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State Estimation with Measurement Error Compensation Using Neural Network

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Abstract

For a system with redundant sensors, the estimated state from the Kalman filter is biased if sensor mounting error existed. To remove this bias, the mounting errors must be compensated first before using the Kalman filter. It is shown that only the projection part of the sensors errors in the measurement space needs to be compensated. If the state of a system is unavailable, a neurofuzzy network can be used to estimate the compensation term. This method is simpler, as it does not require a model for the errors as that proposed in [2]. A sub-optimal Kalman filter with measurement compensation that restrains each row of the Kalman gain matrix to be in the measurement space is also derived. An example is presented to illustrate the performance of the proposed methods.

Keywords: Kalman filter, Redundant sensors, Measurement compensation, Neural networks.

1. Introduction

For a system to have high reliability, not only the reliability of each of its components is high, redundant sensors are often required. In aerospace technology, inertial navigation systems are constructed with redundant sensors that are mounted in orthogonal and skewed positions to improve its reliability. An obvious advantage of using redundant sensors is that sensors with low reliable can be used without jeopardizing unnecessarily the overall reliability of the system. This is the main motivation for developing Fault Detection and Isolation (*FDI*) techniques. Several *FDI* methods are proposed for systems with redundant sensors. The common ones are model-based methods, whilst knowledge-based methods are becoming more popular.

As sensor mounting errors can cause the configuration matrix of the system to deviate from the designed value, *FDI* methods involving residuals generated analytically may give false alarms. To avoid this problem, the measurement, and hence the residuals, must be compensated before it is analyzed. The parity vector compensation for *FDI* using Kalman Filter (*KF*) [2], and the separated-bias estimation method [3] are proposed to solve this problem. In [4], a nonlinear filter is used with a parity vector to estimate the sensor errors. However, these methods assumed that the model of the errors is known, thus limits their application in practice.

Methods to compensate for mounting error are proposed here. If the state of the system is available, then the estimation error of its state can be used directly to compensate for the mounting error. If, however, the state is not available, a neurofuzzy network is proposed to estimate the compensated term. The *KF* using the measurement with mounting error compensation, denoted by *MCKF*, is then applied. The implementation of *MCKF* is presented, and its performance is illustrated by an example.

2. Problem Formulation

Consider a linear discrete system with redundant sensors,

$$x_k = A_k x_{k-1} + B_k u_{k-1} + f_u c_u + w_k \tag{1}$$

$$y_k = H x_k + f_v c_v + v_k \tag{2}$$

where $x_k \in R^n$, $u_k \in R^r$ and $y_k \in R^m$ are respectively the state, control and measurement vectors; w_k *and* v_k are independent white noise with zero mean, and covariance matrices Q_k and R_k respectively; A_k and B_k are constant real matrices of appropriate dimensions, H is the configuration matrix with full column rank, f_u is the actuator fault event vector, and f_v is a sensor fault event vector, which is often, though not always, a unit vector; c_u *and* c_v are time-varying scalar functions of the actuator and the sensor faults respectively [5]. When there are no actuator and sensor faults , i.e., $c_u = c_v = 0$, the well-known standard *KF* gives

$$\hat{x}_{k/k-1} = A_k \hat{x}_{k-1/k-1} + B_k u_k \tag{3}$$

$$P_{k/k-1} = A_k P_{k-1/k-1} A_k' + Q_k \tag{4}$$

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k (y_k - H\hat{x}_{k/k-1}) \tag{5}$$

$$P_{k/k} = (I_n - K_k H) P_{k/k-1} (I_n - K_k H)' + K_k R_k K_k' \tag{6}$$

$$K_k = P_{k/k-1} H' (H P_{k/k-1} H' + R_k)^{-1} \tag{7}$$

where I_n is the $n \times n$ identity matrix. If modeling error, actuator or sensor fault exists, the estimated state from the *KF* is no longer optimal and is biased, as shown below. Let $H_{se} \in R^{m\times m}$ and $H_{me} \in R^{m\times n}$ be respectively the mounting error, and the scaling factor and input misalignment errors, then y_k becomes

