\Sigma}^{(2)}\left(x_1,0\right),\quad x_1<0\,. \tag{20}$$

It is remarked that $\mathbf{U}$ is discontinuous and $\mathbf{\Sigma}$ is equal to zero for $x_1>0$, whilst $\mathbf{u}$ is discontinuous and $\mathbf{t}$ is equal to zero for $x_1<0$.

Correspondingly, the Fourier transform of the generalized singular displacement $\mathbf{U}$ on the boundary can be readily obtained from Eqs. (15) and (16) by replacing $\mathbf{u}$ and $\mathbf{t}$ with $\mathbf{U}$ and $\mathbf{\Sigma}$, respectively, and presented in the following form

$$\tilde{\mathbf{U}}\left(\xi,0^+\right)=\left\{\frac{1}{2\xi}\left(\mathbf{Y}^{(1)}-\bar{\mathbf{Y}}^{(1)}\right)-\frac{1}{2|\xi|}\left(\mathbf{Y}^{(1)}+\bar{\mathbf{Y}}^{(1)}\right)\right\}\mathbf{\Sigma}^{(-)}\left(\xi\right),\quad \xi\in\mathrm{R}\,, \tag{21}$$

$$\tilde{\mathbf{U}}\left(\xi,0^-\right)=\left\{\frac{1}{2\xi}\left(\mathbf{Y}^{(2)}-\bar{\mathbf{Y}}^{(2)}\right)+\frac{1}{2|\xi|}\left(\mathbf{Y}^{(2)}+\bar{\mathbf{Y}}^{(2)}\right)\right\}\mathbf{\Sigma}^{(-)}\left(\xi\right),\quad \xi\in\mathrm{R}\,. \tag{22}$$

According to Eqs. (17) and (18), the symmetric and skew-symmetric weight functions are obtained, respectively, by taking the jump and average of Eqs. (21) and (22)

$$\left[\tilde{\mathbf{U}}\right]^{(+)}\left(\xi\right)=\frac{1}{|\xi|}\left\{\mathrm{i}\,\mathrm{sign}\left(\xi\right)\mathrm{Im}\left(\mathbf{Y}^{(1)}-\mathbf{Y}^{(2)}\right)-\mathrm{Re}\left(\mathbf{Y}^{(1)}+\mathbf{Y}^{(2)}\right)\right\}\mathbf{\Sigma}^{(-)}\left(\xi\right),\quad \xi\in\mathrm{R}\,, \tag{23}$$

$$\left\langle\tilde{\mathbf{U}}\right\rangle\left(\xi\right)=\frac{1}{2|\xi|}\left\{\mathrm{i}\,\mathrm{sign}\left(\xi\right)\mathrm{Im}\left(\mathbf{Y}^{(1)}+\mathbf{Y}^{(2)}\right)-\mathrm{Re}\left(\mathbf{Y}^{(1)}-\mathbf{Y}^{(2)}\right)\right\}\mathbf{\Sigma}^{(-)}\left(\xi\right),\quad \xi\in\mathrm{R}\,. \tag{24}$$

Introducing the following bimaterial matrices

$$\mathbf{H}=\mathbf{Y}^{(1)}+\bar{\mathbf{Y}}^{(2)},\quad \mathbf{L}=\mathbf{Y}^{(1)}-\bar{\mathbf{Y}}^{(2)}, \tag{25}$$

Eqs. (23) and (24) can be rewritten in the following compact form

$$\left[\tilde{\mathbf{U}}\right]^{(+)}\left(\xi\right)=-\frac{1}{|\xi|}\left\{\mathrm{Re}\left(\mathbf{H}\right)-\mathrm{i}\,\mathrm{sign}\left(\xi\right)\mathrm{Im}\left(\mathbf{H}\right)\right\}\mathbf{\Sigma}^{(-)}\left(\xi\right),\quad \xi\in\mathrm{R}\,, \tag{26}$$

$$\left\langle\tilde{\mathbf{U}}\right\rangle\left(\xi\right)=-\frac{1}{2|\xi|}\left\{\mathrm{Re}\left(\mathbf{L}\right)-\mathrm{i}\,\mathrm{sign}\left(\xi\right)\mathrm{Im}\left(\mathbf{L}\right)\right\}\mathbf{\Sigma}^{(-)}\left(\xi\right),\quad \xi\in\mathrm{R}\,. \tag{27}$$

It is remarked that the symmetric and skew-symmetric weight function matrices derived for the general anisotropic MEE bimaterials are formally similar to those for isotropic and anisotropic elastic bimaterials (Piccolroaz et al., 2009; Piccolroaz et al., 2007; Morini et al., 2012; Morini et al., 2013).

3. The Betti formula for MEE materials

Applying general asymmetric load to the crack faces, the generalized traction acting on the material interface can be written as

$$\mathbf{t}_0\left(x_1,0^+\right)=\mathbf{q}^+\left(x_1\right)+\mathbf{t}\left(x_1\right),\quad \mathbf{t}_0\left(x_1,0^-\right)=\mathbf{q}^-\left(x_1\right)+\mathbf{t}\left(x_1\right). \tag{28}$$

According to Willis and Movchan (1995), Morini et al. (2013) and Piccolroaz and Mishuris (2013), the coefficients in the asymptotic representations of the generalized stress field near the crack tip can be obtained by the application of the Betti formula to the MEE field and to weight functions. In the present study, the corresponding relations for the upper and lower MEE half-planes are given as (Morini et al., 2012)

$$\int_{-\infty}^{+\infty}\left\{\mathbf{U}^{\mathrm{T}}\left(x_1'-x_1,0^+\right)\boldsymbol{\Omega}\mathbf{t}_0\left(x_1,0^+\right)-\boldsymbol{\Sigma}^{\mathrm{T}}\left(x_1'-x_1,0^+\right)\boldsymbol{\Omega}\mathbf{u}\left(x_1,0^+\right)\right\}\mathrm{d}x_1=0,\quad x_3=0^+, \tag{29}$$

$$\int_{-\infty}^{+\infty}\left\{\mathbf{U}^{\mathrm{T}}\left(x_1'-x_1,0^-\right)\boldsymbol{\Omega}\mathbf{t}_0\left(x_1,0^-\right)-\boldsymbol{\Sigma}^{\mathrm{T}}\left(x_1'-x_1,0^-\right)\boldsymbol{\Omega}\mathbf{u}\left(x_1,0^-\right)\right\}\mathrm{d}x_1=0,\quad x_3=0^-, \tag{30}$$

where x_1' denotes a shift of the weight functions within the $\left(x_1,x_2\right)$ plane and $\boldsymbol{\Omega}$ is the rotation matrix (Piccolroaz et al., 2007)

$$\boldsymbol{\Omega}=\begin{pmatrix}-1&0&0&0&0\\0&-1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}. \tag{31}$$

Subtracting Eq. (30) from Eq. (29) and using Eq. (28), we have

$$
\begin{aligned}
&\int_{-\infty}^{+\infty}\Big\{\mathbf{U}^{\mathrm{T}}\left(x_1'-x_1,0^+\right)\boldsymbol{\Omega}\mathbf{q}^+\left(x_1\right)-\mathbf{U}^{\mathrm{T}}\left(x_1'-x_1,0^-\right)\boldsymbol{\Omega}\mathbf{q}^-\left(x_1\right)+\mathbf{U}^{\mathrm{T}}\left(x_1'-x_1,0^+\right)\boldsymbol{\Omega}\mathbf{t}\left(x_1\right)\\
&\quad-\mathbf{U}^{\mathrm{T}}\left(x_1'-x_1,0^-\right)\boldsymbol{\Omega}\mathbf{t}\left(x_1\right)-\boldsymbol{\Sigma}^{\mathrm{T}}\left(x_1'-x_1,0^+\right)\boldsymbol{\Omega}\mathbf{u}\left(x_1,0^+\right)-\boldsymbol{\Sigma}^{\mathrm{T}}\left(x_1'-x_1,0^-\right)\boldsymbol{\Omega}\mathbf{u}\left(x_1,0^-\right)\Big\}\mathrm{d}x_1=0,?\,x_3= \quad .
\end{aligned}
\tag{32}
$$

Using Eqs. (17) and (18) as well as the boundary conditions in Eqs. (1), (2), (19) and (20), one deduces (Morini et al., 2013)

$$
\begin{aligned}
&\int_{-\infty}^{+\infty}\Big\{[\mathbf{U}]^{\mathrm{T}}\left(x_1'-x_1\right)\boldsymbol{\Omega}\mathbf{t}^{(+)}\left(x_1\right)-\boldsymbol{\Sigma}^{\mathrm{T}}\left(x_1'-x_1\right)\boldsymbol{\Omega}[\mathbf{u}]^{(-)}\left(x_1\right)\Big\}\mathrm{d}x_1\\
&=-\int_{-\infty}^{+\infty}\Big\{[\mathbf{U}]^{\mathrm{T}}\left(x_1'-x_1\right)\boldsymbol{\Omega}\langle\mathbf{q}\rangle\left(x_1\right)+\langle\mathbf{U}\rangle^{\mathrm{T}}\left(x_1'-x_1\right)\boldsymbol{\Omega}[\mathbf{q}]\left(x_1\right)\Big\}\mathrm{d}x_1,?\,x_3= \quad ,
\end{aligned}
\tag{33}
$$

where $[\mathbf{u}]^{(-)}(x_1)$ is the generalized displacement jump across the interface for $x_1<0$.

Using the convolution with respect to x_1, the aforementioned integral identity, i.e. Eq. (33), can be equivalently rewritten as (Arfken and Weber, 2005)

$$
\left(\boldsymbol{\Omega}[\mathbf{U}]\right)^{\mathrm{T}}*\mathbf{t}^{(+)}-\left(\boldsymbol{\Omega}\boldsymbol{\Sigma}\right)^{\mathrm{T}}*[\mathbf{u}]^{(-)}=-\left(\boldsymbol{\Omega}[\mathbf{U}]\right)^{\mathrm{T}}*\langle\mathbf{q}\rangle-\left(\boldsymbol{\Omega}\langle\mathbf{U}\rangle\right)^{\mathrm{T}}*[\mathbf{q}]\ , \tag{34}
$$

where the symbol "$*$" stands for the convolution. Eq. (34) presents the relationships between the weight functions, the applied generalized load, generalized traction at the interface and the displacement jump across the crack faces. It is useful for the determination of the generalized stress intensity factors.

4. Integral identities

Herein we will derive the integral formulation of a semi-infinite interfacial crack problem based on the results presented in the previous sections. We consider the 2-D problem of monoclinic MEE bimaterials with the special material constants in Eq. (A.1). The symmetry plane of this particular class of anisotropic MEE materials coincides with $x_2=0$. In this situation, $Y_{2i}=Y_{i2}=0\left(i=1,3,4,5\right)$ holds true, where Y_{ij} is the element of bimaterial matrix $\mathbf{Y}$, and the in-plane and out-of-plane deformations are uncoupled (Ting, 1995; Li and Kardomateas, 2007), which will be investigated seperately in the following sections.

4.1 Out-of-plane deformation: Mode III problem

As shown in Appendix A, for out-of-plane deformation, only mechanical stresses σ_{23} and σ_{12} are involved in the constitutive equations. In this case, the traction and the displacement derivatives for both upper and lower half-planes become (Suo, 1990)

$$t_2\left(x_1, x_3\right) = \sigma_{23}\left(x_1, x_3\right) = B_{22} g_2\left(z_2\right) + \bar{B}_{22}\bar{g}_2\left(\bar{z}_2\right), \tag{35}$$

$$u_{2,1}\left(x_1, x_3\right) = A_{22} g_2\left(z_2\right) + \bar{A}_{22}\bar{g}_2\left(\bar{z}_2\right), \tag{36}$$

where $z_2 = x_1 + \mu_2 x_3$.

