

Can accretion disk properties distinguish gravastars from black holes?

Tiberiu Harko*

*Department of Physics and Center for Theoretical and Computational Physics,
The University of Hong Kong, Pok Fu Lam Road, Hong Kong*

Zoltán Kovács†

*Max-Planck-Institute für Radioastronomie, Auf dem Hügel 69, 53121 Bonn, Germany and
Department of Experimental Physics, University of Szeged, Dóm Tér 9, Szeged 6720, Hungary*

Francisco S. N. Lobo‡

*Centro de Física Teórica e Computacional, Faculdade de Ciências da Universidade de Lisboa,
Avenida Professor Gama Pinto 2, P-1649-003 Lisboa, Portugal*

(Dated: September 4, 2009)

Gravastars, hypothetic astrophysical objects, consisting of a dark energy condensate surrounded by a strongly correlated thin shell of anisotropic matter, have been proposed as an alternative to the standard black hole picture of general relativity. Observationally distinguishing between astrophysical black holes and gravastars is a major challenge for this latter theoretical model. This due to the fact that in static gravastars large stability regions (of the transition layer of these configurations) exist that are sufficiently close to the expected position of the event horizon, so that it would be difficult to distinguish the exterior geometry of gravastars from an astrophysical black hole. However, in the context of stationary and axially symmetrical geometries, a possibility of distinguishing gravastars from black holes is through the comparative study of thin accretion disks around rotating gravastars and Kerr-type black holes, respectively. In the present paper, we consider accretion disks around slowly rotating gravastars, with all the metric tensor components estimated up to the second order in the angular velocity. Due to the differences in the exterior geometry, the thermodynamic and electromagnetic properties of the disks (energy flux, temperature distribution and equilibrium radiation spectrum) are different for these two classes of compact objects, consequently giving clear observational signatures. In addition to this, it is also shown that the conversion efficiency of the accreting mass into radiation is always smaller than the conversion efficiency for black holes, i.e., gravastars provide a less efficient mechanism for converting mass to radiation than black holes. Thus, these observational signatures provide the possibility of clearly distinguishing rotating gravastars from Kerr-type black holes.

PACS numbers: 04.50.Kd, 04.70.Bw, 97.10.Gz

I. INTRODUCTION

The Schwarzschild solution has played a fundamental conceptual role in general relativity, and beyond, for instance, regarding event horizons, spacetime singularities, and aspects of quantum field theory in curved spacetimes. However, one still encounters in the literature the existence of misconceptions, as well as a certain ambiguity inherent in the Schwarzschild solution (we refer the reader to [1] for a detailed review). In this context, recently a new final state of gravitational collapse has been proposed, denoted as *gravastars* (*gravitational vacuum stars*) [2], which represent a viable alternative to black holes, and their properties have been extensively investigated. These models consist of a compact object with an interior de Sitter condensate, governed by an equation of state given by $p = -\rho$, matched to a shell of

finite thickness with an equation of state $p = \rho$. The latter is then matched to an exterior Schwarzschild vacuum solution. The thick shell replaces both the de Sitter and the Schwarzschild horizons, therefore, the gravastar model has no singularity at the origin and no event horizon, as its rigid surface is located at a radius slightly greater than the Schwarzschild radius [2]. These configurations are stable from a thermodynamic point of view. The issue of the dynamic stability of the transition layer (an infinitesimally thin shell) against spherically symmetric perturbations was considered in [3], by constructing a model that shares the key features of the gravastar scenario. It was found that there are some physically reasonable equations of state for the transition layer that lead to stability. This latter stability analysis was further generalized to an anti-de Sitter or de Sitter interior and a Schwarzschild (anti)-de Sitter or Reissner-Nordström exterior [4]. Recently, dynamical models of prototype gravastars were constructed and studied [5]. It was found that in some cases the models represent stable gravastars, while in other cases they represent “bounded excursion” stable gravastars, where the thin shell is oscillating between two finite radii. In some other cases they collapse

*Electronic address: harko@hkucc.hku.hk

†Electronic address: zkovacs@mpifr-bonn.mpg.de

‡Electronic address: flobo@cii.fc.ul.pt

until the formation of black holes.

In addition to this, gravastar models that exhibit continuous pressure without the presence of infinitesimally thin shells were introduced in [6] and further analyzed in [7]. By considering the usual TOV equation for static solutions with negative central pressure, it was found that gravastars cannot be perfect fluids and anisotropic pressures in the ‘crust’ of a gravastar-like object are unavoidable. The anisotropic TOV equation can then be used to bound the pressure anisotropy, and the transverse stresses that support a gravastar permit a higher compactness than the Buchdahl-Bondi bound for perfect-fluid stars. A wide variety of gravastar models within the context of nonlinear electrodynamics were also constructed in [8]. Using the F representation, specific forms of Lagrangians were considered describing magnetic gravastars, which may be interpreted as self-gravitating magnetic monopoles with charge g . Using the dual P formulation of nonlinear electrodynamics, electric gravastar models were constructed by considering specific structural functions, and the characteristics and physical properties of the solutions were further explored. Gravastar solutions with a Born-Infeld phantom replacing the de Sitter interior were also analyzed in [9].

It has also been recently proposed by Chapline that this new emerging picture consisting of a compact object resembling ordinary spacetime, in which the vacuum energy is much larger than the cosmological vacuum energy, has been denoted as a ‘dark energy star’ [10]. Indeed, a generalization of the gravastar picture was considered in [11] by considering a matching of an interior solution governed by the dark energy equation of state, $\omega = p/\rho < -1/3$, to an exterior Schwarzschild vacuum solution at a junction interface. Several relativistic dark energy stellar configurations were analyzed by imposing specific choices for the mass function, by assuming a constant energy density, and a monotonic decreasing energy density in the star’s interior, respectively. The dynamical stability of the transition layer of these dark energy stars to linearized spherically symmetric radial perturbations about static equilibrium solutions was also considered, and it was found that large stability regions exist that are sufficiently close to where the event horizon is expected to form. Evolving dark energy stars were explored in [12], where a time-dependent dark energy parameter was considered. The general properties of a spherically symmetric body described through the generalized Chaplygin equation of state were also extensively analyzed in [13]. In the context of cosmological equations of state, in [14] the construction of gravastars supported by a van der Waals equation of state was studied and their respective characteristics and physical properties were further analyzed. It was argued that these *van der Waals quintessence stars* may possibly originate from density fluctuations in the cosmological background. Note that the van der Waals quintessence equation of state is an interesting scenario that describes the late universe, and seems to provide a solution to the puzzle of dark energy,

without the presence of exotic fluids or modifications of the Friedmann equations.

