Robust H_∞ Filtering for Uncertain 2-D Continuous Systems

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***Abstract*—This paper considers the problem of robust H_∞ filtering for uncertain two-dimensional (2-D) continuous systems described by the Roesser state-space model. The parameter uncertainties are assumed to be norm-bounded in both the state and measurement equations. The purpose is the design of a 2-D continuous filter such that for all admissible uncertainties, the error system is asymptotically stable, and the H_∞ norm of the transfer function, from the noise signal to the estimation error, is below a prespecified level. A sufficient condition for the existence of such filters is obtained in terms of a set of linear matrix inequalities (LMIs). When these LMIs are feasible, an explicit expression of a desired H_∞ filter is given. Finally, a simulation example is provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*—H_∞ filtering, linear matrix inequality, 2-D continuous systems, uncertain systems.**

I. Introduction

THE problems of estimation and filter design have received much attention in the past decades. It is known that one of the most popular ways to deal with the filtering problem is the celebrated Kalman filtering approach, which generally provides an optimal estimation of the state variables in the sense that the covariance of the estimation error is minimized [1]. This approach usually requires the exact information on both the external noises and the internal model of the system. However, these requirements are not always satisfied in practical applications. To overcome these difficulties, an alternative approach called H_∞ filtering has been introduced, which aims to determine a filter such that the resulting filtering error system is asymptotically stable, and the $\mathcal{L}_2$-induced norm (for continuous systems) or l_2-induced norm (for discrete systems) from the input disturbances to the filtering error output satisfies a prescribed H_∞ performance level. In contrast to the Kalman filtering approach, the H_∞ filtering approach does not require exact knowledge of the statistical properties of the external noise, which renders this approach very appropriate in many practical applications. A great number of H_∞ filtering results have been reported, and various approaches, such as the linear matrix inequality (LMI) approach [2], polynomial equation approach [10], algebraic Riccati equation approach [19], [22], and frequency domain approach [21], have been proposed in the literature. When parameter uncertainties appear in a system model, the robust H_∞ filtering problem has been investigated, and some results on this topic have been presented; see, e.g., [5], [8], [12], [27], and the references therein. It is worth pointing out that these results were obtained in the context of one-dimensional (1-D) systems.

For two-dimensional (2-D) systems, the H_∞ filtering problem has been studied recently. Based on a proposed bounded real lemma, the H_∞ filtering problem for 2-D systems described by the Roesser model was solved in [7], where H_∞ filters in both the observer-based form and the general state equation form were designed. For 2-D systems in the Fornasini–Marchesini local state-space model, an LMI approach was developed to design H_∞ filters in [25]; these results were further extended to 2-D systems with polytopic parameter uncertainties in the system model in [23]. It is noted that all these mentioned H_∞ filtering results were derived for 2-D discrete systems [13]. Although many stability analysis and control results for 2-D continuous systems have been reported in the literature [9], [14], [17], [18], [20], [24], the H_∞ filtering problem has not been fully investigated, which motivates the present study.

In this paper, we deal with the robust H_∞ filtering problem for uncertain 2-D continuous systems. The parameter uncertainties are assumed to be norm-bounded, appearing in both the state and measurement equations. The class of continuous 2-D systems under consideration is described by the Roesser state-space model. The problem we address is the design of 2-D continuous filters such that for all admissible uncertainties, the error system is asymptotically stable, and the H_∞ norm of the transfer function, from the noise signal to the estimation error, is below a prescribed level. A sufficient condition for the solvability of this problem is obtained in terms of a set of LMIs. A desired H_∞ filter can be constructed by solving these given LMIs. A simulation example is provided to show the effectiveness of the proposed approach.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The superscript "T" represents the

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transpose of a matrix. The symbol $\sigma_{\max}(\cdot)$ denotes the spectral norm of a matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

Consider an uncertain 2-D continuous system described by the following Roesser's state-space model [16]:

$$(\Sigma): \begin{bmatrix} \frac{\partial}{\partial t_1} x^h(t_1,t_2) \\ \frac{\partial}{\partial t_2} x^v(t_1,t_2) \end{bmatrix} = (A+\Delta A)\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + (B+\Delta B)\omega(t_1,t_2) \tag{1}$$

$$y(t_1,t_2) = (C_1+\Delta C_1)\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + (D_1+\Delta D_1)\omega(t_1,t_2) \tag{2}$$

$$z(t_1,t_2) = C\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + D\omega(t_1,t_2) \tag{3}$$

where $x^h(t_1,t_2) \in \mathbb{R}^{n_h}$ and $x^v(t_1,t_2) \in \mathbb{R}^{n_v}$ are the horizontal states and vertical states, respectively; $\omega(t_1,t_2) \in \mathbb{R}^m$ is the exogenous input; $y(t_1,t_2) \in \mathbb{R}^s$ is the measurement output; and $z(t_1,t_2) \in \mathbb{R}^p$ is the signal to be estimated. A, B, C_1, C, D_1, and D are known real constant matrices. ΔA, ΔB, ΔC_1, and ΔD_1 are unknown matrices representing the parameter uncertainties in the system matrices and are assumed to be of the form

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C_1 & \Delta D_1 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F [N_1 \quad N_2] \tag{4}$$

where M_1, M_2, N_1, and N_2 are known real constant matrices, and $F \in \mathbb{R}^{k\times l}$ is an unknown matrix satisfying

$$F^T F \leq I. \tag{5}$$

The uncertain matrices ΔA, ΔB, ΔC_1, and ΔD_1 are said to be admissible if both (4) and (5) hold.