$$y_k = (I_m + H_{se})(H + H_{me}) x_k + b + f_v c_v + v_k \tag{8}$$

where b is the sensor bias vector. Let

$$\overline{H} = (I_m + H_{se})(H + H_{me})$$

$$H_e = H_{se} H + H_{me} + H_{se} \cdot H_{me} = \overline{H} - H$$

$$\overline{b}_k = H_e x_k + b \tag{9}$$

(8) becomes

$$y_k = H x_k + \overline{b}_k + f_v c_v + v_k \tag{10}$$

Let $\tilde{x}_k$ and $\tilde{y}_k$ be respectively the error of the estimated state and the output,

$$\tilde{x}_k = x_k - \hat{x}_{k/k} = (I_n - K_k H)(A_k \tilde{x}_{k-1} + f_u c_u + w_k) - K_k \overline{b}_k - K_k f_v c_v - K_k v_k$$

$$\tilde{y}_k = H\tilde{x}_k + f_v c_v + v_k$$

$\tilde{x}_k$ can be expressed in terms of $\tilde{x}_0$ as

$$\begin{aligned}\tilde{x}_k = &\prod_{i=1}^{k}(I_n - K_iH)A_i\tilde{x}_0 + \sum_{i=1}^{k}\Psi_i(I_n - K_iH)w_i \\ &+ \sum_{i=1}^{k}\Psi_i(I_n - K_iH)f_uc_u - \sum_{i=1}^{k}\Psi_iK_i\bar{b}_i \\ &- \sum_{i=1}^{k}\Psi_iK_if_vc_v - \sum_{i=1}^{k}\Psi_iK_iv_i\end{aligned}$$

where $\Psi_i = \begin{cases} I_n & i=k \\ \prod_{j=i+1}^{k}(I_n - K_jH)A_j & i<k \end{cases}$

If $\tilde{x}_0 = 0$, the expectations of $\tilde{x}_k$ and $\tilde{y}_k$ are

$$\begin{aligned}E[\tilde{x}_k] = &-\sum_{i=1}^{k}\Psi_i(I_n - K_iH)f_uc_u \\ &-\sum_{i=1}^{k}\Psi_iK_i\bar{b}_i - \sum_{i=1}^{k}\Psi_iK_if_vc_v\end{aligned} \tag{11}$$

$$E[\tilde{y}_k] = HE[\tilde{x}_k] + f_vc_v \tag{12}$$

If $c_u = c_v = 0$, then $E[\tilde{x}_k] = -\sum_{i=1}^{k}\Psi_iK_i\bar{b}_i$, which is generally non-zero, indicating that even if there is no actuator or sensor faults, the estimated state from the *KF* is biased, and no longer optimal, as $E[\tilde{x}_k] \neq 0$, and hence $E[\tilde{y}_k] \neq 0$. Clearly, false alarms can arise from mounting error. To reduce the possibility of false alarms, the measurement of the sensors should be suitably compensated first before applying the *KF*, as proposed in this paper.

3. Compensation of modeling error in the measurement space

From (11), only $\bar{b}_i$ given by (9) needs to be compensated, if there are no sensor and actuator faults. There are several approaches to compensate for $\bar{b}_i$. A common approach is to estimate the unknown error, such as the misalignment error of the sensors, the error in the scaling factor, and the sensor bias, or a combination of these errors [2,3,4]. It is shown in [2] that only 3n-9 linear combinations of the 3n elements of H can be determined uniquely from sensor measurement data. Similarly, only n-3 of the n elements of b can be determined. This is because the errors from different sensors may be combined in such a way that the resulting measurement may appear to be without any errors, making it difficult to estimate all the sensor errors. Consequently, only a sub-matrix with a dimension of $(n-3)\times n$, and a sub-vector of dimension n-3 can be estimated using a model of the errors. Assuming the errors can be adequately modeled by a discrete-time Markov process, these estimates can be obtained from the *KF* [2]. Indeed, if H_{me}, H_{se} and b are random variables, then the Extended Kalman Filter (*EKF*) involving augmenting the state variable and the sensor errors can be used to estimate the sensor errors [5]. However, these compensation methods assume the model of the sensor errors existed. In this section, a direct compensation of sensor errors is presented.

