Vertical Integration and Disruptive Cross-Market R&D

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Abstract

We study how vertical market structure affects the incentives of suppliers and customers to develop a new input that will enable the innovator to replace the incumbent supplier. In a vertical setting with an incumbent monopoly upstream supplier and two downstream firms, we show that vertical integration reduces the R&D incentives of the integrated parties, but increases that of the non-integrated downstream rival. Strategic vertical integration may occur whereby the upstream incumbent integrates with a downstream firm to discourage or even preempt downstream disruptive R&D. Depending on the R&D costs, vertical integration may lower the social rate of innovation.

Keywords: innovation, structural change, vertical integration, replacement effect

JEL Codes: L13, L42, O31

1 Introduction

There are many real-life situations in which downstream producers in vertically related industries enter backward into the upstream market as a result of internal R&D or through the acquisition of independent innovating firms. For example, Apple Inc. once considered acquiring Imagination, a major supplier of the graphics processors used in iPhones, but eventually decided to develop the processors in-house to reduce its reliance on Imagination's technology.[1] On the software side, Apple recently launched the mobile payment system Apple Pay, which is viewed by many analysts as posing a direct competitive threat to the incumbent PayPal, the dominant leader in on-line

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[1] "The Apple discount," *Financial Times*, April 8, 2017; https://www.ft.com/content/3d49b76a-1b76-11e7-a266-12672483791a. Similarly, Apple has relied on its main competitor, Samsung, for the production of the chips used in iPhones and iPads, and has been actively developing its own chips in alliance with the Taiwan Semiconductor Manufacturing Company.

payment services.[2] Similarly, in 2012, Dell created its own software division, Dell Software Group, after conducting a series of acquisitions in the software and service sectors.[3]

Motivated by these observations, we aim to address the following questions. How does market structure affect firms' incentives for developing innovations that may disrupt a vertically related industry? What are the effects of such cross-market R&D on incumbent suppliers, downstream producers, and overall level of innovation? What competitive strategies might the affected firms use to fend off such threats? To answer these questions, we develop a model of innovation that consists of an incumbent monopoly supplier and two downstream producers. Initially, the downstream firms produce differentiated products using an input supplied by the upstream monopolist. The input is sold via a two-part tariff contract.[4] Successful R&D results in a new input with higher quality, or equivalently at a lower cost, than the existing input, and enables an innovating downstream firm to enter backward into the upstream market. Central to our model are the structural changes associated with the post-R&D entry into the upstream sector and the incumbent supplier's exit if the innovator is a downstream firm.

We first consider the case in which the market structure is exogenously given as one of two types: vertical separation, where all three firms are independent entities, and vertical integration, where the upstream firm merges with one of the two downstream firms. Our analysis provides the following insights.

First, we show that vertical integration reduces the R&D incentives of the integrated firms but raises that of the non-integrated downstream rival firm. The driving force for this result is the replacement effect of innovation identified by Arrow (1962) and corroborated by extensive studies on horizontal innovations, which show that a firm's R&D incentive is inversely related to its pre-innovation profit (see, e.g., Tirole 1998; Reinganum 1989; Gilbert 2006a and 2006b). If a downstream firm integrates with the incumbent upstream supplier, the downstream firm's innovation incentive will be dampened, because the new input will replace its upstream unit's current business.

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[2] Apple Pay, which is linked with existing credit cards such as Visa, Mastercard, and Union-Pay, enables customers to make payments on their mobile phones at the point of sale in physical stores. See, e.g., "Apple Pay Takes on Paypal with Long-awaited Functions on Websites," *Financial Review*, 13 June, 2016; http://www.afr.com/technology/technology-companies/apple/apple-pay-takes-on-paypal-with-longawaited-function-for-websites-20160613-gpia4f.

[3] "Dell changes focus from hardware to software services," *Dallas Business Journal*, Aug. 7, 2013. Since its creation, Dell Software Group has introduced many new products, including its Operating System 10 (OS10), which is based on a native, unmodified Linux kernel that can support a broad range of applications and services from the Linux ecosystem. "S10 represents an interesting new direction for Dell as it continues to extend and enhance its networking portfolio with innovations in software and hardware," said Brad Casemore, Research Director, Datacenter Networks, IDC. Press Release, Dell, January 20, 2016. http://www/dell/com/learn/us/en/vn/press-releases/2016-01-20-dell-raises-the-bar-for-open-networking.

[4] This assumption removes the usual double-marginalization incentive for vertical integration and enables us to focus on R&D-related factors for vertical integration/separation. Our main findings still hold under linear pricing, as shown in Lin, Zhang, and Zhou (2017).

As the integrated firms reduce their R&D investments, the non-integrated downstream firm steps up its R&D effort in response.

Second, because of these incentives, there are cases in which it pays the incumbent supplier to strategically acquire a downstream firm to pre-empt its disruptive innovation. We call this type of vertical integration strategic vertical integration. This may occur, for example, if the R&D project is inaccessible or very costly to the other downstream firm.

It is worth noting at this point that in contrast to models with linear pricing on input, vertical integration in our setting is never profitable when the R&D incentives are not considered. In particular, with two-part tariff contracts under vertical separation, the input supplier is able to extract the entire monopoly profit of the industry, whereas vertical integration creates some cost distortion in the downstream market and hence lowers the total profits of the integrated entity.[5] In other words, without R&D, vertical integration will never emerge in our model. However, in the presence of input innovation by the downstream firms, which may disrupt the existing upstream input business, the incumbent supplier may choose to merge with an innovating downstream firm to defend and maintain its upstream market position. We also show that there are circumstances under which it pays the incumbent upstream firm not to merge with its target firm to avoid promoting greater R&D investment from the other downstream firm.[6]

Third, we analyze the effects of vertical integration on the overall level of innovation and show that vertical integration reduces the social rate of innovation (the probability that at least one firm succeeds in R&D) if R&D is sufficiently costly. As argued above, vertical integration reduces the R&D incentives of the integrated firms but increases the R&D incentive of the non-integrated firm. When an R&D project is sufficiently costly, the increase in R&D incentive by the non-integrated firm is insufficient to offset the decrease in the R&D incentive for the integrated firms, resulting in a lower social rate of innovation.

Our analysis reveals that vertical mergers, towards which antitrust agencies are usually more lenient than horizontal mergers, can pre-empt R&D and damage innovation. Antitrust authorities should therefore be aware of the possible preemptive motive of vertical integration and assess its possible negative effects on innovation and welfare.[7]

Unlike in a standard horizontal setting (e.g. Gilbert and Newbery, 1982) in which a firm's

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[5] This property also shows up in Chen and Ross's (2003) study of an input joint venture by two vertically integrated firms.

[6] In our model, market power in the form of vertical integration is actually disadvantageous, which generates an interesting tradeoff between softening downstream competition through vertical separation, and reducing innovation through vertical integration. Such a tradeoff may lead to an evolution of the vertical market structure that is quite different from that in a horizontal setting (Gilbert and Newbery, 1982; Reinganum, 1983, 1985, 1989; Tirole, 1988).

[7] In a recent paper, Cunningham, Ederer and Ma (2018) find empirical evidence that incumbent pharmaceutical firms acquire innovative targets for the preemptive motive of terminating their innovative projects to avoid future competition.

innovation leaves the market structure intact, market disruption through innovation may occur in a vertical setting because firms are naturally asymmetric (i.e., some operate in the downstream and some in the upstream) and an innovator has an incentive to enter the vertically related industry. The finding that market power in a vertical setting (in the form of vertical integration) reduces the integrated firms' R&D incentives resonates with Arrow's argument, and part of the driving force is indeed the replacement effect. Nevertheless, the other part of the driving force concerns the reduced ability of the incumbent upstream firm to fully benefit from its innovation because vertical integration forces its downstream unit to compete too aggressively. This is a new force and is specific to vertical settings. Another feature of our model is that we allow multiple firms to conduct R&D simultaneously. This is not only more realistic, but also provides new insights. In particular, vertical integration discourages R&D in some firms but encourages it in others, and the latter effect can dominate so that market power in a vertical setting may increase the overall amount of innovation. This is consistent with Schumpeter's view but again works through a new channel.

Most of the studies on innovation in a vertical setting assume a single innovator that never changes the market structure. Chen and Sappington (2010) find that vertical integration encourages upstream process innovation if the downstream competition is Cournot, as any upstream cost reduction is passed fully onto the downstream unit in vertical integration, which generates a competitive advantage in Cournot competition. Loertscher and Riordan (2019) study the R&D incentives of upstream suppliers that compete to sell to a monopoly buyer. They find that vertical integration encourages the integrated supplier's R&D but discourages the non-integrated suppliers' R&D because the buyer favors its in-house supplier in procurement. Allain, Chambolle, and Rey (2016) demonstrate that vertical integration creates a hold-up problem for an independent downstream firm and therefore discourages its investment. By contrast, vertical separation preserves upstream competition, thus preventing the upstream firms from appropriating downstream firms' investment benefits.[8] In all of these papers, in contrast to our setting, innovation only affects the innovator's competitive profit without changing the market structure.

Our analysis proceeds as follows. In the next section, we set up the model and carry out the preliminary analysis. In Section 3, we consider the standalone incentive for innovation when only

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[8]Some recent papers also investigate the incentive to innovate in a vertical market structure when hold-up is a major concern. Inderst, Jakubovic, and Jovanovic (2015) examine the shift of innovation activity away from manufacturers and towards large retailers and show that there is a hold-up effect when upstream firms innovate and a rent appropriation effect when innovations come from the retailers. Chambolle, Christin, and Meunier (2015) study a situation in which a retailer may either choose to integrate backward with a small firm or rely on a national brand manufacturer to produce its private label. From a different angle and in a horizontal setting, Chen and Schwartz (2013) show that a monopolist may gain more from the invention of an imperfect substitute than a competitive seller because a monopolist has the advantage of being able to coordinate the pricing of the new and old products.

one firm is assumed to conduct R&D. Section 4 presents the results for R&D competition, where we study how exogenous market structures affect individual and overall R&D, and how these effects, in turn, may affect endogenous integration. Finally, Section 5 concludes the paper. All of the proofs and calculations are collected in the Appendix.

2　The Model

Consider a model of two vertically related industries: a downstream and an upstream industry. In the downstream industry, two firms, D_1 and D_2, compete with horizontally differentiated products. The demand function for D_i's product is $p_i(q_i, q_j)$, which satisfies the following properties: $\frac{\partial p_i}{\partial q_j} < 0$ for $i, j = 1, 2$, and $\left|\frac{\partial p_i}{\partial q_i}\right| > \left|\frac{\partial p_i}{\partial q_j}\right|$ for $j \neq i$. An example would be Cournot competition with a linear demand system $p_i = a - q_i - \beta q_j$, where $\beta \in (0, 1)$ represents the degree of product substitution. The production of the final products requires an input supplied initially by an upstream firm U with a constant marginal cost of production, $c > 0$. One unit of the final product requires exactly one unit of the input. The costs of transforming the input into the final product are normalized to zero.