According to Eqs. (26) and (27), the Fourier transform of symmetric and skew-symmetric weight functions for an out-of-plane deformation between two dissimilar monoclinic MEE materials are

$$\left[\hat{U}_2\right]^{(+)}(\xi) = -\frac{H_{22}}{|\xi|}\Sigma_2^{(-)}(\xi), \quad \left\langle\hat{U}_2\right\rangle(\xi) = -\frac{L_{22}}{2|\xi|}\Sigma_2^{(-)}(\xi) = \frac{L_{22}}{2H_{22}}\left[\hat{U}_2\right]^{(+)}(\xi), \tag{37}$$

and the corresponding Betti formula reduces to

$$\left[U_2\right] * t_2^{(+)} - \Sigma_2 * \left[u_2\right]^{(-)} = -\left[U_2\right] * \left\langle q_2\right\rangle - \left\langle U_2\right\rangle * \left[q_2\right]. \tag{38}$$

Applying the Fourier transform with respect to x_1 to the above identity, we obtain

$$\left[\hat{U}_2\right]^{(+)}\hat{t}_2^{(+)} - \hat{\Sigma}_2^{(-)}\left[\hat{u}_2\right]^{(-)} = -\left[U_2\right]^{(+)}\left\langle q_2\right\rangle - \left\langle\hat{U}_2\right\rangle\left[\hat{q}_2\right]. \tag{39}$$

Multiplying both sides of Eq. (39) by $\left(\left[\hat{U}_2\right]^{(+)}\right)^{-1}$ leads to

$$\hat{t}_2^{(+)} - N\left[\hat{u}_2\right]^{(-)} = -\left\langle q_2\right\rangle - M\left[\hat{q}_2\right], \tag{40}$$

where

$$M=\left(\left[\hat{U}_2\right]^{(+)}\right)^{-1}\left\langle U_2\right\rangle=\frac{L_{22}}{2H_{22}},\quad N=\left(\left[\hat{U}_2\right]^{(+)}\right)^{-1}\hat{\Sigma}_2^{(-)}=-\frac{|\xi|}{H_{22}}. \tag{41}$$

If further applying the inverse Fourier transform to Eq. (40), two distinct relationships corresponding to the two cases, $x_1<0$ and $x_1>0$, are derived

$$\left\langle q_2\right\rangle\left(x_1\right)+\mathcal{F}_{x_1<0}^{-1}\left(M\left[\hat{q}_2\right]\right)=\mathcal{F}_{x_1<0}^{-1}\left(N\left[u_2\right]^{(-)}\right),\quad x_1<0, \tag{42}$$

$$t_2^{(+)}\left(x_1\right)=\mathcal{F}_{x_1>0}^{-1}\left(N\left[\hat{u}_2\right]^{(-)}\right),\quad x_1>0. \tag{43}$$

It is worth mentioning that the term $\hat{t}_2^{(+)}$ cancels from Eq. (42) because $t_2\left(x_1\right)=0$ for $x_1<0$, while $\left[\hat{q}_2\right]$ and $\left\langle\hat{q}_2\right\rangle$ cancel from Eq. (43) because $\left[q_2\right]$ and $\left\langle q_2\right\rangle$ vanish for $x_1>0$. The inverse Fourier transform of the function $|\xi|\left[\hat{u}_2\right]^{(-)}$ can be expressed as (Piccolroaz and Mishuris, 2013)

$$\mathcal{F}_{x_1}^{-1}\left(|\xi|\left[\hat{u}_2\right]^{(-)}\right)=\frac{1}{\pi x_1}*\frac{\partial\left[u_2\right]^{(-)}}{\partial x_1}=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{1}{x_1-\eta}\frac{\partial\left[u_2\right]^{(-)}}{\partial\eta}\mathrm{d}\eta. \tag{44}$$

Then we can define the singular operator S and the orthogonal projectors $\mathcal{P}_{\pm}\left(\mathcal{P}_{+}=\mathcal{P}_{-}=\mathcal{I}\right)$ acting on the real axis (Morini et al., 2013)

$$\varpi=S\chi=\frac{1}{\pi x_1}*\chi\left(x_1\right)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\chi(\eta)}{x_1-\eta}\mathrm{d}\eta, \tag{45}$$

$$\mathcal{P}_{\pm}\chi=\begin{cases}\chi\left(x_1\right), & \pm x_1\geq 0,\\ 0, & \text{otherwise.}\end{cases} \tag{46}$$

The operator S is a singular operator of Cauchy type, and it transforms any function χ satisfying the Hölder condition into a new function $S\chi$ which also satisfies this condition (Mushkelishvili, 1946). The properties of the operator S in several functional planes have been described by Prössdorf (1974) in detail.

The integral identities (42) and (43) for a Mode III interfacial crack between two dissimilar monoclinic MEE materials can be further expressed as

$$\langle q_2 \rangle (x_1) + \frac{L_{22}}{2H_{22}} [q_2](x_1) = -\frac{1}{H_{22}} S^{(s)} \frac{\partial [u_2]^{(-)}}{\partial x_1}, \quad x_1 < 0, \tag{47}$$

$$t_2^{(+)}(x_1) = -\frac{1}{H_{22}} S^{(c)} \frac{\partial [u_2]^{(-)}}{\partial x_1}, \quad x_1 > 0, \tag{48}$$

where $S^{(s)} = \mathcal{P}_- S \mathcal{P}_-$ is a singular integral operator, and $S^{(c)} = \mathcal{P}_+ S \mathcal{P}_-$ is a compact integral operator (Gakhov and Cherski, 1978; Krein, 1958; Gohberg and Krein, 1958). These two operators have a similar form, but they are different in essence, since $S^{(s)}$: $\Upsilon(\mathrm{R}_-) \to \Upsilon(\mathrm{R}_-)$, while $S^{(c)}$: $\Upsilon(\mathrm{R}_-) \to \Upsilon(\mathrm{R}_+)$, where $\Upsilon(\mathrm{R}_\pm)$ is some functional plane of functions defined on $\mathrm{R}_\pm$ (Morini et al., 2013).

For better understanding of this point, Eqs. (47) and (48) are rewritten in the extended form

$$\langle q_2 \rangle (x_1) + \frac{L_{22}}{2H_{22}} [q_2](x_1) = -\frac{1}{\pi H_{22}} \int_{-\infty}^{0} \frac{1}{x_1 - \eta} \frac{\partial [u_2]^{(-)}}{\partial \eta} \mathrm{d}\eta, \quad x_1 < 0, \tag{49}$$

$$t_2^{(+)}(x_1) = -\frac{1}{\pi H_{22}} \int_{-\infty}^{0} \frac{1}{x_1 - \eta} \frac{\partial [u_2]^{(-)}}{\partial \eta} \mathrm{d}\eta, \quad x_1 > 0. \tag{50}$$

The integral in Eq. (49) is a Cauchy-type singular integral with a moving singularity, whereas the integral in Eq. (50) has a fixed point singularity (Duduchava, 1976; Duduchava, 1979).

For a homogeneous monoclinic MEE material, the integral identities (47) and (48) will be simplified, since $L_{22} = 0$ holds true, which results in no influence on the generalized skew-symmetric load $\langle q_2 \rangle$. In summary, the integral identities for Mode III interfacial cracks in monoclinic MEE bimaterials are presented as Eqs. (47) and (48). Eq. (47) shows an invertible singular integral relation among the applied mechanical loads $\langle q_2 \rangle$ and $[q_2]$, and the corresponding displacement jump across the crack faces $\left[u_2\right]^{(-)}$, whereas Eq. (48) is an additional relation which makes the definition of the behavior of $\left[u_2\right]^{(-)}$ possible. Considering that the operator $S^{(c)}$ is compact and not invertible, the determination of $\left[u_2\right]^{(-)}$ by inversion of Eq. (47) is necessary to evaluate the traction ahead of the crack tip. More details on the theory of the singular integral equation can be found in Mushkelishvili (1946) and for convenience the inversion of the singular operator $S^{(s)}$ in some specific cases is presented in Appendix B (Piccolroaz and Mishuris, 2013).

4.2 In-plane deformation: Mode I and II problems

Since in-plane deformations are independent of out-of-plane deformations, only the physical quantities and the elements of material matrices corresponding to the in-plane case will be retained; those related to the out-of-plane case will not be involved in this section. Herein new variables will be introduced to distinguish them from those in Section 2; for example, $\tilde{\mathbf{u}} = \left\{u_1,\ u_3,\ \phi,\ \varphi\right\}^{\mathrm{T}}$ and $\tilde{\mathbf{t}} = \left\{\sigma_{31},\ \sigma_{33},\ D_3,\ B_3\right\}^{\mathrm{T}}$ where the tildes denote the corresponding generalized displacement and traction vectors for the in-plane problem. Taking account of the fact that, for plane strain problems in monoclinic MEE bimaterials, Mode I and Mode II are coupled, four linearly independent singular solutions $\mathbf{U}^i = \left\{U_1^i, U_3^i, U_4^i, U_5^i\right\}^{\mathrm{T}}$ and tractions $\mathbf{\Sigma}^i = \left\{\Sigma_1^i, \Sigma_3^i, \Sigma_4^i, \Sigma_5^i\right\}^{\mathrm{T}}$ where $i = 1, 3, 4, 5$ are introduced to define a complete basis of the singular solutions plane (Piccolroaz et al., 2009). Therefore, in this case symmetric and skew-symmetric weight functions $\left[\tilde{\mathbf{U}}\right]$ and $\left\langle\tilde{\mathbf{U}}\right\rangle$, and the associated traction $\tilde{\mathbf{\Sigma}}$ are represented by 4×4 tensors which have the following structures (Morini et al., 2013)

$$\tilde{\mathbf{U}} = \begin{pmatrix} U_1^1 & U_1^3 & U_1^4 & U_1^5 \\ U_3^1 & U_3^3 & U_3^4 & U_3^5 \\ U_4^1 & U_4^3 & U_4^4 & U_4^5 \\ U_5^1 & U_5^3 & U_5^4 & U_5^5 \end{pmatrix},\ \tilde{\mathbf{\Sigma}} = \begin{pmatrix} \Sigma_1^1 & \Sigma_1^3 & \Sigma_1^4 & \Sigma_1^5 \\ \Sigma_3^1 & \Sigma_3^3 & \Sigma_3^4 & \Sigma_3^5 \\ \Sigma_4^1 & \Sigma_4^3 & \Sigma_4^4 & \Sigma_4^5 \\ \Sigma_5^1 & \Sigma_5^3 & \Sigma_5^4 & \Sigma_5^5 \end{pmatrix}. \tag{51}$$

Correspondingly, the rotation matrix degenerates to

$$\tilde{\mathbf{\Omega}} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{52}$$

In this situation, the Fourier transforms of the symmetric and skew-symmetric weight functions are expressed as

$$\left[\hat{\tilde{\mathbf{U}}}\right]^{(+)}(\xi) = -\frac{1}{|\xi|}\left\{\mathrm{Re}\left(\tilde{\mathbf{H}}\right) - \mathrm{i}\,\mathrm{sign}(\xi)\,\mathrm{Im}\left(\tilde{\mathbf{H}}\right)\right\}\tilde{\mathbf{\Sigma}}^{(-)}(\xi), \quad \xi \in \mathrm{R}\,, \tag{53}$$

$$\left\langle\hat{\tilde{\mathbf{U}}}\right\rangle(\xi) = -\frac{1}{2|\xi|}\left\{\mathrm{Re}\left(\tilde{\mathbf{L}}\right) - \mathrm{i}\,\mathrm{sign}(\xi)\,\mathrm{Im}\left(\tilde{\mathbf{L}}\right)\right\}\tilde{\mathbf{\Sigma}}^{(-)}(\xi), \quad \xi \in \mathrm{R}\,, \tag{54}$$

where $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{L}}$ are 4×4 matrices reduced from $\mathbf{H}$ and $\mathbf{L}$, respectively, by deleting their second rows and columns.

