Ambiguity preferences, risk taking and the banking firm

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This paper examines the risk taking behavior of a banking firm facing ambiguity and possessing smooth ambiguity preferences. Ambiguity is modeled by a second-order probability distribution that captures the bank's uncertainty about which of the subjective beliefs govern the return on its loans. Ambiguity aversion is modeled by a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the random loan return. Within this framework, we show that the bank finds it less attractive to take risk in the presence than in the absence of ambiguity. This result extends to the case of greater ambiguity aversion. Given that the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, imposing a more stringent capital requirement to the bank has the desired effect that limits the bank's incentive to take on excessive risk.

JEL classification: D01; D81; G11; G12

Keywords: Ambiguity; Ambiguity aversion; Banking firms; Capital requirements

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Abstract

This paper examines the risk taking behavior of a banking firm facing ambiguity and possessing smooth ambiguity preferences. Ambiguity is modeled by a second-order probability distribution that captures the bank's uncertainty about which of the subjective beliefs govern the return on its loans. Ambiguity aversion is modeled by a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the random loan return. Within this framework, we show that the bank finds it less attractive to take risk in the presence than in the absence of ambiguity. This result extends to the case of greater ambiguity aversion. Given that the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, imposing a more stringent capital requirement to the bank has the desired effect that limits the bank's incentive to take on excessive risk.

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1 Introduction

As Caballero and Krishnamurthy (2008) argue, the 2008 financial crisis was primarily caused by the rise of ambiguity resulting from the common aspects of investor behavior—reevaluation of models, conservatism, and disengagement from risky activities. Ivashina and Scharfstein (2010) document that new loans to large borrowers fell by 47% during the peak period of the financial crisis relative to the prior quarter and by 79% relative to the peak of the credit boom. The purposes of this paper are to examine the effect of ambiguity and ambiguity aversion on bank behavior in general, and the credit freeze in particular.

Klibanoff et al. (2005) have recently developed a powerful decision criterion known as "smooth ambiguity aversion" that is compatible with ambiguity averse preferences under uncertainty (hereafter referred to as the KMM model). The KMM model features the recursive structure that is far more tractable in comparison to other models of ambiguity such as the pioneering maxmin expected utility (or multiple-prior) model of Gilboa and Schmeidler (1989). Specifically, the KMM model represents ambiguity by a second-order probability distribution that captures a decision maker's uncertainty about which of the subjective beliefs govern the underlying risk. The KMM model then measures the decision maker's expected utility under ambiguity by taking the (second-order) expectation of a concave transformation of the (first-order) expected utility of payoff conditional on each plausible subjective distribution of the underlying risk. This recursive structure creates a crisp separation between ambiguity and ambiguity aversion, i.e., between beliefs and tastes, thereby making the conventional techniques used in the decision theory under uncertainty applicable in the context of ambiguity (Alary et al., 2013; Gollier, 2011; Snow, 2010, 2011; Taboga, 2005; Wong, 2015).

We develop a firm-theoretic model of a competitive banking firm (Freixas and Rochet, 2008; Wong, 1997) that possesses smooth ambiguity preferences and faces ambiguity of the uncertain return on its loans. The bank sources its funding from deposits and equity capital and is subject to a capital adequacy requirement. We show that the ambiguity-averse bank optimally lends less in response either to the introduction of ambiguity or to greater ambiguity aversion when ambiguity prevails. Specifically, we show that the ambiguity premium is negative (more negative as the bank becomes more ambiguity averse), which reduces the bank's certainty equivalent marginal revenue of loans. The bank as such is induced to lend less so as to restore the optimality condition that the certainty equivalent marginal revenue of loans is equated to the marginal cost of loans. These findings are consistent with the credit freeze during the 2008 financial crisis. Finally, we show that a more stringent capital requirement is effective in limiting the bank's incentive to take on excessive risk should the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion.

The rest of this paper is organized as follows. Section 2 delineates the model of a competitive banking firm facing ambiguity of the return on its loans. Section 3 examine how ambiguity and ambiguity aversion affect the bank's optimal lending decision. In section 4, we examine the effect of a more stringent capital requirement on the bank's risk taking behavior. The final section concludes.