Remark 1: It should be pointed out that the structure of the uncertainty with the form (4) and (5) has been widely used when dealing with the issues related to both 1-D and 2-D uncertain systems; see, e.g., [6], [15], and the references therein.

The nominal system of (1) and (3) can be written as

$$(\Sigma_n): \begin{bmatrix} \frac{\partial}{\partial t_1} x^h(t_1,t_2) \\ \frac{\partial}{\partial t_2} x^v(t_1,t_2) \end{bmatrix} = A\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + B\omega(t_1,t_2)$$

$$z(t_1,t_2) = C\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + D\omega(t_1,t_2).$$

It can be seen that the transfer function matrix of the 2-D continuous system (Σ_n) is as follows:

$$G(s_1,s_2) = C\left[I(s_1,s_2) - A\right]^{-1} B + D \tag{6}$$

where

$$I(s_1,s_2) = \operatorname{diag}\left(s_1 I_{n_h}, s_2 I_{n_v}\right). \tag{7}$$

Throughout the paper, we adopt the following definition.

Definition 1: The H_∞ norm of the 2-D continuous system (Σ_n) is defined as

$$\|G\|_\infty = \sup_{\theta_1,\theta_2\in\mathbb{R}} \sigma_{\max}\left[G(j\theta_1, j\theta_2)\right].$$

Now, we consider the following 2-D continuous filter for the estimate of $z(t_1,t_2)$:

$$(\Sigma_f): \begin{bmatrix} \frac{\partial}{\partial t_1} \hat{x}^h(t_1,t_2) \\ \frac{\partial}{\partial t_2} \hat{x}^v(t_1,t_2) \end{bmatrix} = \hat{A}_f\begin{bmatrix} \hat{x}^h(t_1,t_2) \\ \hat{x}^v(t_1,t_2) \end{bmatrix} + \hat{B}_f y(t_1,t_2) \tag{8}$$

$$\hat{z}(t_1,t_2) = \hat{C}_f\begin{bmatrix} \hat{x}^h(t_1,t_2) \\ \hat{x}^v(t_1,t_2) \end{bmatrix} \tag{9}$$

where $\hat{x}^h(t_1,t_2) \in \mathbb{R}^{n_h}$ and $\hat{x}^v(t_1,t_2) \in \mathbb{R}^{n_v}$ are the horizontal states and vertical states of the filter, respectively; and $\hat{z}(t_1,t_2) \in \mathbb{R}^p$ is the estimate of $z(t_1,t_2)$. The matrices $\hat{A}_f$, $\hat{B}_f$, and $\hat{C}_f$ are to be selected. Denote

$$\tilde{x}^h(t_1,t_2) = \left[x^h(t_1,t_2)^T \quad \hat{x}^h(t_1,t_2)^T\right]^T$$
$$\tilde{x}^v(t_1,t_2) = \left[x^v(t_1,t_2)^T \quad \hat{x}^v(t_1,t_2)^T\right]^T$$
$$\tilde{z}(t_1,t_2) = z(t_1,t_2) - \hat{z}(t_1,t_2).$$

Then, the filtering error dynamics from the systems (Σ) and (Σ_f) can be obtained as

$$(\tilde{\Sigma}): \begin{bmatrix} \frac{\partial}{\partial t_1} \tilde{x}^h(t_1,t_2) \\ \frac{\partial}{\partial t_2} \tilde{x}^v(t_1,t_2) \end{bmatrix} = (\tilde{A}+\Delta\tilde{A})\begin{bmatrix} \tilde{x}^h(t_1,t_2) \\ \tilde{x}^v(t_1,t_2) \end{bmatrix} + (\tilde{B}+\Delta\tilde{B})\omega(t_1,t_2)$$

$$\tilde{z}(t_1,t_2) = \tilde{C}\begin{bmatrix} \tilde{x}^h(t_1,t_2) \\ \tilde{x}^v(t_1,t_2) \end{bmatrix} + \tilde{D}\omega(t_1,t_2)$$

where

$$\tilde{A} = \Phi\tilde{A}_f\Phi^T, \ \tilde{B} = \Phi\tilde{B}_f, \ \tilde{C} = \tilde{C}_f\Phi^T, \ \tilde{D} = D \tag{10}$$

$$\Delta\tilde{A} = \Phi\Delta\tilde{A}_f\Phi^T, \ \Delta\tilde{B} = \Phi\Delta\tilde{B}_f \tag{11}$$

$$\tilde{A}_f = \begin{bmatrix} A & 0 \\ \hat{B}_f C_1 & \hat{A}_f \end{bmatrix}, \ \tilde{B}_f = \begin{bmatrix} B \\ \hat{B}_f D_1 \end{bmatrix} \tag{12}$$

$$\tilde{C}_f = [C \quad -\hat{C}_f], \ \Delta\tilde{A}_f = \begin{bmatrix} \Delta A & 0 \\ \hat{B}_f \Delta C_1 & 0 \end{bmatrix} \tag{13}$$

$$\Delta\tilde{B}_f = \begin{bmatrix} \Delta B \\ \hat{B}_f \Delta D_1 \end{bmatrix} \tag{14}$$

$$\Phi = \begin{bmatrix} I_{n_h} & 0 & 0 & 0 \\ 0 & 0 & I_{n_h} & 0 \\ 0 & I_{n_v} & 0 & 0 \\ 0 & 0 & 0 & I_{n_v} \end{bmatrix}. \tag{15}$$

The robust H_∞ filtering problem to be addressed in this paper can be formulated as follows: *Given a scalar $\gamma > 0$ and the uncertain 2-D continuous system (Σ), find an asymptotically stable filter (Σ_f) in the form of (8) and (9) such that the filtering error system $(\tilde{\Sigma})$ is asymptotically stable and the transfer function of the error system given as*

$$\tilde{G}(s_1,s_2) = \tilde{C}\left[I(s_1,s_2) - (\tilde{A}+\Delta\tilde{A})\right]^{-1}(\tilde{B}+\Delta\tilde{B}) + \tilde{D}$$

satisfies

$$\|\tilde{G}\|_\infty < \gamma, \tag{16}$$

for all admissible uncertainties.