However, observationally distinguishing between astrophysical black holes and gravastars is a major challenge for this latter theoretical model, as in static gravastars large stability regions exist that are sufficiently close to the expected position of the event horizon. The constraints that present-day observations of well-known black hole candidates place on the gravastar model were discussed in [15]. The heating of neutron stars via accretion is well documented by astronomical observations. A gravastar would acquire most of its mass via accretion, either as part of its birth (e.g., during core collapse in a supernova explosion followed by the rapid accretion of a fallback disk), or over an extended period of time after birth. Similar heating processes have not been observed in the case of black hole candidates. However, the absence of a detectable heating may also be consistent with a gravastar, if the heat capacity is large enough that it requires large amounts of heat to produce small changes in temperature. Nevertheless, some level of accretion heating is unavoidable, and provides some strong constraints on the gravastar model. The large surface redshifts employed in gravastar models implies that the internal energy generated per unit rest mass accreted is very nearly c^2 , and that the radiation emitted from the surface of the object should be an almost perfect black body. The energy evolution of an accreting gravastar is determined by the equation $dU/dt = \dot{M}c^2 - L$, where U is the internal energy, \dot{M} is the mass accretion rate, and L is the luminosity. Assuming that the gravastar is the result of a BEC-like phase transition induced by strong gravity [10], the gravastar which starts at zero temperature and rapidly accretes a mass $\Delta m M_{Sun}$ will be heated to a temperature as observed at infinity of $T_h \approx 3.1 \times 10^6 m^{-2/3} \xi^{1/3} (\Delta m/m)^{1/3}$ K, where m is the mass of the gravastar in solar mass units, and $\xi = l/l_{Pl}$ is the length scale in Planck units at which general relativity fails to adequately describe gravity. In the case of slow accretion, the temperature is given by the equilibrium value $T_{eq} \approx 1.7 \times 10^7 \left(\dot{M}/\dot{M}_{Edd} \right)^{1/4} m^{-1/4}$ K, where $\dot{M}_{Edd} = 2.3 \times 10^{-9} m M_{\odot}/\text{yr}$ is the Eddington mass accretion rate. By comparing these theoretical predictions on the temperature with the upper limits of the temperature of the black hole candidates, obtained through observations, one can find some limits on ξ . Two black hole candidates, known to have extraordinarily low luminosities, namely, the supermassive black hole in the galactic center, Sagittarius A*, and the stellar-mass black hole, XTE J1118 + 480, respectively, were considered in the analysis [15]. For XTE J1118+480, due to the low mass and low heat capacity, the value of ξ is constrained to $\xi \approx 5 \times 10^3$, while for Sagittarius A* values for ξ in the range 10^4 and 10^{11} are excluded. Therefore the length scale for the considered gravastar models must be sub-Planckian. Thus, if any significant fraction of the mass of the gravastars is due to accretion, we should see the ther-

mal emission associated with that phase, which is ruled out for Sagittarius A* and XTE J1118 + 480.

The question of whether gravastars can be distinguished from black holes at all was also considered, from a theoretical point of view, in [16], where two basic questions analyzed were: (i) Is a gravastar stable against generic perturbations and, (ii) if it is stable, can an observer distinguish it from a black hole of the same mass? A general class of gravastars was constructed, and the equilibrium conditions in order to exist as solutions of the Einstein equations were obtained. It was found that gravastars are stable to axial perturbations, and that their quasi-normal modes differ from those of a black hole of the same mass. Thus, these modes can be used to discern, beyond dispute, a gravastar from a black hole. The formation hysteresis effects were ignored in this study. In addition to this, sharp analytic bounds on the surface compactness $2m/r$ that follow from the requirement that the dominant energy condition (DEC) holds at the shell were derived in [17]. In the case of a Schwarzschild exterior, the highest surface compactness is achieved with the stiff shell in the limit of vanishing (dark) energy density in the interior. In the case of a Schwarzschild-de Sitter exterior, it was shown that gravastar configurations with a surface pressure and with a vanishing shell pressure (dust shells), are allowed by the DEC. The causality requirement (sound speed not exceeding that of light) further restricts the space of allowed gravastar configurations.

The ergoregion instability is known to affect very compact objects that rotate very rapidly, and that do not possess an horizon. A detailed analysis on the relevance of the ergoregion instability for the viability of gravastars was presented in [18, 19]. In [18], it was shown that ultra-compact objects with high redshift at their surface are unstable when rapidly spinning, which strengthens the role of black holes as candidates for astrophysical observations of rapidly spinning compact objects. In particular, analytical and numerical results indicate that gravastars are unstable against scalar field perturbations. Their instability timescale is many orders of magnitude stronger than the instability timescale for ordinary stars with uniform density. In the large $l = m$ approximation, suitable for a WKB treatment, gravitational and scalar perturbations have similar instability timescales. In the low- m regime gravitational perturbations are expected to have shorter instability timescales than scalar perturbations. In [19], the analysis shows that not all rotating gravastars are unstable, and stable models can be constructed also with $J/M^2 \sim 1$, where J and M are the angular momentum and mass of the gravastar, respectively. Therefore, the existence of gravastars cannot be ruled out by invoking the ergoregion instability. The gravastar model was extended by introducing an electrically charged component in [20], where the Einstein–Maxwell field equations were solved in the asymptotically de Sitter interior, and a source of the electric field was coupled to the fluid energy density. Two different solutions that satisfy the dominant energy condition were given, and

the equation of state, the speed of sound and the surface redshift were calculated for both models. The dipolar magnetic field configuration for gravastars was studied in [21], and solutions of Maxwell equations in the internal background spacetime of a slowly rotating gravastar were obtained. The shell of the gravastar where the magnetic field penetrated was modeled as a sphere consisting of a highly magnetized perfect fluid, with infinite conductivity. It was assumed that the dipolar magnetic field of the gravastar is produced by a circular current loop symmetrically placed at radius a at the equatorial plane.

The mass accretion around rotating black holes was studied in general relativity for the first time in [22]. By using an equatorial approximation to the stationary and axisymmetric spacetime of rotating black holes, steady-state thin disk models were constructed, extending the theory of non-relativistic accretion [23]. In these models hydrodynamical equilibrium is maintained by efficient cooling mechanisms via radiation transport, and the accreting matter has a Keplerian rotation. The radiation emitted by the disk surface was also studied under the assumption that black body radiation would emerge from the disk in thermodynamical equilibrium. The radiation properties of thin accretion disks were further analyzed in [24, 25], where the effects of photon capture by the hole on the spin evolution were presented as well. In these works the efficiency with which black holes convert rest mass into outgoing radiation in the accretion process was also computed. More recently, the emissivity properties of the accretion disks were investigated for exotic central objects, such as wormholes [26], and non-rotating or rotating quark, boson or fermion stars and brane-world black holes [27, 28, 29, 30, 31, 32]. The radiation power per unit area, the temperature of the disk and the spectrum of the emitted radiation were given, and compared with the case of a Schwarzschild black hole of an equal mass. The physical properties of matter forming a thin accretion disk in the static and spherically symmetric spacetime metric of vacuum $f(R)$ modified gravity models were also analyzed [33], and it was shown that particular signatures can appear in the electromagnetic spectrum, thus leading to the possibility of directly testing modified gravity models by using astrophysical observations of the emission spectra from accretion disks.

It is the purpose of the present paper to consider another observational possibility that may distinguish gravastars from black holes, namely, the study of the properties of the thin accretion disks around rotating gravastars and black holes, respectively. Thus, we consider a comparative study of the thin accretion disks around slowly rotating gravastars and black holes, respectively. In particular, we consider the basic physical parameters describing the disks, such as the emitted energy flux, the temperature distribution on the surface of the disk, as well as the spectrum of the emitted equilibrium radiation. Due to the differences in the exterior geometry, the thermodynamic and electromagnetic properties of the disks (energy flux, temperature distribution and

equilibrium radiation spectrum) are different for these two classes of compact objects, thus giving clear observational signatures, which may allow to discriminate, at least in principle, gravastars from black holes. We would like to point out that the proposed method for the detection of the gravastars by studying accretion disks is an *indirect* method, which must be complemented by *direct* methods of observation of the surface of the considered compact objects.