There are two alternative market structures: vertical separation (S), under which all three firms are independent entities, and vertical integration (I), under which U and D_1 are vertically integrated (into a firm which we denote as UD_1). The input is sold to each independent downstream firm via a two-part tariff contract, $T_i + \omega_i q_i$, where T_i is the lump-sum fee that D_i must pay and ω_i is the marginal cost of obtaining the input. Because the two downstream firms are symmetric, we use $q(y, z)$ to denote the equilibrium output of a downstream firm when its marginal cost of obtaining the input is y and that of its rival is z. Similarly, we use $p(y, z)$ to denote the resulting equilibrium price; that is, $p(y, z) = p_1(q(y, z), q(z, y)) = p_2(q(z, y), q(y, z))$, and use $\pi(y, z)$ to denote the standard duopoly profit (excluding the fixed cost) of a downstream firm. We assume that the usual assumptions for interior equilibrium are satisfied and that the following standard properties hold:

$$\frac{\partial \pi(y, z)}{\partial y} < 0, \ \frac{\partial \pi(y, z)}{\partial z} > 0, \ \text{and} \ \frac{d\pi(y, y)}{dy} < 0.$$

There is an R&D project which, if undertaken successfully, will generate a new input that can be used in place of the existing one. The new input is of a higher quality than the old one. This quality premium can be transformed into a cost premium, i.e., the new input is identical to the old input but has a lower marginal cost of production $c - d$ where the cost differential $d \in [0, c]$ measures the significance of the innovation.

Successful innovation may disrupt the existing industry. For instance, if a downstream firm innovates and invents a new input under market structure S, then it will enter backward into the input market and compete with or even drive out the incumbent supplier U. Under I, if the

non-integrated firm D_2 innovates, then it will not only stop buying the old input from its former supplier, UD_1, but may also choose to supply it with the new input. We refer to this situation as a relationship reversal.

2.1 Pre-innovation under vertical separation

Absent R&D and under S, firm U sells the old input to each downstream firm by offering two-part tariff contracts (ω_i, T_i), or $T_i + \omega_i q$, as total payment for supplying q units of the input. As the only upstream supplier, U solves the following profit-maximization problem:

$$Max_{(\omega_1, T_1, \omega_2, T_2)} \quad \{(\omega_1 - c)\, q\, (\omega_1, \omega_2) + T_1 + (\omega_2 - c)\, q\, (\omega_2, \omega_1) + T_2\}.$$

For any given ω_1 and ω_2, the optimal T_1 is $T_1 = \pi(\omega_1, \omega_2) = (p\,(\omega_1, \omega_2) - \omega_1) q\,(\omega_1, \omega_2)$ and the optimal T_2 is $T_2 = \pi(\omega_2, \omega_1) = (p\,(\omega_2, \omega_1) - \omega_2) q\,(\omega_2, \omega_1)$. Then U's objective becomes

$$Max_{(\omega_1, \omega_2)} \; [p(\omega_1, \omega_2) - c] q\,(\omega_1, \omega_2) + [p(\omega_2, \omega_1) - c] q\,(\omega_2, \omega_1). \tag{1}$$

In equilibrium, U charges the same price to both downstream firms, denoted as $\omega_1 = \omega_2 = \omega^S(c)$ with $\omega^S(c) > c$. The downstream firms earn zero profit, as firm U uses T_i to extract all of the downstream duopoly profits. U's resulting profit is denoted as $\Pi^S(c)$.[9]

2.2 Pre-innovation under vertical integration

Absent R&D and under I, the integrated entity UD_1 supplies the input to its own downstream firm D_1 at the internal cost c, but offers a two-part tariff contract (ω_2, T_2) to D_2. Its optimization problem is

$$Max_{(\omega_2, T_2)} \; \{[p\,(c, \omega_2) - c] q\,(c, \omega_2) + (\omega_2 - c)\, q\,(\omega_2, c) + T_2\}.$$

For any given ω_2, the optimal T_2 is $T_2 = \pi(\omega_2, c) = [p\,(\omega_2, c) - \omega_2] q\,(\omega_2, c)$. Then the objective becomes

$$Max_{\omega_2} \; [p\,(c, \omega_2) - c] q\,(c, \omega_2) + [p(\omega_2, c) - c] q\,(\omega_2, c). \tag{2}$$

Denote the optimal choice as $\omega_2 = \omega^I(c)$. It can be shown that $\omega^I(c) > c$. In equilibrium, D_2 earns zero profit, and the integrated entity UD_1 earns a profit denoted as $\Pi^I(c)$.

Comparing $\Pi^I(c)$ with $\Pi^S(c)$, we find that for any given cost of producing the input, c,

$$\Pi^I(c) \leq \Pi^S(c),$$

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[9]In this paper, we focus on situations in which the two-part tariff contracts are observable to all parties. If contracts are only privately observed, the equilibrium contracts will depend on intricate details about how beliefs are assumed or formed (Rey and Verge, 2004).

with equality only when the final products are perfect substitutes or completely independent. Therefore,

Lemma 1 *The profit of UD_1 under vertical integration is smaller than the joint profits of D_1 and U under vertical separation.*

To understand the (seemingly counter-intuitive) property in Lemma 1, first note that under S and with two-part tariff contracts, firm U is able to achieve the industry monopoly outcome. Because the downstream duopoly equilibrium outcome (under either Cournot or Bertrand competition) is a function of the symmetric marginal cost of purchasing the input, firm U can choose the level of $\omega_1 = \omega_2 = \omega$ to influence the downstream competition. If ω is set at the true marginal cost of producing the input, c, then the resulting symmetric downstream equilibrium price of the final product, $p(c, c)$, will be lower than the corresponding monopoly price implied by c for each final product. Because $p(\omega, \omega)$ is an increasing function of ω, firm U can raise ω such that the corresponding downstream equilibrium price $p(\omega, \omega)$ is equal to the monopoly price of the final product. In this way, under vertical separation, U can implement the industry monopoly outcome, and then use the lump-sum fee to extract all of the downstream profits. The resulting total profit for U, $\Pi^S(c)$, is equal to the industry monopoly profit in the differentiated duopoly market.[10]

Under I, however, the downstream unit of the integrated firm must produce the input at the true marginal cost of c, whereas the non-integrated firm will buy the input it needs to produce the final product at the marginal input price, ω_2, set by U. This constraint limits U's ability to "coordinate" the two downstream firms in making downstream decisions, as in the case of S, as described above. This leads to a lower total profit for U than it would be able to obtain under S.

2.3 Post-innovation under vertical separation

If U innovates, then there is no change in the market structure and U continues to be the sole input supplier. Here, the optimization problem is similar to (1), except that the input cost becomes $c - d$. U's resulting profit is $\Pi^S(c - d)$, with the equilibrium input price being $\omega^S(c - d)$.

If D_1 innovates, then it no longer needs to buy the (old) input from the incumbent supplier U. It will also enter backward into the input market and compete with U for the input purchased from D_2. In particular, U and D_1 simultaneously offer two-part tariff contracts to D_2. Then, D_2 accepts the offer from one of the upstream suppliers and competes with the now vertically integrated D_1 in the downstream market. Upstream competition in this case is similar to standard Bertrand

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[10]This observation was first made by Chen and Ross (2003), who show that an input joint venture set up by downstream rivals can lead to an industry monopoly outcome through the same mechanism described here. However, Chen and Ross do not consider input R&D.

competition, except that each competing supplier has two instruments: the unit price and the lump-sum fee. In equilibrium, the less efficient supplier, U, will be priced out and hence earn zero profit with a contract $(\omega_U, T_U) = (c, 0)$. The more efficient supplier, D_1, solves a problem similar to (2) with marginal cost $c - d$ instead of c, subject to the constraint that its contract must win the competing contract $(c, 0)$ to supply D_2. Using the notation introduced above, the solution to the unconstrained optimization problem is $\omega^I(c - d)$. If $\omega^I(c - d) \leq c$, which will happen if d is sufficiently large, then the presence of U does not constrain D_1's choice, in which case, D_1's payoff equals $\Pi^I(c - d)$, and D_2's payoff is zero. This case is referred to as *drastic innovation*. If, alternatively, $\omega^I(c - d) > c$, then the presence of U does constrain D_1's choice. In particular, if D_2 accepts U's contract, then it will compete with D_1 in the downstream markets and earn a payoff equal to the standard duopoly profit $\pi(c, c - d)$. Therefore, D_1 must offer D_2 a surplus that at least equals $\pi(c, c - d)$, in the form of a lump-sum fee. Therefore, D_1's equilibrium profit will be $\Pi^I(c - d) - \pi(c, c - d)$, and D_2's payoff will be $\pi(c, c - d)$. Such a case is referred to as *non-drastic innovation*.

Under S, D_1 and D_2 are symmetric. Hence, if D_2 is the innovator, the analysis is similar to the above. For both drastic innovation and non-drastic innovation, the innovating downstream firm enters backward into the input market and overthrows the incumbent supplier U. The industry structure changes from S to I.

2.4 Post-innovation under vertical integration

Suppose that U and D_1 are initially integrated. If UD_1 innovates, then the situation is similar to (2), except that the cost of the input is $c - d$ instead of c. As a result, the equilibrium profit for UD_1 is $\Pi^I(c - d)$, and zero for D_2.

If D_2 is the innovator, it is not only able to make the input itself at a marginal cost of $c - d$, it can also sell the input to its former supplier and downstream competitor UD_1. Specifically, D_2 will offer (ω_D, T_D) to UD_1, which has an outside option of using its in-house input at cost c. If the innovation is drastic, the outside option has no value, in which case UD_1's payoff is 0 and D_2's payoff is $\Pi^I(c - d)$. If the innovation is non-drastic, UD_1 receives a payoff that is equal to the duopoly profit it earns if it makes the input it needs at marginal cost c, namely $\pi(c, c - d)$. The innovator D_2 receives a payoff that reflects the compensation to UD_1, namely $\Pi^I(c - d) - \pi(c, c - d)$. In equilibrium, all units of the input are produced using the new $c - d$ technology.

Note that if D_2 innovates, the industry experiences what we call a "relationship reversal:" the innovator D_2, which bought the input from the vertically integrated UD_1 prior to innovation, becomes integrated after innovation and then supplies its former supplier with the innovative production technology. UD_1 shuts down its input production line and ends up being a downstream

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firm. In other words, the invention of the new input leads D_2 and UD_1 to switch their roles as supplier and customer (of course, UD_1's old input product line still constrains D_2's choice of input contract, if the innovation is not drastic.)

The payoffs for all firms in each of these cases are summarized in Table 1 for the case of drastic innovation. (The payoff table for the case of non-drastic innovation can be found in the Appendix.)

Table 1: Payoffs under drastic innovation

Drastic	Under S			Under I	
innovation	U	D_1	D_2	UD_1	D_2
no firm innovates	$\Pi^S(c)$	0	0	$\Pi^I(c)$	0
U innovates	$\Pi^S(c-d)$	0	0	$\Pi^I(c-d)$	0
D_1 innovates	0	$\Pi^I(c-d)$	0		
D_2 innovates	0	0	$\Pi^I(c-d)$	0	$\Pi^I(c-d)$

3 Standalone R&D incentive

In this section, we study how vertical integration affects the R&D incentives when only one firm is capable of conducting R&D, referred to as standalone R&D. This is helpful for understanding the effects of vertical integration when firms compete in R&D, which we analyze in Section 4.