Applying the Fourier transform to Eq. (34), we have

$$\left(\left[\hat{\tilde{\mathbf{U}}}\right]^{(+)}\right)^{\mathrm{T}}\tilde{\mathbf{\Omega}}\hat{\tilde{\mathbf{t}}}^{(+)} - \left(\tilde{\mathbf{\Sigma}}^{(-)}\right)^{\mathrm{T}}\tilde{\mathbf{\Omega}}\left[\hat{\tilde{\mathbf{u}}}\right]^{(-)} = -\left(\left[\hat{\tilde{\mathbf{U}}}\right]^{(+)}\right)^{\mathrm{T}}\tilde{\mathbf{\Omega}}\left\langle\hat{\tilde{\mathbf{q}}}\right\rangle - \left\langle\tilde{\mathbf{U}}\right\rangle^{\mathrm{T}}\tilde{\mathbf{\Omega}}\left[\tilde{\mathbf{q}}\right], \quad \xi \in \mathrm{R}\,. \tag{55}$$

Multiplying both sides by $\tilde{\mathbf{\Omega}}^{-1}\left(\left[\hat{\tilde{\mathbf{U}}}\right]^{(+)}\right)^{-\mathrm{T}}$, the following identity is obtained

$$\hat{\tilde{\mathbf{t}}}^{(+)} - \mathbf{N}\left[\hat{\tilde{\mathbf{u}}}\right]^{(-)} = -\left\langle\hat{\tilde{\mathbf{q}}}\right\rangle - \mathbf{M}\left[\tilde{\mathbf{q}}\right], \tag{56}$$

where

$$\mathbf{M} = \tilde{\mathbf{\Omega}}^{-1}\left(\left[\hat{\tilde{\mathbf{U}}}\right]^{(+)}\right)^{-\mathrm{T}}\left\langle\tilde{\mathbf{U}}\right\rangle^{\mathrm{T}}\tilde{\mathbf{\Omega}}, \quad \mathbf{N} = \tilde{\mathbf{\Omega}}^{-1}\left(\left[\hat{\tilde{\mathbf{U}}}\right]^{(+)}\right)^{-\mathrm{T}}\left(\tilde{\mathbf{\Sigma}}^{(-)}\right)^{\mathrm{T}}\tilde{\mathbf{\Omega}}\,, \tag{57}$$

which can be further written as

$$\mathbf{M} = \mathbf{\Phi} + \mathrm{i}\,\mathrm{sign}(\xi)\mathbf{\Psi}, \quad \mathbf{N} = |\xi|\left\{\mathbf{\Theta} + \mathrm{i}\,\mathrm{sign}(\xi)\mathbf{\Xi}\right\}. \tag{58}$$

The numerical calculation shows that $\mathbf{\Phi}$, $\mathbf{\Psi}$, $\mathbf{\Theta}$ and $\mathbf{\Xi}$ have the following structures

$$\mathbf{\Phi}=\begin{pmatrix}\Phi_{11} & \Phi_{13} & \Phi_{14} & \Phi_{15}\\ \Phi_{31} & \Phi_{33} & \Phi_{34} & \Phi_{35}\\ \Phi_{41} & \Phi_{43} & \Phi_{44} & \Phi_{45}\\ \Phi_{51} & \Phi_{53} & \Phi_{54} & \Phi_{55}\end{pmatrix},\ \mathbf{\Psi}=\begin{pmatrix}\Psi_{11} & \Psi_{13} & \Psi_{14} & \Psi_{15}\\ \Psi_{31} & \Psi_{33} & \Psi_{34} & \Psi_{35}\\ \Psi_{41} & \Psi_{43} & \Psi_{44} & \Psi_{45}\\ \Psi_{51} & \Psi_{53} & \Psi_{54} & \Psi_{55}\end{pmatrix}, \tag{59}$$

$$\mathbf{\Theta}=\begin{pmatrix}\Theta_{11} & \Theta_{13} & \Theta_{14} & \Theta_{15}\\ \Theta_{13} & \Theta_{33} & \Theta_{34} & \Theta_{35}\\ \Theta_{14} & \Theta_{34} & \Theta_{44} & \Theta_{45}\\ \Theta_{15} & \Theta_{35} & \Theta_{45} & \Theta_{55}\end{pmatrix},\ \mathbf{\Xi}=\begin{pmatrix}0 & \Xi_{13} & \Xi_{14} & \Xi_{15}\\ -\Xi_{13} & 0 & \Xi_{34} & \Xi_{35}\\ -\Xi_{14} & -\Xi_{34} & 0 & \Xi_{45}\\ -\Xi_{15} & -\Xi_{35} & -\Xi_{45} & 0\end{pmatrix}, \tag{60}$$

where all the elements of the matrices above are real; $\mathbf{\Theta}$ and $\mathbf{\Xi}$ are, respectively, symmetric and asymmetric matrices. For the transversely isotropic MEE bimaterials, $\mathbf{\Phi}$, $\mathbf{\Psi}$, $\mathbf{\Theta}$ and $\mathbf{\Xi}$ become

$$\mathbf{\Phi}=\begin{pmatrix}\Phi_{11} & 0 & 0 & 0\\ 0 & \Phi_{33} & \Phi_{34} & \Phi_{35}\\ 0 & \Phi_{43} & \Phi_{44} & \Phi_{45}\\ 0 & \Phi_{53} & \Phi_{54} & \Phi_{55}\end{pmatrix},\ \mathbf{\Psi}=\begin{pmatrix}0 & \Psi_{13} & \Psi_{14} & \Psi_{15}\\ \Psi_{31} & 0 & 0 & 0\\ \Psi_{41} & 0 & 0 & 0\\ \Psi_{51} & 0 & 0 & 0\end{pmatrix}, \tag{61}$$

$$\mathbf{\Theta}=\begin{pmatrix}\Theta_{11} & 0 & 0 & 0\\ 0 & \Theta_{33} & \Theta_{34} & \Theta_{35}\\ 0 & \Theta_{34} & \Theta_{44} & \Theta_{45}\\ 0 & \Theta_{35} & \Theta_{45} & \Theta_{55}\end{pmatrix},\ \mathbf{\Xi}=\begin{pmatrix}0 & \Xi_{13} & \Xi_{14} & \Xi_{15}\\ -\Xi_{13} & 0 & 0 & 0\\ -\Xi_{14} & 0 & 0 & 0\\ -\Xi_{15} & 0 & 0 & 0\end{pmatrix}. \tag{62}$$

Furthermore, for a homogeneous monoclinic and transversely isotropic MEE material, $\mathbf{\Phi}=\mathbf{0}$ and $\mathbf{\Xi}=\mathbf{0}$ always hold true, and the corresponding material constant matrices $\mathbf{\Psi}$ and $\mathbf{\Theta}$ take the same form as those in Eqs. (59) and (60), and Eqs. (61) and (62), respectively.

Applying the inverse Fourier transform to Eq. (56), for two cases $x_1<0$ and $x_1>0$, one gets

$$\langle\tilde{\mathbf{q}}\rangle(x_1)+\mathcal{F}_{x_1<0}^{-1}\left(\mathbf{M}\left[\hat{\tilde{\mathbf{q}}}\right]\right)=\mathcal{F}_{x_1<0}^{-1}\left(\mathbf{N}\left[\hat{\tilde{\mathbf{u}}}\right]^{(-)}\right),\quad x_1<0, \tag{63}$$

$$\tilde{\mathbf{t}}^{(+)}(x_1)+\mathcal{F}_{x_1>0}^{-1}\left(\mathbf{M}\left[\hat{\tilde{\mathbf{q}}}\right]\right)=\mathcal{F}_{x_1>0}^{-1}\left(\mathbf{N}\left[\hat{\tilde{\mathbf{u}}}\right]^{(-)}\right),\quad x_1>0. \tag{64}$$

Similar to the case of out-of-plane deformation illustrated in the previous section, the term $\hat{\tilde{\mathbf{t}}}^{(+)}$ in Eq. (56) cancels from Eq. (63) because $\tilde{\mathbf{t}}(x_1)=0$ for the present magnetoelectrically impermeable crack as $x_1<0$, while $\langle\hat{\tilde{\mathbf{q}}}\rangle$ cancels from Eq. (64) because $\langle\tilde{\mathbf{q}}\rangle(x_1)$ vanish as $x_1>0$.

The integral identities for plane strain problems in monoclinic MEE bimaterials, i.e. Eqs. (63) and (64), can be further rewritten as (Morini et al., 2013)

$$\langle\tilde{\mathbf{q}}\rangle(x_1)+\mathbf{M}^{(s)}[\tilde{\mathbf{q}}]=\mathbf{N}^{(s)}\frac{\partial[\tilde{\mathbf{u}}]^{(-)}}{\partial x_1},\quad x_1<0, \tag{65}$$

$$\tilde{\mathbf{t}}^{(+)}(x_1)+\mathbf{M}^{(c)}[\tilde{\mathbf{q}}]=\mathbf{N}^{(c)}\frac{\partial[\tilde{\mathbf{u}}]^{(-)}}{\partial x_1},\quad x_1>0, \tag{66}$$

where matrix operators $\mathbf{M}^{(s)}$ and $\mathbf{N}^{(s)}$: $\Upsilon(\mathrm{R}_-)\to\Upsilon(\mathrm{R}_-)$, as well as $\mathbf{M}^{(c)}$ and $\mathbf{N}^{(c)}$: $\Upsilon(\mathrm{R}_-)\to\Upsilon(\mathrm{R}_+)$, are defined as

$$\mathbf{M}^{(s)}=\mathbf{\Phi}+\mathbf{\Psi}S^{(s)},\ \mathbf{N}^{(s)}=\mathbf{\Theta}S^{(s)}-\mathbf{\Xi},\ \mathbf{M}^{(c)}=\mathbf{\Psi}S^{(c)},\ \mathbf{N}^{(c)}=\mathbf{\Theta}S^{(c)}. \tag{67}$$

Eqs. (65) and (66), as well as the definition of operators in Eq. (67), formulate the system of integral identities for Mode I and Mode II problems in MEE monoclinic bimaterials. Moreover, the integral identities in relation to monoclinic piezoelectric bimaterials can be readily obtained from Eqs. (65) and (66) by neglecting the piezomagnetic phase. Additionally, if we neglect both the piezoelectric and piezomagnetic phases, the identities for the monoclinic elastic bimaterials can also be acquired, which formally agree with the results given by Morini et al. (2013).

As previously mentioned, for a homogeneous transversely isotropic MEE material, $\mathbf{\Phi}=\mathbf{0}$ and $\mathbf{\Xi}=\mathbf{0}$, therefore, Eq. (65) is reduced to the following equation

$$\langle\tilde{\mathbf{q}}\rangle(x_1)+\mathbf{\Psi}S^{(s)}[\tilde{\mathbf{q}}]=\mathbf{\Theta}S^{(s)}\frac{\partial[\tilde{\mathbf{u}}]^{(-)}}{\partial x_1},\quad x_1<0, \tag{68}$$

where $\mathbf{\Psi}$ and $\mathbf{\Theta}$ refer to the forms in Eqs. (61) and (62), respectively. It is remarked that Eq. (65) is composed of four coupled singular integral equations, and they are still coupled for a homogeneous transversely isotropic MEE material according to Eq. (68). This is different from the corresponding results by Morini et al. (2013) in relation to a homogeneous elastic material, which may be owing to the special properties of MEE materials. The solution of Eq. (68) requires the inversion of the singular operator $S^{(s)}$, which has been performed and discussed by Piccolroaz and Mishuris (2013).

5. Illustrative examples: point loads applied on the crack faces

In this section, we will present illustrative examples of the application of the integral identities to the analysis of interfacial cracks in anisotropic MEE bimaterials. Out-of-plane deformation (Fig. 2) and the plane strain problem (Fig. 3) of interfacial cracks in monoclinic MEE bimaterials under the magnetoelectromechanical point load applied on the crack faces are investigated via the proposed integral formulation. The expressions for generalized displacement jump across crack faces and generalized traction ahead of the crack tip subjected to symmetric and skew-symmetric loads are derived.

