

the operator $\mathcal{L}(\cdot)$ to (37), we obtain

$$\tilde{\mathbf{R}}\tilde{\mathbf{x}} = \tilde{\mathbf{y}} \quad (39)$$

which yields

$$\tilde{\mathbf{x}} = \tilde{\mathbf{R}}^{-1}\tilde{\mathbf{y}}. \quad (40)$$

Similarly, because the operator $\mathcal{L}(\cdot)$ exerts exactly the same effect on \mathbf{R} and \mathbf{R}^{-1} (due to Lemma 1), we obtain, from (38)

$$\tilde{\mathbf{x}} = \mathcal{L}(\mathbf{R}^{-1})\tilde{\mathbf{y}}. \quad (41)$$

By comparing (40) and (41), it directly follows that

$$\mathcal{L}(\mathbf{R}^{-1}) = \tilde{\mathbf{R}}^{-1} \implies \mathbf{R}^{-1} = \mathcal{L}^{-1}(\tilde{\mathbf{R}}^{-1}). \quad (42)$$

■

Returning to the MC-CDMA system discussed in this paper, we have $ML = N_c$, and the value of M or L depends on the number of users N_u .

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Throughput Maximization in Linear Multiuser MIMO-OFDM Downlink Systems

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Abstract—In this paper, we study the problem of maximizing the throughput of a multiuser multiple-input-multiple-output orthogonal frequency-division multiplexing (MIMO-OFDM) system in the downlink with a total power constraint using a beamforming approach. An iterative algorithm that takes turns to optimize, jointly among users, the power allocation in the downlink, the transmit and the receive beamforming antenna vectors, and the power allocation in the virtual uplink is proposed. The algorithm is proved to converge, and the throughput increases from one iteration to the next. In addition to the total power constraint, the proposed algorithm is also capable of handling individual users' rate constraints. To reduce complexity, a geometric-programming-based power control in the high signal-to-interference-plus-noise ratio (SINR) region and an orthogonal frequency-division multiple-access scheme in the low SINR region are proposed. Numerical results illustrate that the proposed algorithm significantly outperforms the generalized zero-forcing (GZF) approach.

Index Terms—Beamforming, multiple-input multiple-output (MIMO), multiuser communication, orthogonal frequency-division multiplexing (OFDM), throughput.

I. INTRODUCTION

Physical layer techniques play an important role in providing high-speed reliable wireless communication services. The multiple-input-multiple-output (MIMO) antenna is one of the promising techniques that will substantially increase the spectral efficiency of a wireless system without requiring extra bandwidth [1]–[3]. On the other hand, orthogonal frequency-division multiplexing (OFDM) has become a strong candidate to transmit high-speed data over time-dispersive channels. In this paper, an MIMO-OFDM system in a frequency selective fading channel in the downlink is studied. Perfect channel state information at the transmitter and the receivers is assumed. MIMO-OFDM is attractive for fourth-generation (4G) wireless communications, and its effectiveness has been verified in laboratory experiments, with a recent 4G field experiment by NTT DoCoMo reporting a maximum speed of 2.5 Gb/s [4].

Following the information-theoretic results [5], [6], recent attention on MIMO downlink channels has been turned to the transmit and receive generalized beamforming (TR-GBF) approaches (e.g., [7]–[12]) for the potential of complexity reduction. However, TR-GBF does not make the problem easier to solve, as the problem is not convex, and methods achieving the optimum are still unknown. Although some recent attempts have considered the so-called generalized zero forcing (GZF), which has an advantage of simple handling of users sharing the same radio channel [13], the solution is generally far from optimal.

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Combining MIMO with OFDM gives additional challenges, as it involves both subcarrier and spatial subchannel allocations. In [14], several iterative linear signal processing algorithms were proposed, which were all restricted to fixed subcarrier and spatial subchannel allocations. In [15], the simulated annealing technique was proposed to obtain the globally optimal beamforming solution at the price of huge computational complexity, which offsets the advantages of beamforming.

Our problem of interest is to maximize system throughput (defined as the sum rate) for a multiuser MIMO-OFDM system in the downlink by jointly optimizing the TR-GBF antenna vectors and the power allocation among the users with a total power constraint. The proposed method alternates optimizing the TR-GBF vectors and the power allocation to yield a stable solution. A major contribution of this paper is that the throughput of the proposed algorithm is proved to monotonically increase from one iteration to the next, and hence, its convergence is assured. In addition, the proposed method also has the advantage of handling individual users' rate constraints. Furthermore, to reduce complexity, two simplified power control schemes are also presented with a mild performance loss for high and low signal-to-interference-plus-noise ratio (SINR) regions.

