

Quantum spin field effect transistor

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We propose, theoretically, a type of quantum field effect transistor that operates purely on the flow of spin current in the absence of charge current. This spin field effect transistor (SFET) is constructed without magnetic material, but with the help of a spin flip mechanism provided by a rotating external magnetic field. The SFET generates a *constant* instantaneous spin current that is sensitively controllable by a gate voltage as well as by the frequency and strength of the rotating field. The characteristics of a carbon nanotube based SFET is provided as an example.

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One of the most important possibilities of nanoelectronics¹ is the hope of using spin—in addition to charge—for nonlinear electronic device applications.² So far, progress has been achieved in several areas of spintronics such as the applications of giant magnetoresistive effect,³ the understanding of material properties of magnetic semiconductors,⁴ the improvements of spin injection across a magnetic-nonmagnetic interface,⁵ and optical manipulation of spin degrees of freedom.⁶ On the other hand, despite the fact that it is already more than ten years since the proposal⁷ of field effect transistor (FET) operation on spin-polarized charge current, the spin-FET (SFET) has been an elusive system up to now. The overwhelming majority of actual spintronics devices and proposals up to now are hybrid systems which involve both magnetic and nonmagnetic materials.² Due to differences in chemical bonding and structural properties, these hybrid materials are rather challenging to use. This, together with several other physical factors related to spin transport, has limited a rapid development of nonlinear spintronic devices such as the SFET.

In this paper, we take a different direction by theoretically examining the possibility of SFET operation without involving magnetic materials, and we exploit such a SFET which operates purely on *spin current*. This SFET turns out to be realizable—as we predict, in quantum coherent nanostructures, in the presence of a rotating external magnetic field. Importantly, the rotating field induces a *time-independent* dc spin current, and at the same time generates no charge current. The magnitude of the spin current is critically tunable by a gate voltage so that SFET operation is achieved. The physical principle of our SFET is due to a spin flip mechanism provided by the field. Because no magnetic material is involved in our SFET, any problem that relates to spin injection across a magnetic-nonmagnetic interface is bypassed. Moreover, because there is no charge current involved, our SFET will be less affected by problems of heat dissipation. Since many nanostructures, such as a carbon nanotube, have long spin coherent lengths,⁸ our proposed quantum SFET should be experimentally realizable. To provide a concrete numerical example, we predict the transport characteristics of an all-nanotube based SFET.

Consider a three-probe nonmagnetic device shown in the left inset of Fig. 1 which consists of a scattering region Ohmic-contacted by two leads, while a third lead is a metallic gate capacitively coupled to the scattering region. Here we used a section of an armchair carbon nanotube as the scattering region, but in general it can be a quantum dot, a quantum well, or other mesoscopic conductors. The system can be $2d$ or $3d$. The Hamiltonian of this SFET is ($\hbar = 1$)

$$H = \sum_{k,\sigma,\alpha=L,R} \epsilon_k C_{k\alpha\sigma}^+ C_{k\alpha\sigma} + \sum_{\sigma} [\epsilon + \sigma B_0 \cos \theta] d_{\sigma}^+ d_{\sigma} + H'(t) + \sum_{k,\sigma,\alpha=L,R} [T_{k\alpha} C_{k\alpha\sigma}^+ d_{\sigma} + \text{c.c.}] \quad (1)$$

where $H'(t)$ is the off diagonal part (in spin space) of the Hamiltonian,

$$H'(t) = \gamma [\exp(-i\omega t) d_{\uparrow}^+ d_{\downarrow} + \exp(i\omega t) d_{\downarrow}^+ d_{\uparrow}], \quad (2)$$

with $\gamma = B_0 \sin \theta$. Here, the first term stands for noninteracting electrons in the leads with $C_{k\alpha\sigma}^+$ the creation operators in

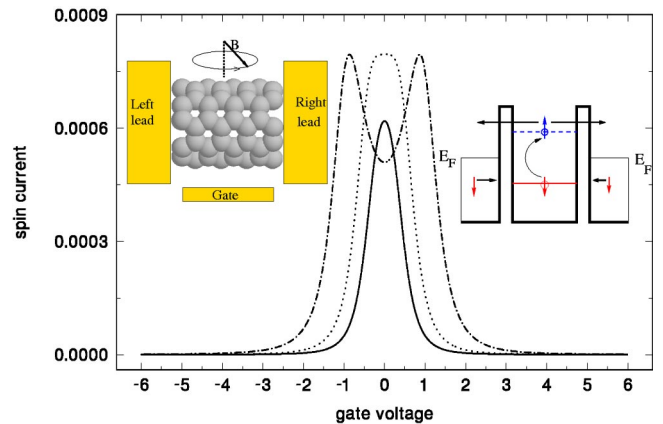


FIG. 1. The pumped spin current I_s vs the gate voltage for different $\gamma = 0.3$ (solid line), 0.5 (dotted line), and 1.0 (dashed line). Left inset: schematic plot of a nanotube SFET device. Right inset: the working principle of SFET. The energy unit in the calculation is 0.035 meV.

