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Theory and Design of Arbitrary-Length Biorthogonal Cosine-Modulated Filter Banks

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Abstract

The design and generalization of Perfect-reconstruction (PR) cosine-modulated filter banks (CMFB) have been studied extensively due to its low design and implementation complexity. In this paper, the theory and design of arbitrary-length biorthogonal CMFB is considered. This is a generalization of the method used in [5] for designing arbitrary length orthogonal CMFB and has the advantage of simple design procedure. We also propose a systematic design method so that biorthogonal CMFB with longer length can be obtained.

1. Introduction

Perfect reconstruction (PR) critically decimated quadrature mirror filter banks (QMF) have many important applications in speech, audio, and image processing. It is well known that efficient FIR filter banks that satisfy PR property can be realized by cosine modulation of a linear phase prototype filter with length $N = 2mM$ [1], [2]. The resulting cosine modulated filter banks (CMFB) are very efficient and relatively ease to design. More recently, it had been shown that [3]-[5] PR CMFB with length other than *2mM* can also be obtained. In [5], it was suggested that instead of deriving the PR conditions directly, it is also possible to obtain arbitrary length CMFB by forcing the coefficients at both ends of the prototype filter for the *2mM* and *(2m+1)M* solutions to be zero. This greatly simplifies the design of the arbitrary length orthogonal CMFB because the same procedure for designing the *2mM* and *(2m+1)M* orthonormal CMFB can also be used. It was first recognized in [6] that if the linear phase property of the prototype filter is relaxed, then more general biorthogonal CMFB can be obtained. In [7], it was shown that the simplicity of the modulated filter bank is closely related to the structural constraints imposed in the polyphase matrix. By exploiting the concept of structural PR, more general cascade linear phase orthogonal and nonorthogonal PR systems can be obtained. It was also shown that the PR conditions of the biorthogonal CMFB is equivalent to the PR conditions of a set of 2-channel PR (biorthogonal) filter banks. However, satisfactory design procedure was not reported. More recently, design procedure for biorthogonal CMFB based on quadratic constrained least squares (QCLS) technique was proposed [8]. The biorthogonal property is used to realize filter banks with different delay. The length of the biorthogonal filter bank is arbitrary.

In this paper, we generalized the approach in [5] to design biorthogonal CMFB with arbitrary-length. The idea is to treat the arbitrary-length biorthogonal CMFB as a biorthogonal CMFB with length N equal to *2mM* but with certain coefficients at the beginning and end of the impulse response being set to zero. We also propose a systematic design method so that biorthogonal CMFB with longer length can be obtained. The method can also be used in designing orthogonal and biorthogonal cosine modulated wavelet bases. For more details, interested reader can consult [10], [11]. The layout of the paper is as follows: In Section 2, we shall review the theory of orthogonal and biorthogonal CMFB. The design procedures are discussed in Section 3 followed by several examples in Section 4.

2. Theory of Orthogonal and Biorthogonal CMFB

In CMFB, the analysis filters $f_k(n)$ are obtained by modulating the prototype filter $h(n)$ by the modulation $c_{k,n}$ where:

$$f_k(n) = h(n) \cdot c_{k,n} \quad (1)$$

$$c_{k,n} = \cos\left((2k+1)\frac{\pi}{2M}(n - \frac{N-1}{2}) + (-1)^k \frac{\pi}{4}\right) \quad (2a)$$

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or $$c_{k,n} = \cos\left((2k+1)\frac{\pi}{2M}(n+\frac{M+1}{2})\right) \quad (2b)$$
$$n=0,\ldots,N-1;\ k=0,\ldots,M-1$$

Here M is the number of channels and N is the length of the filters. Let $H(z)=\sum_{k=0}^{2M-1} z^{-k}H_k(z^{2M})$ be the type-I polyphase decomposition of the prototype filter of length N and $F_k(z)$ the Z-transform of $f_k(n)$. It can shown that:

$$\begin{aligned} f(z) &= U_M[(I_M \pm J_M)h_0(z^{2M}) \\ &\pm z^{-M}(I_M \mp J_M)h_1(z^{2M})]e_M(z) \\ &= E(z^M)e_M(z) \end{aligned} \quad (3)$$

where

$e_M^T(z) = [1 \quad z^{-1} \quad \ldots \quad z^{-(M-1)}]$;
$f(z) = [F_0(z), F_1(z), .., F_{M-1}(z)]^T$;
$h_0(z) = diag[H_0(-z), H_1(-z), \ldots, H_{M-1}(-z)]$;
$h_1(z) = diag[H_M(-z), H_{M+1}(-z), \ldots, H_{2M-1}(-z)]$.
I_M, J_M, U_M are respectively the identity matrix, antidiagonal matrix and some unitary matrix.

