Suboptimal Transmission of Orthogonal Space–Time Block Codes Over Correlated Distributed Antennas

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Abstract—This letter investigates optimal transmission of orthogonal space-time block codes (OSTBCs) over distributed antennas (DAs) in non-ergodic flat Rayleigh fading channels with transmit antenna correlations. A generalized DA topology is considered, where the DAs are grouped into some geographically dispersed ports within each of which the DAs are co-located. Assuming equal power allocation within each port, the outage probability is derived. We find that minimizing the outage probability only requires the feedback of the eigenvalues of the transmit correlation matrix at the transmitter. Since it is computationally intensive to minimize the outage probability, an antenna subset selection with suboptimal power allocation scheme is proposed, whose effectiveness has been demonstrated by numerical results.

Index Terms—Distributed antennas (DAs), orthogonal space–time block codes (OSTBCs), outage probability.

Fig. 1. $(n, l, q_1, \ldots, q_l, m)$ DA topology.

I. INTRODUCTION

INFORMATION theoretic results in [1] and [3] have demonstrated the capacity advantage of distributed antennas (DAs) over point-to-point multiple-input multiple-output (MIMO) channels because of the inherent macroscopic diversity and shortened average access distance. Recently, there have been growing interests in combining space–time codes with DAs. For example, spatial multiplexing schemes are developed in [7] to achieve a high spectral efficiency. When high diversity gain is required, orthogonal space-time block codes (OSTBCs) [5] can be utilized due to their decoding simplicity, e.g., the work in [8].

We focus in this letter on applying OSTBCs over DAs. Without channel state information (CSI) at the transmitter, equal power transmission of OSTBCs over geographically dispersed DAs may result in unacceptable performance because the DAs with severe large-scale fading may waste much power. Therefore, transmit power allocation to the DAs based on some CSI feedback is indispensable. Prior work in [8] has studied the outage probability of OSTBCs in Nakagami fading channels, but the optimal power allocation problem has not been solved. To tackle this problem, we consider a generalized DA topology, i.e., the DAs are grouped into some geographically dispersed ports where the DAs are co-located. This topology seems more practical since deploying fully scattered DAs is generally rather difficult in some scenarios. For simplicity of system implementation, we assume equal power allocation within each port and explore how to optimally allocate transmit power to different ports to minimize the outage probability in non-ergodic flat Rayleigh fading channels with transmit antenna correlations. The contribution of this letter includes the derivation of the outage probability in closed forms and an antenna subset selection with suboptimal power allocation scheme whose performance approaches optimal.

II. SYSTEM MODEL

Consider a single user $(n, l, q_1, \ldots, q_l, m)$ DA topology. As shown in Fig. 1, n DAs are grouped into l geographically dispersed ports with q_j co-located DAs in the jth port, with $\sum_{j=1}^{l} q_j = n$, and m mobile antennas are co-located. All the DAs have independent connections to the processing center that controls the transmit and receive signals of each DA. One example of this mechanism can be found in the distributed wireless communication system [4]. Generally, the macroscopic and microscopic fading experienced at different antenna ports are independent. However, transmit correlations are often observed within each port due to the DA placement or the geometry in the transmission scenarios. We focus on the analysis of transmit correlations and assume the receive antennas are uncorrelated.

At the processing center, OSTBCs' symbols with unit average power are generated by the space–time encoder and are multiplied by the power allocation matrix $\mathbf{P}$ before transmission. We assume the channel is non-ergodic and frequency flat. The received signal can be written as

$$\mathbf{y} = \sqrt{P}\mathbf{H}\mathbf{P}^{1/2}\mathbf{x} + \mathbf{n} = \sqrt{P}\mathbf{H}_w\mathbf{R}_t^{1/2}\mathbf{P}^{1/2}\mathbf{x} + \mathbf{n} \quad (1)$$

