Speed Acquisition*

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Abstract

Speed has become a salient feature of modern financial markets. This paper studies investors' endogenous speed acquisition, alongside their information acquisition. In equilibrium, speed heterogeneity endogenously arises across investors, temporally fragmenting the price discovery process. A deterioration in the long-run price informativeness ensues. Various crowding-out effects drive speed and information to be either substitutes or complements. The model cautions the possible dysfunction of price discovery: An advancing information technology might complement speed acquisition, which fragments the price discovery process, thus hurting price informativeness. Novel predictions are discussed regarding investor composition, fund performance, and trading volume.

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1 Introduction

Price discovery is a fundamental function of financial markets. It involves two steps. First, investors acquire information about the asset. Second, via trading, such information is incorporated into price. The first step determines the amount of information that the price can eventually reflect, i.e. the magnitude of price discovery. The second step is about the process of how the acquired information aggregates into price. Speed is an intrinsic characteristic underlying the price discovery process: Investors race to be the first to reap the information rent.

To date, the literature has mainly emphasized information acquisition, following the pioneering works by Grossman and Stiglitz (1980) and Verrecchia (1982). This paper complements this canonical perspective with an enriched price discovery process, by studying investors' speed acquisition *alongside* their information acquisition.

The notion of speed roots in the course of financial securities trading. There are three aspects: First, investors can reach the same trading strategy sooner, by hiring a larger analyst team or buying more computers to process acquired data. Second, before execution, a trading order needs to journey through the back office for risk management, due diligence, and compliance. The tightening regulatory environment in recent years has encumbered this aspect, slowing down trading, and the associated expenses have been increasing accordingly (Thomson-Reuters, 2017). Third, from trading desks and onward, the order execution speed depends on investments in computational hardware and connection to exchange servers. This last aspect has progressed drastically in the last decade, evidenced by the rise of algorithmic and high-frequency trading technologies.

A set of questions arises regarding the above aspects of trading speed: How much speed technology should different investors acquire? Is investment in speed favored over information? Which securities attract more fast investors? What are the implications for the overall quality of price discovery?

This paper develops a model to address these questions. The model studies an economy populated with atomless investors, who first invest in both speed and information technology and then trade a risky asset. The information technology determines an investor's private signal precision about the asset value, while the speed technology allows him to trade ahead of his peers.[1] The rent-seeking investors in the model have incentive to acquire both technologies. The information technology directly adds to one's information rent. Indirectly via the speed technology, the sooner one trades, the less price discovery has already occurred and the more rent can he extract—a "first-mover advantage". The equilibrium is found where each investor optimally acquires the technologies to maximize his information rent, taking into account the investment costs and the competition from others.

A driving feature of the model is the "temporal fragmentation" effect of the speed technology. Though all investors want to acquire speed to enjoy the "first-mover advantage", not everyone will be equally fast, for otherwise there is no "first-mover" and some will stay slow to save the speed acquisition cost. Speed heterogeneity thus endogenously arises in equilibrium, with investors trading at different times. Accordingly, the price discovery process also temporally splits into parts, e.g., an early fragment with fast investors and a late fragment with the slow.

The fragmented price discovery process delivers novel insights. First, the speed technology inflicts a nonmonotonic impact on price informativeness. With a more advanced (cheaper) speed technology, more investors become fast, boosting the early fragment of price discovery. At the same time, fewer investors remain slow and the late fragment shrinks. The market's eventual price informativeness, therefore, can be either improved or hurt, depending on whether the boost (early) overcomes the decay (late). This result holds even when information acquisition is shut down.

Second, speed and information can be either substitutes or complements. Consider, for example,

[1] To fix the idea, consider a hedge fund for example. Its information acquisition involves, e.g., sending analysts for firm visits, buying various datasets, or developing valuation models. These investments determine the amount and the quality of the data (signal precision). The speed acquisition covers different aspects: The fund can invest in equipment or staff to speed up data processing (reaching the same trading strategy sooner), to streamline the back office, and to expedite order execution at the trading desk. These later investments reduce the time needed to implement trades, improving trading speed but not affecting the quality of the acquired data (or the implied trading strategies).

a positive shock in the information technology, upon which all investors acquire more information. How is the demand for speed affected? The answer depends on the relative change between fast and slow investors' rents. As everyone acquires more information, competition intensifies, attenuating the rents for both the fast and the slow. In addition, the increased early price discovery crowds out the slow investors (if the fast have done all the price discovery, no rent will be left for the slow.) Netting these competition effects, if the fast (the slow) are hurt more, some of them are better off staying slow (becoming fast) instead, in which case speed substitutes (complements) information. Due to such forces, each technology has a nonmonotonic effect on the demand for the other.

Third, an advancement in information technology can still hurt the long-run price informativeness. The key mechanisms at work, as taught by the model, are the temporal fragmentation by the speed technology and its endogenous complementarity with information. Due to complementarity, an improving information technology stimulates demand for both information and speed. While the former improves price informativeness, the latter invokes the temporal fragmentation effect, whereby the early fragment of price discovery improves but the late fragment deteriorates. When the decay in the slow fragment dominates, the overall price informativeness is hurt.

The last result above cautions the dysfunction of information aggregation in financial markets. The "information technology" in the model can be interpreted broadly. For example, recent years have seen strengthened transparency and disclosure requirements by regulators. Policies like Sarbanes-Oxley, Regulation Fair Disclosure, and Rule 10b5-1 have arguably reduced the cost of information acquisition. There is evidence of speed acquisition complementing the accessibility of information. Du (2015) finds that high-frequency traders are constantly crawling the website of U.S. SEC in order to trade on the information in latest company filings. Through such a complementarity channel, this paper argues that transparency and disclosure policies might generate unintended negative impact on price informativeness.

Some recent empirical evidence echoes this view. Weller (2016) shows that algorithmic trading has risen at the cost of long-run price discovery. Gider, Schmickler, and Westheide (2016) shows how high-frequency trading hurts the predictability of earnings in the far future. To emphasize, the mechanism put forward in this paper is new. For example, the argument by Weller (2016), and via equilibrium models by Dugast and Foucault (2017) and by Kendall (2017), is that short-run (early) price discovery can *crowd out* the acquisition of more precise information in the long-run (late)—a substitution effect. In contrast, the mechanism revealed by this paper emphasizes the endogenous *complementarity* between information and speed acquisition. As the information technology advances and incentivizes more investors to acquire speed, the price discovery process fragments at the cost of the (long-run) magnitude.

Different financial assets are exposed to different levels of information and speed technology. The model thus also offers cross-sectional predictions of how technology advancement might affect different assets (e.g., stocks) differently. Bai, Philippon, and Savov (2016) finds a rising trend of the price informativeness of S&P 500 nonfinancial firms in a half-century sample period starting from the 1960s. The finding for firms beyond the S&P 500, however, is the opposite. Farboodi, Matray, and Veldkamp (2017) reproduce the patterns and explain these phenomena through investors' attention-constrained information acquisition. This paper adds to the discussion that the distinction in different technologies—speed v.s. information—is important in determining individual stocks' respective price informativeness over the years.

The model further yields predictions on investor composition across assets. Some assets have lower information acquisition costs (e.g., higher media exposure and analyst coverage) than others, thus attracting investors of different speed. For example, heavily regulated mutual funds and pension funds arguably trade more slowly—due to the time spent on compliance, due diligence, etc.—than lightly regulated hedge funds and proprietary trading firms.[2] This view of hedge funds being faster than mutual funds is consistent with the evidence of information acquisition timing

[2] Hedge funds and proprietary trading firms are also known to invest more in the other two aspects of speed. In terms of information processing, one most salient trend recently is hedge funds' massive investment in machine learning and artificial intelligence; see, e.g., "The Massive Hedge Fund Betting on AI", September 27, 2017, *Bloomberg*. In terms of trading technology, it is well known that many of the high-frequency traders are hedge funds and proprietary trading firms (SEC, 2010, IV.B).

shown in Swem (2017). Thus, the model details which assets will attract more hedge funds (fast) than mutual funds (slow). Specifically, the model predicts that hedge funds' activity (relative to mutual funds') in small, medium, and large stocks has a nonmonotonic pattern, matching the empirical finding by Griffin and Xu (2009). These results further shed light on the implication of technology advancements on trading volume and fund performance.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 sets up the model and Section 4 derives its equilibrium. Section 5 then explores the model implications on investors' technology acquisition and on price informativeness. Discussions on model assumptions, robustness, and extensions are collated in Section 6. Section 7 then concludes.

2 Related literature

This paper builds on a number of models featuring (two) sequential trading rounds: Grundy and McNichols (1989), Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994), Holden and Subrahmanyam (1996, 2002), Brunnermeier (2005), Cespa (2008), Banerjee, Davis, and Gondhi (2017), and Dugast and Foucault (2017). A distinguishing feature of the current model is that investors are allowed to engage in costly speed acquisition, separately from the conventional information acquisition. To compare, in the above, investors either cannot choose their speed at all or do not invest in speed separately from information:

- In Grundy and McNichols (1989), Brunnermeier (2005), and Cespa (2008), all investors trade in both rounds. Hence, there is no speed.
- Investors' speed are exogenously assigned in Froot, Scharfstein, and Stein (1992); Hirshleifer, Subrahmanyam, and Titman (1994); and Banerjee, Davis, and Gondhi (2017).
- In Holden and Subrahmanyam (1996) and Dugast and Foucault (2017), the notion of speed appears as a by-product of, hence not separable from, investors' choice of different information (short- v.s. long-horizon in the former; and raw v.s. processed signals in the latter).

- Section IV of Holden and Subrahmanyam (2002) directly models speed by letting investors choose at a cost to observe and trade on a signal early. However, investors cannot separately invest in information, as the signal is common across investors and across time (in the current paper, this is a special case studied in Section 5.2).

With the separation of speed from information, this paper contributes to the literature by identifying *endogenous* complementarity or substitution between the two technologies.

This paper highlights that lowering information cost can hurt price informativeness, in an environment with *a single information source*.[3] A number of the aforementioned papers share a similar prediction due to some form of "substitution" in different types/sources of information. For example, in Brunnermeier (2005), the existence of an insider who monopolizes the short-run information curbs other analysts' long-run trading aggressiveness; in Dugast and Foucault (2017), a cheaper raw signal can crowd out investment in the processed signal; in Banerjee, Davis, and Gondhi (2017), a public disclosure pushes investors to instead learn about others' beliefs, no longer about fundamentals. Such substitution does not exist in the current model as there is only one information source. Instead, it is the (endogenous) complementarity between information and speed that hurts price informativeness (Section 5.3).[4]

This paper further contributes to three themes of the literature. First, the literature on costly information acquisition largely focuses on the magnitude aspect of price discovery, following the seminal works by Grossman and Stiglitz (1980) and Verrecchia (1982). Recent studies explore other dimensions. To name a few, Peress (2004, 2011) studies the wealth effect on information acquisition. Van Nieuwerburgh and Veldkamp (2009, 2010) analyze information acquisition under

[3] "A single information source" means that there is no public information (Brunnermeier, 2005; Banerjee, Davis, and Gondhi, 2017), no independent fundamentals (Froot, Scharfstein, and Stein, 1992), no short- v.s. long-term information (Holden and Subrahmanyam, 1996), no learning about others' beliefs (Banerjee, Davis, and Gondhi, 2017) or about noise (Froot, Scharfstein, and Stein, 1992), or raw v.s. processed signals (Dugast and Foucault, 2017), etc.

[4] Hirshleifer, Subrahmanyam, and Titman (1994) and Holden and Subrahmanyam (2002) also have a sole information source in their model. While not explicitly studied, cheaper information acquisition cost in those settings would always lead to higher price informativeness. This is precisely because there is no separate investment in speed and information, hence no complementarity.

limited attention. Goldstein and Yang (2015) explore the implication of information diversity. To compare, the above literature assumes that the market always clears with all investors trading at the same time—they have the same speed. With endogenous speed acquisition, this paper allows to study the process of price discovery with investors arriving and trading asynchronously.

Second, the *temporal* fragmentation (due to speed technology) in this paper differs from the existing literature on *spatial* market fragmentation.[5] Regarding the focus on price discovery, an important feature of temporal fragmentation is that information revealed early naturally carries over to the future—the market never forgets. Such natural accumulation of information over time is critical in determining the complementarity or substitution between the two technologies. In a model of multiple venues (spatial fragmentation), there is no naturally directional "flow" of information from one venue to another (more fundamentally, the notion of speed does not apply to a spatial setting). Speed therefore touches upon a novel angle of market fragmentation.

Third, this paper lends equilibrium support to the literature with *endogenous bundling* of speed and information acquisition. The model predicts that fast investors always acquire more information than the slow. This is because price discovery always accumulates over time and the same piece of information has higher marginal benefit the sooner it is traded. This insight justifies a popular connotation for fast traders that they are also more informed. See, e.g., models by Hoffmann (2014), Biais, Foucault, and Moinas (2015), and Budish, Cramton, and Shim (2015); evidence by Brogaard, Hendershott, and Riordan (2014) and Shkilko and Sokolov (2016); and surveys by Biais and Foucault (2014), O'Hara (2015), and Menkveld (2016).

In a different line, investors' speed choice has been studied in limit order models with discrete prices. Examples include Yao and Ye (2017) and Wang and Ye (2017). The main driving feature

[5] For example, Admati (1985), Pasquariello (2007), Boulatov, Hendershott, and Livdan (2013), Goldstein, Li, and Yang (2014), Cespa and Foucault (2014), among many others, study information and cross-market learning of correlated assets. Pagano (1989), Chowdhry and Nanda (1991), and Baruch, Karolyi, and Lemmon (2007) study trading of the same asset on different venues (e.g., dual-listed stocks). More recently, market fragmentation has been theorized in the context of dark v.s. lit trading mechanisms, as in Ye (2011), Zhu (2014), Brolley (2016), and Buti, Rindi, and Werner (2017). Finally, Chao, Yao, and Ye (2017a,b) study the competition among exchanges by zooming in on fee structure and tick size.

is the binding tick size which limits investors' competition on price and as a result they turn to speed competition. The current paper focuses on investors' incentive to acquire speed due to the transitory nature of information advantage.

3 Model

Assets. There is a risky asset and a risk-free numéraire. At the end of the game, each unit of the risky asset will pay off a normally distributed random amount V units of the numéraire. The unconditional expectation $\mathbb{E}V$ is normalized to 0 and let $\mathrm{var}V = \tau_0^{-1}$ (> 0).

Investors. There is a unity continuum of atomless investors, indexed by $i \in [0, 1]$. They have constant absolute risk-aversion (CARA) preference with the same risk-aversion coefficient γ (> 0). There is no endowment or initial inventory position.

Speed technology. An investor i can invest in a speed technology to affect his trading time t_i (see "Timeline" below). Without investing in speed, all investors are slow, trading at $t_i = 2$. One can instead become fast and trade at $t_i = 1$ by paying $1/g_t$ units of the numéraire. The exogenous parameter g_t (> 0) measures the level of speed technology. The larger is g_t, the more advanced (cheaper) is the technology.

Information technology. Before trading, each investor i observes a private signal S_i about the payoff V. Specifically, $S_i = V + \varepsilon_i$, where ε_i is normally distributed with zero mean and variance h_i^{-1} (> 0), independent of V and any other $\varepsilon_{j \neq i}$. The investor i can spend m_i (≥ 0) units of the numéraire on an information technology to improve his private signal precision:

$$h_i = g_h k_h(m_i),$$

where $k_h(\cdot)$ is twice-differentiable, concave, and strictly monotone increasing; and g_h (≥ 0) is a parameter measuring the marginal productivity of this information technology. Without investing in this technology, the investor gets no signal; i.e. $k_h(0) = 0$.

Due to the monotonicity of $k_h(\cdot)$, an investor's information acquisition can be referred to as either h_i (the precision) or m_i (the cost) interchangeably: There exists a weakly convex, monotone increasing information acquisition cost function $c(\cdot)$, so that $\forall h_i \geq 0$,

$$m_i = c(h_i) := {k_h}^{-1}(h_i/g_h).$$

To ensure that there is always some information in the market, let $\dot{c}(0) = 0$; equivalently, $\dot{k}_h(0) \to \infty$.

Timeline. There are four dates in the model: $t \in \{0, 1, 2, 3\}$, illustrated in Figure 1. At $t = 0$, all investors independently invest in technologies t_i and h_i. Time $t \in \{1, 2\}$ are trading rounds. The set of investors $\{i|\, t_i = t\}$ arrive at t together and they independently submit demand schedules $\{x_i(p_t; \cdot)\}$ to trade the risky asset, based on his information set—private signal s_i and the public history of past prices. Specifically, at $t = 1$ only fast investors arrive and trade. Finally, at $t = 3$, the risky asset liquidates at V and all investors consume their terminal wealth.

Trading. In each trading round $t \in \{1, 2\}$ there is noise demand U_t, which is independent of all other random variables and is i.i.d. normally distributed with zero mean and variance τ_{U}^{-1} (> 0). The aggregate demand at t is

$$L_t(p) = \int_{i\in[0,1]} x_i(p; \cdot)\mathbb{1}_{\{t_i=t\}}\mathrm{d}i + U_t. \tag{1}$$

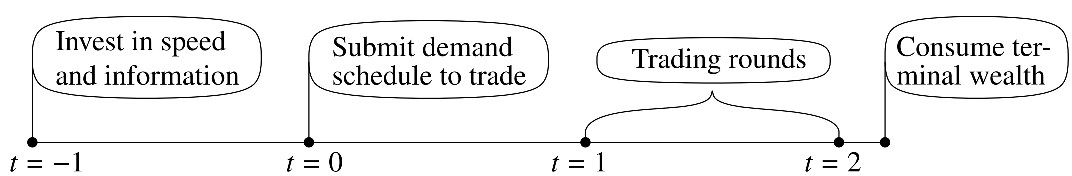

Figure 1: Timeline of the game. The model has four dates: $t \in \{0, 1, 2, 3\}$. At $t = 0$, all investors invest in technology; at $t \in \{1, 2\}$, investors arrive in the market at the time according to their speed technology and submit their demand schedules to trade the risky asset; finally, at $t = 3$ the risky asset liquidates and all investors consume their terminal wealth.

There is a competitive market maker, who clears the market at all times at the efficient price given all historical public information (as in Kyle, 1985). Thus, the trading price in each round t is

$$P_t = \mathbb{E}\Big[V \Big| \{L_r(\cdot)\}_{r\leq t}\Big]. \tag{2}$$

This setup is consistent with, among many others, Hirshleifer, Subrahmanyam, and Titman (1994); Vives (1995); Holden and Subrahmanyam (1996); and Cespa (2008).

Strategy and equilibrium definition. To sum up, each investor maximizes his expected utility over the final wealth by optimizing his demand $x_i(\cdot)$ upon trading; and, backwardly, by choosing his technology pair $(t_i, h_i) \in \{1, 2\} \times [0, \infty)$ at $t = 0$.[6]

Denote by $\pi(t_i, h_i)$ the investor i's ex ante certainty equivalent (whose functional form will be derived below). Define $\mathcal{P} := \{(t_i, h_i)\}_{i\in[0,1]}$ as the collection of all investors' investment policies. A Nash equilibrium is a collection $\mathcal{P}$, such that for any investor $i \in [0, 1]$, fixing $\mathcal{P}\backslash(t_i, h_i)$, he has $\pi(t_i, h_i) \geq \pi(t, h)$, $\forall(t, h) \in \{1, 2\} \times [0, \infty)$.

Remarks regarding the model setup:

Remark 1 (Interpreting speed). The speed technology is fairly stylized in the model, as it only generates two relative speed tiers, $t \in \{1, 2\}$. The time differential (between $t = 1$ and $t = 2$) can be interpreted according to any one of the following three real-world speed levels.

- High-frequency speed (in subseconds to minutes). An investor (institution) can improve his high-frequency speed by investing in the trading desk—algorithms, colocation to exchange servers, optic-fibre cables, and microwave towers.
- Medium-frequency speed (in hours to days). When implementing a trading idea, analysts and managers in an institution are subject to due diligence and regulatory compliance, which can take hours if not days, especially when the trading order is large. This process can be expedited by staffing additional personnel in the back office.

[6] An investor can in fact mix between $t_i \in \{1, 2\}$ by choosing probability $\mu_i \in [0, 1]$ to acquire speed (become fast and trade at $t_i = 1$). Hence, equivalently, an investor's investment decision can be written as $(\mu_i, h_i) \in [0, 1] \times [0, \infty)$.

- Low-frequency speed (in days to weeks). Processing raw data to form a trading idea takes time. For example, a firm's announcement might affect the projections of future cash flows. It can take teams of analysts days or weeks to update such fundamentals. Recruiting more analysts or investing in more advanced computers can speed up this process.

Remark 2 (Who is fast). Based on the above three frequencies of speed, this paper argues that hedge funds fit the fast investors in the model, while the slow investors can be mutual funds or pension funds. Hedge funds' high-frequency speed advantage arises from their investment in trading technology. SEC (2010) defines high-frequency traders as proprietary trading firms or hedge funds. The medium-frequency speed differential can be attributed to different regulatory requirement. Mutual funds are registered with the U.S. Securities and Exchanges Commission (SEC) and are subject to time-consuming regulatory compliance, risk control, and bookkeeping. Hedge funds, on the other hand, are not under such regulatory scrutiny, thus able to expeditiously trade on their signals. Under the low-frequency interpretation, the fast investors process information sooner than the slow and should lead subsequent returns. Swem (2017) documents that hedge funds acquire information ahead of sell-side analysts, who are then followed by other buy-side institutions.

Remark 3 (Modeling choices). Three assumptions are highlighted here: (1) fast investors only trading at $t = 1$; (2) the fixed population size; and (3) the competitive market maker. These assumptions are made purposefully to, and only to, pinpoint the novel economic mechanisms as the analysis proceeds. In Section 6, three extensions relax these assumptions respectively: (1) to allow fast investors trade also more frequently; (2) to endogenize investor participation; (2) to replace the competitive market maker with a fringe of uninformed investors. Both the key mechanisms and the main results of the paper are shown to stand robust to these extensions.

