Fermi-Liquid Description of a Single-Component Fermi Gas with p-Wave Interactions

Shanshan Ding and Shizhong Zhang*
Department of Physics and Center of Theoretical and Computational Physics, The University of Hong Kong, Hong Kong, China

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We study the Fermi liquid properties of a single component Fermi gas with p-wave interactions. In the weak repulsive limit, we obtain exact perturbative expansions for the ground state energy, the chemical potential, and the effective mass of the Landau quasiparticle up to second order in scattering volume a. We also calculated the corresponding Landau functions and Landau parameters and show that they satisfy the general Fermi liquid identities. Using the Landau transport equation, we show that undamped zero sound only appears in the second order in scattering volume, in contrast to the s-wave case.

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Introduction.—The weakly interacting Fermi gas with s-wave interactions provides a paradigmatic model of a Landau Fermi liquid theory [1] where various parameters can be computed exactly and expressed entirely in terms of s-wave scattering length a_s up to the second order [2]. This model has found renewed interest in ultracold Fermi gas where a_s can be tuned by Feshbach resonance and a great many properties, including both static and dynamical, are investigated [3–5].

On the other hand, studies of p-wave Fermi gas have been quite limited due to significant atom loss in actual experiments [6–20]. A crucial difference between the s- and p-wave cases is the existence of a centrifugal barrier in the latter case which tends to support a quasibound state within the barrier that leads to a fast loss of atoms [10,21,22]. However, some progress has been made, in particular, the fast spectroscopic measurements [23] reveal that the p-wave Fermi gas also obeys a set of universal relations in 3D [24–28], as well as in 2D [29,30] and 1D [31–33], similar to the s-wave case. Theoretically, it is known that unlike the s-wave case, the low-energy properties of a p-wave gas requires at least two parameters for its characterization [25,34,35], given by the effective range expansion for the on-shell scattering amplitude $f_p(\mathbf{k},\mathbf{k}') = 3k^2\cos\theta[-1/a - k^2/(2R) - ik^3]^{-1}$. Here, $|\mathbf{k}| = |\mathbf{k}'| = k$, and θ is the angle between incoming $\mathbf{k}$ and outgoing $\mathbf{k}'$. a is the p-wave scattering volume, and R is the p-wave effective range. The relevance of R can be seen from the expression for the binding energy of the p-wave dimers, $E_p = 2R/(ma)$ (we set $\hbar = 1$), which depends both on R and a, in contrast to the s-wave case $E_s = 1/(ma_s^2)$. This has been confirmed in experiments [10,11].

In this Letter, we study Fermi liquid properties of a single component Fermi gas with weak repulsive p-wave interactions ($a > 0$). This corresponds to the Bose-Einstein condensate side of the Feshbach resonance realized in Ref. [23] where measurement of free energy implies that the system remains on the scattering states and no p-wave dimer is present. In addition, it is found that, in this regime, the system is stable against collapse, thus, providing an ideal setting for exploring the physics of a weakly repulsive p-wave gas. To carry out consistent calculation of the Fermi liquid properties to the second order in a, we show that it is also necessary to take into account the effective range correction, in contrast to the s-wave case [2,36]. As in the classic Galistkii calculation [2], we compute the ground state energy, quasiparticle effective mass and lifetime, and also, Landau functions and Landau parameters for the p-wave Fermi gas. We show that, as a result of anisotropic p-wave scattering, undamped zero sound only appears when one takes into account the a^2 order in Landau functions. Our calculations also highlight a few distinct features of p-wave scattering which are common to all high-partial wave scattering, thus, extending and complementing the Galistkii calculation, and provides another nontrivial model of exact calculation in many-body physics. On the other hand, attractive p-wave Fermi gas has been explored extensively theoretically, including various possible pairing states [37–42], superfluid transition temperatures [43–45], and normal state properties [46–49].