where $\mathbf{y}$ and $\mathbf{x}$ are receive and transmit vectors, $\mathbf{n}$ is an $m \times 1$ noise vector with independent identically distributed (i.i.d.) complex $N(0, \sigma^2)$ entries, and P is the total average transmit power. P/σ^2 is the transmit power to receive noise ratio (TSNR) and is denoted by ρ. The composite channel matrix $\mathbf{H}$ is expressed as $\mathbf{H}_w\mathbf{R}_t^{1/2}$ [6], where $\mathbf{H}_w$ is an $m \times n$ matrix with i.i.d. complex $N(0, 1)$ entries, and $\mathbf{R}_t = \mathrm{E}(\mathbf{H}^H\mathbf{H})/m$ is the $n \times n$ transmit correlation matrix with E{.} and superscript H denoting the expectation and Hermitian transpose of a matrix, respectively. Actually, $\mathbf{R}_t = diag(a_1\mathbf{R}_{t,1}, \ldots, a_l\mathbf{R}_{t,l})$, where $\mathbf{R}_{t,j}, j = 1, \ldots, l$ denotes the $q_j \times q_j$ normalized (with diagonal entries of 1) transmit correlation matrix of the jth port, which has $t_j, 1 \leq t_j \leq q_j$ positive eigenvalues $\lambda_{j,k}, k = 1, \ldots, t_j$, and a_j represents the corresponding large-scale fading (shadow fading and path loss). Note that $a_j\lambda_{j,k}, j = 1, \ldots, l$, $k = 1, \ldots, t_j$ are eigenvalues of $\mathbf{R}_t$, and we assume they are known at the transmitter through feedback. At the receiver, accurate CSI is assumed. Assuming equal power allocation within each port, the diagonal power allocation matrix $\mathbf{P}$ can be given as $\mathbf{P} = diag(w_1/q_1, \ldots, w_1/q_1, \ldots, w_l/q_l, \ldots, w_l/q_l)$, where $w_j, j = 1, \ldots, l$ denotes the power allocation weight to the jth port, with $\sum_{j=1}^{l} w_j = 1$. We define transmit antenna subset as any combination of DA ports and denote the possible $2^l - 1$ subsets by $A_1, \ldots, A_{2^l-1}$. For OSTBCs' transmission over subset $A_k(k = 1, \ldots, 2^l - 1)$, we assume only one rate r_k.

Outage capacity is a useful characterization for non-ergodic fading channels where channel matrix $\mathbf{H}$ is random but held constant for each use of the channel. For a given data rate R and TSNR level ρ, our target is to optimally select the antenna subset and allocate power to minimize the outage probability.

III. DERIVATION OF THE OUTAGE PROBABILITY

Following the procedure outlined in [2], for OSTBCs transmitted over subset A_l (assumed to include all l ports) with rate r_l and positive power allocation weights $w_1, \ldots, w_l$, the normalized capacity for a given channel realization can be expressed as

$$C = r_l \log_2(1+\eta) = r_l \log_2 \left(1 + \left\|\mathbf{H}_w\mathbf{R}_t^{1/2}\mathbf{P}^{1/2}\right\|_F^2 \rho\right) \quad (2)$$

where $\|.\|_F$ denotes the Frobenius norm, and η is the SNR at detection, which can be further derived as

$$\eta = \sum_{j=1}^{l} a_j w_j \rho/q_j \sum_{k=1}^{t_j} \lambda_{j,k} \sum_{z=1}^{m} |h_z|^2 = \sum_{j=1}^{l} \sum_{k=1}^{t_j} \eta_{j,k} \quad (3)$$

with i.i.d. complex $N(0,1)$ random variables $h_z, z = 1, \ldots m$. Note η consists of $\sum_{j=1}^{l} t_j$ independent chi-squared $\eta_{j,k}$ with $2m$ degrees of freedom, whose characteristic function (CF) $\Phi_{\eta_{j,k}}(s)$ is given as $\Phi_{\eta_{j,k}}(s) = \int_0^\infty f_{\eta_{j,k}}(x)e^{isx}dx = (1 - sia_j\lambda_{j,k}w_j\rho/q_j)^{-m}$, where $f_{\eta_{j,k}}(x)$ denotes the probability density function (PDF) of $\eta_{j,k}$. Assuming each $a_j\lambda_{j,k}w_j\rho/q_j$ differs, the CF of η can then be obtained and resolved into its partial fractions

$$\Phi_\eta(s) = \prod_{j=1}^{l} \prod_{k=1}^{t_j} \Phi_{\eta_{j,k}}(s)$$
$$= \sum_{j=1}^{l} \sum_{k=1}^{t_j} \sum_{d=1}^{m} q_{j,k,d}(1 - sia_j\lambda_{j,k}w_j\rho/q_j)^{-d} \quad (4)$$