5.1 Mode III problem under symmetric loads

As shown in Appendix A, for out-of-plane deformation, the traction is only dependent on mechanical displacements and is independent of the electric and magnetic potentials. Therefore, herein we consider an interfacial crack under two symmetric point loads which are applied on the crack faces and oriented in the x_2-axis, as shown in Fig. 2a

$$\langle q_2 \rangle (x_1) = -\tau \delta (x_1 + a), \quad [q_2](x_1) = 0, \tag{69}$$

where δ is the Dirac delta function and a denotes the distance between the crack tip and the load position.

Using Eq. (47) and solving the inversion of the operator $S^{(s)}$, we arrive at (Muskhelishvili, 1946;

Rice, 1968; Piccolroaz and Mishuris, 2013)

$$\frac{\partial[u_2]^{(-)}}{\partial x_1} = -\frac{H_{22}\tau}{\pi}\int_{-\infty}^{0}\sqrt{\frac{\eta}{x_1}}\frac{\delta(\eta+a)}{x_1-\eta}\mathrm{d}\eta = -\frac{H_{22}\tau}{\pi}\sqrt{-\frac{a}{x_1}}\frac{1}{x_1+a}. \tag{70}$$

Considering that the displacement jump vanishes at the crack tip and at infinity, the integration of Eq. (70) can be expressed as

$$[u_2](x_1) = \frac{2H_{22}\tau}{\pi}\tanh^{-1}\sqrt{-\frac{x_1}{a}}, \quad -a < x_1 < 0, \tag{71}$$

$$[u_2](x_1) = \frac{2H_{22}\tau}{\pi}\tanh^{-1}\sqrt{-\frac{a}{x_1}}, \quad x_1 < -a. \tag{72}$$

Substituting Eq. (70) into Eq. (50), the explicit expression for the traction ahead of the crack tip is determined as

$$t_2^{(+)}(x_1) = -\frac{1}{\pi H_{22}}\int_{-\infty}^{0}\frac{1}{x_1-\eta}\frac{\partial[u_2]^{(-)}}{\partial\eta}\mathrm{d}\eta = \frac{\tau}{\pi}\sqrt{\frac{a}{x_1}}\frac{1}{x_1+a}. \tag{73}$$

Eq. (73) is very convenient for the evaluation of the stress intensity factor, which is defined and obtained as

$$K_{III} = \lim_{x_1\to 0}\sqrt{2\pi x_1}t_2^{(+)}(x_1) = \sqrt{\frac{2}{\pi a}}\tau. \tag{74}$$

It is worth mentioning that the expressions for the traction ahead of the crack tip and stress intensity factor, i.e. Eqs. (73) and (74), agree with the results by Piccolroaz and Mishuris (2013) and Morini et al. (2013) for isotropic and anisotropic elastic bimaterials, respectively. Additionally, the expressions for displacement jump, i.e. Eqs. (71) and (72), are also formally similar to the results in the aforementioned two papers, except that H_{22} is determined from the material constants of MEE solids. This is easy to understand since the constitutive equations of monoclinic elastic and MEE materials are formally identical for out-of-plane deformation (See Eq. (A.5)).

5.2 Mode III problem under skew-symmetric point loads

As shown in Fig. 2b, we now consider an interfacial crack subjected to two skew-symmetrical point loads on the crack faces and oriented in the x_2-axis

$$\langle q_2 \rangle (x_1) = 0, \quad [q_2](x_1) = -2\tau\delta(x_1 + a), \tag{75}$$

and a denotes the distance between the crack tip and the load position.

Applying the inverse operator $\left(S^{(s)}\right)^{-1}$ to Eq. (47), we obtain

$$\frac{\partial [u_2]^{(-)}}{\partial x_1} = -\frac{L_{22}}{\pi} \tau \int_{-\infty}^{0} \sqrt{\frac{\eta}{x_1}} \frac{\delta(\eta + a)}{x_1 - \eta} \mathrm{d}\eta = -\frac{L_{22}\tau}{\pi} \sqrt{-\frac{a}{x_1}} \frac{1}{x_1 + a}. \tag{76}$$

By integrating Eq. (76), the displacement jump is derived as

$$[u_2](x_1) = \frac{2L_{22}\tau}{\pi} \tanh^{-1} \sqrt{-\frac{x_1}{a}}, \quad -a < x_1 < 0, \tag{77}$$

$$[u_2](x_1) = \frac{2L_{22}\tau}{\pi} \tanh^{-1} \sqrt{-\frac{a}{x_1}}, \quad x_1 < -a. \tag{78}$$

Inserting Eq. (76) into Eq. (50) leads to the traction ahead of the crack tip

$$t_2^{(+)}(x_1) = \frac{L_{22}\tau}{H_{22}\pi} \sqrt{\frac{a}{x_1}} \frac{1}{x_1 + a}. \tag{79}$$

Correspondingly, the stress intensity factor can be obtained as

$$K_{III} = \lim_{x_1 \to 0} \sqrt{2\pi x_1} t_2^{(+)}(x_1) = \frac{L_{22}\tau}{H_{22}} \sqrt{\frac{2}{\pi a}}. \tag{80}$$

Analogously, Eqs. (77)-(80) are formally consistent with the results obtained by Piccolroaz and Mishuris (2013) and Morini et al. (2013) for isotropic and anisotropic elastic bimaterials, respectively, and the only difference is that the parameters H_{22} and L_{22} are calculated from the anisotropic MEE bimaterials.

From Subsections 5.1 and 5.2, we can observe that the explicit expressions for the displacement jump, i.e. Eqs. (71), (72), (77) and (78), and for the traction ahead of the tip, i.e. Eqs. (73) and (79), can be readily acquired only by inversion of the operator $S^{(s)}$ and by a simple integration procedure. This example shows that the integral identities obtained in Section 4 are fairly convenient for solving the out-of-plane deformation of interface crack problems in anisotropic MEE bimaterials.

5.3 Mode I and II problems under magnetoelectromechanical symmetric point loads

In this section, we address the crack problem under the plane strain condition. As shown in Fig. 3a, the generalized loads are assumed to be two magnetoelectromechanical symmetrical point loads applied on the faces, which are defined as

$$\langle \mathbf{q} \rangle (x_1) = -\mathbf{F}\delta(x_1 + a), \quad [\mathbf{q}](x_1) = 0, \tag{81}$$

where $\mathbf{F} = \{\tau_0, \sigma_0, D_0, B_0\}^{\mathrm{T}}$ with τ_0 oriented in the x_1-axis and σ_0, D_0 and B_0 in the x_3-axis; a denotes the distance between the crack tip and the load position.

For the 2-D interfacial crack problem in MEE bimaterials, Modes I and II are usually coupled and generalized stresses show oscillating behaviors near the crack tip owing to the oscillating index ε in Eq. (A.16), defined by the generalized Dundurs parameter β in Eq. (A.17) (Li and Kardomateas, 2007). For simplicity, following Morini et al. (2013), we also assume β to be zero, which implies that both the oscillating index ε and oscillation at the crack tip vanish, and that $\mathrm{Im}(\tilde{\mathbf{H}}) = 0$ and $\mathbf{\Xi} = 0$ hold true. In this particular case, Eq. (65) for $x_1 < 0$ become

$$\langle \mathbf{q} \rangle (x_1) + \left(\mathbf{\Phi} + \mathbf{\Psi} S^{(s)}\right)[\mathbf{q}] = \mathbf{\Theta} S^{(s)} \frac{\partial [\mathbf{u}]^{(-)}}{\partial x_1}, \quad x_1 < 0. \tag{82}$$

Applying the inverse operator $\left(S^{(s)}\right)^{-1}$ to Eq. (82) and using some algebraic manipulations, the following equation is obtained (Morini, et al., 2013)

$$\frac{\partial[\mathbf{u}]^{(-)}}{\partial x_1}=\frac{1}{\pi}\mathbf{\Theta}^{-1}\mathbf{F}\sqrt{-\frac{a}{x_1}}\frac{1}{x_1+a}. \tag{83}$$

Then, after integration, for $-a<x_1<0$ we have

$$[\mathbf{u}]^{(-)}(x_1)=-\frac{2}{\pi}\mathbf{\Theta}^{-1}\mathbf{F}\tanh^{-1}\sqrt{-\frac{x_1}{a}},\quad -a<x_1<0, \tag{84}$$

and for $x_1<-a$

$$[\mathbf{u}]^{(-)}(x_1)=-\frac{2}{\pi}\mathbf{\Theta}^{-1}\mathbf{F}\tanh^{-1}\sqrt{-\frac{a}{x_1}},\quad x_1<-a. \tag{85}$$

The generalized traction components ahead of the crack tip can be determined from Eq. (66) (Morini et al., 2013)

$$\mathbf{t}^{(+)}(x_1)=\frac{\mathbf{F}}{\pi}\sqrt{\frac{a}{x_1}}\frac{1}{x_1+a}. \tag{86}$$

The generalized stress intensity factors are then defined and obtained as

$$\mathbf{K}=\{K_{II}\quad K_I\quad K_D\quad K_B\}^{\mathrm{T}}=\lim_{x_1\to 0}\sqrt{2\pi x_1}\mathbf{t}^{(+)}(x_1)=\sqrt{\frac{2}{\pi a}}\mathbf{F}. \tag{87}$$

For the 2-D plane strain problem subjected to the magnetoelectromechanical symmetric point loads, we can observe that the expressions for the generalized traction ahead of the crack tip and stress intensity factors, i.e. Eqs. (86) and (87), are formally analogous to those by Piccolroaz and Mishuris (2013) and Morini et al. (2013) for isotropic and anisotropic elastic bimaterials, respectively. Additionally, even the oscillating parameter β is assumed to be zero, and the generalized displacement jump across crack faces, i.e. Eqs. (84) and (85), are dependent on both mechanical load and magnetoelectric load, which implies that an arbitrary non-zero load will have an effect on all the components of the generalized displacement jump. A similar phenomenon is also observed in Morini et al. (2013).

In the particular case of transversely isotropic MEE bimaterials, the expressions for generalized displacement jump across crack faces have the same form as Eqs. (84) and (85), and the corresponding constant matrix $\mathbf{\Theta}$ has the structure of Eq. (62). Additionally, if we further assume that the length of the contact zone is zero, the oscillating index $\varepsilon = 0$, and the left crack tip approaches $-\infty$, the corresponding field intensity factors obtained by Herrmann et al. (2010) and Feng et al. (2011) have the same form as Eq. (87), which demonstrates that the integral identities proposed in the present work are correct and effective.