II. SYSTEM MODEL FOR LINEAR MIMO-OFDM

We consider a downlink MIMO-OFDM system with M users, N_f subcarriers, n_T transmit antennas at the base station, and n_{R_m} receive antennas at the m th mobile station. The data for a particular user, for example, m , are transmitted in packets, and denoted as $\mathbf{s}_m[n] \in \mathbb{C}^{N_m}$ for $n = 1, \dots, N_f$, where N_m is the number of spatial subchannels that are offered from the multiple transmit antennas. Since the channels are assumed to be quasi-static fading from one OFDM symbol to another, the time index is omitted for simplicity. We also assume that the elements in $\mathbf{s}_m[n]$ are independent identically distributed (i.i.d.) random variables with $\mathbb{E}[\mathbf{s}_m[n]\mathbf{s}_m^\dagger[n]] = \text{diag}(p_m[1, n], p_m[2, n], \dots, p_m[N_m, n])$, where $p_m[k, n]$ denotes the power that is allocated to the k th spatial subchannel on the n th subcarrier of user m , and the superscript \dagger denotes the Hermitian transposition. Note that, in general, N_m should be a function of n , as the channels at different frequency subcarriers are different. However, this frequency-dependent N_m can be dealt with by allocating zero power to the data if N_m should be smaller for some n . Thus, we set $N_m = \min(n_T, n_{R_m}) \forall m$, which is the maximum possible number of spatial subchannels [3].

With proper guard timing and cyclic prefix, the estimated frequency domain signal is

$$\hat{\mathbf{s}}_m[n] = \mathbf{R}_m^\dagger[n] \left(\mathbf{H}_m[n] \sum_{\tilde{m}=1}^M \mathbf{T}_{\tilde{m}}[n] \mathbf{s}_{\tilde{m}}[n] + \boldsymbol{\eta}_m[n] \right) \quad (1)$$

where $\mathbf{H}_m[n]$ denotes the MIMO channel matrix from the base station to user m at subcarrier n . The data symbol vector $\mathbf{s}_m[n]$ is postmultiplied by the transmit beamforming matrix $\mathbf{T}_m[n] \in \mathbb{C}^{K \times N_m}$ before transmitting from the antennas. Having set the transmit power to be $p_m[k, n]$, the columns of $\mathbf{T}_m[n]$ are normalized to 1, i.e.,

$\|\mathbf{T}_m[k, n]\| = 1 \forall k$, where $\mathbf{T}_m[k, n]$ denotes the k th column vector of $\mathbf{T}_m[n]$. The same holds for the receive beamforming matrix $\mathbf{R}_m[n]$. In addition, the noise $\boldsymbol{\eta}_m[n]$ is assumed to be i.i.d. complex Gaussian with zero mean and variance of σ^2 . The fidelity of the signal $\hat{\mathbf{s}}_m[k, n]$ is measured by its SINR, which is given by

$$\Gamma_m[k, n] = \frac{p_m[k, n] \left| \mathbf{R}_m^\dagger[k, n] \mathbf{H}_m[n] \mathbf{T}_m[k, n] \right|^2}{\sum_{\substack{\tilde{m}=1 \\ (\tilde{m}, \tilde{k}) \neq (m, k)}}^M \sum_{\tilde{k}=1}^{N_{\tilde{m}}} p_{\tilde{m}}[\tilde{k}, n] \left| \mathbf{R}_m^\dagger[k, n] \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k}, n] \right|^2 + \sigma^2} \quad (2)$$

With the assumption that the interference terms in (2) are Gaussian and independent, from the information-theoretic viewpoint, the achievable aggregate rate for user m , which is denoted as v_m , becomes

$$v_m = \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} \log_2 (1 + \Gamma_m[k, n]). \quad (3)$$

Therefore, the system throughput is

$$v = \sum_{m=1}^M v_m = \sum_{m=1}^M \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} \log_2 (1 + \Gamma_m[k, n]). \quad (4)$$

Our aim is to maximize the total throughput v with a total power constraint P_T , i.e.,

$$\max_{\substack{\{p_m[k, n] \geq 0\} \\ \{\mathbf{T}_m[n], \mathbf{R}_m[n]\}}} v \quad \text{s.t.} \quad \sum_{m=1}^M \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} p_m[k, n] \leq P_T. \quad (5)$$

Note that (5) is a nonlinear and nonconvex problem that involves the joint optimization of power allocation $\{p_m[k, n]\}$ among users and spatial subchannels at all subcarriers and joint TR-GBF antenna vectors $\{\mathbf{T}_m[k, n], \mathbf{R}_m[k, n]\}$. The subcarrier and spatial subchannel allocations to users are achieved through power allocation $\{p_m[k, n]\}$. For instance, if $p_m[k, n_1] = 0 \forall k$ for some n_1 , it means that subcarrier n_1 is not allocated to user m .

