

Analysis of Nonlinear Pulse Propagation in an Optical Fiber Laser

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Abstract – Using Fourier Series Analysis Technique and time-domain window function, the formation process of nonlinear laser pulses in a mode-locked fiber laser is analyzed theoretically. Fiber dispersion, Kerr nonlinearity, amplifier gain, pump depletion, Raman contribution to nonlinear polarization, amplitude and phase modulation are also taken into consideration

Erbium-doped Fibers (EDF) are very useful for high bit rate optical communication systems because of their high gain, low noise, polarization insensitive and low coupling loss to optical fibers. Mode-locking of a fiber lasers is useful for short-pulse generation which is compatible with communications and fiber sensor systems.

The laser model consists of a single mode polarization-preserving EDF, an amplitude modulator and a phase modulator. For the generation of laser pulses of pulse widths greater than 100 fs, the propagation equation can be described by the simplified nonlinear Schrodinger equation as follows [1]:

$$\frac{\partial u}{\partial z} = j \frac{1}{2} \frac{\partial^2 u}{\partial T^2} + j |u|^2 u + \frac{1}{2} G u + \frac{1}{2} \gamma_a G \frac{\partial u}{\partial T} - j \gamma_{RU} \frac{\partial |u|^2}{\partial T} \quad (1)$$

where u is the normalized amplitude, T is the normalized time and z is the normalized distance, $\gamma_a \approx 10^2 \text{fs}$, $\gamma_R = 2\beta\mu\delta$, $\mu = (T_0\Omega_R)^{-1}$,

$\delta = (T_2^R \Omega_R)^{-1}$; and Ω_R is the Raman resonant frequency, the characteristic time $T_2^R = 1/(\pi\Delta\nu_R)$; $\Delta\nu_R$ is the Raman line

bandwidth, $G = \frac{G_0}{(1 + \sigma W_l / W_p)}$ is the amplification parameter, W_l is the laser pulse energy, the coefficient σ is proportional to the ratio of pumped and laser transition cross sections, and W_p is the pump energy per period. After the amplification, the pulse is sent through an amplitude and phase modulator. The transmission

functions for both amplitude and phase modulators are as follows:

$$K_a = t_a \exp[-d_a \sin^2(\Omega_a T + \phi_a)] \quad (2)$$

$$K_p = \exp[jd_p \cos(\Omega_p T + \phi_p)] \quad (3)$$

where d_a , Ω_a and ϕ_a are the depth, the frequency, and the phase of the amplitude modulation. d_p , Ω_p and ϕ_p are the depth, the frequency and the phase of the modulation, respectively.

The Fourier series analysis technique (FSAT) [2] is used to analyze the nonlinear pulse formation of the mode locked fiber laser. To apply the FSAT, the nonlinear partial differential equation (1) is first transformed to frequency domain by the use of Fourier series approach. Secondly, by the use of the orthogonal properties of the equation obtained, $2N+1$ first order partial differential equations can be obtained and which can be solved by the Runge-Kutta Method. After being propagated a distance of Δz , results can be obtained, and these results will be used as the initial condition for another propagation distance of Δz . This process should be repeated until the required distance is achieved. When the required propagation distance is achieved, time domain solution can be obtained by inverse Fourier transform. In order to improve the efficiency of the FSAT, a Time-Domain Window Function (TDWF) is also introduced [3]. As a result, by using FSAT and applying the TDWF, the soliton propagation equation can be transformed to a system of first-order partial differential equations as follows:

$$\frac{\partial \hat{u}_n(z)}{\partial z} = \underbrace{[-j\sigma(n) + \frac{1}{2}G]\hat{u}_n(z)}_{\text{linear term}} + j \underbrace{\sum_{\forall \mu - \nu + \lambda = n} \hat{u}_\mu(z) \hat{u}_\nu^*(z) \hat{u}_\lambda(z) + \gamma_{RE} \sum_{\forall p+q=n} q \hat{u}_p(z) \Theta_q(z)}_{\text{nonlinear term}}$$

$$- \sum_{\forall k+L+m=n} \underbrace{G\hat{u}_k(z)}_{\text{time window function term}} \cdot \frac{(-1)^L}{L\varepsilon} \cdot \frac{-(-1)^m + \cos(m\varepsilon T_0) - j \sin(m\varepsilon T_0)}{2\pi m} \quad (4)$$

where $n, \mu, \nu, \lambda, p, q, k, L$ and m are integers of values between $-N$ and N ; $\hat{u}(z)$ and ε are Fourier amplitude coefficient and fundamental frequency parameter, respectively.