The present paper is organized as follows. In Sec. II, we present the fundamental field equations for static and slowly rotating gravastar models. In Sec. III, we review the formalism and the physical properties of the thin disk accretion onto compact objects, for stationary axisymmetric spacetimes. In Sec. IV, we analyze the basic properties of matter forming a thin accretion disk around slowly rotating gravastar spacetimes. We discuss and conclude our results in Sec. V. Throughout this work, we use a system of units so that $c = G = \hbar = k_B = 1$, where k_B is Boltzmann's constant.

II. SLOWLY ROTATING GRAVASTAR AND KERR BLACK HOLES

In order to construct slowly rotating gravastar models we first consider the static case. Then, by assuming that rotation represents a second order perturbation of the static case, a slowly rotating gravastar model can be constructed.

A. Static gravastar models

1. Equations of structure

For a static general relativistic spherically symmetric matter configuration, the interior line element can be taken generally as

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (1)$$

We assume that the star consists of an anisotropic fluid distribution of matter, and is given by

$$T_{\mu\nu} = (\rho + p_{\perp}) U_{\mu} U_{\nu} + p_{\perp} g_{\mu\nu} + (p_r - p_{\perp}) \chi_{\mu} \chi_{\nu}, \quad (2)$$

where U^{μ} is the four-velocity, χ^{μ} is the unit spacelike vector in the radial direction, i.e., $\chi^{\mu} = e^{-\lambda(r)/2} \delta^{\mu}_r$. $\rho(r)$ is the energy density, $p_r(r)$ is the radial pressure measured in the direction of χ^{μ} , and $p_{\perp}(r)$ is the transverse pressure measured in the orthogonal direction to χ^{μ} . Taking into account the above considerations, the stress-energy tensor is given by the following profile: $T^{\mu}_{\nu} = \text{diag}[-\rho(r), p_r(r), p_{\perp}(r), p_{\perp}(r)]$.

We suppose that inside the star $p_r \neq p_{\perp}$, $\forall r \neq 0$, and define the anisotropy parameter as $\Delta = p_{\perp} - p_r$, where Δ is a measure of the deviations from isotropy. If $\Delta > 0$, $\forall r \neq 0$ the body is tangential pressure dominated while

$\Delta < 0$ indicates that $p_r > p_{\perp}$. Note that Δ/r represents a force due to the anisotropic nature of the stellar model, which is repulsive, i.e., being outward directed if $p_{\perp} > p_r$, and attractive if $p_{\perp} < p_r$.

The properties of the anisotropic compact object can be completely described by the gravitational structure equations, which are given by:

$$\frac{dm}{dr} = 4\pi\rho r^2, \quad (3)$$

$$\frac{dp_r}{dr} = -\frac{(\rho + p_r) [m + 4\pi r^3 p_r]}{r^2 (1 - \frac{2m}{r})} + \frac{2\Delta}{r}, \quad (4)$$

$$\frac{d\nu}{dr} = -\frac{2}{\rho + p_r} \frac{dp_r}{dr} + \frac{4\Delta}{r(\rho + p_r)}, \quad (5)$$

where $m(r)$ is the mass inside radius r , and the relationship $e^{-\lambda(r)} = [1 - 2m(r)/r]$ has been used.

A solution of Eqs. (3)-(5) is possible only when boundary conditions have been imposed. As in the isotropic case we require that the interior of any matter distribution be free of singularities, which imposes the condition $m(r) \rightarrow 0$ as $r \rightarrow 0$. Assuming that p_r is finite at $r = 0$, we have $\nu' \rightarrow 0$ as $r \rightarrow 0$. Therefore the gradient dp_r/dr will be finite at $r = 0$ only if Δ vanishes at least as rapidly as r when $r \rightarrow 0$. This requires that the anisotropy parameter satisfies the boundary condition

$$\lim_{r \rightarrow 0} \frac{\Delta(r)}{r} = 0. \quad (6)$$

At the center of the star the other boundary conditions for Eqs. (3)-(5) are $p_r(0) = p_{\perp}(0) = p_c$ and $\rho(0) = \rho_c$, where ρ_c and p_c are the central density and pressure, respectively. The radius a of the star is determined by the boundary condition $p_r(a) = 0$. We do not necessarily require that the tangential pressure p_{\perp} vanishes for $r = a$. Therefore at the surface of the star the anisotropy parameter satisfies the boundary condition $\Delta(a) = p_{\perp}(a) - p_r(a) = p_{\perp}(a) \geq 0$. To close the field equations the equations of state of the radial pressure $p_r = p_r(\rho)$ and of the tangential pressure $p_{\perp} = p_{\perp}(\rho)$ must also be given. To be a gravastar model, we need to impose the equation of state $p_r = -\rho$, so that from the field equations one easily deduces that

$$\nu(r) = -\lambda(r) = \ln \left[1 - \frac{2m(r)}{r} \right], \quad (7)$$

where $m(r)$ is the mass function.

2. Junction interface

We consider models of gravastars by matching an interior solution, governed by an equation of state, $p_r = -\rho$, to an exterior Schwarzschild vacuum solution with $p =$

$\rho = 0$, at a junction interface Σ , with junction radius a . The Schwarzschild metric is given by

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (8)$$

which possesses an event horizon at $r_b = 2M$, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$. To avoid the event horizon, the junction radius lies outside $2M$, i.e., $a > 2M$. We show below that M , in this context, may be interpreted as the total mass of the gravastar.

Using the Darmois-Israel formalism [34], the surface stresses of the thin shell are given by [11]

$$\sigma = -\frac{1}{4\pi a} \left(\sqrt{1 - \frac{2M}{a} + \dot{a}^2} - \sqrt{1 - \frac{2m}{a} + \dot{a}^2} \right), \quad (9)$$

$$\mathcal{P} = \frac{1}{8\pi a} \left(\frac{1 - \frac{M}{a} + \dot{a}^2 + a\ddot{a}}{\sqrt{1 - \frac{2M}{a} + \dot{a}^2}} - \frac{1 - m' - \frac{m}{a} + \dot{a}^2 + a\ddot{a}}{\sqrt{1 - \frac{2m}{a} + \dot{a}^2}} \right), \quad (10)$$

where the overdot denotes a derivative with respect to proper time, τ . σ and \mathcal{P} are the surface energy density and the tangential pressure, respectively [11]. The dynamical stability of the transition layer of these compact spheres to linearized spherically symmetric radial perturbations about static equilibrium solutions was explored using Eqs. (9)-(10) (see [11] for details). Large stability regions were found that exist sufficiently close to where the event horizon is expected to form, so that it would be difficult to distinguish the exterior geometry of these gravastars from astrophysical black holes.

The surface mass of the thin shell is given by $m_s = 4\pi a^2 \sigma$. By rearranging Eq. (9), evaluated at a static solution a_0 , i.e., $\dot{a} = \ddot{a} = 0$, one obtains the total mass of the gravastar, given by

$$M = m(a_0) + m_s(a_0) \left[\sqrt{1 - \frac{2m(a_0)}{a_0}} - \frac{m_s(a_0)}{2a_0} \right]. \quad (11)$$

3. Specific model: Tolman-Matese-Whitman mass function

To gain insight into the problem, it is interesting to present a specific example. For instance, consider the following choice for the mass function, given by

$$m(r) = \frac{b_0 r^3}{2(1 + 2b_0 r^2)}, \quad (12)$$

where b_0 is a non-negative constant, which was extensively analyzed in [11] in the context of dark energy stars. The latter may be determined from the regularity conditions and the finite character of the energy density at the origin $r = 0$, and is given by $b_0 = 8\pi\rho_c/3$, where ρ_c is the energy density at $r = 0$.

