

PHASE TRANSITIONS IN ROTATING NEUTRON STARS: EFFECTS OF STELLAR CRUSTS

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ABSTRACT

As a rapidly rotating neutron star spins down due to the loss of its angular momentum, its central density increases and the nuclear matter in its core converts to quark matter, which leads to a drastic decrease of the stellar moment of inertia, and even results in an era of spin-up of the pulsar (Glendenning, Pei, & Weber 1997). We find that given a certain equation of state in the liquid core, even if the backbending of the moment of inertia as a function of the rotating frequency occurs, an increase of the total moment of inertia by only 1% could carry adequate angular momentum and stop the star spin-up. This small discrepancy in the total moment of inertia might be due to the different properties of subnuclear matter in the crust, especially to different transition density and pressure at the inner boundary of the solid crust between various models. The strong dependence of the phenomenon of backbending on the physical state of the crust provides, in principle, a new observational approach to check and constrain theories on subnuclear matter.

Subject headings: dense matter — stars: evolution — stars: interiors — stars: neutron — stars: rotation

1. INTRODUCTION

The solid crust of a neutron star plays an important role in neutron star evolution and dynamics. For instance, it insulates the neutron star surface from its hot interior, or it releases its gravitational potential energy and heats the neutron star by cracking the crust during the spin-down (Cheng et al. 1992), and therefore affects the star's cooling. Furthermore, transitions of nuclear compositions in the crust result in heat generation during the spin-down, which also significantly influence the cooling of neutron stars (Iida & Sato 1997). The crust serves as one of two independent components of the stellar moment of inertia that is responsible for glitches in pulsar timing (Alpar, Cheng, & Pine 1990; Link, Epstein, & Van Riper 1992). Given the equation of state (EOS) of neutron star matter, the mass and moment of inertia of the crust depend strongly on the transition density, n_t , and pressure, P_t , (Lattimer & Prakash 2001) at its inner boundary. The transition density n_t is around $0.07\text{--}0.10\text{ fm}^{-3}$ (Lorenz, Ravenhall, & Pethick 1993; Krivine, Treiner, & Bohigas 1980; Pethick, Ravenhall, & Lorenz 1995; Cheng, Yao, & Dai 1997; Douchin & Haensel 2000). However, the crust does not significantly affect the gross properties of the star, such as the mass, radius, moment of inertia, and so on, which are determined mainly by the physical state of neutron star matter at densities beyond n_t . This is the reason why the EOS of the crust is often not mentioned in the literature when the gross properties of compact star are discussed (Glendenning et al. 1997).

The composition of matter in the interior of neutron stars is essentially unknown. Some exotic states, such as hyperonic matter (Glendenning 1985), quark matter (Collins & Perry 1975; Baym & Chin 1976; Chapline 1976; Dai, Lu, & Peng 1993; Dai, Peng, & Lu 1995), kaon condensation (Kaplan & Nelson 1986; Brown et al. 1994), or pion condensation (Baym & Pethick 1975, 1979), have been investi-

gated. Having abandoned the assumption of local charge neutrality, Glendenning (1992) put forward that mixed phase of nuclear matter and quark matter, or kaon condensation (Glendenning & Schaffner-Bielich 1998; Glendenning 2001), is favored in the interior of neutron stars. The phase transition might occur during spinning down of an isolated millisecond pulsar. Glendenning et al. (1997) found that a drastic softening of the equation of state, e.g., by a phase transition to pure quark matter, or Bose condensation at a critical angular velocity, alters the stellar moment of inertia and shows up in a backbending moment of inertia as a function of the rotating frequency.