The model.—Let us consider a single component Fermi gas with isotropic short-range potential $V(r)$ which vanishes for $r \gtrsim r_0$. In momentum space, the potential can be written as $V_{\mathbf{k}'\mathbf{k}} = \int d\mathbf{r} V(r)\exp[-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}]$. Here, $\mathbf{k}$ and $\mathbf{k}'$ are the momentum of incoming and outgoing particles in the center of mass frame. Because of rotational invariance, $V_{\mathbf{k}'\mathbf{k}}$ can be further expanded in terms of Legendre polynomials: $V_{\mathbf{k}'\mathbf{k}} = \sum_\ell (2\ell+1)V^\ell_{k'k}P_\ell(\cos\theta)$ where θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$. $k = |\mathbf{k}|$ and $k' = |\mathbf{k}'|$. In particular, the p-wave component of $V^{\ell=1}_{k'k}$ is given by

$$V^{\ell=1}_{k'k} = 4\pi \int r^2 V(r) j_1(kr) j_1(k'r) dr. \tag{1}$$

In low-density Fermi system, the typical magnitude of $\mathbf{k}$ and $\mathbf{k}'$ is of the order of $k_F \equiv (6\pi^2 n)^{1/3} \ll 1/r_0$,

where n is the density. As a result, for $k \ll 1/r_0$ and k' arbitrary, the integration in (1) can be simplified by using the expansion $j_1(kr) \approx kr/3$ and one obtains $V^{\ell=1}_{k'k} = (4\pi/3)k \int r^3 V(r) j_1(k'r) dr = k\bar{g}(k')$. Similarly, we have for $k' \ll 1/r_0$ and k arbitrary, $V^{\ell=1}_{k'k} = k'\bar{g}(k)$. Thus, apart from the regime where both $k, k' \gtrsim 1/r_0$, an appropriate form for the interaction potential can be written in a separable form

$$V^{\ell=1}_{k'k} = \lambda g(k) g(k') k k'. \tag{2}$$

For a small value of k, the function $g(k)$ can be expanded as $g(k) = a_0 + a_1 k^2 + \cdots$ and higher order terms can be neglected. As a result, $V^{\ell=1}_{k'k} = \lambda(a_0 + a_1 k^2)(a_0 + a_1 k'^2) k k'$. As we shall show later, for our calculation, it is only necessary to retain the term up to second order in k^2, and thus, we choose our model potential as follows:

$$V_{\mathbf{k}'\mathbf{k}} = [g_1 + g_2(k^2 + k'^2)] \mathbf{k} \cdot \mathbf{k}'. \tag{3}$$

The parameters g_1 and g_2 can be related to the scattering parameters a and R by calculating the scattering amplitude $f_p(k, \theta)$ perturbatively. For details, see the Supplemental Material [50]

$$g_1 = \frac{12\pi a}{m} + \frac{24 a^2}{m} \int_0^\Lambda dq q^2, \tag{4}$$

$$g_2 = -\frac{3\pi a^2}{Rm} + \frac{12 a^2}{m} \int_0^\Lambda dq, \tag{5}$$

where $\Lambda \sim 1/r_0$ is an appropriate high momentum cutoff. The unphysical high-momentum part of $V_{\mathbf{k}'\mathbf{k}}$ will be renormalized by Eqs. (4) and (5) when the final results are expressed in terms of a and R. We note that, in the weak coupling limit, g_1 is of the order of a and g_2 is of the order of a^2. In our calculation, we shall treat R as a constant, which is appropriate to experiments [23], and focus our attention on the expansion in terms of a.