Remark 4 (Orthogonal technologies). The model assumes that the information technology is orthogonal to the speed technology (g_t and g_h are exogenous parameters, independent of each other). This is an intentional choice, so that the comparative static analyses will help isolate the effect of one technology against the other. In reality, the two technologies will likely affect each other. Section 6.4 studies such interdependence and its implications.

Remark 5 (The amount of noise trading). The amount of noise trading at $t \in \{1, 2\}$ is assumed to be the same (i.i.d. $\{U_t\}$). Depending on the interpretation of "speed", it is possible that the noise trading size changes over time. Section 6.5 accounts for time-varying noise trading and demonstrates the robustness of the results.

4 Equilibrium analysis

Investors' optimal trading is first derived in Section 4.1 and then their technology acquisition studied in Section 4.2. Two benchmarks, respectively with only information and only speed, are also discussed in Section 4.3.

4.1 Optimal trading

Fix all other investors' strategies and consider an investor i with technology (t_i, h_i). At $t = t_i$, he chooses his demand schedule x_i to maximize his expected utility over final wealth:

$$x_i \in \arg\max_{x_i} \mathbb{E}\left[-e^{-\gamma\cdot(V-P_t)x_i} \,|\, V + \varepsilon_i = s_i, P_t, P_{t-1}, \dots\right]$$

where P_t is given by the market maker's efficient pricing as in Equation (2). Note that the investor also observes the price history $\{P_{t-1}, \dots\}$ (with $P_0 = \mathbb{E}V = 0$). Standard conjecture-and-verify analysis as in Vives (1995) yields the following lemma.

Lemma 1 (Trading under "pure speed differential"). *For any technology pair (t_i, h_i), an investor i with signal s_i submits the optimal linear demand schedule at $t = t_i$:*

$$x_i = \frac{h_i}{\gamma}\left(s_i - p_{t_i}\right).$$

His certainty equivalent at the time of technology investment ($t = 0$) is

$$\pi(t_i, h_i) = \frac{1}{2\gamma}\ln\left(1 + \frac{h_i}{\tau_{t_i}}\right) - c(h_i) - \frac{2 - t_i}{g_t}, \tag{3}$$

where the price informativeness $\tau_t := \mathrm{var}[V| \{L_r(\cdot)\}_{\forall r\leq t}]^{-1}$ *satisfies the recursion of*

$$\Delta\tau_t = \tau_t - \tau_{t-1} = \left(\int_{\{t_j=t\}} \frac{h_j}{\gamma}\mathrm{d}j\right)^2 \tau_{\mathrm{U}} \tag{4}$$

with the initial value $\tau_0 = (\mathrm{var}V)^{-1}$. *The equilibrium price* P_t *satisfies the recursion of*

$$\Delta P_t = P_t - P_{t-1} = \frac{\Delta\tau_t}{\tau_t}\left(V + \frac{\gamma U_t}{\int_{\{t_j=t\}} h_j \mathrm{d}j} - P_{t-1}\right) \tag{5}$$

with initial value $P_0 = \mathbb{E}V$ *(= 0).*

An investor's demand x_i scales with the difference between his private signal and the trading price ($s_i - P_{t_i}$), where the scaling factor h_i/γ—his trading aggressiveness—increases with the precision of his signal and decreases with his risk-aversion. His certainty equivalent has three components: The first term represents the information rent due to his private information, while the second and the third term correspond to the cost of information and speed acquisition, respectively.

Note that slow investors ($t_i = 2$) do *not* (directly) trade on the fast round price p_1, thanks to the competitive market maker who sets p_2 while recalling the information from $t = 1$ trading. As such, from a slow investor's perspective, observing only p_2 is as good as observing both p_1 and p_2. This Markov feature inherits from Vives (1995) and dates back to Kyle (1985), where the dynamic equilibrium only uses the contemporaneous price p_t as a state variable, not the entire price history.

4.2 Optimal technology acquisition

The next step is to find investors' optimal technology investment. Consider first the information acquisition h_i by an investor i whose speed is given at $t_i = t$. Since an investor is atomlessly small, his individual information acquisition h_i does not affect the price informativeness τ_t. To maximize his certainty equivalent, he takes τ_t as given and chooses his information precision h_i according to the first-order condition of Equation (3):

$$\frac{1}{2\gamma}\frac{1}{\tau_t + h_i} - \dot{c}(h_i) = 0, \tag{6}$$

which has a unique solution $h(\tau_{t_i})$, satisfying the second-order condition, thanks to the convexity of the cost $c(\cdot)$; see the "Information technology" paragraph in Section 3. By symmetry, therefore, all investors of the same speed $t_i = t \in \{1, 2\}$ acquire the same amount of information: $h_{\mathrm{F}} = h(\tau_1)$ for the fast and $h_{\mathrm{S}} = h(\tau_2)$ for the slow.

The investor's speed acquisition then boils down to comparing the ex ante certainty equivalents:

$$\begin{aligned} \pi_{\mathrm{F}} &= \frac{1}{2\gamma}\ln\left(1 + \frac{h_{\mathrm{F}}}{\tau_1}\right) - c(h_{\mathrm{F}}) - \frac{1}{g_t}; \\ \pi_{\mathrm{S}} &= \frac{1}{2\gamma}\ln\left(1 + \frac{h_{\mathrm{S}}}{\tau_2}\right) - c(h_{\mathrm{S}}). \end{aligned} \tag{7}$$

If $\pi_{\mathrm{F}} > \pi_{\mathrm{S}}$, all investors will acquire speed and become fast (a corner solution); and vice versa. In an interior equilibrium, the break-even condition $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$ must hold so that no investor has incentive to change his speed.

To investigate how $\pi_{\mathrm{F}} \lesseqgtr \pi_{\mathrm{S}}$ is affected by investors' speed acquisition, define

$$\mu_{\mathrm{F}} := \int_{i\in[0,1]} \mathbb{1}_{\{t_i=1\}}\mathrm{d}i \text{ and } \mu_{\mathrm{S}} := \int_{i\in[0,1]} \mathbb{1}_{\{t_i=2\}}\mathrm{d}i$$

as the population sizes for the fast and the slow, respectively. By construction, $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1$. The price informativeness τ_t recursion (Equation 4) can then be rewritten as

$$\Delta\tau_1 = \frac{\tau_{\mathrm{U}}}{\gamma^2}h_{\mathrm{F}}^2\mu_{\mathrm{F}}^2 \text{ and } \Delta\tau_2 = \frac{\tau_{\mathrm{U}}}{\gamma^2}h_{\mathrm{S}}^2\mu_{\mathrm{S}}^2. \tag{8}$$

These *increments*, $\Delta\tau_1$ and $\Delta\tau_2$, are referred to as the "early fragment" and the "late fragment" of price discovery, respectively. In contrast, τ_1 and τ_2 are *cumulative* and are called the "short-run" and the "long-run price informativeness", respectively.[7] The certainty equivalents (3) are therefore affected by the sizes of fast and slow investors.

There are four equilibrium objects: investors' (aggregate) speed acquisition μ_{F} and μ_{S}; and their

[7] The labels of "short-run" v.s. "long-run" are only in a relative sense that $t = 2$ occurs after $t = 1$. See the high-, medium-, and low-frequency interpretations of speed in Remark 1.

information acquisition h_F and h_S. The following proposition states the result.

Proposition 1 (Equilibrium under "pure speed differential"). *There exists a unique equilibrium $\mathcal{P}$, depending on the speed technology g_t relative to a threshold $\hat{g}_t$ (> 0, see the proof):*

Case 1 (corner). *When $g_t \leq \hat{g}_t$, all investors invest in $(t_i, h_i) = (2, h_\mathrm{S})$, where h_S and τ_2 uniquely solve the first-order condition (6) and the recursion (8) with $\mu_\mathrm{F} = 0$ and $\mu_\mathrm{S} = 1$.*

Case 2 (interior). *When $g_t > \hat{g}_t$, a mass $\mu_\mathrm{F} \in (0, 1)$ of investors invest in $(t_i, h_i) = (1, h_\mathrm{F})$, while the rest μ_2 investors invest in $(t_i, h_i) = (2, h_\mathrm{S})$, such that $\{h_\mathrm{F}, h_\mathrm{S}, \mu_\mathrm{F}, \mu_\mathrm{S}\}$ uniquely solve the following equation system:*

$$\begin{aligned}
&\textit{Optimal information acquisition:} && \frac{1}{2\gamma}\frac{1}{\tau_1 + h_\mathrm{F}} - \dot{c}(h_\mathrm{F}) = \frac{1}{2\gamma}\frac{1}{\tau_2 + h_\mathrm{S}} - \dot{c}(h_\mathrm{S}) = 0;\\
&\textit{Indifference in speed:} && \pi_\mathrm{F} = \pi_\mathrm{S};\\
&\textit{Population size identity:} && \mu_\mathrm{F} + \mu_\mathrm{S} = 1;
\end{aligned}$$

where the expressions of τ and π are given by Equations (7) and (8).

The equilibrium depends on the level of speed technology: When $g_t \leq \hat{g}_t$, investing in speed is too costly for any investor and nobody acquires speed in equilibrium. Only for sufficiently advanced speed technology ($g_t > \hat{g}_t$) will there be some investors acquiring speed.[8] In fact, this same intuition holds in the other way:

Corollary 1. *Fixing the speed technology g_t, there exists a threshold $\hat{g}_h$ such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h$.*

That is, when the information technology is too poor, the benefit in information rent of becoming fast is not sufficient to compensate for the cost of acquiring speed. As such, all investors stay slow.

Several immediate features of this equilibrium are worth highlighting. First, the convexity of the information acquisition cost $c(\cdot)$ implies that $h(\tau)$ is decreasing in τ (implicit function theorem

[8] However, there are always non-zero mass of investors staying slow ($\mu_\mathrm{S} > 0$). To see the reason, suppose there is an equilibrium where all investors are fast, i.e., $\mu_\mathrm{F} = 1$ and $\mu_\mathrm{S} = 0$. In this case there is no price discovery in the late fragment, i.e., $\tau_1 = \tau_2$. Equation (7) then suggests that the marginal fast investor is strictly better off if he instead does not invest in the speed technology, saving the speed acquisition cost $1/g_t$. Hence, some fast investors will deviate to staying slow.

applied to Equation 6). As such, fast investors always acquire more information than the slow:

$$h_{\mathrm{F}} \geq h_{\mathrm{S}}. \tag{9}$$

This is because the price discovery process is always cumulative: $\tau_2 \geq \tau_1$, as the market never forgets whatever has been revealed ($\Delta\tau_t \geq 0$ by Equation 8). The earlier an investor can trade, the less price discovery the market has seen and the more valuable is his private information. To take this advantage, fast investors always have stronger incentive to acquire more information. This equilibrium result supports a popular connotation for fast traders that they are also more informed; see Menkveld (2016) for a survey of both theory and evidence.

Second, note that the price discovery $\Delta\tau$ is *non*linear in the population size μ of the trading round. Under the current parametrization, fixing h_{F} and h_{S}, $\Delta\tau$ is convexly increasing in μ. Such convexity, inherent from Grossman and Stiglitz (1980) and Verrecchia (1982), suggests that price discovery has increasing returns to scale: Each marginal informed investor's trading resolves increasingly more uncertainty (from noise trading).

Finally, the population size pair μ_{F} and μ_{S} has an alternative interpretation: Investors' ex ante probability mix between becoming fast or staying slow. At $t = 0$, each investor independently chooses to acquire $(t_i, h_i) = (1, h_{\mathrm{F}})$ with probability μ_{F} or to acquire $(2, h_{\mathrm{S}})$ with probability $\mu_{\mathrm{S}} = 1 - \mu_{\mathrm{F}}$. Under this interpretation, $\mu_{\mathrm{F}} \in [0, 1]$ is an individual investor's demand for speed, while $\mu_{\mathrm{F}} h_{\mathrm{F}} + \mu_{\mathrm{S}} h_{\mathrm{S}}$ is his demand for information.

4.3 Two benchmarks

In order to provide a clear contrast of the results, Section 5 will begin with two constrained versions of the model, where the acquisition of one of the two technologies is shut down. The following two corollaries provide the existence and the uniqueness of equilibrium in these two benchmarks. As both are special cases of Proposition 1, for brevity, their proofs are omitted.

Corollary 2 (Benchmark 1: exogenous speed). *Fix each investor's speed t_i with exogenous μ_{F} and μ_{S} $(= 1 - \mu_{\mathrm{F}})$. Then there exists a unique equilibrium in which fast and slow investors' information acquisition, h_{F} and h_{S}, solve the first-order conditions* (6).

When the speed technology is not available, only the interior case of Proposition 1 is relevant. Further, since the investors cannot choose speed, the indifference condition $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$ becomes irrelevant. Only the "optimal information acquisition" condition remains.

Corollary 3 (Benchmark 2: exogenous information). *Fix fast and slow investors' information acquisition at h_{F} and h_{S}, respectively. Then there exists a unique equilibrium, depending on the speed technology g_t relative to a threshold $\hat{g}_t$ (> 0):*

Case 1 (corner). *When $g_t \leq \hat{g}_t$, all investors stay slow with $\mu_{\mathrm{F}} = 0$ and $\mu_{\mathrm{S}} = 1$.*

Case 2 (interior). *When $g_t > \hat{g}_t$, a mass $\mu_{\mathrm{F}} \in (0, 1)$ of investors acquire speed and become fast, while the rest μ_{S} stay slow. The equilibrium population sizes $\{\mu_{\mathrm{F}}, \mu_{\mathrm{S}}\}$ uniquely solve $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$ and $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1$.*

Corollary 3 is also a special case of Proposition 1, where the "optimal information acquisition" condition is dropped in the interior equilibrium as investors' signal precision are exogenously fixed.

5 Equilibrium properties and implications

This section studies investors' endogenous technology acquisition and the effects on market quality. Three issues stand out: How does an advancement in one technology affect 1) investors' investment in it, 2) investors' investment in the other technology, and 3) the overall price informativeness.

In order to isolate the different implications of the two technologies, the analysis begins by exploring the two benchmarks: Section 5.1 switches off speed acquisition and Section 5.2 information. Section 5.3 then studies the joint effects. As a preview, the results are summarized in Table 1. The shaded cells highlight the key findings: [1] the temporal fragmentation of speed

	(1) Information acquisition			**(2)** Speed acquisition			**(3)** Price informativeness	
	h_{F}	h_{S}	$\int_0^1 h_i \mathrm{d}i$	μ_{F}	μ_{S}	$\int_0^1 \mathbb{1}_{\{t_i=1\}} \mathrm{d}i$	τ_1	τ_2
(a) Exogenous speed and endogenous information (Section 5.1)								
g_h:	$\nearrow$	$\nearrow$	$\nearrow$				$\nearrow$	$\nearrow$
(b) Exogenous information and endogenous speed (Section 5.2)								
g_t:				$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow\nearrow$[1]
(c) Endogenous speed and endogenous information (Section 5.3)								
g_h:	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow\searrow$	$\searrow\nearrow$	$\nearrow\searrow$[2]	$\nearrow$	$\searrow\nearrow$[3]
g_t:	$\searrow$	$\nearrow\searrow$	$\nearrow\searrow$[2]	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow\nearrow$

Table 1: Summary of effects of technology shocks. This table summarizes how technology affects the market in terms of (1) investors' information acquisition; (2) speed acquisition; and (3) price informativeness. Both the short-run (h_{F}, μ_{F}, and τ_1) and the long-run (h_{S}, μ_{S}, and τ_2) effects are shown, together with investors' aggregate demand for information ($\int_0^1 h_i \mathrm{d}i$) and for speed ($\int_0^1 \mathbb{1}_{\{t_i=1\}} \mathrm{d}i$). Three settings are considered: investors (a) have exogenous speed but can endogenously acquire information; (b) have exogenous information but can endogenously acquire speed; and (c) can endogenously acquire both speed and information. Each row represents a positive shock in the respective technology, g_h for information and g_t for speed. A monotone increasing (decreasing) response to the shock is indicated by $\nearrow$ ($\searrow$), while a hump-shape (U-shape) by $\nearrow\searrow$ ($\searrow\nearrow$). Shaded cells highlight the key findings.

(Proposition 4); [2] the complementarity and substitution between speed and information (Proposition 5); and [3] the non-monotone effect of information technology on price informativeness (Proposition 6). A number of model applications are then discussed in Section 5.3.4 in the context of empirical predictions and existing evidence.

5.1 Information acquisition with exogenous speed

This subsection sets the benchmark where investors with fixed speed can only acquire information. The equilibrium corresponds to Corollary 2. That is, there is a fixed mass $\mu_{\mathrm{F}} \in [0, 1]$ of investors who are fast ($t_i = 1$) and the rest $\mu_{\mathrm{S}} = 1 - \mu_{\mathrm{F}}$ investors slow ($t_i = 2$). All results are with respect to the information technology g_h (see Panel (a) of Table 1).

Proposition 2 (Information technology and information acquisition). *Fix the fast and the slow investors' sizes* μ_F *and* μ_S. *As the information technology* g_h *increases, both the fast and the slow investors individually acquire more information:* $\partial h_i / \partial g_h > 0$ *for* $i \in \{F, S\}$.

The result is not surprising. As g_h increases, each investor can acquire more precise information at the same expense. That is, information becomes relatively cheaper and all investors, fast or slow, acquire more of it. Panel (a) of Figure 2 illustrates this effect. The red-dashed line also plots the total information acquisition in the economy, $\int_{i \in [0,1]} h_i \mathrm{d}i = \mu_F h_F + \mu_S h_S$.

As all investors acquire more information, the price becomes more efficient as well:

Corollary 4 (Information technology and price informativeness). *Fix the fast and the slow investors' sizes* μ_F *and* μ_S. *As the information technology* g_h *increases, both the short-run and the long-run price informativeness improve. Mathematically,* $\partial \tau_1 / \partial g_h > 0$ *and* $\partial \tau_2 / \partial g_h > 0$.

Recall from Equation (8) that $\Delta\tau = \tau_U h^2 \mu^2 / \gamma^2$. Because the population sizes $\{\mu_F, \mu_S\}$ are exogenously fixed and because information acquisition h_i monotonically increases with g_h, so does the price discovery $\Delta\tau$. Panel (b) of Figure 2 graphically illustrates the corollary.

5.2 Speed acquisition with exogenous information

This subsection sets the other benchmark, where investors have endowed signals with fixed precision $h_i = h_\circ$ (> 0) but can endogenously acquire speed. The equilibrium corresponds to Corollary 3. All results, summarized in Panel (b) of Table 1, are with respect to the speed technology g_t.

A better speed technology g_t reduces investors' cost to be fast. The usual price effect applies and the demand for speed increases accordingly, as illustrated in Panel (a) of Figure 3.

Proposition 3 (Speed technology and speed acquisition). *Fix all investors' signal precision at* $h_i = h_\circ$ (> 0). *In the interior equilibrium, as the speed technology* g_t *advances, more investors acquire speed:* $\partial \mu_F / \partial g_t > 0$.

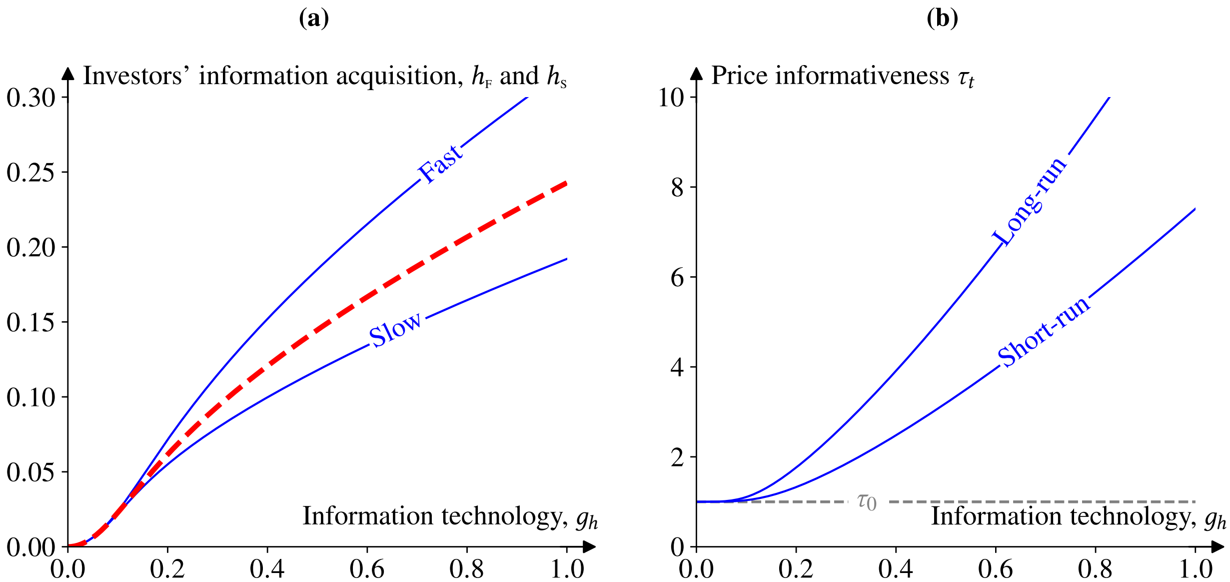

Figure 2: Varying information technology with fixed speed. Panel (a) shows how information technology g_h affects individual investors' information acquisition h_i and Panel (b) price informativeness τ_t. The red-dashed line in Panel (a) plots the aggregate demand for information $\int_{i\in[0,1]} h_i \mathrm{d}i$. The primitive parameters used are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. The fast investor's population size is fixed at $\mu_F = 0.4$; and, hence, $\mu_S = 0.6$.