lead α . We have set the same chemical potential for both leads because a rotating magnetic field will generate a dc spin current without needing a bias voltage. Note that dc charge and spin currents can also be produced without bias using a device called a “quantum parametric pump” where two time-dependent external parameters are cyclically varied in time with a definite phase difference.^{9–11} The second term and $H'(t)$ correspond to the Hamiltonian of the scattering region which is subjected to a rotating magnetic field $\mathbf{B}(t) = B_o[\sin\theta\cos\omega t\mathbf{i} + \sin\theta\sin\omega t\mathbf{j} + \cos\theta\mathbf{k}]$ where B_o is the constant field strength. It is crucial that we use a rotating field (not an oscillating field). For example, a counterclockwise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to absorb a photon and flip to spin-down. This is crucial for the operation of a SFET (see below). Here, for simplicity, we have assumed $\mathbf{B}(t) \neq 0$ only inside the scattering region. This is, however, not a strict condition: as long as $\mathbf{B}(t) \neq 0$ within the spin coherence length and diminishes outside this length scale, the SFET will work. The scattering region is characterized by an energy level $\epsilon = \epsilon_o - qV_g$ which can be controlled by the gate voltage V_g (left inset of Fig. 1). We have only included the coupling between a magnetic field and the spin degrees of freedom. It is, however, not difficult to confirm that the orbital degrees of freedom do not contribute to the current in the presence of a time varying magnetic field for the model above. This is because, in the presence of our magnetic field, the hopping matrix element between sites i and j , t_{ij} , in the tight binding description, will be modified by a phase factor $\exp[i\phi_{ij}]$ with $\phi_{ij} = \mathbf{A} \cdot (\mathbf{r}_i - \mathbf{r}_j)$. But ϕ_{ij} due to our rotating magnetic field in the x - y plane is simply zero, therefore the orbital parameter t_{ij} is not affected by the rotating field. If we allow $\mathbf{B}(t)$ to extend into the leads (but still within the coherence length), some orbital effect may occur but we do not consider this situation in the present work. The third term in Eq. (1) denotes coupling between the scattering region and lead α with coupling matrix elements $T_{k\alpha}$. In the following we solve the transport properties (charge and spin currents) of the model in both adiabatic and nonadiabatic regimes using the standard Keldysh nonequilibrium Green’s function (NEGF) technique.^{12,13}

Adiabatic regime. For this regime ω is small, and the charge with spin σ transported from lead α per unit time is given by¹⁴

$$dQ_{\alpha\sigma}(t)/dt = q \int \frac{dE}{2\pi} (-\partial_{Ef}) [\Gamma_{\alpha} \mathbf{G}^r(t) \mathbf{\Delta} \mathbf{G}^a(t)]_{\sigma\sigma}, \quad (3)$$

where $\mathbf{G}^r(t)$, $\mathbf{G}^a(t)$ are the retarded and advanced Green’s functions. In the adiabatic limit,

$$\mathbf{G}^r(t) = \frac{1}{z} \begin{pmatrix} E - \epsilon_2 & \gamma e^{-i\omega t} \\ \gamma e^{i\omega t} & E - \epsilon_1 \end{pmatrix}, \quad (4)$$

where $z \equiv (E - \epsilon_1)(E - \epsilon_2) - \gamma^2$, $\epsilon_{1,2} \equiv \epsilon \pm B_o \cos\theta - i\Gamma/2$, and $\Gamma = \sum_{\alpha} \Gamma_{\alpha}$ is the linewidth function. We will apply the

wideband limit so that Γ is independent of energy. In Eq. (3), quantity $\mathbf{\Delta} \equiv dH'/dt$ where H' is the 2×2 matrix in spin space given by Eq. (2):

$$H' = \begin{pmatrix} 0 & \gamma e^{-i\omega t} \\ \gamma e^{i\omega t} & 0 \end{pmatrix}. \quad (5)$$