III. PROPOSED SCHEME

A. Optimization Methods

We first summarize the key techniques that will be used in different steps of the proposed algorithm and then present the proposed algorithm.

1) *Receive GBF Optimization* $\{\mathbf{R}_m[k, n]\}$: For given transmit GBF vectors $\{\mathbf{T}_m[k, n]\}$ and power allocation $\{p_m[k, n]\}$, the optimal receive GBF vectors are decoupled and should be designed to maximize the individual SINRs. Therefore, the optimal $\{\mathbf{R}_m[k, n]\}$ satisfies the minimum mean square error (MMSE) criterion [16] and has the form in (6), shown at the bottom of the page, where $\varsigma_m[k, n]$ is chosen such that $\|\mathbf{R}_m[k, n]\| = 1$.

2) *Transmit GBF Optimization* $\{\mathbf{T}_m[k, n]\}$: *Uplink-Downlink Duality*: From the SINR in (2), we can see that the transmit GBF

$$\mathbf{R}_m[k, n] = \varsigma_m[k, n] \left(\sum_{\substack{\tilde{m}=1 \\ (\tilde{m}, \tilde{k}) \neq (m, k)}}^M \sum_{\tilde{k}=1}^{N_{\tilde{m}}} p_{\tilde{m}}[\tilde{k}, n] \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k}, n] \mathbf{T}_{\tilde{m}}^\dagger[\tilde{k}, n] \mathbf{H}_m^\dagger[n] + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}_m[n] \mathbf{T}_m[k, n] \quad (6)$$

vectors $\{\mathbf{T}_m[k, n]\}$ are coupled together and cannot be individually optimized as $\{\mathbf{R}_m[k, n]\}$. This difficulty, nonetheless, can be overcome by performing the optimization in the so-called virtual uplink (VU) channels. In the Appendix, we extend the general uplink-downlink duality in [17] to a MIMO-OFDM system. As a result, $\mathbf{T}_m[k, n]$ becomes the receiver in the VU and should be optimized as in (15). This optimization requires the knowledge of the VU power allocation $\{q_m[k, n]\}$, which will be dealt with in Section III-A3.

3) *VU Power Optimization* $\{q_m[k, n]\}$: To find the VU power allocation, we need the uplink-downlink duality theory in the Appendix. In particular, given the uplink parameters $\{\mathbf{T}_m[k, n], \mathbf{R}_m[k, n]\}$ and $\{p_m[k, n]\}$, the VU power allocation $\{q_m[k, n]\}$ should be chosen as the principal eigenvector of the matrix \mathbf{C}^{UL} in (16) so that the same SINR region is maintained in the downlink and the VU.

4) *Optimal Power (and Subcarrier) Allocation* $\{p_m[k, n]\}$: Given known TR-GBF antenna vectors $\{\mathbf{T}_m[k, n], \mathbf{R}_m[k, n]\}$, (5) reduces to finding the optimal power allocation $\{p_m[k, n]\}$. Similar problems have been extensively studied in the context of digital subscriber lines [18] and code-division multiple-access systems [19], although the optimal solution is still unknown. With all $\{\mathbf{T}_m[k, n]\}$ and $\{\mathbf{R}_m[k, n]\}$ being fixed in (5), the power allocation problem becomes that given in (7), shown at the bottom of the page. This problem is recognized as a signomial programming (SP) problem [19], and it is nonconvex. In [19] and [20], algorithms that achieve a local optimal solution are proposed and will be used in this paper to find the solution to the power allocation.