This choice of the mass function represents a monotonic decreasing energy density in the star interior, and was used previously in the analysis of isotropic fluid spheres by Matese and Whitman [35] as a specific case of the Tolman type-IV solution [36], and later by Finch and Skea [37]. Anisotropic stellar models, with the respective astrophysical applications, were also extensively analyzed in Refs. [38], by considering a specific case of the Matese-Whitman mass function. The numerical results outlined show that the basic physical parameters, such as the mass and radius, of the model can describe realistic astrophysical objects such as neutron stars [38].

The spacetime metric for this solution is provided by

$$ds^2 = - \left(\frac{1+b_0 r^2}{1+2b_0 r^2} \right) dt^2 + \left(\frac{1+2b_0 r^2}{1+b_0 r^2} \right) dr^2 + r^2 d\Omega^2 \quad (13)$$

The stress-energy tensor components are given by

$$p_r = -\rho = - \left(\frac{b_0}{8\pi} \right) \frac{(3 + 2b_0 r^2)}{(1 + 2b_0 r^2)^2}$$

$$p_\perp = - \left(\frac{b_0}{8\pi} \right) \frac{(3 + 2b_0 r^2)}{(1 + 2b_0 r^2)^2} + \left(\frac{b_0^2 r^2}{4\pi} \right) \frac{(5 + 2b_0 r^2)}{(1 + 2b_0 r^2)^3}.$$

The anisotropy factor takes the following form

$$\Delta = \left(\frac{b_0^2 r^2}{4\pi} \right) \frac{(5 + 2b_0 r^2)}{(1 + 2b_0 r^2)^3}, \quad (14)$$

which is always positive, implying that $p_\perp > p_r$, and $\Delta|_{r=0} = 0$ at the center, i.e., $p_\perp(0) = p_r(0)$, as expected.

B. Slowly rotating gravastars

When the equilibrium configuration described in Sec. II A 1 is set into slow rotation, the geometry of spacetime around it and its interior distribution of stress-energy are changed. With an appropriate change of coordinates, the perturbed geometry is described by [39, 40]

$$ds^2 = -e^{\nu_{rot}(r)} \{1 + 2[h_0 + h_2 P_2(\cos\theta)]\} dt^2 + \frac{1 + 2[m_0 + m_2 P_2(\cos\theta)] / (r - 2M)}{1 - 2M/r} dr^2 + r^2 [1 + 2(v_2 - h_2) P_2(\cos\theta)] [d\theta^2 + \sin^2\theta (d\varphi - \omega dt)^2] + O(\Omega_S^3). \quad (15)$$

Here, $P_2(\cos\theta) = (3\cos^2\theta - 1)/2$ is the Legendre polynomial of order two; the angular velocity, ω , of the local inertial frame, is a function of the radial coordinate r , and is proportional to the star's angular velocity Ω_S ; and h_0 ,

h_2, m_0, m_2, v_2 are functions of r that are proportional to Ω_S^2 . Outside the star the metric can be written as [40]

$$\begin{aligned}
ds^2 = & - \left(1 - \frac{2M}{r} + 2\frac{J^2}{r^4}\right) \left\{1 + 2 \left[\frac{J^2}{Mr^3} \left(1 + \frac{M}{r}\right) + \frac{5}{8} \frac{Q - J^2/M}{M^3} Q_2^2(\chi)\right] P_2(\cos\theta)\right\} dt^2 \\
& + \left(1 - \frac{2M}{r} + 2\frac{J^2}{r^4}\right)^{-1} \left\{1 - 2 \left[\frac{J^2}{Mr^3} \left(1 - \frac{5M}{r}\right) + \frac{5}{8} \frac{Q - J^2/M}{M^3} Q_2^2(\chi)\right] P_2(\cos\theta)\right\} dr^2 \\
& + r^2 \left\{1 + 2 \left[-\frac{J^2}{Mr^3} \left(1 + \frac{2M}{r}\right) + \frac{5}{8} \frac{Q - J^2/M}{M^3} \left(\frac{2M}{\sqrt{r(r-2M)}} Q_2^1(\chi) - Q_2^2(\chi)\right)\right] P_2(\cos\theta)\right\} \\
& \times \left\{d\theta^2 + \sin^2\theta \left[d\varphi - \frac{2J}{r^3} dt\right]^2\right\}. \tag{16}
\end{aligned}$$

In Eq. (16) the variable $\chi = r/M - 1$, and the quantities Q_2^1 and Q_2^2 denote associated Legendre polynomials of the second time, so that

$$Q_2^1(\chi) = \sqrt{\chi^2 - 1} \left[\frac{(3\chi^2 - 2)}{\chi^2 - 1} - \frac{3}{2} \chi \ln \frac{\chi + 1}{\chi - 1} \right], \tag{17}$$

and

$$Q_2^2(\chi) = \frac{5\chi - 3\chi^2}{(\chi^2 - 1)} + \frac{3}{2} (\chi^2 - 1) \ln \frac{\chi + 1}{\chi - 1}, \tag{18}$$

respectively. The line element outside the star is determined by three constants: the total mass of the rotating star M , the star's total angular momentum J and the star's mass quadrupole moment Q . The quadrupole moment Q can be expressed in terms of the eccentricity $e = \sqrt{(r_e/r_p)^2 - 1}$ of the star as $e = \sqrt{3Q/Mr^{*2}} + O(1/r^{*2})$, where r^* is a large distance from the origin, and r_e and r_p are the equatorial and polar radius of the star, respectively [40]. The metric given by Eq. (16) is valid in the case of slow rotation, that is, the angular velocity of the star Ω_S must be small enough so that the fractional changes in pressure, energy density, and gravitational field, due to rotation, are all much less than unity. The condition of slow rotation can be formulated as $\Omega_S^2 \ll (c/R)^2 (GM/Rc^2)$, where R is the radius of the static stellar configuration [39]. The critical angular velocity at which mass shedding occurs is given by $\Omega_K^2 = GM/R^3$ [40], and therefore the condition of slow rotation implies $\Omega_S \ll \Omega_K$. All models considered in the present paper satisfy this condition.

The metric given by Eq. (16) is used to determine the electromagnetic signatures of accretion disks around slowly rotating gravastars, which is analyzed in detail below.

C. The Kerr black hole

For self-completeness and self-consistency, we present the Kerr metric, as it will be compared to metric (16) in the electromagnetic signature analysis of accretion disks.

The Kerr metric, describing a rotating black hole, is given in the Boyer-Lyndquist coordinate system by

$$\begin{aligned}
ds^2 = & - \left(1 - \frac{2Mr}{\Sigma_K}\right) dt^2 + 2\frac{2Mr}{\Sigma_K} a \sin^2\theta dt d\phi + \frac{\Sigma_K}{\Delta_K} dr^2 \\
& + \Sigma_K d\theta^2 + \left(r^2 + a^2 + \frac{2Mr}{\Sigma_K} a^2 \sin^2\theta\right) \sin^2\theta d\phi^2, \tag{19}
\end{aligned}$$

where $\Sigma_K = r^2 + a^2 \cos^2\theta$ and $\Delta_K = r^2 + a^2 - 2mr$, respectively. In the equatorial plane, the metric components reduce to

$$\begin{aligned}
g_{tt} &= - \left(1 - \frac{2Mr}{\Sigma_K}\right) = - \left(1 - \frac{2M}{r}\right), \\
g_{t\phi} &= \frac{2Mr}{\Sigma_K} a \sin^2\theta = 2\frac{Ma}{r}, \\
g_{rr} &= \frac{\Sigma_K}{\Delta_K} = \frac{r^2}{\Delta_K}, \\
g_{\phi\phi} &= \left(r^2 + a^2 + \frac{2Mr}{\Sigma_K} a^2 \sin^2\theta\right) \sin^2\theta \\
&= r^2 + a^2 \left(1 + \frac{2M}{r}\right),
\end{aligned}$$

respectively. For the Kerr metric $J = -Ma$ and $Q = J^2/M$, respectively. The latter relationship, i.e., $Q = J^2/M$, between the quadrupole moment and the angular momentum shows the very special nature of the Kerr solution.