3.1 Standalone R&D incentive

We first state the finding in the following proposition, and then explain each result in turn. Regardless of the significance of the innovation (whether drastic or non-drastic), we have the following result.

Proposition 1 *(Standalone R&D incentive)*

(i) If D_1 is the only firm that can conduct R&D, then its innovation incentive is smaller under integration than under separation.

(ii) If U is the only firm that can conduct R&D, then its innovation incentive is smaller under integration than under separation.

(ii) If D_2 is the only firm that can conduct R&D, then its innovation incentive is the same under separation and integration (between D_1 and U).

Consider D_1's standalone R&D incentive. Under I, D_1's profit is $\Pi^I(c)$ before the innovation and $\Pi^I(c-d)$ after the innovation, and the gain from innovation is thus $\Pi^I(c-d) - \Pi^I(c)$. Under S, D_1's pre-innovation profit is 0, and its post-innovation profit depends on the significance of the innovation. In particular, the post-innovation profit is $\Pi^I(c-d)$ if the innovation is drastic, and $\Pi^I(c-d) - \pi(c, c-d)$ if the innovation is non-drastic.

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If the innovation is drastic, we have $\Pi^I(c-d) - \Pi^I(c) < \Pi^I(c-d)$, meaning that vertical integration reduces D_1's innovation incentive. This can be viewed as a replacement effect (Arrow, 1962). After drastic innovation, the outcome is independent of the pre-innovation market structure. Before the innovation, the innovator earns a positive profit under I but zero under S. As a result, the net gain from innovation is smaller under I than under S.

If the innovation is non-drastic, D_1's innovation benefit is $\Pi^I(c-d) - \Pi^I(c)$ under I, and $\Pi^I(c-d) - \pi(c, c-d)$ under S. However, for the pre-innovation profits, we have

$$
\begin{aligned}
\Pi^I(c) &= \pi(c, \omega^I(c)) + \pi(\omega^I(c), c) + (\omega^I(c) - c)q(\omega^I(c), c) \\
&> \pi(c, \omega^I(c)) \\
&> \pi(c, c-d) \quad (\text{as } \omega^I(c) > c > c-d).
\end{aligned}
$$

That is, D_1's pre-innovation profit is higher under I than under S. Because D_1 earns the same post-innovation profit $\left(\Pi^I(c-d)\right)$ under both I and S, the net gain from innovation for D_1 is smaller under S than under I, namely $\Pi^I(c-d) - \Pi^I(c) < \Pi^I(c-d) - \pi(c, c-d)$. This result is also a form of the replacement effect identified by Arrow (1962). Under S, D_1's R&D incentive is restricted by a small post-innovation profit, as it has to give D_2 some surplus due to competition from the existing supplier, U. Under I, by contrast, D_1's R&D incentive is restricted by a large pre-innovation profit. The concession that D_1 gives up in post-innovation under S is the profit of a disadvantaged downstream firm. This must be smaller than the profit of an advantaged downstream firm, which, in turn, is only part of D_1's pre-innovation profit under I.

Consider U's standalone R&D incentive. For any given marginal cost of producing the input, we have shown that vertical integration reduces the integrated firms' profitability (i.e., $\Pi^I(c) < \Pi^S(c)$ and also $\Pi^I(c-d) < \Pi^S(c-d)$), as summarized in Lemma 1. The reduction of profit caused by integration depends on the marginal cost of producing the input. The Appendix shows that the loss induced by integration is a decreasing function of the marginal cost of producing the input, thus, $\Pi^S(c-d) - \Pi^I(c-d) > \Pi^S(c) - \Pi^I(c)$. This inequality is equivalent to

$$
\Pi^S(c-d) - \Pi^S(c) > \Pi^I(c-d) - \Pi^I(c)
$$

which states that vertical integration reduces U's innovation incentive.

Consider the situation in which D_2 is the only innovator. Before the innovation, D_2 obtains its input from an independent supplier, whose two-part tariff extracts all surplus from D_2, whether under I or S. After the innovation, D_2 becomes its own supplier and competes for D_1's business, with either an outside supplier (in the case of S) or an internal supplier (in the case of I), and ends up with a profit that is again the same across the two market structures. Because the market

structure does not affect D_2's profit either before the innovation (the profit is zero) or after it (the profit is $\Pi^I(c-d)$ for drastic innovation and $\Pi^I(c-d) - \pi(c, c-d)$ for non-drastic innovation), vertical integration does not change D_2's innovation incentive.

3.2 Strategic vertical integration

In this subsection, we examine the possibility that the incumbent input supplier U may choose to acquire a downstream firm solely to pre-empt its disruptive innovation. From Proposition 1, vertical integration reduces the integrated downstream firm's R&D incentive. This implies that there may be situations in which the firm will choose to conduct R&D under S but not under I, assuming it is the only firm with access to R&D.

To illustrate this, consider the following extension of the analysis above. Suppose the original market structure is S, and one of the downstream firms, let's say D_1, is the only firm capable of innovation. Suppose that prior to R&D by D_1, U has a chance to make a take-it-or-leave-it offer (with transfer payments) to fully acquire D_1, which D_1 may accept or reject. D_1 then chooses whether or not to conduct R&D, which requires a fixed R&D cost, F.

Suppose the following holds:

$$\Pi^I(c-d) - \Pi^I(c) < F < \Pi = \begin{cases} \Pi^I(c-d) & \text{for drastic innovation} \\ \Pi^I(c-d) - \pi(c, c-d) & \text{for non-drastic innovation} \end{cases}. \quad (3)$$

In this expression, $\Pi^I(c-d) - \Pi^I(c)$, is the R&D incentive for D_1 under I, and the right-hand side, Π, is the R&D incentive for D_1 under S. From Proposition 1, there are values of the R&D cost F that lie between the two values for which D_1 will conduct R&D under S but not under I. We show in the Appendix that when F is in the above range, the total profits of U and D_1 (net of R&D cost) are greater under I than under S. We thus have the following result.

Corollary 1 *(Strategic vertical integration)*

Assume that only D_1 has access to R&D. If $\Pi^I(c-d) - \Pi^I(c) < F < \Pi$, then U and D_1 will integrate and will not conduct R&D.

The above incentive is driven solely by the presence of R&D in our model. Recall Lemma 1, which states that U will never choose to acquire a downstream firm absent R&D, because it can perfectly coordinate the two downstream firms' output decisions through the two-part tariff contracts and integration would weaken its ability to do so. If D_1 is capable of conducting R&D, however, U will be overthrown and replaced by D_1 after its successful R&D. Recognizing that D_1's incentive for R&D is reduced (as shown in Proposition 1), U can purposely acquire D_1 to preempt its disruptive R&D. Corollary 1 shows that such strategic integration succeeds for those R&D projects satisfying $\Pi^I(c-d) - \Pi^I(c) < F < \Pi$.

Strategic vertical integration results in an integrated industry with an input production cost, c. Without such integration, the industry would also become integrated with an input production cost of $c - d$, as D_1 innovates and then replaces the incumbent supplier U. Strategic vertical integration thus deters innovation and lowers welfare. This suggests that in contrast to the relatively lenient treatment of vertical mergers in the EU and the United States, antitrust authorities should take a tougher stance on vertical mergers when innovation is involved.

4   R&D competition

In this section, we investigate R&D competition. Consider the following game. For any given market structure (either S or I), U, D_1, and D_2 simultaneously and independently choose their investments in R&D. We follow recent models of R&D contests (e.g., Kultti, Takalo, and Toikka, 2006, 2007, and Allain, Chambolle, and Rey, 2016) and assume that the cost of R&D is $C(\rho_i) = \frac{\gamma}{2}(\rho_i)^2$ for firm $i \in \{u, 1, 2\}$, where $\rho_i \in [0, 1]$ is i's probability of R&D success (referred to as its R&D investment), and $\gamma > 0$ is a measure of how costly an R&D project is. The outcomes of R&D projects are independent among these firms. If only one firm succeeds in innovation, then that firm will be the only producer of the new input. If two or more firms succeed, then with equal probability, each firm will win a patent that gives it an exclusive right to produce the new input. After R&D competition, the firms interact in the product market as described in Section 2. We assume that the equilibrium in the product market is unique and stable.

For simplicity, in the rest of the paper we only consider linear demand, $p_i = a - q_i - \beta q_j$, and drastic innovation, where $\beta \in (0, 1)$ measures the degree of product substitutability between the two final products. It can then be shown that under Cournot competition, the upstream supplier's pre-innovation profits under S and I are:

$$\Pi^S(c) = \frac{(a-c)^2}{2(\beta+1)}, \quad \text{and } \Pi^I(c) = \frac{\beta^2 - 8\beta + 8}{4(4 - 3\beta^2)}(a-c)^2,$$

respectively, and that the innovation is drastic, i.e., $\omega^I(c - d) < c$, if and only if

$$\frac{d}{a-c} > \frac{\beta(2-\beta)^2}{16 - (\beta+2)(\beta^2+4)} \equiv \tilde{d}.$$

To ensure that the equilibrium probabilities of successful innovation are less than 1, we assume that γ is sufficiently large. Specifically,

$$\gamma > \frac{\Pi^I(c-d)}{2} \equiv \gamma_0.$$

To isolate the complicated forces, we proceed gradually by first studying pairwise R&D competition, in which two firms compete in R&D investments, before moving onto the full three-way competition.

4.1 R&D competition between downstream firms

Suppose that the two downstream firms, D_1 and D_2, engage in R&D competition. In particular, under a given market structure (S or I), D_1 and D_2 simultaneously and independently choose their R&D investments. The payoff matrices (gross of R&D costs) under S and I are as follows.