It is complicated to deal with the rotation of a neutron star with an arbitrary angular velocity in the framework of general relativity. If the angular velocity Ω is small compared to the critical value $\Omega_0 = \sqrt{4\pi G\rho_0}$, where G is the gravitational constant and ρ_0 is the mass density at the center of the star, then the stellar rotation is treated as a perturbation to the metric of the nonrotation case. This is the essential idea of the Hartle's perturbation theory (Hartle 1967; Hartle & Thorne 1968; Chubarian et al. 2000). Glendenning & Weber (1992a, 1992b) improved Hartle's method by considering the effects of centrifugal stretching and frame dragging. On the other hand, there have existed several independent numerical codes for obtaining accurate models of rotating neutron stars in fully general relativity (Stergioulas 1998). Recent work has been based on the BI code (Butterworth & Ipser 1976), the KEH code (Komatsu, Eriguchi, & Hachisu 1989a, 1989b), the WMSHR code (Wu et al. 1991), and the BGSM spectral method (Bonazzola et al. 1993). Using a large number of models and EOSs, an extensive direct comparison of the BGSM, SF (Stergioulas & Friedman 1995) which is based on KEH code, and the original KEH codes were investigated. More than 20 different quantities for each model are compared and the relative differences range from 0.1% to 0.01% or better for smooth EOSs (Nozawa et al. 1998). It is evident that the accuracy of these codes is better than 0.1% to 0.01%.

Glendenning et al. (1997) used their own improved Hartle's method to investigate observational characteristics of phase transition in the interior of a rapidly rotating neutron star, but their method shows large discrepancies

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compared to corresponding models computed with fully rotating schemes. Thus, Hartle's formalism is not suitable for computing models of rapidly rotating relativistic stars with sufficient accuracy (Salgado et al. 1994). Using Stergioulas & Friedman's KEH code (Stergioulas & Friedman 1995; Komatsu, Eriguchi, & Hachisu 1989a, 1989b; Cook, Shapiro, & Teukolsky 1992, 1994a, 1994b), which is available as a public domain code, we reconsider the problem in this paper.

2. QUALITATIVE ANALYSIS

During spin-down of a millisecond neutron star, the central density increases with decreasing centrifugal force; therefore, phase transition from the relatively incompressible nuclear matter to the highly compressible quark matter occurs in the stellar core. After the pure quark matter dominates in the core, the star would contract significantly and its moment of inertia would decrease sharply, which is the common sequence of phase transition from confined to deconfined matter (Glendenning et al. 1997; Chubarian et al. 2000). Observationally, this kind of phase transition might signal at the jump behavior of the brake index of the pulsar. It should be noted that although the neutron star must rotate initially at millisecond intervals before slowing down to the phase transition period (~ 4 ms) in order for its central density to be strongly sensitive to the rotation rate, its initial period need not be close to the break-up period ($\lesssim 1$ ms). However, whether or not there exists an era of spin-up due to phase transition depends upon the characteristics of the nuclear matter and quark matter in the interior. In the original work of Glendenning et al. (1997), nuclear matter consisting of the octet of baryons was described in the relativistic mean-field theory (RMF), and the interaction between baryons in matter through three meson fields (the isoscalar-scalar meson σ , isoscalar-vector meson ω , and isovector-vector meson ρ) and quark matter consisting of three-flavor (u, d, s) quarks was described in the MIT bag model. Constrained by the conservation of total baryon number and electric charge, mixed phase of nuclear matter and quark matter exists in the range of $0.245\text{--}0.859\text{ fm}^{-3}$. Nuclear matter that contains only nucleons, σ , and ω mesons was described in the RMF. Quark matter that contains only two-flavor quarks was described in the MIT model in a recent work of Chubarian et al. (2000) which did not show the backbending of $I(\Omega)$ as a function of Ω .

In our opinion, even though the equations of state of hybrid stars are given, the backbending is also dependent on the properties of the crust and is especially sensitive to the transition density n_t and pressure P_t between the solid crust and the liquid core. The reason is sketched in Figure 1. The solid line represents the result with a backbending of moment of inertia in the region between points a and b (see also Fig. 2 of Glendenning et al. 1997) in a certain model (model A), while the dotted line shows the result of the other model (B) in which spin-up marginally does not exist; that is, the angular velocity at point b' marginally equals that at point a' , $\Omega_{b'} = \Omega_{a'}$. On the other hand, $\Omega_{a'} \gtrsim \Omega_a$ (see below). Model A and Model B differ in their EOSs at subnuclear densities. The disappearance of the spin-up is mainly due to the increase of the moment of inertia contributed by the stellar crust after the pure quark matter dominates the interior of neutron star. After the domination of the deconfined matter, the central pressure is larger than before the phase

