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# Analytical studies on transient groundwater flow induced by land reclamation

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[1] In many coastal areas, land has been reclaimed by dumping fill materials into the sea. Land reclamation may have a significant effect on groundwater regimes, especially when the reclamation is at large scale. Analytical studies on the impact of land reclamation on steady-state ground water flow conditions were conducted previously, but transient analytical solutions are not yet available. Transient analytical solutions are derived to illustrate the temporal change of groundwater systems in response to land reclamation using two hypothetical models: a hillside aquifer and an oceanic elongated island. The analytical solutions show that when time is short, the water level in the reclaimed area increases significantly after reclamation while that in the original aquifer remains almost unchanged. When time is great, the change of water level in the reclaimed site becomes small but the increase of water level propagates into the original aquifer. For the specific parameters and aquifer geometry used in the examples, it takes at least over 100 years for the whole system to approach a new equilibrium. The island example demonstrates that land reclamation on one side of the island will eventually modify the groundwater regimes over the entire island, including the water level, water divide, and submarine groundwater discharge. The degree of the modification of the groundwater system and the time required for the system to approach a new equilibrium depend mainly on the hydraulic conductivity and storativity of the fill materials and the reclamation length. It is suggested that for a large reclamation project, the response of the groundwater regime to reclamation should be studied in detail to evaluate the long-term change of the flow system and the consequent environmental and engineering impacts.

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## 1. Introduction

[2] Land reclamation has been carried out along coastal areas around the world since the 18th century to provide land for various purposes such as agriculture, sea and air ports, and housing development. This practice has become increasingly popular because of the increasing population and booming economies along coastal areas. In coastal cities such as Hong Kong, Shenzhen and Shanghai in China, hundreds of square kilometers have been reclaimed from the sea. The groundwater flow pattern is generally modified by large-scale land reclamation. However the impact of reclamation on groundwater flow systems has not yet been widely recognized probably because the response is often slow and subtle [*Jiao*, 2000].

[3] In recent years, the hydrogeological research group in the University of Hong Kong has carried out a series of studies on the impact of land reclamation on groundwater flow systems [*Jiao*, 2000; *Jiao et al.*, 2001; *Guo and Jiao*, 2007]. *Jiao et al.* [2001] derived analytical solutions to demonstrate how the groundwater level, groundwater divide and submarine groundwater discharge would change with land reclamation near coastal aquifer systems with single-density flow. *Guo and Jiao* [2007] derived analytical solutions for the same aquifer system but considered the interface between the salt water and fresh water. However all these solutions can be used only to study the long-term impact of land reclamation on groundwater flow because they are steady state.

[4] The transient response to land reclamation is far more interesting and important. After a reclamation project is completed, it would be very useful to predict the temporal change of the groundwater level and how long it will take for the system to approach a new equilibrium. This requires transient analytical solutions, but these solutions are hard to achieve because of the difficulty of solving the non-linear equations.

[5] As an extension of the previous study by *Jiao et al.* [2001], this paper derives the transient solution for groundwater flow induced by land reclamation. Two conceptual models are used. In the first model, reclamation occurs along a hillside aquifer with assumption that the groundwater divide does not move after reclamation. The second model is an elongated oceanic island where the reclamation

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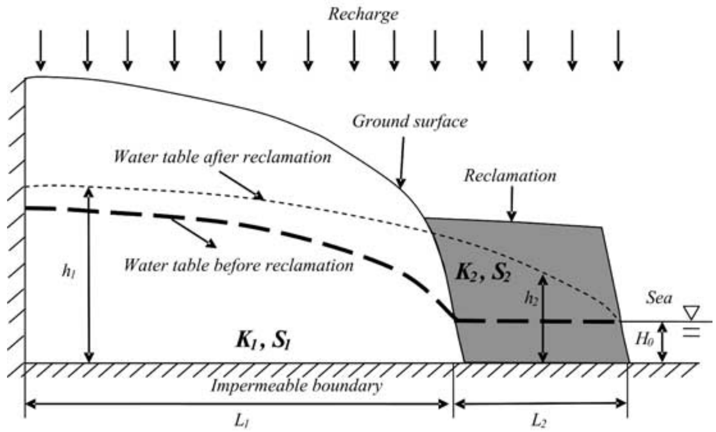


**Figure 1.** A unconfined aquifer system in a coastal hillside before and after reclamation (the grey area represents the reclaimed land).

on one side of the island may affect the ground water regime in the entire area of an island.

[6] It is assumed that precipitation is uniform and the only recharge and the groundwater flow before reclamation is steady-state. The aquifer is relatively thin compared to its lateral extent, so that the vertical resistance to flow can be neglected (the Dupuit assumption). Both the original aquifer and fill materials are assumed to be homogeneous and isotropic. The groundwater in the systems has uniform density.

[7] Modern land reclamation is usually carried out rapidly. For example, 9.4 $km^2$ of the Hong Kong International Airport was reclaimed from the sea within about 32 months and at the peak of activity, land was being formed at the rate of 20,000 $m^2$/day [*Plant et al.*, 1998; *Pickles and Tosen*, 1998]. As will be discussed, land reclamation is usually completed over a period which usually is much shorter than the transient response of the groundwater flow system induced by land reclamation. The land reclamation therefore is assumed to occur instantaneously.

## 2. Impact of Reclamation on Ground Water in a Coastal Hillside

[8] Assume that the aquifer is unconfined, with hydraulic conductivity of $K_1$ (L/T) (as shown in Figure 1), and that the groundwater flow satisfies the Dupuit assumption [*Fetter*, 1994]. The left boundary is a groundwater divide and represented as a no-flow boundary, and the right boundary is constant head boundary controlled by the sea level. The distance from the divide to the sea is $L_1$ (L). The uniform recharge rate is $R$ (L/T). The datum of the water table is the horizontal impermeable bottom of the unconfined aquifer. The average sea level is assumed to be $H_0$ (L). Because of the reclamation, the coastline moves toward the sea by $L_2$ (L), which is called the reclamation length. The hydraulic conductivity of the fill materials is $K_2$ (L/T). The storativities of original media and fill materials are $S_1$ and $S_2$ respectively. A total length is define as $L = L_1 + L_2$.

### 2.1. Analytical Solution for Groundwater Flow After Reclamation

[9] Assume that the groundwater flow is steady-state before reclamation, and that reclamation is completed instantly. The mathematical model of the unconfined aquifer system immediately after land reclamation can be described by equation (1).

$$\begin{cases} \dfrac{\partial}{\partial x}\left(Kh\dfrac{\partial h}{\partial x}\right) + R = S\dfrac{\partial h}{\partial t} & \\ \left.\dfrac{\partial h}{\partial x}\right|_{x=0} = 0 & t > 0 \\ h|_{x=L_1+L_2} = H_0 & t > 0 \\ h|_{x=L_1^+} = h|_{x=L_1^-} & t > 0 \\ K_1\left.\dfrac{\partial h}{\partial x}\right|_{x=L_1^+} = K_2\left.\dfrac{\partial h}{\partial x}\right|_{x=L_1^-} & t > 0 \\ h|_{t=0} = \sqrt{H_0^2 + R\left(L_1^2 - x^2\right)/K_1} & 0 \le x \le L_1 \\ h|_{t=0} = H_0 & L_1 \le x \le L \end{cases} \tag{1}$$

A function $\varphi$ is introduced and defined as

$$\varphi = h^2/2$$

The governing equation in equation (1) is nonlinear and does not admit general solutions. Therefore in most studies the hydraulic approach is simplified by the introduction of additional approximations. Simple linearization should be more broadly applicable, because it involves fewer approximations [*Brutsaert*, 1994]. Linearizing the Boussinesq equation in equation (1) in terms of $h^2$ or $h$ is discussed by *Chapman* [1995] and *Brutsaert* [1995]. when $L \gg D$, where $D$ is the thickness of the saturated aquifer, adoption of $h^2$ as the dependent variable, instead of $h$, does not appear particularly useful or meaningful [*Brutsaert*, 1995], but for solving the governing equation in equation (1) easily, the form of $h^2$ is adopted. In most cases, the linearization of head is in the form $\overline{h} = p \times D$. $p$ is a constant introduced to compensate for the approximation resulting from the linearization and is specified through the comparison between the analytical and numerical results. In this paper, arithmetic average is applied to obtain the average hydraulic transmissivity. Although this simplification causes some error, the results will be compared with the results of a numerical model and be checked whether it is applicable.

