Decision Analysis for the Emission-limited Manufacturer with Option Contracts under Demand Uncertainty

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Abstract: Emission options have been introduced to relieve the emission-dependent manufacturers' low-carbon pressure and contribute to the long-run success of the Emission Trading Scheme (ETS). Yet few research works have studied the manufacturer's behaviors and performance with/without emission options to achieve sustainability. This paper seeks to bridge this research gap and investigate the manufacturer's optimal emission purchasing and product pricing strategy under the ETS. Newsvendor models are adopted with originality in the use of the Lagrange Multipliers and KKT conditions to achieve optimality subject to emission constraints. Although this approach has rarely been used in the emission-constrained production, it is found effective to achieve optimality subject to emission constraints, especially when economic instruments like options are considered. Mathematical analysis and numerical results show that options increase customer demand and profitability of the firm in a stringent emission market when the price-sensitivity is not too high, and that reasonable option pricing is vital to the emission trading market. As such, a new method is developed for achieving profitability and emission reduction via option contracts.

Keywords: Emission Purchasing Strategy, Pricing Problem, Emission Trading System (ETS), Option Contracts, Lagrange Multipliers, KKT Conditions

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I. INTRODUCTION

Atmospheric carbon dioxide, a dominant cause of global warming, has surged 40% in recent decades over the pre-industrial era level (International Energy Agency, 2017). To cap the rise of atmospheric temperature below 2 ℃, it is imperative to cut emissions substantially. A 40-70% emission reduction target in 2050 lower than 2010 has been put forward by the 43rd Session of the Intergovernmental Panel on Climate Change (IPCC) (Intergovernmental Panel on Climate Change, 2014).

Some policy instruments have been introduced to help mitigate emissions, among which the emission trading scheme (ETS) is the most popular in the world. The regulatory body of the ETS sets aggregate emission limits and issues emission permits to individual firms, which are required to hold enough permits for their emissions. If a firm emits over its limit, it is required to buy extra permits from the carbon market, whereas a less polluting firm can sell its spare permits for profit. The emission permits represent a cost of production for the shortages and a by-product for the leftovers.

Mitigation of manufacturing emissions is key to achieving low-carbon development, as the manufacturing industry accounts for about 36% of the global emissions (International Energy Agency, 2017). However, emission reduction poses huge challenges and burdens for manufacturers.

Under emission constraints, a firm needs to schedule its production under, at, or over the allotted emission cap to optimise profit. If under the cap, the firm can benefit from selling the spare emission permits while suppressing its product sales revenue; if over the cap, the firm can continue its production to increase sales revenue by paying for the extra emission permits; an extreme case exists that the firm just produces the emission-capped products. Since the market becomes more competitive and the emission regulation more stringent, sustaining competitiveness and profitability becomes more dependent on the low-carbon strategy.

Regarding emission permits as a kind of essential raw material for production, the production planning under emission constraints becomes a procurement problem in which a certain amount of emission is free. The complexity of procurement decision-making is due to the demand risk. Demand uncertainty that deviates from the production schedule often causes profit haemorrhage. Failure to meet the vital demand will either lead to product shortage or to redundant emission cost. Hence, an objective of such a procurement strategy is to decide the emission permits required out of demand uncertainty.

Normally, customers are rational and inclined to buy products at bargain prices. Yet, the financial pressure from emission reduction will likely drive up the selling price which will subsequently be passed on to the downstream customers. Therefore, it is of vital importance for the manufacturer to balance the selling price and the price-sensitive demand in order to gain its optimal customer surplus and to relieve the pressure of emission restriction. To this end, it is essential to develop a joint pricing and procurement strategy to gain competitiveness in the carbon-restricted market.

Emission price volatility is considered as the primary ETS-related risk for industries (Chevallier et al., 2011), since it threatens the final customer surplus achieved in a tradable permit market. Fortunately, this volatility can be effectively hedged by taking advantage of financial instruments, such as options.

Academic studies and industrial practices have proven that option contracts are strongly resilient against economic, political, and financial uncertainty (Xu, 2010). Some researchers have explored the impact of option instruments on the volatility in the ETS system. However, few have attempted to use it to reduce the manufacturing emission cost.

Among common option contracts, a call option is a contracted right to purchase a certain amount of emission quotas at a certain price, regardless of the future emission price before the expiration date. To get the call option right, a premium is prepaid for the emission reservation. It effectively hedges the demand risk and then reduces the emission cost from the extra production over the emission cap. The manufacturer decides whether to exercise the options or not at a pre-negotiated price when the demand uncertainty is resolved.

This paper takes into account the call option contract to improve the manufacturer's profitability and flexibility under emission constraints. It proposes and compares two emission ordering and product pricing policies, with and without option. The without-option policy is a benchmark scenario that the manufacturer makes routine decisions. Under it, profit loss occurs out of spare emission and unsatisfied orders from demand mis-forecast. If options are available, the manufacturer can simultaneously purchase the emission credits and options. This alleviates the impacts of demand mis-forecast, but a higher unit emission cost is paid for the emission options. A key question is to find out the best emission and pricing pattern to maximise the profitability in a carbon-limited environment. We address this issue by combining the newsvendor model with the Lagrange Multipliers, which are respectively a classical technique for considering demand uncertainty and a strategy for finding optimality, to investigate the decision behaviours and performance of the manufacturer with the call option contract under emission constraints.

The objective of this research is to investigate the firm's decision behaviours with an emission option contract to gain optimality under emission limitations and demand uncertainty. Its contributions are listed as follows:

(1) A new mathematical method is explored for better solving the pricing and production problem under demand uncertainty, which helps the manufacturer thrive in the low-carbon and price-sensitive environment.
(2) Option contracts under the ETS can facilitate the manufacturer's decision-making and improve profitability in most cases.
(3) It provides an insight for the policy-maker to impose reasonable option and emission prices to develop a healthy emission market.

This research first adopts the call option contract to release the emission pressure under demand uncertainty. Moreover, it adopts the newsvendor model with originality in using the Lagrange Multipliers and KKT conditions to achieve optimality subject to emission constraints. Although this method is famous in the economics research, it has rarely been used in the field of emission-constrained production. However, it is effective to solve this production problem under the emission limitations, especially when economic instruments, like options, are available.

The rest of this paper is organized as follows: a literature basis is summarized in Section II. Section III presents the assumptions and notations, and formulates and analyses the newsvendor model under the ETS with options. Section IV conducts numerical studies to demonstrate and validate the proposed model, while Section V draws conclusions and highlights managerial insights.

II. LITERATURE REVIEW

This section briefly reviews the previous literature from two dimensions: (1) emission trading scheme (ETS), and (2) pricing and ordering problem with option contracts.

2.1 Emission Trading Scheme (ETS)

The ETS is a government-regulated mechanism based on the cap-and-trade principle that limits carbon emission through economic incentives. It has gained popularity in the field of emission reduction since 1970s (Burton & Sanjour, 1970). The success of the ETS may be attributed to its effectiveness and efficiency in meeting the emission mitigation targets (Montgomery, 1972). The regulatory bodies decide the stringency of the overall emission cap (Demailly & Quirion, 2008). Extra emissions are required if a firm emits over its limit, while spare emissions can be traded to the market if a firm emits less. Emission cost occurs when there are shortages and emission revenue gains from the emission leftovers (Wang & Choi, 2019b). A robust evidence of industrial emission reduction (10% to 26%) under the Phase II of the European Union emission trading system (EU ETS) was given by Martin et al. (2015).

The ETS is economically considered a cost-effective way to curtail industrial emissions. Firms over emission caps own two choices to meet their emission reduction obligations: making emission abatement investment or purchasing emission credits (Clarkson et al., 2015). In a well-functioning carbon market, a firm will likely invest in the abatement level when the marginal cost of green upgrades is less than the credit price (Matisoff, 2010). Otherwise, it will procure the emission credits from the open market to fulfill its production yield. Purchasing emission credits is an easy and direct way commonly used to expand a firms's production under the emission constraints.

The ETS is being widely researched and have attracted positive attitudes from most of the researchers in different countries or industries. The major ETS markets include the European Union, the United Kingdom, Australia, New Zealand, Japan, India, and China (Lin & Jia, 2017). The types of coverage industry are increasing and evolving with the maturity of the ETS. The current ETS mainly covers the industries of electricity, oil, steel, cement, glass, paper, air traffic, commercial, chemical fertilizer, railway, aluminum and organic chemistry, etc (Gans & Hintermann, 2013; Liu et al., 2017; Passey et al., 2008; Zhang et al., 2017). The ETSs in Japan, Chicago and the UK are voluntary accession, while others are mandatory emission reduction markets (Lin & Jia, 2017). Reilly and Paltsev (2005) pointed out that a long-term experiment with the ETS is necessary to tell whether to establish an international emission trading market that could cover much of the world. Marin et al. (2018) found the mixed conclusion of the effectiveness of the ETS and thus its impact needs further research and experiment. Geissler (2018) believes that the EU ETS must be continued for tackling climate change. In line with the EU targets, researchers also discuss the effectiveness and futures of the ETSs in other countries considering different industries. The UK's climate change program introduced the ETS and covers over 40 industrial sectors. Smith and Swierzbinski (2007) examined the economic and environmental results of the ETS with industries in the UK and advocated this market mechanism in UK environmental policy, as David Pearce did. Villoria-Sáez et al. (2016) analyzed the effectiveness of the ETS in six regions: the European Union, Australia, New Zealand, Japan, the United States of America, and Canada. They found that the EU-ETS is effective to reach its targets, while the ETS implementation period is not long enough to arise the results for other regions. The ETS needs to be continued and further analysis is necessary. China's ETS was launched in 2017, which may take some time to adjust for effective implementation (Wang & Choi, 2020). Jiang et al. (2016) concluded that a nationwide ETS will be established based on its achievements from the ETS under its implementation period. All in all, most researchers believe that countries implementing the ETS will benefit in the following years.