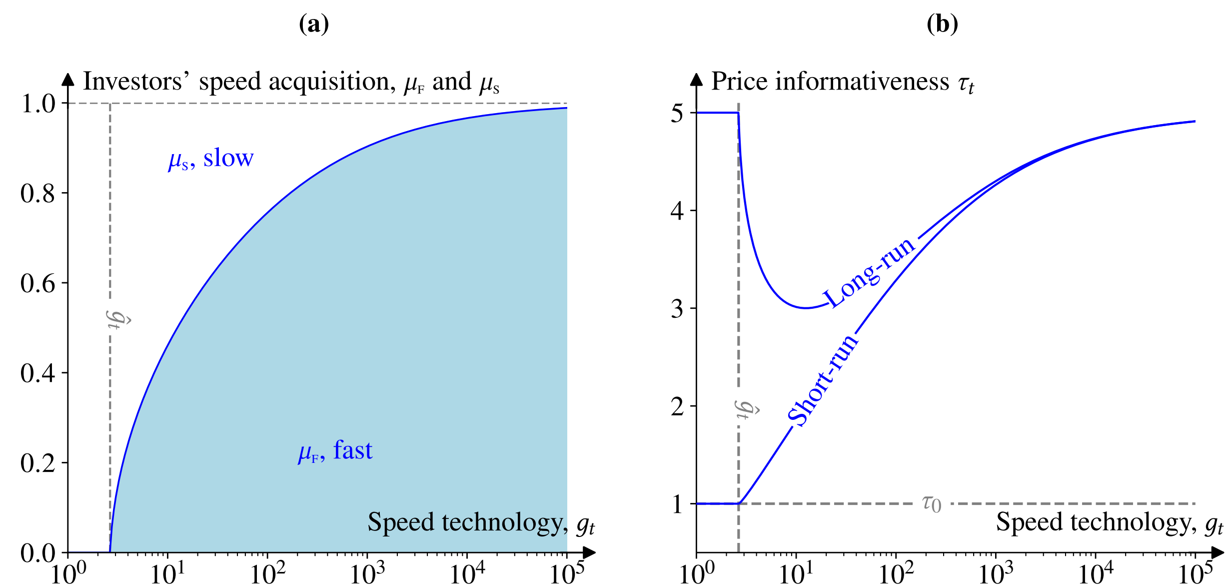

Figure 3: Varying speed technology with fixed information. Panel (a) shows how speed technology g_t affects individual investors' speed acquisition t_i and Panel (b) price informativeness τ_t. To the right of the vertical dashed line, the equilibrium is interior (with both fast and slow investors). The primitive parameters are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. The common signal precision is fixed at $h_\circ = 0.1$.

As more investors acquire speed, the short-run price informativeness τ_1 increases. However, the speed technology exerts a non-monotone effect on the long-run τ_2, as shown in Panel (b) of Figure 3. This is because the speed technology *temporally fragments* price discovery. When the speed technology is affordable (beyond the threshold $\hat{g}_t$), a fraction μ_F of the investors become fast and trade at $t = 1$, while the rest μ_S ($= 1 - \mu_\mathrm{F}$) stay slow and trade at $t = 2$. The price discovery process accordingly fragments into an early $\Delta\tau_1$ and a late $\Delta\tau_2$. From Equation (8), the early fragment increases with g_t:

$$\Delta\tau_1 = \frac{\tau_\mathrm{U}}{\gamma^2} h_\circ^2 \mu_\mathrm{F}^2,$$

as μ_F is increasing with g_t (Proposition 3). However, the late fragment drops:

$$\Delta\tau_2 = \frac{\tau_\mathrm{U}}{\gamma^2} h_\circ^2 \mu_\mathrm{S}^2 = \frac{\tau_\mathrm{U}}{\gamma^2} h_\circ^2 \cdot (1 - \mu_\mathrm{F})^2.$$

The long-run $\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2$ is subject to the joint force of both fragments of price discovery and, therefore, exhibits a nonmonotonic trend in the speed technology g_t. This result is highlighted in "[1]" in Table 1 and formally stated below.

Proposition 4 (Speed technology and price informativeness). *Fix all investors' signal precision at $h_i = h_\circ$ (> 0). In the interior equilibrium, as the speed technology g_t advances, the short-run price informativeness τ_1 monotonically increases, while the long-run τ_2 first decreases and then increases. Mathematically, $\partial\tau_1/\partial g_t > 0$; and $\partial\tau_2/\partial g_t < 0$ (> 0) for small (large) g_t.*

In particular, Proposition 4 describes that the long-run price informativeness τ_2 as U-shape in g_t. This U-shape arises from the fact that each fragment of price discovery, $\Delta\tau$, is a *convex function* in the population size μ. That is, price discovery has increasing returns to scale, consistent with, e.g., Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). Therefore, the impact of a marginal change in μ (due to speed technology) on τ depends on the initial level of μ. For example, when μ_F close to zero and μ_S to one (when $g_t \downarrow \hat{g}_t$), a small increase in speed $\mathrm{d}g_t$ prompts a small population $\mathrm{d}\mu_\mathrm{F}$ to move from slow to fast. The resulting loss in the late fragment $\Delta\tau_2$ is much larger than the gain in the early $\Delta\tau_1$:

$$\text{(10)} \qquad d\tau_2 = \frac{\partial \tau_2}{\partial \mu_F} d\mu_F = \left(\frac{\partial \Delta\tau_1}{\partial \mu_F} + \frac{\partial \Delta\tau_2}{\partial \mu_F} \right) d\mu_F = \frac{\tau_U}{\gamma^2} h_\circ^2 \underbrace{\frac{\partial}{\partial \mu_F} \left(\mu_F^2 + \mu_S^2 \right)}_{=2(2\mu_F - 1) < 0 \text{ for } \mu_F \text{ close to } 0} d\mu_F < 0.$$

The reverse holds true when μ_F is close to one and μ_S close to zero.

To sum up, Proposition 4 builds on three robust elements. First, price discovery has increasing returns to scale, i.e., $\Delta\tau_t$ being convex in μ_t, which is a generic feature in the rational expectations equilibrium literature. Second, the fast investors only contribute to the early fragment $\Delta\tau_1$, not the late $\Delta\tau_2$. Section 6.1 shows that this separation does not depend on the assumption that the fast only trade at $t = 1$. Even when allowed to trade at both dates, they still only trade on private signals at $t = 1$ (and uninformatively rebalance at $t = 2$). The intuition is that hoarding information in a competitive setting is suboptimal as the value of the signal is eroded overtime. Third, when μ_F increases, μ_S drops. This is by construction in the current setting as the population size is fixed. Section 6.2 shows this is robust even when investor participation is endogenous (free entry).

Section IV of Holden and Subrahmanyam (2002) studies a similar model to this benchmark. The temporal fragmentation effect exists in their setting as well but is unflagged (they do not study price informativeness). Compared to the setup in Section 3, the key difference is that investors' speed acquisition is fixed and bundled with information in their setting. The separation of information and speed is key in this paper and generates novel implications, as explored in Section 5.3 below.

5.3 Interaction between speed and information technology

Both speed and information are now made available to investors, and Proposition 1 holds together with Corollary 1. Suppose there is an advancement in one technology. The discussion below focuses three effects: 1) investors' acquisition in this technology; 2) investors' acquisition in the other technology; and 3) the price discovery function of the financial market.

5.3.1 Own-price effect: Acquisition in the advancing technology

When a technology advances (cheaper), the first-order effect is its own-price effect: increased demand responding to a lower price, consistent with Proposition 2 and 3.

> **Proposition 2 (continued).** *Whether investors' speed acquisition is exogenous or endogenous, in the interior equilibrium, investors' information acquisition monotonically increases with the information technology: $\partial h_i/\partial g_h > 0$ for $i \in \{F, S\}$.*

> **Proposition 3 (continued).** *Whether investors' information acquisition is exogenous or endogenous, in the interior equilibrium, as the speed technology advances, more investors acquire speed: $\partial \mu_{\mathrm{F}}/\partial g_t > 0$.*

Panel (a) and (b) of Figure 4 illustrate this intuitive own-price effect of the speed and the information technology. The patterns are qualitatively similar with Panel (a) in Figure 2 and in 3.

5.3.2 Cross-price effect: Are speed and information substitutes or complements?

The cross-price effects are graphed in Panel (c) and (d) in Figure 4. In both panels, it can be seen that the aggregate demand for one technology is first increasing but eventually decreasing when the other improves: The technologies can be *either complements or substitutes*.

> **Proposition 5 (Complementarity and substitution between speed and information).** *In the interior equilibrium, as one technology increases, fixing the other, investors' aggregate speed and information acquisition are initially complements but eventually substitutes. Mathematically, $\partial \mu_{\mathrm{F}}/\partial g_h > 0$ (< 0) for small (large) g_h; and $\partial(\mu_{\mathrm{F}} h_{\mathrm{F}} + \mu_{\mathrm{S}} h_{\mathrm{S}})/\partial g_t > 0$ (< 0) for small (large) g_t. In addition, $\partial h_{\mathrm{F}}/\partial g_t < 0$; but $\partial h_{\mathrm{S}}/\partial g_t > 0$ (< 0) for small (large) g_t.*

Such non-monotone cross-price effects are driven by various crowding-out forces—an increase in price informativeness τ_t hurts the certainty equivalent of whoever trades at or after t (Equation 7; Grossman and Stiglitz, 1980). Consider Panel (c) for example. An advancement in information g_h

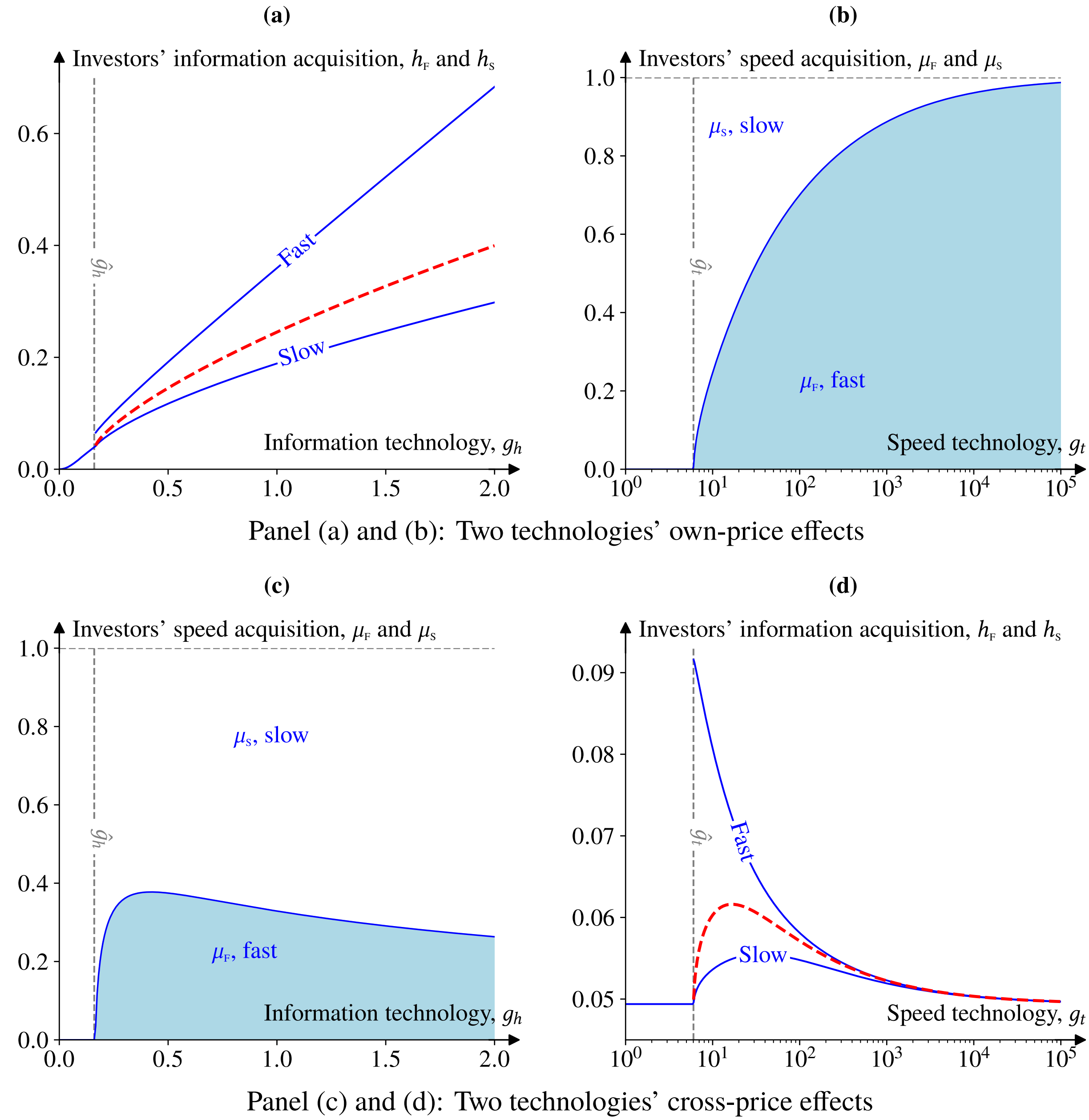

Figure 4: Technology acquisition. This figure illustrates how investors' technology acquisition (demand for speed and for information) are affected differently by levels of technologies. Panel (a) and (b) show the technologies' own-price effect. Panel (c) and (d) show the cross-price effect. The vertical dashed lines indicate the thresholds of the corresponding technology, below which all investors stay slow. The red-dashed lines in Panel (a) and (d) are the aggregate demand for information in the economy, $\int_{i\in[0,1]} h_i \mathrm{d}i$. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (b) and (d), $g_h = 0.2$. For Panel (a) and (c), $g_t = 10.0$.

stimulates all investors to acquire more information (Proposition 2). Three crowding-out effects arise: (1) *intra*temporally the fast at $t = 1$ crowd out each other; (1) similarly the slow at $t = 2$ crowd out each other; and (3) *inter*temporally the fast crowd out the slow. The first effect hurts fast investors' rent π_{F}, making them less willing to acquire speed—reducing demand for speed. The second and the third effects hurt the slow, pushing them to compete with the fast at $t = 1$ instead—raising demand for speed. It is these countervailing crowding-out effects that drive the net demand for the two technologies.[9]

This is the second contribution of the model, highlighted in "[2]" in Table 1. It reveals that the intrinsic complementarity and substitution between the two technologies are driven by various crowding-out effects, both intratemporal and interteporal.

5.3.3 Technology and price discovery

The effects of the technologies on price informativeness τ_t are illustrated in Figure 5. The patterns shown in Panel (a) are qualitatively similar to those shown in Panel (b) of Figure 3.

Proposition 4 (continued). *Whether investors' information acquisition is exogenous or endogenous, in the interior equilibrium, as the speed technology g_t advances, the short-run price informativeness τ_1 monotonically increases, while the long-run τ_2 initially decreases but eventually increases. Mathematically, $\partial\tau_1/\partial g_t > 0$; and $\partial\tau_2/\partial g_t < 0$ (> 0) for small (large) g_t.*

This suggests that even with endogenous information acquisition, the speed technology's temporal fragmentation effect dominates.

Turning to the information technology, Panel (b) of Figure 5 contrasts Panel (b) of Figure 2. Notably, information technology can hurt overall price informativeness τ_2.

[9] When the information technology g_h is low (close to $\hat{g}_h$), there are very few fast investors (μ_{F} close to zero). Therefore, Effect (2) dominates, stimulating slow investors to acquire speed and move to $t = 1$. As more investors have acquired speed, Effect (3) strengthens and the remaining slow investors have growing incentive to become fast. These two forces result in complementarity between speed and information. However, when there are too many fast investors, Effect (1) dominates: Each individual fast investor's rent is hurt too much by advancement in g_h. When it is no longer profitable to acquire speed, information substitutes speed, as shown in Panel (c). Panel (d) can be explained with these three crowding-out effects similarly.

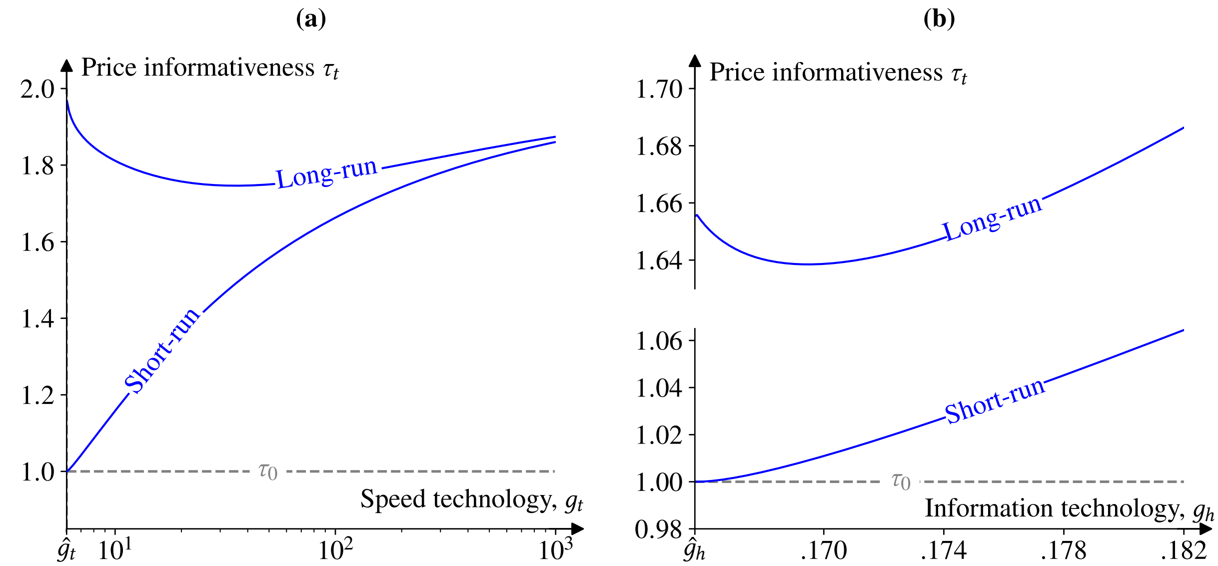

Figure 5: Price informativeness. This figure illustrates how the aggregate price informativeness τ_t is affected differently by different technologies. Panel (a) shows the response to varying speed technology g_t and Panel (b) to information technology g_h. To manifest the patterns, only the range with interior equilibrium is shown; i.e. $g_t > \hat{g}_t$ in Panel (a) and $g_h > \hat{g}_h$ in Panel (b). Further, the vertical axis in Panel (b) is split into two ranges, respectively, for the long-run and the short-run price informativeness. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_{\mathrm{U}} = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

Proposition 6 (Information technology and price informativeness). *In the interior equilibrium, advancement in the information technology always improves short-run informativeness τ_1. However, with endogenous speed acquisition, long-run informativeness τ_2 is initially hurt but eventually improved. Mathematically, $\partial\tau_1/\partial g_h > 0$; and $\partial\tau_2/\partial g_h < 0$ (> 0) for small (large) g_h.*

To see how information technology can hurt price informativeness, recall: [1] that speed technology temporally fragments price discovery; and [2] that the two technologies can exhibit complementarity (g_h close to the threshold $\hat{g}_h$). Therefore, when g_h improves from $\hat{g}_h$, due to the complementarity, investors acquire both speed and information. The temporal fragmentation of the price discovery process ensues, hurting the long-run price informativeness τ_2.

To see the above intuition mathematically, decompose τ_2 into

$$\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2 = \tau_0 + \frac{\tau_U}{\gamma^2}\Big(\mu_F^2 h_F^2 + \mu_S^2 h_S^2\Big).$$

There are four equilibrium objects affected by a (positive) shock in g_h: $\{h_F, h_S, \mu_F, \mu_S\}$. The direct effects on information acquisition are $\frac{\partial h_F}{\partial g_h} > 0$ and $\frac{\partial h_S}{\partial g_h} > 0$, as shown in Panel (a) of Figure 4 (Proposition 2). Indirectly, speed acquisition complements information when g_h is close to $\hat{g}_h$, i.e., $\frac{\partial \mu_F}{\partial g_h} > 0$ and $\frac{\partial \mu_S}{\partial g_h} < 0$, as shown in Panel (c) of Figure 4 (Proposition 5). It turns out that the dominating effect is the the drop in μ_S:

$$\frac{\partial \tau_2}{\partial g_h} \approx \frac{\partial \tau_2}{\partial \mu_F}\frac{\partial \mu_F}{\partial g_h} + \frac{\partial \tau_2}{\partial \mu_S}\frac{\partial \mu_S}{\partial g_h} = \frac{2\tau_U}{\gamma^2} h_F^2 \underbrace{\mu_F \frac{\partial \mu_F}{\partial g_h}}_{\downarrow 0} + \frac{2\tau_U}{\gamma^2} h_S^2 \underbrace{\mu_S \frac{\partial \mu_S}{\partial g_h}}_{<0} < 0,$$

as when g_h is close to $\hat{g}_h$, μ_F close to zero and μ_S to one. Note that the above is essentially the same temporal fragmentation effect analyzed in Equation (10).

This is the third key finding of the model, highlighted in "[3]" in Table 1. It cautions against how information technology might negatively impact price informativeness, *through the channel of speed acquisition*. A number of recent studies share qualitatively similar caveats. For example, Dugast and Foucault (2017) and Kendall (2017) show that the acquisition of raw information can crowd out processed information, thus hurting the overall price informativeness. Banerjee, Davis, and Gondhi (2017) show that a public announcement could worsen price informativeness, because investors would switch to learning about others' beliefs instead of the fundamental. Both mechanisms feature some *substitution* between information of different sources. To compare, the novel mechanism revealed here is due to the joint effect of *the endogenous complementarity* between information and speed (Proposition 5) and the temporal fragmentation of speed (Proposition 4). Note that the effect does not exist if speed acquisition is shutdown; c.f. Panel (a) of Table 1 and Panel (b) of Figure 2.

5.3.4 Applications

The propositions developed above yield a number of applied results. The applications build on the interpretation (Remark 1 and 2) that the fast investors can be thought of as hedge funds, who trade on information sooner than mutual funds at various frequencies.

Investor composition. The contrast between Panel (b) and (c) of Figure 4 produces testable predictions on investor composition—hedge funds' participation in different securities' trading. In the equity market, for example, some stocks have higher analyst coverage and more media exposure than others. Investors' information acquisition cost in these stocks should be relatively lower. An empiricist can sort stocks according to their analyst coverage or media exposure. The proportion of fast investors (hedge funds or proprietary trading firms) should exhibit a hump-shape, similar to the pattern outlined in Panel (c). To the extent that large stocks have higher analyst coverage and media exposure, the predicted hump-shape exactly matches the empirical finding by Griffin and Xu (2009, Figure 3). Following speed technology boosts (e.g., the democratization of microwave transmission in late 2012; see Shkilko and Sokolov, 2016), one should see holdings by fast investors, by and large, increase as shown in Panel (b).