2 The model

Consider a competitive banking firm (hereafter referred to as the bank) that makes decisions in a single period horizon with two dates, 0 and 1. At date 0, the bank has the following balance sheet:

$$L = D + E, \tag{1}$$

where $L > 0$ is the amount of loans, $D > 0$ is the quantity of deposits, and $E > 0$ is the stock of equity capital. By regulation, the bank is subject to the following capital adequacy requirement:

$$\alpha L \leq E, \tag{2}$$

where $\alpha \in (0, 1)$ is the minimum capital-to-loan ratio.

The bank's loans belong to a single homogeneous class, which mature at date 1. The gross return on loans, $\tilde{R}$, is stochastic and distributed according to an objective cumulative distribution function (CDF), $F^{\circ}(R)$, over support $[\underline{R}, \overline{R}]$ with $0 < \underline{R} < \overline{R}$.[1] The bank's deposits are insured by a government-funded deposit insurance scheme. The supply of deposits is perfectly elastic at the fixed one-plus deposit rate, $R_D \geq 1$. The bank's shareholders contribute equity capital with a required gross return, R_E, on their investment, where $R_E > R_D$, reflecting the scarcity of shareholders' wealth. We assume that the unconditional expected gross return on loans is no less than R_E, and thereby is greater than R_D, so that the bank has incentives to extend loans by raising deposits and equity capital.

[1] Throughout the paper, random variables have a tilde (~) while their realizations do not.

The bank's shareholders receive the following net worth at date 1:

$$\tilde{W} = \tilde{R}L - R_D D - R_E E - C(L), \tag{3}$$

where $C(L)$ is the cost function of servicing loans such that $C(0) = C'(0) = 0$ and that $C'(L) > 0$ and $C''(L) > 0$ for all $L > 0$. The bank possesses a von Neumann-Morgenstern utility function, $u(W)$, defined over the net worth of its shareholders at date 1, W, with $u'(W) > 0$ and $u''(W) \leq 0$. The bank is risk neutral or risk averse, depending on whether $u(W) = W$ or $u''(W) < 0$, respectively.

Given that $R_E > R_D$, the bank would like to rely on deposits rather than equity capital to finance loans, thereby rendering the capital adequacy requirement to be binding. Substituting the initial balance sheet constraint, Eq. (1), and the binding capital adequacy requirement, Eq. (2), into Eq. (3) yields the following net worth of the bank's shareholders at date 1:

$$\tilde{W} = (\tilde{R} - R_C)L - C(L), \tag{4}$$

where $R_C = \alpha R_E + (1 - \alpha)R_D$ is the bank's weighted average cost of capital.

The bank faces ambiguity in that it is uncertain about the objective CDF, $F^\circ(R)$. Succinctly, the bank has a continuum of priors, $\{F(R|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$, where $F(R|\theta)$ denotes a plausible first-order CDF of $\tilde{R}$ over support $[\underline{R}, \overline{R}]$, which is sensitive to a parameter, θ, whose value is not known ex ante. Based on its subjective information, the bank associates a second-order CDF, $G(\theta)$, over the continuum of priors, i.e., over support $[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} < \overline{\theta}$. This captures the bank's uncertainty about which of the first-order CDF, $F(R|\theta)$, governs the gross return on loans, $\tilde{R}$. Following Snow (2010, 2011) and Wong (2015), we assume that the bank's ambiguous beliefs are unbiased in the following sense:

$$\int_{\underline{\theta}}^{\overline{\theta}} F(R|\theta)\mathrm{d}G(\theta) = F^\circ(R), \tag{5}$$

for all $R \in [\underline{R}, \overline{R}]$.[2] We denote $\mathrm{E}_F[\cdot|\theta]$, $\mathrm{E}_G[\cdot]$, and $\mathrm{E}_{F^\circ}[\cdot]$ as the expectation operators with

[2]This assumption is motivated by the premise that the behavior of an ambiguity-neutral decision maker should be unaffected by the introduction of, or changes in, ambiguity.

respect to the first-order CDF, $F(R|\theta)$, the second-order CDF, $G(\theta)$, and the objective CDF, $F^\circ(R)$, respectively.