Herein some numerical results are presented to reveal the variation of the crack opening displacement and generalized tractions along the material interface for Mode I and II problems under different symmetric loads. A semi-infinite interface crack between two transversely isotropic $CoFe_2O_4$-$BaTiO_3$ composites poled in x_3 direction is considered. Material properties of the MEE materials as volume percentage (or volume fraction V_f) of $BaTiO_3$-$CoFe_2O_4$ are listed in Table 1 (Sih and Song, 2003). After a series of normalized treatment for these material constants (Ma et al., 2015b), the matrices $\mathbf{\Phi}$, $\mathbf{\Psi}$, $\mathbf{\Theta}$ and $\mathbf{\Xi}$ are found as

$$\mathbf{\Phi} = 10^{-2}\begin{pmatrix} -1.2056 & 0 & 0 & 0 \\ 0 & -0.1912 & -0.4200 & 0.0335 \\ 0 & -1.9336 & 20.5944 & -0.0713 \\ 0 & 0.2513 & 0.2592 & -18.7063 \end{pmatrix}, \quad \mathbf{\Psi} = \begin{pmatrix} 0 & -0.1267 & 0.0064 & 0.0002 \\ 0.1046 & 0 & 0 & 0 \\ 1.2169 & 0 & 0 & 0 \\ 0.2045 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{\Theta} = \begin{pmatrix} -0.1535 & 0 & 0 & 0 \\ 0 & -0.1480 & -0.4129 & -0.1375 \\ 0 & -0.4129 & 21.2841 & 0.2523 \\ 0 & -0.1375 & 0.2523 & 70.9149 \end{pmatrix}, \quad \mathbf{\Xi} = 10^{-3}\begin{pmatrix} 0 & 5.8756 & 6.6908 & -1.2321 \\ -5.8756 & 0 & 0 & 0 \\ -6.6908 & 0 & 0 & 0 \\ 1.2321 & 0 & 0 & 0 \end{pmatrix}.$$

In this example, $\tau_0 = 0$ and σ_0 is always applied. The electric and magnetic loads are define by $D_0 = \lambda_D d_0$, $B_0 = \lambda_B b_0$, respectively, where $d_0 = \sigma_0 e_{33}^{(1)} / c_{11}^{(1)}$, $b_0 = \sigma_0 h_{33}^{(1)} / c_{11}^{(1)}$; the superscript "(1)" corresponds to Material 1. Since we have assumed that $\varepsilon = 0$, the displacement jump $[u_1]$ and stress

σ_{13} will not be given herein. Figs. 4 and 5 show the variation of normalized generalized crack opening displacement along crack surface including displacement jump $\pi[u_3]/2\sigma_0(-\Theta_{22}^{-1})$, electric potential jump $\pi[\phi]/2d_0\Theta_{33}^{-1}$ and magnetic potential jump $\pi[\varphi]/2b_0\Theta_{44}^{-1}$, as well as the normalized generalized tractions ahead of the crack tip including stress $a\pi\sigma_{33}/\sigma_0$, electric displacement $a\pi D_3/d_0$ and magnetic induction $a\pi B_3/b_0$ with respect to x_1/a under different symmetric loads, where $\Theta_{ii}^{-1} = \mathbf{\Theta}^{-1}(i,i)$, $i = 2,3,4$, Θ_{22}^{-1} is negative whereas Θ_{33}^{-1} and Θ_{33}^{-1} are positive. Fig. 4 demonstrates that an increase in symmetric electric load leads to an increase in displacement jump whereas the influence of symmetric magnetic load on the displacement jump is small. Additionally, increasing symmetric electric and magnetic loads cause significant variation of electric and magnetic potential jumps, respectively. All these phenomena are consistent with those in Ma et al. (2013) and Ma et al. (2015b). In Fig.4b, it can be seen that when a symmetric electric load is applied, the magnitude of electric potential jump decreases. Therefore, if many pairs of symmetric point electric loads with the same magnitude are uniformly applied on the crack faces, which can be regarded as a distributed load, the electric potential jump will become negative and its magnitude will further increase as the distributed symmetric electric load increases. This fully agrees with the observation in Ma et al. (2015b). Similar conclusion can also be extended to the magnetic potential jump in Fig.4c. Fig.5 shows the variation of the generalized tractions ahead of the crack tip, and as expected, all of them decrease rapidly with the increase of x_1/a.

5.4 Mode I and II problems under magnetoelectromechanical skew-symmetric point loads

As shown in Fig. 3b, two magnetoelectromechanical skew-symmetric point loads are imposed on crack faces, at a distance *a* from the crack tip and

$$\langle \mathbf{q} \rangle (x_1) = 0, \quad [\mathbf{q}](x_1) = -2\mathbf{F}\delta(x_1 + a) \tag{88}$$

where $\mathbf{F}=\{\tau_0,\sigma_0,D_0,B_0\}^{\mathrm{T}}$ with τ_0 oriented in the x_1-axis and the other three components in the x_3-axis.

Similar to the previous subsection, we also set β as zero. Applying the inverse operator $\left(S^{(s)}\right)^{-1}$ to Eq. (65) for $x_1<0$, we obtain

$$\frac{\partial[\mathbf{u}]^{(-)}}{\partial x_1}=2\mathbf{\Theta}^{-1}\left\{\frac{\mathbf{\Phi}}{\pi}\sqrt{-\frac{a}{x_1}}\frac{1}{x_1+a}-\mathbf{\Psi}\delta\left(x_1+a\right)\right\}\mathbf{F}. \tag{89}$$

Then integrating Eq. (89), one gets

$$[\mathbf{u}]^{(-)}\left(x_1\right)=-\frac{4}{\pi}\mathbf{\Theta}^{-1}\mathbf{\Phi}\mathbf{F}\tanh^{-1}\sqrt{-\frac{x_1}{a}},\quad -a<x_1<0, \tag{90}$$

$$[\mathbf{u}]^{(-)}\left(x_1\right)=-2\mathbf{\Theta}^{-1}\left(\frac{2\mathbf{\Phi}}{\pi}\tanh^{-1}\sqrt{-\frac{a}{x_1}}-\mathbf{\Psi}\right)\mathbf{F},\quad x_1<-a. \tag{91}$$

The generalized traction ahead of the crack tip becomes

$$\mathbf{t}^{(+)}\left(x_1\right)=\frac{\mathbf{\Phi}\mathbf{F}}{\pi}\sqrt{\frac{a}{x_1}}\frac{1}{x_1+a}. \tag{92}$$

Eqs. (90) and (91) show that the generalized displacement jump across crack faces is related to all components of the magnetoelectromechanical skew-symmetric loads, which agrees with the results of the case of symmetric loads in Section 5.3. Moreover, Eq. (92) reveals that generalized traction ahead of the crack tip is also dependent on all components of the applied generalized load. A similar phenomenon is observed in Morini et al. (2013) for anisotropic elastic bimaterials.

The corresponding generalized stress intensity factors can then be evaluated as

$$\mathbf{K}=\{K_{II}\quad K_I\quad K_D\quad K_B\}^{\mathrm{T}}=\lim_{x_1\to 0}\sqrt{2\pi x_1}\,\mathbf{t}^{(+)}\left(x_1\right)=\sqrt{\frac{2}{\pi a}}\mathbf{\Phi}\mathbf{F}. \tag{93}$$

In the particular case of transversely isotropic MEE bimaterials under magnetoelectromechanical skew-symmetric loads, the corresponding generalized traction and intensity factors take the same form as Eqs. (92) and (93), and the constant matrix $\mathbf{\Phi}$ has the structure of Eq. (61).

The numerical results corresponding to this subsection under skew-symmetric loads are presented in Figs. 6 and 7. The material combination and load parameters which define the magnitude of applied load are same as those in Subsection 5.3. The skew-symmetric shear load τ_0 is set as zero and D_0, B_0, d_0 and b_0 have the same form as the previous subsection. The results shown in Fig.6 reveal that under skew-symmetric loads, increasing electric and magnetic loads, respectively, lead to a moderate increase and a slight increase in the magnitude of displacement jump. Similar to Figs. 4 and 5, electric and magnetic potential jumps as well as the electric displacement and magnetic induction in Figs.6 and 7 are sensitive to electric and magnetic loads, respectively, for the case of skew-symmetric loads. Fig.7a indicates that compared with skew-symmetric magnetic load, electric load has a larger effect on the stress ahead of the crack tip. Moreover, in Fig.7, the magnitude of generalized tractions also decreases rapidly as x_1/a increases, which agrees with the results in relation to the case of symmetric loads.

In this section, we present illustrative examples regarding the magnetoelectromechanical symmetric and skew-symmetric loads. As previously mentioned, since an arbitrary load combination, which is self-balanced, can be expressed by the superposition of symmetric and skew-symmetric parts, the generalized stress intensity factors can be readily obtained from the superposition of the corresponding stress intensity factors related to symmetric and skew-symmetric loads. The examples in this section show that the generalized displacement jump across crack faces and the generalized traction ahead of the tip can be derived without using Green's functions, which requires challenging calculations. For this reason, the singular integral formulation proposed in this study may provide a very suitable technique for the fracture analysis of 2-D interfacial crack problems in anisotropic MEE bimaterials.

6. Conclusions

In this paper, we first derive the symmetric and skew-symmetric weight functions for a semi-infinite interfacial crack in anisotropic MEE bimaterials under 2-D deformation condition. By extending the application of the Betti formula to the MEE field and to the aforementioned weight functions, the integral identities, which describe the relationship between the applied magnetoelectromechanical load and the generalized displacement jump across crack faces, are further obtained for both out-of-plane and in-plane deformation problems in anisotropic MEE bimaterials. Detailed derivations of the identities have been conducted for monoclinic MEE bimaterials, which are the most general class of anisotropic MEE media allowing decoupling between in-plane and out-of-plane deformations (Li and Kardomateas, 2007). The validality of the present integral formulation is demonstrated by its application to the Mode III problem and the plain strain crack problem subjected to symmetric and skew-symmetric magnetoelectromechanical loads, in which the expressions for the generalized displacement jump across crack faces and generalized traction ahead of the tip are presented and the corresponding numerical results are also provided. It is shown that the derived integral identities provide a very powerful tool for solving interfacial crack problems in the MEE bimaterial, which is a multi-field coupled system, within the framework of linear fracture mechanics. Another advantage of the method proposed in the present work is that it avoids the use of Green's function, which can lead to challenging numerical calculations. Additionally, from a mathematical point of view, the integral identities herein also have their own value since, to the authors' best knowledge, they are not available in the existing literature related to MEE bimaterials.

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Appendix A

The complex variable representation for stress and displacement field in anisotropic elastic materials was firstly proposed by Stroh (1962) and further extended to anisotropic MEE materials by Gao et al. (2003) and Li and Kardomateas (2007). By referring their work, the magnetoelectromechanical field in anisotropic MEE materials subjected to 2-D deformations is reported in this appendix.

For monoclinic MEE materials with a symmetry plane at $x_2 = 0$ in the absence of body force, electric charge and electric current, the following conditions regarding the material constants in contracted notation should be satisfied in order to decouple out-of-plane and in-plane deformations (Li and Kardomateas, 2007)

$$c_{14} = c_{16} = c_{34} = c_{36} = c_{54} = c_{56} = 0,\ e_{14} = e_{16} = e_{34} = e_{36} = 0,\ h_{14} = h_{16} = h_{34} = h_{36} = 0, \tag{A.1}$$

where c_{ij}, e_{ij} and h_{ij} are the elastic, piezoelectric and piezomagnetic constants, respectively. In this case, the explicit form of the constitutive equations for the plane strain problem, i.e. $u_{2,2} = 0$, $\varphi_{,2} = 0$ and $\phi_{,2} = 0$, can be written as

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{33} \\ \sigma_{13} \end{Bmatrix} = \begin{pmatrix} c_{11} & c_{13} & c_{15} \\ c_{13} & c_{33} & c_{35} \\ c_{15} & c_{25} & c_{55} \end{pmatrix} \begin{Bmatrix} u_{1,1} \\ u_{3,3} \\ u_{1,3} + u_{3,1} \end{Bmatrix} + \begin{pmatrix} e_{11} & e_{31} \\ e_{13} & e_{33} \\ e_{15} & e_{35} \end{pmatrix} \begin{Bmatrix} \varphi_{,1} \\ \varphi_{,3} \end{Bmatrix} + \begin{pmatrix} h_{11} & h_{31} \\ h_{13} & h_{33} \\ h_{15} & h_{35} \end{pmatrix} \begin{Bmatrix} \phi_{,1} \\ \phi_{,1} \end{Bmatrix}, \tag{A.2}$$

$$\begin{Bmatrix} D_1 \\ D_3 \end{Bmatrix} = \begin{pmatrix} e_{11} & e_{13} & e_{15} \\ e_{31} & e_{33} & e_{35} \end{pmatrix} \begin{Bmatrix} u_{1,1} \\ u_{3,3} \\ u_{1,3} + u_{3,1} \end{Bmatrix} - \begin{pmatrix} \alpha_{11} & \alpha_{13} \\ \alpha_{13} & \alpha_{33} \end{pmatrix} \begin{Bmatrix} \varphi_{,1} \\ \varphi_{,3} \end{Bmatrix} - \begin{pmatrix} d_{11} & d_{13} \\ d_{13} & d_{33} \end{pmatrix} \begin{Bmatrix} \phi_{,1} \\ \phi_{,1} \end{Bmatrix}, \tag{A.3}$$

$$\begin{Bmatrix} B_1 \\ B_3 \end{Bmatrix} = \begin{pmatrix} h_{11} & h_{13} & h_{15} \\ h_{31} & h_{33} & h_{35} \end{pmatrix} \begin{Bmatrix} u_{1,1} \\ u_{3,3} \\ u_{1,3} + u_{3,1} \end{Bmatrix} - \begin{pmatrix} d_{11} & d_{13} \\ d_{13} & d_{33} \end{pmatrix} \begin{Bmatrix} \varphi_{,1} \\ \varphi_{,3} \end{Bmatrix} - \begin{pmatrix} \mu_{11} & \mu_{13} \\ \mu_{13} & \mu_{33} \end{pmatrix} \begin{Bmatrix} \phi_{,1} \\ \phi_{,1} \end{Bmatrix}, \tag{A.4}$$

$$\begin{Bmatrix} \sigma_{12} \\ \sigma_{23} \end{Bmatrix} = \begin{pmatrix} c_{46} & c_{66} \\ c_{44} & c_{46} \end{pmatrix} \begin{Bmatrix} u_{2,3} + u_{3,2} \\ u_{1,2} + u_{2,1} \end{Bmatrix}, \tag{A.5}$$

where σ_{ij}, D_i and B_i are the components of mechanical stress, electric displacement and magnetic induction, respectively; u_i, φ and ϕ are the mechanical displacement, electric potential and magnetic potential, respectively. d_{ij}, α_{ij} and μ_{ij} are the electromagnetic constants, dielectric permittivities and magnetic permeabilities, respectively. It is remarked that Eqs. (A.2)-(A.4) are related to in-plane deformation whereas Eq. (A.5) is related to out-of-plane deformation. It is interesting that Eq. (A.5) is identical to that in relation to monoclinic elastic materials.