	Under S				Under I	
$D_1 \setminus D_2$	Successful	Not Successful		$UD_1 \setminus D_2$	Successful	Not Successful
Successful	$\frac{\Pi^I(c-d)}{2}, \frac{\Pi^I(c-d)}{2}$	$\Pi^I(c-d),\, 0$		Successful	$\frac{\Pi^I(c-d)}{2}, \frac{\Pi^I(c-d)}{2}$	$\Pi^I(c-d),\, 0$
Not Successful	$0,\, \Pi^I(c-d)$	$0,\, 0$		Not Successful	$0,\, \Pi^I(c-d)$	$\Pi^I(c),\, 0$

Under S, the R&D equilibrium is derived by

$$\max_{\rho_1} E\pi_1^S \equiv \rho_1\left[\rho_2\frac{\Pi^I(c-d)}{2} + (1-\rho_2)\Pi^I(c-d)\right] - \frac{\gamma}{2}(\rho_1)^2, \tag{4}$$

$$\max_{\rho_2} E\pi_2^S \equiv \rho_2\left[\rho_1\frac{\Pi^I(c-d)}{2} + (1-\rho_1)\Pi^I(c-d)\right] - \frac{\gamma}{2}(\rho_2)^2. \tag{5}$$

The Nash equilibrium R&D probabilities $\left(\rho_1^S, \rho_2^S\right)$ are given by the first-order conditions:

$$\left(1-\rho_2^S\right)\Pi^I(c-d) + \rho_2^S\frac{\Pi^I(c-d)}{2} = \gamma\rho_1^S \tag{6}$$

$$\left(1-\rho_1^S\right)\Pi^I(c-d) + \rho_1^S\frac{\Pi^I(c-d)}{2} = \gamma\rho_2^S. \tag{7}$$

On the left-hand side of (6), $\Pi^I(c-d)$ in the first term measures the value of D_1's R&D investment (i.e., its profit from success over failure) given that D_2 does not succeed, i.e., D_1's *standalone incentive* for R&D that was studied in the previous section. By contrast, $\frac{\Pi^I(c-d)}{2}$ in the second term measures the value of D_1's investment given that D_2 succeeds ($\frac{1}{2}$ indicates the probability that D_1 wins the patent), and is therefore referred to as D_1's *competitive incentive* for R&D. (6) states that a firm's optimal investment must equate the marginal cost of R&D with the marginal benefit, which is a weighted average of its standalone and competitive incentives. Because D_1 and D_2 are symmetric under S, (7) is similarly expressed. In Figure 1, the two solid lines illustrate the two firms' best responses under S. The lines are both downward sloping (as $\frac{\partial^2 E\pi_i^S}{\partial\rho_i\partial\rho_j} = -\frac{\Pi^I(c-d)}{2} < 0$), meaning that D_1 and D_2's investments are strategic substitutes under S. The R&D equilibrium under S is illustrated by point A in Figure 1. It can be shown that the equilibrium R&D probabilities under S are

$$\rho_1^S = \rho_2^S = \frac{2}{2\sigma + 1},$$

where $\sigma = \frac{\gamma}{\Pi^I(c-d)}$.

Under I, the equilibrium R&D probabilities $\left(\rho_1^I, \rho_2^I\right)$ are determined by

$$\max_{\rho_1} E\pi_1^I \equiv \rho_1\left[\rho_2\frac{\Pi^I(c-d)}{2} + (1-\rho_2)\Pi^I(c-d)\right] + (1-\rho_1)(1-\rho_2)\Pi^I(c) - \frac{\gamma}{2}(\rho_1)^2 \tag{8}$$

$$\max_{\rho_2} E\pi_2^I \equiv \rho_2\left[\rho_1\frac{\Pi^I(c-d)}{2} + (1-\rho_1)\Pi^I(c-d)\right] - \frac{\gamma}{2}(\rho_2)^2. \tag{9}$$
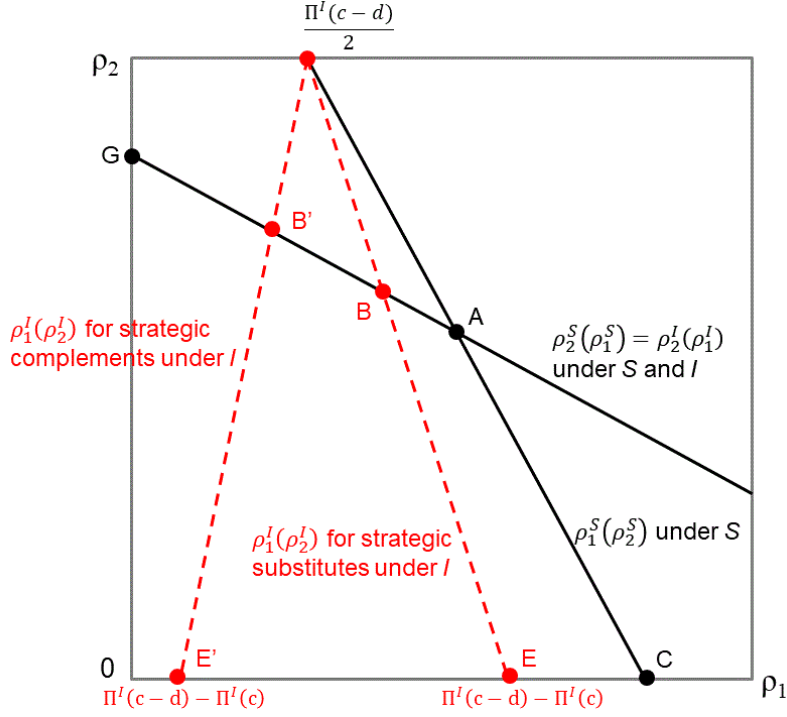
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Figure 1: Best responses in R&D competition between D_1 and D_2

The first-order conditions are

$$\left(1 - \rho_2^I\right)\left[\Pi^I\left(c - d\right) - \Pi^I\left(c\right)\right] + \rho_2^I \frac{\Pi^I(c-d)}{2} = \gamma \rho_1^I \tag{10}$$

$$\left(1 - \rho_1^I\right)\Pi^I\left(c - d\right) + \rho_1^I \frac{\Pi^I(c-d)}{2} = \gamma \rho_2^I. \tag{11}$$

Similar to the previous case, each firm's R&D incentive consists of a standalone component and a competitive component. Note that the competitive incentive is the same in all four equations. D_2's standalone incentive is also the same in (7) and (11), but D_1's standalone incentive is smaller under I than under S (i.e., $\Pi^I\left(c - d\right) - \Pi^I\left(c\right) < \Pi^I\left(c - d\right)$), as per Proposition 1. In Figure 1, therefore, D_2's best response is exactly the same under S and I, whereas D_1's best response under I is a clockwise rotation of its best response under S, becoming one of the two dashed lines depending on which of its two incentives is larger. If $\Pi^I\left(c\right) < \frac{\Pi^I(c-d)}{2}$ (which corresponds to a relatively large d),[11] then the standalone incentive is larger, with $\frac{\partial^2 E\pi_1^I}{\partial\rho_1\partial\rho_2} = \Pi^I\left(c\right) - \frac{\Pi^I(c-d)}{2} < 0$, meaning that D_1's investment under I is a strategic substitute of D_2's. If, by contrast, $\Pi^I\left(c\right) > \frac{\Pi^I(c-d)}{2}$, then the competitive incentive is larger. In that case, $\frac{\partial^2 E\pi_1^I}{\partial\rho_1\partial\rho_2} > 0$, meaning that D_1's investment under I is

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[11] $\Pi^I\left(c\right) < \frac{\Pi^I(c-d)}{2}$ if and only if $\frac{d}{a-c} > \sqrt{2} - 1 \equiv \hat{d}$. Recall that drastic innovation requires $\frac{d}{a-c} > \tilde{d}$. Because $\tilde{d}$ depends on β (in fact, it increases with β) whereas $\hat{d}$ does not, drastic innovation (i.e., $\frac{d}{a-c} > \tilde{d}$) can lead to both strategic complements and strategic substitutes as long as $\beta < 2(\sqrt{2} - 1) \approx 0.828$.

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a strategic complement of D_2's. The R&D equilibrium is illustrated in Figure 1 as points B or B' depending on the slope of D_2's R&D best response function.

We can show that the equilibrium R&D probabilities under I are

$$\rho_2^I = 2\frac{2\sigma + x - 1}{4\sigma^2 + 2x - 1} \quad \text{and} \quad \rho_1^I = 2(1 - \sigma\rho_2^I)$$

where $x = \frac{\Pi^I(c)}{\Pi^I(c-d)} < 1$.

4.1.1 Effects of vertical integration

By comparing the equilibrium R&D investments under S (i.e., point A in Figure 1) with those under I (i.e., point B in the case of strategic substitute, or point B' in the case of strategic complement), we can conclude that the vertical integration between U and D_1 reduces D_1's investment but raises D_2's. Define the social rate of innovation as the probability of at least one firm succeeding in R&D, i.e.,

$$\theta^t \equiv 1 - (1 - \rho_1^t)(1 - \rho_2^t),$$

under market structure $t = I, S$.

Proposition 2 *(Effects of vertical integration under R&D competition)*

When D_1 and D_2 compete in R&D, the following results hold:

(i) the R&D investment of UD_1 under integration is smaller than that of D_1 under separation;

(ii) the R&D investment of D_2 is larger under integration (between D_1 and U) than under separation; and

(iii) integration between D_1 and U raises the social rate of innovation ($\theta^I > \theta^S$) if and only if

$$\frac{\Pi^I(c)}{\Pi^I(c-d)} > \frac{1}{2}\left[\frac{2\gamma}{\Pi^I(c-d)} + 1\right]\left[\frac{2\gamma}{\Pi^I(c-d)} - 1\right]^2,$$

which is more likely to hold when γ is smaller, β is smaller, or d is larger.

Proposition 2(i) extends our result from standalone R&D to R&D competition. The driving force is still Arrow's replacement effect. As D_1 lowers its R&D investment, D_2 responds by raising its investment because the marginal likelihood of winning the patent for D_2 increases.

Proposition 2(ii) states that the probability of at least one of the firms succeeding in R&D may be larger or smaller after the integration of U and D_1, depending on the cost of the R&D project (and the degree of product differentiation in the downstream market and the magnitude of innovation). In particular, overall innovation is more likely to occur after vertical integration when R&D is less costly (γ is smaller), products are less similar (β is smaller), or innovation is more significant (d is larger).

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4.1.2 Endogenous vertical integration

In Section 3.2, we illustrated how firms can carry out integration to preempt disruptive R&D by a downstream firm. However, we only considered the situation in which one downstream firm is capable of conducting R&D. Thus, the question remains whether such a strategic incentive for integration still exists when both downstream firms invest in R&D. To explore this, we consider the following game, which is similar to the one introduced in Section 3.2. In the first stage, U makes a take-it-or-leave-it offer to acquire D_1 for some lump-sum fee, which D_1 may accept or reject. In the second stage, D_1 and D_2 carry out R&D competition as described earlier in this section. Because the firms already face R&D costs, $C(\rho_i)$, we assume away the fixed cost of R&D for simplicity. Other than the R&D competition, this setting differs from that in Section 3.2 in that R&D incurs a marginal cost with probabilistic outcomes rather than a fixed cost with deterministic outcomes. As a result, the influence of innovation on the equilibrium market structure depends on the likelihood of R&D success (as captured by γ). The pre-innovation outcome also matters because there is always a positive probability that neither firm succeeds.