The recursive structure of the KMM model implies that we can compute the bank's expected utility under ambiguity in three steps. First, we calculate the bank's expected utility for each first-order CDF of $\tilde{R}$:

$$U(L,\theta) = \mathrm{E}_F[u(\tilde{W})|\theta], \tag{6}$$

where $\tilde{W}$ is given by Eq. (4). Second, we transform each first-order expected utility obtained in the first step by an ambiguity function, $\phi(U)$, where $\phi'(U) > 0$ and U is the bank's utility level. Finally, we take the expectation of the transformed first-order expected utility obtained in the second step with respect to the second-order CDF of $\tilde{\theta}$. The bank's ex-ante decision problem as such is given by

$$\max_{L\geq 0} \int_{\underline{\theta}}^{\overline{\theta}} \phi\Big(U(L,\theta)\Big)\mathrm{d}G(\theta), \tag{7}$$

where $U(L,\theta)$ is given by Eq. (6). Inspection of the objective function of program (7) reveals that the effect of ambiguity, represented by the second-order CDF, $G(\theta)$, and the effect of ambiguity preferences, represented by the shape of the ambiguity function, $\phi(U)$, can be separated and thus studied independently.

The bank is said to be ambiguity averse if, for a given amount of loans, L, the objective function of program (7) decreases when the bank's ambiguous beliefs, specified by $G(\theta)$, change in a way that induces a mean-preserving spread in the distribution of the bank's expected utility. According to this definition, Klibanoff et al. (2005) show that ambiguity aversion implies concavity for $\phi(U)$, and that a concave transformation of $\phi(U)$ results in greater ambiguity aversion.[3] To see this, we define the ambiguity aversion premium, P, as the solution to the following equation:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi\Big(U(L,\theta)\Big)\mathrm{d}G(\theta) = \phi\bigg(\int_{\underline{\theta}}^{\overline{\theta}} U(L,\theta)\mathrm{d}G(\theta) - P\bigg). \tag{8}$$

[3]When $\phi(U) = [1-\exp(-\eta U)]/\eta$, Klibanoff et al. (2005) show that the maxmin expected utility model of Gilboa and Schmeidler (1989) is the limiting case as the constant absolute ambiguity aversion, η, approaches infinity under some conditions.

Hence, P measures the "pain" the bank is willing to suffer in order to get rid of ambiguity. It follows from Eq. (8) and Jensen's inequality that $P > 0$ if, and only if, $\phi''(U) < 0$. To compare ambiguity aversion, we define the ambiguity aversion premium, P_i, as the solution to the following equation:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_i\Big(U(L,\theta)\Big)\mathrm{d}G(\theta) = \phi_i\bigg(\int_{\underline{\theta}}^{\overline{\theta}} U(L,\theta)\mathrm{d}G(\theta) - P_i\bigg), \tag{9}$$

for $i = 1$ and 2. It follows from Eq. (9) and Pratt that $P_1 < P_2$ if, and only if, $-\phi_1''(U)/\phi_1'(U) < -\phi_2''(U)/\phi_2'(U)$, which is equivalent to $\phi_2(U)$ being a concave transformation of $\phi_1(U)$. To disentangle the effect of ambiguity aversion vis-à-vis that of risk aversion on the bank's risk taking behavior, we assume that the bank is risk neutral, i.e., $u(W) = W$, and ambiguity averse, i.e., $\phi''(U) < 0$, throughout the paper.

Since $u(W) = W$, the first-order condition for program (7) is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi'\Big([R(\theta) - R_C]L^* - C(L^*)\Big)[R(\theta) - R_C - C'(L^*)]\mathrm{d}G(\theta) = 0, \tag{10}$$

where $R(\theta) = \mathrm{E}_F[\tilde{R}|\theta]$ and an asterisk (*) signifies an optimal level. Differentiating the objective function of program (7) with $u(W) = W$ twice yields

$$\frac{\partial^2}{\partial L^2}\int_{\underline{\theta}}^{\overline{\theta}} \phi\Big([R(\theta) - R_C]L - C(L)\Big)\mathrm{d}G(\theta)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \phi''\Big([R(\theta) - R_C]L - C(L)\Big)[R(\theta) - R_C - C'(L)]^2\mathrm{d}G(\theta)$$

$$- \int_{\underline{\theta}}^{\overline{\theta}} \phi'\Big([R(\theta) - R_C]L - C(L)\Big)C''(L)\mathrm{d}G(\theta) = \Delta(L) < 0, \tag{11}$$

for all $L \geq 0$, where the inequality follows from the assumed properties of $\phi(U)$ and $C(L)$. It then follows from Eq. (11) that Eq. (10) is both necessary and sufficient for L^* to the unique maximum solution to program (7) with $u(W) = W$.