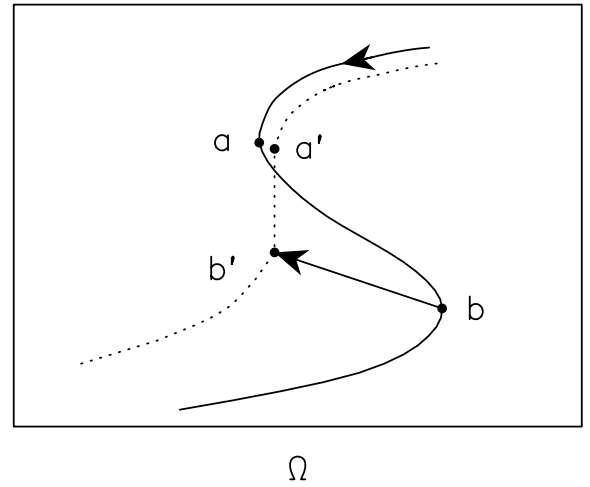


FIG. 1.—Sketch of the evolution of moment of inertia as a function of rotational angular velocity from large to small $I(\Omega)$. The solid line shows the result of a model (A) in which the spin-up of the pulsar takes place (between points a and b). The dotted line shows the result of the other model (B) in which the spin-up marginally does not occur due to the variation of the moment of inertia. Model A and Model B differ in their EOSs at subnuclear densities.

transition, and the difference between the moment of inertia of the cores in different models can be overlooked. Hence, $\delta I(\Omega_{b,b'}) = \delta I_{\text{core}}(\Omega_{b,b'}) + \delta I_{\text{crust}}(\Omega_{b,b'}) \simeq \delta I_{\text{crust}}(\Omega_{b,b'}) > 0$; here δ denotes the differences of the quantities between model B and model A. Before the domination of the deconfined matter, the central pressure is far less than that of the static star and is closer to the transition pressure P_t , so the difference of the moment of inertia of the cores comes near to and even goes beyond that of the crusts, i.e., $-\delta I_{\text{core}}(\Omega_{a,a'}) \gtrsim \delta I_{\text{crust}}(\Omega_{a,a'})$. Thus, $\delta I(\Omega_{a,a'}) \lesssim 0$.

Keeping the angular momentum at b' equal to that at b , $I(\Omega_{b'})\Omega_{b'} = (I(\Omega_b) + \delta I(\Omega_{b,b'}))\Omega_{b'} = I(\Omega_b)\Omega_b$, point b would be moved to point b' . The moment of inertia, $\delta I(\Omega_{b,b'})$, which should be added to the total at point b is obtained as

$$\frac{\delta I(\Omega_{b,b'})}{I(\Omega_b)} = \frac{\Omega_b - \Omega_{b'}}{\Omega_{b'}} \lesssim \frac{\Delta\Omega_{ba}}{\Omega_a}. \quad (1)$$

According to Glendenning et al. (1997), $\Omega_b \sim 1376\text{ rad s}^{-1}$, and $\Delta\Omega_{ba} \sim 15\text{ rad per second}$. Therefore,

$$\frac{\delta I(\Omega_{b,b'})}{I(\Omega_b)} \simeq 1.1\%. \quad (2)$$

As in the above discussions, $\Omega_{a'} \gtrsim \Omega_a$, because $\delta I(\Omega_{a,a'}) \lesssim 0$. Consequently, if the total moment of inertia $I(\Omega)$ in other models differs by 1.1% from that in the model of Glendenning et al. (1997), the backbending phenomena of Glendenning et al. (1997) might disappear. The effects of the crust are estimated as follows: The ratio of the moment of inertia of the crust to the total one at point b or b' is approximately written as (Lattimer & Prakash 2001)

$$\begin{aligned} \frac{I_{\text{crust}}}{I}(\Omega_{b,b'}) &\simeq \frac{I_{\text{crust}}}{I}(\Omega = 0) \\ &\simeq \frac{28\pi P_t R^3}{3Mc^2} \frac{(1 - 1.67\beta - 0.6\beta^2)}{\beta} \\ &\times \left[1 + \frac{2P_t(1 + 5\beta - 14\beta^2)}{n_t m_b c^2 \beta^2} \right]^{-1}, \quad (3) \end{aligned}$$