B. Proposed Iterative Algorithm

Based on the above results, an iterative algorithm is devised as follows.¹

- 1) Set $i = 1$, and initialize the vectors $\{\mathbf{T}_m^{(i,1)}[k, n], \mathbf{R}_m^{(i,1)}[k, n]\}$ using GZF [13] and the power allocation $\{p_m^{(i,1)}[k, n]\}$ by solving the SP problem using the single condensation method [19].
- 2) For given $\{\mathbf{T}_m^{(i,1)}[k, n]\}$ and $\{\mathbf{R}_m^{(i,1)}[k, n]\}$, maximize the system throughput (or sum rate) using SP with a power initialization $\{p_m^{(i,1)}[k, n]\}$ to obtain the new power allocation $\{p_m^{(i,2)}[k, n]\}$.
- 3) Update $\{\mathbf{R}_m^{(i,3)}[k, n]\}$ as the MMSE downlink receiver using (6).
- 4) Calculate the achievable downlink SINR region $\{\Gamma_m^{(i,4)}[k, n]\}$ based on $\{\mathbf{T}_m^{(i,1)}[k, n], \mathbf{R}_m^{(i,3)}[k, n]\}$ and $\{p_m^{(i,2)}[k, n]\}$. Then, find the VU power allocation $\{q_m^{(i,4)}[k, n]\}$ as the principal eigenvector of \mathbf{C}^{UL} in (16) such that each subchannel achieves the same SINR as the downlink channel.

¹The superscript (i, j) is used to refer to a given variable at the j th step of the i th iteration.

- 5) Using the duality theory in the Appendix, obtain $\{\mathbf{T}_m^{(i,5)}[k, n]\}$ using (15). Then, compute the achievable uplink SINR region $\{\Gamma_m^{(i,5)}[k, n]\}$ based on $\{\mathbf{T}_m^{(i,5)}[k, n], \mathbf{R}_m^{(i,3)}[k, n]\}$ and $\{q_m^{(i,4)}[k, n]\}$. Next, find the downlink power allocation $\{p_m^{(i,5)}[k, n]\}$ as the principal eigenvector of \mathbf{C}^{DL} in (19) such that each subchannel achieves the same SINR as the VU channel. Moreover, the power allocation $\{p_m^{(i,5)}[k, n]\}$ will be used as the initialization of the SP approach at step 2 of the next iteration, i.e., $\{p_m^{(i+1,1)}[k, n] = p_m^{(i,5)}[k, n]\}$.
- 6) Set $i = i + 1$, and go back to step 2 until it converges.

C. Proof of Convergence

In the i th iteration, although the globally optimal solution at step 2 is not guaranteed from the SP power allocation, it has been shown in [19] that using the single condensation method, a local optimal solution that is guaranteed better than the initial solution can be found. Thus, we have

$$v^{(i,2)} \geq v^{(i,1)}. \quad (8)$$

At steps 3 and 5, $\{\mathbf{R}_m[k, n]\}$ and $\{\mathbf{T}_m[k, n]\}$ are found to maximize the individual SINRs, and, hence, $v^{(i,3)} \geq v^{(i,2)}$, and $v^{(i,5)} \geq v^{(i,4)}$. Step 4 further refines the VU power allocation so that both links have the same SINR region; therefore, $v^{(i,4)} = v^{(i,3)}$. At step 2 in the $(i + 1)$ th iteration, similarly to (8), we have $v^{(i+1,2)} \geq v^{(i,5)}$. As such, the system throughput is monotonically increasing from one iteration to the next; the proposed algorithm converges to a limit when $i \rightarrow \infty$. It is noted that because the original problem is not convex, we are not able to prove whether this algorithm can achieve the globally optimal solution and what exactly it will converge to.

D. Simplified Schemes for High and Low SINR Regions

1) *Simplified Power Control Solution in the High SINR Region Geometric Programming (GP)*: In the high SINR region, using $\log_2(1 + \text{SINR}) \approx \log_2 \text{SINR}$ simplifies the problem as that given in (9), shown at the bottom of the next page. Both the objective function and the constraints are polynomials, and (9) is recognized as GP [20], which can be solved optimally and much more efficiently than SP.

2) *Simplified Scheme in the Low SINR Region Orthogonal Frequency-Division Multiple Access (OFDMA)*: In the low SINR region, noise is the dominating factor, and the interference among users may be neglected. Furthermore, it is shown in [21] that OFDMA is asymptotically optimal in achieving the sum rate of the channel at low SINRs. This strategy can be used to simplify problem (5) by assigning the n th subcarrier to the strongest user $u(n)$. The optimal TR-GBF vectors will be given by the single-user optimal eigenbeamforming solution [3]. Denoting $\lambda_u[k, n]$ as the k th largest eigenvalue of