[10] Equation (1) can be linearized as

$$\begin{cases} \dfrac{K_1\overline{h_1}}{S_1}\dfrac{\partial^2\varphi_1}{\partial x^2} + \dfrac{R\overline{h_1}}{S_1} = \dfrac{\partial\varphi_1}{\partial t} & 0 \le x \le L_1 \\ \dfrac{K_2\overline{h_2}}{S_2}\dfrac{\partial^2\varphi_2}{\partial x^2} + \dfrac{R\overline{h_2}}{S_2} = \dfrac{\partial\varphi_2}{\partial t} & L_1 \le x \le L \\ \left.\dfrac{\partial\varphi_1}{\partial x}\right|_{x=0} = 0 & t > 0 \\ \varphi_2|_{x=L} = H_0^2/2 & t > 0 \\ \varphi_1|_{x=L_1^+} = \varphi_2|_{x=L_1^-} & t > 0 \\ K_1\left.\dfrac{\partial\varphi_1}{\partial x}\right|_{x=L_1^+} = K_2\left.\dfrac{\partial\varphi_2}{\partial x}\right|_{x=L_1^-} & t > 0 \\ \varphi_1|_{t=0} = H_0^2/2 + R\left(L_1^2 - x^2\right)/(2K_1) & 0 \le x \le L_1 \\ \varphi_2|_{t=0} = H_0^2/2 & L_1 \le x \le L. \end{cases} \quad (2)$$

Where $\overline{h_1}$ and $\overline{h_2}$ are linearizations of the arithmetic average of groundwater level, and are defined as:

$$\overline{h_1} = \left(h|_{x=0} + h|_{x=L_1}\right)/2.0 \quad (3)$$

$$\overline{h_2} = \left(h|_{x=L_1} + h|_{x=L}\right)/2.0 = H_0 \quad (4)$$

When there is no recharge, the equations are classic heat conduction equations. These topics about composite media have been widely discussed [e.g., *Carslaw and Jaeger*, 1959; *Monte*, 2000, 2006].The mathematical equations can be solved by the method of separation of variables. The difficulty lies in how to apply the initial conditions in solving the equations and the key is to establish the relationship between the eigenvalues for the different media. *Monte* [2000] developed a new type of orthogonality relationship to obtain the final complete series solution. The detailed derivation of this solution can be seen in Appendix A.

[11] Introduce the following dimensionless variables. $x_D = x/L_1, L_D = L_2/L_1, K_D = K_2/K_1, R_D = RL_1^2/K_1/H_0^2, t_D = a_2\, t/L_1^2, a_D$ $a_D = a_1/a_2$, $h_D = h/H_0$, where $a_1 = K_1\, \overline{h_1}/S_1$, $a_2 = K_2\, \overline{h_2}/S_2$. The final solutions of the mathematical equations in dimensionless form can be expressed as:

For $0 \le x_D \le 1$:

$$h_D^2(x_D, t_D) = \begin{cases} 1 + \dfrac{R_D}{K_D}\left[(1+L_D)^2 - 1\right] + R_D\left(1 - x_D^2\right) + 2\displaystyle\sum_{m=1}^{\infty} b_D \cos\lambda_D(x_D - 1)\exp\left(-\lambda_D^2 a_D t_D\right) & \sin\lambda_D = 0 \\ 1 + \dfrac{R_D}{K_D}\left[(1+L_D)^2 - 1\right] + R_D\left(1 - x_D^2\right) - 2K_D\sqrt{a_D}\displaystyle\sum_{m=1}^{\infty} c_D \dfrac{\cos(\lambda_D x_D)}{\sin\lambda_D}\exp\left(-\lambda_D^2 a_D t_D\right) & \sin\lambda_D \ne 0 \end{cases} \quad (5)$$

For $1 \le x_D \le 1 + L_D$:

$$h_d^2(x_D, t_D) = \begin{cases} 1 + \dfrac{R_D}{K_D}\left((1+L_D)^2 - x_D^2\right) + 2\displaystyle\sum_{m=1}^{\infty} b_D \cos(\lambda_D\sqrt{a_D}(x_D - 1)\exp\left(-\lambda_D^2 a_D t_D\right) & \sin\lambda_D = 0 \\ 1 + \dfrac{R_D}{K_D}\left((1+L_D)^2 - x_D^2\right) + 2\displaystyle\sum_{m=1}^{\infty} c_D \dfrac{\sin\left(\lambda_D\sqrt{a_D}(x_D - 1 - L_D)\right)}{\cos\left(\lambda_D L_D\sqrt{a_D}\right)}\exp\left(-\lambda_D^2 a_D t_D\right) & \sin\lambda_D \ne 0 \end{cases} \quad (6)$$

$b_D$, $c_D$, $N_D$, and $\lambda_D$ are group parameters, and they can be calculated by the following system of equations.

$$b_D = \frac{2R_D}{K_D(1+L_D)a_D{\lambda_D}^2}\left(-1 - \frac{\sin\left(\lambda_D L_D\sqrt{a_D}\right)}{\lambda_D\sqrt{a_D}}\right) \quad (7)$$

$$c_D = \frac{R_D}{N_D{\lambda_D}^2}\left(\frac{1}{\lambda_D\sqrt{a_D}\cos\left(\lambda_D L_D\sqrt{a_D}\right)} - \frac{1}{\lambda_D\sqrt{a_D}} + \tan(\lambda_D L_D\sqrt{a_D})\right) \quad (8)$$

$$N_D = \frac{K_D a_D}{2}\left[\frac{K_D}{\sin^2\lambda_D} + \frac{L_D}{\cos^2\left(\lambda_D L_D\sqrt{a_D}\right)}\right] \quad (9)$$

$$\lambda_D = \lambda_{1m} L_1 \quad (10)$$

$$(1 - K_D\sqrt{a_D})\cos(1 - L_D\sqrt{a_D})\lambda_D = (1 + K_D\sqrt{a_D})\cos(1 + L_D\sqrt{a_D})\lambda_D \; (m = 1, 2, 3, \ldots) \quad (11)$$

$$(m-1)\pi < (1 + L_D\sqrt{a_D})\lambda_D \le m\pi \; (m = 1, 2, 3, \ldots) \quad (12)$$

When $t \to \infty$, the time-variant dimensionless items in equations (5) and (6) approach zero and the two equations will be identical to the steady state solutions of equations (12) and (13) in *Jiao et al.* [2001], respectively.

[12] When $0 \leq x \leq L_1$, the increase of water table ($\Delta h = h_D - 1$) can be derived from equation (5) and the initial condition of the mathematical model, which will be used in the following discussion.

$$\Delta h = \begin{cases} \sqrt{1 + \frac{R_D}{K_D}\left[(1+L_D)^2 - 1\right] + R_D\left(1 - x_D^2\right) + 2\sum_{m=1}^{\infty} b_D \cos\lambda_D(x_D - 1)\exp\left(-\lambda_D^2 a_D t_D\right)} - \sqrt{1 + R_D(1 - x_D^2)} & \sin\lambda_D = 0 \\ \sqrt{1 + \frac{R_D}{K_D}\left[(1+L_D)^2 - 1\right] + R_D\left(1 - x_D^2\right) - 2K_D\sqrt{a_D}\sum_{m=1}^{\infty} c_D \frac{\cos(\lambda_D x_D)}{\sin\lambda_D}\exp\left(-\lambda_D^2 a_D t_D\right)} - \sqrt{1 + R_D(1 - x_D^2)} & \sin\lambda_D \neq 0. \end{cases} \quad (13)$$

## 2.2. Comparison of Analytical and Numerical Solutions in a Coastal Hillside Aquifer

[13] To assess the appropriateness of the linearization used in developing the above equations, the groundwater finite-element modeling software FEFLOW [*Diersch*, 2005] is applied to re-calculate the change of water level in response to land reclamation in the coastal hillside aquifer. Automatic time step control via predictor-corrector schemes (AB/TR time integration scheme) is adopted. The convergence criteria of water level is $10^{-5}$ m. and maximum number of iterations per time step is set at 100. Figure 2a shows the changes of water level with distance obtained from numerical and analytical solutions using dimensionless format. There are only minor differences in the region of $x = L_1$, which could be due to the linearization in the analytical model or the discretization of the numerical model. Figures 2b and 2c shows the increases of water level at the original coastline with time in response to land reclamation for different $K_D$ and $L_D$, respectively. There is some difference between the two approaches, especially when $K_D$ is small and $L_D$ is large, but overall, the analytical solution can describe the dynamic response of the aquifer system induced by land reclamation.