where M is the gravitational mass, R is the corresponding radius for the static star, $\beta \equiv GM/Rc^2$, and m_b denotes the baryon mass. The above approximation is good, because the pure quark core dominates in the interior when $0 \leq \Omega \leq \Omega_{b,b'}$ and the gross properties of the neutron star are insensitive to Ω . For instance, for a hybrid star with $M = 1.413 M_\odot$, $R = 11.15$ km, we choose $n_t = 0.0725 \text{ fm}^{-3}$, $P_t = 0.229 \text{ MeV fm}^{-3}$ for model A, and $n_t = 0.0957 \text{ fm}^{-3}$, $P_t = 0.392 \text{ MeV fm}^{-3}$ for model B, thus $\delta I_{\Omega,b,b'}/I \sim 1.0\%$. Observationally, the glitch constraint $I_{\text{crust}}/I \geq 1.4\%$ (Link, Epstein, & Lattimer 1999). It is evident that the equation of state for subnuclear matter could provide a sufficient moment of inertia to affect the timing structure of pulsar spin-down in some situations.

3. NUMERICAL RESULTS

In our calculation, the physical state of the interior neutron star matter is the same as that described in Glendenning et al. (1997). For the description of neutron star matter at subnuclear densities, we make three comparisons (shown in Fig. 2). The first is labeled as BPS. In this scenario, at the “low” densities $n_b < 0.01 \text{ fm}^{-3}$, where n_b denotes the baryon number density, we choose the EOS of BPS (Baym, Pethick, & Sutherland 1971; Baym, Bethe, & Pethick 1971) which is matched to the equation of state of nuclear matter described in the relativistic mean field theory at high densities $n_b \geq 0.01 \text{ fm}^{-3}$. This example is the same as that in the monograph of Glendenning (1997), but it should be noted that this kind of example might be unrealistic. Compared to the other choices, the resulting EOS is much softer, in the range of $0.01 \text{ fm}^{-3} < n_b < 0.1 \text{ fm}^{-3}$. The second scenario is labeled FPS in Figure 2. In this situation, in the outer crust $n_b \leq 8.5 \times 10^{-4} \text{ fm}^{-3}$, the EOS of BPS (Baym, Pethick, & Sutherland 1971) is chosen, while in the inner crust $8.5 \times 10^{-4} \text{ fm}^{-3} < n_b \leq 0.0957 \text{ fm}^{-3}$ the EOS of FPS is used (Lorenz, Ravenhall, & Pethick 1993). At the inner boundary of the solid crust, the pressures of the subnuclear matter and the liquid core are smoothly

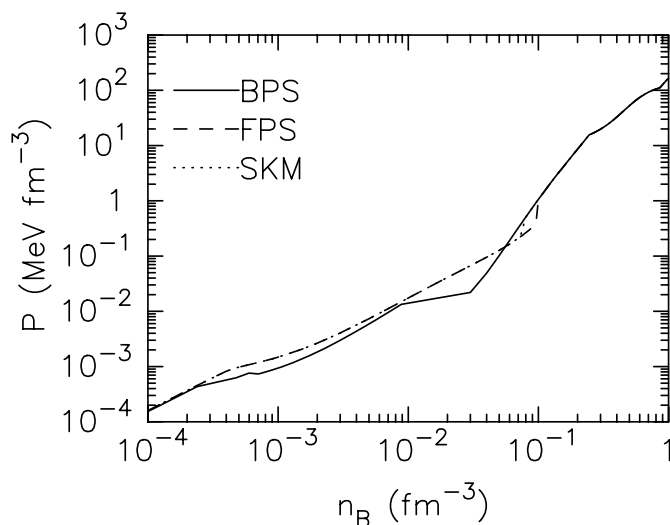


FIG. 2.—EOSs of neutron star matter in the inner crust and the liquid core. The difference between these EOSs is only in the inner crust consisting of the subnuclear matter. The solid line corresponds to the BPS model, the dashed line to the FPS model, and the dotted line to the SKM model. The difference between SKM and FPS models is mainly due to the uncertainty of the transition density n_t .

matched to each other in a narrow range of the baryon number density. The third situation is labeled as SKM in Figure 2. The difference between SKM and FPS models is mainly due to the uncertainty of the transition density n_t . In the SKM model, $n_t = 0.0725 \text{ fm}^{-3}$ and the corresponding transition pressure $P_t = 0.229 \text{ MeV}$ (Krivine, Treiner, & Bohigas 1980). In the FPS model, $n_t = 0.0957 \text{ fm}^{-3}$ and $P_t = 0.392 \text{ MeV}$ (Lorenz, Ravenhall, & Pethick 1993). Given the EOS of a neutron star, Stergiou & Friedman’s (1995) KEH code which is named “rns” is applied to calculate the gross properties of a rapidly rotating neutron star.