Investment in information. Similarly, the comparison between Panel (a) and (d) of Figure 4 yields predictions on investors' information acquisition. Notably, upon a speed technology shock, fast investors like hedge funds reduce their individual information acquisition, while slow ones like mutual funds' responses are uncertain. This prediction complements the literature in two ways. First, it specifically establishes an *endogenous* link between speed and investors' information acquisition (the two technologies are by construction orthogonal; Remark 4). To compare, for example, Holden and Subrahmanyam (2002) bundle speed and information, while Dugast and Foucault (2017) assume an exogenous tradeoff between raw (fast) and processed information (slow). Second, it looks at an investor's *individual* signal precision, conditional on his speed. For example, in Holden and Subrahmanyam (1996, 2002), investors learn precisely about the asset payoff by paying a fixed cost, rather than choosing the amount of information to affect signal precision. Similarly, in Dugast and Foucault (2017), neither a fast or a slow investor can invest more to acquire higher precision.

To test this prediction, an empiricist should focus on an exogenous speed shock and needs to observe investors' individual information acquisition before and after. Under the high-frequency interpretation of speed, for example, a speed technology shock can be the technology upgrade by an exchange. While directly observing institutions' information acquisition expenditure might be difficult, the profit and loss can serve as a rough proxy as presumably there is a monotone relationship between information precision and trading profit.

Fund performance. The model predicts that hedge fund (fast) and mutual fund (slow) performances are affected differently by technologies. An investor's (a fund's) trading performance can be measured as

$$\mathbb{E}[(V - P_1)x_i(s_i, P_1)] = \frac{h_{\mathrm{F}}}{\gamma\tau_1}, \text{ if } i \text{ is fast } (t_i = 1);$$
$$\mathbb{E}[(V - P_2)x_i(s_i, P_2)] = \frac{h_{\mathrm{S}}}{\gamma\tau_2}, \text{ if } i \text{ is slow } (t_i = 2).$$

Note that the performance can be equivalently interpreted as the *return predictability* of funds' holdings: $\mathrm{cov}[V - P_t, x_i] = \mathbb{E}[(V - P_t)x_i]$. Intuitively, a fund's performance (information rent) is higher if and only if it predicts future return more precisely.

Panel (a) of Figure 6 shows that the speed technology monotonically hurts fast funds ' performance. That is, the return predictability of their holdings lowers with the speed technology. This is due to the intensified crowding out effect among the fast at $t = 1$. Instead, slow funds' performance is non-monotonically affected: The initial increase is due to the reduced competition at $t = 2$ (as some investors have acquired speed and traded early). The eventual decrease is because there have been too many fast trading, extracting most of the informational rent, and by the time the slow investors trade there is not much rent left.

Panel (b) shows that both fast and slow investors' performance exhibit hump-shapes in infor-

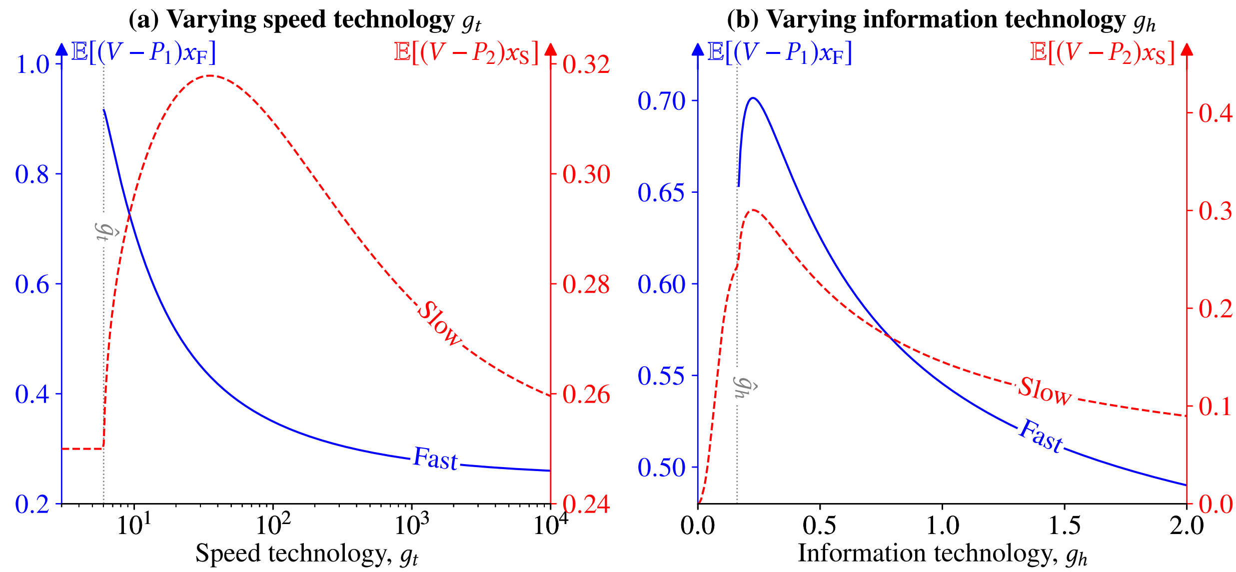

Figure 6: Fund performance. This figure illustrates fast and slow funds' performance. The level of the speed technology g_t varies in Panel (a), while the level of the information technology g_h varies in Panel (b). In each panel, the blue-solid (the red-dashed) line shows the expected trading profit for the fast (the slow). The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

mation technology g_h. Due to the initial complementarity, more traders acquire both speed and information. This reduces the *intra*temporal competition among the slow, improving their performance. In the meantime, the increasing information technology overcomes the mild competition among the fast, also improving their performance. However, as all traders acquire more and more information, prices become very revealing, eventually crowding out everyone's rent. Both the fast and the slow funds' performance worsens.

Trading volume. The aggregate trading volume across investors of speed-t is given by

$$\int_{i\in[0,1]} \mathbb{1}_{\{t_i=t\}} |x(s_i, p_t)| \mathrm{d}i = \mu_F \mathbb{E}\left|\frac{h_t}{\gamma}(s_i - p_t)\right| = \frac{\mu_t h_t}{\gamma}\sqrt{\frac{2}{(\tau_t + h_t)\pi}},$$

where μ_t and h_t are the population size and the individual information acquisition of speed-t investors. Figure 7 below shows how the aggregate trading volume is affected by technologies.

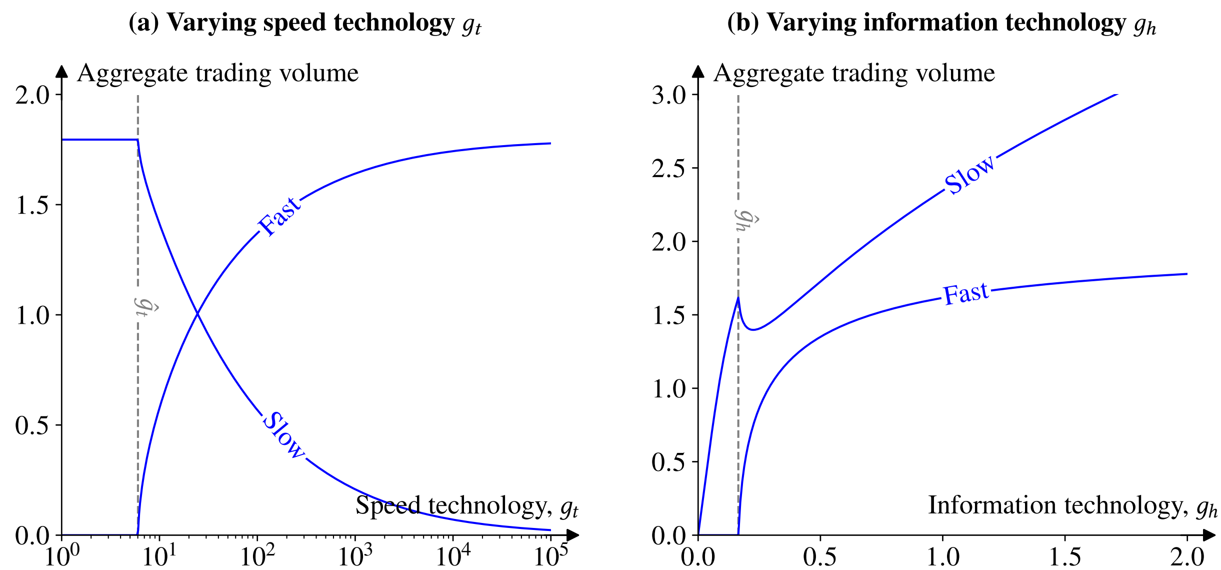

Figure 7: Trading volume and technologies. This figure illustrates how trading volume is affected differently by different technologies. The level of the speed technology g_t varies in Panel (a), while the level of the information technology g_h varies in Panel (b). Fast and slow investors' aggregate volume are separately plotted in each panel. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

As either technology improves, hedge funds' (fast) total trading volume monotonically increases. The main driver is the fast population size μ_F. In the case of an increasing g_t, this is a direct effect of cheaper speed technology. In the case of an increasing g_h, μ_F grows initially due to the complementarity. While eventually information starts to substitute speed (μ_F starts to decrease with g_h), each fast investors' trading aggressiveness h_F/γ increases and dominates.

The pattern, however, differs for mutual funds' (slow) total trading volume. As speed technology g_t increases, more investors become fast and fewer remain slow, reducing the total trading volume. As information technology g_h increases, mutual funds' volume is first increasing, then decreasing, and finally increasing again. The increasing ranges—when $g_h < \hat{g}_h$ and eventually g_h sufficiently large—are due to improved signal precision h_S, which make each individual investor trade more aggressively. The decreasing range is due to the complementarity between information and speed: Better information prompts more investors to acquire speed and become fast.

6 Discussion and robustness

This section studies some model extensions (Remark 3) and demonstrates robustness of the results. Section 6.1 allows the fast investors to trade more frequently. Section 6.2 endogenizes population size. Section 6.3 replaces the competitive market maker with a fringe of uninformed investors. Section 6.4 studies the interdependence between the two technologies. Section 6.5 considers time-varying noise trading.

6.1 Frequent fast trading

There are two aspects of being fast: to trade early and to trade frequently. The baseline model focuses on trading early, restricting the fast to trade only at $t = 1$. This appears to be critical for the temporal fragmentation effect (Section 5.2): If the fast trade at $t \in \{1, 2\}$ (more frequently), will they contribute to price discovery "smoothly" overtime, overturning the fragmentation effect?

The answer is no. Section A in Supplementary Material studies such an extension, keeping all other model structure as in the baseline. It is shown that all results regarding investors' trading stated in Lemma 1 remain the same. Notably, an investor's *cumulative* demand in round t holds in the same form of $x_{it} = \frac{h_i}{\gamma}(s_i - p_2)$; that is, he is trading on the signal-price difference and amplifies/dampens this difference with his risk-aversion adjusted signal precision. As such, a fast investor's *net* demand at $t = 2$ is $x_{i2} - x_{i1} = \frac{h_i}{\gamma}(p_1 - p_2)$, which does not depend on his private signal s_i. He simply rebalances his position based on the new price p_2 and reveals no new information at $t = 2$. Intuitively, hoarding information in a competitive setting is suboptimal, as the value of the signal is eroded overtime. As a result, the recursions of price informativeness τ_t and of the price p_t remain exactly the same as in Equation (4) and (5). The temporal fragmentation effect (Proposition 4) still holds.

In fact, the only difference this extension creates lies in the fast investors' certainty equivalent:

$$\pi_{\mathrm{F}} = \frac{1}{2\gamma} \ln\left(1 + \frac{h_{\mathrm{F}}}{\tau_1} + \frac{h_{\mathrm{F}}^2}{\tau_2^2} \frac{\Delta\tau_2}{\tau_1}\right) - c(h_{\mathrm{F}}) - \frac{1}{g_t}. \tag{11}$$

Compared to Equation (7), there arises an extra term inside the $\ln(\cdot)$ operator. This positive term represents fast investors' additional information rent from trading again at $t = 2$. This only difference does not alter any of the key intuition in the baseline. All results studied in Section 5 go through this extension. Figure 8 graphically demonstrate the main results (corresponding to the highlighted ones in Table 1).

6.2 Endogenous population size

In the baseline model, the population size is fixed at $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1$. This appears to "mechanically" create the temporal fragmentation effect: As the speed technology prompts investors to acquire speed, a higher μ_{F} implies a lower μ_{S}, thus temporally fragmenting price discovery.

An extension in Section B in Supplementary Material studies the robustness of the results by endogenizing total investor population size. The analysis is briefly summarized here. Compared to the baseline, the only modification is the set of investors: There is a continuum of investors indexed on $i \in [0, \infty)$, and following the literature (see, e.g., Bolton, Santos, and Scheinkman, 2016), they are sorted according to their reservation value $R(i)$ for not trading. Specifically, if investor i chooses not to trade, he obtains a certainty equivalent of $R(i)$, which is monotone increasing in i. Equivalently, $R(i)$ can be interpreted as investor i's entry cost and $\forall i < j$, investor i has higher comparative advantage in trading. To ensure participation, normalize $R(0) = 0$.

As no other model assumptions are changed, investors trade just like in the baseline and Lemma 1 holds. Further, the (interior) equilibrium is pinned down by conditions similar to those stated in Proposition 1. Each investor optimizes his signal precision according to first-order conditions $\partial\pi_{\mathrm{F}}/\partial h_{\mathrm{F}} = \partial\pi_{\mathrm{S}}/\partial h_{\mathrm{S}} = 0$; and should be indifferent between fast or slow: $\pi_{\mathrm{F}} - R(i) = \pi_{\mathrm{S}} - R(i)$.

The condition that determines the total population size is

$$R(\mu_{\mathrm{F}} + \mu_{\mathrm{S}}) = \pi_{\mathrm{F}} = \pi_{\mathrm{S}},$$

which says that the marginal investor is indifferent between trading or not. Equivalently, if the monotonicity of $R(\cdot)$ is strict, $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = R^{-1}(\pi_{\mathrm{F}}) = R^{-1}(\pi_{\mathrm{S}})$. To compare, the population size condition under the baseline is $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1$.

Consider a speed technology shock in g_t, after which μ_{F} increases (more fast investors). To illustrate intuition clearly, assume information acquisition is exogenous as in Section 5.2. More fast investors imply more price discovery in the short-run; i.e., $\Delta\tau_1$ increases. This crowds out slow investors because $\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2$. Having resolved more price discovery at $t = 1$, there is less information rent left for the slow. Therefore, there will be fewer slow investors, μ_{S} decreases—the temporal fragmentation effect.[10]

The key intuition behind the robustness of temporal fragmentation is simply that *the fast and the slow compete for the same piece of pie*. When the speed technology benefits the fast, some of the slow are hurt (relative to the fast) and therefore must be crowded out from $t = 2$. Either they become fast as in the benchmark where the total population is fixed, or they stay out of trading under endogenous entry. Figure 9 graphically reproduces the main results of the baseline model.

6.3 The market clearing mechanism

Investors' demand schedules are cleared by a competitive market maker, who can take any position at the efficient price. The purpose of having such a market maker is that he helps ensure the trading price p_t is always semi-strong efficient (as in Kyle, 1985; and Vives, 1995) and this suits the focus on price informativeness of this study.

[10] More rigorously, the result can be proved by contradiction. Suppose μ_{S} (weakly) also increases with g_t. Then the marginal investors' certainty equivalent $\pi_{\mathrm{S}} = \pi_{\mathrm{F}} = R(\mu_{\mathrm{F}} + \mu_{\mathrm{S}})$ must increase to support the additional entry. Notably, a higher π_{S} can only be achieved with a lower τ_2 (less price discovery, hence more information rent left; Equation 7). But this leads to a contradiction as the increasing μ_{F} and μ_{S} imply a higher τ_2: more informed investors, more price discovery. Thus, a higher μ_{F} must be accompanied by a lower μ_{S}.

A market maker is not the only way to facilitate trading. An alternative is to determine the price p_t by market clearing, as in Grossman and Stiglitz (1980) and Verrecchia (1982). An extension in Section C of Supplementary Material re-examines the model with the competitive market maker replaced by a fringe of uninformed investors of mass n. All other model specifications remain the same as in Section 3. It is shown that an informed investor i's *cumulative* demand at round t always has the well-known form

$$x_{it} = \frac{h_i}{\gamma}(s_i - m_t) - a_{it} \cdot (p_t - m_t)$$

where p_t is the market clearing price, $m_t := E[V | p_t, p_{t-1}, \dots]$ is the semi-strong efficient price, and $a_{i,t}$ is some constant up to investor i and time t. Note that this generalizes the baseline demand, because when the competitive market maker exists and sets $p_t = m_t$, the above demand reduces to $x_{it} = \frac{h_i}{\gamma}(s_i - p_t)$ as seen in Lemma 1.

It turns out that under this setup all results studied in Section 5 remain robust. Figure 10 graphically illustrate the main ones. Notably, as investors still trade on private signals s_i with the same aggressiveness as in the baseline, the price discovery recursion of $\Delta\tau_t$ remains the same as stated in Equation (8). It should be emphasized that allowing frequent fast trading in this setting still does not affect price discovery. This is because fast investors' *net* demand at $t = 2$, $x_{F2} - x_{F1}$, does not depend on the private signal s_i—they only contribute to the early fragment of price discovery $\Delta\tau_1$. This is consistent with Section 6.1: At $t = 2$, a fast investor trades again only to rebalance his holding according to the new price p_2, not to recycle his private information.

6.4 Dependence between the two technologies

In the model, the acquisition of one technology does not affect the cost of the other (Remark 4). Such independence need not necessarily be the case. The two can complement each other, for example, because both technologies require common hardware (CPUs, cables, and optical fiber, etc.). Having invested for one technology can reduce the cost for the other (e.g., g_h increases in g_t).

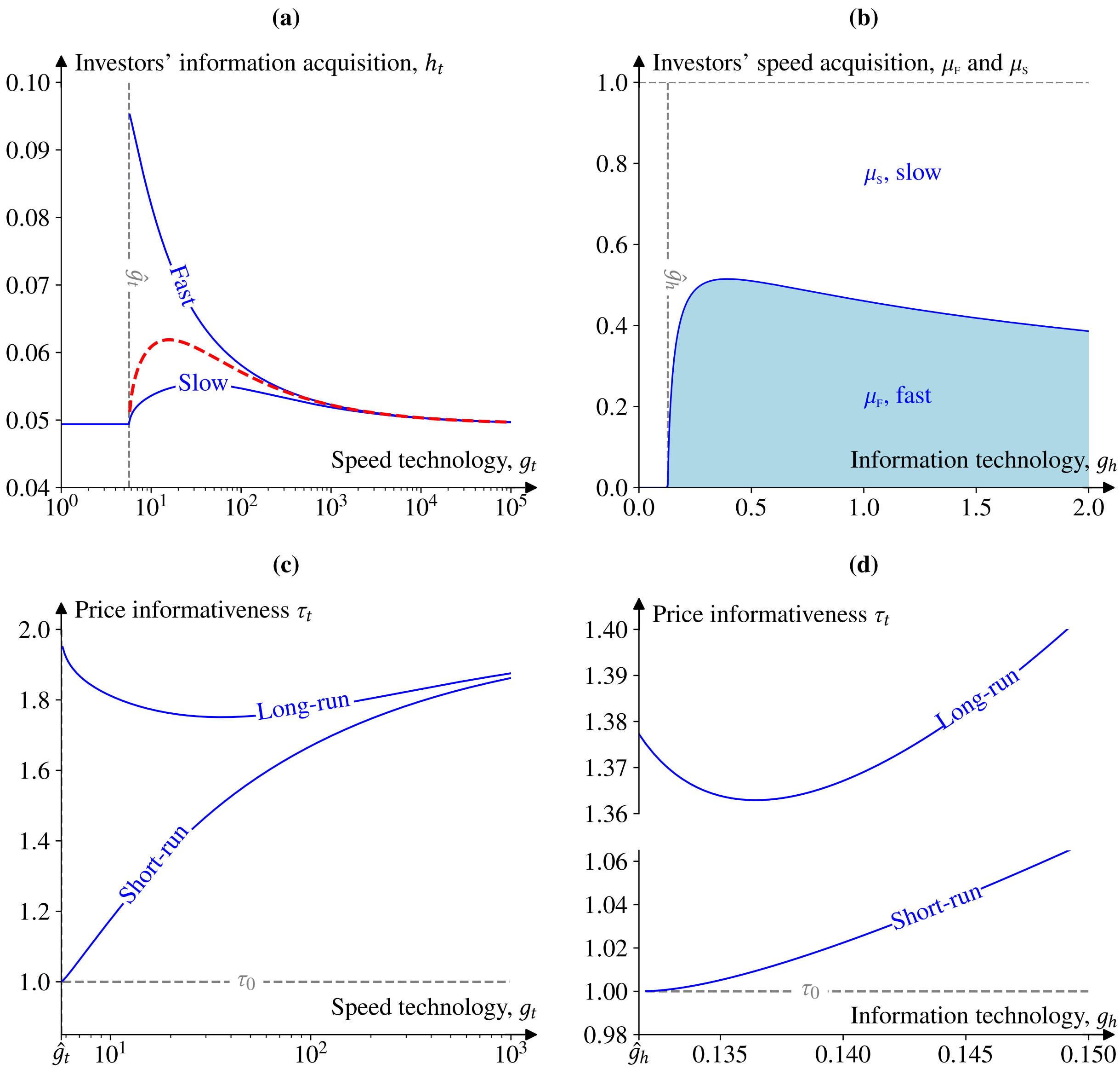

Figure 8: Robustness: Main results under frequent fast trading. This figure demonstrates that under frequent fast trading, the three key results stand robust: Panel (a) and (b) replicate Panel (c)d and (d) of Figure 4 and show that the two technologies can be either complements or substitutes. Panel (c) and (d) replicate Figure 5, showing that both technologies can hurt price informativeness. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a) and (c), $g_h = 0.2$. For Panel (b) and (d), $g_t = 10.0$.