$$\begin{aligned} \min_{\{p_m[k, n] \geq 0\}} & \prod_{m=1}^M \prod_{n=1}^{N_m} \prod_{k=1}^{N_f} \left(\frac{\sum_{\substack{\tilde{m}=1 \\ (\tilde{m}, \tilde{k}) \neq (m, k)}}^M \sum_{\tilde{k}=1}^{N_{\tilde{m}}} p_{\tilde{m}}[\tilde{k}, n] |\mathbf{R}_m[k, n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k}, n]|^2 + \sigma^2}{\sum_{\tilde{k}=1}^{N_{\tilde{m}}} \sum_{\tilde{m}=1}^M p_{\tilde{m}}[\tilde{k}, n] |\mathbf{R}_m[k, n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k}, n]|^2 + \sigma^2} \right) \\ \text{s.t. } & P = \sum_{m=1}^M \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} p_m[k, n] \leq P_T \end{aligned} \quad (7)$$

the matrix $\mathbf{H}_u[n]\mathbf{H}_u[n]^\dagger$, the throughput maximization problem is formulated as

$$\begin{aligned} \max_{\{p_{u(n)}[k,n] \geq 0\}} & \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} \log_2 \left(1 + \frac{\lambda_{u(n)}[k,n] p_{u(n)}[k,n]}{\sigma^2} \right) \\ \text{s.t.} & \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} p_{u(n)}[k,n] \leq P_T \end{aligned} \quad (10)$$

which can be efficiently solved, and its solution has a water-filling interpretation.

As shown in [21], $u(n)$ can be chosen by

$$\begin{aligned} u(n) &= \arg \max_{m \in \{1,2,\dots,M\}} v_m[n] \\ &= \arg \max_{m \in \{1,2,\dots,M\}} \lambda_m[1,n]. \end{aligned} \quad (11)$$

It can be seen that for each selected user at the n th subcarrier, the optimal power strategy is to pour all power that is available to its best spatial dimension.

E. Handling Individual Rate Constraints

Maximizing the system throughput without concerning individual users' achievable rates may result in an unfair resource allocation, particularly when the channel statistics of some users are consistently inferior to those of the others. This can be corrected by imposing additional individual users' rate constraints C_m for user m , i.e.,

$$\max_{\substack{\{p_m[k,n] \geq 0\} \\ \{\mathbf{T}_m[n], \mathbf{R}_m[n]\}}} v \quad \text{s.t.} \quad \begin{cases} v_m \geq C_m \forall m, \\ \sum_{m=1}^M \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} p_m[k,n] \leq P_T. \end{cases} \quad (12)$$

Incorporating the additional constraints into the previous formulations results in the SP formulation

$$\begin{aligned} \prod_{k=1}^{N_m} \prod_{n=1}^{N_f} & \left(\frac{\sum_{\substack{\tilde{k}=1 \\ (\tilde{m}, \tilde{k}) \neq (m,k)}}^{N_m} \sum_{\tilde{m}=1}^M p_{\tilde{m}}[\tilde{k},n] |\mathbf{R}_m[k,n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k},n]|^2 + \sigma^2}{\sum_{\tilde{k}=1}^{N_m} \sum_{\tilde{m}=1}^M p_{\tilde{m}}[\tilde{k},n] |\mathbf{R}_m[k,n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k},n]|^2 + \sigma^2} \right) \\ & \leq \frac{1}{2^{C_m}} \end{aligned} \quad (13)$$

the GP formulation

$$\begin{aligned} & \prod_{k=1}^{N_m} \prod_{n=1}^{N_f} \left(p_m[k,n] |\mathbf{R}_m[k,n]^\dagger \mathbf{H}_m[n] \mathbf{T}_m[k,n]|^2 \right)^{-1} \\ & \times \left(\sum_{\substack{\tilde{k}=1 \\ (\tilde{m}, \tilde{k}) \neq (m,k)}}^{N_m} \sum_{\tilde{m}=1}^M p_{\tilde{m}}[\tilde{k},n] |\mathbf{R}_m[k,n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k},n]|^2 + \sigma^2 \right) \leq \frac{1}{2^{C_m}} \end{aligned}$$

and the OFDMA formulation

$$\sum_{n=1}^{N_f} \sum_{k=1}^{N_m} \log_2 \left(1 + \frac{\lambda_{u(n)}[k,n] p_{u(n)}[k,n]}{\sigma^2} \right) \geq C_m \quad (14)$$

where $u(n)$ is found using (11).

It can be easily shown that with these additional individual rate constraints, the structure of the power optimization and the TR-GBF vector optimization remains the same. Therefore, all of the previously proposed algorithms can be readily used.