Figure 3 shows the moment of inertia as a function of the rotating frequency. The phase transition from the mixed phase of nuclear matter and quark matter to pure quark matter takes place at two critical frequencies which are chosen as two representative cases: $\Omega_c = 1400 \text{ s}^{-1}$ and 1640 s^{-1} . It is evident that the moment of inertia $I(\Omega)$ drops sharply after the phase transition at a certain frequency, which is characteristic of this kind of phase transition. From this point, our result is consistent with that of Glendenning et al. (1997). The local magnifications of Figure 3 at two inflections are shown in Figure 4 ($\Omega_c = 1400 \text{ s}^{-1}$) and Figure 5 ($\Omega_c = 1640 \text{ s}^{-1}$), respectively. When phase transition occurs at a lower frequency (see Fig. 4), we do not see the backbending of $I(\Omega)$ as described in Glendenning et al. (1997); the difference might be due to discrepancies between Hartle’s perturbation methods and the models in the framework of fully general relativity. In the original Hartle’s theory, when the central density is given, the stellar moment of inertia is a constant and does not change with angular velocity! It is obvious that the original Hartle’s theory is inadequate here. Glendenning & Weber (1992a, 1992b) improved Hartle’s theory by including the effects of centrifugal stretching and frame dragging. Unfortunately, their improved Hartle’s model shows great discrepancies compared to corresponding models computed with fully rotating schemes (Salgado et al. 1994). As we mentioned above, the accuracy of the numerical codes we require here is 1% or better. Our results in Figure 4 also evidently show that the accuracy of Hartle’s method must be better than 1%. The “S” curve of $I(\Omega)$ as a function of Ω just appears in Figure 5 for BPS and SKM models. In Figure 5, $\Omega_b = 1593$

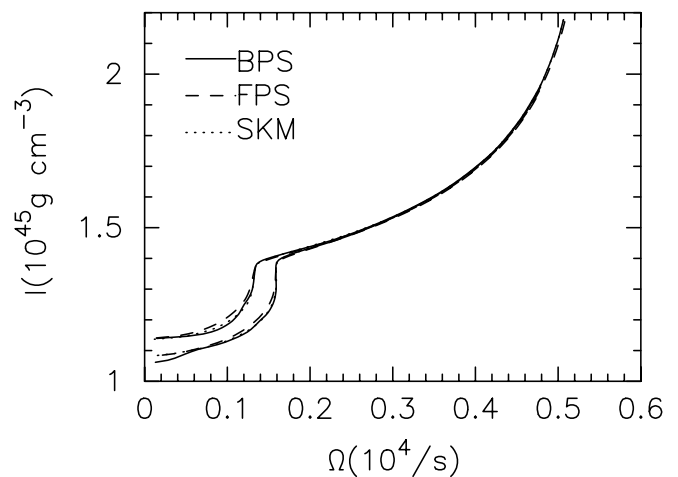


FIG. 3.—Moment of inertia $I(\Omega)$ vs. rotational angular velocity Ω . The pure quark matter appears at two critical frequencies: $\Omega_c = 1400$ and 1640 s^{-1} . The solid line corresponds to the BPS model, the dashed line to the FPS model, and the dotted line to the SKM model.