For 2-D problems, the generalized displacement vector $\mathbf{u} = \{u_1, u_2, u_3, \phi, \varphi\}^{\mathrm{T}}$ and generalized stress vector $\mathbf{t} = \{\sigma_{31}, \sigma_{32}, \sigma_{33}, D_3, B_3\}^{\mathrm{T}}$ depend only on x_1 and x_3, and the general solution takes the form (Stroh, 1962; Gao et al., 2003)

$$\mathbf{u}_{,1} = \mathbf{A}\mathbf{g}(z) + \bar{\mathbf{A}}\bar{\mathbf{g}}(\bar{z}), \tag{A.6}$$

$$\mathbf{t} = \mathbf{B}\mathbf{g}(z) + \bar{\mathbf{B}}\bar{\mathbf{g}}(\bar{z}), \tag{A.7}$$

where $\mathbf{g}(z) = \{\mathbf{g}_1(z_1), \mathbf{g}_2(z_2), \mathbf{g}_3(z_3), \mathbf{g}_4(z_4), \mathbf{g}_5(z_5)\}^{\mathrm{T}}$ is an arbitrary analytic vector function; $z_j = x_1 + p_j x_3 \; (j = 1, 2, \cdots, 5)$ and $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}^{\mathrm{T}}$, where p_j and $\mathbf{a}_j = \{a_{1j}, a_{2j}, a_{3j}, a_{4j}, a_{5j}\}^{\mathrm{T}}$ are, respectively, an eigenvalue and an eigenvector of the system (Stroh, 1962)

$$\left\{\mathbf{Q} + p_j\left(\mathbf{R} + \mathbf{R}^{\mathrm{T}}\right) + p_j^2 \mathbf{T}\right\} \mathbf{a}_j = 0, \tag{A.8}$$

where the 5×5 matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{T}$ for the aforementioned monoclinic MEE materials are defined as

$$\mathbf{Q}=\begin{pmatrix} c_{11} & 0 & c_{15} & e_{11} & h_{11} \\ 0 & c_{66} & 0 & 0 & 0 \\ c_{15} & 0 & c_{55} & e_{15} & h_{15} \\ e_{11} & 0 & e_{15} & -\alpha_{11} & -d_{11} \\ h_{11} & 0 & h_{15} & -d_{11} & -\mu_{11} \end{pmatrix}, \mathbf{R}=\begin{pmatrix} c_{15} & 0 & c_{13} & e_{31} & h_{31} \\ 0 & c_{46} & 0 & 0 & 0 \\ c_{55} & 0 & c_{35} & e_{35} & h_{35} \\ e_{15} & 0 & e_{13} & -\alpha_{13} & -d_{13} \\ h_{15} & 0 & h_{13} & -d_{13} & -\mu_{13} \end{pmatrix}, \mathbf{T}=\begin{pmatrix} c_{55} & 0 & c_{35} & e_{35} & h_{35} \\ 0 & c_{44} & 0 & 0 & 0 \\ c_{35} & 0 & c_{33} & e_{33} & h_{33} \\ e_{35} & 0 & e_{33} & -\alpha_{33} & -d_{33} \\ h_{35} & 0 & h_{33} & -d_{33} & -\mu_{33} \end{pmatrix}. \tag{A.9}$$

The 5×5 matrix $\mathbf{B}$ can be found by the formulas

$$\mathbf{B}=\mathbf{R}^{\mathrm{T}}\mathbf{A}+\mathbf{TAP}, \tag{A.10}$$

where $\mathbf{P}=\mathrm{diag}\{p_1,\ p_2,\ p_3,\ p_4,\ p_5\}$.

Introducing the following vector function

$$\mathbf{h}(z)=\begin{cases}\mathbf{Bg}(z), & \mathrm{Im}(z)\geq 0,\\ -\bar{\mathbf{B}}\bar{\mathbf{g}}(\bar{z}), & \mathrm{Im}(z)<0.\end{cases} \tag{A.11}$$

Then Eq. (A.7) leads to a non-homogeneous Riemann-Hilbert problem as $x_3\to 0^{\pm}$

$$\mathbf{h}(x_1,0^+)-\mathbf{h}(x_1,0^-)=\mathbf{t}(x_1),\quad x_1\in\mathrm{R}. \tag{A.12}$$

For a semi-infinite interfacial along the negative semi-axis $x_1<0$, if the magnetoelectrically impermeable condition is adopted, $\mathbf{t}(x_1)=0$ for $x_1<0$. Considering the continuity of the generalized stresses ahead of the interface crack, the following equations are obtained (Suo, 1990; Suo et al., 1992)

$$\mathbf{h}(x_1,0^+)+\bar{\mathbf{H}}^{-1}\mathbf{Hh}(x_1,0^-)=\mathbf{t}(x_1),\quad x_1>0, \tag{A.13}$$

$$\mathbf{h}(x_1,0^+)+\bar{\mathbf{H}}^{-1}\mathbf{Hh}(x_1,0^-)=0,\quad x_1<0. \tag{A.14}$$

Assuming the generalized stresses vanish at infinity, the solution of Eq. (A.14) can be written as $\mathbf{h}(z)=\mathbf{v}z^{-1/2+\mathrm{i}\varepsilon}$, and $\mathbf{v}$ is a solution of the following eigenvalue problem (Li and Kardomateas, 2007)

$$\bar{\mathbf{H}}\mathbf{v}=e^{2\pi\varepsilon}\mathbf{Hv}, \tag{A.15}$$

where ε is the oscillating index. As previously mentioned, for the 2-D interfacial crack problem satisfying Eq. (A.1), the out-of-plane and in-plane deformations are undecoupled and, correspondingly, they have different singularities. Li and Kardomateas (2006) and Li and Kardomateas (2007) have shown that the out-of-plane problem exhibits the classical inverse square root singularity, i.e. $-1/2\left(\varepsilon=0\right)$, whereas the in-plane problem possesses oscillating singularity defined by $-1/2\pm\mathrm{i}\varepsilon$, and

$$\varepsilon=\frac{1}{2\pi}\ln\frac{1-\beta}{1+\beta}, \tag{A.16}$$

where

$$\beta_1=\sqrt{c_2+\sqrt{\left(c_2\right)^2-c_4}},\ \ \beta_2=\sqrt{c_2-\sqrt{\left(c_2\right)^2-c_4}}, \tag{A.17}$$

$$c_2=-\frac{1}{4}\mathrm{tr}\left(\tilde{\mathbf{D}}^{-1}\tilde{\mathbf{W}}\right)^2,\ c_4=\left|\tilde{\mathbf{D}}^{-1}\tilde{\mathbf{W}}\right|. \tag{A.18}$$

$$\tilde{\mathbf{D}}=\mathrm{Re}\left(\tilde{\mathbf{H}}\right)=\mathrm{Re}\left(\tilde{\mathbf{Y}}^{(1)}+\bar{\tilde{\mathbf{Y}}}^{(2)}\right),\ \tilde{\mathbf{W}}=\mathrm{Im}\left(\tilde{\mathbf{H}}\right)=\mathrm{Im}\left(\tilde{\mathbf{Y}}^{(1)}+\bar{\tilde{\mathbf{Y}}}^{(2)}\right). \tag{A.19}$$

Additionally, for transversely isotropic MEE bimaterials, $c_4=\left|\tilde{\mathbf{D}}^{-1}\tilde{\mathbf{W}}\right|$ holds true and the corresponding singularity parameters become $-1/2\pm\mathrm{i}\varepsilon_1$ and $-1/2$ (Herrmann et al., 2010; Feng et al., 2011; Feng et al., 2012). Certainly, for practical MEE materials, $c_2\geq 0$, $c_4\geq 0$ and $\left(c_2\right)^2\geq c_4$ are satisfied (Li and Kardomateas, 2007).

Appendix B

Referring to Piccolroaz and Mishuris (2013), the inversion of the 2-D singular operator $S^{(s)}$ in Section 4 is presented herein. The inversion formula for the singular integral equation

$$\varpi(x_1) = S^{(s)}\chi(x_1) = \frac{1}{\pi}\int_{-\infty}^{0} \frac{\chi(\eta)}{x_1-\eta}\mathrm{d}\eta, \quad x_1 < 0\,, \tag{B.1}$$

is strongly dependent on the properties of the known function $\varpi(x_1)$. Assuming $\varpi(x_1)$ has compact support $-b \le x_1 \le -c$, where b and c are positive constants belonging to a Hölder class, the inversion formula can be written as (Muskhelishvili, 1946; Rice, 1968)

$$\chi(x_1) = \left(S^{(s)}\right)^{-1}\varpi(x_1) = -\frac{1}{\pi}\int_{-\infty}^{0}\sqrt{\frac{\eta}{x_1}}\frac{\chi(\eta)}{x_1-\eta}\mathrm{d}\eta, \quad x_1 < 0\,. \tag{B.2}$$

Using such assumptions

$$\chi(x_1) \sim \frac{K_0}{\sqrt{-x_1}}, \quad x_1 \to 0^-\,, \tag{B.3}$$

$$\chi(x_1) \sim \frac{K_\infty}{(-x_1)^{3/2}}, \quad x_1 \to -\infty\,, \tag{B.4}$$

where K_0 is the so-called stress intensity factor; K_0 and K_∞ are determined as

$$K_0 = -\frac{1}{\pi}\int_{-\infty}^{0}\frac{\varpi(\eta)}{\sqrt{-\eta}}\mathrm{d}\eta, \quad K_\infty = \frac{1}{\pi}\int_{-\infty}^{0}\varpi(\eta)\sqrt{-\eta}\,\mathrm{d}\eta\,. \tag{B.5}$$

In cases where $\varpi(x_1)$ extends over the whole negative semi-axis and has the following characteristic

$$\varpi(x_1) \sim \frac{\varpi_0}{(-x_1)^{\alpha_0}}, \quad x_1 \to 0^-\,, \tag{B.6}$$

$$\varpi(x_1) \sim \frac{\varpi_\infty}{(-x_1)^{\alpha_\infty}}, \quad x_1 \to -\infty . \tag{B.7}$$

If $\alpha_0 < 1/2$ and $\alpha_\infty > 3/2$, Eq. (B.2) still holds true and leads to the asymptotic Eqs. (B.3) and (B.4). In the case of $\alpha_0 < 1/2$ and $1/2 < \alpha_\infty < 3/2$, Eq. (B.2) remains valid, and the function $\varpi(x_1)$ in the limit of zero takes the same form as Eq. (B.3), whereas Eq. (B.4) at infinity becomes

$$\chi(x_1) \sim \frac{-\pi\varpi_\infty \sin(\pi\alpha_\infty)}{(-x_1)^{\alpha_\infty}}, \quad x_1 \to -\infty . \tag{B.8}$$