The essence of the ETS is that the regulatory body collectively sets an acceptable emission abatement level. It ensures that carbon-emitting firms have compliance liability for emission reduction, and that they will suffer from negative valuation out of their excess emissions (Clarkson et al., 2015), so as to work towards low-carbon production. In practice, the European Union launched in 2005 its emission trading regime, the EU ETS, which is now the largest scheme worldwide (Demailly & Quirion, 2008). Around 45% of the total EU emissions are regulated by the EU ETS, and a 43% reduction target is projected by 2030 compared to 2005 (European Commission, 2017). New Zealand broached the NZ ETS in 2008 to realize the transition to a low-emission and climate-resilient society. The NZ ETS is aimed to reduce emission by 30% below the 2005 emission levels by 2030. Australia also established in 2008 an economic-wide emission trading scheme with the target to curb an unconditional 5-15% emission by 2020 compared to 2000 (Jotzo & Betz, 2009). The California ETS program, launched in 2013, is expected to achieve a 16% emission reduction by 2020 and an additional 40% by 2030. The Republic of Korea announced to reduce 30% emission by 2020 and implemented its ETS system from 2015 onwards (Park & Hong, 2014). The Chinese government authorized in 2011 seven pilot cities and provinces for a trial run of its ETS, and officially started the operation of its national ETS at the end of

2017. Its target is to reduce emission intensity by 40% to 45% from 2005 to 2020 (Jotzo & Löschel, 2014). To fulfill these emission reduction targets, both the regulatory bodies and the emission-related industries put great efforts on green implementation and investment.

Although the ETS allows a firm to buy or sell carbon credits that it needs or not, ambitious emission reduction targets decide that the majority of firms suffer from the emission shortage. Despite the green technology, purchasing emission credits is essential for most firms to thrive in the low-carbon market.

2.2 Pricing and Ordering Problem with Option Contracts

It has been proven that the option contract is strongly resilient against economic, political, and financial risks, and its introduction has inspired extensive academic research and practical works on the purchasing and pricing problem (Zhang et al., 2015). Indeed, more and more manufacturing enterprises have been turning to use the option contract to hedge risks from demand uncertainty, price fluctuation, and unreliable suppliers. For instance, it was reported that option contracts accounted for 35% of Hewlett-Packard’s (HP) procurement value (Chen et al., 2014), and that HP reaped 425 million dollars in cumulative cost saving out of options in 2008 (Nagali et al., 2008).

Tucker (2001) has pointed out that selling the option for emission credits instead of the credits can be a way to circumvent the difficulties of pricing and timing strategies. It means that the flexibility of purchasing emission options is more desirable than outright purchases, as it serves as an insurance to countries and their industries to answer the emission reduction claim. Some researchers, including Uhrig-Homburg and Wagner (2008), have further studied the impact of the option contract on the long-run success of the ETS, while the European Climate Exchange (ECX) has practically implemented the option contract to hedge the risk of credit price. To realize the expected emission reduction, many ETS schemes impose stringent emission targets under which the firms tend to downsize their operations by 10% in response to carbon pricing (Martin et al., 2014a, 2014b; Martin et al., 2015). This is bound to hamper the economic progress out of production cuts. To address this issue, the option contract is introduced to balance the emission reduction and the production capacity. Wang et al. (2017) used options to solve the ordering and pricing problem without the consideration of emission reduction, which is first discussed by Wang et al. (2018) under a flexible cap-and-trade system, but it ignored the impact of the common ETS which owns more academic significance in emission reduction. Also, its optimal results can only be obtained under some specific constraints. Wang and Choi (2019a) filled this gap by discussing the emission ordering strategy under the common ETS system with customer awareness on environmental protection, but it ignored the impact of economic instrument – options, to relieve the emission pressure and increase the final profit. This paper substantially extends these previous researches by considering the options to gain the manufacturer optimality under the largest emission reduction policy – the ETS system. More importantly, it can achieve the general analytical results and offers guidance as whether to produce under, at, or over the emission cap.

This emission procurement research faces the common purchasing difficulties, such as demand and credit price uncertainty. The advent of the ETS introduces emission quotas as a new tradable asset required to carry out production activities as a dominant raw material. Besides, companies tend to pass on the cost of the emission credits to their downside customers. It is documented by Alexeeva-Talebi (2011) that European refineries fully passed on the credit price to their petrol customers via the retail price during the first phase of the EU ETS. The extra cost of emission credits makes the procurement and pricing problem more intricate, taking the manufacturer more efforts to rival its competitors. Purchasing emission options is key to tackle these pricing and purchasing difficulties, since procurement flexibility and pricing easiness are provided through the use of option contracts. However, few research works have considered the buyer's option purchasing and pricing strategy with low-carbon restrictions, which is vital to the environment protection and sustainable production. This paper therefore tries to bridge this research gap and analyse the manufacturer's decision behaviours and performance considering the demand uncertainty under the ETS system.

III. PROBLEM FORMULATION

This section elaborates a mathematical model, which incorporates the newsvendor model with the call option contract, to analyse the decision behaviours and profit performance of a manufacturer with demand uncertainty under the ETS system.

3.1 Basic Model Description

This research solves the procurement and pricing problem by the newsvendor model in an emission-tradable market, in which an emission-dependent manufacturer schedules batch production upon receiving sufficient emission quotas from the regulatory body and credit suppliers. A call option contract enables the manufacturer to prepay a premium for the emission reservation. Upon receiving larger orders, the manufacturer can exercise these emission options to produce more, so as to avoid losing orders and hence the brand and profit.

Random demand makes it difficult to determine the production quantity and emission credits needed, causing price volatility that further harms a firm's end profit. Moreover, since the selling price is a primary consideration, price discovery is crucial to achieving profit optimality.

The newsvendor model is therefore established and integrated with the Lagrange multipliers to solve this joint procurement and pricing problem in the emission-reliable industry facing uncertain demand. The parameter notations and the decision variables used in this research are presented in Table 1 and Table 2, respectively.

Table 1. Notations for Parameters

Demand Function	Description
$D(p,\varepsilon)$	The price-driven demand function considering uncertainties, which is continuous and differentiable. $D(p,\varepsilon)=y(p)+\varepsilon$.
$y(p)$	The decreasing and deterministic demand function for selling price. $y(p)=a-bp$.
p	The unitary selling price
a	The market scale. $a>0$.
b	The price sensitivity to the demand. $b>0$.
ε	The random variable for the demand uncertainty. $\varepsilon\in[A,B], E(\varepsilon)=\mu$. $A>-a$.
$f(\cdot)$	The probability density function for ε .
$F(\cdot)$	The non-negative, invertible distribution function for ε .

Parameters	Description
e	The emission level of the product
K	The emission cap
c	The unitary cost without emissions, including production, inventory, and managerial cost, etc.
s	The resold price of the spare emission credits
g	The goodwill cost out of the unsatisfied demand
p_o	The emission option price
p_b	The emission credit price
p_e	The emission option exercising price
λ	The Lagrange Multipliers
η^2	The slack variables
x^+	The larger value comparing zero with x , $x^+=\max(0,x)$
$p>c+p_o+p_e>c+p_b$, $p_o+p_e>p_b>p_o+s$, $p_e>s$	

Table 2. Notations for Decision Variables

Decision Variables	Description
p	The unitary selling price of the product
q	The total emission quantity needed, including the emission options and credits
q_o	The emission option quantity

q_b	The emission credit quantity
q_s	The spare emission credit quantity
Q	The production quantity
r	The stocking factor when $q \geq 0$, $r = \frac{q+K}{e} - y(p)$, $q = q_o + q_b$
z	The stocking factor when $q \leq 0$, $z = Q - y(p)$

Assumptions for the development of the proposed newsvendor model are made as follows:

Assumption 1: No capacity limit is considered.

Assumption 2: It assumes the positive demand and the non-negative profit.

$$p > c + p_o + p_e > c + p_b$$

Assumption 3: The demand function is additive and it is price-driven considering uncertainty.

$$D(p,\varepsilon) = y(p) + \varepsilon\ ,\ y(p) = a - bp\ \ (a > 0, b > 0), \varepsilon \in [A,B], A > -a\ , E(\varepsilon) = \mu\ .$$

Assumption 4: The unused emission can be resold at a lower price, $p_e > s$.

The manufacturer cannot benefit from exercising all the options out of the resold price.

Assumption 5: The manufacturer can purchase both emissions quotas and options from the emission permit supplier, and $p_o + p_e > p_b > p_o + s$.

3.2 Option-Available Scenario

3.2.1 Model Description

This scenario assumes that the manufacturer can purchase both emission credits and options. The batch production is scheduled upon the emission credits received. If the emission credits are insufficient for production, exercising the emission options can solve this urgent need for larger orders. A premium is repaid to hold this emission reservation.

The firm orders q_b emission credits and q_o emission options to produce and sells its products at the unitary selling price p . The maximum emission can be put into the production process is $q = q_b + q_o$. The emission reservation from options charges p_o option price for a premium and exercising it claims p_e exercising price for one-unit emission credit. This call option contract raises the unitary cost of the emission credits by $p_o + p_e > p_b$. The unused emission credits can be resold at s , but the spare emission options cannot. The option-available profit model is given as:

$$\begin{cases} \Pi(q_o,q,p)=(p-c)\cdot\min\left[D,\dfrac{q+K}{e}\right]+s\cdot[q-q_o+K-e\cdot D]^{+}-p_o\cdot q_o-p_b\cdot(q-q_o) \\ \qquad\qquad -p_e\cdot\left[\min\left[e\cdot D-K-(q-q_o),q_o\right]\right]^{+}-g\cdot\left[D-\dfrac{q+K}{e}\right]^{+} \\ s.t.\quad q\geq 0,\ q_o\geq 0,\ p>0 \end{cases} \tag{1}$$

A stocking factor r is used to solve this model, similar to the works by Petruzzi and Dada (1999) and Wang and Chen (2015). This scenario defines it as $r=\dfrac{q+K}{e}-y(p)$, and it is the riskless leftover which neglects the impact of demand uncertainty. Compared with the demand uncertainty variable ε, credit leftover occurs when $r>\varepsilon$, and shortage when $r<\varepsilon$.