where $q_{j,k,d}$ are derived by solving a system of linear equations

$$q_{j,k,d} = \frac{(-ia_j\lambda_{j,k}w_j\rho/q_j)^{-(m-d)}}{(m-d)!} \frac{\partial^{m-d}}{\partial s^{m-d}}$$
$$\times \left(\prod_{u=1,u\neq j}^{l} \prod_{v=1,v\neq k}^{t_j} (1-sia_u\lambda_{u,v}w_u\rho/q_u)^{-m}\right)_{s=q_j/ia_j\lambda_{j,k}w_j\rho}. \quad (5)$$

The PDF of η can therefore be obtained by inverse transformation to $\Phi_\eta(s)$

$$f_\eta(x) = \sum_{j=1}^{l} \sum_{k=1}^{t_j} \sum_{d=1}^{m} \frac{q_{j,k,d}}{\Gamma(d)} \left(\frac{a_j\lambda_{j,k}w_j\rho}{q_j}\right)^{-d} x^{d-1} e^{-\frac{xq_j}{a_j\lambda_{j,k}w_j\rho}} \quad (6)$$

with Gamma function $\Gamma(x)$. Finally, the outage probability P_o with a data rate R can be calculated

$$P_o = P(C < R) = P(\eta < 2^{R/r_l} - 1) = \int_0^{2^{R/r_l}-1} f_\eta(x)dx$$
$$= \sum_{j=1}^{l} \sum_{k=1}^{t_j} \sum_{d=1}^{m} \frac{q_{j,k,d}}{\Gamma(d)} \Gamma\left(d, \frac{(2^{R/r_l}-1)q_j}{a_j\lambda_{j,k}w_j\rho}\right) \quad (7)$$

where $\Gamma(a, x) = \int_0^x y^{a-1}e^{-y}dy$ is the lower incomplete Gamma function. For OSTBCs transmitted over any other subset, the outage probability can also be obtained. We observe that only with the eigenvalues of $\mathbf{R}_t$ at the transmitter, we can minimize the outage probability of each subset and then select the subset with the best outage performance for transmission. Since directly optimizing the outage probability is difficult, we resort to the following suboptimal scheme.

IV. ANTENNA SUBSET SELECTION WITH SUBOPTIMAL POWER ALLOCATION SCHEME

At different TSNR levels, the optimal transmission of OSTBCs may employ different subsets. Intuitively, at high TSNR levels, subset A_l is optimal due to its high diversity order, while at some lower TSNR levels, fewer ports may perform better

because DAs with severe large-scale fading may waste power. At TSNR level ρ, we assume subset A_l with positive weights $w_1, \ldots, w_l$ is optimal. The optimal outage probability P_{o,A_l}^{opt} can be written in terms of $\Phi_\eta(s)$

$$
\begin{aligned}
P_{o,A_l}^{opt} &= \int_0^{2^{R/r_l}-1} f_\eta(x)dx \\
&= \int_0^{2^{R/r_l}-1} \left(\int_{-\infty}^{+\infty} \Phi_\eta(s)e^{-isx}ds \right) dx \\
&= \int_0^{2^{R/r_l}-1} \int_{-\infty}^{+\infty} \prod_{j=1}^l \prod_{k=1}^{t_j} \left(\left(1 - \frac{sia_jw_j\rho\lambda_{j,k}}{q_j}\right)^{-m \sum_{j=1}^l t_j} \right. \\
&\qquad \left. \times e^{-isx} \right)^{\overline{\sum_{j=1}^l t_j}} dsdx. \quad (8)
\end{aligned}
$$

Since A_l is assumed optimal, the contribution of each DA port is similar. Denote $m \sum_{j=1}^l t_j$ by D, P_{o,A_l}^{opt} can be approximated by

$$
\begin{aligned}
P_{o,A_l}^{opt} &\approx \int_0^{2^{R/r_l}-1} \prod_{j=1}^l \prod_{k=1}^{t_j} \left(\int_{-\infty}^{+\infty} \left(1 - \frac{sia_j\lambda_{j,k}w_j\rho}{q_j}\right)^{-D} \right. \\
&\qquad \left. \times e^{-isx}ds \right)^{\frac{m}{D}} dx \\
&= \int_0^{2^{R/r_l}-1} \frac{x^{D-1}}{\Gamma(D)} \prod_{j=1}^l \prod_{k=1}^{t_j} \left(\left(\frac{a_j\lambda_{j,k}w_j\rho}{q_j}\right)^{-m} \right. \\
&\qquad \left. \times e^{-\frac{q_jx}{a_j\lambda_{j,k}w_j\rho}\frac{m}{D}} \right) dx = P_{o,A_l}^{Approx.}.
\end{aligned}
$$
$$(9)$$