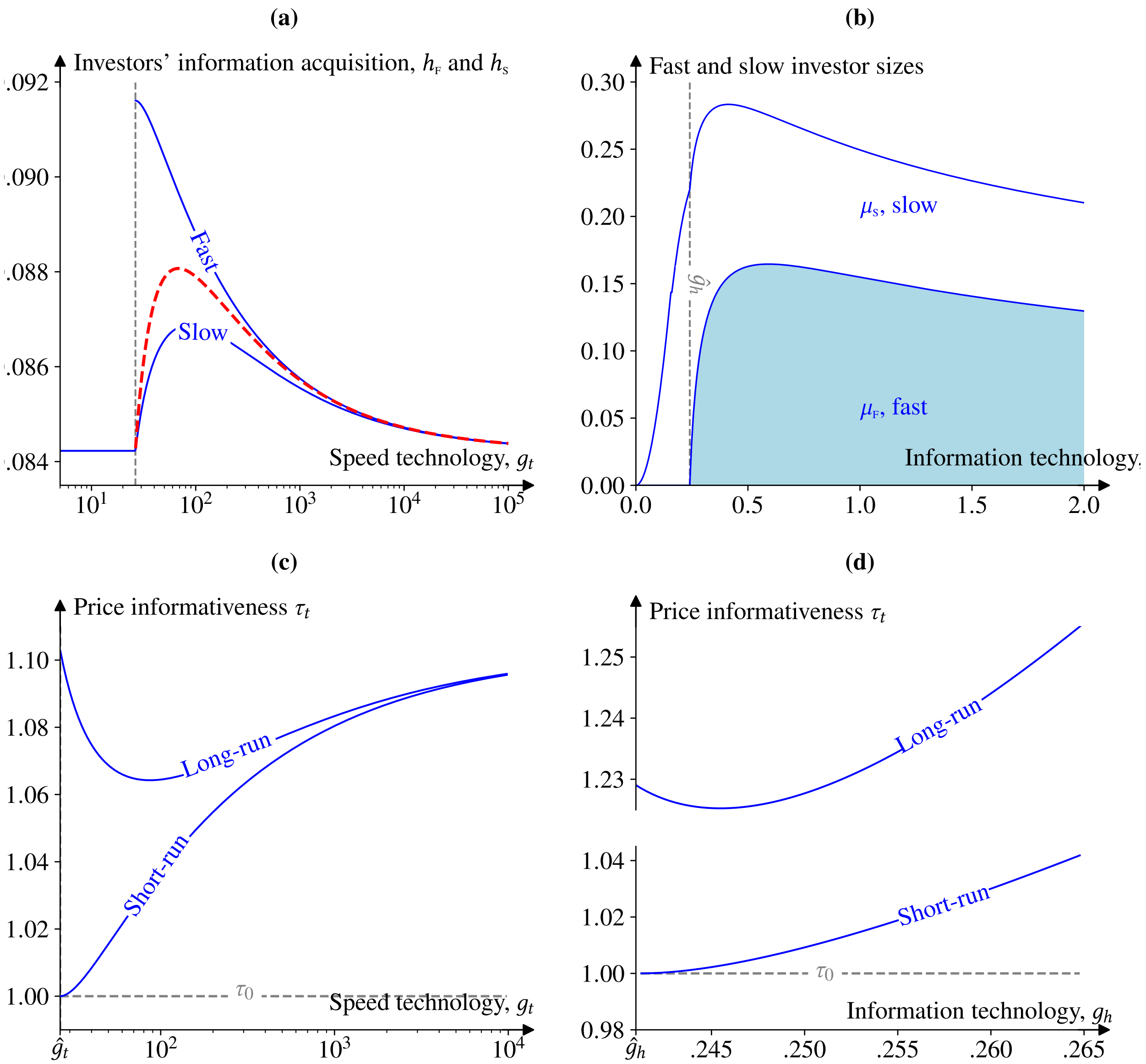

Figure 9: Robustness: Main results under endogenous population size. This figure demonstrates that under endogenous population size, the three key results stand robust: Panel (a) and (b) replicate Panel (c)d and (d) of Figure 4 and show that the two technologies can be either complements or substitutes. Panel (c) and (d) replicate Figure 5, showing that both technologies can hurt price informativeness. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, $k_h(m) = \sqrt{m}$, and $R(i) = i$. For Panel (a) and (c), $g_h = 0.2$. For Panel (b) and (d), $g_t = 10.0$.

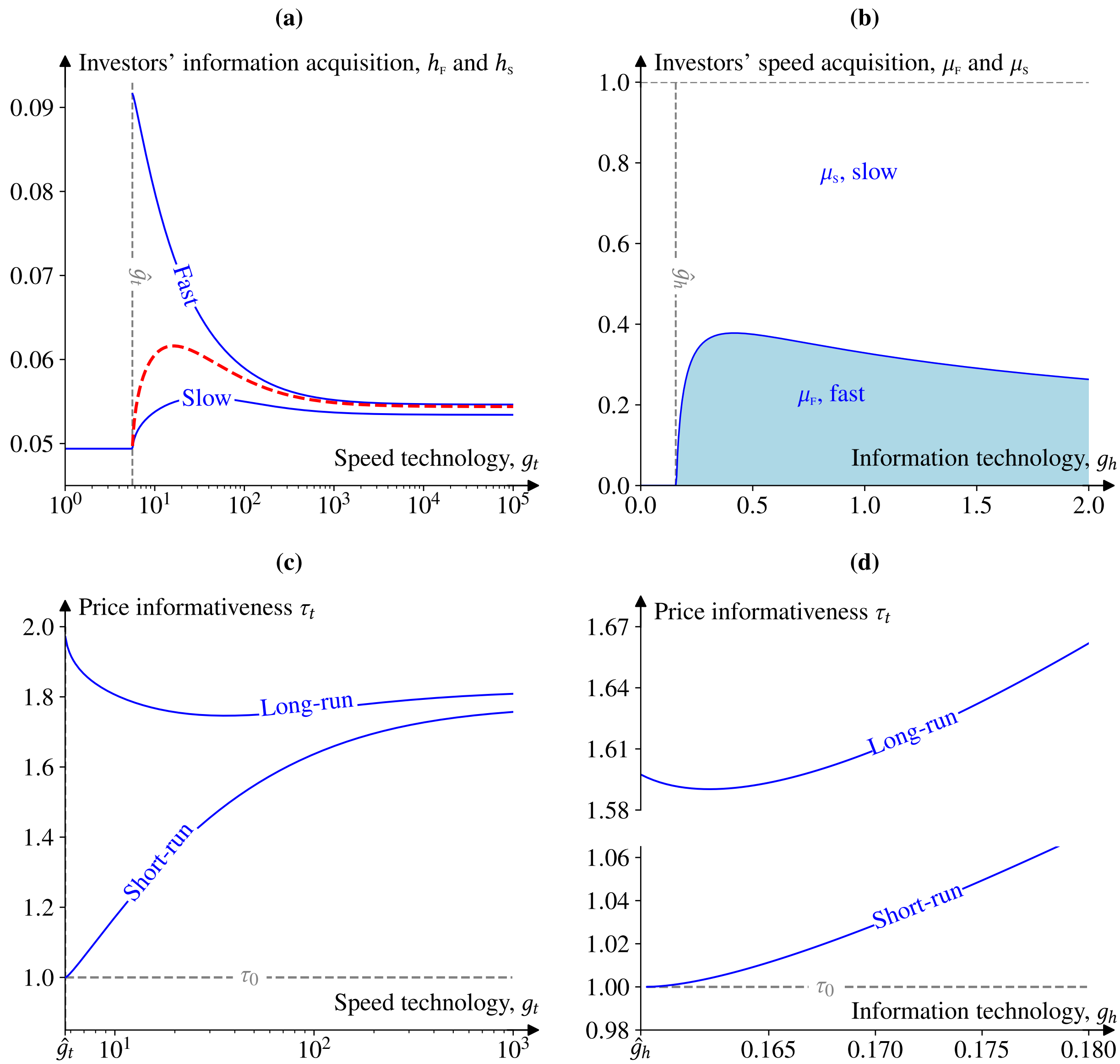

Figure 10: Robustness: Main results with uninformed investors replacing the competitive market maker. This figure demonstrates that with uninformed investors replacing the competitive market maker, the three key results stand robust: Panel (a) and (b) replicate Panel (c)d and (d) of Figure 4 and show that the two technologies can be either complements or substitutes. Panel (c) and (d) replicate Figure 5, showing that both technologies can hurt price informativeness. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, $k_h(m) = \sqrt{m}$, and $n = 1.0$. For Panel (a) and (c), $g_h = 0.2$. For Panel (b) and (d), $g_t = 10.0$.

This feature is often seen in the algorithmic and high-frequency trading literature, where investors pay for a "bundled" advantage in both information and speed (see Menkveld, 2016 for a review).

Substitution between the two is also possible. Dugast and Foucault (2017) argue that because processing information takes time, investors trading on "processed" information are intrinsically slower than those trading on "raw" information. That is, investing in one technology might increase the (marginal) cost for the other (e.g., g_h decreases in g_t).

Exactly how speed and information technologies interfere with each other is perhaps a question of engineering and computer science. The current model sets a baseline with independent technologies—an agnostic view. The outcomes of the model, therefore, offer a clean set of predictions on investors' *endogenous* demand for the two technologies, as opposed to the exogenous, built-in substitution/complementarity.

One can use the current model as a starting point to study implications of built-in substitution or complementarity between the two technologies. Figure 11 plots price informativeness τ_1 (blue-solid line) and τ_2 (red-dashed line) on a contour of (g_t, g_h).[11] When there is complementarity, the effect of an increase in one technology can be examined by, e.g., the left (green) arrow in the figure (g_h increases from 0.15 to 0.16, while g_t increases from about 10 to 100). If instead the substitution of the technologies dominates, the effect can be seen from, e.g., the right (blue) arrow (g_h mildly increases from 0.125 to 0.135, while g_t drops sharply from about 4,000 to 50). In both examples, note that the long-run price informativeness τ_2 (blue-solid contour lines) drops. Note that the right (blue) arrow is consistent with Dugast and Foucault (2017) and Kendall (2017), who show that when processing information takes time, better information might hurt price informativeness.

[11] Note the pattern shown is consistent with Figure 5: Moving right on a horizontal cut of Figure 11, the information technology g_h is fixed and as the speed technology g_t improves, the short-run price informativeness τ_1 monotonically increases, while the long-run price informativeness τ_2 first decreases and then increases. Moving upward on a vertical cut, g_t is fixed and as g_h increases, τ_1 monotonically increases but τ_2 first decreases and then increases.

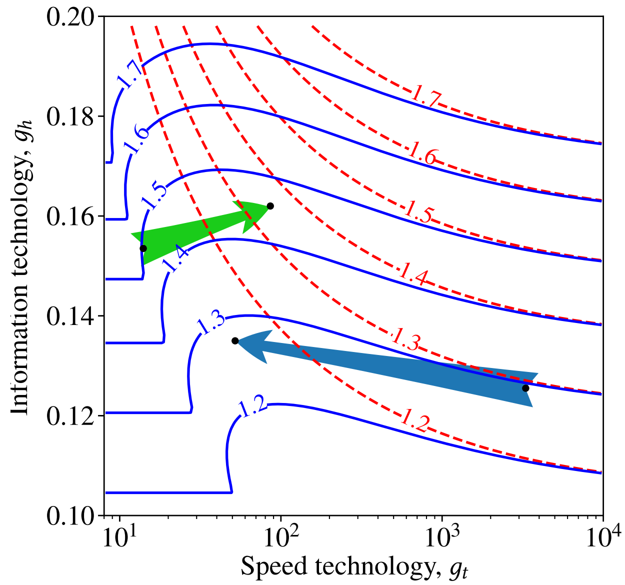

Figure 11: Price informativeness plotted against both technologies. This contour graph plots how the long-run price informativeness τ_2, in blue-solid line, and the short-run price informativeness τ_1, in red-dashed line, vary with the two technologies, g_t and g_h. The two arrows illustrates the different effects of an information technology advancement. The left arrow (green) shows complementarity between the two, while the right arrow (blue) shows substitution. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$.

6.5 The amount of noise trading

Introducing noise trading U_t in each round is a standard practice to avoid a fully revealing equilibrium. The current setup assumes the same magnitude for both noises: $\text{var}[U_t] = \tau_U^{-1}$ for all $t \in \{1, 2\}$. It is straightforward to account for time-varying noise trading by allowing time-dependent $\text{var}[U_t] = \tau_{U_t}^{-1}$. The key economic insights of the model are unaffected. First, irrespective of noise trading sizes, the speed technology creates "temporal fragmentation": Adapting the price informativeness recursion (Equation 8) yields $\tau_2 = \tau_0 + \frac{\tau_{U1}}{\gamma^2}\mu_F^2 h_F^2 + \frac{\tau_{U2}}{\gamma^2}\mu_S^2 h_S^2$. The price discovery process is still fragmented into the early and the late parts, but with time-varying τ_U in each. Second, the complementarity and substitution between the technologies depend only on the

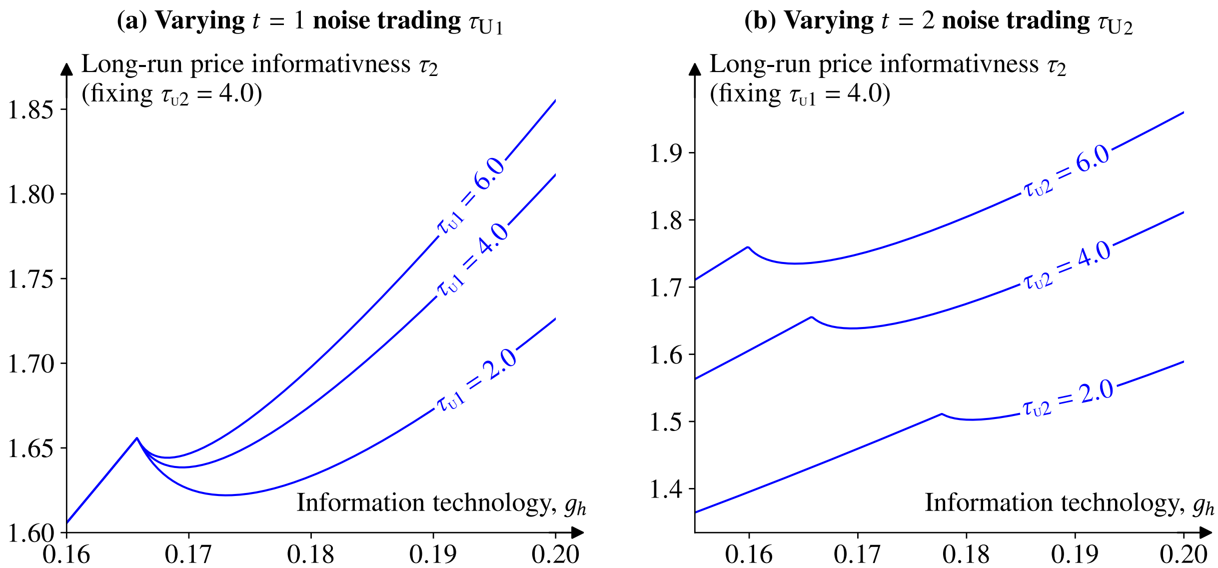

Figure 12: Time varying noise trading. This figure illustrates how different amount of noise trading τ_{U1} and τ_{U2} affects the long-run price discovery, τ_2, when the information technology varies. Three levels of noise trading are illustrated: $\tau_{Ut} \in \{2.0, 4.0, 6.0\}$. Panel (a) varies τ_{U1} while fixing $\tau_{U2} = 4.0$. Panel (b) varies τ_{U2} while fixing $\tau_{U1} = 4.0$. The other primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$.

relative strength of various crowding-out effects. Having different sizes of noises only affects the thresholds of when and which effect dominates. Indeed, all analyses in Section 5 qualitatively go through. For example, Figure 12 illustrates that the qualitative predictions of 6 remain robust about the long-run informativeness τ_2. (Other results are omitted for brevity.)

The underlying assumption for such (possibly time-varying) exogenous noise trading is that some investors in the economy (unmodeled) have no flexibility at all in terms of how much and when to trade. Endogenizing such "noise trading", making such "noise" demand either price-elastic or timing sensitive, will lead to richer predictions. For example, the noise may arise from investors' hedging demand (Diamond and Verrecchia, 1981). Such extensions are are left for future research.

7 Conclusion

There are two aspects of price discovery: the magnitude and the process. The magnitude aspect (investors' information acquisition) has been a key focus of the extant literature. This paper studies a model with investors' endogenous speed acquisition, alongside their information acquisition. The focus is turned to the process of price discovery, i.e., the process through which acquired information is incorporated into price.

The analysis reveals that these two aspects of price discovery are intrinsically connected via investors' competition. There are two key mechanisms at work: First, investors endogenously acquire heterogeneous speed and participate in the market at different times. The price discovery process is accordingly fragmented over time. Second, investors' information and speed acquisition can be either complements or substitutes of each other, depending on the relative strengths of competition effects (crowding-out forces). Based on the interaction of these two mechanisms, the model generates testable implications for how technologies could affect various market quality. Most notably, when either the speed or the information technology improves, the price informativeness can be hurt. This provides a cautionary tale of the disruptive effects of how technological advancement, as seen in recent years, might negatively affect the price discovery function of financial markets.

Appendix

Proofs

For notation simplicity, the proofs will often use $\mu_1 = \mu_{\mathrm{F}}$, $\mu_2 = \mu_{\mathrm{S}}$, $h_1 = h_{\mathrm{F}}$, $h_2 = h_{\mathrm{S}}$, $\pi_1 = \pi_{\mathrm{F}}$, and $\pi_2 = \pi_{\mathrm{S}}$. This way, the subscript $t = 1$ can handily refer to both the time $t = 1$ and the "F"ast investors; and similarly, $t = 2$ refers to both the time $t = 2$ and the "S"low investors.

Lemma 1

Proof. The proof proceeds by conjecture-and-verify (as in Vives, 1995). Conjecture that a fast investor i's demand schedule is $x_i = a_{i,1}s_i - b_{i,1}p_1$ and that a slow investor i's demand schedule is $x_i = a_{i,2}s_i - b_{i,2}p_1 - c_{i,2}p_2$. At $t = 1$, with only the fast investors, the aggregate demand is

$$L_1(p_1) = \int_{i\in[0,1]} x_i(p_1, s_i)\mathbb{1}_{\{t_i=1\}}\mathrm{d}i + U_1 = \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)V - \left(\int_{t_i=1} b_{i,1}\mathrm{d}i\right)p_1 + U_1,$$

where the convention $\int \varepsilon_i \mathrm{d}i = 0$ is used. From the market maker's perspective, the sufficient summary statistic, therefore, is the intercept of the above linear demand, which can be transformed into $z_1 := V + U_1/\left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)$. Therefore, using standard property of normal distribution,

$$\tau_1 = \mathrm{var}[V|\, L_1(\cdot)]^{-1} = \tau_0 + \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}. \tag{12}$$

The incremental price discovery is $\Delta\tau_1 = \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}$. The maker maker sets the efficient price

$$p_1 = \mathbb{E}[V|\, L_1(\cdot)] = \mathbb{E}[V||\, z_1\,] = \frac{\tau_0}{\tau_1}p_0 + \frac{\Delta\tau_1}{\tau_1}z_1. \tag{13}$$

As such, the trading price p_1 is an equivalent statistic of z_1. From a fast investor's perspective, $\mathrm{var}[V|\, s_i, p_1\,]^{-1} = \mathrm{var}[V|\, s_i, z_1\,]^{-1} = h_i + \tau_1$ and $\mathbb{E}[V|\, s_i, p_1\,] = \mathbb{E}[V|\, s_i, z_1\,] = (\tau_0 p_0 + h_i s_i + \Delta\tau_1 z_1)/(\tau_1 + h_i)$. Using the above, a CARA fast investor i's optimal demand is

$$x_i = \frac{\mathbb{E}[V|\, s_i, p_1\,] - p_1}{\gamma \mathrm{var}[s_1, p_1]} = \frac{1}{\gamma}(h_i s_i + \Delta\tau_1 z_1 - (\tau_0 + h_i + \Delta\tau_1)p_1) = \frac{h_i}{\gamma}(s_i - p_1).$$

(Recall the normalization $p_0 = 0$.) The conjectured linear demand $x_i = a_{i,1}s_i - b_{i,1}p_1$ for fast investors has thus been verified with coefficients $a_{i,1} = b_{i,1} = h/\gamma$.

At $t = 2$, only slow investors trade and the aggregate demand is

$$\begin{aligned} L_2(p_2; p_1) &= \int_{i\in[0,1]} x_i(p_2, s_i; p_1)\mathbb{1}_{\{t_i=2\}}\mathrm{d}i + U_2 \\ &= \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)V - \left(\int_{t_i=2} b_{i,2}\mathrm{d}i\right)p_1 - \left(\int_{t_i=2} c_{i,2}\mathrm{d}i\right)p_2 + U_2, \end{aligned}$$

Recalling p_1, the market maker updates his information set to $\{p_1, z_2\}$, where $z_2 := V + U_2/\left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)$ summarizes the new information in $L_2(\cdot)$. Then,

$$\tau_2 = \mathrm{var}[V \mid p_1, L_2(\cdot)]^{-1} = \mathrm{var}[V \mid z_1, z_2]^{-1} = \tau_1 + \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}, \tag{14}$$

where the incremental price discovery $\Delta\tau_2 = \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}$. The market maker then sets the efficient price

$$p_2 = \mathbb{E}[V \mid p_1, L_2(\cdot)] = \mathbb{E}[V \mid z_1, z_2] = \frac{\tau_0}{\tau_2}p_0 + \frac{\Delta\tau_1}{\tau_2}z_1 + \frac{\Delta\tau_2}{\tau_2}z_2. \tag{15}$$

A slow investor updates $\mathrm{var}[V \mid s_i, p_1, p_2]^{-1} = \mathrm{var}[V \mid s_i, z_1, z_2]^{-1} = h_1 + \tau_2$ and $\mathbb{E}[V \mid s_i, p_1, p_2] = \mathbb{E}[V \mid s_i, z_1, z_2] = (\tau_0 p_0 + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 + h_i s_i)/(\tau_2 + h_i)$. Solving a quadratic optimization problem, a CARA slow investor's optimal demand is

$$x_i = \frac{\mathbb{E}[V \mid s_i, p_1, p_2] - p_2}{\gamma \mathrm{var}[s_1, p_1, p_2]} = \frac{1}{\gamma}(h_i s_i + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 - (\tau_0 + \Delta\tau_1 + \Delta\tau_2 + h_i)p_2) = \frac{h_i}{\gamma}(s_i - p_2).$$

Thus the conjectured linear demand for slow investors is also verified with coefficients $a_{i,2} = c_{i,2} = h_i/\gamma$ and $b_{i,2} = 0$. That is, the slow investor's demand is independent of p_1.