IV. SIMULATION RESULTS

A. Channel Model and Simulation Setup

Simulations are conducted to evaluate the performance of the proposed method for frequency-selective fading channels with the channel impulse response between a pair of transmit and receive antennas for a user being modeled by $h(t) = \sum_{l=0}^{L-1} \beta_l \delta(t - \tau_l)$, where L is the channel order, which we assume to be $L = 3$, β_l denotes the complex channel gain for the l th path and is modeled as a zero-mean complex Gaussian random variable with variance σ_l^2 and $\sum_{l=0}^{L-1} \sigma_l^2 = 1$, and τ_l is the time delay of the l th path. For simplicity, we consider only the paths with delays of less than five normalized rms delay spreads and specifically assume that $\tau_l = 5Dl/(L-1)$, where the normalized rms delay spread D is set to be 0.5 in the simulations. The channels for different users and antenna pairs are i.i.d.; therefore, the user and antenna indexes are omitted for convenience. In the frequency domain, for example, at the n th subcarrier, the channel is, therefore, given as $G[n] = \sum_{l=0}^{L-1} \beta_l e^{-j2\pi n \tau_l / N}$. The notation M -user (n_T, n_R) is used to denote a downlink system with M users, n_T transmit antennas, and n_R antennas at all mobile receivers. The number of subcarriers N_f is 16. The throughput, which is defined as v/N_f , will be considered as the performance measure against the system budget P_T/σ^2 .

To evaluate the throughput performance of the proposed algorithm, we compare it with the GZF scheme that is proposed in [13]. For comparison purposes, we assume that in GZF, each subcarrier is shared by all of the users and that only one spatial subchannel is turned on by

$$\begin{aligned} \min_{\{p_m[k,n] \geq 0\}} & \prod_{m=1}^M \prod_{n=1}^{N_f} \prod_{k=1}^{N_m} \left(\sum_{\substack{\tilde{k}=1 \\ (\tilde{m}, \tilde{k}) \neq (m,k)}}^{N_m} \sum_{\tilde{m}=1}^M p_{\tilde{m}}[\tilde{k},n] |\mathbf{R}_m[k,n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k},n]|^2 + \sigma^2 \right) p_m^{-1}[k,n] \\ \text{s.t.} & P = \sum_{m=1}^M \sum_{n=1}^{N_f} \sum_{k=1}^{N_m} p_m[k,n] \leq P_T \end{aligned} \quad (9)$$

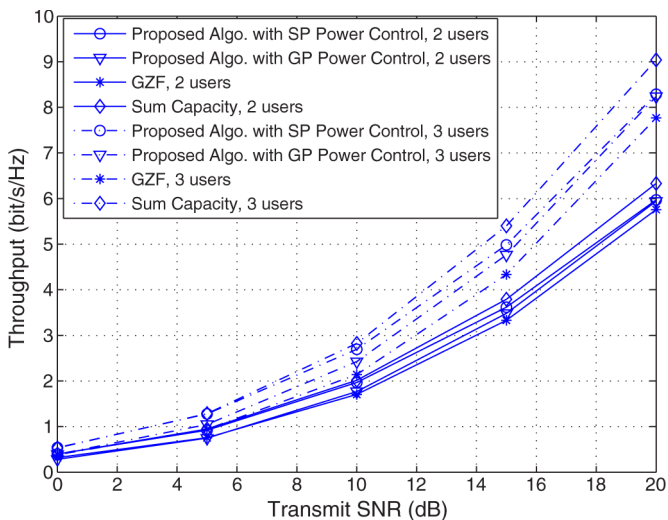


Fig. 1. Throughput results for two-user (2, 2) and three-user (3, 2) systems.

each user at a subcarrier. Moreover, we also compare the throughputs of the proposed algorithms with the maximum sum rate of the channel [5], [6], which can be served as an upper bound.

B. Results

Fig. 1 shows the throughput results of the proposed algorithms, the GZF solution, and the sum capacity against the transmit signal-to-noise ratio (SNR) P_T/σ^2 for a two-user (2, 2) system and a three-user (3, 2) system. As can be seen, the proposed scheme with the SP power control outperforms the GP and GZF algorithms, whereas in the high SINR region, the performance gap between SP and GP becomes negligible, as expected. For the three-user (3, 2) system, when transmit SNR = 15 dB, and the throughput using GZF is about 4.2 b/s/Hz, an increase of 20% can be obtained using the proposed scheme with the SP power control. In the low SINR region, however, the performance of the GP algorithm degrades and is similar to that of GZF. This is because of the fact that the GP algorithm is forced to operate all the spatial subchannels of the users at all subcarriers.