Before proceeding further, the concept of measurement space is introduced first. Let *S(H)* be a measurement space spanned by all the column of *H*, and $S(V) = S^{\perp}(H) = \{v | v'H = 0\}$, its orthogonal complement or the parity space, where the column vectors of *V* are the parity vector [7]. As *H* is of full column rank, the orthogonal projection matrices of *S(H)* and *S(V)* are: $P_H = H(H'H)^{-1}H'$, and $P_V = I_m - P_H$ respectively. Let $z_k = y_k + \xi_k$ be the new measurement vector, where ξ_k is the measurement compensation vector. Methods to determine ξ_k for both known and unknown state of the system are presented below. Assuming R_k^{-1} exists, the Kalman filter gain (*KGM*)given in (7) can be expressed as

$$K_k = P_{k/k}H'R_k^{-1} \tag{13}$$

For simplicity, the sensors are identical with the same variance of σ^2. Then R_k becomes

$$R_k = \sigma^2 \cdot I_m \tag{14}$$

Substituting (14) into (13) yields

$$K_k = \sigma^{-2} \cdot P_{k/k}H' \tag{15}$$

Rewrite ξ_k as

$$\xi_k = \xi_{Hk} + \xi_{Vk} \tag{16}$$

where $\xi_{Hk} \in S(H)$, $\xi_{Vk} \in S(V)$. The state updated using the new measurement is

$$\begin{aligned}\hat{x}_{k/k} &= \hat{x}_{k/k-1} + K_k(y_k + \xi_k - H\hat{x}_{k/k-1}) \\ &= \hat{x}_{k/k-1} + K_k(y_k + \xi_{Hk} - H\hat{x}_{k/k-1})\end{aligned} \tag{17}$$

Clearly, only the projection of $\bar{b}_k$ in the measurement space need to be compensated. Note that $K_k\xi_{Vk} = 0$ from the definition of *S(V)*, the one-step ahead estimate of the state, $\hat{x}_{k/k-1}$ is no longer unbiased, as $E[\tilde{x}_{k/k-1}] \neq 0$. Let $\xi_k = H\tilde{x}_{k/k-1}$, which is a vector in *S(H)* and can be considered as a measurement compensation with known state as shown in Fig.1. Equations (3) to (7) remain unchanged, except (5) now becomes

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k(y_k + \xi_k - H\hat{x}_{k/k-1}) \tag{18}$$

If the state is not available, the compensation can be achieved by a B-spline neural network, as discussed in the next section.

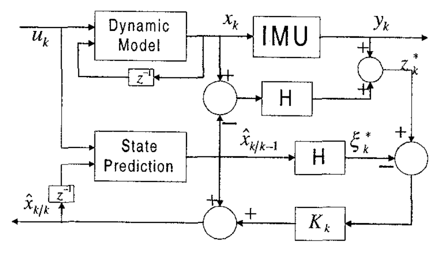

Fig.1: Measurement compensation based on the available state

4. B-spline neurofuzzy network

To estimate the state, neurofuzzy networks based on B-spline functions, denoted by *BSNN*, are used. The *BSNN* is shown in Fig. 2, and its output, $\hat{y}(t)$,is given by

$$\hat{y}(t) = \sum_{j=1}^{q} w_j s_j(x_t) \tag{19}$$

where x is the input, and w_j, $j=1,\ldots,q$, the weights of the hidden layer, and $s(x) = (s_1(x) \;\; \cdots \;\; s_q(x))'$ is the multivariate basis function given by tensor product [9].

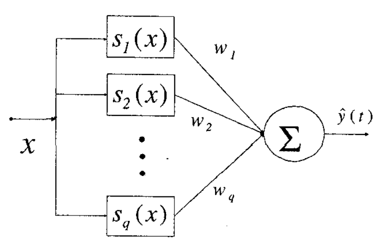

Fig.2: *BSNN* for scalar output

To compute the multi-step ahead prediction of dynamic systems, a *BSNN* with a recurrent structure (*BSRNN*), as shown in Figs. 3 and 4, is used. The system shown in Fig. 3 is based on available measurement, whilst that in Fig. 4, the state is estimated by a *BSRNN* with the state x_k, and the control u_k as input, and the next state (Fig. 3), or the measurement (Fig. 4) as output. To train the *BSRNN*, the following performance index is used.