III. ELECTROMAGNETIC RADIATION PROPERTIES OF THIN ACCRETION DISKS IN STATIONARY AXISYMMETRIC SPACETIMES

To set the stage, we present the general formalism of electromagnetic radiation properties of thin accretion disks in stationary axisymmetric spacetimes.

A. Stationary and axially symmetric spacetimes

In this work we analyze the physical properties and characteristics of particles moving in circular orbits

around general relativistic compact spheres in a stationary and axially symmetric geometry given by the following metric

$$ds^2 = g_{tt} dt^2 + 2g_{t\phi} dt d\phi + g_{rr} dr^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2. \quad (20)$$

Note that the metric functions g_{tt} , $g_{t\phi}$, g_{rr} , $g_{\theta\theta}$ and $g_{\phi\phi}$ only depend on the radial coordinate r in the equatorial approximation, i.e., $|\theta - \pi| \ll 1$, which is the case of interest here. In the following we denote the square root of the determinant of the metric tensor by $\sqrt{-g}$.

To compute the flux integral given by Eq. (39), we determine the radial dependence of the angular velocity Ω , of the specific energy \tilde{E} and of the specific angular momentum \tilde{L} of particles moving in circular orbits around compact spheres in a stationary and axially symmetric geometry through the geodesic equations. The latter take the following form

$$\frac{dt}{d\tau} = \frac{\tilde{E}g_{\phi\phi} + \tilde{L}g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad (21)$$

$$\frac{d\phi}{d\tau} = -\frac{\tilde{E}g_{t\phi} + \tilde{L}g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad (22)$$

$$g_{rr} \left(\frac{dr}{d\tau} \right)^2 = -1 + \frac{\tilde{E}^2 g_{\phi\phi} + 2\tilde{E}\tilde{L}g_{t\phi} + \tilde{L}^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}. \quad (23)$$

One may define an effective potential term defined as

$$V_{eff}(r) = -1 + \frac{\tilde{E}^2 g_{\phi\phi} + 2\tilde{E}\tilde{L}g_{t\phi} + \tilde{L}^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}. \quad (24)$$

For stable circular orbits in the equatorial plane the following conditions must hold: $V_{eff}(r) = 0$ and $V_{eff,r}(r) = 0$. These conditions provide the specific energy, the specific angular momentum and the angular velocity of particles moving in circular orbits for the case of spinning general relativistic compact spheres, given by

$$\tilde{E} = -\frac{g_{tt} + g_{t\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}, \quad (25)$$

$$\tilde{L} = \frac{g_{t\phi} + g_{\phi\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}, \quad (26)$$

$$\Omega = \frac{d\phi}{dt} = \frac{-g_{t\phi,r} + \sqrt{(g_{t\phi,r})^2 - g_{tt,r}g_{\phi\phi,r}}}{g_{\phi\phi,r}}. \quad (27)$$

The marginally stable orbit around the central object can be determined from the condition $V_{eff,rr}(r) = 0$, which provides the following important relationship

$$\tilde{E}^2 g_{\phi\phi,rr} + 2\tilde{E}\tilde{L}g_{t\phi,rr} + \tilde{L}^2 g_{tt,rr} - (g_{t\phi}^2 - g_{tt}g_{\phi\phi})_{,rr} = 0. \quad (28)$$

By inserting Eqs. (25)-(27) into Eq. (28) and solving this equation for r , we obtain the marginally stable orbit for the explicitly given metric coefficients g_{tt} , $g_{t\phi}$ and $g_{\phi\phi}$.

For a Kerr black hole the geodesic equation (23) for r becomes

$$\frac{r^2}{\Delta_K} \left(\frac{dr}{d\tau} \right)^2 = V_{eff}(r) \quad (29)$$

with the effective potential given by

$$V_{eff}(r) = -1 + \left\{ \tilde{E}^2 [r^2(r^2 + a^2) + 2ma^2r] + 4\tilde{E}\tilde{L}mar - \tilde{L}^2 (r^2 - 2mr) \right\} / [r^2 (r^2 - 2mr + a^2)]. \quad (30)$$

Note that these relationships may be rewritten in the following manner

$$r^4 \left(\frac{dr}{d\tau} \right)^2 = V(r) \quad (31)$$

with $V(r)$ given by

$$V(r) = r^2 \Delta_K V_{eff}(r) = r^2 (r^2 - 2mr + a^2) V_{eff}(r). \quad (32)$$

where the relationship $\Delta_K = g_{t\phi}^2 - g_{tt}g_{\phi\phi} = r^2 - 2mr + a^2$ along the equatorial plane has been used.

B. Properties of thin accretion disks

For the thin accretion disk, it is assumed that its vertical size is negligible, as compared to its horizontal extension, i.e, the disk height H , defined by the maximum half thickness of the disk in the vertical direction, is always much smaller than the characteristic radius r of the disk, defined along the horizontal direction, $H \ll r$. The thin disk is in hydrodynamical equilibrium, and the pressure gradient and a vertical entropy gradient in the accreting matter are negligible. The efficient cooling via the radiation over the disk surface prevents the disk from cumulating the heat generated by stresses and dynamical friction. In turn, this equilibrium causes the disk to stabilize its thin vertical size. The thin disk has an inner edge at the marginally stable orbit of the compact object potential, and the accreting plasma has a Keplerian motion in higher orbits.

In steady state accretion disk models, the mass accretion rate \dot{M}_0 is assumed to be a constant that does not change with time. The physical quantities describing the orbiting plasma are averaged over a characteristic time scale, e.g. Δt , over the azimuthal angle $\Delta\phi = 2\pi$ for a total period of the orbits, and over the height H [22, 23, 24].

The particles moving in Keplerian orbits around the compact object with a rotational velocity $\Omega = d\phi/dt$ have a specific energy \tilde{E} and a specific angular momentum \tilde{L} , which, in the steady state thin disk model, depend only on the radii of the orbits. These particles, orbiting with the four-velocity u^μ , form a disk of an averaged surface

density Σ , the vertically integrated average of the rest mass density ρ_0 of the plasma. The accreting matter in the disk is modeled by an anisotropic fluid source, where the density ρ_0 , the energy flow vector q^μ and the stress tensor $t^{\mu\nu}$ are measured in the averaged rest-frame (the specific heat was neglected). Then, the disk structure can be characterized by the surface density of the disk [22, 24],

$$\Sigma(r) = \int_{-H}^H \langle \rho_0 \rangle dz, \quad (33)$$

with averaged rest mass density $\langle \rho_0 \rangle$ over Δt and 2π and the torque

$$W_\phi{}^r = \int_{-H}^H \langle t_\phi{}^r \rangle dz, \quad (34)$$

with the averaged component $\langle t_\phi{}^r \rangle$ over Δt and 2π . The time and orbital average of the energy flux vector gives the radiation flux $\mathcal{F}(r)$ over the disk surface as $\mathcal{F}(r) = \langle q^z \rangle$.