III. Main Results

In this section, an LMI approach will be developed to solve the robust H_∞ filtering problem formulated in the previous section. Before giving the main results, we first present the following results, which will be used in the following development.

Lemma 1: [9], [20] The 2-D continuous system

$$\begin{bmatrix} \frac{\partial}{\partial t_1} x^h(t_1,t_2) \\ \frac{\partial}{\partial t_2} x^v(t_1,t_2) \end{bmatrix} = A \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix}$$

is asymptotically stable if there exist matrices $P_h > 0$ and $P_v > 0$ satisfying the following LMI:

$$A^T P + PA < 0,$$

where $P = \operatorname{diag}(P_h, P_v)$.

Lemma 2: [26] Let $\mathcal{D}$, $\mathcal{S}$, and F be real matrices of appropriate dimensions with F satisfying $F^T F \le I$. Then, for any scalar $\epsilon > 0$

$$\mathcal{D}F\mathcal{S} + (\mathcal{D}F\mathcal{S})^T \le \epsilon^{-1}\mathcal{D}\mathcal{D}^T + \epsilon \mathcal{S}^T\mathcal{S}.$$

Theorem 1: Given a scalar $\gamma > 0$. The 2-D continuous system (Σ_n) is asymptotically stable and satisfies the H_∞ performance $\|G\|_\infty < \gamma$ if there exists a matrix $P = \operatorname{diag}(P_h, P_v) > 0$ with $P_h \in \mathbb{R}^{n_h}$ and $P_v \in \mathbb{R}^{n_v}$ such that the following LMI holds:

$$\begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0. \tag{17}$$

Proof: By (17), we have

$$A^T P + PA < 0$$

which, together with Lemma 1, implies that system (Σ) is asymptotically stable. Next, we show the H_∞ performance. By applying the Schur complement formula to (17), we obtain

$$V := \gamma^2 I - D^T D > 0$$

and

$$A^T P + PA + \gamma^{-1} C^T C + \\ \gamma(PB + \gamma^{-1}C^T D)V^{-1}(B^T P + \gamma^{-1} D^T C) < 0.$$

Multiplying this inequality by γI yields

$$A^T(\gamma P) + (\gamma P)A + C^T C + \\ \left[(\gamma P)B + C^T D\right] V^{-1} \left[B^T(\gamma P) + D^T C\right] < 0. \tag{18}$$

Let $Q = \gamma P > 0$; then, (18) can be rewritten as

$$A^T Q + QA + C^T C + (QB + C^T D)V^{-1}(B^T Q + D^T C) < 0.$$

Therefore, there exists a matrix $U > 0$ such that

$$-A^T Q - QA - C^T C > \Lambda \tag{19}$$

where

$$\Lambda = (QB + C^T D)V^{-1}(B^T Q + D^T C) + U.$$

Set

$$\Omega(j\theta_1, j\theta_2) = I(j\theta_1, j\theta_2) - A.$$

Then, it can be verified that

$$QI(j\theta_1, j\theta_2) + I(-j\theta_1, -j\theta_2)^T Q = 0. \tag{20}$$

By (19) and (20), we have

$$\begin{aligned} &\Omega(-j\theta_1, -j\theta_2)^T Q + Q\Omega(j\theta_1, j\theta_2) - C^T C \\ &\quad = -A^T Q - QA - C^T C > \Lambda. \end{aligned} \tag{21}$$

Since system (Σ) is asymptotically stable, we have

$$\det\left[I(j\theta_1, j\theta_2) - A\right] \ne 0$$

for all θ_1, $\theta_2 \in \mathbb{R}$. Therefore, $\Omega(j\theta_1, j\theta_2)^{-1}$ is well defined for all θ_1, $\theta_2 \in \mathbb{R}$. Now, pre- and post-multiplying (21) by $B^T\Omega(-j\theta_1, -j\theta_2)^{-T}$ and $\Omega(j\theta_1, j\theta_2)^{-1}B$, respectively, we have that for all θ_1, $\theta_2 \in \mathbb{R}$

$$\begin{aligned} &B^T\Omega(-j\theta_1, -j\theta_2)^{-T} \\ &\quad \times \left[\Omega(-j\theta_1, -j\theta_2)^T Q + Q\Omega(j\theta_1, j\theta_2) - C^T C\right] \\ &\quad \times \Omega(j\theta_1, j\theta_2)^{-1}B \\ &\qquad \ge B^T\Omega(-j\theta_1, -j\theta_2)^{-T}\Lambda\Omega(j\theta_1, j\theta_2)^{-1}B. \end{aligned}$$