## 2.3. Discussion of the Solutions Using a Hypothetical Example

[14] To provide the readers a sense of the actual time scales of the unsteady groundwater response to land reclamation in a realistic reclamation project, a hypothetical example is presented here using typical aquifer parameters and the reclamation length in Hong Kong. The hydraulic conductivity in a reclamation site is extremely unpredictable and varies with the nature of the fill materials and the method of placement. More discussions of hydraulic conductivity of fill materials or case studies on land reclamation can be found in previous publications [*Jiao et al.*, 2001; *Li et al.*, 2002; *Jiao and Li*, 2004; *Jiao et al.*, 2006, 2008]. For this hypothetical study, the hydraulic conductivity of the aquifer ($K_1$) is set to be 0.1 m/day and the distance from the groundwater divide to the coastline ($L_1$) is 1000 m. The storativity of the original aquifer ($S_1$) and fill materials ($S_2$) is 0.1. The recharge rate ($R$) is 0.0006 m/day and $H_0$ = 10 m.

[15] Figure 3 shows the change of the groundwater level in the aquifer ($h$) with distance from the groundwater divide ($x$) for different time ($t$) after reclamation. It can be seen that the change of the groundwater level ($h$) is rapid inside the reclamation area but in the original aquifer change is relatively slow when time is small. As time increases, the increase of water level gradually propagates upstream in the original aquifer. After about 10 years, the water level change inside the reclamation site becomes small but the water level increase in the original aquifer, especially toward the left boundary, continues at a noticeable rate.

[16] The water levels for $t$ = 100 and 1000 years are very similar. This suggests that the system is close to steady state after about 100 years. How long it will take for the system to approach a quasi-steady state depends on the error tolerance ($\varepsilon$), which is defined as the absolute difference between the calculated water level at the time $t$ and the steady state water level ($h_s$) when $t \to \infty$, divided by $h_s$. In the following discussion, the time required to approach a quasi-steady state is calculated using $\varepsilon$ equals 95% and 99%.

[17] Figure 4a shows the increase of water level ($\Delta h$) with time ($t$) at the original coastline for different values of $K_2$, when $L_2$ = 500 m, $K_1$ = 0.1m/d and $S_1 = S_2 = 0.1$. For low $K_2$, the buildup of the water level at the interface is large and sensitive to the value of $K_2$. When $K_2$ is high, the buildup of the water level is less sensitive to $K_2$. The buildup of the water level is most significant when the hydraulic conductivity of the fill materials is lower than that of the background aquifer, as shown by the dark solid line.

[18] Figure 4b shows the increase of water level ($\Delta h$) with time ($t$) at the original coastline for different values of storativity when $L_2$ = 500 m and $K_1 = K_2$ = 0.1 m/d. For low storativity, the system shows quicker response to reclamation and the buildup of the water level at the interface is large.

[19] When $K_2$ is high, the time required for the water level to approach steady state is short. Table 1 presents the time for the system to approach a quasi-steady state using different $\varepsilon$. When $\varepsilon$ is 95%, the time for the system to approach a quasi-steady state ranges from about 1 year

**Figure 2.** Comparisons of the analytical and numerical results in an unconfined aquifer system in a hillside near the coast. (a) Dimensionless groundwater level ($h_D$) in the aquifer with dimensionless distance from dimensionless groundwater divide ($x_D$) for different dimensionless times ($t_D$) when $K_D$ = 5, $L_D$ = 0.5; (b) increase of groundwater level ($\Delta h = h_D - 1$) at the original coastline with dimensionless time ($t_D$) for different dimensionless hydraulic conductivity ($K_D$) when $L_D$ = 0.5; (c) increase of groundwater level ($\Delta h = h_D - 1$) in the aquifer with dimensionless time ($t_D$) at the original coastline for different dimensionless reclamation length ($L_D$) when $K_D$ = 5.

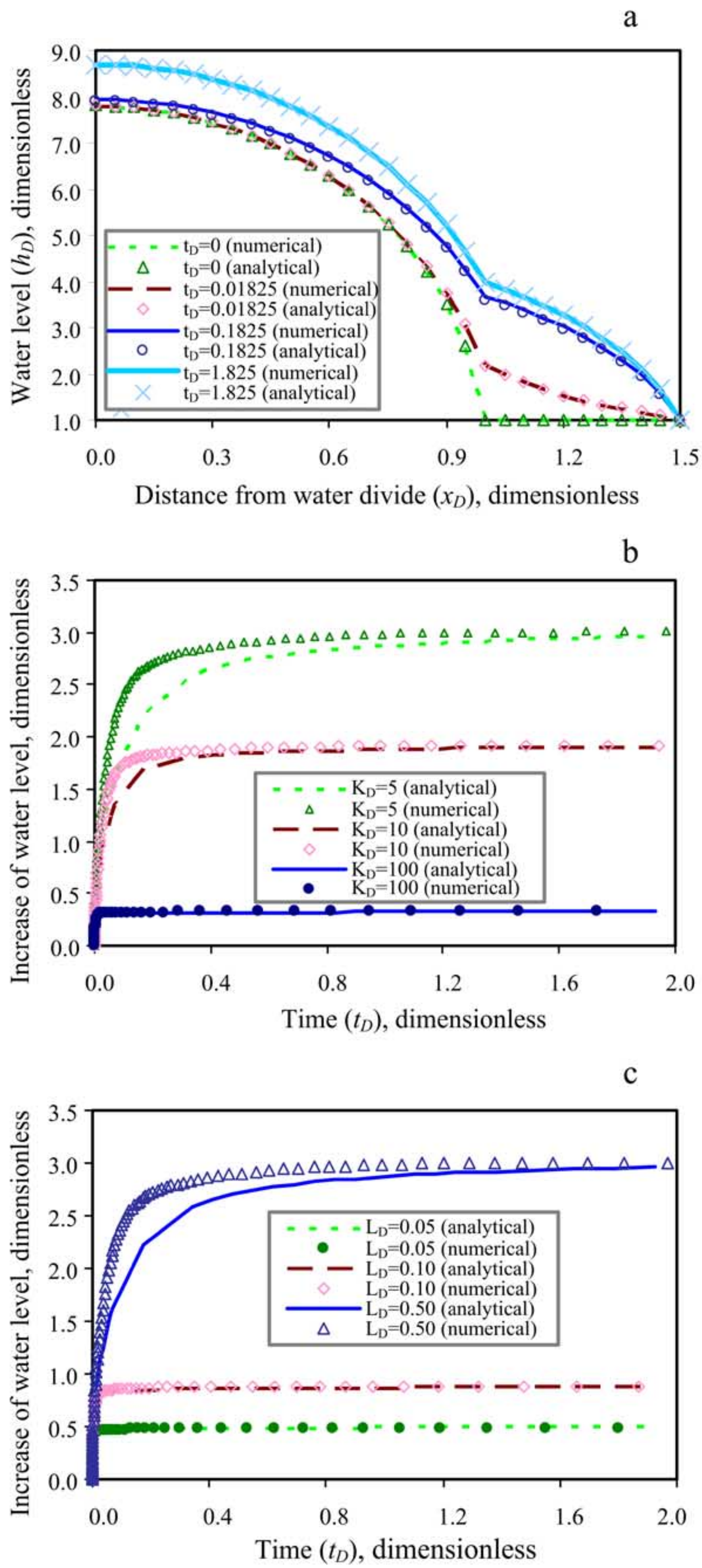
a
Water level ($h_D$), dimensionless
Distance from water divide ($x_D$), dimensionless
$t_D$=0 (numerical)
$t_D$=0 (analytical)
$t_D$=0.01825 (numerical)
$t_D$=0.01825 (analytical)
$t_D$=0.1825 (numerical)
$t_D$=0.1825 (analytical)
$t_D$=1.825 (numerical)
$t_D$=1.825 (analytical)
b
Increase of water level, dimensionless
Time ($t_D$), dimensionless
$K_D$=5 (analytical)
$K_D$=5 (numerical)
$K_D$=10 (analytical)
$K_D$=10 (numerical)
$K_D$=100 (analytical)
$K_D$=100 (numerical)
c
Increase of water level, dimensionless
Time ($t_D$), dimensionless
$L_D$=0.05 (analytical)
$L_D$=0.05 (numerical)
$L_D$=0.10 (analytical)
$L_D$=0.10 (numerical)
$L_D$=0.50 (analytical)
$L_D$=0.50 (numerical)