At this point, it is useful to point out a crucial difference between the s-wave and p-wave scattering that will have profound implications for many-body physics. In the s-wave case, as is well known, the on-shell scattering amplitude is given by $f_s(k) = (-a_s^{-1} - ik)^{-1}$, where a_s is related to the bare coupling constant g by $g = 4\pi a_s/m + 8a_s^2/m \int dq$ [36]. What is special about the s-wave case is that this renormalization condition is sufficient to remove divergences in both the on-shell and off-shell elements of scattering vertex function, to the second order in a_s [36]. On the contrary, the p-wave case is more complicated. The renormalization conditions (4) and (5) are fixed via scattering amplitude $f_p(k, \theta)$, which, in general, are not guaranteed to remove the divergence in the off-shell elements of scattering vertex function $\Gamma(\mathbf{k}, \mathbf{k}', \mathbf{P}, i\Omega)$ (see below), in contradiction to the s-wave case [36,50]. To remove the ambiguity of off-shell divergence in Γ, it is necessary to include extra parameters, in addition to a and R, that characterize the low-energy scattering properties of few-body systems [51,52]. At low-energy and density, this leads to higher order corrections in a and, thus, will be neglected. To the second order in a, we will compute the physical quantities that depend only on the low-energy scattering parameters a and R, or equivalently, the dimensionless parameters $\alpha = k_F^3 a$ and $\zeta = k_F R$.

Now, we investigate the many-body problem and calculate the self-energy $\Sigma(\mathbf{p}, i\omega)$ to the second order in a. We will work in Matsubara formalism and take the temperature $T \to 0$ limit. For that, we need to compute the many-body vertex function $\Gamma(\mathbf{k}, \mathbf{k}', \mathbf{P}, i\Omega)$ up to second order in g_1 and first order in g_2. Explicitly, this is given by

$$\begin{aligned}\Gamma(\mathbf{k}, \mathbf{k}', \mathbf{P}, i\Omega) &= V_{\mathbf{k}'\mathbf{k}} - \frac{1}{\beta V} \sum_{\mathbf{q}, iq_0} V_{\mathbf{q}\mathbf{k}} G_0\left(\frac{\mathbf{P}}{2} + \mathbf{q}, iq_0\right) \\ &\times G_0\left(\frac{\mathbf{P}}{2} - \mathbf{q}, i\Omega - iq_0\right) V_{\mathbf{k}'\mathbf{q}},\end{aligned} \tag{6}$$

where $P = (\mathbf{P}, i\Omega)$ is the total four momentum, and $\mathbf{k}$ and $\mathbf{k}'$ are the relative incoming and outgoing momentum of the colliding fermions. $G_0(\mathbf{q}, iq_0) = (iq_0 - \xi_{\mathbf{q}})^{-1}$ is the free fermion Green's function. $\xi_{\mathbf{q}} = \mathbf{q}^2/(2m) - \epsilon_F$ where $\epsilon_F = k_F^2/(2m)$ is the Fermi energy for a noninteracting Fermi gas. $\beta = 1/(k_B T)$ is the inverse temperature. It is explicitly shown in the Supplemental Material [50] that the on-shell element $\Gamma(\mathbf{k}, \mathbf{k}', \mathbf{P}, (k^2 + k'^2)/(2m) + \mathbf{P}^2/(4m) - 2\epsilon_F)$ is finite by using the renormalization equations (4) and (5), while the off-shell elements of $\Gamma(\mathbf{k}, \mathbf{k}', \mathbf{P}, i\Omega)$ in general diverge. The self-energy $\Sigma(\mathbf{p}, i\omega)$ is given by [$\mathbf{k} = (\mathbf{p} - \mathbf{q})/2$, $\mathbf{P} = \mathbf{p} + \mathbf{q}$ and $i\Omega = i\omega + iq_0$]

$$\Sigma(\mathbf{p}, i\omega) = \frac{2}{\beta V} \sum_{\mathbf{q}, iq_0} G_0(\mathbf{q}, iq_0) \Gamma(\mathbf{k}, \mathbf{k}, \mathbf{P}, i\Omega). \tag{7}$$

One can write $\Sigma(\mathbf{p}, i\omega) = \Sigma_1(\mathbf{p}, i\omega) + \Sigma_2(\mathbf{p}, i\omega)$, where $\Sigma_{1,2}$ gives the self energy to order a and a^2, respectively.