$$E=\sum_{i=1}^{L}(y_i-\hat{y}(i))^2 \tag{20}$$

where $\hat{y}(i)$ is the output vector of the *BSRNN*. The weights can be updated using the steepest decent algorithm as given below.

$$w(k)=w(k-1)+\eta S'\Delta\hat{Y}^{(k-1)} \tag{21}$$

where $S=(s(x_1)\ \cdots\ s(x_L))'$, $\Delta\hat{Y}^{(k)}=Y-\hat{Y}^{(k)}$, $Y=(y_1\ \cdots\ y_L)'$, $\hat{Y}^{(k)}=[\hat{y}^{(k)}(1)\ \cdots\ \hat{y}^{(k)}(L)]'$. To improve the convergence rate in the training of the network, the learning rate η is updated at each iteration as follows [10].

$$\hat{\eta}=\|S'\Delta\hat{Y}^{(k-1)}\|^2/\|S'\Delta\hat{Y}^{(k-1)}\|_G^2 \tag{22}$$

where $G=S'S$.

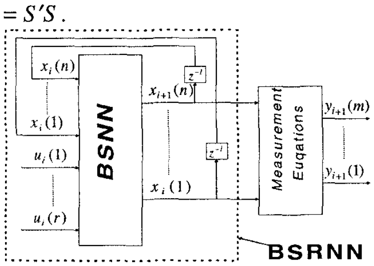

Fig. 3: *BSRNN* for known measurement

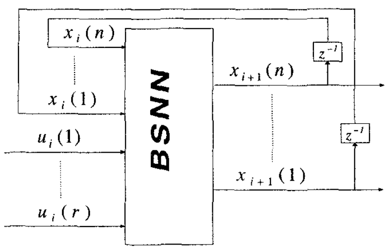

Fig. 4: ***BSRNN*** for estimated state

5. Compensation for measurement error

Let x_k^*, and $\xi_k^*=H(x_k^*-\hat{x}_k)$ be respectively the state, and the measurement compensation using the *BSRNN*, where ξ_k^* can be considered as an estimate of the projection of $\bar{b}_k$ in the measurement space. Then z_k is given by

$$z_k=y_k+\xi_k^*=Hx_k+v_k \tag{23}$$

The implementation of *MCKF* is shown in Fig.5. Let ξ_k^* be a noise sequence that is uncorrelated with the dynamic noise w_k, the measurement noise v_k and the estimate of the initial state estimate $\hat{x}_0$. Its mean and covariance matrix are

$$E[\xi_k^*]=m_k \tag{24}$$

$$E[(\xi_k^*-m_k)(\xi_k^*-m_j)']=\Omega_k\delta_{kj} \tag{25}$$

where δ_{kj} is the Kronecker function. Replacing ξ_k^* by m_k, the modified measurement is now given by

$$\begin{aligned} z_k^* &= y_k+m_k=(y_k+\xi_k^*)-(\xi_k^*-m_k) \\ &= Hx_k+v_k^* \end{aligned} \tag{26}$$

where $v_k^*=v_k-(\xi_k^*-m_k)$ is a zero-mean noise with a covariance matrix of $(R_k+\Omega_k)$. If m_k,Ω_k are unknown, they can be estimated by

$$\hat{m}=1/L\cdot\sum_{i=1}^{L}H(x_i^*-\hat{x}_{i/i-1})$$

$$\hat{\Omega}=1/(L-1)\cdot\sum_{i=1}^{L}H(x_i^*-\hat{x}_{i/i-1})(x_i^*-\hat{x}_{i/i-1})'H'$$

where L is the number of training data. The *MCKF* is given below, and its implementation is shown in Fig. 6, whilst the network for computing the *KGM* with measurement compensation is shown in Fig. 7.