The stress-energy tensor is decomposed according to

$$T^{\mu\nu} = \rho_0 u^\mu u^\nu + 2u^{(\mu} q^{\nu)} + t^{\mu\nu}, \quad (35)$$

where $u_\mu q^\mu = 0$, $u_\mu t^{\mu\nu} = 0$. The four-vectors of the energy and angular momentum flux are defined by $-E^\mu \equiv T^\mu_\nu (\partial/\partial t)^\nu$ and $J^\mu \equiv T^\mu_\nu (\partial/\partial \phi)^\nu$, respectively. The structure equations of the thin disk can be derived by integrating the conservation laws of the rest mass, of the energy, and of the angular momentum of the plasma, respectively [22, 24]. From the equation of the rest mass conservation, $\nabla_\mu(\rho_0 u^\mu) = 0$, it follows that the time averaged rate of the accretion of the rest mass is independent of the disk radius,

$$\dot{M}_0 \equiv -2\pi\sqrt{-g}\Sigma u^r = \text{constant}. \quad (36)$$

The conservation law $\nabla_\mu E^\mu = 0$ of the energy has the integral form

$$[\dot{M}_0 \tilde{E} - 2\pi\sqrt{-g}\Omega W_\phi{}^r]_{,r} = 4\pi\sqrt{-g}F\tilde{E}, \quad (37)$$

which states that the energy transported by the rest mass flow, $\dot{M}_0 \tilde{E}$, and the energy transported by the dynamical stresses in the disk, $2\pi\sqrt{-g}\Omega W_\phi{}^r$, is in balance with the energy radiated away from the surface of the disk, $4\pi\sqrt{-g}F\tilde{E}$. The law of the angular momentum conservation, $\nabla_\mu J^\mu = 0$, also states the balance of these three forms of the angular momentum transport,

$$[\dot{M}_0 \tilde{L} - 2\pi r W_\phi{}^r]_{,r} = 4\pi\sqrt{-g}F\tilde{L}. \quad (38)$$

By eliminating $W_\phi{}^r$ from Eqs. (37) and (38), and applying the universal energy-angular momentum relation $dE = \Omega dJ$ for circular geodesic orbits in the form $\tilde{E}_{,r} = \Omega \tilde{L}_{,r}$, the flux F of the radiant energy over the disk

can be expressed in terms of the specific energy, angular momentum and of the angular velocity of the central compact object [22, 24],

$$F(r) = -\frac{\dot{M}_0}{4\pi\sqrt{-g}} \frac{\Omega_{,r}}{(\tilde{E} - \Omega\tilde{L})^2} \int_{r_{ms}}^r (\tilde{E} - \Omega\tilde{L})\tilde{L}_{,r} dr. \quad (39)$$

Another important characteristics of the mass accretion process is the efficiency with which the central object converts rest mass into outgoing radiation. This quantity is defined as the ratio of the rate of the radiation of energy of photons escaping from the disk surface to infinity, and the rate at which mass-energy is transported to the central compact general relativistic object, both measured at infinity [22, 24]. If all the emitted photons can escape to infinity, the efficiency is given in terms of the specific energy measured at the marginally stable orbit r_{ms} ,

$$\epsilon = 1 - \tilde{E}_{ms}. \quad (40)$$

For Schwarzschild black holes the efficiency ϵ is about 6%, whether the photon capture by the black hole is considered, or not. Ignoring the capture of radiation by the hole, ϵ is found to be 42% for rapidly rotating black holes, whereas the efficiency is 40% with photon capture in the Kerr potential [25].

The accreting matter in the steady-state thin disk model is supposed to be in thermodynamical equilibrium. Therefore the radiation emitted by the disk surface can be considered as a perfect black body radiation, where the energy flux is given by $F(r) = \sigma_{SB} T^4(r)$ (σ_{SB} is the Stefan-Boltzmann constant), and the observed luminosity $L(\nu)$ has a redshifted black body spectrum [28]:

$$L(\nu) = 4\pi d^2 I(\nu) = \frac{8}{\pi} \cos\gamma \int_{r_i}^{r_f} \int_0^{2\pi} \frac{\nu_e^3 r d\phi dr}{\exp(\nu_e/T) - 1}. \quad (41)$$

Here d is the distance to the source, $I(\nu)$ is the Planck distribution function, γ is the disk inclination angle, and r_i and r_f indicate the position of the inner and outer edge of the disk, respectively. We take $r_i = r_{ms}$ and $r_f \rightarrow \infty$, since we expect the flux over the disk surface vanishes at $r \rightarrow \infty$ for any kind of general relativistic compact object geometry. The emitted frequency is given by $\nu_e = \nu(1+z)$, where the redshift factor can be written as

$$1+z = \frac{1 + \Omega r \sin\phi \sin\gamma}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}} \quad (42)$$

where we have neglected the light bending [41, 42].

IV. ELECTROMAGNETIC AND THERMODYNAMIC SIGNATURES OF ACCRETION DISKS AROUND SLOWLY ROTATING GRAVASTARS AND BLACK HOLES

A. Electromagnetic and thermodynamic properties of the disks

In this section we compare the radiation properties of thin accretion disks around gravastars and black holes in the slowly rotating case when the spin parameter $a_* = J/M^2$ has the maximal value of 0.5. In Fig. 1 we present the time averaged energy flux $F(r)$ radiated by the disk for both types of central objects with the total mass M of $10^6 M_\odot$ and increasing spin parameter from 0.1 to 0.5. The quadrupole moment Q of the gravastar models runs between $0.1M^3$ and $2M^3$. Here the mass accretion rate \dot{M}_0 is set to $2.5 \times 10^{-5} M_\odot/\text{yr}$, which is in the typical range for super massive central objects.

If we compare the flux emerging from the surface of the thin accretion disk around black holes and gravastars, we find that its maximal value is systematically lower for gravastars, independently of the values of the spin parameter or the quadrupole momentum. For very slow rotation, ($a_* = 0.1$), and a relative small value of the quadrupole moment ($Q = 0.1M^3$), the radial distribution of the disk radiation is close to each other for the two types of compact central objects. The maximal flux for gravastars is roughly 90% of the black hole's flux, and the maximum of the inner edge of the accretion disk is located at somewhat higher radii for gravastars. With increasing rotational frequency of the central object, the flux values also increase, but the increment is higher for black holes than for gravastars. For $a_* = 0.5$ the flux maximum for black holes is almost twice the maximal flux value for gravastars. The more rapid rotation does not cause strong effect on the location of the inner disk edge for gravastars, as we find a slight decrease in the value of r_{ms} as the rotation of central object increases (see in Tab II). For black holes, the effect of the rotation is also stronger here. Although the error of the approximation applied for the slow rotation is rapidly increasing in the regime around $a_* \sim 0.5$, this picture is still definitely adequate for lower values of a_* .

The variation of the quadrupole moment causes considerable changes in both the maximal value of disk radiation and the location of the inner edge of the disk. As we increase Q , the maximal flux decreases and r_{ms} increases. These effects are presented in Fig. 2, showing the disk temperatures, although the differences are somewhat less striking.

The disk spectra, presented in Fig. 3, have similar features in the dependence of the disk radiation on the rotation parameter and the quadrupole momentum. The amplitudes and the cut-off frequencies of the spectra for gravastars are always lower than those for black holes. For higher rotational velocity, the amplitudes are somewhat higher but do not exhibit much change. The cut-off

frequency for black holes increases moderately, whereas it has only a negligible increment for gravastars. This makes the differences in the spectral properties more acute for higher values of the spin parameter ($a_* \gtrsim 0.3$). The increase in the quadrupole moment somewhat lowers the amplitude of the spectra but causes a stronger decrease in the cut-off frequencies.

