# PERFORMANCE OF GENERALIZED LAPPED TRANSFORM (GLT) BASED CDMA SYSTEM IN A MULTIPATH FADING CHANNEL

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## **ABSTRACT**

n this paper, we investigate the performance of a new class of spreading codes in a Direct Sequence Spread Spectrum (DSSS/CDMA) system. The preading and despreading codes are generated from a biorthogonal filter bank whose outputs are of continuous values. The performance of the proposed codes is compared with the gold codes in a Rayleigh ading multipath channel. Simulation results show hat the proposed codes yield lower BER performance than the gold codes when the number of users is resonably large.

#### 1. INTRODUCTION

There are two common types of interferences in a nobile communication channel. They are multiuser nd multipath interference. Multiuser interference occurs when more than one user attempting to use he same channel. Multipath interference occurs due o multiple propagation paths of transmitted signal.

In order to minimize these types of nterferences, the user transmission can be made rthogonal to each other. In a DSSS/CDMA system, I the codes of different users are orthogonal, all sers can transmit information at the same time sing the same bandwidth. Traditionally, Direct equence Spread Spectrum (DSSS)/CDMA systems tilize maximal length sequences (m-sequences) as ne spreading codes [3]. The m-sequences were hosen because of their good correlation property nd their simple generation using shift registers.

In this paper, we investigate another nethod of generating spreading codes using rthogonal functions which can assume continuous

values. We shall be using the synthesis filter coefficients and the analysis filter coefficients of a biorthogonal filter bank as the spreading codes and the despreading codes, respectively. In particular, we shall investigate the performance of using a class of biorthogonal filter bank called the Generalized Lapped Transform (GLT). The advantage of such filter bank is the availability of fast algorithms in generating basis functions and they are flexible.

The paper is organized as follows. In Section 2, the structure of the GLT is described. Section 3 is devoted to the design of GLT based spreading codes. Channel model and simulation results are given in section 4 and section 5. Finally conclusion is given in section 6.

## 2. GENERALIZED LAPPED TRANSFORM

Fig1 shows the structure of an M-channel uniform filter bank with  $f_i(n)$  and  $g_i(n)$  the analysis and synthesis filters, respectively. The incoming signal x(n) is split into M frequency bands by filtering with the analysis filters. Each subband signal is then maximally decimated by a factor of M. After processing in the subband domain, the M decimated signals will be interpolated, filtered by the synthesis filters and added together to reconstruct the signal.

In a perfect reconstruction (PR) filter bank the input and output are equal except for a delay (i.e.  $y(n) = x(n - n_d)$ ). For perfect reconstruction,  $f_i(n)$  and  $g_i(n)$  have to satisfy certain conditions. Let  $F_{i,k}(z^M)$  and  $G_{i,k}(z^M)$  be the type-1 and type-2 polyphase components of  $F_i(z)$  and  $G_i(z)$ , respectively.

$$F_i(z) = \sum_{k=0}^{M-1} z^{-k} F_{i,k}(z^M) \text{ and } G_i(z) = \sum_{k=0}^{M-1} z^k G_{i,k}(z^M)$$

(1) The filter bank is PR if [4]:

$$R(z)E(z) = z^{-d}I \tag{2}$$

where d is a constant and E(z), R(z) are the polyphase matrices of the analysis and synthesis filters and are given by:

$$[E(z)]_{i,k} = F_{i,k}(z);$$
  $[R(z)]_{i,k} = G_{i,k}(z)$ 
(3)

A biorthogonal PR filter bank or biorthogonal lapped transform of length 2M called the Generalized Lapped Transform (GLT) was introduced in [8]. The polyphase matrix of the GLT is given by:

$$E(z) = \frac{1}{2} P \begin{bmatrix} U_{00} & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & U_{11} \end{bmatrix} \begin{bmatrix} I_{M/2} & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & \left( C_{M/2}^{II} \mathbf{S}_{M/2}^{IV} \right)^{T} \end{bmatrix} \cdot \begin{bmatrix} I_{M/2} & I_{M/2} \\ I_{M/2} & -I_{M/2} \end{bmatrix} \begin{bmatrix} I_{M/2} & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & z^{-1} I_{M/2} \end{bmatrix} R_{1}$$
(4)

where

 $U_{00}$  and  $U_{11}$  are block diagonal invertible matrices,  $R_1 = P' diag\{B_2 \cdot \dots \cdot B_2\} DC_M^H J_M$  with D a diagonal matrix,