With an initial pulse of the shape $u(0,T)=\text{sech}(T)$ (i.e. fundamental soliton pulse shape), Fig.1a and b shows the 3-dimensional propagation behavior of the initial pulse inside a mode-locked fiber laser in both time and frequency domain. The parameter values are: $d_p = 0.2$, $d_a = 0.4$, $\Omega_a = 8.3 \times 10^{-2}$, $\Omega_p = 3.4$, $\phi_a = 0.0$, $\phi_p = \pi/2$, $\gamma_a = 0.2$, $\delta = 0.3$, $\mu = 0.025$, $\varepsilon_R = 0.003$, $t_a = 0.9$, $G_0 = 2$, $\alpha = 1$, $L_F = 0.5$. And the slope of the TDWF is 5. After a propagation distance of $z=30$, stable narrow pulses are obtained in the time domain and spectral broadening is observed in the frequency domain. The pulse widths obtained at $z=10$ and 150 are about the same and both are much narrower than the input pulse width. The peak power of the narrow pulse obtained at $z=10$ is about 11 times greater than the input peak power. The spectral broadening is a consequence of the time dependence of nonlinear phase shift. The self-phase modulation (SPM) produces new frequency components and broadens the pulse spectrum. A temporally varying phase implies that the instantaneous optical frequency differs across the pulse from its central value. This frequency chirp occurs mainly near the leading and trailing edges. As these edges become steeper, the tails extend over a longer frequency range.

The formation of subpicosecond laser pulses in an active mode-locked fiber lasers has been investigated by the use of FSAT and TDWF. Both 3D time and frequency domain results have been obtained. The effects of various parameters, such as energy loss coefficient, gain parameter, pump energy and active fiber length have been considered. This is our first time of applying FSAT to analyze a mode-locked fiber laser in both time and frequency domain. It is found that the improved FSAT, is very powerful and general. The compatibility of the program is very well - the program can easily be modified to investigate the

behavior of different mode-locking schemes of fiber lasers. The numerical results are justified by doubling the sampling points. The results of the effect of different parameters on the propagation behavior of nonlinear soliton laser can be used as a reference purpose for further research on the soliton laser with mode-locking.

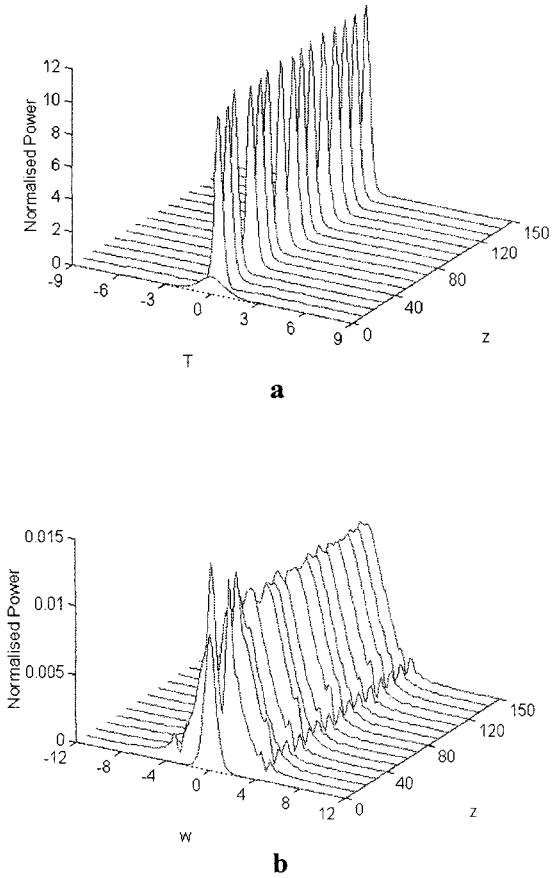


Fig 1 The propagation of a fundamental soliton pulse in a mode-locked fiber laser. (a) 3D time domain and (b) 3D frequency-domain

References

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