By inserting this stocking factor r and the demand function $D(p,\varepsilon)=y(p)+\varepsilon$, the option-available profit model becomes:

$$\begin{cases} \Pi(q_o,r,p)=(p-c)\cdot\min\left[y(p)+\varepsilon,y(p)+r\right]+s\cdot\left[e\cdot(r-\varepsilon)-q_o\right]^{+}-p_o\cdot q_o \\ \qquad\qquad -p_b\cdot\left[e\cdot\left(y(p)+r\right)-K-q_o\right]-p_e\cdot\left[\min\left[e\cdot(\varepsilon-r)+q_o,q_o\right]\right]^{+} \\ \qquad\qquad -g\cdot[\varepsilon-r]^{+} \\ s.t.\quad K-e\cdot\left(y(p)+r\right)\leq 0 \end{cases} \tag{2}$$

For simplification of calculation, we define $\Lambda(q_o,r)=\int_A^{r-\frac{q_o}{e}}\left(r-\dfrac{q_o}{e}-x\right)f(x)dx$ for the expected credit leftovers, $\Omega(q_o,r)=\int_{r-\frac{q_o}{e}}^{r}(r-x)f(x)dx$ for expected option leftovers, and $\Gamma(r)=\int_r^B(x-r)f(x)dx$ for the excepted emission shortage. Then we have the expected profit, denoted as $E\left[\Pi(q_o,r,p)\right]$, as follows:

$$E\left[\Pi(q_o,r,p)\right]=\psi(p)-\chi(q_o,r,p) \tag{3}$$

where

$$\begin{aligned} \psi(p)&=(p-c-p_b\cdot e)\cdot\left[y(p)+\mu\right]+p_b\cdot K \\ \chi(q_o,r,p)&=p_o\cdot q_o\cdot\int_A^{r-\frac{q_o}{e}}f(x)dx+(p_o-p_b+p_e)\cdot q_o\cdot\int_{r-\frac{q_o}{e}}^{B}f(x)dx \\ &\quad+(p_b-s)\cdot e\cdot\Lambda(q_o,r)+(p_b-p_e)\cdot e\cdot\Omega(q_o,r) \\ &\quad+(p-c+g-p_b\cdot e)\cdot\Gamma(r) \end{aligned} \tag{4}$$

$\psi(p)$ refers to the riskless profit ignoring the demand uncertainty. $\chi(r,p)$ represents the demand-risk-related profit loss. The first expression $p_o \cdot q_o \cdot \int_A^{r-\frac{q_o}{e}} f(x)dx$ in the profit loss function n means the possibility cost for all the options are leftovers, as the unused options cannot be resold. The second form $(p_o - p_b + p_e)\cdot q_o \cdot \int_{r-\frac{q_o}{e}}^{B} f(x)dx$ estimates the profit loss for exercising the options due to the higher option full price $(p_o + p_e)$. The spare emission credits charge $(p_b - s)\cdot e$ and the emission shortage charges $(p - c + g - p_b \cdot e)$. For the fourth expression in $\chi(r,p)$, we need to go back to the riskless profit function $\psi(p)$, which formulates the unitary profit by $(p - c - p_b \cdot e)$. However, the actual unitary profit from the products by exercising the options is $\left[p - c - (p_o + p_e)\cdot e\right]$. Thus, this incremental value should be lessened as $(p_b - p_e)\cdot e \cdot \Omega(q_o, r)$. The option-available problem can be written as:

$$\begin{cases} E\left[\Pi(q_o, r, p)\right] = \psi(p) - \chi(q_o, r, p) \\ s.t. \quad K - e\cdot\left(y(p) + r\right) \le 0 \end{cases} \tag{5}$$

3.2.2 Problem Solving

The method of Lagrange Multipliers is explored to solve this problem, and it is a widely used strategy for finding the local maxima and minima of a function subject to equality constraints. A slack variable η^2 is adopted to build the equality constraints as $K - e\cdot\left(y(p) + r\right) + \eta^2 = 0$. The quadratic form η^2 ensures this slack variable is non-negative. Then the problem can be re-written as:

$$\begin{cases} L(q_o, r, p, \eta_1, \lambda_1) = -\left[p - c - p_b \cdot e\right]\cdot\left[y(p) + \mu\right] - p_b \cdot K + \left[p - c + g - p_b \cdot e\right]\cdot\Gamma(r) \\ \qquad + (p_b - s)\cdot e \cdot \Lambda(q_o, r) + (p_b - p_e)\cdot e\cdot\Omega(q_o, r) \\ \qquad + p_o \cdot q_o \cdot \int_A^{r-\frac{q_o}{e}} f(x)dx + (p_o - p_b + p_e)\cdot q_o \cdot \int_{r-\frac{q_o}{e}}^{B} f(x)dx \\ \qquad + \lambda_1 \cdot \left[K - e\cdot\left(y(p) + r\right) + \eta_1^2\right] \\ s.t. \quad \lambda_1 \ge 0 \end{cases} \tag{6}$$

The Karush-Kuhn-Tucker (KKT) conditions are the first-order necessary conditions for a solution in a non-linear programming to get optimality, given some regularity conditions are satisfied. This option-available problem with Lagrange Multipliers can be solved by the KKT conditions, as follows:

$$
\begin{cases}
\dfrac{\partial L}{\partial p} = 2bp - (c + p_b \cdot e) \cdot b - a - \mu + \Gamma(r) + \lambda_1 \cdot b \cdot e = 0 \\
\dfrac{\partial L}{\partial q_o} = (p_o - p_b + p_e) - (p_e - s) \cdot F\left(r - \dfrac{q_o}{e}\right) = 0 \\
\dfrac{\partial L}{\partial r} = -(p - c + g - p_b \cdot e) + (p - c + g - p_e \cdot e) \cdot F(r) \\
\qquad + (p_e - s) \cdot e \cdot F\left(r - \dfrac{q_o}{e}\right) - \lambda_1 \cdot e = 0 \\
\dfrac{\partial L}{\partial \eta_1} = 2\lambda_1 \cdot \eta_1 = 0 \\
\dfrac{\partial L}{\partial \lambda_1} = K - e \cdot \left(y(p) + r\right) + \eta_1^2 = 0
\end{cases}
\tag{7}
$$

The above equations reach their optimality when $\lambda_1 = 0$ and $\eta_1 = 0$.

(1) When $\lambda_1 = 0$, no spare emission credits exist, and extra emissions are needed.

The KKT conditions with $\lambda_1 = 0$ can be given as:

$$
\begin{cases}
\dfrac{\partial L}{\partial p} = 2bp - (c + p_b \cdot e) \cdot b - a - \mu + \Gamma(r) = 0 \\
\dfrac{\partial L}{\partial q_o} = (p_o - p_b + p_e) - (p_e - s) \cdot F\left(r - \dfrac{q_o}{e}\right) = 0 \\
\dfrac{\partial L}{\partial r} = -(p - c + g - p_b \cdot e) + (p - c + g - p_e \cdot e) \cdot F(r) + (p_e - s) \cdot e \cdot F\left(r - \dfrac{q_o}{e}\right) = 0 \\
\dfrac{\partial L}{\partial \lambda_1} = K - e \cdot \left(y(p) + r\right) + \eta_1^2 = 0
\end{cases}
\tag{8}
$$

Thus, the solutions under $\lambda_1 = 0$ are obtained. The optimal r^* that can be obtained by solving the following equation:

$$
\left(\bar{p} - \frac{\Gamma(r)}{2b} - c + g - p_e \cdot e\right) \cdot \bar{F}(r) = p_o \cdot e \tag{9}
$$

Where $\bar{p} = \dfrac{a + (c + p_b \cdot e) \cdot b + \mu}{2b}$.

With the optimal r^*, the required optimal solutions are given as:

$$
\left(\bar{p} - \frac{\Gamma(r)}{2b} - c + g - p_e \cdot e\right) \cdot \bar{F}(r) = p_o \cdot e \tag{10}
$$

Where $\bar{p} = \dfrac{a + (c + p_b \cdot e) \cdot b + \mu}{2b}$.

With the optimal r^*, the required optimal solutions are given as:

$$\begin{cases} p^* \equiv p\left(r^*\right) = \overline{p} - \dfrac{\Gamma\left(r^*\right)}{2b} \\ q^* = \eta_1^2 = e \cdot \left(y\left(p^*\right) + r^*\right) - K \\ F\left(r^* - \dfrac{q_o}{e}\right) = \dfrac{\left(p_o - p_b + p_e\right)}{\left(p_e - s\right)} \\ q_b^* = q^* - q_o^* \end{cases} \tag{11}$$

The optimal emission option quantity can be obtained by solving the third equation.

Lemma 1: When $\lambda_1 = 0$, the optimal stocking factor r^* is uniquely determined by the equation $\left(\overline{p} - \dfrac{\Gamma(r)}{2b} - c + g - p_e \cdot e\right) \cdot \overline{F}(r) = p_o \cdot e$. Thus, the firm requires $q^* = \eta_1^2 = e \cdot \left(y\left(p^*\right) + r^*\right) - K$ emissions from the emission permits supplier and sells its products at the unitary selling price $p^* \equiv p\left(r^*\right) = \overline{p} - \dfrac{\Gamma\left(r^*\right)}{2b}$ where $\overline{p} = \dfrac{a + (c + p_b \cdot e) \cdot b + \mu}{2b}$.

Lemma 2: When $\lambda_1 = 0$, there exists an optimal value of emission option quantity q_o^* by solving the equation $F\left(r^* - \dfrac{q_o}{e}\right) = \dfrac{\left(p_o - p_b + p_e\right)}{\left(p_e - s\right)}$. The optimal emission credit quantity q_b^* is determined by $q_b^* = q^* - q_o^*$.

(2) When $\eta_1 = 0$, no extra emissions are required.

The firm produces under the emission-capped quantity, and benefits from selling the spare emission credits. A new model is built as follows:

$$\begin{cases} \Pi(Q, p) = (p - c) \cdot \min[D, Q] + s \cdot [K - e \cdot Q]^+ - g \cdot [D - Q]^+ \\ s.t. \quad e \cdot Q \le K \end{cases} \tag{12}$$

This model can be solved by the KKT conditions with the Lagrange Multiplier λ_2 and slack variable η_2, then we can have:

a. When $\lambda_2 = 0$, spare emission credits exist.