$$\hat{x}_{k/k-1}=A_k\hat{x}_{k-1/k-1}+B_ku_k$$

$$P_{k/k-1}=A_kP_{k-1/k-1}A_k'+Q_k$$

$$z_k^*=y_k+\hat{m}$$

$$\hat{x}_{k/k}=\hat{x}_{k/k-1}+K_k(z_k^*-H\hat{x}_{k/k-1})$$

$$\begin{aligned} P_{k/k} &= (I_n-K_kH)P_{k/k-1}(I_n-K_kH)' \\ &\quad +K_kR_kK_k'+K_k\hat{\Omega}K_k' \end{aligned}$$

$$K_k=P_{k/k-1}H'(HP_{k/k-1}H'+R_k+\hat{\Omega})^{-1}$$

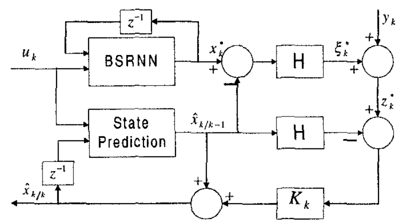

Fig. 5 *KF* with neural network-based compensation

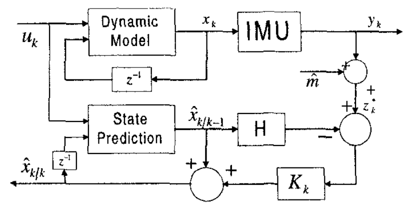

Fig.6 Implementation of *MCKF*

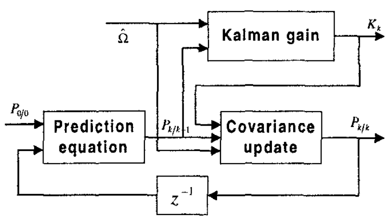

Fig.7 Computing *KGM* with measurement compensation

6. Sub-optimal Kalman filter

From (14), each row of K_k belongs to the measurement space for the special case that the measurement accuracy of all sensors are identical. In practice, it is enough to constrain each row of the *KGM* to be in *S(H)* for a system with redundant sensor. Let

$$\lambda_k=y_k-H\hat{x}_{k/k-1} \tag{27}$$

the so-called innovation sequence. In the ideal case, (27) can be written as

$$\lambda_k=H\tilde{x}_{k/k-1}+v_k \tag{28}$$

Substituting (28) into (3) yields

$$\begin{aligned} \hat{x}_{k/k} &= \hat{x}_{k/k-1}+K_k\lambda_k \\ &= \hat{x}_{k/k-1}+K_kH\tilde{x}_{k/k-1}+K_kv_k \end{aligned} \tag{29}$$

As v_k is the measurement noise, the third term on the right hand of (29) is much smaller than the first two terms. The update of the state estimate is mainly from the second term, suggesting that only the projection of the rows of K_k in *S(H)* need to be considered in the *MCKF*. The resulting *KF* is referred to as a sub-optimal *KF* for convenience. To constrain each row of the *KGM* to be in the measurement space, the *KGM* is first expressed as

$$K_k = D_k H' \tag{30}$$

where $D_k \in R^{n\times n}$, is random. Each of its row is called a coordinate vector of its corresponding row of K_k in *S(H)* that forms a basis of all the columns of *H*. It can be shown that D_k is given by

$$D_k = P_{k/k-1} G(GP_{k/k-1}G' + H'(R_k + \Omega)H)^{-1} \tag{31}$$

where $G = H'H$. The minimum variance estimate of x_k conditioned on y_k, $\hat{x}_{k/k}$, has the following form

$$\hat{x}_{k/k} = K_k y_k + d_k \tag{32}$$

When each row of K_k is changed in the whole m-dimensional space R^m, the resulting optimal estimate is the standard *KF* estimate. For the constrained condition given by (30), (32) can be rewritten as

$$\hat{x}_{k/k} = D_k y_k^* + d_k \tag{33}$$

where $y_k^* = H' y_k$. From (26), the measurement with measurement compensation is given by

$$y_k^* = Gx_k + v_k^{**}$$

where v_k^{**} is a zero-mean noise with a covariance matrix of $H'(R_k + \Omega_k)H$. From the standard *KF*, (31) can be obtained readily. The computation of the KGM with the constrained condition (30) is shown in Fig. 8.