Vertical integration takes place if and only if U and D_1's joint profit is higher under I than under S. The expected profits for D_1 are given by expressions (4) and (8) under S and I, respectively, whereas U's expected profit under S is $E\pi_u^S = (1 - \rho_1^S)(1 - \rho_2^S)\Pi^S(c)$. Then the total expected profits of U and D_1 (under S and I respectively) are

$$E\pi_1^S + E\pi_u^S = \rho_1^S(1 - \tfrac{1}{2}\rho_2^S)\Pi^I(c-d) + (1 - \rho_1^S)(1 - \rho_2^S)\Pi^S(c) - \frac{\gamma}{2}(\rho_1^S)^2 \qquad (12)$$

$$E\pi_1^I = \rho_1^I(1 - \tfrac{1}{2}\rho_2^I)\Pi^I(c-d) + (1 - \rho_1^I)(1 - \rho_2^I)\Pi^I(c) - \frac{\gamma}{2}(\rho_1^I)^2. \qquad (13)$$

We find that integration takes place if and only if γ is not too large. When D_1 and D_2 compete in R&D, the integration between U and D_1 has four effects on the integrated firms' joint profit. Two forces work against integration: integration reduces D_1's success rate, and it also reduces the integrated firms' joint profit when neither firm succeeds in R&D (Lemma 1). However, there are also two forces in favor of integration: it may increase the probability of unsuccessful R&D and thus help maintain U's incumbency, and it can save D_1's R&D costs. The two benefits of integration stem from the fact that integration reduces D_1's R&D investment. Although integration raises D_2's investment, the effect on D_1 can still dominate, in which case integration is an equilibrium choice.

Proposition 3 *(Endogenous vertical integration under R&D competition)*

When D_1 and D_2 compete in R&D, there exists a unique $\tilde{\gamma} > \gamma_0$ such that U and D_1 integrate when $\gamma_0 < \gamma < \tilde{\gamma}$, and remain separate when $\gamma \geq \tilde{\gamma}$.

4.1.3 R&D competition and strategic separation

We now consider how the presence of R&D competition from D_2 affects the strategic integration incentive between U and D_1. As demonstrated in the analysis of standalone R&D in Section 3, there are situations in which U strategically integrates with D_1 to deter D_1 from undertaking disruptive input R&D. Proposition 2 states that when the non-integrated firm D_2 is also capable of conducting R&D, integration between U and D_1 will stimulate greater R&D effort by D_2, thus increasing the risk that the integrated firm will be overthrown by the innovation from the non-integrated firm. This suggests that when facing R&D competition from the other downstream firm, the strategic incentive for vertical integration is likely to be weakened.

Proposition 4 *(Effects of R&D competition on the incentive for vertical separation)*

Assume that only D_1 can conduct R&D. There exists a unique $\bar{\gamma} > \gamma_0$ such that U and D_1 integrate if and only if $\gamma_0 < \gamma < \bar{\gamma}$. Furthermore, $\bar{\gamma} > \tilde{\gamma}$.

That $\bar{\gamma} > \tilde{\gamma}$ implies that U and D_1 will be less likely to integrate when D_2 can also conduct R&D because integration would stimulate more intensive R&D from D_2, which would hurt the integrated entity.

Loertscher and Riordan (2019) have found that a monopoly customer may remain vertically separate to encourage innovation by competitive upstream suppliers because the integrated firm tends to rely more on its internal supplier, thus reducing the R&D reward to an independent supplier. Our result provides a new explanation for why vertical disintegration can discourage innovation by downstream rivals.[12]

4.2 R&D competition between the integrated firms

Now we assume that D_2 has no access to R&D, and U and D_1 engage in R&D competition. The payoffs (gross of R&D cost) under S are

$U \setminus D_1$	Successful	Not Successful
Successful	$\frac{\Pi^S(c-d)}{2}$, $\frac{\Pi^I(c-d)}{2}$	$\Pi^S(c-d)$, 0
Not Successful	0, $\Pi^I(c-d)$	$\Pi^S(c)$, 0

The equilibrium in R&D investments, denoted as $\left(\rho_u^S, \rho_1^S\right)$, are

$$\left(1 - \rho_u^S\right)\Pi^I(c-d) + \rho_u^S \frac{\Pi^I(c-d)}{2} = \gamma\rho_1^S,$$
$$\left(1 - \rho_1^S\right)\left[\Pi^S(c-d) - \Pi^S(c)\right] + \rho_1^S \frac{\Pi^S(c-d)}{2} = \gamma\rho_u^S.$$

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[12]In a model without R&D, Lin (2006) shows that strategic separation enables the once-integrated firm to credibly increase its supply to downstream rivals, thereby reducing the market shares of upstream rivals.

Under I, there is a single innovator, UD_1, whose payoff is $\Pi^I(c-d)$ if its R&D is successful and $\Pi^I(c)$ if it is not. Then UD_1 chooses ρ^I to maximize its expected profit. The first-order condition is

$$\Pi^I(c-d) - \Pi^I(c) = \gamma \rho^I.$$

The derivation of how integration affects each firm's R&D incentive and the social rate of innovation is provided in the Appendix. The results are summarized as follows:

Proposition 5 *(R&D competition between integrated firms)*

When U and D_1 compete in R&D, the following results hold:

(i) if $\tilde{d} < \frac{d}{a-c} \leq \sqrt{2} - 1$, the R&D investment of UD_1 under integration is smaller than that of D_1 or U under separation, and the integration between D_1 and U reduces the social rate of innovation;

(ii) if $\frac{d}{a-c} > \sqrt{2} - 1$ and R&D is not very costly, the R&D investment of UD_1 under integration is larger than that of D_1 or U under separation, and the integration between D_1 and U raises the social rate of innovation; and

(iii) the two firms integrate if and only if R&D is not very costly.

The integrated entity's R&D incentive, $\Pi^I(c-d) - \Pi^I(c)$, is always smaller than D_1's stand-alone incentive, $\Pi^I(c-d)$. When $\frac{d}{a-c}$ is small, the integrated entity's R&D incentive $\Pi^I(c-d) - \Pi^I(c)$ is also smaller than D_1's competitive incentive, $\frac{\Pi^I(c-d)}{2}$. Putting the two together, integration reduces the overall R&D investment of D_1. As with the comparison between UD_1's incentive and U's incentive, if $\frac{d}{a-c}$ is large, then UD_1's R&D incentive is greater than D_1's competitive incentive under S. As the latter is weighted by U's success rate, if γ is small, then U is likely to succeed, which means that D_1's overall R&D incentive under S is small. In that case, the integrated firm's R&D investment can exceed D_1's investment under S. For the same reason, it can also exceed U's investment under S.

When the innovation is significant (i.e., $\frac{d}{a-c}$ is large), a firm's major concern in R&D competition is that it may not reap the full benefit of the innovation when its rival also succeeds in R&D. This reduces each firm's expected return on R&D and hence its equilibrium R&D investment. Vertical integration eliminates concerns about cannibalization because the R&D investor is able to fully benefit from an innovation. This may encourage R&D investment to the extent that the social rate of innovation may be higher under I than under S, which happens when R&D is not very costly.

Finally, we consider endogenous vertical integration. Suppose U and D_1 are originally separated and have a chance to integrate. Consider their expected joint profits under S and I, taking into

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consideration the endogenous investments in R&D:

$$
\begin{aligned}
E\pi_1^S + E\pi_u^S &= \rho_1^S \left(1 - \tfrac{\rho_u^S}{2}\right) \Pi^I (c-d) + \rho_u^S \left(1 - \tfrac{\rho_1^S}{2}\right) \Pi^S (c-d) \\
&\quad + \left(1 - \rho_u^S\right)\left(1 - \rho_1^S\right) \Pi^S (c) - \tfrac{\gamma}{2}(\rho_1^S)^2 - \tfrac{\gamma}{2}(\rho_u^S)^2 \\
E\pi^I &\equiv \rho^I \Pi^I (c-d) + \left(1 - \rho^I\right) \Pi^I (c) - \tfrac{\gamma}{2}(\rho^I)^2.
\end{aligned}
$$

The major trade-offs are as follows. On the one hand, integration reduces the joint profit: successful innovation leads to a profit of $\Pi^S (c-d)$ under S (when U succeeds), but a smaller profit $\Pi^I (c-d)$ under I; unsuccessful innovation leads to a profit of $\Pi^S(c)$ under S, and a smaller profit $\Pi^I(c)$ under I. This tends to discourage integration. On the other hand, integration internalizes the negative externality of each firm's R&D on the other firm under S, and therefore reduces the joint R&D investment of the two firms. When the R&D cost is low, each firm tends to invest heavily in R&D competition under S, causing great damage to each other. In that case, there is a great need to integrate to reduce the damage. The second effect thus dominates and the equilibrium choice is integration.

4.3 R&D competition between the upstream and the non-integrated downstream firm

Consider pairwise R&D competition between U and D_2, and how each firm's R&D incentive is affected by whether U and D_1 (which is not capable of conducting R&D) are separate or integrated. The profit expressions can be found in the Appendix. Under S, the first-order conditions are:

$$
\begin{aligned}
\left(1 - \rho_2^S\right) \left[\Pi^S (c-d) - \Pi^S (c)\right] + \rho_2^S \tfrac{\Pi^S(c-d)}{2} &= \gamma \rho_u^S \\
\left(1 - \rho_u^S\right) \Pi^I (c-d) + \rho_u^S \tfrac{\Pi^I(c-d)}{2} &= \gamma \rho_2^S.
\end{aligned}
$$

Under I, the first-order conditions are:

$$
\begin{aligned}
\left(1 - \rho_2^I\right) \left[\Pi^I (c-d) - \Pi^I (c)\right] + \rho_2^I \tfrac{\Pi^I(c-d)}{2} &= \gamma \rho_u^I \\
\left(1 - \rho_u^I\right) \Pi^I (c-d) + \rho_u^I \tfrac{\Pi^I(c-d)}{2} &= \gamma \rho_2^I.
\end{aligned}
$$

After integration, the upstream firm's standalone incentive drops from $\Pi^S (c-d) - \Pi^S (c)$ to $\Pi^I (c-d) - \Pi^I (c)$, whereas its competitive incentive drops from $\tfrac{\Pi^S(c-d)}{2}$ to $\tfrac{\Pi^I(c-d)}{2}$. As a result, U's best response curve moves to the left (see the best responses in Figure 2 in the Appendix). Because D_2's best response curve does not shift, we immediately conclude that integration reduces U's R&D investment but raises D_2's investment. Calculation shows that integration raises the social rate of innovation when γ is below some threshold, for a reason similar to that in the previous case. Finally, when integration is endogenized, it will never happen in equilibrium. This is easy to understand:

integration hurts U and D_1 by reducing their joint profit for any given input cost (Lemma 1) and by raising D_2's R&D effort.

Proposition 6 *(R&D competition between U and D_2)*

When U and D_2 compete in R&D, the following results hold:

(i) the R&D investment of UD_1 under integration is smaller than that of U under separation;

(ii) the R&D investment of D_2 is larger under integration (between D_1 and U) than under separation;

(iii) integration between D_1 and U raises social rate of innovation if and only if R&D is not very costly; and

(iv) U and D_1 never integrate with one another.