$$\Sigma_1(\mathbf{p}, i\omega) = \Sigma_1(\mathbf{p}) = \frac{6}{\pi} \epsilon_F \alpha \left[\frac{1}{5} + \frac{1}{3}\left(\frac{|\mathbf{p}|}{k_F}\right)^2\right], \tag{8}$$

which is independent of frequency $i\omega$, as in the s-wave case [36]. Unlike the s-wave case, however, it contains explicit momentum dependence arising from the momentum dependent p-wave scattering. The second order term $\Sigma_2(\mathbf{p}, i\omega)$ is more complicated and is given in the Supplemental Material [50].

To the second order in a, the single particle Green's function $G(\mathbf{p}, i\omega)$ can be written as

$$G(\mathbf{p}, i\omega) = \frac{1}{i\omega - \xi_{\mathbf{p}} - \Sigma_1(\mathbf{p}) - \Sigma_2(\mathbf{p}, i\omega)}. \quad (9)$$

The quasiparticle energy and lifetime are given by the pole of G, $i\omega = \tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}}$, and this leads to the implicit equation $\tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}} = \xi_{\mathbf{p}} + \Sigma_1(\mathbf{p}) + \Sigma_2(\mathbf{p}, \tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}})$. We are only interested in $\tilde{\xi}_{\mathbf{p}}$ and $\gamma_{\mathbf{p}}$ to the second order in a; thus, we can replace $\tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}}$ by $\xi_{\mathbf{p}} + i\eta$ (η is a positive infinitesimal) in Σ_2 in the above equation, obtaining the explicit expression

$$\tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}} = \xi_{\mathbf{p}} + \Sigma_1(\mathbf{p}) + \Sigma_2(\mathbf{p}, \xi_{\mathbf{p}} + i\eta). \quad (10)$$

Effective mass of the quasiparticle.—The effective mass m^* of the quasiparticle is determined by $\tilde{\xi}_{\mathbf{p}}$, $k_F/m^* = \partial\tilde{\xi}_{\mathbf{p}}/\partial|\mathbf{p}|$ when $|\mathbf{p}| = k_F$. This leads to the explicit expression

$$\frac{m}{m^*} = 1 + \frac{2}{\pi}\alpha - \left[\frac{1}{\pi\zeta} - \frac{8(313 - 426\ln 2)}{315\pi^2}\right]\alpha^2. \quad (11)$$

The term $(2/\pi)\alpha$ corresponds to the mean field correction to the effective mass. This is different from the s-wave case where momentum independent scattering leads to zero correction to effective mass at the mean field level [see, also, Eq. (8)]. The term $\alpha^2/(\pi\zeta)$ arises essentially from the expansion of scattering amplitude $f_p(k, \theta)$ to order a^2/R and does not include the medium scattering effects, while the last term is due to medium scattering. We note that the effective mass only relates to the on-shell self-energy in Eq. (10). On the other hand, if we want to calculate the quasiparticle residue $Z^{-1} = 1 - \partial \mathrm{Re}\Sigma(\mathbf{p}, \omega + i\eta)/\partial\omega$ at the Fermi surface $|\mathbf{p}| = k_F$, it is necessary to know the off-shell self-energy which is divergent in our calculation. This means that, in general, Z depends on more details of the interatomic potential than given by a and R [50]. This is in direct contrast to the s-wave case where the on-shell renormalization condition automatically removes the divergence in the off-shell self-energy [36]. Finally, we note that the density of state at the Fermi surface $\nu(0) = m^* k_F/(2\pi^2)$, is entirely determined by m^*.