 $C_M^k$ ,  $S_M^k$  denote the type-k length-M discrete cosine and sine transform,

P is a permutation matrix which permutes the k and (k+M/2) rows to 2k and (2k+1) rows respectively, (k=0,...M/2-1),

P' is a permutation matrix which permutes the 2k and (2k+1) rows to the k and (k+M/2) rows respectively, (k=0,...M2-1),

We can parameterize  $U_{ii}$  by products of block diagonal  $(2 \times 2)$  invertible matrices.  $U_{ii}$  can then be written as,

$$U_{ii} = \prod_{k=1}^{M/2-1} v_k^i$$

(5)

where

$$v_k^i = \begin{bmatrix} I_{k-1} & 0 \\ x_k^i & y_k^i \\ y_k^i & x_k^i \\ 0 & I_{M-k-1} \end{bmatrix}$$
 and

$$\left( \mathbf{v}_{k}^{i} \right)^{-1} = \frac{1}{\left( \left( \mathbf{x}_{k}^{i} \right)^{2} - \left( \mathbf{y}_{k}^{i} \right)^{2} \right)} \begin{bmatrix} \mathbf{I}_{k-1} & & & \mathbf{0} \\ & \mathbf{x}_{k}^{i} & -\mathbf{y}_{k}^{i} & \\ & -\mathbf{y}_{k}^{i} & \mathbf{x}_{k}^{i} & \\ \mathbf{0} & & & \mathbf{I}_{M-k-1} \end{bmatrix}$$

Other product forms for  $U_{ii}$  can be used but the present choice has the advantage that  $U_{ii}$  will be diagonal dominance. Parameters  $x_k^i$ ,  $y_k^i$ ,  $d_{ii}$  can be used to minimise different objective functions. For example, in source coding or compression applications, the coding gain is maximised. But in spread spectrum applications, cross-correlation between the basis functions should be minimised for all relative time shifts. At the same time, auto correlation should be constrained to a delta function. Such Objective function is derived in the following section. The signal flow graphs of the forward Generalized Lapped Transform (despreading codes) and the inverse Generalized Lapped Transform (spreading codes) are shown in Fig 2a. and Fig 2b.

# 3. DESIGN OF GLT BASED SPREADING CODES

In many situations, the multiuser channel can be modelled as a synchronous communication channel, i.e., all user transmissions are at the same starting time. Examples of such a channel transmultiplexer and base to mobile link in cellular system. Under synchronous condition, each of the codes (basis functions of GLT) is orthogonal. There will be no interference between different users. However, in some situations, the system is asynchrounous (e.g. mobile to base link in cellular system). Under these circumstances, the crosscorrelation between different codes will have nonzero value. This causes multi-user interference. In this situation, the cross-correlation between the different codes has to be minimized for all relative time shifts. Assuming the time delay between two different users is m chips, there are two different cases, as shown in fig 3, due to the polarity of the transmitted bits.

For case 1, sum of the cross-correlation between the codes for a given delay m can be written as follows,

$$r_{1}(m) = \sum_{k=1}^{M} \sum_{l=1,l\neq k}^{M} \sum_{n=0}^{m-1} s_{k}(n) h_{k}(n+m) + \sum_{k=1}^{M} \sum_{l=1,l\neq k}^{M} \sum_{n=m}^{2M-1} s_{k}(n) h_{k}(n+m)$$

(6)

Similarly for case 2, sum of the cross-correlation between the codes for a given delay m can be written as follows,

$$r_{2}(m) = \sum_{k=1}^{M} \sum_{l=1, l \neq k}^{M} \sum_{n=0}^{m-1} s_{k}(n) h_{k}(n+m)$$

$$- \sum_{k=1}^{M} \sum_{l=1, l \neq k}^{M} \sum_{n=m}^{2M-1} s_{k}(n) h_{k}(n+m)$$

(7)

where  $s_k(n)$  and  $h_k(n)$  are the impulse responses of the  $k^{th}$  synthesis filter and analysis filter and  $h_k(n+m)$  is the cyclic shift of  $h_k(n)$  by m.