**Figure 2**

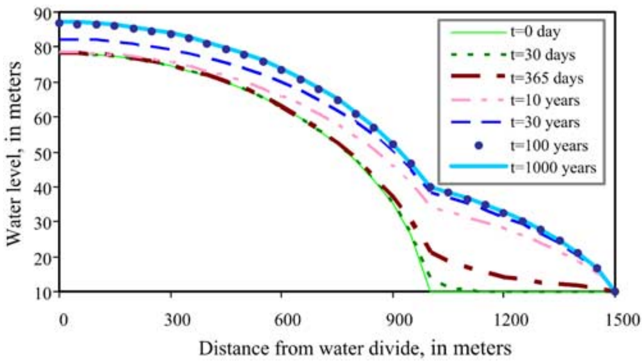


**Figure 3.** Groundwater level in the aquifer with distance from the groundwater divide for different times when $K_2$ = 0.5 m/d, $L_2$ = 500 m.

**Table 1.** Time Required for the Groundwater System to Approach a Quasi-Steady State at the Original Coastline for Different Hydraulic Conductivity of the Fill Materials ($K_2$) and Reclamation Length ($L_2$) When $\varepsilon$ is 95% and 99%

| $L_2$ (m) | $K_2$ (m/d) | $t$ (years, $\varepsilon$ = 95%) | $t$ (years, $\varepsilon$ = 99%) |
|---|---|---|---|
| 500 | 0.5 | 35.02 | 86.94 |
| | 1 | 15.24 | 56.61 |
| | 5 | 2.10 | 13.82 |
| | 7.5 | 1.34 | 7.24 |
| | 10 | 0.99 | 5.16 |
| 50 | 0.5 | 0.68 | 13.53 |
| 100 | | 2.29 | 29.30 |
| 300 | | 16.26 | 60.56 |
| 500 | | 35.00 | 86.79 |
| 1000 | | 94.50 | 191.77 |

when $K_2$ = 10 m/day to about 35 years when $K_2$ = 0.5 m/day if $L_2$ = 500 m. However, when $\varepsilon$ is 99%, the time for the system to approach a quasi-steady state is three to five times larger than for $\varepsilon$ = 95%.

[20] Figure 5 shows the increase of water level ($\Delta h$) with time ($t$) at the original coastline for different $L_2$ when $L_1$ = 1000 m, $K_1$ = 0.1 m/d and $K_2$ = 0.5 m/d. The final water level buildup is about 5 m when $L_2$ = 50 m, but is over 50 m when $L_2$ is longer than 1000 m. The water level is increased with larger $L_2$ because the flow path to the sea is extended and there is additional recharge in the newly reclaimed land. Table 1 also shows that the time for the system to approach a quasi-steady state ($\varepsilon$ = 95%) ranges from 0.68 year when $L_2$ = 50 m to about 94.5 years when $L_2$ = 1000 m.

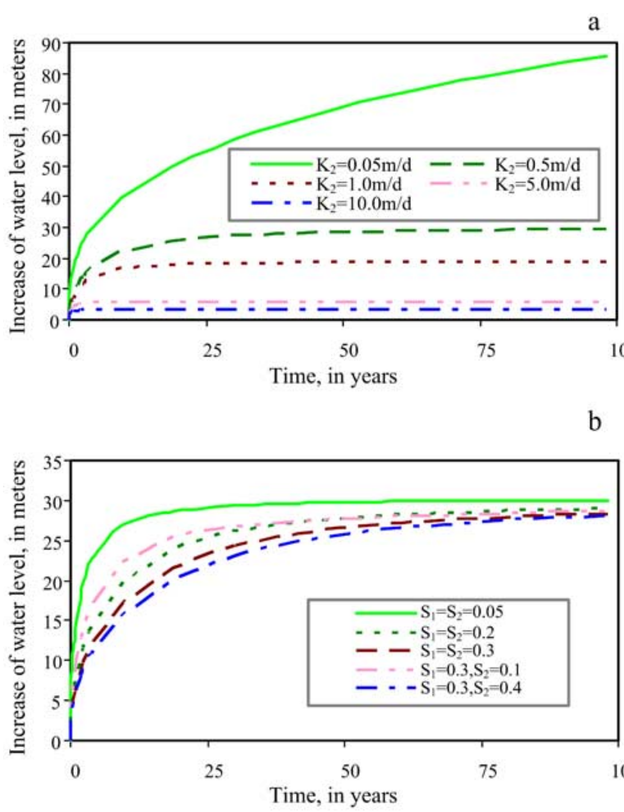


**Figure 4.** Increase of groundwater level at the original coastline with time for different hydraulic conductivity values of the fill when $L_2$ = 500 m, $K_1$ = 0.1 m/d; $S_1$ = $S_2$ = 0.1 (a) and for different storativity values when $L_2$ = 500 m, $K_1$ = $K_2$ = 0.1 m/d (b).

## 3. Impact of Reclamation on Ground Water in an Oceanic Elongated Island

[21] Many coastal groundwater models assume that the ground water divide is so far from the coastline that it remains unchanged after reclamation [*Gingerich and Voss*, 2005]. In an oceanic elongated island, the water divide often moves when the reclamation length is significant compared to the width of the island, as often occurred in Hong Kong [*Jiao*, 2000]. In such cases, the reclamation on one side of the island may cause displacement of the ground water divide and eventually change ground water conditions on the other side. How the water level changes and the water divide moves under the influence of coastal reclamation will now be discussed.

[22] Figure 6 shows the schematic profile of an unconfined aquifer system in an elongated island near the coast. Reclamation occurs on the right coast. Assume the island is bounded below by a horizontal impermeable layer, the groundwater flow is in a steady state before reclamation, and the reclamation is completed instantly.

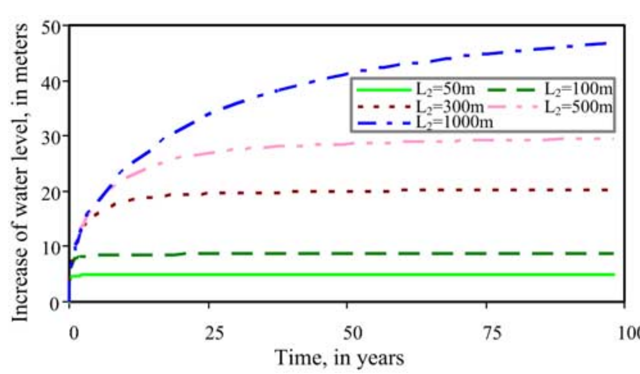


**Figure 5.** Increase of groundwater level in the aquifer with time at the original coastline for different reclamation length ($L_2$) when $L_1$ = 1000 m, $K_1$ = 0.1 m/d and $K_2$ = 0.5 m/d.