4.4 Three-way R&D competition

Now we consider R&D competition among all three firms. Under S, the probability that firm i will succeed in R&D and win the patent is

$$\phi_i = \rho_i \left[\left(1 - \rho_j\right)\left(1 - \rho_k\right) + \frac{1}{2}\rho_j(1 - \rho_k) + \frac{1}{2}\left(1 - \rho_j\right)\rho_k + \frac{1}{3}\rho_j\rho_k \right]$$

for $i, j, k \in \{u, 1, 2\}$, and the probability that all three firms will fail in R&D is $\phi_0 = (1 - \rho_i)\left(1 - \rho_j\right)(1 - \rho_k)$. Then the expected profit of firm U is $E\pi_u^S = \phi_u \Pi^S(c - d) + \phi_0 \Pi^S(c) - \frac{\gamma}{2}(\rho_u)^2$, whereas the expected profit of D_1 is $E\pi_1^S = \phi_1 \Pi^I(c - d) - \frac{\gamma}{2}(\rho_1)^2$. The expected profit of D_2 can be similarly derived. Let $(\rho_1^S, \rho_2^S, \rho_u^S)$ denote the equilibrium investments of R&D competition under S. As D_1 and D_2 are symmetric under S, we have $\rho_1^S = \rho_2^S = \rho^S$. The first-order conditions are

$$\left(1 - \rho^S\right)^2 \left[\Pi^S(c - d) - \Pi^S(c)\right] + \left[\rho^S(1 - \rho^S) + \frac{1}{3}(\rho^S)^2\right] \Pi^S(c - d) = \gamma \rho_u^S$$

$$\left[\left(1 - \rho_u^S\right)(1 - \rho^S) + \frac{\rho_u^S(1 - \rho^S) + \left(1 - \rho_u^S\right)\rho^S}{2} + \frac{\rho_u^S \rho^S}{3}\right] \Pi^I(c - d) = \gamma \rho^S.$$

Under I, the two competing innovators are UD_1 and D_2. The expected profit of the integrated firm UD_1 is $E\pi_1^I \equiv \rho_1 \left[(1 - \rho_2) + \frac{\rho_2}{2}\right] \Pi^I(c - d) + (1 - \rho_1)(1 - \rho_2)\Pi^I(c) - \frac{\gamma}{2}(\rho_1)^2$, whereas the expected profit of D_2 is $E\pi_2^I \equiv \rho_2 \left[(1 - \rho_1) + \frac{\rho_1}{2}\right] \Pi^I(c - d) - \frac{\gamma}{2}(\rho_2)^2$. Let (ρ^I, ρ_2^I) denote the equilibrium investments of R&D competition under I. The first-order conditions are

$$\left(1 - \rho_2^I\right) \left[\Pi^I(c - d) - \Pi^I(c)\right] + \rho_2^I \frac{\Pi^I(c - d)}{2} = \gamma \rho^I$$

$$\left[\left(1 - \rho^I\right) + \frac{\rho^I}{2}\right] \Pi^I(c - d) = \gamma \rho_2^I.$$

The Appendix proves the following analytical results.

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Proposition 7 *(Three-way R&D competition)*

When all three firms compete in R&D, the following results hold:

(i) the R&D investment of D_2 is larger under integration (between D_1 and U) than under separation;

(ii) the R&D investment of UD_1 under integration is smaller than that of D_1 under separation if $\tilde{d} < \frac{d}{a-c} < \sqrt{2} - 1$ and γ is sufficiently large; and

(iii) the R&D investment of UD_1 under integration is smaller than that of U under separation if $\tilde{d} < \frac{d}{a-c} < \sqrt{\frac{3}{2}} - 1$ and γ is sufficiently large.

This main intuition of Proposition 7 can be understood by starting from the case of pair-wise competition between D_1 and D_2, and then adding U to the game. Under vertical integration (between U and D_1), the three-firm economy becomes one of two, with D_2 competing with the vertically integrated firm UD_1 in R&D. Under S, and in the three-way R&D game, D_2 competes in R&D with both D_1 and U, whereas under S, and in the pair-wise R&D game between D_1 and D_2, D_2 competes with D_1 only. Hence, other things being equal, D_2's probability of succeeding in R&D and winning the patent is lower in the three-way R&D game than in the pair-wise R&D game. This implies that the incentive for D_2 to invest in R&D is lower in the three-way game than in the pair-wise game. As shown earlier, D_2's R&D incentive rises in the pairwise R&D game once U and D_1 integrate. It follows that D_2's R&D incentive rises (from a lower level) in the three-way R&D game if U and D_1 integrate, as stated in part (i) of Proposition 7.

Regarding part (ii) and part (iii), the basic force is still the replacement effect associated with vertical integration: U and D_1's R&D incentives are reduced once they integrate because innovating the new input replaces their old input. However, there is another force at work. Under S, D_1 faces R&D competition from U under three-way competition but not under the pair-wise competition. Vertical integration eliminates this competition, which increases D_1's probability of winning the patent and hence its R&D incentive. Likewise, vertical integration reduces U's R&D incentive because of the replacement effect. Of course, D_2's incentive also changes here. We are able to show that for the parameter values stated in (ii) and (iii) of the proposition (which both require that d is not too large), D_1 and U's R&D incentives are reduced under vertical integration.

5 Conclusion

In this paper, we have studied firm R&D incentives in a vertical setting in which a new input can be developed not only by the incumbent input supplier but also by existing customers. Unlike in most studies in a horizontal or vertical setting, such cross-market innovation usually results in structural changes. In particular, in a vertically separated industry, downstream input R&D transforms the

industry into a vertically integrated one as the innovating downstream firm enters backward into the upstream industry. In a vertically integrated industry, input R&D by the non-integrated firm leads to a relationship reversal, whereby the innovating non-integrated firm supplies the new input to its former supplier after successful R&D. We show that vertical integration reduces the integrated firms' R&D incentives, but raises the non-integrated firm's R&D incentive, which may lead to a higher rate of overall innovation if the latter's R&D investment is substantial. The concern about the structural changes caused by disruptive downstream R&D can produce strategic incentives for both vertical integration and separation. We show that situations exist in which an upstream supplier may takeover a downstream firm to preempt its otherwise disruptive R&D, which would overthrow the supplier. Similarly, there are cases in which the incumbent supplier foregoes its downstream target firm and instead commits to separation for fear of increasing the R&D incentive of the other downstream competitor. To the best of our knowledge, such incentives for strategic integration and separation have not been identified in the literature.

The input in our model is sold to the downstream firms through two-part tariff contracts.[13] Under separation, these contracts enable the upstream incumbent to implement the industry monopoly outcome by choosing the appropriate marginal cost of obtaining the input for the downstream firms. Hence, vertical integration between the upstream supplier and a downstream firm never occurs in the absence of R&D. Yet, we show that in the presence of R&D, vertical integration can occur for certain parametric values, driven specifically and solely by the above mentioned strategic incentive. If the vertical transaction takes the form of linear pricing, vertical integration will be beneficial because it will eliminate double marginalization. Nevertheless, the incumbent firm may strategically choose not to acquire its downstream target firm to avoid stimulating R&D investment by the other downstream firm (Lin, Zhang, and Zhou, 2017).

The focus of our paper is on how vertical integration and separation can affect firms' incentives for cross-market R&D. As a result, we have abstracted from the possibility that the upstream supplier might use the input contract strategically to influence the R&D incentives of the downstream firms. For example, the upstream firm may deliberately lower the unit and lump-sum prices to "bribe" the downstream firms to reduce their R&D investments. It would be interesting to analyze whether the strategic outcomes highlighted in this paper can be achieved through strategic contracting rather than mergers. Such input contracts must be long-term (as R&D projects usually are long-term), and must be terminated if any R&D project succeeds, in which case the industry structure will necessarily change as the innovator replaces the incumbent supplier. Because the R&D success dates are uncertain, it may be difficult for a contract to specify termination conditions based on the outcome of an unidentified party's R&D success. Future research along this line

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[13]We thank a referee and a co-editor for suggesting this setting.

may be promising.

We also abstracted away patent licensing in the current model.[14] If licensing of the patented input is possible, a downstream innovator, e.g., D_1, may have an incentive to license its new innovation to the upstream supplier U, instead of entering backward into the upstream market directly and overthrowing U. Licensing enables the firms to realize the returns to R&D without having to integrate with one another and, hence, avoid the disadvantages associated with vertical integration as stated in Lemma 1. In other words, cross-market licensing of innovation may serve as a device against disruptive innovation. Future research along this line seems warranted.

6 Appendix

6.1 Payoff table under non-drastic innovation

Table 2: Payoffs under non-drastic innovation

Non-drastic	Under S			Under I	
innovation	U	D_1	D_2	UD_1	D_2
no firm innovates	$\Pi^S(c)$	0	0	$\Pi^I(c)$	0
U innovates	$\Pi^S(c-d)$	0	0	$\Pi^I(c-d)$	0
D_1 innovates	0	Δ	$\pi(c,c-d)$		
D_2 innovates	0	$\pi(c,c-d)$	Δ	$\pi(c,c-d)$	Δ

where $\Delta \equiv \Pi^I(c-d) - \pi(c,c-d)$.

6.2 Proof of Proposition 1

The effects on D_1 and D_2 are straightforward and are provided in the text. Here, consider the effect of integration on U's standalone incentive. Let $\delta(c) \equiv \Pi^S(c) - \Pi^I(c) > 0$ denote the difference between the equilibrium profits under S and I for any given input production cost c. We have established that $\Pi^S(c) = \max_{\omega_1,\omega_2} \Pi(\omega_1,\omega_2,c) \equiv [p(\omega_1,\omega_2) - c]q(\omega_1,\omega_2) + [p(\omega_2,\omega_1) - c]q(\omega_2,\omega_1)$, whereas $\Pi^I(c) = \max_{\omega_1,\omega_2} \Pi(\omega_1,\omega_2,c)$ subject to the constraint $\omega_1 = c$. Therefore $\Pi^I(c) = \Pi(c) - \lambda(\omega_1 - c)$, where $\lambda > 0$ is the usual Lagrangian. Now apply the Envelope Theorem, $\delta'(c) = \frac{\partial \Pi^S(c)}{\partial c} - \frac{\partial \Pi^I(c)}{\partial c} = \frac{\partial \Pi(\omega_1,\omega_2,c)}{\partial c} - \left[\frac{\partial \Pi(\omega_1,\omega_2,c)}{\partial c} + \lambda\right] = -\lambda < 0$, meaning that when c is lower, $\Pi^S(c) - \Pi^I(c)$ will be larger. Given that $c - d < c$, we have $\Pi^S(c-d) - \Pi^I(c-d) > \Pi^S(c) - \Pi^I(c)$.

6.3 Proof of Corollary 1

In the second, innovation stage, D_1 chooses to conduct R&D if and only if the benefit from R&D (which is its standalone R&D incentive) is greater than the R&D cost. Given (3), which simply

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[14]We thank a referee for pointing out the possibility of patent licensing.

states that the R&D cost is smaller than the R&D benefit under S, but greater than the R&D benefit under I, D_1's optimal choice is to conduct R&D under S, but not under I.