The analysis so far has proved the investors' optimal demand as stated in the lemma. In the meantime, Equations (12) through (15) verify the recursion systems of p_t and $\Delta\tau_t$. It remains to compute the investors' ex ante certainty equivalent. Consider a fast investor. Before accounting for the technology acquisition cost, his expected utility at $t = 0$ is $-\mathbb{E}\left[\exp\left\{-\frac{[\mathbb{E}[V \mid s_i, p_1] - p_1]^2}{2\mathrm{var}[V \mid s_i, p_1]}\right\}\right]$. The expressions derived earlier yield the following: $\mathbb{E}[V \mid s_i, p_1] - p_1 = \frac{h_i}{\tau_1 + h_i}\left(\frac{\tau_0}{\tau_1}V + \varepsilon_i - \frac{\Delta\tau_1}{\tau_1}\frac{U_1}{\int_{t_j=1}(h_j/\gamma)\mathrm{d}j}\right)$ and $\mathrm{var}[V \mid s_i, p_1]^{-1} = \tau_1 + h_i$. Plug the above into the $t = 0$ expected utility for a fast investor, simplify, and the resulting ex ante certainty equivalent *before technology acquisition costs* is $\frac{1}{2\gamma}\ln\left(1 + \frac{h_i}{\tau_1}\right)$. Subtracting the information acquisition cost and the speed acquisition cost gives the expression stated in the lemma. The calculation for slow investors repeats the above and is omitted. □

Proposition 1

Proof. The proof begins by writing investors' certainty equivalent π_1 and π_2 as functions of the fast population size μ_1 in $[0, 1]$. To do this, first note that from the first-order condition (6), investors' endogenous choice of h_i can be written as a monotone function of τ_{t_i}. By Lemma 1, $\tau_1 = \tau_0 + \Delta\tau_1$ and $\tau_2 = \tau_0 + \Delta\tau_2$, where $\Delta\tau_t = \tau_U \mu_t^2 h_t^2 / \gamma^2$. Hence, τ_1 is effectively a function of μ_1, while τ_2 of both μ_1 and μ_2. Finally, note that $\mu_2 = 1 - \mu_1$. As such, investors' certainty equivalent π_t are functions of μ_1. Then, depending on μ_1, there are three cases.

Case 1: First, suppose $\mu_1 = 1$ and $\mu_2 = 0$; i.e. all investors pay the speed technology cost $1/g_t$ and become fast. If this is the case, then in equilibrium $\pi_1 \geq \pi_2$ must hold. Consider an investor i's unilateral deviation to not investing in the speed technology, saving the cost of $1/g_t$ and becomes slow. By Equation (4), the price informativeness remains the same, $\tau_1 = \tau_2$, because a single investor's deviation has zero population measure. Then i's optimal technology investment h_i, by the first-order condition (6), remains the same as if he were fast: $h_i = h(\tau_2) = h(\tau_1) = h_1$. As a result, his certainty equivalent $\pi_2 = \pi_1 + 1/g_t > \pi_1$ and he indeed will deviate. Such a case of $\mu_1 = 1$ and $\mu_2 = 0$, therefore, can never be an equilibrium.

Case2: Second, consider the case of $\mu_1 = 0$ and $\mu_2 = 1$. (This will correspond to the corner equilibrium stated in the proposition.) If this is an equilibrium, it has to be the case that $\pi_1 \leq \pi_2$, i.e., all investors stay slow. The argument below shows that fixing all other exogenous parameters, $\pi_1 \leq \pi_2$ holds if and only if $g_t < \hat{g}_t$, for some threshold $\hat{g}_t$. At $\mu_1 = 0$, $\tau_1 = \tau_0 < \tau_2$ and thus a slow investor's unilateral deviation to fast yields $\pi_1|_{\mu_1=0} = \frac{1}{2\gamma} \ln\left(1 + \frac{h_1}{\tau_0}\right) - \dot{c}(h_1) - \frac{1}{g_t}$, where h_1 is the unique solution implied by the first-order condition (6) with $\tau_1 = \tau_0$. By envelope theorem, $\partial\pi_1/\partial g_t = 1/g_t^2 > 0$. Therefore, $\pi_1|_{\mu_1=0}$ is monotone increasing in g_t with limits $\lim_{g_t \downarrow 0} \pi_1 = -\infty < 0 < \pi_2 < \lim_{g_t \uparrow \infty} \pi_1$. (Note that $\pi_2|_{\mu_1=0}$ is a finite number unaffected by g_t.) By continuity, therefore, there exists a unique $\hat{g}_t$ such that $\pi_1 = \pi_2$ when $\mu_1 = 0$. As such, $\pi_1 \leq \pi_2$, supporting $\mu_1 = 0$ and $\mu_2 = 1$, if and only if $g_t \leq \hat{g}_t$. When instead $g_t > \hat{g}_t$, this corner equilibrium does not exist.

Case3: Third, consider the interior case of $\mu_1 \in (0, 1)$, implying $\pi_1 = \pi_2$. The key is to show the following result: The difference $\pi_1 - \pi_2$ strictly decreases in μ_1. Evaluate the partial derivative of $\pi_1 - \pi_2$ with respect to μ_1 and after some simplification,

$$\frac{\partial(\pi_1 - \pi_2)}{\partial\mu_1} \cdot 2\gamma = \left[\frac{h_2/\tau_2}{\tau_2 + h_2} - \frac{h_1/\tau_1}{\tau_1 + h_1}\right]\frac{\partial\tau_1}{\partial\mu_1} + \frac{h_2/\tau_2}{\tau_2 + h_2}\frac{\partial\Delta\tau_2}{\partial\mu_1}.$$

Note that the term in the square-brackets is non-positive, because $\tau_2 \geq \tau_1$ by construction and because $h_t = h(\tau_t)$ decreases in τ_t as implied by the first-order condition (6).

One still needs to sign both $\partial\tau_1/\partial\mu_1$ and $\partial\Delta\tau_2/\partial\mu_1$. To do so, rearrange the first-order condition (6) for $t = 1$ as $(\tau_0 + \Delta\tau_1 + g_h k_h(m_1))/\dot{k}_h(m_1) = g_h/(2\gamma)$ with $\Delta\tau_1 = \mu_1^2 g_h^2 k_h(m_1)^2 \tau_U/\gamma^2$ following Equation (4). It can then immediately be concluded that m_1 strictly decreases in μ_1, as otherwise the left-hand side of the above equation is always increasing in μ_1, unable to maintain the equality. (Recall that $k_h(\cdot)$ is concavely increasing.) Similarly, it is also known that τ_1 $(= \tau_0 + \Delta\tau_1)$ decreases in m_1. Hence, τ_1 (and $\Delta\tau_1$) increases in μ_1. For $t = 2$, $(\tau_0 + \Delta\tau_1 + \Delta\tau_2 + h_2)/\dot{k}_h(m_2) = g_h/(2\gamma)$ with $\Delta\tau_2 = (1-\mu_1)^2 g_h^2 k_h(m_2)^2 \tau_U/\gamma^2$. Note that $\frac{\partial\Delta\tau_2}{\partial\mu_1} = \left(-2(1-\mu_1)h_2^2 + 2(2-\mu_1)^2 h_2 \frac{\partial h_2}{\partial\mu_1}\right)\frac{\tau_U}{\gamma^2}$. As such, if $\Delta\tau_2$ increases in μ_1, then it has to be the case that $\partial h_2/\partial\mu_1 > 0$. Because $h_2 = g_h k_h(m_2)$, m_2 is also increasing in μ_1. It then follows that the left-hand side of the above equation strictly increases in μ_1—$\Delta\tau_1$, $\Delta\tau_2$, and m_2 all increase with μ_1, invalidating the equality. Therefore, it must be $\Delta\tau_2$ decreases in μ_1.

As τ_1 increases in μ_1 but $\Delta\tau_2$ decreases in μ_1, one can conclude from the above partial derivative that the difference $\pi_1 - \pi_2$ indeed strictly decreases in μ_1.

To sum up, recall from the first cases that at $\mu_1 = 1$, $\pi_1 < \pi_2$. From the second case, at $\mu_1 = 0$, $\pi_1 > \pi_2$ if and only if $g_t > \hat{g}_t$. Hence, when $g_t \leq \hat{g}_t$, the equilibrium with interior μ_1 does not exist due to the above monotonicity of $\pi_1 - \pi_2$ in μ_1. When $g_t > \hat{g}_t$, there exists a unique $\mu_1 \in (0, 1)$ such that $\pi_1 = \pi_2$, sustaining this equilibrium. This completes the proof of this proposition. □

Proposition 2 and Corollary 4

Proof. First, the following shows that h_t is monotonically increasing in g_h for both $t = 1$ and $t = 2$. The first-order condition (6) can be written as $\dot{k}_h(m_t)/(2\gamma) - k_h(m_t) = \tau_t/g_h$, which uniquely solves m_t. Holding g_h (and γ) constant,

$$\frac{\partial m_t}{\partial \tau_t} = \frac{1}{g_h}\left(\frac{\ddot{k}_h(m_t)}{2\gamma} - \dot{k}_h(m_t)\right)^{-1} \leq 0 \tag{16}$$

where the inequality follows the concavity of $k_h(m)$. (Note that this also implies that $m_1 \geq m_2$ because $\tau_2 \geq \tau_1$.) In addition,

$$\frac{\partial m_t}{\partial g_h} = -\frac{\tau_t}{g_h^2}\left(\frac{\ddot{k}_h(m_t)}{2\gamma} - \dot{k}_h(m_t)\right)^{-1} = -\frac{\tau_t}{g_h}\frac{\partial m_t}{\partial \tau_t} \geq 0. \tag{17}$$

From the definition of $h_t = g_h k_h(m_t)$, $\partial h_t/\partial g_h = k_h(m_t) + g_h \dot{k}_h(m_t)\partial m_t/\partial g_h \geq 0$. Therefore, in any case, the equilibrium h_t is increasing in the information technology g_h.

The rest of this proof only deals with the case of exogenous speed acquisition, i.e., with fixed μ_1

and μ_2. The proof for the case of endogenous μ_t is deferred to proof of Proposition 6. Consider the short-run of $t = 1$. While g_h increases, h_1 increases to satisfy the first-order condition, as shown above. It then follows that $\partial\tau_1/\partial g_h > 0$ because $\tau_1 = \tau_0 + \tau_{\mathrm{U}} h_1^2 \mu_1^2/\gamma^2$ with μ_1 exogenous.

Consider the long-run of $t = 2$ now. Suppose the opposite, $\partial\tau_2/\partial g_h < 0$, is true. Then h_2 should be decreasing with g_h because $\tau_2 = \tau_1 + \tau_{\mathrm{U}} h_2^2 \mu_2^2/\gamma^2$ with τ_1 is increasing in g_h. However, the transformation of first-order condition 6, $g_h/(2\gamma) = (\tau_2 + h_2){k_h}^{-1}(h_2/g_h)$, shows that it is impossible for both τ_2 and h_2 to be decreasing with g_h at the same time. Thus, the assumed inequality is wrong and τ_2 increases with g_h. □

Proposition 3

Proof. To avoid repetition, the proof only considers the full equilibrium where the information acquisition is available. A similar argument can be constructed for the special case where all investors have the same exogenous information $h_\circ$. In the interior equilibrium, $\pi_1 - \pi_2 = 0$ and, following the proof of Proposition 1, the equality implies an implicit function of μ_1 in terms of the speed technology g_t, which implies: $\frac{\mathrm{d}\mu_1}{\mathrm{d}g_t} = -\frac{\partial\pi_1/\partial g_t}{\partial(\pi_1-\pi_2)/\partial\mu_1}$, where the denominator of the fraction is negative as shown in Case 3 of the proof for Proposition 1. The numerator equals $1/g_t^2 > 0$ by envelope theorem. Therefore, μ_1 increases in g_t. □

Proposition 4

Proof. This proof deals with two cases. The first case is where all investors' information precision is exogenously given at $h_\circ$. The second case is where investors endogenously acquire information.

In the first case, as shown in the proof of Proposition 3, μ_1 is increasing with g_t, which directly implies that τ_1 is increasing with g_t. For the long-run informativeness τ_2, by the implicit function theorem, $\partial\tau_2/\partial\mu_1 = 2\tau_{\mathrm{U}} h_o^2 \mu_1/\gamma^2 - 2\tau_{\mathrm{U}} h_o^2 \mu_2/\gamma^2$, or $\partial\tau_2/\partial g_t = 2(\tau_{\mathrm{U}} h_o^2 \mu_1/\gamma^2 - 2\tau_{\mathrm{U}} h_o^2 \mu_2/\gamma^2)(\partial g_t/\partial\mu_1)$. It is clear that $\partial\tau_2/\partial g_t < 0$ when μ_1 is close to zero and μ_2 close to one (i.e., g_t is small), and $\partial\tau_2/\partial g_t > 0$ when μ_1 is close to one and μ_2 close to zero (i.e., g_t is large).

For the second case, two steps are involved. The first step is to prove that $\partial\tau_1/\partial g_t > 0$. In the interior equilibrium, the first-order condition (6) for $t = 1$, together with $\tau_1 = \tau_0 + \tau_{\mathrm{U}} h_1^2 \mu_1^2/\gamma^2$, implies an implicit function of $h_1 = g_h k_h(m_1)$ and μ_1, from which $\partial h_1/\partial\mu_1 = -\frac{2\tau_{\mathrm{U}}\mu_1 h_1^2/\gamma^2}{2\tau_{\mathrm{U}}\mu_1^2 h_1/\gamma^2+1-\ddot{k}_h(m_1)/\dot{k}_h(m_1)} < 0$, where the inequality follows because $k_h(\cdot)$ is concavely increasing. From the effect of speed technology and population of sizes, $\partial\mu_1/\partial g_t > 0$. Therefore, by chain rule, $\partial h_1/\partial g_t < 0$. The first-order condition (6) also implies that τ_1 decreases with m_1 and, hence, also with h_1, yielding $\partial\tau_1/\partial g_t > 0$.

The second step is to prove that τ_2 first decreases and then increases with g_t. In the interior equilibrium, the first-order condition (6) for $t = 2$ always holds. Recall $\tau_2 = \tau_0 + \tau_U \tau_1^2 \mu_1^2/\gamma^2 + \tau_U \tau_2^2 \mu_2^2/\gamma^2$. By implicit function theorem, it implies

$$\frac{\partial h_2}{\partial \mu_2} = -\frac{4\tau_U}{\gamma} \frac{\mu_2 h_2^2 - \mu_1 h_1^2 - \mu_1^2 h_1 \partial h_1/\partial \mu_1}{-\ddot{k}_h(m_2)/\dot{k}_h(m_2) + 2\gamma + 4\tau_U \mu_2^2 \tau_2/\gamma}.$$

As done in the proof of step 1, the idea is to first sign the above partial derivative and then sign $\partial h_2/\partial g_t$ using chain rule: $\frac{\partial h_2}{\partial g_t} = \frac{\partial h_2}{\partial \mu_2}\frac{\partial \mu_2}{\partial \mu_1}\frac{\partial \mu_1}{\partial g_t}$, where $\partial \mu_2/\partial \mu_1 = -1$ following the identity $\mu_1 + \mu_2 = 1$ and $\partial \mu_1/\partial g_t > 0$. In particular, consider the limits of $\partial h_2/\partial \mu_2$ as $g_t \uparrow \infty$ and $g_t \downarrow \hat{g}_t$, respectively. To evaluate these limits, one needs to show that h_1, h_2, and $\partial h_1/\partial \mu_1$ are have finite bounds.

The finite bounds for h_t can be easily established by noting from the first-order condition (6) that τ_t in equilibrium is monotone decreasing in τ_t. From the model setting, it is known that τ_t has strictly positive lower bound τ_0. Therefore, both h_1 and h_2 have finite upper bounds. (They also have lower bounds of zero by construction.) Finally, from the expression of $\partial h_1/\partial \mu_1$ derived in the proof of the previous step, it can be seen that $\mu_1 \cdot (\partial h_1/\partial \mu_1) = -\frac{2\tau_U \mu_1^2 h_1/\gamma^2}{2\tau_U \mu_1^2 \tau_1/\gamma^2 + 1 - \ddot{k}_h(m_1)/\dot{k}_h(m_1)} h_1 > -h_1$ is also bounded.

Now the limits can be evaluated. When speed technology $g_t \uparrow \infty$, almost all investors become fast and $\mu_2 \downarrow 0$ and $\lim_{\mu_2 \downarrow 0}(\frac{\partial h_2}{\partial \mu_2}) = -\frac{4\tau_U}{\gamma}\frac{-\mu_1 h_1^2 - \mu_1^2 h_1 \partial h_1/\partial \mu_1}{-\ddot{k}_h(m_2)/\dot{k}_h(m_2) + 2\gamma} > 0$. Similarly, when speed technology $g_t \downarrow \hat{g}_t$, almost all investors stay slow, $\mu_1 \downarrow 0$, and $\lim_{\mu_1 \downarrow 0}(\frac{\partial h_2}{\partial \mu_2}) = -\frac{4\tau_U}{\gamma}\frac{\mu_2 h_2^2}{-\ddot{k}_h(m_2)/\dot{k}_h(m_2) + 2\gamma + 4\tau_U \mu_2^2 h_2/\gamma} < 0$. As the above shows, for sufficiently large (low) g_t, h_2 increases (decreases) in μ_2 and hence decreases (increases) in g_t by the chain rule expression above. The first-order condition (6) implies that τ_2 decreases with τ_2 and the stated results are proved. □

Proposition 5

Proof. **Fixing g_t, g_h increases from $\hat{g}_h$ to ∞.** The aggregate demand for speed in the economy is $\int_{[0,1]} \mathbb{1}_{\{t_i=1\}} \mathrm{d}i = \mu_1$. From $\Delta\tau_1 = \tau_U h_1^2 \mu_1^2/\gamma^2$, by implicit function theorem,

$$\frac{\partial \mu_1}{\partial g_h} = \frac{\gamma^2}{2\tau_U \mu_1 h_1^2}\left(\frac{\partial \Delta\tau_1}{\partial g_h} - \frac{2\tau_U \mu_1^2 h_1}{\gamma^2}\frac{\partial h_1}{\partial g_h}\right). \tag{18}$$

Hence, the sign of $\partial \mu_1/\partial g_h$ depends on the difference between the two terms in the brackets. Consider first the case of a very small g_h. Corollary 1 establishes the existence of a lower bound $\hat{g}_h$ for g_h, such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h$. In particular, when $g_h \downarrow \hat{g}_h$, the marginal investor is just indifferent between becoming fast or not, implying $\mu_1 \downarrow 0$. The first-order condition (6) at this limit gives $1/(2(\tau_0 + h_1)\gamma) - \dot{c}(h_1) = 0$, which has interior solution of $0 < h_1 < \infty$, thanks to the assumption of $\dot{c}(0) = 0$. By differentiability, therefore, $\partial h_1/\partial g_h$ is finite in this limit as well. Taken together, the second term in the above brackets has limit zero as $\mu_1 \downarrow 0$, when $g_h \downarrow \hat{g}_h$. The remaining term is $\partial \Delta\tau_1/\partial g_h$, which is shown by Proposition 6 to be strictly positive. Thus, $\partial \mu_1/\partial g_h$ is positive in the case of a very small g_h, close to the lower bound of $\hat{g}_h$.