For simplicity, we assume that the second term inside the square bracket in (3) takes on the plus sign. The polyphase matrix can be written as:

$$E(z) = U_M[p(z) + J \cdot q(z)] = U_M\Lambda \quad (4)$$

where, $p(z) = h_0(z^2) + z^{-1}h_1(z^2)$
$q(z) = \pm[h_0(z^2) - z^{-1}h_1(z^2)]$

As $p(z)$ and $q(z)$ are diagonal matrices, matrix Λ is zero except only along its diagonal and antidiagonal entries. Therefore, row (columns) k and M-k are orthogonal to all other rows (columns). Without loss of generality, we assume that M is even. The results for M odd is similar. Let $p_{k,k}(z)$ and $q_{k,k}(z)$ be the k diagonal elements of matrices $p(z)$ and $q(z)$ respectively. If U_M is an invertible matrix, then it is necessary for the determinant of the following submatrices to be a delay to achieve PR.

$$S_k = \begin{bmatrix} p_{k,k}(z) & q_{M-k-1,M-k-1}(z) \\ q_{k,k}(z) & p_{M-k-1,M-k-1}(z) \end{bmatrix},$$
$$k = 0,1,\ldots,M/2-1 \quad (5)$$

or equivalently,

$$\det S_k = \beta_k z^{-\alpha_k}. \quad (6)$$

If S_k is lossless then, the PR condition becomes:

$$\tilde{H}_k(z^2)H_k(z^2) + \tilde{H}_{M+k}(z^2)H_{M+k}(z^2) = 1 \quad (7)$$

where ~ denotes conjugate transpose and z replaced by z^{-1}.

For general values of $p(z)$ and $q(z)$, we have the PR condition for the biorthogonal CMFB as follows:

$$H_k(z)H_{2M-k-1}(z) + H_{M+k}(z)H_{M-k-1}(z) = z^{-n_k} \quad (8)$$

which is equivalent to the PR conditions of a set of 2-channel PR (biorthogonal) filter banks. Usually n_k are chosen to be the same for all k and it determines the delay of the filter bank. If the 2-channel filter bank admits lattice structure, then the CMFB can also be implemented as a set of 2 channel lattice structure. This has been done for the orthogonal CMFB [1].

The PR conditions for arbitrary length orthogonal CMFB had been derived in [3]-[5]. In [5], it was suggested that arbitrary length CMFB can be considered as a *2mM* or *(2m+1)M* filter design problem with certain coefficients of the prototype filter at the ends being set to zero. This greatly simplifies the design of the arbitrary length orthogonal CMFB because the same procedure for designing the *2mM* and *(2m+1)M* orthonormal CMFB can be used except that the variables at the ends of the prototype filter have to be zero and do not appear as the optimization variables. Here, we employ the same approach to generate the arbitrary length biorthogonal filter banks. Since the prototype filter is no longer linear phase, we can either force the coefficients at the beginning or end of the impulse response to be zero.

3. Design Procedure

For the CMFB, the analysis filters will be frequency shifted version of the prototype filter. Therefore, the objective function is reduced to:

$$\Phi = \int_{\omega_s}^{\pi} |H(e^{i\omega})|^2 d\omega \quad (9)$$

where ω_s is the stopband cutoff frequency whose value should be between $\frac{\pi}{2M}$ and $\frac{\pi}{M}$. Larger ω_s leads to larger stopband attenuation but the overlap between adjacent analysis filters will also increase at the same time. It is also possible to replace the integral in (9) by a summation. This has the advantage of being able to put different weighting to different parts of the stopband and provides more control over the stopband attenuation. We shall consider the design of arbitrary length orthogonal and biorthogonal CMFB separately.

Arbitrary Length Orthogonal CMFB

The design problem is formulated as the following constrained optimization problem:

$$\min_{h} \int_{\omega_s}^{\pi} \left|H(e^{j\omega})\right|^2 d\omega \qquad (10)$$

subjected to the constrains in (7).

where $\boldsymbol{h}$ is the vector containing the impulse response of the prototype filter.

The design procedure is similar to the one that was introduced in [5] and the optimization is performed using the DNCONF subroutine of the IMSL library. For small value of m and M, we use a normalized linear phase lowpass filter as initial guess. To design CMFB with longer length, we make use of the interpolation technique introduced in [9]. The idea is based on the observation that for M a power of two, the envelop of the prototype filters with the same value of m is very similar though the number of channels and the scaling factors are different. Hence it is possible to interpolate the impulse response of a prototype filter with M channels to obtain the initial guess for the prototype filter with $2M$ channels. The arbitrary length orthogonal CMFB can readily be obtained by setting appropriate coefficients in the impulse response of the prototype filter to zero during the constrained optimization.

Arbitrary Length Biorthogonal CMFB

The design problem is also formulated as a constrained optimization similar to (10) except that the constrains are given by (8). To design a biorthogonal CMFB with length N equal to $2mM$ and a given delay s, we need an appropriate nonlinear phase filter as initial guess of $\boldsymbol{h}$. Here, the nonlinear phase filter is obtained through a unconstrained optimization. Our procedure starts by designing a linear phase lowpass filter with the same length. The impulse response of this filter is then appropriately shifted to provide the initial guess to the unconstrained optimization that minimize the stopband attenuation of the nonlinear phase filter. The solution can then be used as the initial guess to the constrained optimization.