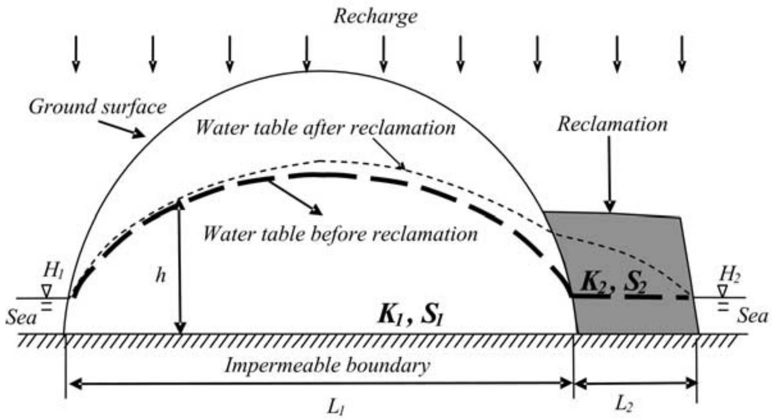


**Figure 6.** Groundwater in an unconfined aquifer system in an oceanic elongated island near the coast.

### 3.1. Analytical Solutions for Groundwater Flow After Reclamation

[23] The mathematical model of the unconfined aquifer system immediately after the land reclamation can be described as:

$$\begin{cases} \dfrac{\partial}{\partial x}\left(Kh\dfrac{\partial h}{\partial x}\right) + R = S\dfrac{\partial h}{\partial t} & \\ h|_{x=0} = H_1 & t > 0 \\ h|_{x=L_1+L_2} = H_2 & t > 0 \\ h|_{x=L_1^+} = h|_{x=L_1^-} & t > 0 \\ K_1\dfrac{\partial h}{\partial x}\Big|_{x=L_1^+} = K_2\dfrac{\partial h}{\partial x}\Big|_{x=L_1^-} & t > 0 \\ h|_{t=0} = \sqrt{H_1^2 + x\left(H_2^2 - H_1^2\right)/L_1 + Rx(L_1 - x)/K_1} & 0 \le x \le L_1 \\ h|_{t=0} = H_2 & L_1 \le x \le L \end{cases} \tag{14}$$

The following dimensionless variables are introduced:

$$x_D = x/L_1,\ L_D = L_2/L_1,\ K_D = K_2/K_1,\ R_D = RL_1^2/K_1/H_1^2,$$
$$t_D = a_2t/L_1^2,\ a_D = a_1/a_2,\ H_{1D} = H_2/H_1,\ h_D = h/H_1$$

Using similar methods as in the previous hillside case, the solutions are obtained as:

[24] For $0 \le x_D \le 1$:

$$h_D^2(x_D, t_D) = \begin{cases} -R_Dx_D^2 + K_D\sigma_Dx_D + 1 + 2\sum_{m=1}^{\infty} d_D\cos\lambda_D(x_D - 1)\exp\left(-\lambda_D^2a_Dt_D\right) & \cos\lambda_D = 0 \\ -R_Dx_D^2 + K_D\sigma_Dx_D + 1 + 2K_D\sqrt{a_D}\sum_{m=1}^{\infty} e_D\dfrac{\sin(\lambda_Dx_D)}{\cos\lambda_D}\exp\left(-\lambda_D^2a_Dt_D\right) & \cos\lambda_D \ne 0 \end{cases} \tag{15}$$

For $1 \le x_D \le 1 + L_D$:

$$h_D^2(x_D, t_D) = \begin{cases} \dfrac{R_D}{K_D}\left((1+L_D)^2 - x_D^2\right) + \sigma_D(x_D - 1 - L_D) + H_{1D}^2 + 2\sum_{m=1}^{\infty} d_D\cos\left(\sqrt{a_D}\lambda_D(x_D - 1)\right)\exp\left(-\lambda_D^2a_Dt_D\right) & \cos\lambda_D = 0 \\ \dfrac{R_D}{K_D}\left((1+L_D)^2 - x_D^2\right) + \sigma_D(x_D - 1 - L_D) + H_{1D}^2 + 2\sum_{m=1}^{\infty} e_D\dfrac{\sin\left(\sqrt{a_D}\lambda_D(x_D - 1 - L_D)\right)}{\cos\left(\sqrt{a_D}\lambda_DL_D\right)}\exp\left(-\lambda_D^2a_Dt_D\right) & \cos\lambda_D \ne 0 \end{cases} \tag{16}$$

$d_D$, $e_D$, $F_m$, $\sigma_D$, $\lambda_D$ are group parameters, which are calculated by the following equations.

$$d_D = \frac{1}{(1+L_D)\lambda_D^2}\left\{H_{1D}^2 - 1 + R_D - K_D\sigma_D + \frac{\sigma_D}{a_D} - \frac{2R_D}{K_Da_D}\left(1 + \frac{\sin\left(\sqrt{a_D}\lambda_DL_D\right)}{\sqrt{a_D}\lambda_D}\right)\right\} \tag{17}$$

$$e_D = \frac{\sqrt{a_D}}{2F_D\lambda_D}\left\{K_D(H_{1D}^2 - 1 + R_D - K_D\sigma_D)\left(\frac{\tan\lambda_D}{\lambda_D} - 1\right) + K_D\sigma_D\left(L_D - \frac{\tan\left(\sqrt{a_D}\lambda_DL_D\right)}{\sqrt{a_D}\lambda_D}\right) + R_D(1 - (1+L_D)^2 + \frac{2}{\sqrt{a_D}\lambda_D}\left(\frac{1}{\sqrt{a_D}\lambda_D\cos\left(\sqrt{a_D}\lambda_DL_D\right)} - \frac{1}{\sqrt{a_D}\lambda_D} + \tan(\sqrt{a_D}\lambda_DL_D)\right)\right)\right\} \tag{18}$$

$$F_D = \frac{K_Da_D}{2}\left[\frac{K_D}{\cos^2\lambda_D} + \frac{L_D}{\cos^2\left(\lambda_DL_D\sqrt{a_D}\right)}\right] \tag{19}$$

$$\sigma_D = \frac{1}{K_D + L_D}\left(R_D + \frac{R_D}{K_D}\left((1+L_D)^2 - 1\right) + H_{1D}^2 - 1\right) \tag{20}$$

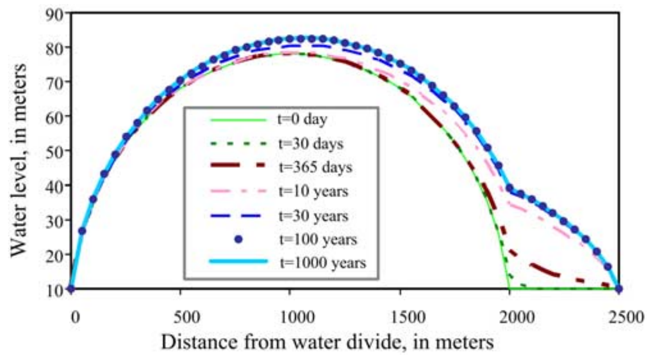


**Figure 7.** Groundwater level in the aquifer with distance from the left coastline for different time with $K_2$ = 0.5 m/d, $L_2$ = 500 m.

$$(1 + K_D\sqrt{a_D})\sin(1 + \sqrt{a_D}L_D)\lambda_D - (1 - K_D\sqrt{a_D})\sin(1 - \sqrt{a_D}L_D)\lambda_D = 0 (m = 1, 2, 3, \ldots) \quad (21)$$

$$\frac{2m-1}{2}\pi < (1 + \sqrt{a_D}L_D)\lambda_D \le \frac{2m+1}{2}\pi (m = 1, 2, 3, \ldots) \quad (22)$$

The groundwater flux in the original aquifer can be calculated using the Darcy equation:

$$q_D(x) = -h_D\frac{dh_D}{dx_D} = \begin{cases} R_D x_D - \frac{K_D\sigma_D}{2} + \sum_{m=1}^{\infty} d_D\lambda_D \sin\lambda_D(x_D - 1)\exp\left(-\lambda_D^2 a_D t_D\right) & \cos\lambda_D = 0 \\ R_D x_D - \frac{K_D\sigma_D}{2} - K_D\sqrt{a_D}\sum_{m=1}^{\infty} e_D\lambda_D \frac{\cos(\lambda_D x_D)}{\cos\lambda_D}\exp\left(-\lambda_D^2 a_D t_D\right) & \cos\lambda_D \ne 0 \end{cases} \quad (23)$$

By setting $q_D$ $(x)$ = 0, this can be solved for $x$ at the groundwater divide. Equation (23) then can be used to estimate the displacement of the water divide with respect to the original divide.

### 3.2. Discussion of the Solutions Using a Hypothetical Example

[25] The hypothetical example represents an oceanic elongated island. The heads on both sides of the island are equal ($H_1$ = $H_2$). The original distance between the left and right coastlines is 2000 m ($L_1$). All other parameters remain the same as the previous example. The groundwater levels in the entire island in response to reclamation can be calculated by equations (15) and (16) and the results can be seen in Figure 7. As in the previous example, when time is short the water level increase is most significant inside the reclamation site, and it takes time for the increase to propagate into the original coastal aquifer. Again, it takes over 100 years for the system to approach a quasi-steady state for this example. The time for the system to reach a steady state varies with the hydraulic conductivity of the fill materials and the reclamation length.