B. Conversion efficiency of the accreting mass

We also present the conversion efficiency ϵ of the accreting mass into radiation, measured at infinity, which is given by Eq. (40), for the case where the photon capture by the slowly rotating central object is ignored. The value of ϵ measures the efficiency of energy generating mechanism by mass accretion. The amount of energy released by matter leaving the marginally stable orbit, and falling down the black hole, is the binding energy \tilde{E}_{ms} of the black hole potential. In Tabs. I and II, the marginally stable orbits r_{ms} and ϵ are given for black holes and gravastars with the parameters a_* and Q in the range used for the plots presenting the radiation properties of the accretion disks.

a_*	$r_{ms} [M]$	$\epsilon [10^{-2}]$
0.1	5.67	6.06
0.2	5.33	6.46
0.3	4.98	6.94
0.4	4.62	7.51
0.5	4.24	8.21

TABLE I: The marginally stable orbit and the efficiency for slowly rotating Kerr black holes with different spin parameters.

These values demonstrate the variation in the location of the inner disk edge with the changing spin parameter and quadrupole momentum, as we have seen in the discussion on the radial distribution of the flux. The higher these values are, the closer the marginally stable orbits are to the center in the dimensionless radial scale. For slowly rotating black holes, the conversion efficiency is still close to 6%, which is the value obtained for Schwarzschild black holes. Up to $a_* = 0.5$ it increases to 8%, which is still much lower than the one for extreme Kerr black holes. For gravastars, ϵ has a smaller variation, and always remains smaller than the conversion efficiency for black holes. For very slow rotation, the efficiency is approximately 5.8%, which is close to the one for the static black hole, and it decreases to 5% as the quadrupole moment increases to $2M^3$. For a higher spin parameter ($a_* \sim 0.5$), ϵ is still around 6.5% but it becomes smaller than 6% as Q increases. We conclude that the conversion efficiency is higher for more rapidly rotating gravastars, but this is moderated by the increment

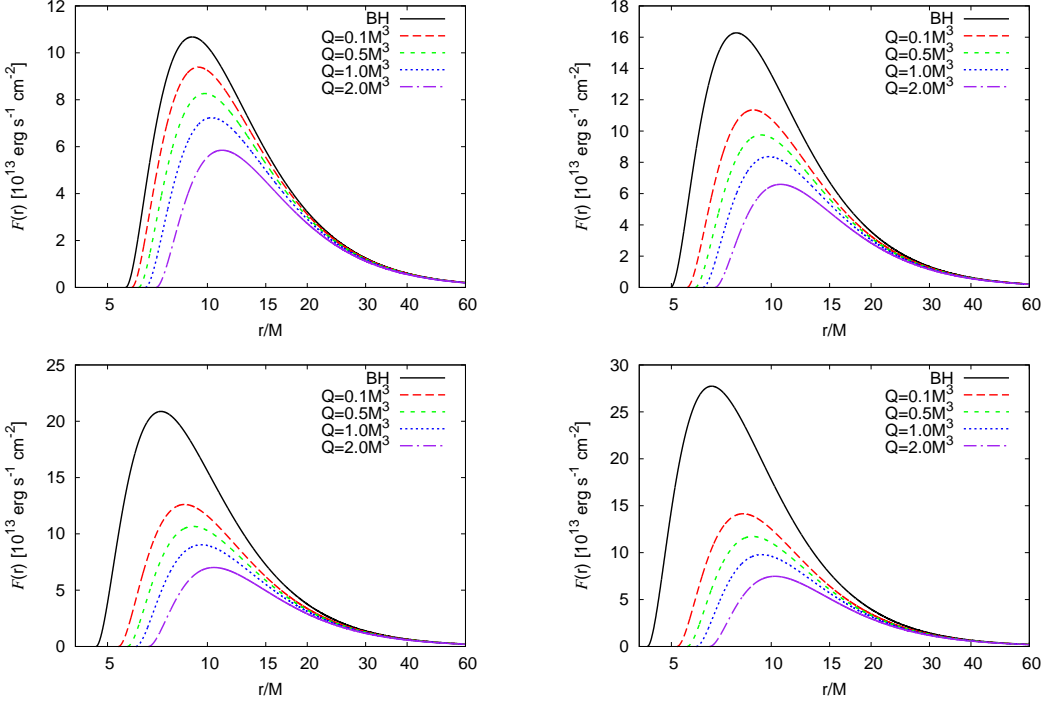


FIG. 1: The energy flux emerging from the accretion disk of slowly rotating gravastars and black holes for the spin parameter $a_* = 0.1$ (upper left hand plot), $a_* = 0.3$ (upper right hand plot), $a_* = 0.4$ (lower left hand plot), and $a_* = 0.5$ for (lower right hand plot). All the plots are given for the total mass $M = 10^6 M_\odot$, the quadrupole moments $Q = 0.1, 0.5, 1.0, 2.0$ times M^3 , and the mass accretion rate $2.5 \times 10^{-5} M_\odot/\text{yr}$.

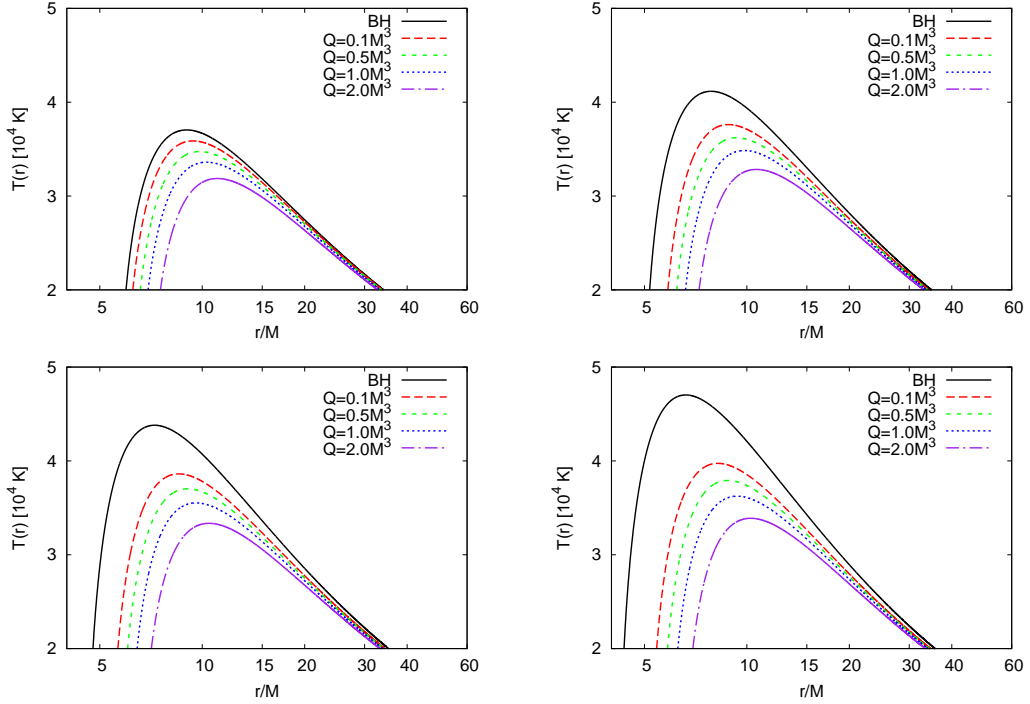


FIG. 2: The disk temperature of slowly rotating gravastars and black holes for the spin parameter $a_* = 0.1$ (upper left hand plot), $a_* = 0.3$ (upper right hand plot), $a_* = 0.4$ (lower left hand plot), and $a_* = 0.5$ for (lower right hand plot). All the plots are given for the total mass $M = 10^6 M_\odot$, the quadrupole moments $Q = 0.1, 0.5, 1.0, 2.0$ times M^3 , and the mass accretion rate $2.5 \times 10^{-5} M_\odot/\text{yr}$.