Thus, by noting (6), we have

$$\begin{aligned} &\gamma^2 I - G(-j\theta_1, -j\theta_2)^T G(j\theta_1, j\theta_2) \\ &\quad = (\gamma^2 I - D^T D) + B^T\Omega(j\theta_1, j\theta_2)^{-T}QB \\ &\qquad + B^T Q\Omega(j\theta_1, j\theta_2)^{-1}B \\ &\qquad - B^T\Omega(-j\theta_1, -j\theta_2)^{-T}C^T C\Omega(j\theta_1, j\theta_2)^{-1}B \\ &\qquad - B^T\Omega(-j\theta_1, -j\theta_2)^{-T}(QB + C^T D) \\ &\qquad - (B^T Q + D^T C)\Omega(j\theta_1, j\theta_2)^{-1}B \\ &\quad \ge V + B^T\Omega(-j\theta_1, -j\theta_2)^{-T}\Lambda\Omega(j\theta_1, j\theta_2)^{-1}B \\ &\qquad - B^T\Omega(-j\theta_1, -j\theta_2)^{-T}(QB + C^T D) \\ &\qquad - (B^T Q + D^T C)\Omega(j\theta_1, j\theta_2)^{-1}B \\ &\quad \ge V - (B^T Q + D^T C)\Lambda^{-1}(QB + C^T D). \end{aligned} \tag{22}$$

Now, observe that

$$\Lambda - (QB + C^T D)V^{-1}(B^T Q + D^T C) = U > 0.$$

Then, by the Schur complement formula, we have

$$\begin{bmatrix} V & B^TQ + D^TC \\ QB + C^TD & \Lambda \end{bmatrix} > 0$$

which, by the Schur complement formula again, gives

$$V - (B^TQ + D^TC)\Lambda^{-1}(QB + C^TD) > 0. \qquad (23)$$

Then, it follows from (22) and (23) that for all $\theta_1, \theta_2 \in \mathbb{R}$

$$\gamma^2 I - G(-j\theta_1, -j\theta_2)^T G(j\theta_1, j\theta_2) > 0.$$

Hence, by Definition 1, we have $\|G\|_\infty < \gamma$. This completes the proof. □

Remark 2: Theorem 1 provides an LMI condition for the 2-D continuous system (Σ) to be asymptotically stable and satisfy a specified H_∞ performance level. Theorem 1 can be regarded as an extension of existing results on bounded realness for 1-D continuous systems [11] to the 2-D case. It is noted that the bounded lemma for spatially interconnected systems was reported in [4], which cannot include Theorem 1 as a special case.

Now, we are in a position to present the solvability condition for the robust H_∞ filtering problem.

Theorem 2: Given a scalar $\gamma > 0$ and the uncertain 2-D continuous system (Σ). Then, the robust H_∞ filtering problem is solvable if there exists a scalar $\epsilon > 0$ and matrices Z, Θ, Ψ, $X = \mathrm{diag}(X_h, X_v) > 0$, and $Y = \mathrm{diag}(Y_h, Y_v) > 0$ with X_h, $Y_h \in \mathbb{R}^{n_h}$, X_v, and $Y_v \in \mathbb{R}^{n_v}$ satisfying the LMIs in (24) and (25), shown at the bottom of the page, where

$$\begin{aligned} J_1 &= YA + A^TY + \epsilon N_1^T N_1 \\ J_2 &= XA + A^TX + \Psi C_1 + C_1^T \Psi^T + \epsilon N_1^T N_1 \\ H_1 &= YA + A^TX + C_1^T\Psi^T + Z^T + \epsilon N_1^T N_1 \\ H_2 &= YB + \epsilon N_1^T N_2 \\ H_3 &= XB + \Psi D_1 + \epsilon N_1^T N_2. \end{aligned}$$

In this case, a desired 2-D continuous filter in the form of (8) and (9) can be chosen with parameters as follows:

$$\hat{A}_f = X_{12}^{-1} Z Y^{-1} Y_{12}^{-T} \qquad (26)$$

$$\hat{B}_f = X_{12}^{-1}\Psi \qquad (27)$$

$$\hat{C}_f = \Theta Y^{-1} Y_{12}^{-T} \qquad (28)$$

where

$$X_{12} = \begin{bmatrix} X_{h12} & 0 \\ 0 & X_{v12} \end{bmatrix}, \quad Y_{12} = \begin{bmatrix} Y_{h12} & 0 \\ 0 & Y_{v12} \end{bmatrix} \qquad (29)$$

in which X_{h12}, X_{v12}, Y_{h12}, and Y_{v12} are any nonsingular matrices satisfying

$$X_{12}Y_{12}^T = I - XY^{-1}. \qquad (30)$$

Proof: Let

$$\bar{Y}_h = Y_h^{-1}, \quad \bar{Y}_v = Y_v^{-1}, \quad \bar{Y} = Y^{-1}.$$

Then, by (25), it is easy to see that $I - X\bar{Y}$ is nonsingular. Therefore, there always exist nonsingular matrices X_{h12}, X_{v12}, Y_{h12}, and Y_{v12} such that (30) is satisfied, that is

$$X_{h12}Y_{h12}^T = I - X_h\bar{Y}_h \qquad (31)$$

$$X_{v12}Y_{v12}^T = I - X_v\bar{Y}_v. \qquad (32)$$

Set

$$\begin{aligned} \Pi_{h1} &= \begin{bmatrix} \bar{Y}_h & \\ Y_{h12}^T & 0 \end{bmatrix}, \quad \Pi_{h2} = \begin{bmatrix} I & X_h \\ 0 & X_{h12}^T \end{bmatrix} \\ \Pi_{v1} &= \begin{bmatrix} \bar{Y}_v & \\ Y_{v12}^T & 0 \end{bmatrix}, \quad \Pi_{v2} = \begin{bmatrix} I & X_v \\ 0 & X_{v12}^T \end{bmatrix} \\ \Pi_1 &= \begin{bmatrix} \Pi_{h1} & 0 \\ 0 & \Pi_{v1} \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} \Pi_{h2} & 0 \\ 0 & \Pi_{v2} \end{bmatrix}. \end{aligned}$$