[26] Before reclamation, the groundwater divide is located at the middle of the island. After reclamation, the water divide shifts toward the right as expected. Figure 8 shows the position of the groundwater divide over time for different $K_2$. The displacement of the water divide is small when $K_2$ is large, but when $K_2$ is lower the movement is more significant. Meanwhile, the change in the groundwater discharge to the sea on the left equals the displacement of the water divide times the recharge rate. Consequently, if the water divide moves, seaward groundwater discharge to the left will be proportionally increased. This example demonstrates that reclamation on one side of the island will eventually change the whole groundwater regime in the entire island, including an increase in groundwater level and shifts in the divide and seaward groundwater discharge.

## 4. Summary and Conclusions

[27] Transient one-dimensional analytical solutions for groundwater flow under the influence of land reclamation near both a hillside and an oceanic elongated island are obtained using the method of separation of variables based on simplified aquifer systems. These solutions depict changing groundwater regimes in response to land reclamation, which are the major advantages of these solutions over the previous steady-state analytical solutions by *Jiao et al.* [2001]. For two hypothetical examples analyzed, in a short time after reclamation the increase in the water level inside the reclamation site is significant while the change inside the original aquifer is very small; after a long time, the water level buildup then gradually propagates into the original aquifer system while the water level inside the reclamation site approaches a quasi-steady state. The oceanic elongated island example demonstrates that land reclamation on one side of the island will eventually modify the groundwater regimes over the entire island, including water level, water divide and seaward groundwater discharge. The degree of the modification of the groundwater system by land reclamation and the time required for the system to approach a new equilibrium depend on the hydraulic conductivity and storativity of the original and fill materials and the reclamation length.

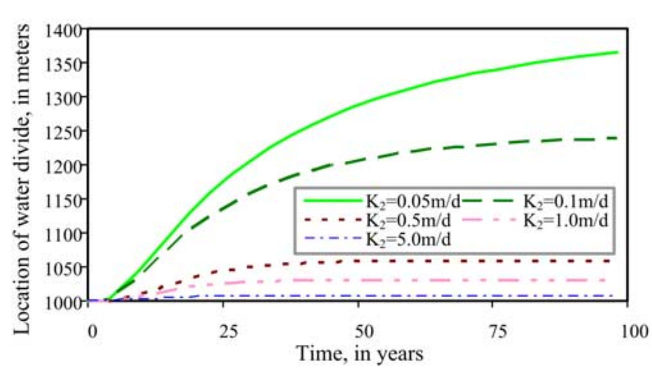


**Figure 8.** Position of the groundwater divide in the aquifer with time for different hydraulic conductivity ($K_2$) when $K_1$ = 0.1 m/d, $L_2$ = 500 m.

[28] Because the change in the original aquifer system may be slow, taking tens of years, it is possible that, by the time the impact of the reclamation becomes significant in the areas beyond the reclamation site, people would not link the effects to the long-forgotten reclamation, as speculated by *Jiao* [2000]. It is suggested that for a large reclamation site, the response of the groundwater regime to reclamation should be studied in detail to evaluate the long-term change of the flow system and the consequent environmental and engineering impacts. The hydraulic conductivity of the fill material and the reclamation length can be carefully selected to minimize the impact of the land reclamation on the flow system.

[29] The analytical studies here have many significant simplifying assumptions. Because of the difficulty in solving the non-linear equation, a simple linearization method is used. The analytical results match well with numerical model results that account for nonlinearities, but there exist some small differences, which is caused by the simple linearization method. The reclamation is assumed to occur instantly. This assumption is reasonable because reclamation projects usually are completed within one or two years. In most cases, this is much shorter than the time required for the groundwater system to approach a new equilibrium. The hydraulic conductivity in the reclamation site is assumed to be constant, while the soil in the real reclamation site, especially the mud at the original sea bottom, will go through a slow and time-variant consolidation process, which will decrease $K_2$ over time, slowing the response. In the analytical solutions presented in this paper, it is also assumed that groundwater flow satisfies the Dupuit assumption with constant recharge rate and the paper does not include the heterogeneity of the aquifer system, the topography at the coast, and the fresh/salt water density difference. In a real coastal reclamation sites with the low topography, springs and seepage may develop, as is commonly observed at the contact areas between the original coast and the newly reclaimed site in Hong Kong. In such cases, the water level increase calculated from the analytical solutions will be overestimated.

## Appendix A: Derivation of the Solutions of Groundwater Flow in a Hillside Aquifer With Land Reclamation

[30] Equation (2) is a non-homogeneous equation and requires homogenization. The two functions $w_1(x,t)$ and $w_2(x,t)$ are introduced here.

$$w_1(x) = \frac{H_0^2}{2} + \frac{R}{2K_2}\left(L^2 - L_1^2\right) + \frac{R}{2K_1}\left(L_1^2 - x^2\right) \tag{A1}$$

$$w_2(x) = \frac{H_0^2}{2} + \frac{R}{2K_2}\left(L^2 - x^2\right) \tag{A2}$$

The form of solutions is written as

$$\varphi_1(x,t) = v_1(x,t) + w_1(x) \tag{A3}$$

$$\varphi_2(x,t) = v_2(x,t) + w_2(x) \tag{A4}$$

Here $v_1$ $(x,t)$ and $v_2$ $(x,t)$ are introduced functions. In addition, applying a shift transformation, let $x' = x - L_1$, equation (2) can be expressed as

$$\begin{cases} a_1 \dfrac{\partial^2 v_1}{\partial x'^2} = \dfrac{\partial v_1}{\partial t} & 0 \le \mathrm{x} \le \mathrm{L}_1 \\ a_2 \dfrac{\partial^2 v_2}{\partial x'^2} = \dfrac{\partial v_2}{\partial t} & \mathrm{L}_1 \le \mathrm{x} \le \mathrm{L} \\ \left.\dfrac{\partial v_1}{\partial x'}\right|_{x'=-L_1} = 0 & \mathrm{t} > 0 \\ v_2|_{x'=L_2} = 0 & \mathrm{t} > 0 \\ v_1|_{x'=0^+} = v_2|_{x'=0^-} & \mathrm{t} > 0 \\ K_1 \left.\dfrac{\partial v_1}{\partial x'}\right|_{x'=0^+} = K_2 \left.\dfrac{\partial v_2}{\partial x'}\right|_{x'=0^-} & \mathrm{t} > 0 \\ v_1|_{t=0} = \dfrac{R}{2K_2}\left(L_1^2 - L^2\right) & 0 \le \mathrm{x} \le \mathrm{L}_1 \\ v_2|_{t=0} = \dfrac{R}{2K_2}\left(\left(x' + L_1\right)^2 - L^2\right) & \mathrm{L}_1 \le \mathrm{x} \le \mathrm{L} \end{cases} \tag{A5}$$

Here

$$a_1 = K_1 \overline{h_1} / S_1 \tag{A6}$$

$$a_2 = K_2 \overline{h_2} / S_2 \tag{A7}$$

The general form of governing equations in equation (A5) is

$$a_I \frac{\partial^2 v_i}{\partial x'^2} = \frac{\partial v_i}{\partial t} \qquad i = 1, 2 \tag{A8}$$

Solving the equations using the method of separation of variables, the dependent variables $v_i$ may be separated in the form

$$v_i(x', t) = X_i(x') T_i(t) \tag{A9}$$

Equation (A9) is introduced into equation (A8), which leads to

$$\frac{X_i''(x')}{X_i(x')} = \frac{T_i'(t)}{a_i T_i(t)} = -\lambda_i^2 \tag{A10}$$

Here $\lambda_i$ are the so-called separation constants for solving the governing equation in equation (A5) mathematically and has no physical interpretation. So the general solutions of equation (A10) is

$$X_i(x') = b_i \cos(\lambda_i x') + c_i \sin(\lambda_i x') \tag{A11}$$

$$T_i(t) = \exp\left(-\lambda_i^2 a_i t\right) \tag{A12}$$

Where $b_i$, $c_i$ are the integration constants. Here, Let $\lambda_i > 0$.