In Figs. 2 and 3, we provide some examples of the optimized power allocation (from SP, GP, or OFDMA) in the space–frequency domain to demonstrate how subcarrier and spatial subchannel allocations for users are achieved. Fig. 2 shows the comparison of the power allocations for SP and OFDMA at SNR = 0 dB (i.e., low SINR regimes). As can be seen, for most subcarriers, only one user is assigned by SP, which is consistent with the OFDMA approach. By and large, the solutions from SP and OFDMA coincide with each other with only a very small difference. Power allocations for high SINRs are investigated in Fig. 3, where 20 dB of SNR is assumed. Results indicate that each subcarrier is usually occupied by more than one user (or one user but with many subchannels sometimes). In addition, a close observation reveals that the total power is approximately equally allocated among the users in the case when the users have the same channel statistics. Moreover, it is observed that the GP-based power control goes to the extreme to force an equal power allocation at all subcarriers for all the users.

V. CONCLUSION

This paper has studied the throughput maximization problem of a multiuser MIMO–OFDM beamforming system in the downlink with a total power constraint when the channel state information of the users is known at the transmitter and the receivers. An iterative algorithm

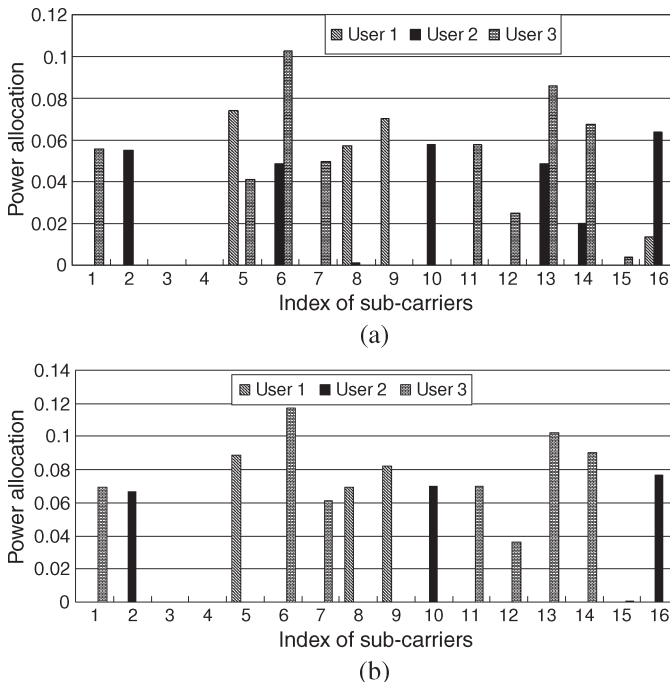


Fig. 2. Power allocation for a three-user (3, 2) system with transmit SNR = 0 dB. (a) SP-based power allocation. (b) Simplified OFDMA-based power allocation.

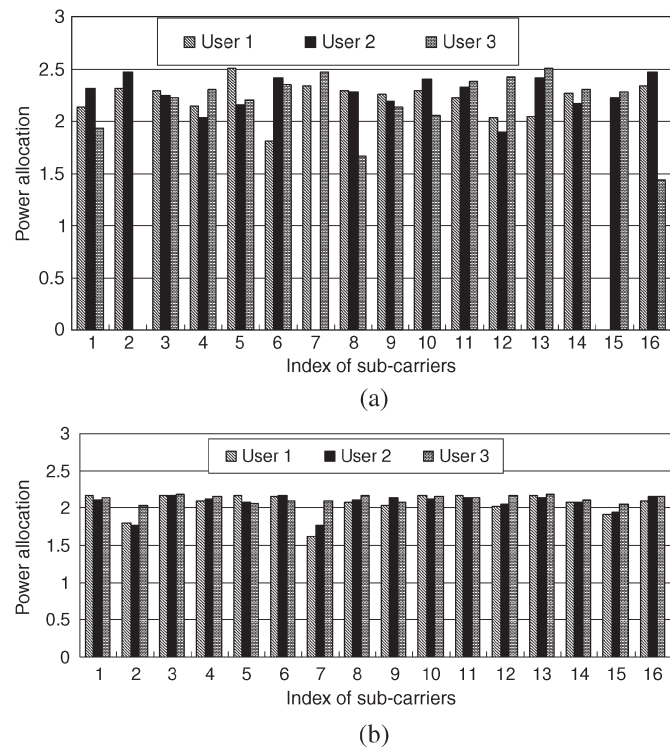


Fig. 3. Power allocation for a six-user (6, 3) system using the adaptive spatial subchannel allocation with transmit SNR = 20 dB. (a) SP-based power allocation. (b) GP-based power allocation.

that takes turns optimizing the power allocation and the TR-GBF antenna vectors of the users has been devised. It has been proved that the throughput monotonically increases from one iteration to the next, and hence, the algorithm is convergent. The proposed method

also has the ability to handle users' individual rate constraints. Two simplified optimization strategies have been presented to reduce the processing complexity. Simulation results have demonstrated a considerable performance gain of the proposed methods when compared with the existing GZF method.