Then, by some calculations, it can be verified that

$$\tilde{P} := \Pi_2\Pi_1^{-1} = \begin{bmatrix} \tilde{P}_h & 0 \\ 0 & \tilde{P}_v \end{bmatrix} \qquad (33)$$

where

$$\begin{aligned} \tilde{P}_h &= \begin{bmatrix} X_h & X_{h12} \\ X_{h12}^T & X_{h12}^T(X_h - Y_h)^{-1}X_{h12} \end{bmatrix} \\ \tilde{P}_v &= \begin{bmatrix} X_v & X_{v12} \\ X_{v12}^T & X_{v12}^T(X_v - Y_v)^{-1}X_{v12} \end{bmatrix}. \end{aligned}$$

Considering (25), we can deduce that $\tilde{P}_h > 0$ and $\tilde{P}_v > 0$. Now, pre- and post-multiplying (24) by $\mathrm{diag}(\bar{Y}, I, I, I, I)$ yields the first equation shown at the bottom of the next page, which, by the Schur complement formula, implies (34), shown at the bottom of the next page, where $\hat{A}_f$, $\hat{B}_f$, and $\hat{C}_f$ are given in (26)–(28). By (33), the inequality (34) can be rewritten as (35), shown at the bottom of the next page, where Φ is given in (15), and

$$\tilde{M}_1 = \begin{bmatrix} M_1 \\ \hat{B}_f M_2 \end{bmatrix}, \quad \tilde{N}_1 = [N_1 \quad 0], \quad \tilde{N}_2 = N_2.$$

$$\begin{bmatrix} J_1 & H_1 & H_2 & C^T - \Theta^T & YM_1 \\ H_1^T & J_2 & H_3 & C^T & XM_1 + \Psi M_2 \\ H_2^T & H_3^T & \epsilon N_2^T N_2 - \gamma I & D^T & 0 \\ C - \Theta & C & D & -\gamma I & 0 \\ M_1^T Y & M_1^T X + M_2^T \Psi^T & 0 & 0 & -\epsilon I \end{bmatrix} < 0 \qquad (24)$$

$$X - Y > 0 \qquad (25)$$

Pre- and postmultiplying (35) by $\mathrm{diag}(\Pi_1^{-T}\Phi^{-T}, I, I)$ and $\mathrm{diag}(\Phi^{-1}\Pi_1^{-1}, I, I)$ results in

$$\begin{bmatrix} \tilde{P}\tilde{A}+\tilde{A}^T\tilde{P} & \tilde{P}\tilde{B} & \tilde{C}^T \\ \tilde{B}^T\tilde{P} & -\gamma I & \tilde{D}^T \\ \tilde{C} & \tilde{D} & -\gamma I \end{bmatrix} + \epsilon^{-1}\begin{bmatrix} \tilde{P}\Phi\tilde{M}_1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \tilde{P}\Phi\tilde{M}_1 \\ 0 \\ 0 \end{bmatrix}^T + \epsilon\begin{bmatrix} \Phi\tilde{N}_1^T \\ \tilde{N}_2^T \\ 0 \end{bmatrix}\begin{bmatrix} \Phi\tilde{N}_1^T \\ \tilde{N}_2^T \\ 0 \end{bmatrix}^T < 0 \quad (36)$$

where the relationship $\Phi^T = \Phi^{-1}$ is used, and $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, and $\tilde{D}$ are given in (10). Now, noting

$$[\Delta\tilde{A}_f \quad \Delta\tilde{B}_f] = \tilde{M}_1 F [\tilde{N}_1 \quad \tilde{N}_2]$$

and using Lemma 2, we have

$$\begin{bmatrix} \tilde{P}\Phi\Delta\tilde{A}_f\Phi^T + (\tilde{P}\Phi\Delta\tilde{A}_f\Phi^T)^T & \tilde{P}\Phi\Delta\tilde{B}_f & 0 \\ \Delta\tilde{B}_f^T\Phi^T\tilde{P} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \le \epsilon^{-1}\begin{bmatrix} \tilde{P}\Phi\tilde{M}_1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \tilde{P}\Phi\tilde{M}_1 \\ 0 \\ 0 \end{bmatrix}^T + \epsilon\begin{bmatrix} \Phi\tilde{N}_1^T \\ \tilde{N}_2^T \\ 0 \end{bmatrix}\begin{bmatrix} \Phi\tilde{N}_1^T \\ \tilde{N}_2^T \\ 0 \end{bmatrix}^T.$$

This, together with (36), gives

$$\begin{bmatrix} \tilde{P}(\tilde{A}+\Delta\tilde{A}) + (\tilde{A}+\Delta\tilde{A})^T\tilde{P} & \tilde{P}(\tilde{B}+\Delta\tilde{B}) & \tilde{C}^T \\ (\tilde{B}+\Delta\tilde{B})^T\tilde{P} & -\gamma I & \tilde{D}^T \\ \tilde{C} & \tilde{D} & -\gamma I \end{bmatrix} < 0.$$

Finally, by Theorem 1, it follows that the error system $(\tilde{\Sigma})$ is asymptotically stable, and the transfer function of the error system satisfies (16). This completes the proof. □

Remark 3: Theorem 2 provides a sufficient condition for the solvability of the robust H_∞ filtering problem for 2-D continuous systems. A desired filter can be constructed by solving the LMIs in (24) and (25), which can be implemented by using recently developed interior-point methods, and no tuning of parameters is required [3].