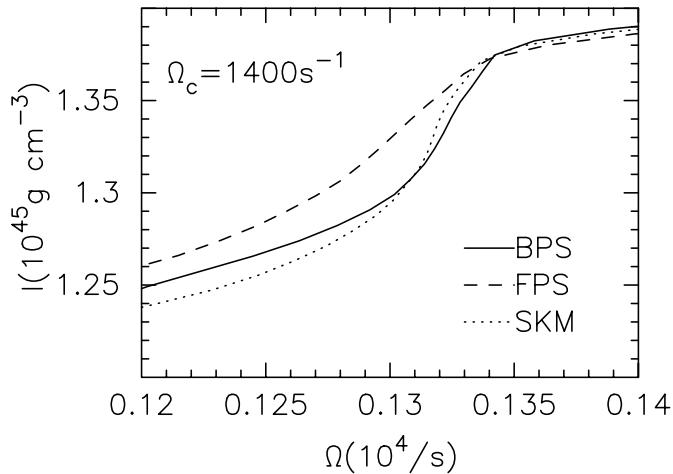


FIG. 4.—Same as Figure 3, but only the case when $\Omega_c = 1400 \text{ s}^{-1}$ is presented in a narrower range of Ω , at which the pure quark core grows up.

s^{-1} and $\Delta\Omega_{ba} = 2.4 \text{ s}^{-1}$ for the SKM model, thus $\Delta I(\Omega_b)/I(\Omega_b) \sim 0.15\%$, according to equation (1). The polytropic indices of the EOSs of the inner crust for SKM and FPS models are the same, but the difference between these two models is due to the different crust transition densities and therefore different transition pressures. According to equation (3), the contribution of the crust to the total moment of inertia is 2.57% in the SKM model and 1.6% in the FPS model, respectively. The difference in the total moment of inertia between these two models should be of the same order. Namely, in the FPS model, the crust contributes an additional 1% to the total moment of inertia, which is larger than the δI that needs to be added to stop the spin-up of the pulsar in the SKM model. The numerical results are consistent with those based on our qualitative analysis.

The distinctive decrease of the moment of inertia after phase transition is reflected in the timing structure of the pulsar spin-down ($\Omega, \dot{\Omega}$); that is, the jump behavior of the braking index, $n(\Omega)$, of the pulsars. The braking index is defined as follows (Glendenning et al. 1997):

$$n(\Omega) = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} = 3 - \frac{3I'(\Omega)\Omega + I''(\Omega)\Omega^2}{2I(\Omega) + I'(\Omega)\Omega}, \quad (4)$$

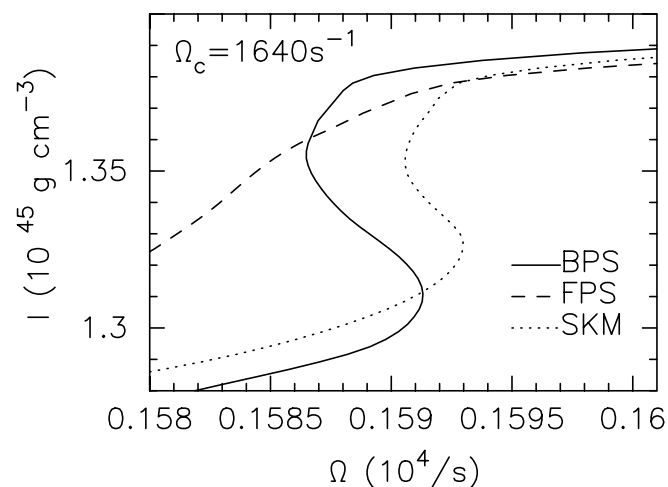


FIG. 5.—Same as Figure 4 for case $\Omega_c = 1640 \text{ s}^{-1}$

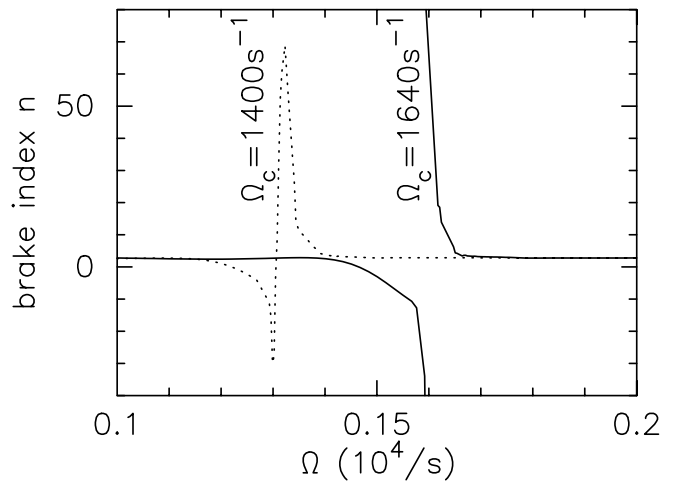


FIG. 6.—The braking index $n(\Omega)$ vs. rotational frequency Ω for the SKM model at two critical frequencies, $\Omega_c = 1400$ and 1640 s^{-1} when the phase transition to pure quark matter occurs.