   We can get the optimal z^* by solving

$$(p - c + g)\overline{F}(z) = s \cdot e \tag{13}$$

   With the optimal z^*, the required optimal solutions are given as:

$$\begin{cases} p^{S*} = \bar{p} - \dfrac{\Upsilon\left(z^*\right)}{2b} \\ q_s^{S*} = \eta_2^2 = K - e \cdot \left(a - b \cdot p^{S*} + z^*\right) \\ Q^{S*} = a - b \cdot p^{S*} + z^* \end{cases} \tag{14}$$

Lemma 3: When $\lambda_2 = 0$, there exists an optimal value of the stocking factor z^* which is determined by $(p - c + g)\bar{F}(z) = s \cdot e$. And the optimal selling price is uniquely determined by a function of z^*, and $p^{S*} = \bar{p} - \dfrac{\Upsilon\left(z^*\right)}{2b}$, where $\bar{p} = \dfrac{a + (c + s \cdot e) \cdot b + \mu}{2b}$.

Lemma 4: When $\lambda_2 = 0$, the firm produces the emission-capped quantity $Q^{S*} = a - b \cdot p^{S*} + z^*$ and benefits from selling the spare emission credits which is determined by $q_s^{S*} = \eta_2^2 = K - e \cdot \left(a - b \cdot p^{S*} + z^*\right)$.

b. When $\eta_2 = 0$, the firms just produced emission-capped quantity.

We can get the optimal z^* by solving

$$\left(a + z - \frac{K}{e} - b \cdot c + b \cdot g\right) \cdot F(z) + \Upsilon(z) + z = b \cdot g + u + \frac{K}{e} \tag{15}$$

With the optimal z^*, the required optimal solutions are given as:

$$\begin{cases} p^{K*} = \dfrac{\left(a + z^*\right) \cdot e - K}{b \cdot e} \\ Q^{K*} = \dfrac{K}{e} \end{cases} \tag{16}$$

Lemma 5: When $\eta_2 = 0$, the firm just produces the emission-capped quantity $Q^{K*} = \dfrac{K}{e}$ to sell at the unit price p^{K*}, and $p^{K*} = \dfrac{\left(a + z^*\right) \cdot e - K}{b \cdot e}$, which is uniquely determined by z^* that satisfies $\left(a + z - \dfrac{K}{e} - b \cdot c + b \cdot g\right) \cdot F(z) + \Upsilon(z) + z = b \cdot g + u + \dfrac{K}{e}$.

3.3 Option-Void Scenario

3.3.1 Model Description

Under the option-void scenario, the firm can only purchase emission credits. No options are tradable in the emission market. This scenario acts as a referencing benchmark, as it reflects the basic emission ordering strategy adopted by most manufacturers.

The firm orders q^n emission credits for producing $\frac{q^n + K}{e}$ product units sold at the unitary selling price p^n. The key to assuring production is to hold sufficient emission credits, and this is of high possibility of spare credits that can be resold at s. Without options, the total emission needed is equivalent to the emission credits quantity, that is, $q^n = q_b^n$. The option-void profit model is given as:

$$\begin{cases} \Pi\left(q^n, p^n\right) = \left(p^n - c\right) \cdot \min\left[D, \frac{q^n + K}{e}\right] + s \cdot \left[q^n + K - e \cdot D\right]^+ - p_b \cdot q^n - g \cdot \left[D - \frac{q^n + K}{e}\right]^+ \\ s.t.\ q^n \geq 0,\ p^n > 0 \end{cases} \tag{17}$$

This scenario defines the stocking factor as $r^n = \frac{q^n + K}{e} - y\left(p^n\right)$. By inserting r^n and $D\left(p^n, \varepsilon\right) = y\left(p^n\right) + \varepsilon$, the option-void profit model is built like:

$$\begin{cases} \Pi\left(r^n, p^n\right) = \left(p^n - c\right) \cdot \min\left[y\left(p^n\right) + \varepsilon, y\left(p^n\right) + r^n\right] + s \cdot e \cdot \left[r^n - \varepsilon\right]^+ \\ \qquad - p_b \cdot \left[e \cdot \left(y\left(p^n\right) + r^n\right) - K\right] - g \cdot \left[\varepsilon - r^n\right]^+ \\ s.t.\ q^n \geq 0, p^n \geq 0 \end{cases} \tag{18}$$

We define $\Lambda\left(r^n\right) = \int_A^{r^n} \left(r^n - x\right) f\left(x\right) dx$ for the expected product leftover and $\Gamma\left(r^n\right) = \int_{r^n}^{B} \left(x - r^n\right) f\left(x\right) dx$ for the expected product shortage. Then we have the expected profit, denoted $E\left[\Pi\left(r^n, p^n\right)\right]$, as follows:

$$E\left[\Pi\left(r^n, p^n\right)\right] = \psi\left(p^n\right) - \chi\left(r^n, p^n\right) \tag{19}$$

Where

$$\begin{aligned} \psi\left(p^n\right) &= \left[p^n - c - p_b \cdot e\right] \cdot \left[y\left(p^n\right) + \mu\right] + p_b \cdot K \\ \chi\left(r^n, p^n\right) &= \left(p_b - s\right) \cdot e \cdot \Lambda\left(r^n\right) + \left(p^n - c + g - p_b \cdot e\right) \cdot \Gamma\left(r^n\right) \end{aligned} \tag{20}$$

$\psi\left(p^n\right)$ ignores the demand uncertainty and refers to the riskless profit. $\chi\left(r^n, p^n\right)$ represents the profit loss out of the demand risk, where the expression $\left(p_b - s\right) \cdot e \cdot \Lambda\left(r^n\right)$ is the overage cost out of the spare emission credits and the expression $\left(p^n - c + g - p_b \cdot e\right) \cdot \Gamma\left(r^n\right)$ is the shortage cost out of the unsatisfied orders. Thus, the option-void problem can be written as:

$$\begin{cases} E\left[\Pi\left(r^n, p^n\right)\right] = \psi\left(p^n\right) - \chi\left(r^n, p^n\right) \\ s.t. \quad K - e \cdot \left(y\left(p\right) + r\right) \le 0 \end{cases} \tag{21}$$

3.3.2 Problem Solving

To solve this problem, the Lagrange Multipliers are explored and a slack variable η^2 is adopted. Then the problem can be re-written as:

$$\begin{cases} L\left(r^n, p^n, \eta_3, \lambda_3\right) = -\left(p^n - c - p_b \cdot e\right) \cdot \left[y\left(p^n\right) + \mu\right] - p_b \cdot K \\ \qquad + \left(p_b - s\right) \cdot e \cdot \Lambda\left(r^n\right) + \left(p^n - c + g - p_b \cdot e\right) \cdot \Gamma\left(r^n\right) \\ \qquad + \lambda_3 \cdot \left[K - e \cdot \left(y\left(p^n\right) + r\right) + \eta_3^2\right] \\ s.t. \ \lambda_3 \ge 0 \end{cases} \tag{22}$$

This option-void problem can be solved by the KKT conditions, as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p^n} = 2bp^n - a - u - \left(c + p_b \cdot e\right) \cdot b + \Gamma\left(r^n\right) + \lambda_3 \cdot b \cdot e = 0 \\ \dfrac{\partial L}{\partial r^n} = \left(p_b - s\right) \cdot e \cdot \int_A^{r^n} f\left(x\right) dx - \left(p^n - c + g - p_b \cdot e\right) \cdot \int_{r^n}^B f\left(x\right) dx - \lambda_3 \cdot e = 0 \\ \dfrac{\partial L}{\partial \eta_3} = 2\eta_3 \cdot \lambda_3 = 0 \\ \dfrac{\partial L}{\partial \lambda_3} = K - e \cdot \left(y\left(p^n\right) + r\right) + \eta_3^2 = 0 \end{cases} \tag{23}$$

The above equations reach their optimality when $\lambda_3 = 0$ and $\eta_3 = 0$.

(1) When $\lambda_3 = 0$, no spare emission credits exist.

The KKT conditions can be given as:

$$\begin{cases} \dfrac{\partial L}{\partial p^n} = 2bp^n - a - u - \left(c + p_b \cdot e\right) \cdot b + \Gamma\left(r^n\right) = 0 \\ \dfrac{\partial L}{\partial r^n} = \left(p_b - s\right) \cdot e \cdot \int_A^{r^n} f\left(x\right) dx - \left(p^n - c + g - p_b \cdot e\right) \cdot \int_{r^n}^B f\left(x\right) dx = 0 \\ \dfrac{\partial L}{\partial \lambda_3} = K - e \cdot \left(y\left(p^n\right) + r\right) + \eta_3^2 = 0 \end{cases} \tag{24}$$

Thus, the solutions under $\lambda_3 = 0$ are obtained. The optimal r^{n*} that can be obtained by solving the following equation:

$$\left(\overline{p}^n - \frac{\Gamma\left(r^n\right)}{2b} - c + g - s \cdot e\right) \cdot \overline{F}\left(r^n\right) = \left(p_b - s\right) \cdot e \tag{25}$$

Where $\overline{p}^n = \dfrac{a + \left(c + p_b \cdot e\right) \cdot b + u}{2b}$.

With the optimal r^{n*}, the required optimal solutions are given as:

$$\begin{cases} p^{n*} \equiv p\left(r^{n*}\right) = \overline{p}^n - \dfrac{\Gamma\left(r^{n*}\right)}{2b} \\ q^{n*} = \eta_3^2 = e \cdot \left(y\left(p^{n*}\right) + r^{n*}\right) - K \end{cases} \tag{26}$$

Lemma 6: When $\lambda_3 = 0$, the firm orders $q^{n*} = \eta_3^2 = e \cdot \left(y\left(p^{n*}\right) + r^{n*}\right) - K$ emission credits from the emission permits supplier and sells its products at the unitary selling price $p^{n*} \equiv p\left(r^{n*}\right) = \overline{p}^n - \dfrac{\Gamma\left(r^{n*}\right)}{2b}$ where $\overline{p}^n = \dfrac{a + \left(c + p_b \cdot e\right) \cdot b + u}{2b}$. The optimal stocking factor r^{n*} is uniquely determined by $\left(\overline{p}^n - \dfrac{\Gamma\left(r^n\right)}{2b} - c + g - s \cdot e\right) \cdot \overline{F}\left(r^n\right) = \left(p_b - s\right) \cdot e$.

(2) When $\eta_3 = 0$, no extra emission credits are required.

This situation derives another model as mentioned in the former option-available section. And the optimal solutions are results achieved when $\lambda_2 = 0$ or when $\eta_2 = 0$.

3.4 Results

The analytical results are compared in Table 3.