Lifetime of the quasiparticle.—Because of scattering, the quasiparticle is only well defined in the vicinity of the Fermi surface according to Landau Fermi liquid theory. In p-wave Fermi gas, this is also the case and one finds, for momentum close to the Fermi surface,

$$\gamma_{\mathbf{p}} = -\frac{24\alpha^2}{5\pi m}(|\mathbf{p}| - k_F)^2. \quad (12)$$

The lifetime of the quasiparticle is correspondingly given by $\tau_{\mathbf{p}} = -1/(2\gamma_{\mathbf{p}}) = (5\pi m\alpha^{-2}/48)(|\mathbf{p}| - k_F)^{-2}$, which diverges at the Fermi surface. We note that the mean field terms (α and α^2/ζ) do not contribute to $\tau_{\mathbf{p}}$, and the only contribution (α^2) arises from medium scattering, as it should.

Chemical potential.—By definition, the chemical potential μ represents the minimal energy required to add an extra particle to the system; in the case of a Fermi liquid, at the Fermi surface, $\mu = \tilde{\xi}_{k_F} + \epsilon_F$. Therefore,

$$\mu = \epsilon_F\left[1 + \frac{16}{5\pi}\alpha - \left(\frac{6}{7\pi\zeta} - \frac{2066 - 312\ln 2}{315\pi^2}\right)\alpha^2\right]. \quad (13)$$

Similar discussions regarding different contributions, as in the case of effective mass, can be performed. The velocity of the first sound can be calculated straightforwardly as $c_1^2 = (n/m)\partial\mu/\partial n$,

$$c_1^2 = \frac{k_F^2}{3m^2}\left[1 + \frac{8\alpha}{\pi} - \left(\frac{3}{\pi\zeta} - \frac{8(1033 - 156\ln 2)}{315\pi^2}\right)\alpha^2\right]. \quad (14)$$

In the limit $\alpha \to 0$, c_1 tends to $k_F/(\sqrt{3}m)$, as in the s-wave case. It is also consistent with the general result from Fermi liquid theory that $c_1^2 = k_F^2(1 + F_0)/(3mm^*)$ where F_0 is the Landau parameter calculated below in Eq. (22).

Ground state energy per particle.—According to thermodynamic identity, the ground state energy E can be obtained by integrating μ over N, the number of particles: $E_0 = \int_0^N dN'\mu(N')$ for a fixed volume at zero entropy. By using Eq. (13), the ground state energy per particle up to the a^2 order is given by

$$\frac{E_0}{N} = \epsilon_F\left[\frac{3}{5} + \frac{6\alpha}{5\pi} - \left(\frac{9}{35\pi\zeta} - \frac{2066 - 312\ln 2}{1155\pi^2}\right)\alpha^2\right]. \quad (15)$$

From here, one can calculate the two p-wave contacts. Using the adiabatic relations $C_a = -2m\partial E_0/\partial a^{-1}|_R$ and $C_R = -4m\partial E_0/\partial R^{-1}|_a$ [25]

$$C_a = \frac{2N}{k_F}\left[\frac{3}{5\pi}\alpha^2 - \left(\frac{9}{35\pi\zeta} - \frac{2066 - 312\ln 2}{1155\pi^2}\right)\alpha^3\right], \quad (16)$$

$$C_R = \frac{18}{35\pi}\alpha^2 N k_F. \quad (17)$$

C_a and C_R can be measured via either momentum distribution or radio-frequency spectroscopy [23].

Landau functions and parameters.—The formulation of Fermi liquid theory for a collection of spinless fermions is much simpler than the usual Fermi liquid theory because of the absence of spin degrees of freedom. As usually defined, the Landau f function is given by the functional derivative with respect to the distribution function

$$f(\mathbf{p}, \mathbf{p}') = \frac{\delta\tilde{\xi}_{\mathbf{p}}}{\delta n_{\mathbf{p}'}} = \left.\frac{\delta \mathrm{Re}\Sigma(\mathbf{p}, \tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}})}{\delta n_{\mathbf{p}'}}\right|_{|\mathbf{p}| = |\mathbf{p}'| = k_F}. \quad (18)$$