where $I' \equiv dI/d\Omega$ and $I'' \equiv d^2I/d\Omega^2$. The constant 3 in the above equation is due to magnetic dipole radiation. We plot the braking index as a function of angular velocity for the SKM model in Figure 6. It shows clearly that the braking index $n(\Omega)$ comes to departure significantly from the canonical value 3 after emergence of the pure quark core. When phase transition happens at lower frequency, $\Omega_c = 1400 \text{ s}^{-1}$, there is not an era of spin-up, which means that $n(\Omega)$ reaches a very large value but is finite. However, when the phase transition happens at higher frequency, $\Omega_c = 1640 \text{ s}^{-1}$, there is an era of spin-up of the pulsar, which shows that $n(\Omega)$ switches from $+\infty$ to $-\infty$ at two turning points (see Fig. 1, Fig. 5). The braking indices of other models, which are not shown here, are similar to those of the SKM model.

Finally, it is interesting to estimate the fraction of the lifetime of pulsar which is spent in the spin-up phase. The duration of backbending is $\Delta T \simeq -\Delta\Omega_{ba}/\dot{\Omega}$, where $\Delta\Omega_{ba}$ is the frequency interval of spin-up phase (see § 3). For instance, $\Delta\Omega_{ba} = 2.4 \text{ s}^{-1}$ in the SKM model, and a typical period derivative of millisecond pulsars $\dot{P} \sim 10^{-19} \text{ s}$ per second is taken, then $\Delta T \sim 2 \times 10^6 \text{ yr}$. The dipole age of the pulsars is about 10^9 yr . Thus, the spin-up would last for 0.1% of a typical active pulsar lifetime. At present, more than 1000 isolated pulsars have been discovered and only one of them might be in an era of spin-up ($\dot{P} < 0$). In a word, the probability of finding the phenomenon of “backbending” is presently small, but this effect might be detected in the future with an increase in the number of known pulsars—especially of millisecond pulsars.

4. SUMMARY AND DISCUSSION

We have investigated the change of the internal structure of a hybrid star due to the decrease of its rotating velocity. Our calculation is based on Stergioulas & Friedman’s (1995) KEH code which is developed to compute the gross properties of rapidly rotating neutron stars in fully general relativity. Though the softening of the EOS due to phase transition in the core of a neutron star must lead to a drastic decrease of its moment of inertia, it was found that the so-called analogous phenomenon of “backbending” is mainly dependent on the EOS of the interior neutron star matter (Chubarian et al. 2000). We have found that even

when the EOS of the stellar core is given, the physical state of the crust, including its EOS, its transition density, n_t , and pressure, P_t , significantly affect the evolution and dynamics of the star in some situations, such as when the spin-up of the pulsar takes place during the rotation of the compact star slowing down. The reason is whether or not the pulsar spins up depends on about 1% or even less of a change in the magnitude of the stellar moment of inertia. It is well known that glitches of pulsars provide a lower limit on I_{crust}/I and further provide a lower limit on the transition density of the inner crust (Link, Epstein, & Van Riper 1992). On the other hand, in our opinion, if there is an era of spin-up after phase transition, the moment of inertia of the crust should be not so large, which might provide an upper limit on I_{crust}/I .

It is further noted that the accuracy of some perturbative methods of computing the structure of rotating neutron stars might exceed 1% or even more compared to fully

rotating scenarios. Therefore, in order to understand the physical processes that happen possibly in a millisecond pulsar, the accurate numerical codes should be applied. It should also be pointed out that if the millisecond pulsars are formed by accretion-induced spin-up of a neutron star, the accreted material may be a source of variability in the moment of inertia from star to star. However, specific effects will be verified by careful calculation in the future.

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