APPENDIX

UPLINK-DOWNLINK DUALITY FOR MIMO-OFDM

Here, we extend the duality between the uplink and the downlink of a linear multiuser MIMO system that is established in [17] to a linear multiuser MIMO-OFDM system, where mobile stations have multiple antennas, and each user has, generally, multiple spatial subchannels. In this setting, at a particular subcarrier n , the M -user MIMO system with multiple streams $\{N_m\}$ can be viewed as an M_T -user ($M_T = \sum_{m=1}^M N_m$) multiple-input-single-output single-stream system, and the duality theory in [17] can be applied. Therefore, $\mathbf{T}_m[k, n]$ has the following form:

$$\mathbf{T}_m[k, n] = \zeta_m[k, n] \left(\sum_{\substack{\tilde{m}=1 \\ (\tilde{m}, \tilde{k}) \neq (m, k)}}^M \sum_{\tilde{k}=1}^{N_{\tilde{m}}} q_{\tilde{m}}[\tilde{k}, n] \mathbf{H}_m^\dagger[n] \mathbf{R}_{\tilde{m}}^\dagger \times [\tilde{k}, n] \mathbf{R}_{\tilde{m}}[\tilde{k}, n] \mathbf{H}_m[n] + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}_m^\dagger[n] \mathbf{R}_m[k, n] \quad (15)$$

where $\zeta_m[k, n]$ is chosen such that $\|\mathbf{T}_m[k, n]\| = 1$. The power allocation in the VU is, therefore, obtained by choosing the principal eigenvector of the uplink coupled matrix \mathbf{C}_n^{UL} , which is defined as follows:

$$\mathbf{C}_n^{\text{UL}} \triangleq \begin{bmatrix} \mathbf{D}_n \Psi_n^T & \mathbf{D}_n \mathbf{1} \\ \frac{1^T \mathbf{D}_n \Psi_n^T}{Q_n} & \frac{1^T \mathbf{D}_n \sigma}{Q_n} \end{bmatrix} \quad (16)$$

where $\mathbf{1}$ is a column vector with all ones, Q_n is the total transmit power at the n th subcarrier, \mathbf{D}_n is given by

$$\mathbf{D}_n = \text{diag} \left\{ \frac{\Gamma_1[1, n]}{|\mathbf{R}_1[1, n]^\dagger \mathbf{H}_1[n] \mathbf{T}_1[1, n]|^2}, \dots, \frac{\Gamma_M[N_M, N_f]}{|\mathbf{R}_M[N_M, N_f]^\dagger \mathbf{H}_M[n] \mathbf{T}_M[N_M, N_f]|^2} \right\} \quad (17)$$

and

$$[\Psi_n]_{\tilde{N}_m+k, \tilde{N}_{\tilde{m}}+\tilde{k}} = \begin{cases} \frac{|\mathbf{R}[k, n]^\dagger \mathbf{H}_m[n] \mathbf{T}_{\tilde{m}}[\tilde{k}, n]|^2}{\sigma^2}, & (\tilde{m}, \tilde{k}) \neq (m, k) \\ 0, & (\tilde{m}, \tilde{k}) = (m, k) \end{cases} \quad (18)$$

where $\tilde{N}_m = \sum_{s=1}^{m-1} N_s$. This process is repeated for all of the subcarriers to obtain all the uplink power allocations. The downlink power

allocation can be found in the same way by choosing the principal eigenvector of the downlink coupled matrix \mathbf{C}_n^{DL} given by

$$\mathbf{C}_n^{\text{DL}} \triangleq \begin{bmatrix} \mathbf{D}_n \Psi_n & \mathbf{D}_n \mathbf{1} \\ \frac{1^T \mathbf{D}_n \Psi_n}{Q_n} & \frac{1^T \mathbf{D}_n \sigma}{Q_n} \end{bmatrix} \quad (19)$$

such that the same SINR region is achieved for both links with the same total power.

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