Table 3. Comparison of Analytical Results

Results	Strategy	Option-Available Scenario	Option-Void Scenario
Selling Price	Over K	$p^* \equiv p\left(r^*\right) = \overline{p} - \dfrac{\Gamma\left(r^*\right)}{2b}$ with $\overline{p} = \dfrac{a + \left(c + p_b \cdot e\right) \cdot b + \mu}{2b}$	$p^{n*} \equiv p\left(r^{n*}\right) = \overline{p}^n - \dfrac{\Gamma\left(r^{n*}\right)}{2b}$ with $\overline{p}^n = \dfrac{a + \left(c + p_b \cdot e\right) \cdot b + u}{2b}$

	At K	$p^{K*} = \frac{(a+z^*)\cdot e - K}{b \cdot e}$	$p^{K*} = \frac{(a+z^*)\cdot e - K}{b \cdot e}$
	Under K	$p^{S*} = \bar{p} - \frac{\Upsilon(z^*)}{2b}$ with $\bar{p} = \frac{a+(c+s\cdot e)\cdot b+\mu}{2b}$	$p^{S*} = \bar{p} - \frac{\Upsilon(z^*)}{2b}$ with $\bar{p} = \frac{a+(c+s\cdot e)\cdot b+\mu}{2b}$
Stocking Factor	Over K	$\left(\bar{p} - \frac{\Gamma(r^*)}{2b} - c + g - p_e \cdot e\right)\cdot \bar{F}(r^*) = p_o \cdot e$	$\left(\bar{p}^n - \frac{\Gamma(r^{n*})}{2b} - c + g - s \cdot e\right)\cdot \bar{F}(r^{n*}) = (p_b - s)\cdot e$
	At K	$\left(a + z^* - \frac{K}{e} - b\cdot c + b\cdot g\right)\cdot F(z^*) + \Upsilon(z^*) + z^* = b\cdot g + u + \frac{K}{e}$	$\left(a + z^* - \frac{K}{e} - b\cdot c + b\cdot g\right)\cdot F(z^*) + \Upsilon(z^*) + z^* = b\cdot g + u + \frac{K}{e}$
	Under K	$(p - c + g)\bar{F}(z^*) = s\cdot e$	$(p - c + g)\bar{F}(z^*) = s\cdot e$
Total Emission Required	Over K	$q^* = e\cdot\left(y(p^*) + r^*\right) - K$	$q^{n*} = e\cdot\left(y(p^{n*}) + r^{n*}\right) - K$
	At K	0	0
	Under K	0	0
Emission Options Required	Over K	$F\left(r^* - \frac{q_o^*}{e}\right) = \frac{(p_o - p_b + p_e)}{(p_e - s)}$	0
	At K	0	0
	Under K	0	0
Emission Credits Required	Over K	$q_b^* = q^* - q_o^*$	$q^{n*} = e\cdot\left(y(p^{n*}) + r^{n*}\right) - K$
	At K	0	0
	Under K	0	0
Spare Emission Credits	Over K	0	0
	At K	0	0
	Under K	$q_s^{S*} = K - e\cdot\left(a - b\cdot p^{S*} + z^*\right)$	$q_s^{S*} = K - e\cdot\left(a - b\cdot p^{S*} + z^*\right)$
Production Quantity	Over K	$Q^* = \frac{q_b^* + exercised\ q_o}{e}$	$Q^{n*} = \frac{q^{n*}}{e}$
	At K	$Q^{K*} = \frac{K}{e}$	$Q^{K*} = \frac{K}{e}$
	Under K	$Q^{S*} = a - b\cdot p^{S*} + z^*$	$Q^{S*} = a - b\cdot p^{S*} + z^*$

From the above analytical results and lemmas, we have the following propositions:

Proposition 1: There exists the optimal emission ordering and product pricing policy when the options are available, and the firm can achieve its option-available optimality when $\lambda_1 = 0$, $\lambda_2 = 0$ or $\eta_2 = 0$.

By comparing the resulted profits when $\lambda_1 = 0$, $\lambda_2 = 0$ or $\eta_2 = 0$, the firms can obtain its optimal option-available profit.

Proposition 2: There exists the optimal emission ordering and product pricing policy when the options are void, and the firm can achieve its option-void optimality when $\lambda_3 = 0$, $\lambda_2 = 0$ or $\eta_2 = 0$.

By comparing the resulted profits when $\lambda_3 = 0$, $\lambda_2 = 0$ or $\eta_2 = 0$, the firms can obtain its optimal option-available profit.

Proposition 3: The firm can achieve its final optimality by comparing the resulted option-available profit and option-void profit.

Proposition 4: The demand uncertainty lowers the optimal selling price of the product.

Since $\frac{\Gamma\left(r^*\right)}{2b}$, $\frac{\Gamma\left(z^*\right)}{2b}$, $\frac{\Gamma\left(r^{n*}\right)}{2b}$ is non-negative, the optimal selling price is the riskless price less the price loss, $p^* \leq \overline{p}$, $p^{n*} \leq \overline{p}^n$.

IV. NUMERICAL ANALYSIS

Previous research works show that the fertilizer industry is a heavy energy user and carbon emitter, which is estimated at between 2-3% of the total GHG emissions (Brentrup et al., 2016). Hence, the fertilizer industry bears social responsibilities and legal obligations to fulfill the emission reduction targets. China, as the largest fertilizer producer and consumer, is essential to the global efforts in reducing carbon emissions. Therefore, the following data are collected from the Chinese fertilizer industry to conduct numerical studies to validate the proposed model and reveal a firm's performance under different scenarios. Moreover, the impact of decision variables on the resulting optimal profit is separately discussed.

The data used in the option-available and option-void scenarios are shown below:

Table 4. Data Values

Emission level of product e	Emission cap K	Unitary cost of product c	Resell price of emission credits s	Unitary goodwill cost g	Order price of emission credits p_b	Option price of emission credits p_o	Exercising price of emission credits p_e

0.9 ton/ton	0.8 100ton	200 USD/ton	5 USD/ton	10 USD/ton	10 USD/ton	4 USD/ton	8 USD/ton

Under this data set, a firm produces under an emission cap of 800 ton per month. The total production cost of fertilizer is 200USD/ton with 0.9 ton of carbon emission. The emission price is set at 10USD/ton. 4USD need to be prepaid for one-ton emission option and 8USD for exercising them. The goodwill cost for unsatisfied demand is 10USD/ton and the spare emission credits can be resold at 5USD.

Besides, a normal distribution is adopted to build the demand risk, which is measured by the coefficient of variation (CV), similar to the works by Wang and Chen (2015). The decreasing and deterministic demand function for selling price is formed by $y(p) = 400 - p$, and the demand uncertainty is established with $\mu = 50$ and $\sigma = 25$ when $\varepsilon \in [-200, 200]$.

4.1 Result Comparison

Based on the above dataset, we have the numerical results in Table 5.

Table 5 shows that the option-available scenario earns more than the option-void scenario, and that both scenarios choose to purchase extra emissions to produce more than the emission-capped quantity. The option-available scenario requires more emissions including the options and credits, however, its initial emission cost is less than that in the option-void one, due to the lower option premium in the initial period. This smaller initial emission cost relieves the financial pressure and increases the fund liquidity, which is vital to a firm's development in the competitive market. Moreover, purchasing options lowers the unitary selling price which can attract more customer demand. In the option-available scenario, the options account for 47% of the whole emission reservation. Although the firm may burden more possible emission cost, the options make it more profitable due to risk aversion.

Table 5. Comparison of Numerical Results

	Unit	The Option-Available Scenario	The Option-Void Scenario
Selling Price	USD/ton	328.47	329.34
Total Extra Emission Required	100ton	72.5796	69.8484
Emission Credit Quantity	100ton	39.0694	69.8484

Emission Option Quantity	100ton	33.5102	0
Emission Cost (without exercising cost)	1000USD	52.4735	69.8484
Profit	1000USD	1508.7409	1506.9744
Strategy	——	Over K	Over K
		Options & Credits	Credits

Brief Summary 1: Options enable less initial cost which relieves the financial pressure and increases the fund liquidity.

4.2 The Influence of Demand Uncertainty

The decision behaviours and profit performance of a fertilizer manufacturer under a specific market have been studied. We now discuss how they are influenced by the demand uncertainty. Three production strategies, namely producing by, below, and over the emission cap, are proposed and they are represented by 1, 2, and 3 in the right top sub-figure of the following figures, respectively.

Obviously, as shown in Figure1, the demand uncertainty lowers the firm's profit, and purchasing extra emissions help the firm achieve better performance. Moreover, the firm can win more earning from buying emission options, while the profit gap between option-available and option-void profits is enlarged by the increasing demand uncertainty.

It is rational to reserve more emissions when the demand suffers higher uncertainty, as the larger emission reservation gives the firm more flexibility to solve the demand fluctuation. Options that owns lower initial emission cost enable the firm to hold more carbon liabilities, and this is one reason why option-available is more profitable under this dataset.

The trends of unitary selling price are intricate with increasing demand uncertainty, decreasing and then increasing, because of the synergetic impact of price-sensitive demand and risk cost. And option-void scenario charges more due to the demand uncertainty.