Figure and Table captions

Fig. 1. (a) Physical problem configuration and (b) weight function configuration for a semi-infinite interfacial crack in an anisotropic MEE bimaterial

Fig. 2. Mode III problem of a semi-infinite interfacial crack under (a) symmetric loads and (b) skew-symmetric loads

Fig. 3. Mode I and II problems of a semi-infinite interfacial crack under (a) magnetoelectromechanical symmetric loads and (b) magnetoelectromechanical skew-symmetric loads

Fig. 4. The normalized (a) displacement jump; (b) electric potential jump; (c) magnetic potential jump with respect to x_1/a under magnetoelectromechanical symmetric loads

Fig. 5. The normalized (a) mechanical stress; (b) electric displacement; (c) magnetic induction ahead of the crack tip with respect to x_1/a under magnetoelectromechanical symmetric loads

Fig. 6. The normalized (a) displacement jump; (b) electric potential jump; (c) magnetic potential jump with respect to x_1/a under magnetoelectromechanical skew-symmetric loads

Fig. 7. The normalized (a) mechanical stress; (b) electric displacement; (c) magnetic induction ahead of the crack tip with respect to x_1/a under magnetoelectromechanical skew-symmetric loads

Table 1 Material properties of $CoFe_2O_4$-$BaTiO_3$ composites (Sih and Song, 2003)

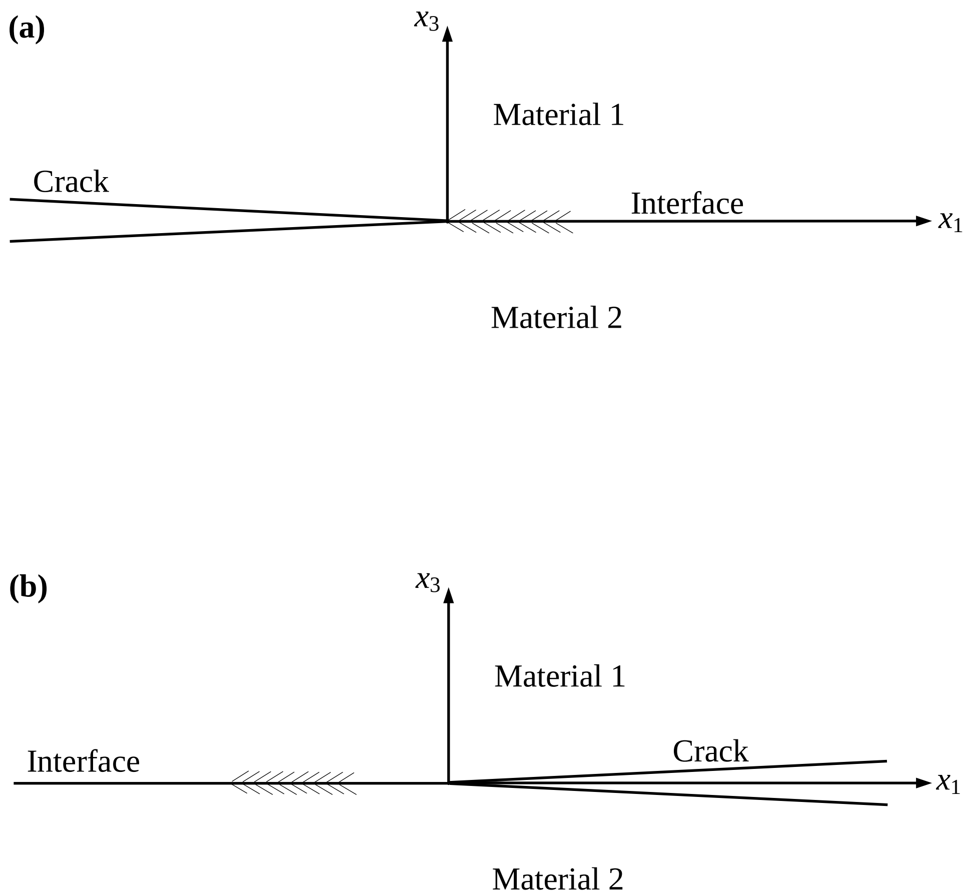

Fig. 1. (a) Physical problem configuration and (b) weight function configuration for a semi-infinite interfacial crack in an anisotropic MEE bimaterial

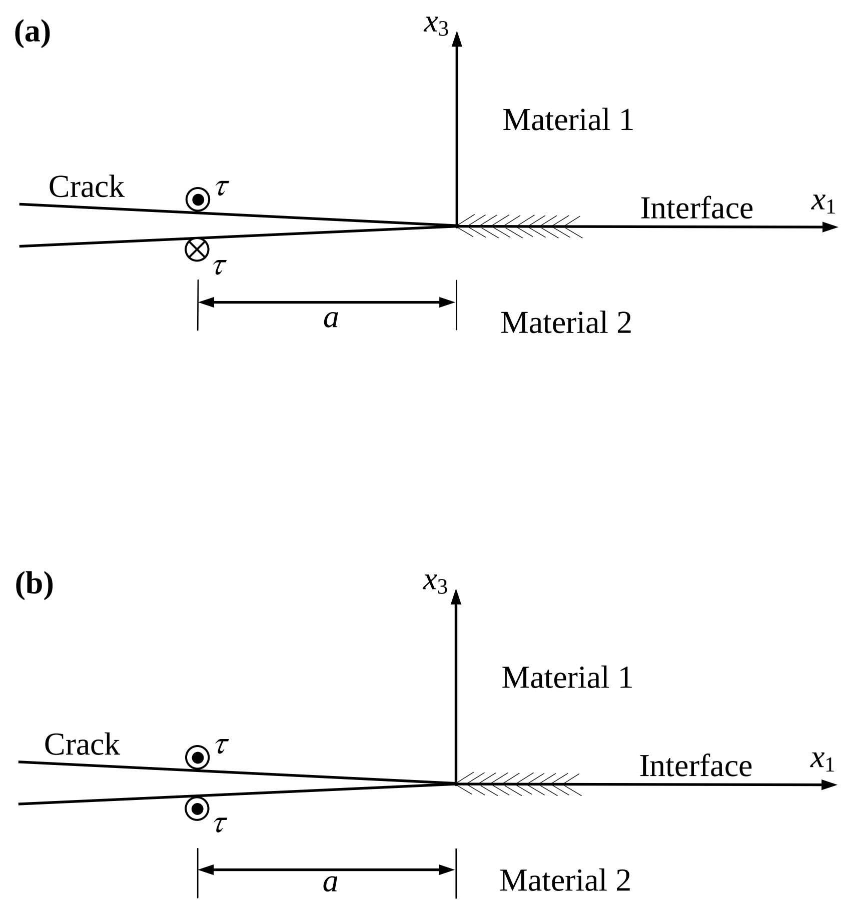

Fig. 2. Mode III problem of a semi-infinite interfacial crack under (a) symmetric loads and (b) skew-symmetric loads

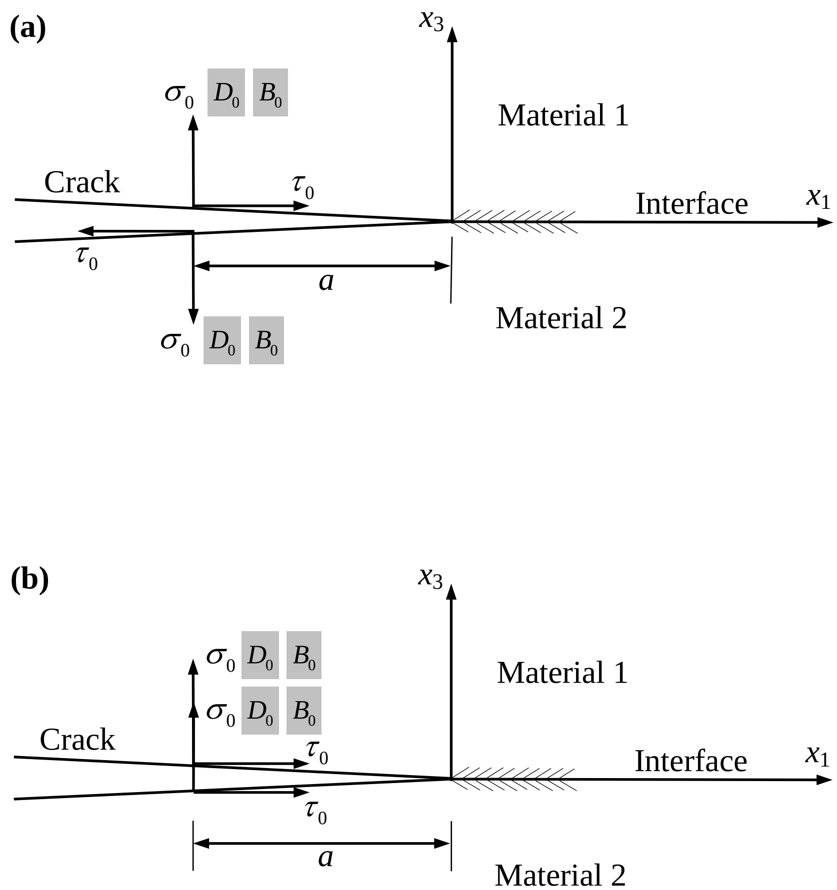

Fig. 3. Mode I and II problems of a semi-infinite interfacial crack under (a) magnetoelectromechanical symmetric loads and (b) magnetoelectromechanical skew-symmetric loads

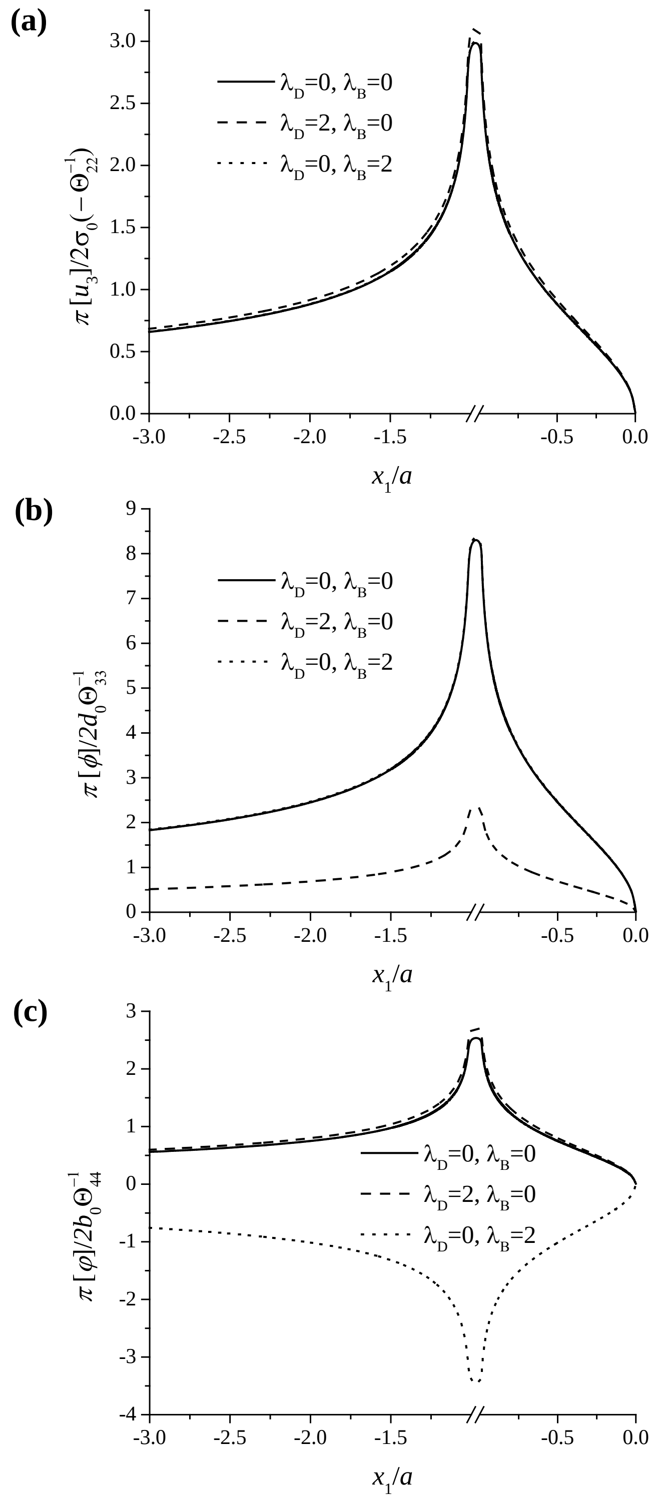

Fig. 4. The normalized (a) displacement jump; (b) electric potential jump; (c) magnetic potential jump with respect to x_1/a under magnetoelectromechanical symmetric loads