Consider next the case of a very large g_h, i.e. $g_h \uparrow \infty$. First, there exists an upper bound for investors' expense on information acquisition, m_t. To see this, note from the first-order condition (6):

$$\text{(19)} \qquad \frac{1}{2\gamma}\dot{k}_h(m_t) > \frac{1}{2\gamma}\dot{k}_h(m_t) - \frac{1}{g_h}\tau_t = k_h(m_t) \geq k_h(0) + m_t\dot{k}_h(m_t) = m_t\dot{k}_h(m_t)$$

where the first inequality holds because $\tau_t \geq \tau_0 > 0$ and the last inequality holds by concavity of $k_h(\cdot)$ and by $k_h(0) = 0$. Therefore, for $t \in \{1, 2\}$, there exists an upper bound for $m_t \leq 1/(2\gamma)$, an upper bound for $k_h(m_t) \leq k_h(1/(2\gamma))$, and a lower bound for $\dot{k}_h(m_t) \geq \dot{k}_h(1/(2\gamma)) > 0$. Second, in the limit of $g_h \uparrow \infty$, the equilibrium is always interior (following Corollary 1). Hence, the limit of the fast investor's ex ante certainty equivalent $\lim_{g_h\uparrow\infty} \pi_1 = \frac{1}{2\gamma} \lim_{g_h\uparrow\infty} \ln\left(1 + \frac{h_1}{\tau_1}\right) - \lim_{g_h\uparrow\infty} m_1 - \frac{1}{g_t}$ exists and must be nonnegative to sustain the interior equilibrium. Since m_1 is bounded from above, it follows that $\lim_{g_h\uparrow\infty}(h_1/\tau_1)$ also exists and is strictly positive. That is, there exists some $a \in (0, \infty)$, such that $\lim_{g_h\uparrow\infty}(\tau_1/h_1) = a$. Equivalently, as τ_0 is a finite constant, $\lim_{g_h\uparrow\infty}(\Delta\tau_1/h_1) = a$. Further, a fast investor's first-order condition (6) can be rewritten as $\frac{1}{2\gamma}\frac{g_h}{\tau_1+h_1} - \dot{c}(h_1/g_h) = 0$. Since the above holds under $g_h \uparrow \infty$, it follows that $h_1 \sim g_h$; or $\lim_{g_h\uparrow\infty}(h_1/g_h) = b \in (0, \infty)$. (If h_1 is of higher magnitude than g_h, the limit of the first term above falls to zero, while the limit of the second term is strictly positive as $c(\cdot)$ is strictly convex. If instead h_1 is of lower magnitude than g_h, the limit of the first term approaches infinity, while the second term falls to zero.) Now consider the limit of the difference in the brackets of Equation (18):

$$\lim_{g_h\uparrow\infty}\left(\frac{\partial\Delta\tau_1}{\partial g_h} - 2\frac{\tau_U\mu_1^2 h_1}{\gamma^2}\frac{\partial h_1}{\partial g_h}\right) = \lim_{g_h\uparrow\infty}\left(\frac{\partial\Delta\tau_1}{\partial g_h} - 2\frac{\Delta\tau_1}{h_1}\frac{\partial h_1}{\partial g_h}\right) = (ab - 2ab) < 0$$

where the last equality follows L'Hôpital's rule. Therefore, in the limit of $g_h \uparrow \infty$, $\partial\mu_1/\partial g_h < 0$. Finally, consider the value of μ_1 in this limit. Note that $\Delta\tau_1 = \tau_0 + \tau_U\mu_1^2 h_1^2/\gamma^2$. Therefore, in order for $\lim_{g_h\uparrow\infty}(\Delta\tau_1/h_1) = a \in (0, \infty)$ to hold, it must be such that $\lim_{g_h\uparrow\infty}(\mu_1^2 h_1) = c \in (0, \infty)$, i.e., μ_1 in this limit is of magnitude $h_1^{-1/2}$. As $h_1 \uparrow \infty$, this also implies that $\mu_1 \downarrow 0$ in this limit.

Fixing g_h, g_t increases from $\hat{g}_t$ to ∞. The aggregate demand for information is $\bar{h} := \mu_1 h_1 + \mu_2 h_2$. Since μ_1 is monotone in g_t (Proposition 3), it is sufficient to examine the partial derivative of the above aggregate demand with respect to μ_1: $\partial\bar{h}/\partial\mu_1 = h_1 - h_2 + \mu_1 \cdot (\partial h_1/\partial\mu_1) - \mu_2 \cdot (\partial h_2/\partial\mu_2)$. At the initial extreme of $g_t \downarrow \hat{g}_t$, the proof of Proposition 4 has shown that 1) $\mu_1 \downarrow 0$, 2) $\mu_1 \cdot \partial h_1/\partial\mu_1$

is bounded, and 3) $\partial h_2/\partial\mu_2 < 0$. Taking these into the above partial derivative yields $\partial\bar{h}/\partial\mu_1 \to h_1 - h_2 - \mu_2 \cdot (\partial h_2/\partial\mu_2) > 0$, recalling that $h_1 \geq h_2$ from Equation (9). At the eventual extreme of $g_t \uparrow \infty$, the proof of Proposition 4 has shown that 1) $\mu_2 \downarrow 0$, 2) $\partial h_1/\partial\mu_1 < 0$, and 3) $\partial h_2/\partial\mu_2 > 0$. In addition, since $\mu_2 \downarrow 0$, $\Delta\tau_2 = \mu_2^2 h_2^2 \tau_{\mathrm{U}}/\gamma^2 \downarrow 0$ (h_2 is bounded), implying $\tau_2 \downarrow \tau_1$ and 4) $h_2 \uparrow h_1$. Taking the above into $\bar{h}$ yields $\partial\bar{h}/\partial\mu_1 \to \mu_1 \cdot (\partial h_1/\partial\mu_1) - \mu_2 \cdot (\partial h_2/\partial\mu_2) < 0$. □

Proposition 6

Proof. By construction, $\tau_1 = \tau_0 + \Delta\tau_1$ and $\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2$. The first-order condition implicitly has m_1 and m_2 as functions of $m_1(\Delta\tau_1)$ and $m_2(\Delta\tau_1, \Delta\tau_2)$. Further, $\Delta\tau_t = \tau_{\mathrm{U}} g_h^2 k_h(m_t)^2 \mu_t^2/\gamma^2$, or $\mu_t = \frac{\gamma}{\sqrt{\tau_{\mathrm{U}}}} \frac{\sqrt{\Delta\tau_t}}{g_h k_h(m_t)}$. Therefore, the unconstrained equilibrium (with endogenous acquisition of both speed and information) is pinned down by a two-equation, two-unknown system: $\pi_1 - \pi_2 = 0$ and $\mu_1 + \mu_2 - 1 = 0$; or, equivalently, with a vector function $F(\Delta\tau_1, \Delta\tau_2; g_h)$,

$$F = \begin{bmatrix} \left(\frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_1)}{\tau_1}\right) - m_1 - \frac{1}{g_t}\right) - \left(\frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_2)}{\tau_2}\right) - m_2\right) \\ \frac{\sqrt{\Delta\tau_1}}{k_h(m_1)} + \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} g_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{20}$$

where $\{m_t\}_{t\in\{1,2\}}$ are functions of $\Delta\tau_1$ and $\Delta\tau_2$ following the first-order condition (6), which can be rewritten as $\dot{k}_h(m_t)/(2\gamma) - k_h(m_t) = \tau_t/g_h$.

Take total derivatives with respect to g_h on the equilibrium condition $F = 0$ to get $\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}\begin{bmatrix} \mathrm{d}\Delta\tau_1 \\ \mathrm{d}\Delta\tau_2 \end{bmatrix} + \begin{bmatrix} F_{1g} \\ F_{2g} \end{bmatrix}\mathrm{d}g_h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. One can easily evaluate, using envelope theorem,

$$F_{1g} = \frac{1}{2\gamma}\frac{k_h(m_1)}{\tau_1 + g_h k_h(m_1)} - \frac{1}{2\gamma}\frac{k_h(m_2)}{\tau_2 + g_h k_h(m_2)} = \frac{1}{g_h}\left(\frac{k_h(m_1)}{\dot{k}_h(m_1)} - \frac{k_h(m_2)}{\dot{k}_h(m_2)}\right) > 0,$$

where the second equality follows the first-order condition (6), while the last inequality follows the concavity of $k_h(m)$, knowing that $m_1 > m_2$. Also,

$$\begin{aligned} F_{2g} &= -\frac{\sqrt{\Delta\tau_1}}{k_h(m_1)^2}\dot{k}_h(m_1)\frac{\partial m_1}{\partial g_h} - \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)^2}\dot{k}_h(m_2)\frac{\partial m_2}{\partial g_h} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} \\ &= -\frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma}\mu_1 g_h \frac{\dot{k}_h(m_1)}{k_h(m_1)}\frac{\partial m_1}{\partial g_h} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma}\mu_2 g_h \frac{\dot{k}_h(m_2)}{k_h(m_2)}\frac{\partial m_2}{\partial g_h} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} < -\frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} < 0, \end{aligned}$$

where the equality uses the expression of μ_t and the inequality holds because $\partial m_t/\partial g_h$ is derived earlier to be positive (inequality 17).

The elements in the Jacobian matrix can also be evaluated. Using envelope theorem,

$$F_{11} = -\frac{k_h(m_1)}{\dot{k}_h(m_1)\tau_1} + \frac{k_h(m_2)}{\dot{k}_h(m_2)\tau_2} \leq 0$$

where the inequality holds because $k_h(m_1)/\dot{k}_h(m_1) \geq k_h(m_2)/\dot{k}_h(m_2)$ (concavity) and $\tau_1 \leq \tau_2$. Similarly,

$$F_{12} = \frac{k_h(m_2)}{\dot{k}_h(m_2)\tau_2} > 0.$$

Now consider the partial derivatives with respect to F_2:

$$\begin{aligned} F_{21} &= \frac{1}{2\sqrt{\Delta\tau_1}k_h(m_1)} - \frac{\sqrt{\Delta\tau_1}}{k_h(m_1)^2}\dot{k}_h(m_1)\frac{\partial m_1}{\partial \tau_1}\cancel{\frac{\partial \tau_1}{\partial \Delta\tau_1}} - \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)^2}\dot{k}_h(m_2)\frac{\partial m_2}{\partial \tau_2}\cancel{\frac{\tau_2}{\Delta\tau_1}} \\ &= \frac{1}{2\sqrt{\Delta\tau_1}k_h(m_1)} - \frac{\sqrt{\tau_U}}{\gamma}\mu_1 g_h\frac{\dot{k}_h(m_1)}{k_h(m_1)}\frac{\partial m_1}{\partial \tau_1} - \frac{\sqrt{\tau_U}}{\gamma}\mu_2 g_h\frac{\dot{k}_h(m_2)}{k_h(m_2)}\frac{\partial m_2}{\partial \tau_2} > 0 \end{aligned}$$

where the equality follows the expression of μ_t and the inequality holds because $\partial m_t/\partial \tau_t \leq 0$ as shown before (inequality 16). Similarly,

$$F_{22} = \frac{1}{2\sqrt{\Delta\tau_2}k_h(m_2)} - \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)^2}\dot{k}_h(m_2)\frac{\partial m_2}{\partial \tau_2}\cancel{\frac{\partial \tau_2}{\partial \Delta\tau_2}} > 0.$$

By Cramer's rule,

$$\frac{\partial \Delta\tau_1}{\partial g_h} = \frac{\begin{vmatrix} -F_{1g} & F_{12} \\ -F_{2g} & F_{22} \end{vmatrix}}{\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}} \text{ and } \frac{\partial \Delta\tau_2}{\partial g_h} = \frac{\begin{vmatrix} F_{11} & -F_{1g} \\ F_{21} & -F_{2g} \end{vmatrix}}{\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}}.$$

The the denominator is easy to sign: $F_{11}F_{22} - F_{12}F_{21} < 0$. It remains to examine the numerators. For τ_1, it can be seen that $-F_{1g}F_{22} + F_{12}F_{2g} < 0$; hence $\partial\tau_1/\partial g_h = \partial\Delta\tau_1/\partial g_h > 0$.

To sign $\partial\tau_2/\partial g_h$ is equivalent to signing the sum of the numerators of $\partial\Delta\tau_1/\partial g_h$ and $\partial\Delta\tau_2/\partial g_h$:

$$(-F_{1g}F_{22} + F_{12}F_{2g}) + (-F_{11}F_{2g} + F_{1g}F_{21}) = (F_{21} - F_{22})F_{1g} + (F_{12} - F_{11})F_{2g}.$$

To prove the statement made in the proposition, the objective is to show that under the limits of $g_h \uparrow \infty$ and of $g_h \downarrow \hat{g}_h$, the sign of the above term is negative and positive, respectively (recall that the determinant for the denominator is negative). The proof of Proposition 5 shows that in the upper limit, $\mu_1 \downarrow 0$ and $\mu_2 \uparrow 1$. The proof of Corollary 1 shows that in the lower limit, investors are just indifferent between acquiring the speed or not, implying again $\mu_1 \downarrow 0$ and $\mu_2 \uparrow 1$. Using these limiting values of μ_1 and μ_2, the above simplifies to

$$\left(\frac{1}{2\sqrt{\Delta\tau_1}k_{h1}} - \frac{1}{2\sqrt{\Delta\tau_2}k_{h2}}\right)F_{1g} + \frac{k_{h1}}{\dot{k}_{h1}\tau_1}F_{2g}, \tag{21}$$

where, simplifying the notation, $k_h(\cdot)$ and $\dot{k}_h(\cdot)$ are replaced by subscripts of $t \in \{1,2\}$.

Consider the limit of $g_h \uparrow \infty$ first. Equation (21) satisfies the following inequality:

$$\left(\frac{1}{2\sqrt{\Delta\tau_1}k_{h1}} - \frac{1}{2\sqrt{\Delta\tau_2}k_{h2}}\right)F_{1g} + \frac{k_{h1}}{\dot{k}_{h1}\tau_1}F_{2g} < \frac{F_{1g}}{2\sqrt{\Delta\tau_1}k_{h1}}$$

because $F_{1g} > 0$ and $F_{2g} < -\sqrt{\tau_U}/\gamma < 0$. The proof of Proposition 5 establishes that $\Delta\tau_1 \to \infty$. In addition, the inequality (19) establishes that in equilibrium, both m_1 and m_2 have finite upper and lower bounds, implying that both k_{h1} and F_{1g} is also finite (since $k_h(\cdot)$ is twice-differentiable). Therefore, $\lim_{g_h \uparrow \infty}(F_{1g}/(2\sqrt{\Delta\tau_1}k_{h1}) = 0$ and

$$\lim_{g_h \uparrow \infty}\left[\left(\frac{1}{2\sqrt{\Delta\tau_1}k_{h1}} - \frac{1}{2\sqrt{\Delta\tau_2}k_{h2}}\right)F_{1g} + \frac{k_{h1}}{\dot{k}_{h1}\tau_1}F_{2g}\right] < \lim_{g_h \uparrow \infty}\frac{F_{1g}}{2\sqrt{\Delta\tau_1}k_{h1}} = 0.$$

This proves that in this upper limit, τ_2 is increasing with g_h.

Finally, consider the limit of $g_h \downarrow \hat{g}_h$. As $g_h \downarrow \hat{g}_h$, clearly F_{1g} and F_{2g} are finite. However, $\mu_1 \downarrow 0$, $\Delta\tau_1 \downarrow 0$, and the first term of Equation (21) approaches $+\infty$. The sum of numerators above therefore has a positive sign. Given the negative sign of the denominator, it can be concluded that $\partial\tau_2/\partial g_h < 0$ in the limit of $g_h \downarrow \hat{g}_h$. □

Corollary 1

Proof. Consider the threshold $\hat{g}_t$, at which the benefit of investing in speed to trade at $t = 1$ is small enough, so that the marginal investor is just willing to stay slow. Therefore, at this threshold $\mu_1 = 0$ and $\mu_2 = 1$, implying $\pi_1 = \frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_1)}{\tau_0}\right) - m_1 - \frac{1}{\hat{g}_t}$ and $\pi_2 = \frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_2)}{\tau_2}\right) - m_2$, where $\tau_2 = \tau_0 + \tau_U g_h^2 k_h(m_2)^2/\gamma^2$. In equilibrium, it has to be such that $\pi_1 = \pi_2 = \pi^*$, which implies $\tau_2/(\tau_2 + g_h k_h(m_2)) > \tau_0/(\tau_0 + g_h k_h(m_1))$ because $m_1 + 1/\hat{g}_t > m_1 > m_2$. Subtract by 1 on both sides and rearrange to get $k_h(m_2)/(\tau_2 + g_h k_h(m_2)) < k_h(m_1)/(\tau_0 + g_h k_h(m_1))$.

Next, from the expression of π_1, by envelope theorem, $\frac{\partial\pi^*}{\partial g_h} = \frac{1}{2\gamma}\frac{k_h(m_1)}{\tau_0 + g_h k_h(m_1)} + \frac{1}{\hat{g}_t^2}\frac{\partial\hat{g}_t}{\partial g_h}$. Similarly, from the expression of π_2, $\frac{\partial\pi^*}{\partial g_h} = \frac{1}{2\gamma}\frac{1}{\tau_2 + g_h k_h(m_2)}\left(1 - \frac{2hg_h^2 k_h(m_2)^2}{\gamma^2\tau_2}\right)k_h(m_2) < \frac{1}{2\gamma}\frac{k_h(m_2)}{\tau_2 + g_h k_h(m_2)} < \frac{1}{2\gamma}\frac{k_h(m_1)}{\tau_0 + g_h k_h(m_1)} = \frac{\partial\pi^*}{\partial g_h} - \frac{1}{\hat{g}_t^2}\frac{\partial\hat{g}_t}{\partial g_h}$. Therefore, $\partial\hat{g}_t/\partial g_h < 0$.

Further, consider the extremes of $g_h \downarrow 0$ and $g_h \uparrow \infty$. Toward the lower bound 0, from the expression of π_1 it can be seen that the first term in π_1 drops down to zero. Since an investor always has the option not to trade, π_1 is bounded below by zero. This leads to $m_1 \downarrow 0$ and $1/\hat{g}_t \downarrow 0$, implying $\lim_{g_h \downarrow 0} \hat{g}_t = \infty$. On the other hand, the first-order condition (6) applied to π_1 implies $0 = \frac{\dot{k}_h(m_1)}{2\gamma} - k_h(m_1) - \frac{\tau_0}{g_h} < \left(\frac{1}{2\gamma} - m_1\right)\dot{k}_h(m_1)$, where the inequality follows because $\tau_0/g_h > 0$ and because $k_h(m) \geq \dot{k}_h(m)m$ by concavity. Hence, m_1 is always bounded from above by $1/(2\gamma)$. From the first-order condition, with τ_1 fixed at τ_0, it follows the concavity of $k_h(\cdot)$ that m_1 monotone increases in g_h, and so does $k_h(m_1)$. Taken together, $\lim_{g_h \uparrow \infty} \pi_1 > \frac{1}{2\gamma} \lim_{g_h \uparrow \infty} \ln\left(1 + \frac{g_h k_h(m_1)}{\tau_0}\right) - \frac{1}{2\gamma} - \lim_{g_h \uparrow \infty} \frac{1}{\hat{g}_t}$. If $\lim_{g_h \uparrow \infty} \hat{g}_t > 0$, then the above limit of π_1 shoots to infinity. In that case, the assumed equilibrium will not hold, however, because all slow investors will have incentive to acquire speed by paying $1/\hat{g}_t$ to earn infinite profit. Therefore, it has to be the case that $\lim_{g_h \uparrow \infty} \hat{g}_t = 0$.

Finally, the above concludes that $\hat{g}_t$ is a strictly decreasing function in g_h, with $\hat{g}_t(0) \to \infty$ and $\hat{g}_t(\infty) \to 0$. As the strict monotonicity implies invertibility, there exists $\hat{g}_h(g_t)$ for all $g_t \in (0, \infty)$ such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h(g_t)$. □

References

Admati, Anat R. 1985. "A Noisy Rational Expectations Equilibrium for Multiple Asset Securities Markets." *Ecnometrica* 53 (3):629–657.

Bai, Jennie, Thomas Philippon, and Alexi Savov. 2016. "Have Financial Markets Become More Informative?" *Journal of Financial Economics* 122 (3):625–654.

Banerjee, Snehal, Jesse Davis, and Naveen Gondhi. 2017. "When Transparency Improves, Must Prices Reflect Fundamentals Better?" Working paper.

Baruch, Shmuel, G. Andrew Karolyi, and Michael L. Lemmon. 2007. "Multimarket Trading and Liquidity: Theory and Evidence." *The Journal of Finance* 62 (5):2169–2200.

Biais, Bruno and Theirry Foucault. 2014. "HFT and Market Quality." *Bankers, Markets & Investors* (128).

Biais, Bruno, Thierry Foucault, and Sophie Moinas. 2015. "Equilibrium Fast Trading." *Journal of Financial Economics* 116 (2):292–313.

Bolton, Patrick, Tano Santos, and Jose A. Scheinkman. 2016. "Cream-Skimming in Financial Markets." *The Journal of Finance* 71 (2):709–736.

Boulatov, Alex, Terrence Hendershott, and Dmitry Livdan. 2013. "Informed Trading and Portfolio Returns." *Review of Economic Studies* 80 (1):35–72.

Brennan, Michael J. and H. Henry Cao. 1996. "Information, Trade, and Derivative Securities." *The Review of Financial Studies* 9 (1):163–208.

Brogaard, Jonathan, Terrence Hendershott, and Ryan Riordan. 2014. "High Frequency Trading and Price Discovery." *The Review of Financial Studies* 27 (8):2267–2306.

Brolley, Michael. 2016. "Should Dark Pools Improve Upon Visible Quotes?" Working paper.

Brunnermeier, Markus K. 2005. "Information Leakage and Market Efficiency." *The Review of Financial Studies* 18 (2):417–457.

Budish, Eric, Peter Cramton, and John Shim. 2015. "The High-Frequency Trading Arms Race: Frequent Batch Auctions as a Market Design Response." *Quarterly Journal of Economics* 14 (3):1547–1621.

Buti, Sabrina, Barbara Rindi, and Ingrid M. Werner. 2017. "Dark Pool Trading Strategies, Market Quality and Welfare." *Journal of Financial Economics* 124 (2):244–265.

Cespa, Giovanni. 2008. "Information Sales and Insider Trading with Long-Lived Information." *The Journal of Finance* 53 (2):639–672.

Cespa, Giovanni and Thierry Foucault. 2014. "Illiquidity Contagion and Liquidity Crashes." *The Review of Financial Studies* 27 (6):1615–1660.

Chao, Yong, Chen Yao, and Mao Ye. 2017a. "Discrete Pricing and Market Fragmentation: a Tale of Two-Sided Markets." *American Economic Review: Papers and Proceedings* 107 (5):196–199.

———. 2017b. "Why Discrete Price Fragments U.S. Stock Exchanges and Disperses Their Fee Structures." Working paper.

Chowdhry, Bhagwan and Vikram Nanda. 1991. "Multimarket Trading and Market Liquidity." *Review of Financial Studies* 4 (3):483–511.

Diamond, Douglas W. and Robert E. Verrecchia. 1981. "Information Aggregation in a Noisy Rational Expectations Economy." *Journal of Financial Economics* 9:221–235.

Du, Zhenduo. 2015. "Endogenous Information Acquisition: Evidence from Web Visits to SEC Filings of Insider Trdes." Working paper.

Dugast, Jérôme and Thierry Foucault. 2017. "Data Abundance and Asset Price Informativeness." *Journal of Financial Economics* Forthcoming.

Farboodi, Maryam, Adrien Matray, and Laura Veldkamp. 2017. "Where Has All the Big Data Gone?" Working paper.

Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein. 1992. "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation." *The Journal of Finance* 42 (4):1461–1484.