Combining these two cases, the objective function can be written as follows,

$$y = \sum_{m=1}^{2M-1} r_1(m) + r_2(m)$$

(8)

By minimizing the objective function structural parameters of the GLT is determined.

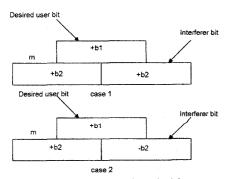


Fig 3. Data Bit Polarities

#### 4. CHANNEL MODEL

Several multipath models have been used in the literature, varying from the very comprehensive model to the simple tapped delay line model [3]. It has been shown that the performance of the SS-CDMA system is robust with respect to channel models. In this paper, the multipath fading channel is modelled as consisting of a continuum of multipaths. We also assume that the channel consists of a fixed number of paths. Utilizing power control among the CDMA users, the channel is assumed to be statistically identical for all users. The lowpass equivalent impulse response of the pass band fading channel is given by [3],

$$h(t) = \sum_{l=1}^{L} \beta_l \delta(t - \tau_l) e^{\tau_l}$$

(9)

where, L is the number of paths,  $\beta_l$ ,  $\tau_l$  and  $\gamma_l$  are respectively the path gain, delay and phase of the lth path. The channel auto-covariance function is easily found to be,

$$\mu(t) = \sum_{l=1}^{L} \sigma_l^2 \delta(t - \tau_l)$$

(10)

and the unit energy constraint on the fading process covariance function implies,

$$\sum_{l=1}^{L} \sigma_l^2 = 1$$

(11)

In this paper, we consider exponential multipath power profile, it has

$$\sigma_l^2 = \sigma_1^2 e^{-\frac{l-1}{L}}$$

(12)

The parameter L' decides the rate of power decay on the successive paths. Applying (11) for the exponential profile and arbitrarily taking L' = L one can get,

$$\sigma_1^2 = \frac{1 - e^{-1/L}}{1 - e^{-1}} \tag{13}$$

## 5. SIMULATION RESULTS

The simulation result is shown in fig 4. Here M, the number of codes is taken as 16. Rayleigh fading channel with 2-ray exponential profile is considered.

To generate the simulation results, each user in the channel is given a random starting time which is uniformly distributed over the bit interval. Each data point on the figure is the average of multiple simulations with different starting time. Gold codes based DSSS/CDMA is used as a comparison since it is the most popular code used at the moment in multiuser environment. The probability of bit error is plotted against the energy to noise ratio (Eb/No). Since we optimized the structural parameters of the GLT for the different relative shifts of basis functions, GLT based codes become more stable than the Gold codes when the number of users is increased. When the bit energy to noise ratio is increased, such improvement of the GLT based codes over the Gold codes is increased.

#### 6. CONCLUSION

This paper investigates the performance of a new class of spreading codes in a Direct Sequence Spread Spectrum (DSSS/CDMA) system. The spreading and despreading codes are generated from a class of biorthogonal filter bank called Generalized Lapped Transform (GLT). The structural parameters of the GLT are obtained by minimizing the cross-correlation between the basis functions under multiuser environment for all relative time shifts. Simulation results show that the proposed system yield lower BER performance than the Gold codes under Rayleigh fading channel.

#### **ACKNOWLEDGEMENT**

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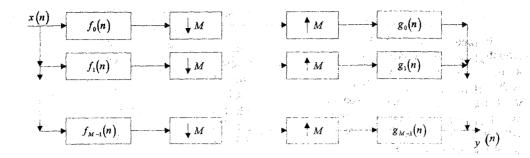


Fig 2a. Flow graph of forward Generalized Lapped Transform (Generation of Despreading codes)

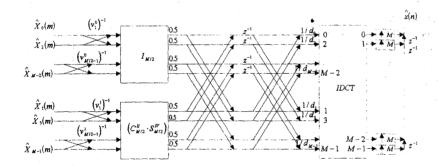


Fig 2b. Flow graph of inverse Generalized Lapped Transform (Generation of Spreading codes)

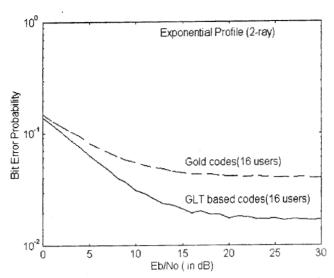


Fig 4. Performance of GLT based codes and Gold codes for 16 users under Rayleigh fading with 2-ray exponential profile.