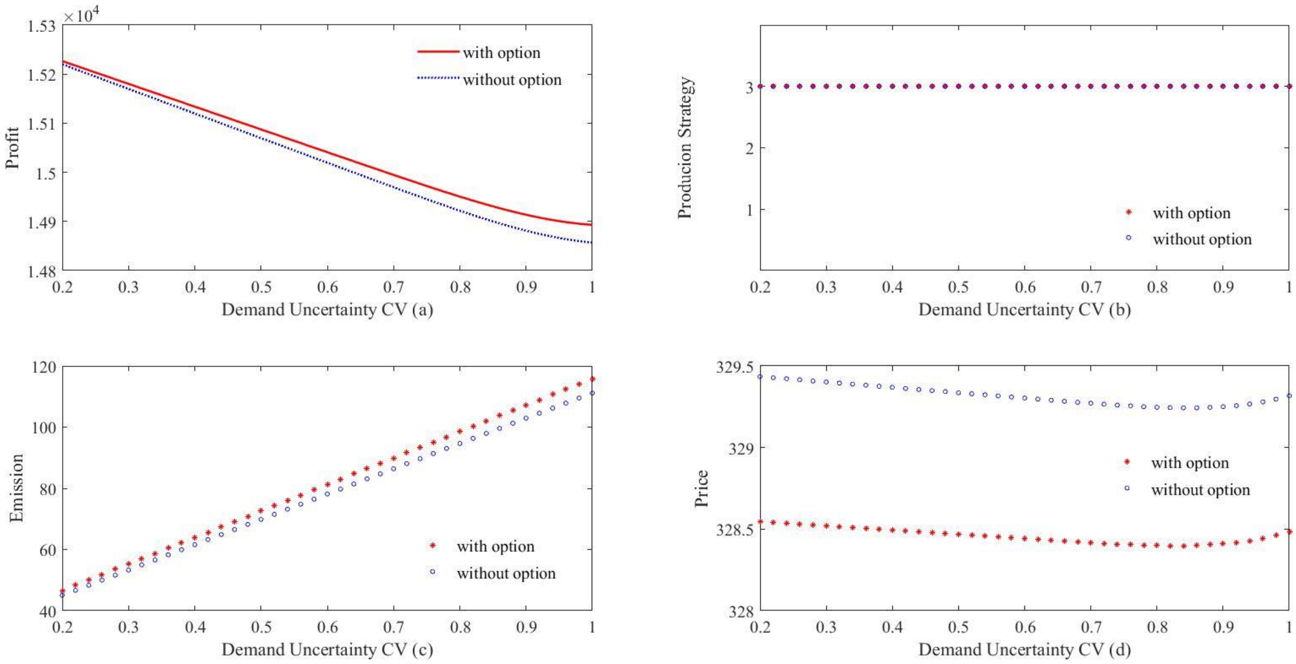

Figure 1. The Trends of Decisions and Performance when coefficient of variation CV varies

Brief Summary 2: Options achieve better performance and more emissions are required under higher risk; option-void scenario charges more selling price due to the demand uncertainty.

4.3 The Influence of Emission Options and Credits

Figure 2 and Figure 3 show the trends of decision behaviours and profit performance when the emission option and exercising prices vary, respectively. Figure 4 shows their trends when the emission credit price changes. As known, the emission-void scenario cannot be affected by the changes of option prices, thus its related decision variables remain the same.

From Figure 2, the option-available profit decreases from 3 to 4.8, and then experience a sharp plunge from 4.8 to 6. This is caused by the production strategy changing from 3 to 1. This plunge makes the optimal strategy change to not buying emission options. The emission required shows the same trend, and no emission is needed when the option price is between 4.8 and 6 in the option-available scenario. The unitary selling price in the option-available scenario is smaller than the option-void scenario from 3 to 4.8, and then surges to higher after 4.8.

From Figure 3, the option-available profit decreases from 7 to 10 with the increasing option exercising price. The production strategy remains at 3. The emission required drops down as well, and the unitary selling price increases and then exceeds the option-void selling price after 9.8.

From Figure 4, when the emission credit price increases, the profits from both scenarios decrease. The firm tends to buy options to earn more. The production strategy remains over K in the option-available model, and changes from over K to at K in the option-void model. The emission required decreases and the option scenario realizes a larger emission reservation, while the selling price increases due to the higher emission cost.

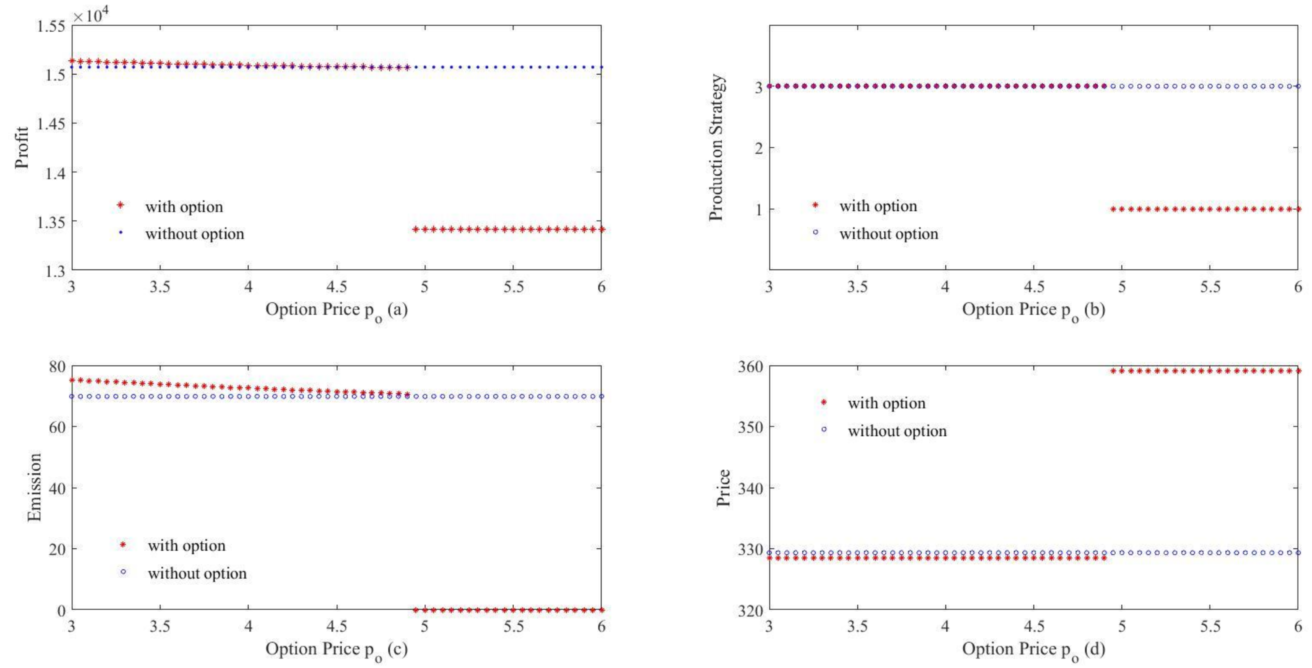

Figure 2. The Trends of Decisions and Performance when emission option price p_o varies

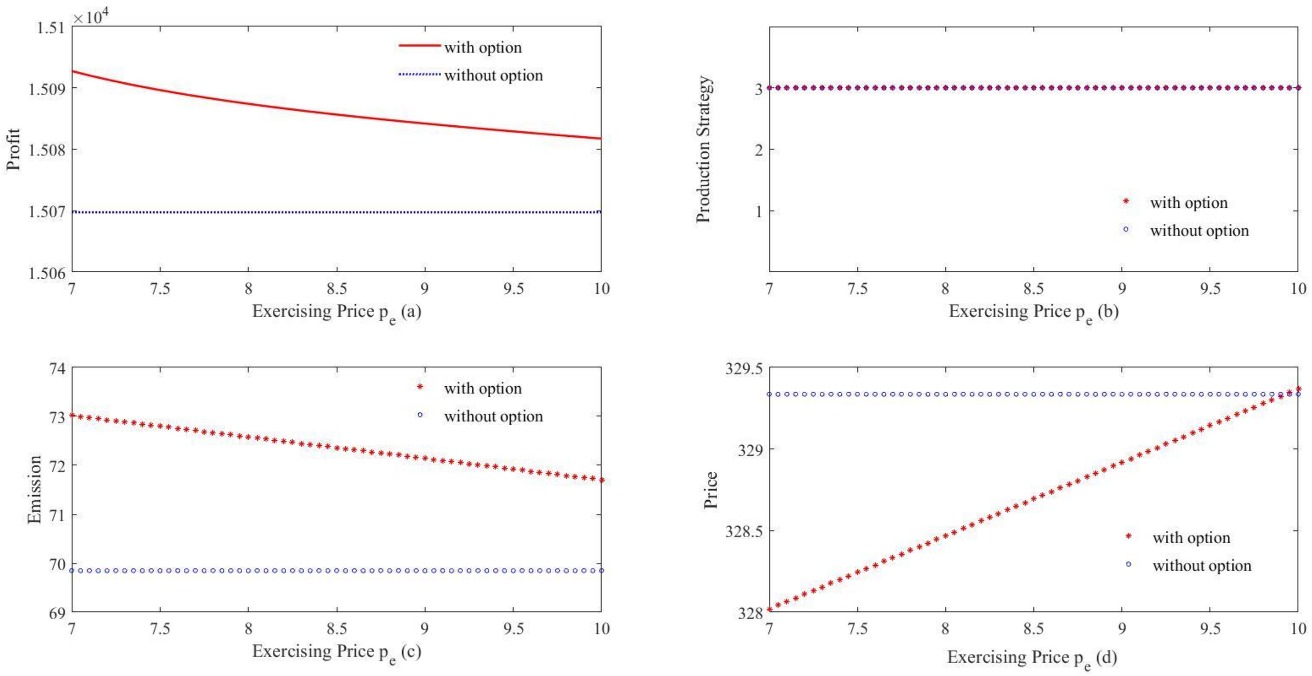

Figure 3. The Trends of Decisions and Performance when option exercising price p_e varies