Using Eqs. (3), (5), and (4), the instantaneous electric current is found to be (Fermi energy and temperature are set to zero)

$$\frac{dQ_{\alpha\uparrow}}{dt} = -\frac{dQ_{\alpha\downarrow}}{dt} = \frac{q\omega\Gamma_{\alpha}\Gamma\gamma^2}{(2\pi|\epsilon_1\epsilon_2 - \gamma^2|^2)}. \quad (6)$$

The spin current is defined as ($\hbar = 1$) $I_s = (I_{\uparrow} - I_{\downarrow})/(2q) = dQ_{\uparrow}/dt/q$, and the electric current $I = I_{\uparrow} + I_{\downarrow} = 0$. The physics of this result is depicted in the right inset of Fig. 1. Due to Zeeman splitting, the energy level ϵ is split into $\epsilon_{\downarrow} = \epsilon - B_o \cos\theta$ and $\epsilon_{\uparrow} = \epsilon + B_o \cos\theta$. A spin-down electron can tunnel into ϵ_{\downarrow} from the left lead, and due to the rotating field it *absorbs* a photon and transits to the ϵ_{\uparrow} level where its spin is flipped. This spin-up electron then tunnels out of the scattering region with certain probabilities to the left and right leads. Exactly the same happens to spin-down electrons in the right lead, and the average outcome is that there is a spin-up electron flowing away from the scattering region. This way, with spin-down electrons flowing toward the scattering region and an equal number of spin-up electrons flowing away from it [see Eq. (6)], a spin current is established without charge current. An originally spin-up electron in the lead may also enter the scattering region, but due to the rotation direction of $\mathbf{B}(t)$, it can only *emit* a photon and go down in energy. Since the levels below ϵ_{\downarrow} are all filled, this process practically does not occur so that incoming spin-up electrons do not contribute to spin current. If the rotation direction and z component of $\mathbf{B}(t)$ are reversed, the flow of spin current will also reverse.

The maximum spin current in the adiabatic regime is obtained by setting $\theta = \pi/2$ and $\Gamma_{\alpha} = \gamma = \Gamma/2$; we have

$$I_{s\alpha} = \frac{\omega}{4\pi} \frac{\Gamma^4/4}{\epsilon^4 + \Gamma^4/4}. \quad (7)$$

This line shape—involving the fourth power of the relevant quantities, is ideal for SFET operation: $I_{s\alpha}$ is sensitive to the energy level position which is controlled by the gate voltage. For instance, at resonance $\epsilon = 0$ the spin current reaches its maximum value $\omega/4\pi$. However, when ϵ is varied by V_g to $10(\Gamma/\sqrt{2})$, the spin current is reduced by a factor of 10^4 . Since $I_s = s/\tau$, with $\tau = 2\pi/\omega$ being the period of the rotating magnetic field, we therefore conclude that, at resonance, the SFET outputs exactly one spin through the left or right lead in one field rotation. This quantization of the spin is substantially easier to realize than that of the charge^{15,16} in a parametric charge pump. If there is only one lead connected to the scattering region, the spin current is given by Eq. (7) multiplied by a factor of 2: in this case the SFET can be viewed as a nonmagnetic version of spin battery.¹⁷

Nonadiabatic regime. The electric and spin current beyond the adiabatic approximation can be calculated exactly using NEGF. It is convenient to define the particle current operator in spin space:

$$\hat{J}_{\alpha,\sigma\sigma'} = -i \sum_k [T_{k\alpha} C_{k\alpha\sigma}^+ d_{\sigma'} - T_{k\alpha}^* d_{\sigma}^+ C_{k\alpha\sigma'}]. \quad (8)$$

Then the electric current operator is $\hat{I}_{\alpha q} = q \sum_{\sigma} \hat{J}_{\alpha,\sigma\sigma}$ and the spin current operator is $\mathbf{I}_{s\alpha} = \sum_{\sigma\sigma'} \hat{J}_{\alpha,\sigma\sigma'} \mathbf{s}_{\sigma\sigma'}$ where $\mathbf{s} = \boldsymbol{\sigma}/2$. From this we compute particle current

$$J_{\alpha\sigma\sigma'}(t) \equiv \langle \hat{J}_{\alpha,\sigma\sigma'}(t) \rangle \\ = - \sum_k [T_{k\alpha} G_{d\sigma,k\alpha\sigma'}^<(t,t) - T_{k\alpha}^* G_{k\alpha\sigma',d\sigma}^<(t,t)] \quad (9)$$