[31] Equations (A11) and (A12) satisfy the boundary conditions at the left and right boundaries and the continuity conditions at the interface between the original aquifer and the reclamation site. The following set of algebraic equations is obtained.

$$c_1 \cos \lambda_1 L_1 = -b_1 \sin \lambda_1 L_1 \quad \text{(A13)}$$

$$b_2 \cos \lambda_2 L_2 = -c_2 \sin \lambda_2 L_2 \quad \text{(A14)}$$

$$b_1 \exp\left(-\lambda_1^2 a_1 t\right) = b_2 \exp\left(-\lambda_2^2 a_2 t\right) \quad \text{(A15)}$$

$$K_1 c_1 \lambda_1 \exp\left(-\lambda_1^2 a_1 t\right) = K_2 c_2 \lambda_2 \exp\left(-\lambda_2^2 a_2 t\right) \quad \text{(A16)}$$

According to equations (A15) and (A16), and for continuity of groundwater flow:

$$\lambda_1^2 a_1 = \lambda_2^2 a_2 \quad \text{(A17)}$$

Rearranging equations (A13), (A14), (A15), and (A16) based on equation (A17) leads to

$$b_1 = b_2 \quad \text{(A18)}$$

$$c_1 = \frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}} c_2 \quad \text{(A19)}$$

$$\left(1 - \frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}}\right)\cos\left(L_1 - \sqrt{\frac{a_1}{a_2}} L_2\right)\lambda_1 = \left(1 + \frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}}\right)\cos\left(L_1 + \sqrt{\frac{a_1}{a_2}} L_2\right)\lambda_1 \quad \text{(A20)}$$

It can be proved that the solutions (eigenvalues) are infinite, distinct and real: $\lambda_1 < \lambda_2 < \ldots \lambda_m < \ldots$ ($m$ = 1, 2, 3, ...). Here $\lambda_m$ satisfied the following equations:

$$(m-1)\pi < \left(L_1 + \sqrt{\frac{a_1}{a_2}} L_2\right)\lambda_m \leq m\pi (m = 1, 2, 3, \ldots) \quad \text{(A21)}$$

$c_{2m}$ or $b_{2m}$ will be obtained using initial conditions as follows.

[32] (a) If sin ($\lambda_{1m}$ $L_1$) = 0, there exist cos ($\lambda_{2m}$ $L_2$) = 0 and $c_{1m}$ = $c_{2m}$ = 0.

[33] The solutions can be written as

$$v_1(x', t) = \sum_{m=1}^{\infty} b_{2m} \cos(\lambda_{1m} x') \exp\left(-\lambda_{1m}^2 a_1 t\right) \quad \text{(A22)}$$

$$v_2(x', t) = \sum_{m=1}^{\infty} b_{2m} \cos(\lambda_{2m} x') \exp\left(-\lambda_{1m}^2 a_1 t\right) \quad \text{(A23)}$$

It can be proven that

$$\int_{-L_1}^{0} \cos(\lambda_{1m} x)\cos(\lambda_{1n} x) dx + \int_{0}^{L_2} \cos(\lambda_{2m} x)\cos(\lambda_{2n} x) dx = \begin{cases} 0 & \text{m} \neq \text{n} \\ \text{L}/2 & \text{m} = \text{n} \end{cases} \quad \text{(A24)}$$

According to the initial conditions,

$$\frac{R}{2K_2}\left(L_1^2 - L^2\right) = \sum_{\text{m}=1}^{\infty} b_{2m} \cos(\lambda_{1m} x') \qquad -L_1 \leq x' \leq 0 \quad \text{(A25)}$$

$$\frac{R}{2K_2}\left(\left(x' + L_1\right)^2 - L^2\right) = \sum_{\text{m}=1}^{\infty} c_{2m} \cos(\lambda_{2m} x') \qquad 0 \leq x' \leq L_2 \quad \text{(A26)}$$

Equation (A25) can be multiplied by cos ($\lambda_{1n}$ $x'$) and equation (A26) can be multiplied by cos ($\lambda_{2n}$ $x'$). The resulting expression is integrated with respect to $x'$ from $-L_1$ to 0, and 0 to $L_2$:

$$\int_{-L_1}^{0} \frac{R}{2K_2}\left(L_1^2 - L^2\right)\cos(\lambda_{1n} x) dx = \sum_{\text{m}=1}^{\infty} b_{2m} \int_{-L_1}^{0} \cos(\lambda_{1m} x) \cdot \cos(\lambda_{1n} x) dx \quad \text{(A27)}$$

$$\int_{0}^{L_2} \frac{R}{2K_2}\left(\left(x + L_1\right)^2 - L^2\right)\cos(\lambda_{2n} x) dx = \sum_{\text{m}=1}^{\infty} b_{2m} \int_{0}^{L_2} \cos(\lambda_{2m} x)\cos(\lambda_{2n} x) dx \quad \text{(A28)}$$

Summing up the above expressions:

$$b_{2m} = \frac{2R}{K_2 L \lambda_{2m}^2}\left(-L_1 - \frac{\sin(\lambda_{2m} L_2)}{\lambda_{2m}}\right) \quad \text{(A29)}$$

(b) If sin ($\lambda_{1m}$ $L_1$) ≠ 0, the solutions can be written as

$$v_1(x', t) = \sum_{m=1}^{\infty} c_{2m} \frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}} \frac{-\cos(\lambda_{1m}(x' + L_1))}{\sin \lambda_{1m} L_1} \exp\left(-\lambda_{1m}^2 a_1 t\right) \quad \text{(A30)}$$

$$v_2(x', t) = \sum_{m=1}^{\infty} c_{2m} \frac{\sin(\lambda_{2m}(x' - L_2))}{\cos \lambda_{2m} L_2} \exp\left(-\lambda_{1m}^2 a_1 t\right) \quad \text{(A31)}$$

Here $c_{2m}$ is an unknown parameter, and it can be specified by the initial condition.

[34] Let

$$X'_{1,m}(x) = \frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}} \frac{-\cos(\lambda_{1m}(x' + L_1))}{\sin \lambda_{1m} L_1} \quad \text{(A32)}$$

$$X'_{2,m}(x) = \frac{\sin(\lambda_{2m}(x' - L_2))}{\cos \lambda_{2m} L_2} \quad \text{(A33)}$$

It can be proven that the eigenfunctions $X'_{1,m}(x')$ and $X'_{2,m}(x')$ satisfy the new type of orthogonality relationship.

$$\int_{-L_1}^{0} X'_{1,m}(x)X'_{1,n}(x)dx + \frac{K_2}{K_1}\frac{a_1}{a_2}\int_{0}^{L_2} X'_{2,m}(x)X'_{2,n}(x)dx = \begin{cases} 0 & \mathrm{m} \neq \mathrm{n} \\ N_m & \mathrm{m} = \mathrm{n} \end{cases} \quad \text{(A34)}$$

Where $N_m$ is the normalization integral and is derived from the equation (A35):

$$N_m = \frac{K_2 a_1}{2K_1 a_2}\left[\frac{K_2 L_1}{K_1 \sin^2 \lambda_{1m} L_1} + \frac{L_2}{\cos^2 \lambda_{2m} L_2}\right] \quad \text{(A35)}$$

According to the initial conditions, when $t = 0$, there exists

$$\frac{R}{2K_2}\left(L_1^2 - L^2\right) = \sum_{\mathrm{m}=1}^{\infty} c_{2m} X'_{1,m}(x') \qquad -L_1 \leq x' \leq 0 \quad \text{(A36)}$$

$$\frac{R}{2K_2}\left((x' + L_1)^2 - L^2\right) = \sum_{\mathrm{m}=1}^{\infty} c_{2m} X'_{2,m}(x') \qquad 0 \leq x' \leq L_2 \quad \text{(A37)}$$

Equation (A36) can be multiplied by $X'_{1,n}(x')$ and equation (A37) can be multiplied by $\frac{K_2}{K_1}\frac{a_1}{a_2} X'_{2,n}(x')$, and the resulting expression is integrated with respect to $x'$ from $-L_1$ to 0, and 0 to $L_2$, so

$$\int_{-L_1}^{0} \frac{R}{2K_2}\left(L_1^2 - L^2\right)X'_{1,n}(x)dx = \sum_{\mathrm{m}=1}^{\infty} c_{2m}\int_{-L_1}^{0} X'_{1,m}(x)X'_{1,n}(x)dx \quad \text{(A38)}$$

$$\frac{K_2}{K_1}\frac{a_1}{a_2}\int_{0}^{L_2} \frac{R}{2K_2}\left((x + L_1)^2 - L^2\right)X'_{2,n}(x)dx = \frac{K_2}{K_1}\frac{a_1}{a_2}\sum_{\mathrm{m}=1}^{\infty} c_{2m}\int_{0}^{L_2} X'_{2,m}(x)X'_{2,n}(x)dx \quad \text{(A39)}$$

Summing up the above expressions leads to

$$c_{2m} = \frac{1}{N_m}\left\{\int_{-L_1}^{0} \frac{R}{2K_2}\left(L_1^2 - L^2\right)X'_{1,m}(x)dx + \frac{K_2}{K_1}\frac{a_1}{a_2}\int_{0}^{L_2}\frac{R}{2K_2}\left((x + L_1)^2 - L^2\right)X'_{2,m}(x)dx\right\} \quad \text{(A40)}$$

hence

$$c_{2m} = \frac{R}{K_1\lambda_{1m}^2 N_m}\left(\frac{1}{\lambda_{2m}\cos\lambda_{2m}L_2} - \frac{1}{\lambda_{2m}} + L_1\tan(\lambda_{2m}L_2)\right) \quad \text{(A41)}$$

Hence the final solution of this problem can be expressed as the following.