Note $e^{-x} \le 1$ for $x \ge 0$, the upper bound of $P_{o,A_l}^{Approx.}$ is derived to be

$$
\begin{aligned}
P_{o,A_l}^{Approx.} &\le \left(\frac{2^{R/r_l}-1}{\rho}\right)^D \frac{D^{-1}}{\Gamma(D)} \\
&\quad \times \prod_{j=1}^l \prod_{k=1}^{t_j} \left(\frac{a_jw_j\lambda_{j,k}}{q_j}\right)^{-m} = P_{o,A_l}^{ub}. \quad (10)
\end{aligned}
$$

The suboptimal power allocation vector $\mathbf{w}^* = [w_1^*, \ldots, w_l^*]$ can be obtained by minimizing P_{o,A_l}^{ub}, thus reducing to the following optimization problem:

$$
w^* = \arg\max \prod_{j=1}^l (w_j)^{t_j}, \quad \text{subject to} \sum_{j=1}^l w_j = 1. \quad (11)
$$

Using the Lagrangian method, we obtain w_j^* as

$$
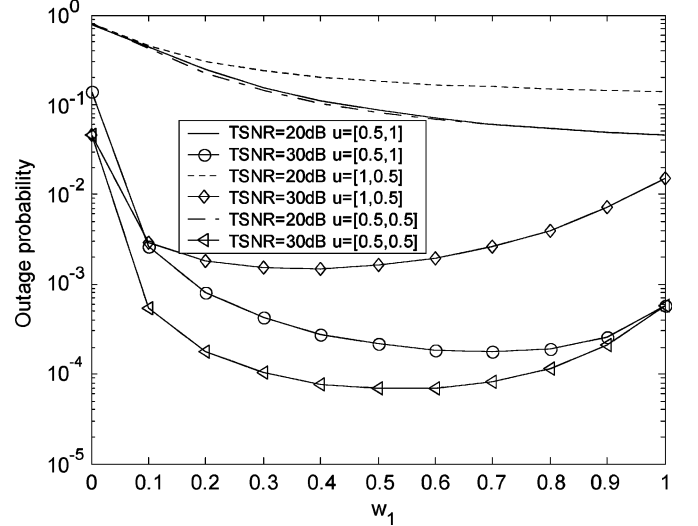w_j^* = t_j / \sum_{j=1}^l t_j, \ j = 1, \ldots, l. \quad (12)
$$

Fig. 2. Outage probability versus w_1 (power allocation weight to Port 1) of 3/4 rate OSTBCs.

Note that the above power allocation scheme is accurate in the high TSNR region because P_{o,A_l}^{ub} is tight when ρ is high enough. Substituting (12) into (7), the suboptimal outage probability P_{o,A_l}^{sub} can be calculated. Employing the above method to any optimal antenna subset, a similar suboptimal power allocation scheme can be easily derived, i.e., to allocate transmit power proportional to the rank of the transmit correlation matrix of each port. To obtain the optimal subset, we apply the suboptimal power allocation scheme to any possible optimal subset $A_k(k = 1, \ldots, 2^l - 1)$ to obtain the corresponding suboptimal outage probability P_{o,A_k}^{sub}. The subset with the best outage performance shall be selected. Consequently, the suboptimal outage probability of OSTBCs transmitted over the $(n, l, q_1, \ldots, q_l, m)$ DA topology, P_o^{sub}, is obtained as

$$
P_o^{sub} = \min\left(P_{o,A_1}^{sub}, P_{o,A_2}^{sub}, \ldots, P_{o,A_{2^l-1}}^{sub}\right). \quad (13)
$$

The above power allocation results can also be obtained at the receiver and fed back to the transmitter. However, complexity of receiver is increased, and we do not discuss this case here.