where the NEGFs are defined as $G_{d\sigma,k\alpha\sigma'}^<(t,t') = i \langle C_{k\alpha\sigma'}^+(t') d_{\sigma}(t) \rangle$, $G_{k\alpha\sigma',d\sigma}^<(t,t') = i \langle d_{\sigma'}^+(t') C_{k\alpha\sigma}(t) \rangle$. They are calculated by the Keldysh equation $\mathbf{G}^< = \mathbf{G}^r \Sigma^< \mathbf{G}^a$ in standard fashion.^{12,13} Therefore, the transport problem is reduced to the calculation of the retarded Green's function $G_{\sigma\sigma'}^r(t,t')$.

In general, a perturbation theory is needed to solve a time-dependent problem. Fortunately, for the time-dependent Hamiltonian considered here, $G_{\sigma\sigma'}^r(t,t')$ can be solved *exactly* as follows. It is simple to obtain the retarded Green's function for the diagonal part (in spin space) of Hamiltonian (1):

$$\mathbf{G}^{0r}(t-t') = -i \theta(t-t') \begin{pmatrix} e^{-i\epsilon_1(t-t')} & 0 \\ 0 & e^{-i\epsilon_2(t-t')} \end{pmatrix}.$$

The full Green's function of Hamiltonian (1) is then calculated by the Dyson equation in spin space,

$$\mathbf{G}^r(t,t') = \mathbf{G}^{0r}(t-t') + \int dt_x \mathbf{G}^{0r}(t-t_x) H'(t_x) \mathbf{G}^{0r}(t_x-t) \\ + \dots,$$

where H' is given by Eq. (5). After applying the double-time Fourier transform, the Dyson equation can be summed up exactly to obtain the exact Green's function of model (1),

$$G_{\sigma\sigma}^r(E,E') = \frac{2\pi \delta(E-E') G_{\sigma\sigma}^{0r}(E)}{1 - \gamma^2 g(E)},$$

$$G_{\sigma\sigma'}^r(E,E') = 2\pi \delta(E + \bar{\sigma}\omega - E') \frac{\gamma g(E)}{1 - \gamma^2 g(E)},$$

where $g(E) \equiv G_{\sigma\sigma}^{0r}(E) G_{\sigma\sigma}^{0r}(E + \bar{\sigma}\omega)$, $\bar{\sigma} = -\sigma$, and $\sigma = (\uparrow\downarrow) = \pm 1$.

Using these relations, it is straightforward to obtain the particle current from Eq. (9),

$$J_{L\uparrow\uparrow} = -J_{L\downarrow\downarrow} = - \int \frac{dE}{2\pi} \Gamma_L \Gamma [f(E) - f(E_-)] \\ \times \frac{\gamma^2 |G_{\uparrow\uparrow}^{0r}(E)|^2 |G_{\downarrow\downarrow}^{0r}(E_-)|^2}{|1 - \gamma^2 G_{\uparrow\uparrow}^{0r}(E) G_{\downarrow\downarrow}^{0r}(E_-)|^2}, \quad (10)$$

and $J_{L\uparrow\downarrow} = 0$, where $E_- \equiv E - \omega$. This result allows us to conclude that the charge current is still identically zero while the spin current is given by

$$\mathbf{I}_{sL} = J_{L\uparrow\uparrow} \mathbf{k}, \quad (11)$$

which is independent of time. These qualitative features are the same as those of the adiabatic limit discussed above. However, the nonadiabatic result [Eq. (10)] involves processes with energies $E \pm \omega$, as shown by the arguments of the Green's functions. This indicates that in the general nonadiabatic situation, many *single* photon processes are participating the operation of the SFET device. Furthermore, other spin-independent scattering processes can be easily included in the result Eq. (10): they only modify the Green's function $G_{\sigma\sigma}^{0r}$.