In the case when there is no parameter uncertainty in system (Σ), that is, (Σ) reduces to the following 2-D continuous system

$$\begin{bmatrix} \frac{\partial}{\partial t_1}x^h(t_1,t_2) \\ \frac{\partial}{\partial t_2}x^v(t_1,t_2) \end{bmatrix} = A\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + B\omega(t_1,t_2) \quad (37)$$

$$y(t_1,t_2) = C_1\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + D_1\omega(t_1,t_2) \quad (38)$$

$$z(t_1,t_2) = C\begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + D\omega(t_1,t_2) \quad (39)$$

by Theorem 2, we have the following corollary.

$$\begin{bmatrix} \bar{Y}J_1\bar{Y} & \bar{Y}H_1 & \bar{Y}H_2 & \bar{Y}C^T - \bar{Y}\Theta^T & M_1 \\ H_1^T\bar{Y} & J_2 & H_3 & C^T & XM_1+\Psi M_2 \\ H_2^T\bar{Y} & H_3^T & \epsilon N_2^TN_2-\gamma I & D^T & 0 \\ C\bar{Y}-\Theta\bar{Y} & C & D & -\gamma I & 0 \\ M_1^T & M_1^TX+M_2^T\Psi^T & 0 & 0 & -\epsilon I \end{bmatrix} < 0$$

$$\begin{bmatrix} A\bar{Y}+\bar{Y}A^T & A+\bar{Y}A^TX+\bar{Y}C_1^T\hat{B}_f^TX_{12}^T+Y_{12}\hat{A}_f^TX_{12}^T & B & \bar{Y}C^T-Y_{12}\hat{C}_f^T \\ A^T+XA\bar{Y}+X_{12}\hat{B}_fC_1\bar{Y}+X_{12}\hat{A}_fY_{12}^T & XA+A^TX+X_{12}\hat{B}_fC_1+(X_{12}\hat{B}_fC_1)^T & XB+X_{12}\hat{B}_fD_1 & C^T \\ B^T & B^TX+D_1^T\hat{B}_f^TX_{12}^T & -\gamma I & D^T \\ C\bar{Y}-\hat{C}_fY_{12}^T & C & D & -\gamma I \end{bmatrix} + \epsilon^{-1}\begin{bmatrix} M_1 \\ XM_1+X_{12}\hat{B}_fM_2 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} M_1 \\ XM_1+X_{12}\hat{B}_fM_2 \\ 0 \\ 0 \end{bmatrix}^T + \epsilon\begin{bmatrix} \bar{Y}N_1^T \\ N_1^T \\ N_2^T \\ 0 \end{bmatrix}\begin{bmatrix} \bar{Y}N_1^T \\ N_1^T \\ N_2^T \\ 0 \end{bmatrix}^T < 0 \quad (34)$$

$$\begin{bmatrix} \Phi^T\Pi_1^T\tilde{P}\Phi\tilde{A}_f\Phi^T\Pi_1\Phi + \Phi^T\Pi_1^T\Phi\tilde{A}_f^T\Phi^T\tilde{P}\Pi_1\Phi & \Phi^T\Pi_1^T\tilde{P}\Phi\tilde{B}_f & \Phi^T\Pi_1^T\Phi\tilde{C}_f^T \\ \tilde{B}_f^T\Phi^T\tilde{P}\Pi_1\Phi & -\gamma I & D^T \\ \tilde{C}_f\Phi^T\Pi_1\Phi & D; & -\gamma I \end{bmatrix} + \epsilon^{-1}\begin{bmatrix} \Phi^T\Pi_1^T\tilde{P}\Phi\tilde{M}_1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \Phi^T\Pi_1^T\tilde{P}\Phi\tilde{M}_1 \\ 0 \\ 0 \end{bmatrix}^T + \epsilon\begin{bmatrix} \Phi^T\Pi_1^T\Phi\tilde{N}_1^T \\ \tilde{N}_2^T \\ 0 \end{bmatrix}\begin{bmatrix} \Phi^T\Pi_1^T\Phi\tilde{N}_1^T \\ \tilde{N}_2^T \\ 0 \end{bmatrix}^T < 0 \quad (35)$$

Corollary 1: Consider the 2-D continuous system in (37)–(39). Then, the H_∞ filtering problem for this system is solvable if there exist matrices Z, Θ, Ψ, $X = \mathrm{diag}(X_h, X_v) > 0$, and $Y = \mathrm{diag}(Y_h, Y_v) > 0$ with $X_h, Y_h \in \mathbb{R}^{n_h}$, X_v, and $Y_v \in \mathbb{R}^{n_v}$ satisfying the LMIs in (40) and (41), shown at the bottom of the page. In this case, a desired 2-D continuous filter in the form of (8) and (9) can be chosen with parameters as given in (26)–(28).

IV. Simulation Example

In this section, we provide a simulation example to illustrate the application of the proposed method in this paper.