V. NUMERICAL RESULTS

Consider a $(4,2,2,2,1)$ DA topology where 4 four DAs are grouped into two ports and the receiver has one antenna, with the normalized large-scale fading parameters $a_1 = 1$ and $a_2 = 0.1$. The targeted capacity is 3 bits/s/Hz. The antenna subsets include A_1 (Port 1 DAs), A_2 (2 ports DAs), and A_3 (Port 2 DAs). The 3/4 rate OSTBCs are transmitted on A_2, and full rate OSTBCs are transmitted on A_1 and A_3. The transmit correlation coefficients are denoted by $\mathbf{u} = [u_1, u_2]$, where u_1 and u_2 denote the correlation coefficient between the two DAs of Ports 1 and 2, respectively. $\mathbf{R}_{t,1}$ and $\mathbf{R}_{t,2}$ are then given as $\mathbf{R}_{t,1} = [1\ u_1; u_1\ 1]$ and $\mathbf{R}_{t,2} = [1\ u_2; u_2\ 1]$.

For 3/4 rate OSTBCs over subset A_2, Fig. 2 depicts the outage probability versus power allocation weight to Port 1 (w_1) in three correlation scenarios, i.e., $\mathbf{u} = [0.5, 0.5]$, $\mathbf{u} = [0.5, 1]$,
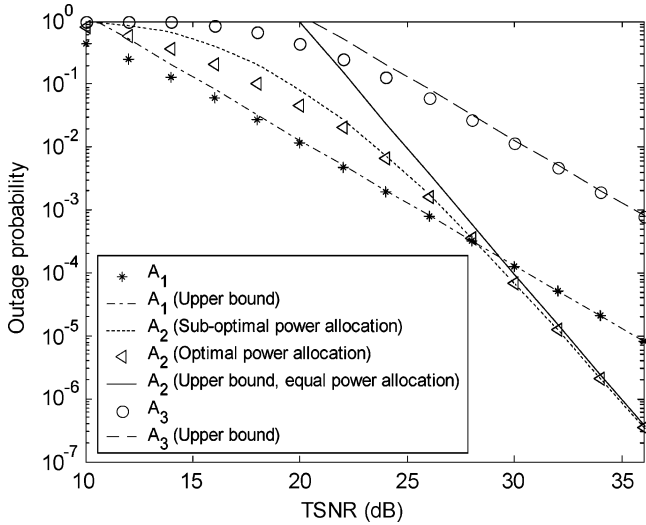
Fig. 3. Outage probability versus TSNR of OSTBCs over different subsets.

and $\mathbf{u} = [1, 0.5]$. When TSNR is 20 dB, the outage curves decrease monotonously as w_1 increases, indicating the optimality of transmission over A_1. However, at TSNR level of 30 dB, A_2 is optimal, with the optimal w_1 being about 1/2, 2/3, and 1/3 for the three scenarios, respectively. These results verify that the suboptimal power allocation scheme is very close to optimal. For example, according to (12), when A_1 is optimal, the suboptimal w_1^* is 1, which is exactly the optimal value, and when A_2 is optimal, the suboptimal w_1^* is 1/2, 2/3, and 1/3 for the three scenarios, which are very close to optimal.

Antenna subset selection with suboptimal power allocation scheme is also illustrated by the above system in the scenario where $\mathbf{u} = [0.5, 0.5]$. Fig. 3 shows the outage probability of A_1, A_3, A_2 with suboptimal power allocation (i.e., equal power allocation in this case) and A_2 with optimal power allocation

obtained from numerical optimization. The upper bounds on the outage probability approximations of A_1, A_3, and A_2 with equal power allocation are also depicted. We observe that these upper bounds are tight in high TSNR regions. Also, we find the suboptimal performance of A_2 is very close to the optimal performance at high TSNR levels. Subset selection is easy but effective, e.g., A_1 is selected when TSNR is below 28 dB, and A_2 is selected when TSNR is beyond 28 dB. With subset selection, the suboptimal performance in (13) is very close to optimal at any TSNR level. The benefit of subset selection compared with equal power transmission over A_2 is obvious, e.g., a 3-dB reduction in TSNR at a 10^{-2} outage level. Finally, we gladly observe that subset selection based on the upper bounds of outage probability approximation yields almost the same result but with significantly reduced complexity.

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