To design filter banks with larger number of channels, it is found that the interpolation technique as described above is also effective. To design the arbitrary length biorthogonal filter, the appropriate coefficients of the prototype filter are set to zero throughout the constrained optimization. Nguyen and Heller [8] have generalized the biorthogonal CMFB using the modulation (2a) with N equal to the length of the arbitrary length filter.

4. Design Examples

Here we shall give several design examples on the design of orthogonal and biorthogonal CMFB. Figure 1 shows the impulse and frequency response of the prototype filter of an orthogonal CMFB with $m = 8$, $M = 4$ and $N = 70$. It is found that for certain values of m like $m = 2$, there are discontinuity at both ends in the impulse response of the prototype filter. This can be reduced if an arbitrary length orthogonal CMFB with slightly larger or smaller length are used.

Figure 2 shows the impulse and frequency response of the prototype filter of a biorthogonal CMFB with $m = 8$, $M = 4$, $N = 76$ and $s = 4$.

Figure 3 shows the impulse and frequency response of the prototype filter of a biorthogonal CMFB with $m = 5$, $M = 16$, $N = 160$ and $s = 4$. This is generated using the interpolation technique mentioned in Section 3.

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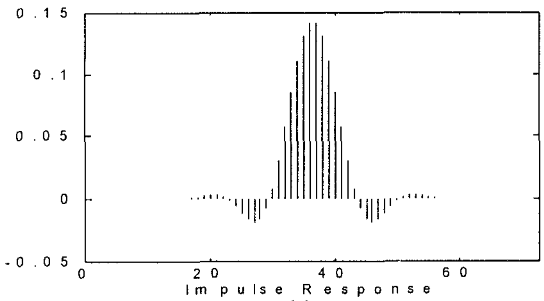
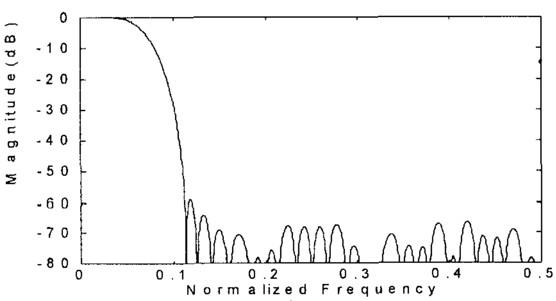


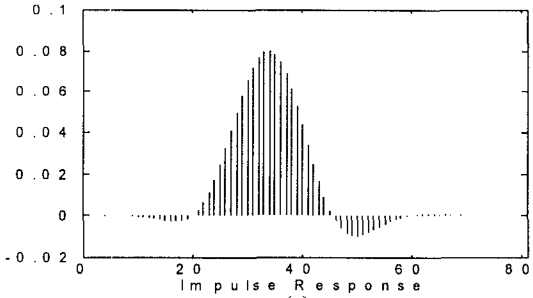
(a)



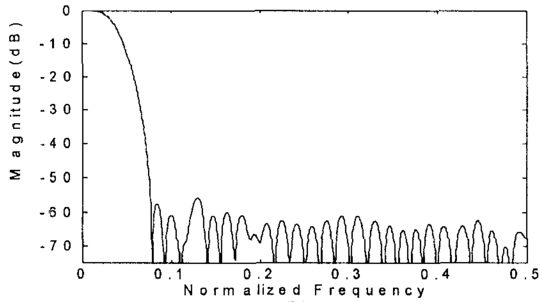
(b)

Fig 1. Orthogonal CMFB prototype filter with the arbitrary length $N = 2mM + \beta = 2 \times 8 \times 4 + 6$

(a) Impulse response (b) Frequency response



(a)

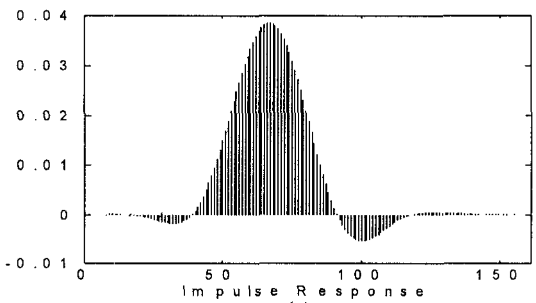


(b)

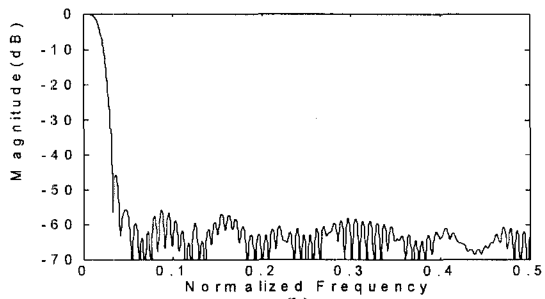
Fig 2. Biorthogonal CMFB prototype filter with the arbitrary length

$N = 2mM + \beta = 2 \times 4 \times 8 + 12$

(a) Impulse response (b) Frequency response.



(a)



(b)

Fig 3. Biorthogonal CMFB prototype filter with the length

$N = 2mM = 2 \times 5 \times 16$

(a) Impulse response (b) Frequency response