3 Ambiguity and risk taking

In this section, we examine the effect of ambiguity on the bank's optimal lending decision. To this end, we consider the benchmark case wherein there is no ambiguity in that the bank knows the objective CDF of $\tilde{R}$, i.e., $F(R|\theta) = F^\circ(R)$ for all $R \in [\underline{R}, \overline{R}]$ and $\theta \in [\underline{\theta}, \overline{\theta}]$. Eq. (10) as such reduces to

$$\mathrm{E}_{F^\circ}[\tilde{R}] - R_C - C'(L^\circ) = 0, \tag{12}$$

where a nought ($^\circ$) indicates an optimal level in the absence of ambiguity. In this benchmark case, Eq. (12) is the usual optimality condition that the risk-neutral bank equates the expected marginal revenue of loans, $\mathrm{E}_{F^\circ}[\tilde{R}]$, to the marginal cost of loans, $R_C + C'(L^\circ)$. Using Eq. (5), we can write Eq. (12) as

$$\mathrm{E}_G[R(\tilde{\theta})] - R_C - C'(L^\circ) = 0. \tag{13}$$

Differentiating the objective function of program (7) with respect to L, and evaluating the resulting derivative at $L = L^\circ$ yields

$$\frac{\partial}{\partial L} \int_{\underline{\theta}}^{\overline{\theta}} \phi\Big([R(\theta) - R_C]L - C(L)\Big)\mathrm{d}G(\theta)\Big|_{L=L^\circ}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \phi'\Big([R(\theta) - R_C]L^\circ - C(L^\circ)\Big)[R(\theta) - R_C - C'(L^\circ)]\mathrm{d}G(\theta). \tag{14}$$

Using Eq. (13), we can write Eq. (14) as[4]

$$\frac{\partial}{\partial L} \int_{\underline{\theta}}^{\overline{\theta}} \phi\Big([R(\theta) - R_C]L - C(L)\Big)\mathrm{d}G(\theta)\Big|_{L=L^\circ}$$

$$= \mathrm{Cov}_G\Big[\phi'\Big([R(\tilde{\theta}) - R_C]L^\circ - C(L^\circ)\Big), R(\tilde{\theta})\Big], \tag{15}$$

which $\mathrm{Cov}_G[\cdot,\cdot]$ is the covariance operator with respect to the CDF, $G(\theta)$. Since $\phi''(U) < 0$, the covariance term on the right-hand side of Eq. (15) is negative. Our first proposition follows immediately from Eqs. (10) and (11).

[4]For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

Proposition 1. *Introducing ambiguity to the ambiguity-averse bank always reduces the optimal amount of loans, i.e., $L^* < L^\circ$.*

To see the intuition for Proposition 1, we can write the right-hand side of Eq. (14) as

$$\mathrm{E}_G[R(\tilde{\theta})] + \frac{\mathrm{Cov}_G\Big[\phi'\Big([R(\tilde{\theta}) - R_C]L^\circ - C(L^\circ)\Big), R(\tilde{\theta})\Big]}{\mathrm{E}_G\Big[\phi'\Big([R(\tilde{\theta}) - R_C]L^\circ - C(L^\circ)\Big)\Big]} - R_C - C'(Q^\circ). \tag{16}$$

We can interpret the second term of expression (16) as the ambiguity premium demanded by the bank to compensate for its exposure to ambiguity at $L = L^\circ$. Given that $\phi''(U) < 0$, the covariance term is negative and thereby the ambiguity premium is negative. To restore the optimality condition that the certainty equivalent marginal revenue of loans is equated to the marginal cost of loans, the ambiguity-averse bank is induced to lend less than L° in response to the introduction of ambiguity.

We now examine the effect of greater ambiguity aversion on the bank's optimal lending decision when ambiguity prevails. To this end, we follow the comparative notion of "more ambiguity aversion" defined by Klibanoff et al. (2005). The more ambiguity-averse bank's ex-ante decision problem is given by

$$\max_{L \geq 0} \int_{\underline{\theta}}^{\overline{\theta}} K\Big(\phi\Big([R(\theta) - R_C]L - C(L)\Big)\Big)\mathrm{d}G(\theta), \tag{17}$$

where $K(\cdot)$ satisfies that $K'(\cdot) > 0$ and $K''(\cdot) < 0$. The first-order condition for program (17) is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} K'\Big(\phi\Big([R(\theta) - R_C]L^\dagger - C(L^\dagger)\Big)\Big)$$

$$\times\phi'\Big([R(\theta) - R_C]L^\dagger - C(L^\dagger)\Big)[R(\theta) - R_C - C'(L^\dagger)]\mathrm{d}G(\theta) = 0, \tag{18}$$

where a dagger (†) indicates an optimal level. It is evident that Eq. (11) remains valid when we replace $\phi(U)$ by $K\Big(\phi(U)\Big)$.