For systems with reflection symmetry, $f(\mathbf{p},\mathbf{p}') = f(-\mathbf{p},-\mathbf{p}')$. Unlike the s-wave case, here, only the equivalent of the symmetric Landau function f_s exists. For the isotropic case, $f(\mathbf{p},\mathbf{p}')$ depends only on the angle χ between $\mathbf{p}$ and $\mathbf{p}'$ and can be expanded in Legendre polynomials, $f(\chi) = \sum_\ell f_\ell P_\ell(\cos\chi)$. To compute the functional derivative in Eq. (18), we need to obtain the expression of $\mathrm{Re}\Sigma(\mathbf{p}, \tilde{\xi}_{\mathbf{p}} + i\gamma_{\mathbf{p}})$ in terms of Fermi distribution function; this is readily given in the Matsubara formalism as given in Eqs. (17), (20)–(22) in the Supplemental Material [50]. Up to the order a^2, one finds

$$f(\chi) = \frac{\pi}{mk_F}\left\{12\alpha(1-\cos\chi) - \frac{3\alpha^2}{\zeta}(1-\cos\chi)^2 + \frac{\pi\alpha^2}{16}\csc\frac{\chi}{2}\left[\left(1428\sin\frac{\chi}{2} - 674\sin\frac{3\chi}{2} + 330\sin\frac{5\chi}{2}\right) + \frac{3}{2}\ln\frac{1+\sin\frac{\chi}{2}}{1-\sin\frac{\chi}{2}}(153\cos\chi - 30\cos 2\chi + 55\cos 3\chi - 18)\right]\right\}. \quad (19)$$

The terms of order α and α^2/ζ again arise from the mean field correction. To see that explicitly, consider two quasiparticles, with momenta $\mathbf{p}$ and $\mathbf{p}'$ on the Fermi surface, that undergo forward scattering. In this case, the relevant matrix element at the mean field level is given by

$$\left[\frac{12\pi a}{m} - \frac{3\pi a^2}{mR}2\left(\frac{\mathbf{p}-\mathbf{p}'}{2}\right)^2\right]\left(\frac{\mathbf{p}-\mathbf{p}'}{2}\right)^2. \quad (20)$$

Using $(\mathbf{p}-\mathbf{p}')^2 = 2k_F^2(1-\cos\chi)$, this produces the first two terms in $f(\chi)$, apart from an overall constant. The remaining terms in $f(\chi)$ are second order in α and come from medium scattering. In particular, one can see that a logarithm singularity occurs when $\chi = \pi$. This is the same as in the s-wave case and its removal requires the summation of the principal terms exhibiting logarithm divergences [53].

In a Galilean invariant system, a well-known relation between the effective mass m^* and f function can be established [1,53]. In the case of p-wave Fermi gas, this takes the simpler form

$$\frac{1}{m^*} = \frac{1}{m} - \frac{k_F}{(2\pi)^3}\int f(\chi)\cos\chi d\Omega, \quad (21)$$

where $d\Omega$ denotes the integration over solid angles. One can readily verify that Eq. (21) is satisfied by using Eq. (19) and comparing with Eq. (11). Below, we give the explicit expression for several dimensionless Landau parameters $F_\ell = \nu(0) f_\ell$.

$$F_0 = \frac{1}{\pi}\left[6\alpha - \left(\frac{2}{\zeta} - \frac{48\ln 2 + 44}{7\pi}\right)\alpha^2\right], \quad (22)$$

$$F_1 = \frac{1}{\pi}\left[-6\alpha + \left(\frac{3}{\zeta} + \frac{4(852\ln 2 - 311)}{105\pi}\right)\alpha^2\right], \quad (23)$$

$$F_2 = \frac{1}{\pi}\left(-\frac{1}{\zeta} + \frac{8(66\ln 2 - 13)}{21\pi}\right)\alpha^2. \quad (24)$$

We note that the linear term in a or in α appears in both F_0 and F_1 for reasons already explained above. For $\ell \geq 2$, F_ℓ will be of, at least, second order in scattering volume a. We note that the values of Landau parameters are such that stability conditions, $F_\ell > -(2\ell+1)$ are all satisfied.