Now we move back to the acquisition stage. Given that U can acquire D_1 at a lump-sum fee, which is a transfer between the two firms, the acquisition (i.e., U makes an offer and D_1 accepts it) takes place if and only if the two firms' total profits are greater under I than under S, taking into consideration how each market structure will affect D_1's R&D incentive. As per the previous analysis, under I, D_1 does not conduct R&D, and the integrated entity (i.e., the two firms combined) earns a profit of $\Pi^I(c)$. Under S, D_1 conducts R&D, and the two firms' total profit is $\Pi - F$ (with U's profit being 0). Given the left side of (3), we have $\Pi^I(c) > \Pi^I(c-d) - F = \Pi - F$ (for drastic innovation) and consequently $\Pi^I(c) > \Pi^I(c-d) - \pi(c, c-d) - F = \Pi - F$ (for non-drastic innovation), meaning that the two firms' joint profit is larger under I than under S. In equilibrium, then, U acquires D_1.

6.4   Profits under linear demand and Cournot competition

The demand is $p_i = a - q_i - \beta q_j$. Given (ω_i, ω_j), downstream Cournot equilibrium leads to

$$q_i(\omega_i, \omega_j) = \frac{(2-\beta)a - 2\omega_i + \beta\omega_j}{4-\beta^2}, \quad \text{and} \quad p_i(\omega_i, \omega_j) = \frac{(2-\beta)a - (2-\beta^2)\omega_i + \beta\omega_j}{4-\beta^2}.$$

Under S, the upstream firm U chooses ω_1 and ω_2 to maximize $\Pi(\omega_1, \omega_2) = (p_1 - c)q_1 + (p_2 - c)q_2$, and the solution is

$$\omega^S(c) = \frac{\beta(a-c)}{2(\beta+1)} + c, \quad \text{and} \quad \Pi^S(c) = \frac{(a-c)^2}{2(\beta+1)}.$$

Under I, UD_1 supplies its downstream subsidiary at $\omega_1 = c$ and chooses the supply price ω_2 to maximize $\Pi(\omega_2) = (p_1 - c)q_1 + (p_2 - c)q_2$, and the solution is

$$\omega^I(c) = \frac{\beta(2-\beta)^2(a-c)}{8 - 6\beta^2} + c, \quad \text{and} \quad \Pi^I(c) = \frac{\beta^2 - 8\beta + 8}{4(4 - 3\beta^2)}(a-c)^2.$$

The innovation is drastic, i.e., $\omega^I(c-d) < c$, if and only if

$$\frac{d}{a-c} > \frac{\beta(2-\beta)^2}{8 - 4\beta - 2\beta^2 - \beta^3} \equiv \tilde{d}.$$

Also, $\Pi^I(c) < \frac{\Pi^I(c-d)}{2}$ if and only if

$$\frac{d}{a-c} > \sqrt{2} - 1 \equiv \hat{d}.$$

Note that $\tilde{d}$ depends on β whereas $\hat{d}$ does not. Drastic innovation (i.e., $\frac{d}{a-c} > \tilde{d}$) can thus lead to both strategic complements and strategic substitutes as long as $\beta < 2(\sqrt{2} - 1) \approx 0.828$.

6.5   Proof of Proposition 2

Given the first-order conditions in the text, we can easily solve

$$\rho_1^S = \rho_2^S = \frac{2}{2\sigma + 1}; \ \ \rho_2^I = 2\frac{2\sigma + x - 1}{4\sigma^2 + 2x - 1}, \ \ \rho_1^I = 2(1 - \sigma\rho_2^I), \tag{14}$$

where $\sigma = \frac{\gamma}{\Pi^I(c-d)}$ and $x = \frac{\Pi^I(c)}{\Pi^I(c-d)} < 1$. Note that $\sigma > \frac{1}{2}$ ensures that these equilibrium ρs are less than one. Then, it is straightforward to show that $\rho_1^S < \rho_2^I$. Moreover,

$$\rho_1^I = 2(1 - \sigma\rho_2^I) < 2(1 - \sigma\frac{2}{2\sigma + 1}) = \rho_1^S.$$

Furthermore, $\theta^I > \theta^S$ if and only if

$$x > \frac{1}{2}(2\sigma + 1)(2\sigma - 1)^2. \tag{15}$$

The right-hand side of (15) increases in σ for $\sigma > \frac{1}{2}$, such that it is more likely to hold when σ is smaller, i.e., when γ is smaller. Conversely, $x = \frac{\Pi^I(c)}{\Pi^I(c-d)} = \frac{(a-c)^2}{(a-c+d)^2}$, which is independent of β, whereas $\sigma = \frac{\gamma}{\Pi^I(c-d)} = \frac{\gamma}{(a-c+d)^2}\frac{4(4-3\beta^2)}{\beta^2-8\beta+8}$, which increases in β. When β is larger, σ increases while x does not change. Then (15) is less likely to hold. We can also show that a larger d makes (15) more likely to hold.

6.6   Proof of Proposition 3

The equilibrium investments were solved in (14). These can be plugged into the expressions of joint profits (12) and (13) to establish that $E\pi_1^I > E\pi_1^S + E\pi_u^S$ if and only if $H(\gamma) > 0$. The closed-form analytical expression for $H(\gamma)$ (which also contains other parameters, such as $a - c$, d, and β) is available but not reported here as the expression is tedious. We can show that there exists a unique $\tilde{\gamma}$ such that $H(\tilde{\gamma}) = 0$, with $H(\gamma) > 0$ for $\gamma < \tilde{\gamma}$, and $H(\gamma) < 0$ for $\gamma > \tilde{\gamma}$. In addition, $H(\gamma_0) > 0$ so $\tilde{\gamma} > \gamma_0$.

6.7   Proof of Proposition 4

When only D_1 can conduct R&D, we can show that $E\pi_1^I > E\pi_1^S + E\pi_u^S$ if and only if $\gamma < \Pi^I(c-d) + \frac{\Pi^I(c)^2}{2[\Pi^S(c)-\Pi^I(c)]} \equiv \bar{\gamma}$. Note that $\bar{\gamma} > \gamma_0 \equiv \frac{\Pi^I(c-d)}{2}$.

After plugging into the profit expressions, we have

$$\bar{\gamma} = \frac{(a-c)^2}{4}\frac{\beta^3 + 8\beta(1-\beta)}{4-3\beta^2}\left[\frac{d}{a-c}\left(\frac{d}{a-c} + 2\right) + \frac{8 - 5\beta^2 - \beta^3}{2\beta^2(1-\beta)}\right].$$

It is clear that $\bar{\gamma}$ increases in $(a - c)$ and d, but is inversely U-shaped in β. We can also show that $H(\bar{\gamma}) < 0 = H(\tilde{\gamma})$, which means $\bar{\gamma} > \tilde{\gamma}$.

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6.8 Proof of Proposition 5

Under S, the R&D competition is

$$\max_{\rho_1} E\pi_1^S \equiv \rho_1 \left[(1 - \rho_u) \Pi^I (c - d) + \rho_u \frac{\Pi^I(c-d)}{2} \right] - \frac{\gamma}{2}(\rho_1)^2$$

$$\max_{\rho_u} E\pi_u^S \equiv \rho_u \left[(1 - \rho_1) \Pi^S (c - d) + \rho_1 \frac{\Pi^S(c-d)}{2} \right] + (1 - \rho_u)(1 - \rho_1) \Pi^S (c) - \frac{\gamma}{2}(\rho_u)^2.$$

Given the first-order conditions in the text, the solutions are solved as

$$\rho_1^S = 2 \frac{2\sigma - \eta}{4\sigma^2 + 1 - 2\eta}, \quad \rho_u^S = 2(1 - \sigma \rho_1^S),$$

where $\sigma = \frac{\gamma}{\Pi^I(c-d)}$ and $\eta = \frac{\Pi^S(c-d) - \Pi^S(c)}{\Pi^I(c-d)}$.

Under I, the solution is $\rho^I = \frac{\tau}{\sigma}$, where $\tau = \frac{\Pi^I(c-d) - \Pi^I(c)}{\Pi^I(c-d)} < 1$. We know that $\sigma > \frac{1}{2}$, and $0 < \tau < \eta$. Note that to ensure that ρ_1^S and ρ_u^S are between 0 and 1, we must have $\eta < \sigma + \frac{1}{2}$. As a result, the signs of the numerator and denominator of ρ_1^S are $4\sigma^2 + 1 - 2\eta > 0$ and $2\sigma - \eta > 0$.

(i) D_1's marginal benefit from R&D investment is $(1 - \rho_u^S) \Pi^I (c - d) + \rho_u^S \frac{\Pi^I(c-d)}{2} = \Pi^I(c-d) - \rho_u^S \frac{\Pi^I(c-d)}{2}$ under S, and $\Pi^I (c - d) - \Pi^I (c)$ under I. If $\Pi^I(c) > \frac{\Pi^I(c-d)}{2}$, then the former is larger. Because the marginal cost curve is the same, a larger marginal benefit will lead to a larger R&D investment, meaning that $\rho_1^S > \rho^I$. Similarly, if $\Pi^I(c) > \frac{\Pi^I(c-d)}{2}$, which corresponds to $\frac{d}{a-c} < \sqrt{2}-1$, then we also have $\Pi^S(c) > \frac{\Pi^S(c-d)}{2}$. As a result, $(1 - \rho_1^S) \left[\Pi^S (c - d) - \Pi^S (c) \right] + \rho_1^S \frac{\Pi^S(c-d)}{2} > (1 - \rho_1^S) \left[\Pi^S (c - d) - \Pi^S (c) \right] + \rho_1^S \left[\Pi^S (c - d) - \Pi^S (c) \right] = \Pi^S (c - d) - \Pi^S (c) > \Pi^I (c - d) - \Pi^I (c)$, where the first expression is firm U's marginal benefit from R&D investment under S, and the last expression is its marginal benefit under I. As before, a larger marginal benefit under S means a larger investment: $\rho_u^S > \rho^I$.

Now suppose $\frac{d}{a-c} > \sqrt{2} - 1$ so that $\Pi^I(c) < \frac{\Pi^I(c-d)}{2}$ (i.e., $\tau > \frac{1}{2}$) and $\Pi^S(c) < \frac{\Pi^S(c-d)}{2}$. Given the equilibrium expressions, $\rho^I > \rho_1^S$ if and only if $\tau > \frac{2\sigma(2\sigma - \eta)}{4\sigma^2 + 1 - 2\eta} \equiv \tau_1(\sigma)$. We can show that $\tau_1(\sigma)$ increases in σ, with $\tau_1(\frac{1}{2}) = \frac{1}{2}$. Then as long as $\tau > \frac{1}{2}$, there exists a critical $\hat{\sigma}_1$ such that $\tau > \tau_1(\sigma)$ if and only if $\sigma < \hat{\sigma}_1$. As σ increases in γ, $\sigma < \hat{\sigma}_1$ is equivalent to $\gamma < \hat{\gamma}_1$ for some $\hat{\gamma}_1$. Similarly, $\rho^I > \rho_u^S$ if and only if $\tau > \frac{2\sigma[1 + 2\eta(\sigma - 1)]}{4\sigma^2 + 1 - 2\eta} \equiv \tau_u(\sigma)$. Again, $\tau_u(\sigma)$ increases in σ, and $\tau_u(\frac{1}{2}) = \frac{1}{2}$. Then there exists a critical $\hat{\sigma}_u$ such that $\tau > \tau_u(\sigma)$ if and only if $\sigma < \hat{\sigma}_u$, which is equivalent to $\gamma < \hat{\gamma}_u$ for some $\hat{\gamma}_u$.