[35] For $0 \leq x \leq L_1$:

$$h^2(x,t) = \begin{cases} H_0^2 + \frac{R}{K_2}\left(L^2 - L_1^2\right) + \frac{R}{K_1}\left(L_1^2 - x^2\right) + 2\sum_{m=1}^{\infty} b_{2m}\cos\lambda_{1m}(x - L_1)\exp\left(-\lambda_{1m}^2 a_1 t\right) & \sin\lambda_{1m}L_1 = 0 \\ H_0^2 + \frac{R}{K_2}\left(L^2 - L_1^2\right) + \frac{R}{K_1}\left(L_1^2 - x^2\right) - \frac{2K_2}{K_1}\sqrt{\frac{a_1}{a_2}}\sum_{m=1}^{\infty} c_{2m}\frac{\cos(\lambda_{1m}x)}{\sin\lambda_{1m}L_1}\exp\left(-\lambda_{1m}^2 a_1 t\right) & \sin\lambda_{1m}L_1 \neq 0 \end{cases} \quad \text{(A42)}$$

For $L_1 \leq x \leq L$:

$$h^2(x,t) = \begin{cases} H_0^2 + \frac{R}{K_2}\left(L^2 - x^2\right) + 2\sum_{m=1}^{\infty} b_{2m}\cos\lambda_{2m}(x - L_1)\exp\left(-\lambda_{1m}^2 a_1 t\right) & \sin\lambda_{1m}L_1 = 0 \\ H_0^2 + \frac{R}{K_2}\left(L^2 - x^2\right) + 2\sum_{m=1}^{\infty} c_{2m}\frac{\sin(\lambda_{2m}(x - L))}{\cos\lambda_{2m}L_2}\exp\left(-\lambda_{1m}^2 a_1 t\right) & \sin\lambda_{1m}L_1 \neq 0 \end{cases} \quad \text{(A43)}$$

$b_{2m}$, $c_{2m}$, $N_m$, $\lambda_{1m}$, $\lambda_{2m}$ are group parameters, and they can be calculated by the following equations.

$$b_{2m} = \frac{2R}{K_2 L\lambda_{2m}^2}\left(-L_1 - \frac{\sin(\lambda_{2m}L_2)}{\lambda_{2m}}\right) \quad \text{(A44)}$$

$$c_{2m} = \frac{R}{K_1\lambda_{1m}^2 N_m}\left(\frac{1}{\lambda_{2m}\cos\lambda_{2m}L_2} - \frac{1}{\lambda_{2m}} + L_1\tan(\lambda_{2m}L_2)\right) \quad \text{(A45)}$$

$$N_m = \frac{K_2 a_1}{2K_1 a_2}\left[\frac{K_2 L_1}{K_1\sin^2\lambda_{1m}L_1} + \frac{L_2}{\cos^2\lambda_{2m}L_2}\right] \quad \text{(A46)}$$

$$\lambda_{1m}^2 a_1 = \lambda_{2m}^2 a_2 \quad (m = 1, 2, 3, \ldots) \quad \text{(A47)}$$

$$\left(1-\frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}}\right)\cos\left(L_1-\sqrt{\frac{a_1}{a_2}}L_2\right)\lambda_{1m}=\left(1+\frac{K_2}{K_1}\sqrt{\frac{a_1}{a_2}}\right)\cdot\cos\left(L_1+\sqrt{\frac{a_1}{a_2}}L_2\right)\lambda_{1m}\qquad(m=1,2,3,\ldots)\tag{A48}$$

$$(m-1)\pi<\left(L_1+\sqrt{a_1/a_2}L_2\right)\lambda_{1m}\leq m\pi\qquad(m=1,2,3,\ldots)\tag{A49}$$

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## References

Brutsaert, W. (1994), The unit response of groundwater outflow from a hillslope, *Water Resour. Res.*, *30*(10), 2759–2763.

Brutsaert, W. (1995), Reply to comment on "The unit response of groundwater outflow from a hillslope" by Tom Chapman, *Water Resour. Res.*, *31*(9), 2379–2380.

Carslaw, H. S., and J. C. Jaeger (1959), *Conduction of Heat in Solids,* 2nd ed., 510 pp., Oxford Univ. Press, London, U.K.

Chapman, T. (1995), Comment on "The unit response of groundwater outflow from a hillslope" by Wilfried Brutsaert, *Water Resour. Res.*, *31*(9), 2377–2378.

Diersch, H. J. G. (2005), *WASY Software FEFLOW (R)-Finite Element Subsurface Flow and Transport Simulation System: Reference Manual*, WASY GmbH Institute for Water Resources Planning and Systems Research, Berlin, Germany.

Fetter, C. W. (1994), *Applied Hydrogeology*, 3rd ed., 691 pp., Prentice-Hall Inc., N. J.

Gingerich, S. B., and C. I. Voss (2005), Three-dimensional variable-density flow simulation of a coastal aquifer in southern Oahu, Hawaii, USA, *Hydrogeol. J.*, *13*(2), 436–450.

Guo, H. P., and J. J. Jiao (2007), Impact of coastal land reclamation on groundwater level and the sea water interface, *Ground Water*, *45*(3), 362–367.

Jiao, J. J. (2000), Modification of regional groundwater regimes by land reclamation, *Hong Kong Geol.*, *6*, 29–36.

Jiao, J. J., and H. L. Li (2004), Breathing of coastal vadose zone induced by sea level fluctuations, *Geophys. Res. Lett.*, *31*(13), L11502, doi:10.1029/2004GL019572.

Jiao, J. J., S. Nandy, and H. Li (2001), Analytical studies on the impact of reclamation on groundwater flow, *Ground Water*, *39*(6), 912–920.

Jiao, J. J., X. S. Wang, and S. Nandy (2006), Preliminary assessment of the impacts of deep foundations and land reclamation on groundwater flow in a coastal area in Hong Kong, China, *Hydrogeol. J.*, *14*(1–2), 100–114.

Jiao, J. J., C. M. Leung, and G. P. Ding (2008), Change of groundwater system from 1888 to present in a highly-urbanized coastal area in Hong Kong, *Hydrogeol. J.*, doi:10.1007/s10040-008-0332-z, in press.

Li, H., J. J. Jiao, M. Luk, and K. Cheung (2002), Tide-induced groundwater level fluctuation in coastal aquifers bounded by L-shaped coastlines, *Water Resour. Res.*, *38*(3), 1024, doi:10.1029/2001WR000556.

Monte, F. D. (2000), Transient heat conduction in one-dimensional composite slab. A "natural" analytic approach, *Int. J. Heat Mass Transfer*, *43*, 3607–3619.

Monte, F. D. (2006), Multi-layer transient heat conduction using transition time scales, *Int. J. Thermal Sci.*, *45*, 882–892.

Pickles, A. R., and R. Tosen (1998), Settlement of reclaimed land for the new Hong Kong International Airport, *Proc. Inst. Civ. Eng.-Geotech. Eng.*, *131*(4), 191–209.

Plant, G. W., C. S. Covil, and R. A. Hughes (1998), Site preparation for the new Hong Kong International Airport, in *Design, Construction and Performance of the Airport Platform*, 600 pp., Thomas Telford, London.

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