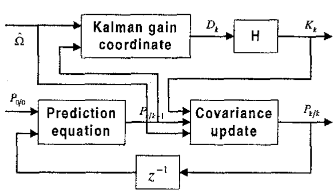

Fig.8: Calculation of the *KGM* of the Sub-optimal *KF* with measurement compensation

7. Example

Consider a system consisting of five sensors, some of which are redundant sensors. The exact configuration matrix *H* is,

$$H = \begin{bmatrix} \sin\alpha & 0 & \cos\alpha \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \\ -\sin\alpha\cos\beta & \sin\alpha\sin\beta/2 & \cos\alpha \\ -\sin\alpha\cos\beta & -\sin\alpha\sin\beta/2 & \cos\alpha \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix}$$

where (α,β) are the exact angles. With sensor mounting errors, the configuration matrix becomes

$$H + H_{me} = \begin{bmatrix} \sin\alpha_1 & \sin(\beta_1 - \beta) & \cos\alpha_1 \\ \sin\alpha_2\cos\beta_2 & \sin\alpha_2\sin\beta_2 & \cos\alpha_2 \\ -\sin\alpha_3\cos\beta_3 & \sin\alpha_3\sin\beta_3/2 & \cos\alpha_3 \\ -\sin\alpha_4\cos\beta_4 & -\sin\alpha_4\sin\beta_4/2 & \cos\alpha_4 \\ \sin\alpha_5\cos\beta_5 & -\sin\alpha_5\sin\beta_5 & \cos\alpha_5 \end{bmatrix}$$

where (α_i,β_i) $(i=1,\ldots,5)$ are the actual mounting angles of the sensors, and can be written as follows.

$$\alpha_i = \alpha + \Delta\alpha_i,\ \beta_i = \beta + \Delta\beta_i \text{, for } i=1,\ldots,5.$$

where $\Delta\alpha_i \sim N(0,\sigma_\alpha^2)$, and $\Delta\beta_i \sim N(0,\sigma_\beta^2)$. Let $\alpha = \sin^{-1}\sqrt{2/3} = 54.7356^\circ$ and $\beta = 72\pi/180 = 72^\circ$, and $H_{se} = diag(h_{11} \ \cdots \ h_{mm})$, where $h_{ii} \sim N(0,\sigma_h^2)$. The specifications of the system are given in Table 1, which is the same gyro used in [2].

Table 1: Nominal Ring-Laser Gyro Parameters

Parameter	Value
Scale factor	131 328 pulses/rad
Misalignment	5×10^{-5} rad (1σ)
Bias	0.01deg/h (1σ)
Scale-factor error	5ppm (1σ)

Let $Q = \sigma_x^2 I_n$, $R = \sigma_y^2 I_m$, $A_k = I_n$, $B_k = 0$, $\sigma_x = \sigma_y = 1.e-5$, $c_u = c_v = 0$ and

$$\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\\ \alpha_5\end{bmatrix} = \begin{bmatrix}54.8363^\circ\\ 54.5361^\circ\\ 55.2215^\circ\\ 54.7525^\circ\\ 54.2504^\circ\end{bmatrix},\ \begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\end{bmatrix} = \begin{bmatrix}72.0757^\circ\\ 72.2497^\circ\\ 72.5857^\circ\\ 71.7991^\circ\\ 72.3569^\circ\end{bmatrix}$$

The initial state, its estimate and the covariance matrix are

$$x_0 = [1.1650 \quad 0.6268 \quad 0.0751]'$$

$$\hat{x}_{0/0} = (-7.9669 \quad -4.4799 \quad 0.5725)$$

$$P_{0/0} = 100 I_3$$

The actual and the estimated values of the first element of the state vector from the standard *KF* with no measurement compensation are shown in Fig. 9, for *k* from 1 to 100, showing clearly a bias. The effect of compensation using the *MCKF* for the case that the state variable is available, is shown in Fig.10. The improvement over that without measurement compensation is clearly seen.

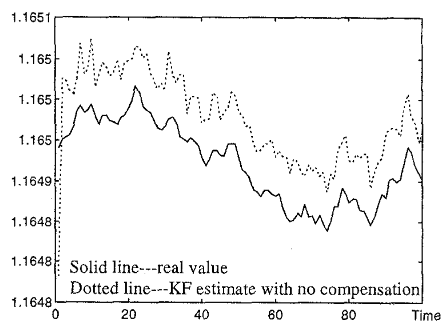

Fig. 9: Comparison between the real value and *KF* estimate with no compensation

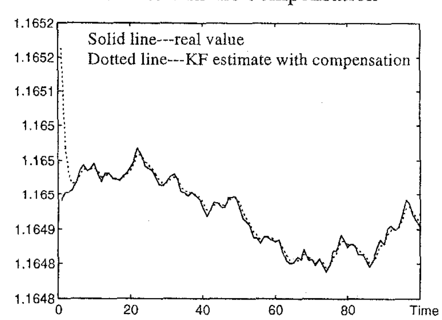

Fig.10: Comparison between the real value and *MCKF* estimate for the available state