Consider the uncertain 2-D continuous system (Σ) with parameters as follows:

$$A = \left[\begin{array}{cc|cc} -1 & 0.2 & -0.5 & 0.3 \\ 0.6 & -2 & 0.2 & 0 \\ \hline -0.3 & 0.7 & -2.3 & 0.6 \\ -0.1 & 0.2 & 0.1 & -1.8 \end{array}\right], \; B = \left[\begin{array}{c} 0.2 \\ -0.5 \\ \hline -0.8 \\ 0.3 \end{array}\right]$$

$$M_1 = \left[\begin{array}{c} 0.1 \\ 0.2 \\ \hline 0.2 \\ -0.1 \end{array}\right], \; C_1 = \left[\begin{array}{cc|cc} 0.8 & -0.9 & 0.2 & -0.1 \\ 0.5 & -0.3 & 0 & 0.5 \end{array}\right]$$

$$D_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \; M_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$$

$$C = [\,0.6 \quad 0.1 \quad | \quad -0.8 \quad 0.5\,], \; D = 0.5$$

$$N_1 = [\,0.2 \quad 0.1 \quad | \quad -0.2 \quad 0.3\,], \; N_2 = 0.2.$$

It can be verified that the nominal system is asymptotically stable. The purpose of this example is to design a 2-D continuous filter in the form of (8) and (9) such that the error system is asymptotically stable and satisfies a prescribed H_∞ performance level γ, which is assumed to be 0.6 in this example.

Now, by resorting to the Matlab LMI Control Toolbox, we obtain the solution to the LMIs in (24) and (25) as follows:

$$X = \mathrm{diag}\left(\begin{bmatrix} 12.8826 & 2.1510 \\ 2.1510 & 11.4955 \end{bmatrix}, \begin{bmatrix} 6.7699 & 2.5628 \\ 2.5628 & 10.4701 \end{bmatrix} \right)$$

$$Y = \mathrm{diag}\left(\begin{bmatrix} 3.9910 & 1.9438 \\ 1.9438 & 3.2938 \end{bmatrix}, \begin{bmatrix} 1.3692 & 1.2243 \\ 1.2243 & 4.8107 \end{bmatrix} \right)$$

$$Z = \begin{bmatrix} 15.4356 & -1.0227 & 5.2852 & 1.9771 \\ 1.9798 & 18.6566 & 1.4903 & -17.0024 \\ -2.5998 & -3.1151 & 19.5865 & -7.3993 \\ -1.2987 & -1.3306 & 4.4731 & 30.7549 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 2.1019 & -7.5892 \\ -19.1418 & 23.9874 \\ -5.0247 & 19.0491 \\ 6.6869 & -7.3625 \end{bmatrix}$$

$$\Theta = [0.5744 \quad -1.0098 \quad -1.5076 \quad 1.1908]$$

$$\epsilon = 1.2391.$$

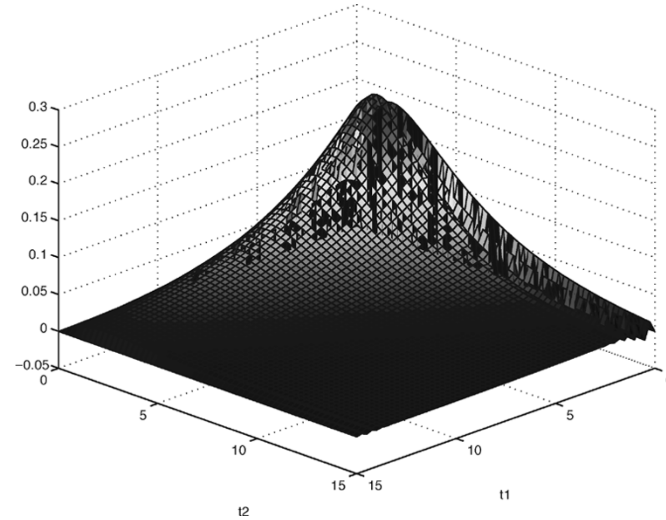

Fig. 1. Response of $\tilde{x}_1^h(t_1, t_2)$.

To construct a desired filter, we further choose

$$X_{h12} = \begin{bmatrix} 1 & 0.5 \\ -1 & 5 \end{bmatrix}, \quad X_{v12} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \tag{42}$$

$$Y_{h12} = \begin{bmatrix} -2.9514 & -0.2645 \\ 1.9109 & -0.3081 \end{bmatrix} \tag{43}$$

$$Y_{v12} = \begin{bmatrix} -2.0366 & 0.7109 \\ 0.2310 & -0.4773 \end{bmatrix}. \tag{44}$$

It can be verified that the matrices X_{h12}, X_{v12}, Y_{h12}, and Y_{v12} chosen in (42)–(44) are nonsingular and satisfy (30). Thus, from Theorem 2, a desired filter can be chosen as

$$\left[\begin{array}{c} \frac{\partial}{\partial t_1}\hat{x}^h(t_1,t_2) \\ \hline \frac{\partial}{\partial t_2}\hat{x}^v(t_1,t_2) \end{array}\right] = \left[\begin{array}{cc|cc} -1.9260 & 1.0154 & -1.9360 & -0.4679 \\ 0.0349 & -2.0843 & -0.1687 & 2.0796 \\ \hline -0.0404 & 0.9828 & -3.2490 & 2.2923 \\ 0.0081 & -0.0395 & -0.4045 & -6.5123 \end{array}\right] \times \left[\begin{array}{c} \hat{x}^h(t_1,t_2) \\ \hline \hat{x}^v(t_1,t_2) \end{array}\right] + \left[\begin{array}{cc} 3.6510 & -9.0799 \\ -3.0982 & 2.9815 \\ \hline -1.1982 & 7.1121 \\ 2.6284 & -4.8249 \end{array}\right] y(t_1,t_2)$$

$$\hat{z}(t_1,t_2) = [\;-0.1923 \quad 0.5904 \quad | \quad 0.4102 \quad -1.2329\;] \times \left[\begin{array}{c} \hat{x}^h(t_1,t_2) \\ \hline \hat{x}^v(t_1,t_2) \end{array}\right].$$