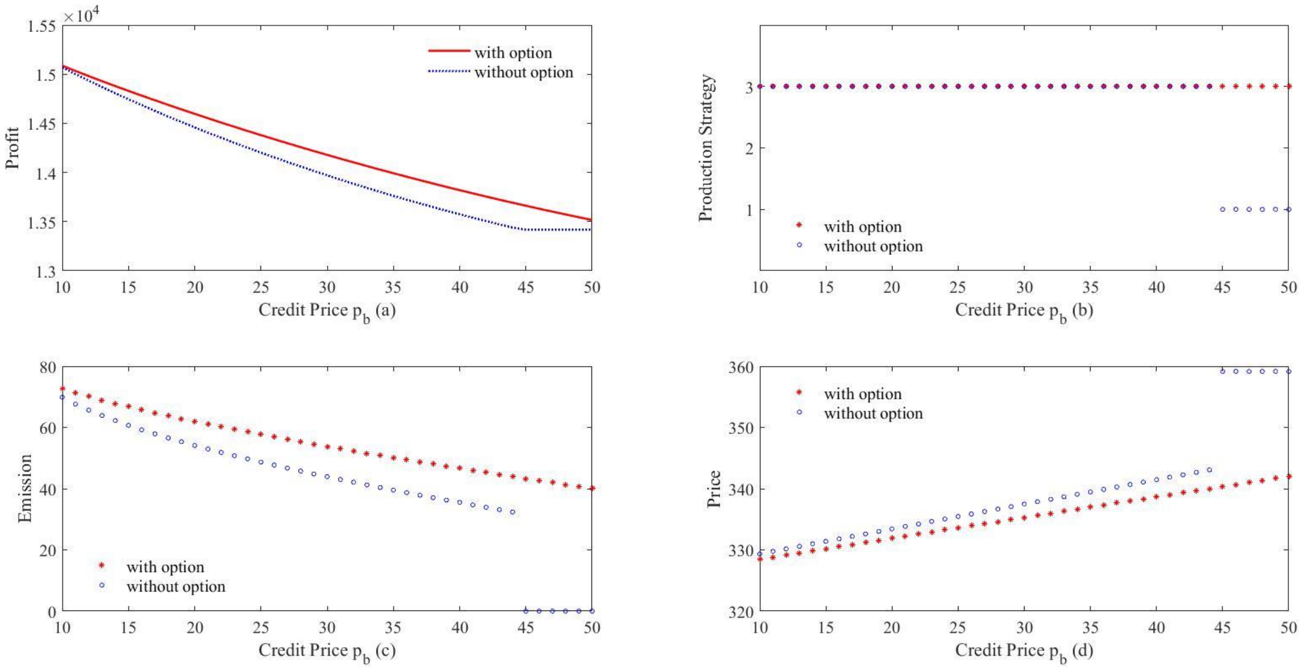

Figure 4. The Trends of Decisions and Performance when emission credit price p_b varies

Brief Summary 3: Higher emission-related prices shrink the profit and the emission required, while higher selling price is charged to compensate for the higher emission cost.

4.4 Sensitivity Analysis for Parameters

This section discusses how the firm behaves when the price-sensitivity and the emission cap change. Figure 5 elaborates that profit decreases with increasing price-sensitivity to demand. The firm tends to purchase emissions at a lower price-sensitivity level and just produce emission-capped quantity at a higher level. Both the emission required and the selling price decline with a higher price-sensitivity. From Figure 6, purchasing options earns more at a lower price-sensitivity and less at a higher price-sensitivity. The price-sensitivity to demand significantly affects the firm's decision-making in terms of the options.

The trends of decision variables are intricate when the emission cap changes as shown in Figure 7. When the emission cap is too loose at more than 152, the firm produces below the emission cap and sells the spare emissions to the market. No difference between the option-available and option-void scenario exists after 152. From 40 to 152, the option-void profit increases and the option-available profit first increases and then experiences a sharp plunge after 120, as its production strategy changes from 1 to 3. The emission required decreases. The selling price stays the same from 40 to 120 and from 150 to 180 if buying options, and it remains unchanged from 40 to 150 and from 150 to 180 if not buying. From Figure 8, we can see the option-available scenario enjoys higher profits with lower selling prices.

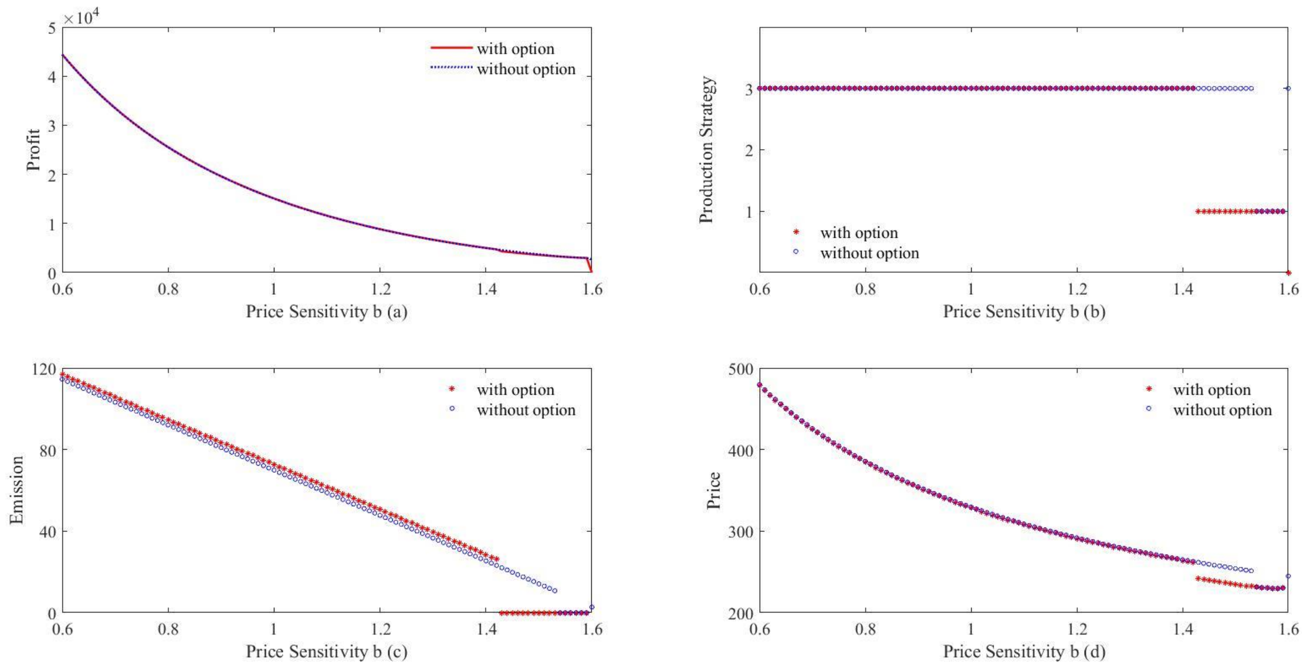

Figure 5. The Trends of Decisions and Performance when price-sensitivity to demand b varies