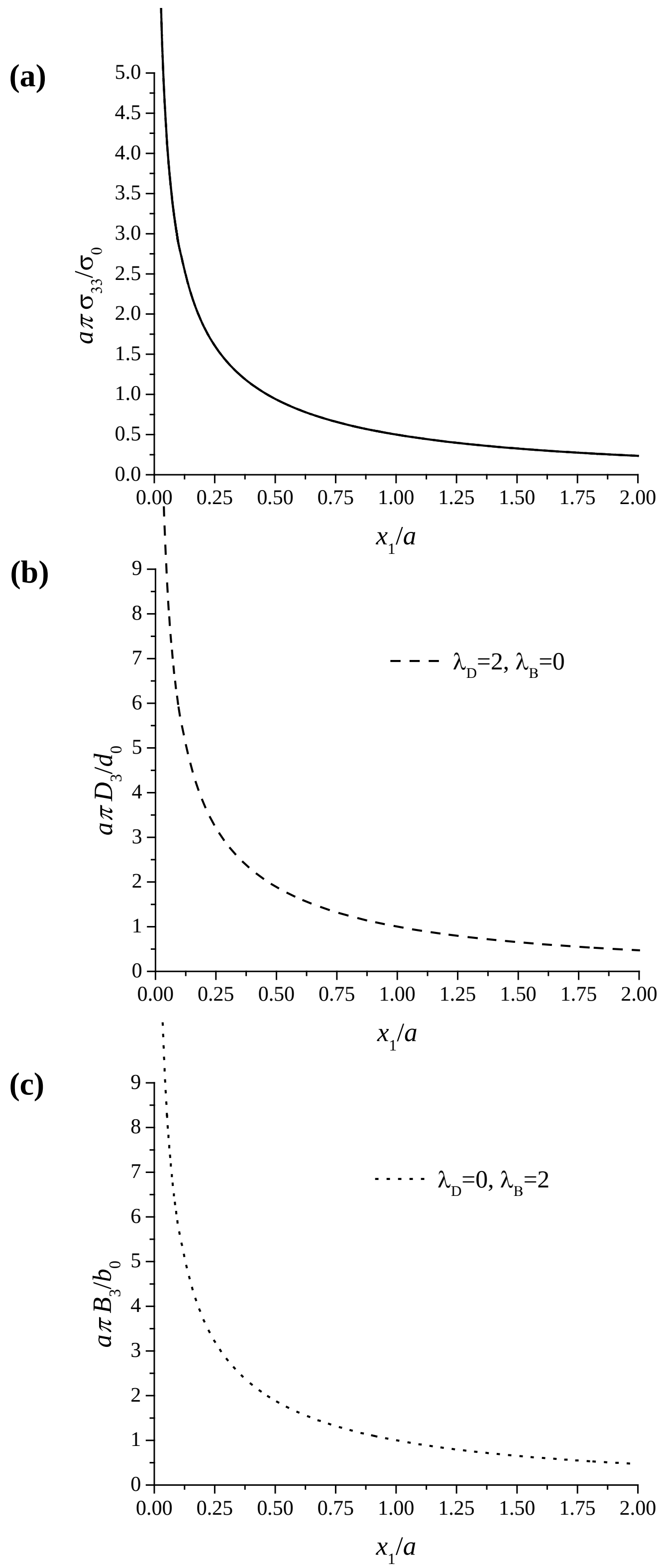

Fig. 5. The normalized (a) mechanical stress; (b) electric displacement; (c) magnetic induction ahead of the crack tip with respect to x_1/a under magnetoelectromechanical symmetric loads

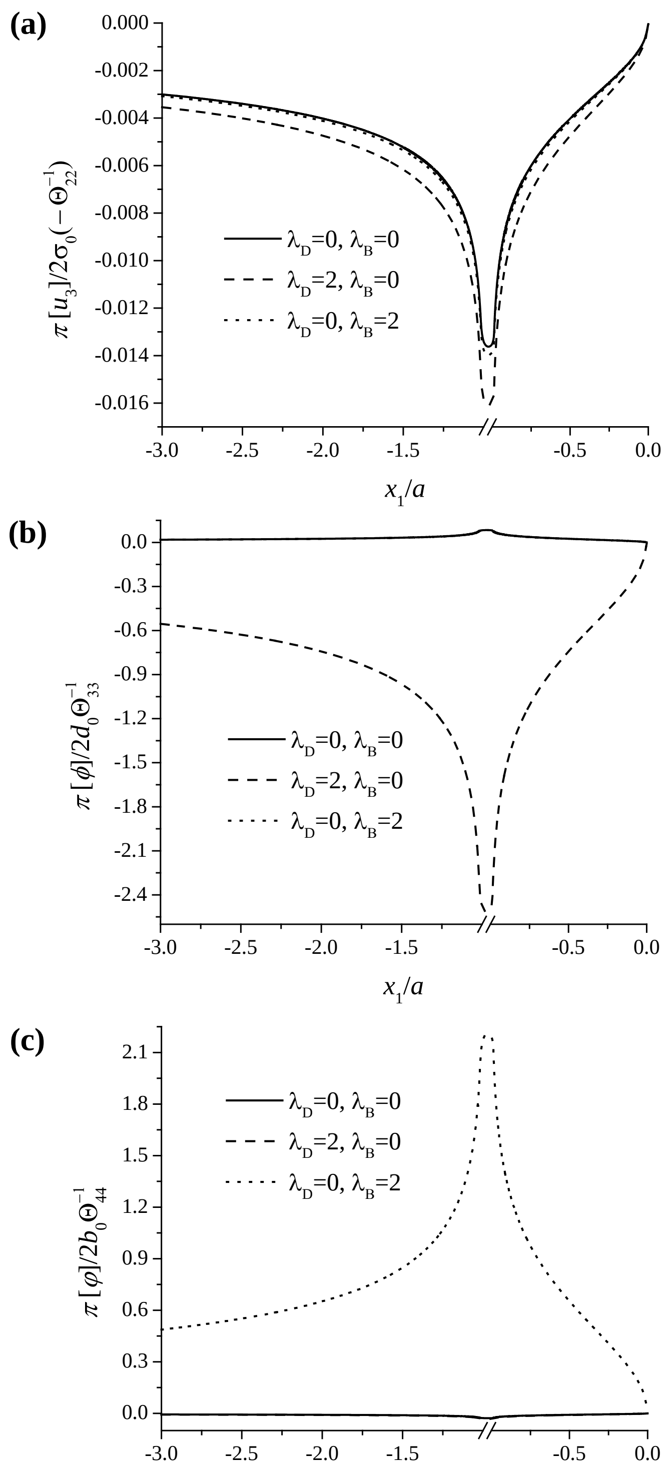

Fig. 6. The normalized (a) displacement jump; (b) electric potential jump; (c) magnetic potential jump with respect to x_1/a under magnetoelectromechanical skew-symmetric loads

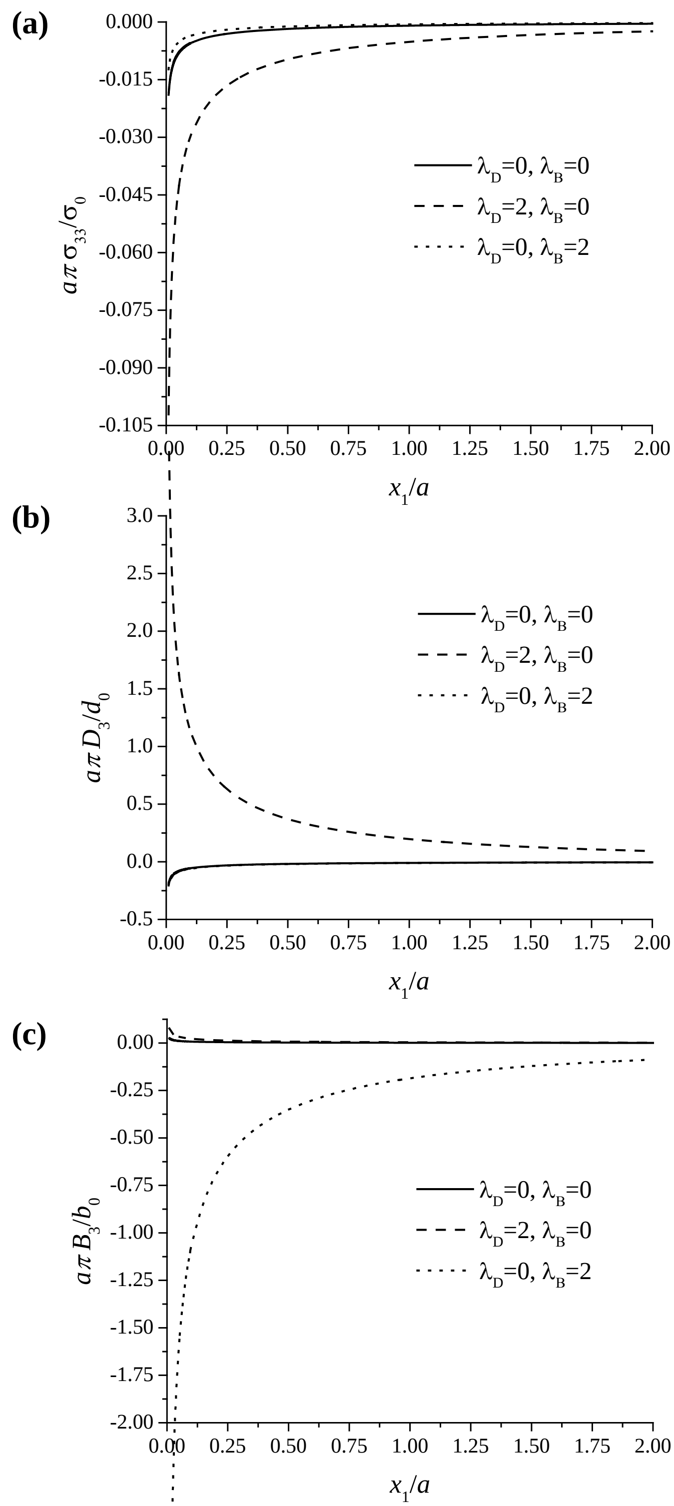

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Table 1 Material properties of $CoFe_2O_4$-$BaTiO_3$ composites (Sih and Song, 2003)

(c_{ij} in 10^9 N/m^2, e_{ij} in C/m^2, α_{ij} in 10^{-10}C/Vm, h_{ij} in N/Am, μ_{ij} in 10^{-6}Ns^2/ C^2).

	c_{11}	c_{13}	c_{33}	c_{44}	e_{15}	e_{31}	e_{33}
Material 1 (V_f=0.3)	250.0	142.7	237.3	44.6	3.48	-1.32	5.58
Material 2 (V_f=0.7)	202.0	105.0	194.2	43.7	8.12	-3.08	13.02
	h_{15}	h_{31}	h_{33}	α_{11}	α_{33}	μ_{11}	μ_{33}
Material 1 (V_f=0.3)	385.0	406.2	489.8	34.2	38.5	414.5	112.9
Material 2 (V_f=0.7)	174.1	165.0	209.9	78.6	88.5	180.5	54.1