When the state is not available, 100 data points were used to train the *BSRNN* yielding,

$$\hat{m} = 10^{-4} \times$$
$$[0.1013 \quad 0.4068 \quad 0.6941 \quad 0.7076 \quad 0.4287]'$$

$$\hat{\Omega} = 10^{-9} \times$$
$$\begin{bmatrix} 0.7286 & 0.1127 & -0.0950 & 0.2374 & 0.6505 \\ 0.1127 & 0.0632 & 0.0235 & 0.0279 & 0.0703 \\ -0.0950 & 0.0235 & 0.0544 & -0.0182 & -0.0940 \\ 0.2374 & 0.0279 & -0.0182 & 0.1186 & 0.2493 \\ 0.6505 & 0.0703 & -0.0940 & 0.2493 & 0.6258 \end{bmatrix}$$

The estimate of the first element of the state using the proposed *MCKF* for unknown state is shown in Fig. 11.

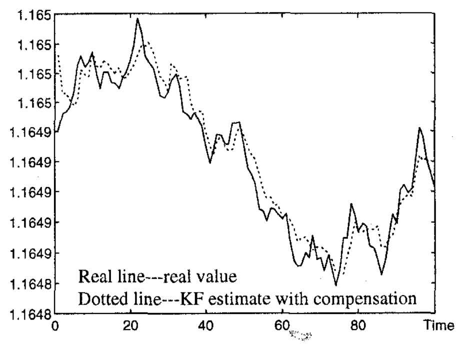

Fig.11: Comparison between the real value and *MCKF* estimate for the unavailable state

The result obtained using the proposed sub-optimal *MCKF* method with an initial estimate value of $\hat{x}_{0/0} = (0.7842 \;\; -1.4725 \;\; 1.1741)$, for k from 200 to 400 is shown in Fig. 12. The estimated error between the sub-optimal *MCKF* and the ordinary *MCKF* is plotted in Fig. 13, showing that the ordinary *MCKF* can only compensate for the projection of the modeling error in the measurement space.

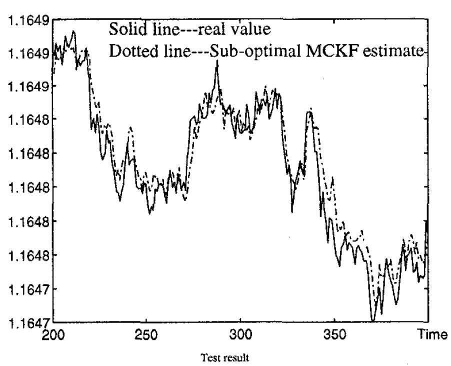

Fig.12: Comparison between the real value and the sub-optimal *MCKF* estimate for the unavailable state

8. Conclusion

It is shown that the conventional *KF* gives a biased estimate and its corresponding state residual vector and measurement residual vector are biased if mounting errors of the sensors existed. Consequently, it may lead to false alarms, when it is used in fault detection. To overcome this problem, it is proposed that the mounting error of the sensors are compensated first before the *KF* is applied. For a system with redundant sensors, each row of *KGM* can be constrained in the measurement space so that only a vector in the measurement space can be considered as the desired measurement compensation term. The performance of the proposed sub-optimal *MCKF* is illustrated by an example.

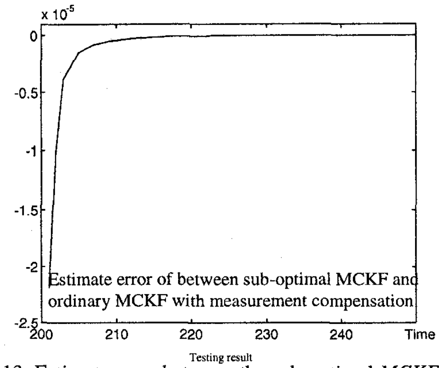

Fig.13: Estimate error between the sub-optimal *MCKF* and ordinary *MCKF* for the unavailable state

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