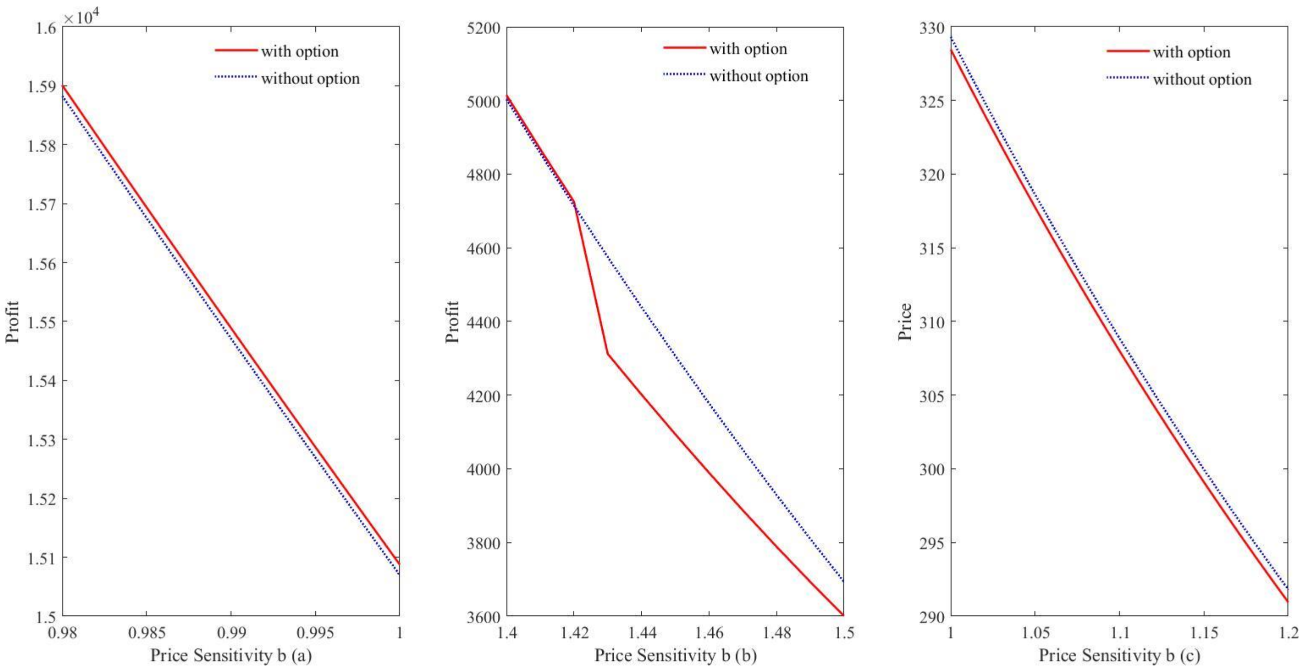

Figure 6. The Profit Trends when price-sensitivity to demand b varies

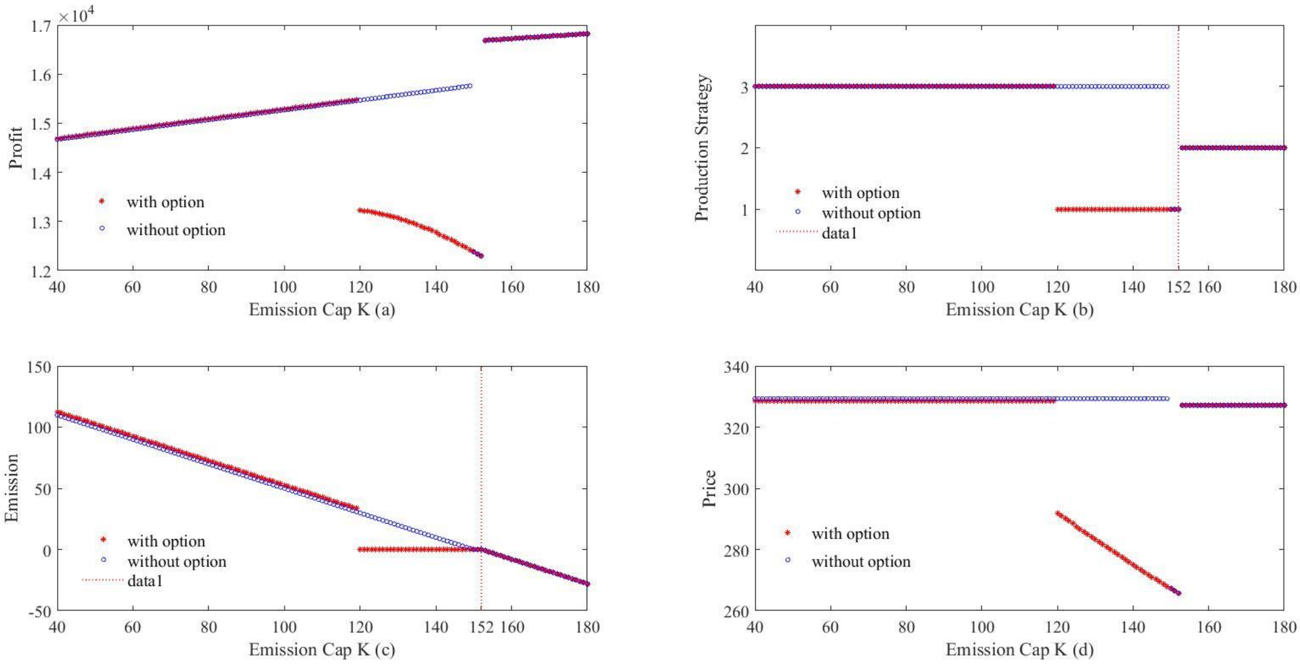

Figure 7. The Trends of Decisions and Performance when emission cap K varies

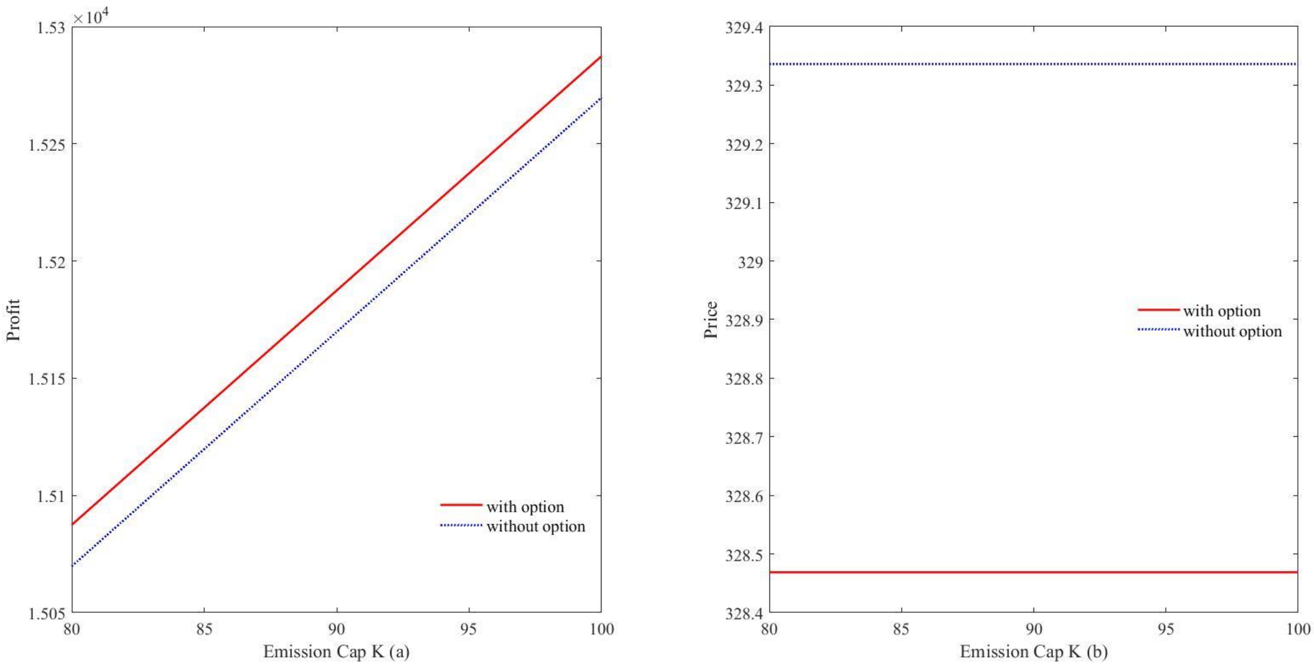

Figure 8. The Trends of profit and selling price when emission cap K varies

Brief Summary 4: The price-sensitivity to demand significantly affects the firm's decision-making on options, and the firm easily thrives in the loose emission market.

4.5 Sensitivity Analysis for Emission Options

We have studied the trends of emission reservation including options and credits under both scenarios when the parameters vary. One more question is how many options are required to set up this reservation. With an increasing price-sensitivity to demand, Figure 9 shows that the emission option decreases and drops to zero at 1.42. Besides, a higher emission cost (emission option and exercising price) lowers the option quantity. The option price gives a similar trend to price-sensitivity, that is, the emission options declines to zero after 4.8. More options are purchased to hedge higher demand risk, and loose emission cap trims down the options to zero level. When the credit price goes up, emission options first rises and then falls. Given the credit price is lower than 20, buying options lead to less initial emission cost and higher fund flexibility, and given more than 20, this large emission cost shrinks the production. This is the reason why the emission options follow this trend.

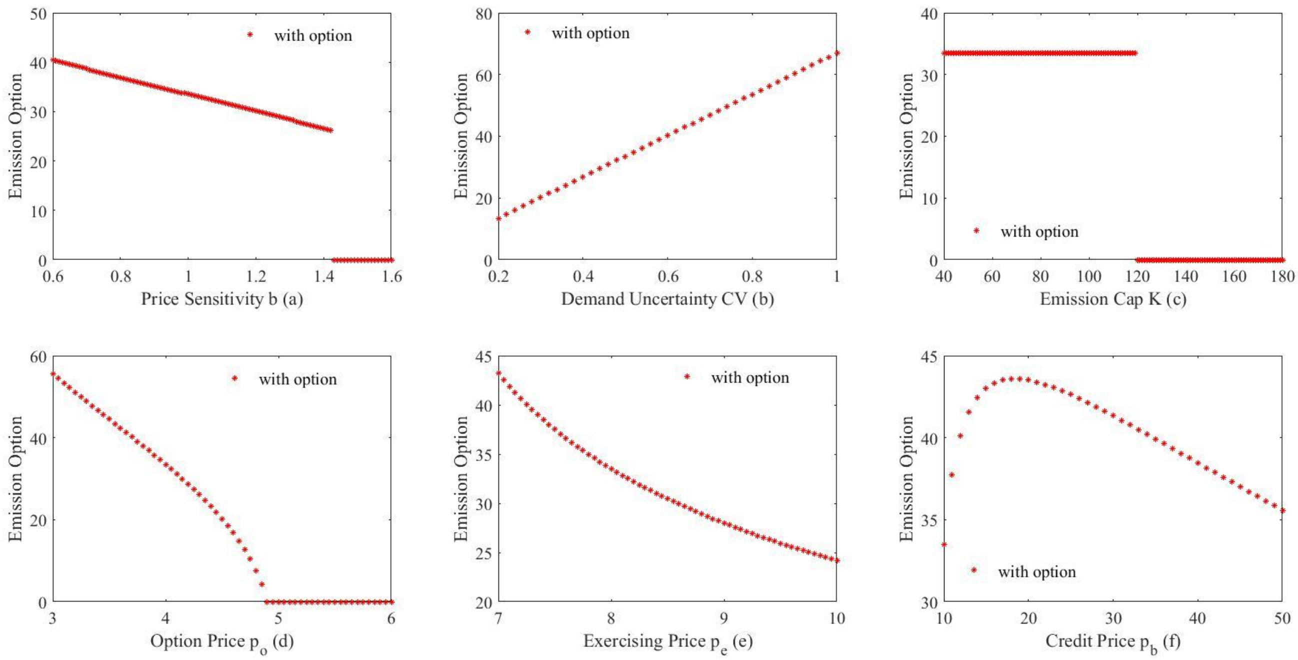

Figure 9. The Trends of Emission Options

Brief Summary 5: Options with reasonable unitary cost helps hedge the demand uncertainty when the price-sensitivity is none too high.

According to the results and brief summaries from this numerical study, we have the following managerial insights:

Managerial Insight 1: Options help attract more customer demand due to the lower selling price.

Managerial Insight 2: The firm achieves more profitability from buying options in a stringent emission market when the price-sensitivity is none too high.

Managerial Insight 3: Reasonable option pricing is vital to the emission permit suppliers and the emission-dependent manufacturer.

Managerial Insight 4: The firm easily thrives in the loose emission market, but this cannot achieve effective emission reduction.

V. CONCLUSION

This research investigates the manufacturer's decision behaviours and profit performance in an emission option-available market facing demand uncertainty. Its novelty lies in the incorporation of the call option contract in terms of emission permits which simultaneously realize emission reduction and demand risk aversion.

The newsvendor model is established to study how the manufacturer behaves under different market environments, and whether or not the emission options help the manufacturer achieve outperformance. Besides, the impact of emission options and its proportion of the required emission reservation are also discussed in this research.

From the analytical results, we can reach the optimal pricing and emission purchasing strategy by comparing the profitability gotten in the option-available and option-void scenario. The demand uncertainty lowers the optimal selling price of the product, and this helps attract more customers to compensate for the risk cost.

The numerical results provide several managerial insights for the manufacturing decision-making. Options achieve better performance in a stringent emission market with higher demand risk when the price-sensitivity is none too high. More emissions are required under higher demand certainty or lower related emission cost. Moreover, reasonable pricing of options is vital to the emission permit suppliers and the emission-dependent manufacturer to balance the environmental protection and economic development, as a firm easily thrives in the loose emission market, but this cannot achieve effective emission reduction.

These findings call for the manufacturer to make better decisions for achieving both the optimality and the emission reduction, as well as for the emission permit suppliers to price the emission credits and options within a reasonable range for the health of the carbon market.

There are some limitations in this paper that deserve further research. It only considers one-period production. In reality, the production process is a multi-period activity, and thus the multi-period production problem with demand uncertainty under emission restrictions needs more research. Second, only a call option contract is discussed, and the future works can explore some other contracts, such as pull option and future contract. Finally, we assume a two-stage supply chain consisting of one manufacturer and one emission permit supplier. However, the emission reduction and profit maximum require the cooperation of all the supply chain members. It would be worthwhile to develop a more complex model composed of more supply chain members in a more practical market.

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