Collective modes.—The fact that F_0 and F_1 cancel each other to leading order in a has the consequences that undamped zero sound and transverse sound do not exist in a single component repulsive Fermi gas to leading order in a. In the s-wave case, for spin-1/2 repulsive Fermi gas, the dominance of a positive $F_0^s = 4\pi a_s/m$ guarantees the existence of zero sound since all other Landau parameters are of the order a_s^2 and can be neglected in the calculation of zero sound dispersion [53]. However, for the p-wave case to obtain an undamped zero sound, it is necessary to keep the α^2 order in the Landau function, Eq. (19). Let us denote the displacement of the Fermi surface as $u(\theta,\phi)$ where θ and ϕ denote the direction of momentum $\mathbf{p}$ at the Fermi surface. Then a similar transport equation can be written down [53]

$$(\cos\theta - \lambda)u(\theta,\phi) + \frac{\cos\theta}{4\pi}\int d\Omega' F(\chi)u(\theta',\phi') = 0, \quad (25)$$

where χ is the angle between $\mathbf{p}$ and $\mathbf{p}'$. The parameter $\lambda = \omega/(qv_F)$ is the ratio between the phase velocity of the collective wave to the Fermi velocity $v_F = k_F/m^*$. For an undamped collective mode to exist, it is necessary that $\lambda > 1$ [53].

Zero sound.—In this case, $u(\theta,\phi) = u(\theta)$, independent of the azimuthal angle ϕ. It is straightforward to show that to the first order in α, there is no well-defined zero sound mode [50], and it is necessary to keep second order terms in α. Defining $\nu(\theta) = (\cos\theta - \lambda)u(\theta)$, one can rewrite Eq. (25) as

$$\nu(\theta) + \frac{\cos\theta}{4\pi}\int d\Omega' F(\chi)\frac{\nu(\theta')}{\cos\theta' - \lambda} = 0. \quad (26)$$

When $F(\chi) \to 0$, it is easy to see from Eq. (25) that $\lambda \to 1$, so the integration is dominated by the region where $\theta' \approx 0$. In that case, one can replace $F(\chi)$ by $F(0) = 48\alpha^2$ at $\theta = 0$ and one obtains $\lambda = 1 + 2\exp[-1/(24\alpha^2)]$. Namely, the zero sound only exists if the high order term in α is retained, similar to the conclusion in the two-dimensional dipolar gas [54].

Transverse sound.—Similarly, one can investigate the transverse sound mode for which $u(\theta,\phi)=\tilde{u}(\theta)\exp(i\phi)$. Substituting this into Eq. (25), one finds that, in order to have $\lambda>1$, one must have $F_1>6$ [50,55], which is obviously not satisfied by F_1 in Eq. (23). As a result, no well-defined transverse sound mode exists.

Conclusions.—In this Letter, we investigated the Fermi liquid properties of a single component p-wave Fermi gas with weak repulsive interactions to the second order in the scattering volume a, treating the effective range R as a constant. A major difference, as compared with the s-wave case, is the divergence of an off-shell element in the many-body scattering vertex function which requires more details of the interatomic potential other than a and R. In addition, we showed that an undamped zero sound mode only exists due to higher order terms in the scattering volume, in contrast to the s-wave case. Given the stability of p-wave Fermi gas in the weakly repulsive side of the Feshbach resonance [23], it should be possible to verify the predictions made above, especially the quantitative prediction of p-wave contacts. Furthermore, single component Fermi gas offers an ideal setting for exploring the effects of anisotropic scattering in transport phenomena which can be readily dealt with in the framework of Landau Fermi liquid theory.

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*shizhong@hku.hk

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