Now, we choose $F = 0.8$, and then, the responses $\tilde{x}_1^h(t_1,t_2)$ and $\tilde{x}_2^h(t_1,t_2)$ of the error system are shown in Figs. 1 and 2, respectively. Fig. 3 gives the response of the error $\tilde{z}(t_1,t_2)$. The frequency response of the error system is given in Fig. 4, and the

$$\begin{bmatrix} YA + A^TY & YA + A^TX + C_1^T\Psi^T + Z^T & YB & C^T - \Theta^T \\ A^TY + XA + \Psi C_1 + Z & XA + A^TX + \Psi C_1 + C_1^T\Psi^T & XB + \Psi D_1 & C^T \\ B^TY & B^TX + D_1^T\Psi^T & -\gamma I & D^T \\ C - \Theta & C & D & -\gamma I \end{bmatrix} < 0 \tag{40}$$

$$X - Y > 0. \tag{41}$$

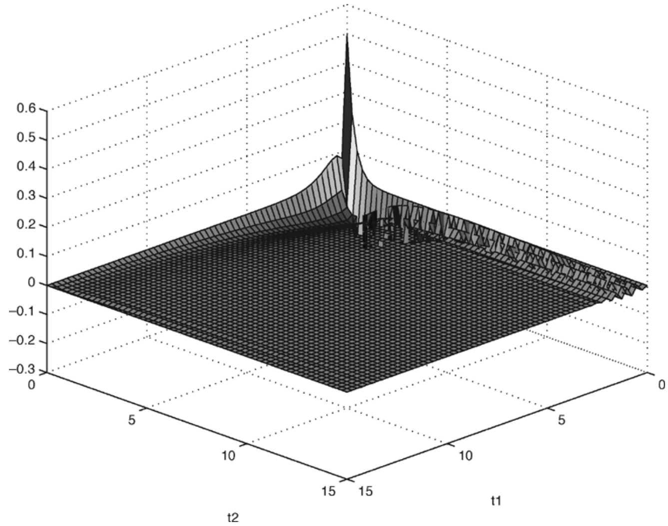

Fig. 2. Response of $\tilde{x}_2^h(t_1, t_2)$.

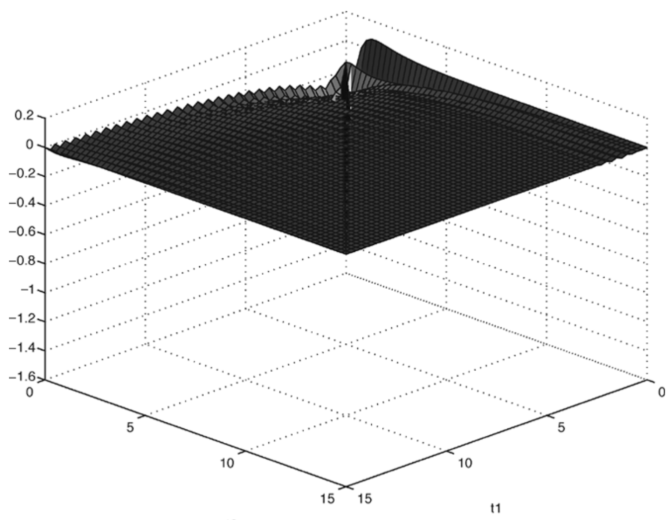

Fig. 3. Filtering error response of $\tilde{z}(t_1, t_2)$.

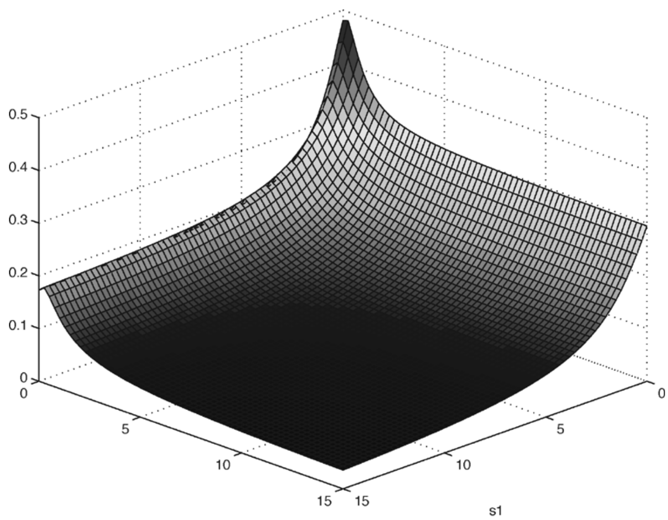

Fig. 4. Frequency response of error system.

achieved H_∞ norm is approximately 0.4834, which compares well with the value $\gamma = 0.6$ used. The simulation result shows the effectiveness of the designed filter.

V. Conclusions

In this paper, we have studied the problem of robust H_∞ filtering for 2-D continuous systems described by Roesser's state-space model with norm-bounded parameter uncertainties in the state and measurement equations. An LMI approach for designing a 2-D continuous filter, which ensures asymptotic stability of the error system and reduces the H_∞ norm of the transfer function from the noise signal to the estimation error to a prescribed level for all admissible uncertainties, has been proposed. A desired filter can be constructed through a convex optimization problem that has been investigated fully in the literature. A simulation example has been provided to demonstrate the effectiveness of the proposed method.

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