Differentiating the objective function of program (17) with respect to L, and evaluating the resulting derivative at $L = L^*$ yields

$$\frac{\partial}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} K\Big(\phi\Big([R(\theta)-R_C]L - C(L)\Big)\Big)\mathrm{d}G(\theta)\Big|_{L=L^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} K'\Big(\phi\Big([R(\theta)-R_C]L^* - C(L^*)\Big)\Big)$$

$$\times\phi'\Big([R(\theta)-R_C]L^* - C(L^*)\Big)[R(\theta)-R_C-C'(L^*)]\mathrm{d}G(\theta). \tag{19}$$

If the right-hand side of Eq. (19) is negative (positive), it follows from Eqs. (11) and (18) that $L^\dagger < (>)\ L^*$. In the following proposition, we derive sufficient conditions under which the right-hand side of Eq. (19) is negative.

Proposition 2. *Given that the conditional expected loan return, $R(\theta)$, is monotonic in the state variable, θ, making the ambiguity-averse bank more ambiguity averse always reduces the optimal amount of loans, i.e., $L^\dagger < L^*$.*

Proof. Given that $R'(\theta) \geq (\leq)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, it follows from Eq. (10) that there exists a point, $\theta^* \in (\underline{\theta}, \overline{\theta})$, at which $R(\theta^*) = R_C + C'(L^*)$. Note that

$$\frac{\partial}{\partial\theta}K'\Big(\phi\Big([R(\theta)-R_C]L^* - C(L^*)\Big)\Big)$$

$$= K''\Big(\phi\Big([R(\theta)-R_C]L^* - C(L^*)\Big)\Big)\phi'\Big([R(\theta)-R_C]L^* - C(L^*)\Big)R'(\theta), \tag{20}$$

which is negative (positive) given that $R'(\theta) \geq (\leq)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and $K''(\cdot) < 0$. Using Eq. (10), we have

$$\mathrm{Cov}_G\Big[K'\Big(\phi\Big([R(\tilde\theta)-R_C]L^* - C(L^*)\Big)\Big), \phi'\Big([R(\tilde\theta)-R_C]L^* - C(L^*)\Big)[R(\tilde\theta)-R_C-C'(L^*)]\Big]$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big[K'\Big(\phi\Big([R(\theta)-R_C]L^* - C(L^*)\Big)\Big) - K'\Big(\phi\Big([R(\theta^*)-R_C]L^* - C(L^*)\Big)\Big)\Big]$$

$$\times\phi'\Big([R(\theta)-R_C]L^*-C(L^*)\Big)[R(\theta)-R_C-C'(L^*)]\mathrm{d}G(\theta)<0, \tag{21}$$

where the inequality follows from $R'(\theta) \geq (\leq)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and Eq. (20). From Eq. (21), the right-hand side of Eq. (19) is negative so that $L^\dagger < L^*$. □

To see the intuition for Proposition 2, we use Eq. (16) to compare the ambiguity premium under $\phi(U)$ and that under $K\Big(\phi(U)\Big)$, both of which are evaluated at $L = L^*$:

$$\frac{\mathrm{Cov}_G\Big[\phi'\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big), R(\tilde{\theta})\Big]}{\mathrm{E}_G\Big[\phi'\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big]}$$

$$-\frac{\mathrm{Cov}_G\Big[K'\Big(\phi\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big)\phi'\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big), R(\tilde{\theta})\Big]}{\mathrm{E}_G\Big[K'\Big(\phi\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big)\phi'\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big]}$$

$$=-\frac{\mathrm{Cov}_G\Big[K'\Big(\phi\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big), \phi'\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)[R(\tilde{\theta})-R_C-C'(L^*)]\Big]}{\mathrm{E}_G\Big[K'\Big(\phi\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big)\phi'\Big([R(\tilde{\theta})-R_C]L^*-C(L^*)\Big)\Big]}, \tag{22}$$

where we have used Eq. (10). It follows from Eq. (21) that the covariance term on the right-hand side of Eq. (22) is negative given that $R(\theta)$ is monotonic in θ. In this case, the ambiguity premium is more negative under $K\Big(\phi(U)\Big)$ than under $\phi(U)$. Greater ambiguity aversion as such reduces the certainty equivalent marginal revenue of loans. To restore the optimality condition that the certainty equivalent marginal revenue of loans is equated to the marginal cost of loans, the more ambiguity-averse bank finds it optimal to lend less than L^*.