Gider, Jasmin, Simon N. M. Schmickler, and Christian Westheide. 2016. "High-Frequency Trading and Fundamental Price Efficiency." Working paper.

Goldstein, Itay, Yan Li, and Liyan Yang. 2014. "Speculation and Hedging in Segmented Markets." *The Review of Financial Studies* 27 (3):881–922.

Goldstein, Itay and Liyan Yang. 2015. "Information Diversity and Complementarities in Trading and Information Acquisition." *Journal of Finance* 70 (4):1723–1765.

Griffin, John M. and Jin Xu. 2009. "How Smart Are the Smart Guys? A Unique View from Hedge Fund Stock Holdings." *The Review of Financial Studies* 22 (7):2532–2570.

Grossman, Sanford J. and Joseph E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70 (3):393–408.

Grundy, Bruce D. and Maureen McNichols. 1989. "Trade and the Revelation of Information through Prices and Direct Disclosure." *The Review of Financial Studies* 2 (4):495–526.

Hellwig, Martin F. 1980. "On the Aggregation of Information in Competitive Markets." *Journal of Economic Theory* 22 (3):477–498.

Hirshleifer, David, Avanidhar Subrahmanyam, and Sheridan Titman. 1994. "Security Analysis and Trading Patterns When Some Investors Receive Information Before Others." *The Journal of Finance* 49 (5):1665–1698.

Hoffmann, Peter. 2014. "A Dynamic Limit Order Market with Fast and Slow Traders." *Journal of Financial Economics* 113 (1):156–169.

Holden, Craig W. and Avanidhar Subrahmanyam. 1996. "Risk aversion, Liquidity, and Endogenous Short Horizons." *The Review of Financial Studies* 9 (2):691–722.

———. 2002. "News Events, Information Acquisition, and Serial Correlation." *Journal of Business* 75 (1):1–32.

Kendall, Chad. 2017. "The Time Cost of Information in Financial Markets." Working paper.

Kyle, Albert S. 1985. "Continuous Auctions and Insider Trading." *Econometrica* 53 (6):1315–1336.

Menkveld, Albert J. 2016. "The Economics of High-frequency Trading: Taking Stock." *Annual Review of Financial Economics* 8:1–24.

O'Hara, Maureen. 2015. "High Frequency Market Microstructure." *Journal of Financial Economics* 116 (2):257–270.

Pagano, Marco. 1989. "Trading Volume and Asset Liquidity." *The Quarterly Journal of Economics* 104 (2):255–274.

Pasquariello, Paolo. 2007. "Imperfect Competition, Information Heterogeneity, and Financial Contagion." *The Review of Financial Studies* 20 (2):391–426.

Peress, Jo el. 2004. "Wealth, Information Acquisition, and Portfolio Choice." *The Review of Financial Studies* 17 (3):879–914.

———. 2011. "Wealth, Information Acquisition, and Portfolio Choice: A Correction." *The Review of Financial Studies* 24 (9):3187–3195.

SEC. 2010. "Concept Release on Equity Market Structure." Report, SEC. Release No. 34-61358; File No. S7-02-10.

Shkilko, Andriy and Konstantin Sokolov. 2016. "Every Cloud has a Silver Lining: Fast Trading, Microwave Connectivity, and Trading Costs." Working paper.

Swem, Nathan. 2017. "Information in Financial Markets: Who Gets It First?" Workign paper.

Thomson-Reuters. 2017. "Cost of Compliance 2017." Report.

Van Nieuwerburgh, Stijn and Laura Veldkamp. 2009. "Information Immobility and the Home Bias Puzzle." *The Journal of Finance* 64 (3):1187–1215.

———. 2010. "Information Acquisition and Under-Diversification." *The Review of Economic Studies* 77 (2):779–805.

Verrecchia, Robert E. 1982. "Information Acquisition in a Noisy Rational Expectations Economy." *Econometrica* 50 (6):1415–1430.

Vives, Xavier. 1995. "Short-term Investment and the Informational Efficiency of the Market." *Review of Financial Studies* 8 (1):125–160.

Wang, Xin and Mao Ye. 2017. "Who Supplies Liquidity, and When?" Working paper.

Weller, Brian M. 2016. "Efficient Prices at Any Cost: Does Algorithmic Trading Deter Information Acquisition?" Working paper.

Yao, Chen and Mao Ye. 2017. "Why Trading Speed Matters: A Tale of Queue Rationing under Price Controls." Workign paper.

Ye, Mao. 2011. "A Glimpse into the Dark: Price Formation, Transaction Costs and Market Share of the Crossing Network." Working paper.

Zhu, Haoxiang. 2014. "Do Dark Pools Harm Price Discovery?" *Review of Financial Studies* 27 (3):747–789.

Supplementary material for

"Speed Acquisition"

This note adds to the paper by studying three extensions and verifying the robustness of the main results. The certainty equivalents and equilibrium conditions are characterized in this note to help the numerical studies in the Section 6.

A Frequent fast trading

The main model restricts to "trading-early" feature of the speed technology, i.e., the fast investors only trade at $t = 1$, sooner than slow investors at $t = 2$. This extension considers an alternative setup of "frequent fast trading", where fast investors can trade at both $t \in \{1, 2\}$. That is, the speed technology in addition allows fast investors to trade more frequently. This is the only modification to the main model. The equilibrium analysis proceeds in the following steps: (1) investors' optimal demand functions; (2) the recursion of $\{\tau_t\}$ and $\{p_t\}$; (3) investors' certainty equivalent expressions; and (4) optimal technology acquisition and equilibrium conditions.

Consider investors' demand at $t = 2$ first. All investors, $i \in [0, 1]$, submit their demand schedule $x_{i2}(\cdot)$. Specifically, an arbitrary investor i solves

$$\max_{x_{i2}} E\big[-\exp\{-\gamma \cdot [(p_2 - p_1)x_{i1} + (V - p_2)x_{i2}]\}\big| x_{i1}, s_{i,} p_1, p_2\big],$$

or, equivalently,

$$\max_{x_{i2}} \exp\{-\gamma \cdot [(p_2 - p_1)x_{i1}]\} E\big[-\exp\{-\gamma \cdot [(V - p_2)x_{i2}]\}|s_{i,} p_1, p_2\big].$$

(For a slow investor, who only trades at $t = 2$, $x_{i1} = 0$.) Hence, the optimization problem reduces to the same one as the one faced by the slow investors in the main model. The same conjecture-and-verify analysis as in Lemma 1 applies and gives the optimal linear *cumulative* demand,

$$x_{i2} = \frac{h_i}{\gamma}(s_i - p_2),$$

for both the fast and the slow investors.

It remains to solve for the fast investors trading at $t = 1$. A fast investor i's terminal wealth is $(p_2 - p_1)x_{i1} + (V - p_2)x_{i2}$, where x_{i1} is to be solved and x_{i2} follows the above optimal demand. Recalling that $p_2 = \mathbb{E}[V|\,p_1, p_2\,]$ (competitive market making), therefore, a fast investor's $t = 1$ optimization becomes

$$\max_{x_{i1}} \left[-\exp\left\{ -\gamma \cdot (p_2 - p_1)x_{i1} - \frac{h_i^2}{2(\tau_2 + h_i)}(s_i - p_2)^2 \right\} \Big|\, s_i, p_1 \right],$$

or, equivalently,

$$\max_{x_{i1}} \left[-\exp\left\{ \gamma \cdot (s_i - p_2)x_{i1} - \gamma(s_i - p_1)x_{i1} - \frac{h_i^2}{2(\tau_2 + h_i)}(s_i - p_2)^2 \right\} \Big| s_i, p_1 \right].$$

To simplify notations, let $z_i = s_i - p_2$ and it follows:

$$\mathbb{E}[z_i|\,s_i, p_1\,] = s_i - \frac{\tau_1}{\tau_2}p_1 - \frac{\Delta\tau_2}{\tau_2}\left(\frac{h_i}{\tau_1 + h_i}s_i + \frac{\tau_1}{\tau_1 + h_i}p_1\right);$$
$$\mathrm{var}[z_i|\,s_i, p_1\,] = \left(\frac{\Delta\tau_2}{\tau_2}\right)^2\left(\frac{1}{\tau_1 + h_i} + \frac{1}{\Delta\tau_2}\right).$$

Denote also by

$$\mu := \mathbb{E}[z_i|\,s_i, p_1\,] = \left(1 - \frac{\Delta\tau_2}{\tau_2}\frac{h_i}{\tau_1 + h_i}\right)(s_i - p_1);$$
$$\beta := \mathrm{var}[z_i|\,s_i, p_1\,] = \left(\frac{\Delta\tau_2}{\tau_2}\right)^2\left(\frac{1}{\tau_1 + h_i} + \frac{1}{\Delta\tau_2}\right).$$

The above $t = 1$ optimization problem reduces to:

$$\max_{x_{i1}} \frac{-1}{\sqrt{1 + \frac{h_i^2}{(\tau_2 + h_i)}\mathrm{var}(z_i|s_i, p_1)}} \cdot \exp\left\{ \gamma \cdot \mu x_{i1} - \gamma(s_i - p_1)x_{i1} - \frac{h_i^2}{2(\tau_2 + h_i)}\mu^2 + \frac{1}{2}\frac{(\gamma x_{i1} - \frac{h_i^2}{(\tau_2 + h_i)}\mu)^2\beta}{1 + \frac{h_i^2}{(\tau_2 + h_i)}\beta} \right\}$$

or after some simplification,

$$\max_{x_{i1}} \frac{-1}{\sqrt{1+\frac{h_i^2}{(\tau_2+h_i)}\text{var}(z_i|s_i,p_1)}} \cdot \exp\left\{-\gamma(s_i-p_1)x_{i,1} + \frac{1}{2}\frac{(\gamma x_{i1})^2\beta + 2\gamma\cdot\mu x_{i1} - \frac{h_i^2}{(\tau_2+h_i)}\mu^2}{1+\frac{h_i^2}{(\tau_2+h_i)}\beta}\right\}$$

The first-order condition with respect to $x_{i,1}$ is

$$-(s_i-p_1) + \frac{x_{i1}\beta\gamma+\mu}{1+\frac{h_i^2}{(\tau_2+h_i)}\beta} = 0,$$

which uniquely pins down x_{i1}. Substituting in μ and β and simplifying yield

$$x_{i1} = \frac{h_i}{\gamma}(s_i-p_1),$$

which is exactly the same form as in the main model.

Next consider the recursions of τ_t and p_t. They can be found using the above optimal demand functions. At $t=1$, since the fast investors' optimal demand is the same as shown in Lemma 1, the same results hold: $\Delta\tau_1 = \tau_1 - \tau_0 = \left(\int_{\{t_j=1\}} \frac{h_j}{\gamma}\mathrm{d}j\right)^2 \tau_U$ and $p_1 = p_0 + \frac{\Delta\tau_1}{\tau_1}\left(V + \frac{\gamma U_1}{\int_{\{t_j=1\}} h_j \mathrm{d}j}\right)$. At $t=2$, the market maker observes the aggregate demand

$$\begin{aligned} L_2(p_2) &= \int_{\{t_j=1\}} \left(x_{j2}(s_j,p_2) - x_{j1}(s_j,p_1)\right)\mathrm{d}j + \int_{\{t_j=2\}} x_{j2}(s_j,p_2)\mathrm{d}j + U_2 \\ &= p_1 \int_{\{t_j=1\}} \frac{h_j}{\gamma}\mathrm{d}j - p_2 \int_{\forall j} \frac{h_j}{\gamma}\mathrm{d}j + V\int_{\{t_j=2\}} \frac{h_j}{\gamma}\mathrm{d}j + U_2, \end{aligned}$$

where the second equality follows the optimal demand schedules derived earlier. Observe how the fast investors' signals are exactly offset, not contributing to the price discovery in the second fragment ($t=2$). The market maker then sets the price exactly the same as in Lemma 1 and the resulting recursions hold: $\Delta\tau_2 = \tau_2 - \tau_1 = \left(\int_{\{t_j=2\}} \frac{h_j}{\gamma}\mathrm{d}j\right)^2 \tau_U$ and $p_2 = p_1 + \frac{\Delta\tau_2}{\tau_2}\left(V + \frac{\gamma U_2}{\int_{\{t_j=2\}} h_j\mathrm{d}j} - p_1\right)$.

Finally, consider investors' ex ante certainty equivalent. Since slow investors only trade once at $t=2$, they expect the same certainty equivalent as solved in Lemma 1:

$$\pi_S = \frac{1}{2\gamma}\ln\left(1+\frac{h_S}{\tau_2}\right) - c(h_S).$$

A fast investor i's unconditional expected utility, before paying the technology cost, is

$$\begin{aligned}&\mathbb{E}[-\exp\{-\gamma\cdot(p_2-p_1)x_{i1}-\gamma\cdot(V-p_2)x_{i2}\}]\\=&\mathbb{E}\left[-\exp\left\{-h_i\cdot(s_i-p_1)^2+h_i\cdot(s_i-p_2)(s_i-p_1)-\frac{h_i^2}{2(\tau_2+h_i)}(s_i-p_2)^2\right\}\right]\end{aligned}$$

where the equality follows the optimal demand $x_{i1}(\cdot)$ and x_{i2} derived above. Define $Y := [s_i-p_1; s_i-p_2]$ as a bivariate normal (column) random vector, with

$$\mathbb{E}Y=\begin{bmatrix}0\\0\end{bmatrix}\quad\text{and}\quad\text{var}Y=\begin{bmatrix}\tau_1^{-1}+h_i^{-1} & \tau_2^{-1}+h_i^{-1}\\ \tau_2^{-1}+h_i^{-1} & \tau_2^{-1}+h_i^{-1}\end{bmatrix}.$$

Then the above expected utility can be rewritten as $E[-e^{Y^TAY}]$ where the coefficient matrix A is given by $A=[-h_i, h_i/2; h_i/2, -h_i^2/(2(\tau_2+h_i))]$. Evaluating the expectation with the density of the bivariate normal Y yields the expected utility of $-\tau_1\tau_2/\sqrt{\tau_1\cdot(h_i+\tau_2)(-h_i\tau_1+(h_i+\tau_1)\tau_2)}$. Solving the certainty equivalent yields

$$\pi_{\mathrm{F}}=\frac{1}{2\gamma}\ln\left(1+\frac{h_{\mathrm{F}}}{\tau_1}+\frac{h_{\mathrm{F}}}{\tau_2^2}\frac{\Delta\tau_2}{\tau_1}\right)-c(h_{\mathrm{F}})-\frac{1}{g_t}.$$

Compared to the π_{F} in the main model, it can be seen that there arises an extra term in the $\ln(\cdot)$ when the fast investors are allowed to trade more frequently at $t=2$.

Considering investors' ex-ante technology acquisition, the equilibrium are then characterized by the following conditions:

Optimal information acquisition:	$\partial\pi_{\mathrm{F}}/\partial h_{\mathrm{F}}=\partial\pi_{\mathrm{S}}/\partial h_{\mathrm{S}}=0$;
Indifference in speed:	$\pi_{\mathrm{F}}=\pi_{\mathrm{S}}$;
Population size identity:	$\mu_{\mathrm{F}}+\mu_{\mathrm{S}}=1$.

(It is easy to verify the second-order conditions for optimal information acquisition hold.) The numerical illustrations in Section 6.1 in the main paper are constructed using these results.

B Endogenous population size

The main model fixes the population size at $\mu_\mathrm{F} + \mu_\mathrm{S} = 1$. This extension studies the free entry of investors. Let there be a continuum of investors indexed on $i \in [0, \infty)$, and following the literature (see, e.g., Bolton, Santos, and Scheinkman, 2016), they are sorted according to their reservation value $R(i)$ for not trading. Specifically, if investor i chooses not to trade, he obtains a certainty equivalent of $R(i)$, which is monotone increasing in i. To ensure at least some participation, normalize $R(0) = 0$.

As no other model assumptions are changed, investors trade just like in the baseline and Lemma 1 holds. Investors' certainty equivalents are also in the same form as in Equations (7) in the main paper. It only remains to stipulate the equilibrium conditions characterizing investors' technology acquisition:

$$
\begin{aligned}
&\text{Optimal information acquisition:} && \partial\pi_\mathrm{F}/\partial h_\mathrm{F} = \partial\pi_\mathrm{S}/\partial h_\mathrm{S} = 0;\\
&\text{Indifference in speed:} && \pi_\mathrm{F} = \pi_\mathrm{S};\\
&\text{Indifference in entry} && R(\mu_\mathrm{F} + \mu_\mathrm{S}) = \pi_\mathrm{F} = \pi_\mathrm{S}.
\end{aligned}
$$

Compared with Proposition 1, the only different condition is the last condition that determines the population size. Under the main model, the population is fixed, hence $\mu_\mathrm{F} + \mu_\mathrm{S} = 1$. Here, due to investors' free entry, the marginal investor, i.e., the $(\mu_\mathrm{F} + \mu_\mathrm{S})$-th investor, must be indifferent from trading or not. The numerical analyses presented in Section 6.2 are built upon the above characterization.

C The market clearing mechanism

In the main model, there is always a competitive market maker who clears the market at the efficient price. This extension studies an alternative setting where market clears by a set of uninformed investors, as in, e.g., Grossman and Stiglitz (1980). Specifically, the market maker is replaced by a continuum of uninformed investors in each period with mass n. All investor, informed and uninformed, have the same constant absolute risk-aversion utility with a risk-aversion coefficient γ. All other assumptions are the same as in the main model. In particular, all investors only trade once (buy-and-hold investors). Note that in absence of the market maker, the trading price is determined by the market clearing condition for each period:

$$\int_{i\in[0,1]} x_i \mathbb{1}_{\{t_i=t\}} \mathrm{d}i + U_t + n x_{Ut} = 0,$$

where x_{Ut} is the time-t uninformed investors' demand.

This alternative setting is essentially a static version of Hellwig (1980). The equilibrium trading therefore follows the analysis there. The prices and demand functions are characterized as follows:

$$p_1 = a_{11} z_1, \; p_2 = a_{21} z_1 + a_{22} z_2;$$

where $z_1 = V + \frac{1}{\int_{\{t_j=1\}} \frac{h_j}{\gamma} dj} U_1$, $z_2 = V + \frac{1}{\int_{\{t_j=2\}} \frac{h_j}{\gamma} dj} U_2$; and

$$a_{11} = \frac{\int_{\{t_j=1\}} \frac{h_j}{\gamma} dj + (\mu_F + n)\Delta\tau_1 \Delta\tau_1}{\int_{\{t_j=1\}} \frac{h_j}{\gamma} dj + (\mu_F + n)(\tau_0 + \Delta\tau_1)},$$

$$a_{21} = \frac{(\mu_S + n)\Delta\tau_1}{\int_{\{t_j=2\}} \frac{h_j}{\gamma} dj + (\mu_S + n)(\tau_0 + \Delta\tau_1 + \Delta\tau_2)}, a_{22} = \frac{\int_{\{t_j=2\}} \frac{h_j}{\gamma} dj + (\mu_S + n)\Delta\tau_2}{\int_{\{t_j=2\}} \frac{h_j}{\gamma} dj + (\mu_S + n)(\tau_0 + \Delta\tau_1 + \Delta\tau_2)}.$$

The demand functions of different types of investors are:

$$\begin{aligned}
\text{fast informed:} \quad & x_{i1} = \frac{1}{\gamma}[h_i s_i + \Delta\tau_1 z_1 - (\tau_0 + h_i + \Delta\tau_1) p_1], \\
\text{slow informed:} \quad & x_{i2} = \frac{1}{\gamma}[h_i s_i + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 - (\tau_0 + h_i + \Delta\tau_1 + \Delta\tau_2) p_2], \\
\text{fast uninformed:} \quad & x_{U1} = \frac{1}{\gamma}[\Delta\tau_1 z_1 - (\tau_0 + \Delta\tau_1) p_1], \\
\text{slow uninformed:} \quad & x_{U2} = \frac{1}{\gamma}[\Delta\tau_1 z_1 + \Delta\tau_2 z_2 - (\tau_0 + \Delta\tau_1 + \Delta\tau_2) p_1].
\end{aligned}$$

where $\Delta\tau_1 = \left(\int_{\{t_j=1\}} \frac{h_j}{\gamma} dj\right)^2 \tau_{\mathrm{U}}$ and $\Delta\tau_2 = \left(\int_{\{t_j=2\}} \frac{h_j}{\gamma} dj\right)^2 \tau_{\mathrm{U}}$. Brennan and Cao (1996) have studied investors' expected utilities in such an environment. Following their Section 1, investors' certainty equivalents at the time of technology investment ($t = 0$) are

$$\pi_{\mathrm{F}} = \frac{1}{2\gamma}\ln(\tau_1 + h_{\mathrm{F}}) + \frac{1}{2\gamma}\ln(\mathrm{var}[V - p_1]) - c(h_{\mathrm{F}}) - \frac{1}{g_t};$$
$$\pi_{\mathrm{S}} = \frac{1}{2\gamma}\ln(\tau_2 + h_{\mathrm{S}}) + \frac{1}{2\gamma}\ln(\mathrm{var}[V - p_2]) - c(h_{\mathrm{S}}).$$

Compared to the main model, the difference lies in the second $\ln(\cdot)$ term. When the asset price is set by the market maker, $p_t = \mathbb{E}[V | p_t, p_{t-1}, \dots]$ and $\mathrm{var}[V - p_t] = 1/\tau_t$, under which the above certainty equivalents reduce to those stated in the main paper (Equation 7). When such market maker is replaced by uninformed liquidity providers, $\mathrm{var}[V - p_t]$ no longer takes such simple form and investors certainty equivalents change accordingly. (Intuitively, as the price is no longer efficient, informed investors acquire additional trading gains from providing liquidity to noise trading.)

Finally, the equilibrium conditions remain the same as before:

Optimal information acquisition:	$\partial\pi_{\mathrm{F}}/\partial h_{\mathrm{F}} = \partial\pi_{\mathrm{S}}/\partial h_{\mathrm{S}} = 0;$
Indifference in speed:	$\pi_{\mathrm{F}} = \pi_{\mathrm{S}};$
Population size identity:	$\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1.$

The numerical results in Section 6.3 are constructed based on the above analysis.