(ii) $\theta^S = 1 - (1 - \rho_1^S)(1 - \rho_u^S)$, and $\theta^I = \rho^I$. We can show that $\theta^I > \theta^S$ if and only if $G(\sigma) > 0$. There exists a unique σ^* such that $G(\sigma^*) = 0$, with $G(\sigma) > 0$ when $\sigma < \sigma^*$, and $G(\sigma) < 0$ when $\sigma > \sigma^*$. In addition, $G(\frac{1}{2}) > 0$ whenever $\tau > \frac{1}{2}$.

(iii) By plugging into the equilibrium investments, we find that $E\pi^I > E\pi_1^S + E\pi_u^S$ if and only if γ is below some critical level.
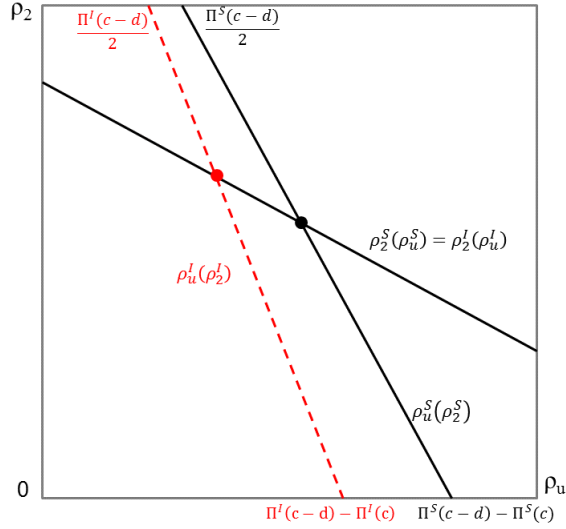
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Figure 2: Best responses in R&D competition between U and D_2

6.9 Proof of Proposition 6

When U and D_2 compete in R&D, the payoff matrix is as follows.

	Under S				Under I	
$U \backslash D_2$	Successful	Not Successful		$UD_1 \backslash D_2$	Successful	Not Successful
Successful	$\frac{\Pi^S(c-d)}{2}$, $\frac{\Pi^I(c-d)}{2}$	$\Pi^S(c-d)$, 0		Successful	$\frac{\Pi^I(c-d)}{2}$, $\frac{\Pi^I(c-d)}{2}$	$\Pi^I(c-d)$, 0
Not Successful	0, $\Pi^I(c-d)$	$\Pi^S(c)$, 0		Not Successful	0, $\Pi^I(c-d)$	$\Pi^I(c)$, 0

Under S, the equilibrium is given by

$$\max_{\rho_u} E\pi_u^S \equiv \rho_u \left[(1-\rho_2)\Pi^S(c-d) + \rho_2 \frac{\Pi^S(c-d)}{2} \right] + (1-\rho_u)(1-\rho_2)\Pi^S(c) - \frac{\gamma}{2}(\rho_u)^2$$

$$\max_{\rho_2} E\pi_2^S \equiv \rho_2 \left[(1-\rho_u)\Pi^I(c-d) + \rho_u \frac{\Pi^I(c-d)}{2} \right] - \frac{\gamma}{2}(\rho_2)^2.$$

Under I, the equilibrium is given by

$$\max_{\rho_u} E\pi_u^V \equiv \rho_u \left[(1-\rho_2)\Pi^I(c-d) + \rho_2 \frac{\Pi^I(c-d)}{2} \right] + (1-\rho_u)(1-\rho_2)\Pi^I(c) - \frac{\gamma}{2}(\rho_u)^2$$

$$\max_{\rho_2} E\pi_2^S \equiv \rho_2 \left[(1-\rho_u)\Pi^I(c-d) + \rho_u \frac{\Pi^I(c-d)}{2} \right] - \frac{\gamma}{2}(\rho_2)^2.$$

The best responses are shown in Figure 2.

Given the first-order conditions in the text, the equilibrium investments can be solved as

$$\rho_2^S = 2\frac{2\sigma + \varphi - \kappa}{4\sigma^2 + 2\varphi - \kappa}, \quad \rho_u^S = 2(1 - \sigma\rho_2^S),$$

$$\rho_2^I = 2\frac{2\sigma + x - 1}{4\sigma^2 + 2x - 1}, \quad \rho_u^I = 2(1 - \sigma\rho_2^I),$$

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where $\sigma = \frac{\gamma}{\Pi^I(c-d)}$, $x = \frac{\Pi^I(c)}{\Pi^I(c-d)}$, $\varphi = \frac{\Pi^S(c)}{\Pi^I(c-d)}$, and $\kappa = \frac{\Pi^S(c-d)}{\Pi^I(c-d)} > \varphi$.

It can be shown that $\theta^I > \theta^S$ if and only if

$$x > \frac{2\sigma - 1}{2}\left[\frac{4\sigma^2(4\sigma^2 - 4\sigma + \kappa)}{4\sigma^2 - 4\sigma(\kappa - \varphi) + \kappa - 2\varphi} - 1\right].$$

The right-hand side increases in σ, which in turn increases in γ, such that it is easier to satisfy when γ is sufficiently small.

Because D_1's profit is always 0, the joint profit between U and D_1 is given by $E\pi_u^S$ and $E\pi_u^V$ above. Because $\Pi^I(c-d) < \Pi^S(c-d)$, $\Pi^I(c) < \Pi^S(c)$, and $\rho_2^I > \rho_2^S$, using the Envelope Theorem, it is straightforward that $E\pi_u^V < E\pi_u^S$ always holds.

6.10   Proof of Proposition 7

Under S, from the first-order conditions, we have

$$\rho^S = \frac{1 - \frac{\rho_u^S}{2}}{\sigma + \frac{1}{2} - \frac{\rho_u^S}{3}}, \quad \text{and } \rho_u^S = \frac{\kappa}{\sigma}\left[\left(1 - \rho^S\right)^2 \tau + \rho^S - \frac{2}{3}(\rho^S)^2\right]. \tag{16}$$

Under I, the closed-form equilibrium R&D investments can be solved as:

$$\rho^I = \frac{2\sigma\tau + 1 - 2\tau}{2\sigma^2 + \frac{1}{2} - \tau}, \quad \text{and } \rho_2^I = \frac{2\sigma - \tau}{2\sigma^2 + \frac{1}{2} - \tau}.$$

(i) Note that $\frac{d\rho^S}{d\rho_u^S} = \frac{-\frac{1}{2}\sigma + \frac{1}{12}}{\left(\sigma + \frac{1}{2} - \frac{\rho_u^S}{3}\right)^2} < 0$ because $\sigma > \frac{1}{2}$. Hence,

$$\rho^S < \frac{1}{\sigma + \frac{1}{2}} < \frac{2\sigma - \tau}{2\sigma^2 + \frac{1}{2} - \tau} = \rho_2^I$$

because $1 < \frac{(2\sigma - \tau)\left(\sigma + \frac{1}{2}\right)}{2\sigma^2 + \frac{1}{2} - \tau} = \frac{2\sigma^2 + \frac{1}{2} - \tau + (1 - \tau)\left(\sigma - \frac{1}{2}\right)}{2\sigma^2 + \frac{1}{2} - \tau}$.

(ii) Note that $\rho^S > \frac{\frac{1}{2}}{\sigma + \frac{1}{6}}$ since $\frac{d\rho^S}{d\rho_u^S} < 0$. Thus, $\rho^S > \rho^I$ if $\frac{\frac{1}{2}}{\sigma + \frac{1}{6}} > \frac{2\sigma\tau + 1 - 2\tau}{2\sigma^2 + \frac{1}{2} - \tau}$ or equivalently,

$$\left(2\sigma^2 - \frac{5}{3}\sigma + \frac{1}{6}\right)\tau < \sigma^2 - \sigma + \frac{1}{12}. \tag{17}$$

A sufficient condition for (17) to hold is

$$\tau < b(\sigma) \equiv \frac{\sigma^2 - \sigma + \frac{1}{12}}{2\sigma^2 - \frac{5}{3}\sigma + \frac{1}{6}} \quad \text{for} \quad \sigma > 0.91. \tag{18}$$

Notice that, for $\sigma > 0.91$, $b(\sigma)$ increases with σ. Moreover, γ increases with σ. Hence, for $\tau < \frac{1}{2}$ (which is equivalent to $\frac{d}{a-c} < \sqrt{2} - 1$), (18) holds when γ is sufficiently large. Furthermore, it can be verified that for $\beta < 0.83$, $\tilde{d} = \frac{\beta(2-\beta)^2}{8 - 4\beta - 2\beta^2 - \beta^3} < \sqrt{2} - 1$. Therefore, for $\tilde{d} < \frac{d}{a-c} < \sqrt{2} - 1$, $\rho^S > \rho^I$ if γ is sufficiently large.

(iii) From (16),

$$\rho_u^S (\rho_S) = \frac{\kappa}{\sigma} \left[-\left(\frac{2}{3} - \tau \right) (\rho_S)^2 + (1 - 2\tau) \rho_S + \tau \right].$$

If $\tau < \frac{1}{3}$, the axis of symmetry for $h(\rho_S) = -\left(\frac{2}{3} - \tau \right) (\rho_S)^2 + (1 - 2\tau) \rho_S + \tau$ is $\frac{1-2\tau}{2\left(\frac{2}{3}-\tau\right)} > \frac{1}{2}$ and thus $\rho_u^S > \rho_u^S (0) = \frac{\kappa}{\sigma} \tau$ given that $\rho_S \in (0, 1)$. Hence, $\rho_u^S > \rho^I$ if

$$\kappa\tau > \mu(\sigma, \tau) \equiv \frac{2\sigma^2 \tau + \sigma(1 - 2\tau)}{2\sigma^2 + \frac{1}{2} - \tau}. \tag{19}$$

Note that

$$\frac{\partial \mu(\sigma, \tau)}{\partial \sigma} = -2(2\sigma - 1)(1 - 2\tau) \frac{2\sigma - 2\tau + 1}{(4\sigma^2 - 2\tau + 1)^2} < 0$$

since $\sigma > \frac{1}{2}$ and $\tau < \frac{1}{3}$. Moreover, $\mu(\sigma, \tau)$ approaches τ when σ is sufficiently large. It follows that (19) will hold if $\tau < \frac{1}{3}$ (which is equivalent to $\frac{d}{a-c} < \sqrt{\frac{3}{2}} - 1$) and σ is sufficiently large because $\kappa > 1$. Furthermore, it can be verified that for $\beta < 0.54$, $\tilde{d} < \sqrt{\frac{3}{2}} - 1$. Therefore, for $\tilde{d} < \frac{d}{a-c} < \sqrt{\frac{3}{2}} - 1$, $\rho_u^S > \rho^I$ if γ is sufficiently large.

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