4 Capital requirements and risk taking

In this section, we examine the effect of a more stringent capital requirement on the bank's lending decision. To this end, we totally differentiate Eq. (10) with respect to α to yield

$$\frac{\mathrm{d}L^*}{\mathrm{d}\alpha} = -\frac{1}{\Delta(L^*)}\int_{\underline{\theta}}^{\overline{\theta}} \phi'\Big([R(\theta) - R_C]L^* - C(L^*)\Big)(R_D - R_E)\mathrm{d}G(\theta)$$

$$-\frac{1}{\Delta(L^*)}\int_{\underline{\theta}}^{\overline{\theta}} \phi''\Big([R(\theta) - R_C]L^* - C(L^*)\Big)[R(\theta) - R_C - C'(L^*)](R_D - R_E)L^*\mathrm{d}G(\theta), \tag{23}$$

where $\Delta(L^*) < 0$ is given by Eq. (11). Given that $R_D < R_E$, an increase in α makes R_C higher so that the bank is induced to lend less. This is the substitution effect, which is captured by the first term on the right-hand side of Eq. (23) that is negative. An increase in α also reduces the net worth of the bank's shareholders at date 1 by $(R_E - R_D)L^*$, which creates an income effect that depends on the bank's smooth ambiguity preferences. We say that the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion if the bank's coefficient of absolute ambiguity aversion, $-\phi''(U)/\phi'(U)$, is a non-increasing function of U. In the following proposition, we derive sufficient conditions under which the income effect is negative so that $\mathrm{d}L^*/\mathrm{d}\alpha < 0$.

Proposition 3. *Given that the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion and the conditional expected loan return, $R(\theta)$, is monotonic in the state variable, θ, imposing a more stringent capital requirement to the ambiguity-averse bank always reduces the optimal amount of loans, i.e.,* $\mathrm{d}L^*/\mathrm{d}\alpha < 0$.

Proof. Consider the function:

$$\Phi(\theta) = -\frac{\phi''\Big([R(\theta) - R_C]L^* - C(L^*)\Big)}{\phi'\Big([R(\theta) - R_C]L^* - C(L^*)\Big)}. \tag{24}$$

Given that $R'(\theta) \geq (\leq)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, it follows from Eq. (10) that there exists a point, $\theta^* \in (\underline{\theta}, \overline{\theta})$, at which $R(\theta^*) = R_C + C'(L^*)$. Since $-\phi''(U)/\phi'(U)$ is a non-increasing function of U, Eq. (24) implies that $\Phi'(\theta) \leq (\geq)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ given that $R'(\theta) \geq (\leq)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Using Eq. (10), we have

$$\int_{\underline{\theta}}^{\overline{\theta}} [\Phi(\theta) - \Phi(\theta^*)]\phi'\Big([R(\theta) - R_C]L^* - C(L^*)\Big)[R(\theta) - R_C - C'(L^*)]\mathrm{d}G(\theta) < 0. \tag{25}$$

It then follows from Eq. (25) that the second term on the right-hand side of Eq. (23) is negative so that $\mathrm{d}L^*/\mathrm{d}\alpha < 0$. □

The intuition for Proposition 3 is as follows. Since the bank regards equity capital to be more expensive than deposits, raising the capital-to-loan ratio, α, reduces the net worth of the bank's shareholders at date 1. Since the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, the bank becomes more ambiguity averse when the capital requirement is more stringent. It then follows from Proposition 2 that the bank is induced to lend less in response to a more stringent capital requirement so that the income effect reinforces the substitution effect.

5. Conclusion

In this paper, we examine the risk taking behavior of a competitive banking firm that possesses smooth ambiguity preferences as developed by Klibanoff et al. (2005). The KMM model represents ambiguity by a second-order probability distribution that captures the bank's uncertainty about which of the subjective beliefs govern the return on its loans. On the other hand, the KMM model specifies ambiguity aversion by a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the random loan return. We show that the ambiguity-averse bank optimally lends less in response either to the introduction of ambiguity or to greater ambiguity aversion when ambiguity prevails. Finally, we show that a more stringent capital requirement is effective in limiting the bank's incentive to take on excessive risk should the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion.

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