Nanotube SFET. We now apply the general principle discussed above to a (5,5) armchair single wall carbon nanotube (CNT) with 200 unit cells which is contacted by two leads and gated by a third (left inset of Fig. 1). For simplicity, the CNT is modeled with the nearest-neighbor π -orbital tight-binding model with bond potential $V_{pp\pi} = -2.75$ eV for the carbon atoms. This model is known to give a reasonable, qualitative description of the electronic and transport properties of carbon nanotubes.¹⁸ Using Eq. (10) the spin current flowing out of the CNT SFET in the adiabatic regime can be written as $I_s = (\omega/4\pi)T$, where

$$T = \frac{\Gamma^2 \gamma^2}{(\epsilon^2 + \Gamma^2/4 - \gamma^2)^2 + \Gamma^2 \gamma^2}. \quad (12)$$

Clearly, if $\gamma \leq \Gamma/2$, there is only one peak with $T \leq 1$. If $\gamma > \Gamma/2$, there are two peaks with $T = 1$. It is interesting to note that Eq. (12) has the form as that of Andreev reflection coefficient in the presence of superconducting lead (NS system).¹⁹ Figure 1 shows the spin current I_s versus the gate voltage V_g for different γ with $\omega = 0.01$ (corresponds to 86 MHz in our units) and $\theta = 88^\circ$. Here $\gamma = 0.1$ corresponds to $B = 0.06$ Tesla. Very similar results are obtained for other θ . The SFET operation is clearly seen: I_s increases from practically zero to large values under the control of V_g . Figure 2 displays the spin current versus frequency using the nonadiabatic result Eq. (10), with $\theta = 50^\circ$, $\gamma = 0.5$, and $V_g = 0.0$. Finally the inset of Fig. 2 depicts spin current as a function of θ with $\omega = 0.01$, $\gamma = 0.5$, and $V_g = 0.0$. The spin current is rather substantial for a wide range of angles. These numerical results were obtained at zero temperature limit. The temperature scale is set by the linewidth parameter Γ . Therefore, if one wishes to achieve the maximum spin current, one sets $\Gamma = \gamma$ [Eq. (7)], and with $\Gamma = 0.035$ meV the temperature scale is 0.42 K which is achievable. A larger Γ allows a higher temperature scale, although making $\Gamma > \gamma$ will reduce the value of spin current.

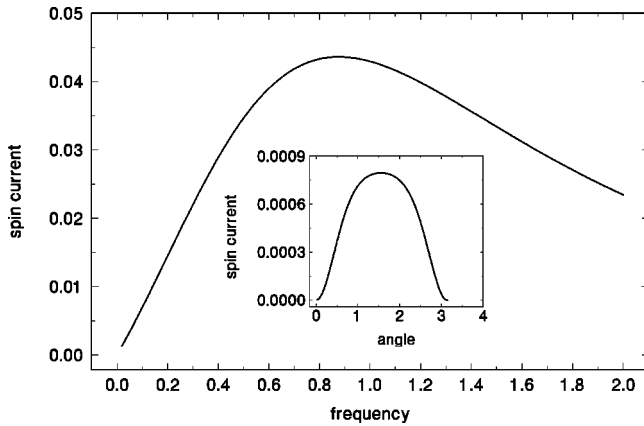


FIG. 2. I_s vs frequency. Inset: I_s vs the angle θ .

In summary, we have demonstrated that a rotating magnetic field induces a spin current without a charge current, in coherent quantum conductors without needing magnetic material. The spin current is critically tunable through the control of a resonance level in the system by an external gate voltage, thereby generating a field effect transistor operation. The physics behind this phenomenon is the spin-flip mecha-

nism by the external field. Because spin current can be detected using an idea proposed by Hirsch,²⁰ the rotating frequency of the field needs not to be large, and the device structure is quite typical, we believe the SFET should be experimentally realizable. Finally, we briefly comment that the physical mechanism of our SFET can be viewed from another line of thought. As pointed out in Ref. 11, in a *quantum parametric charge pump*, the pumped charge per cycle is related to the Berry's phase.²¹ This argument can also be generalized to the case of spin current discussed here. In fact, using the spinor $|\Psi\rangle = \begin{pmatrix} s_{11} \\ s_{21} \end{pmatrix}$, with s_{ij} the scattering matrix, the output charge can be obtained¹¹ from the definition of Berry's phase $\gamma = \int_0^\tau \bar{\gamma}(t) dt$ where $\bar{\gamma}(t) = i\langle \Psi[\mathbf{R}(t)] | \dot{\Psi}[\mathbf{R}(t)] \rangle$, $\mathbf{R}(t)$ labels the slowly varying system parameters, and τ is the period of variation. Note that in the case of charge pumping, $\bar{\gamma}(t)$ corresponds to the instantaneous pumped charge. Setting $T_{k\alpha} = 0$ in Eq. (1), it is easy to verify that $\bar{\gamma}(t)$ (instantaneous phase) is independent of time.

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