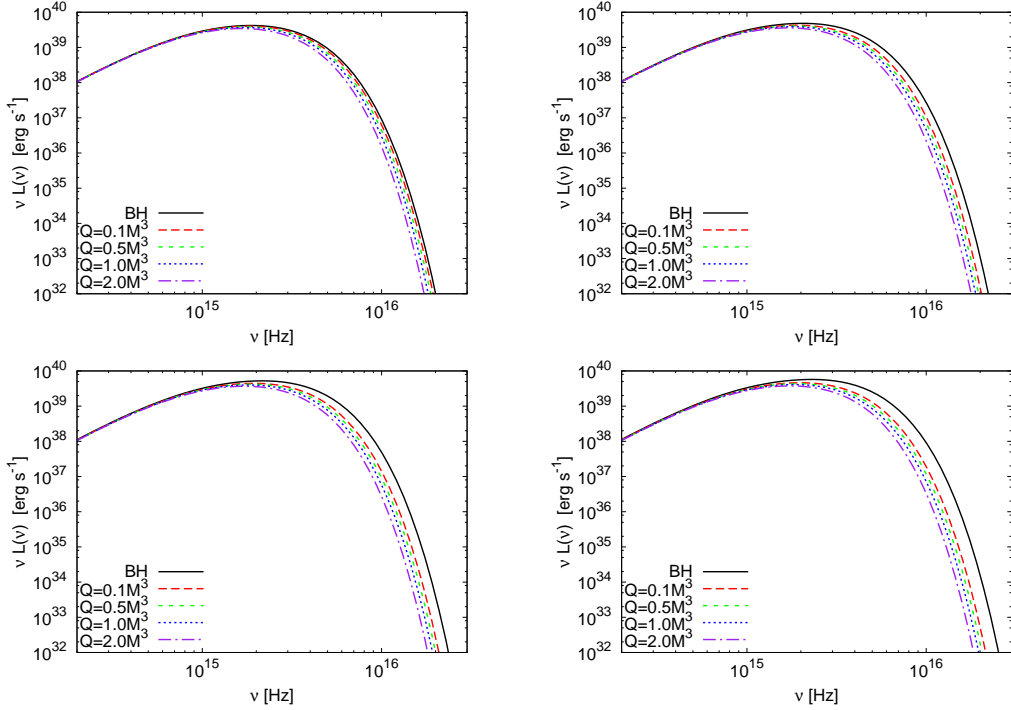


FIG. 3: The disk spectra of slowly rotating gravastars and black holes for the spin parameter $a_* = 0.1$ (upper left hand plot), $a_* = 0.3$ (upper right hand plot), $a_* = 0.4$ (lower left hand plot), and $a_* = 0.5$ for (lower right hand plot). All the plots are given for the total mass $M = 10^6 M_\odot$, the quadrupole moments $Q = 0.1, 0.5, 1.0, 2.0$ times M^3 , and the mass accretion rate $2.5 \times 10^{-5} M_\odot/\text{yr}$.

a_*	$Q [M^3]$	$r_{ms} [M]$	$\epsilon [10^{-2}]$
0.1	0.1	5.91	5.82
0.1	0.5	6.20	5.60
0.1	1.0	6.50	5.37
0.1	2.0	7.00	5.02
0.2	0.1	5.75	6.00
0.2	0.5	6.05	5.75
0.2	1.0	6.37	5.50
0.2	2.0	6.77	5.23
0.3	0.1	5.58	6.19
0.3	0.5	5.89	5.90
0.3	1.0	6.23	5.63
0.3	2.0	6.89	5.12
0.4	0.1	5.39	6.40
0.4	0.5	5.74	6.08
0.4	1.0	6.09	5.77
0.4	2.0	6.65	5.33
0.5	0.1	5.20	6.64
0.5	0.5	5.58	6.27
0.5	1.0	5.95	5.93
0.5	2.0	6.52	5.45

TABLE II: The marginally stable orbit and the efficiency for slowly rotating gravastars with different spin parameters and quadrupole moments.

in its quadrupole moment. In addition to this, it is always smaller than the ϵ for black holes, i.e., gravastars provide a less efficient mechanism for converting mass to radiation than black holes.

In order that our proposal for discriminating gravastars from black holes by using the electromagnetic emission properties of accretion disks could be effectively applied in concrete observational cases, it is necessary to know at least the value of the mass and of the spin parameter of the central rotating compact object. To estimate the spins of stellar-mass black holes in X-ray binaries, one has to fit the continuum X-ray spectrum of the radiation from the accretion disk, by using the standard thin disk model, and extract the dimensionless spin parameter $a_* = a/M$ of the black hole as a parameter of the fit [44]. Recently, a number of precise black hole spin determinations have been reported. By using Chandra and Gemini-North observations of the eclipsing X-ray binary M33 X-7, precise values of the mass of its black hole primary and of the system's orbital inclination angle have been obtained. The distance to the binary is also known to a few percent. By using these precise results, from the analysis of 15 Chandra and XMM-Newton X-ray spectra, and a fully relativistic accretion disk model, one can find that the dimensionless spin parameter of the black hole primary is $a_* = 0.77 \pm 0.05$ [45]. Therefore, even that presently there are severe observational limitations for the application of our proposal, with the future improve-

ments of the observational techniques, the observation of the emission spectra of accretion disks could be effectively used to discriminate between gravastars and black holes.

V. DISCUSSIONS AND FINAL REMARKS

If gravastars are surrounded by a thin shell of matter, the presence of a turning point for matter (the point where the motion of the infalling matter suddenly stops) at the surface of the gravastars may have important astrophysical and observational implications. Since the velocity of the matter at the gravastar surface is zero, matter can be captured and deposited on the surface of gravastar. Therefore gravastars may have a gaseous surface, formed from a thin layer of dense and hot gas. Moreover, because matter is accreted continuously, the increase in the size and density of the surface will ignite some thermonuclear reactions [29]. The ignited reactions are usually unstable, causing the accreted layer of gas to burn explosively within a very short period of time. After the nuclear fuel is consumed, the gravastar reverts to its accretion phase, until the next thermonuclear instability is triggered. Thus, gravastars may undergo a semi-regular series of explosions, called type I thermonuclear bursts, discovered first for X-ray binaries [46, 47].

The observational signatures indicating the presence of X-ray bursts from gravastars are similar to those of standard neutron stars, and are the gravitational redshift of a surface atomic line, the touchdown luminosity of a radius-expansion burst, and the apparent surface area during the cooling phases of the burst [48].

If the thermal radiation with wavelength λ_e emitted by the matter at the surface of the gravastar has absorption or emission features characteristic of atomic transitions, these features will be detected at infinity at a wavelength λ_o , gravitationally redshifted with a value

$$z_{grav} = \frac{\lambda_o - \lambda_e}{\lambda_e} = e^{-\nu(R)/2} - 1, \quad (43)$$

where we have assumed, for simplicity, that the gravastar is static, and that the exterior metric can be approximated by the standard Schwarzschild metric, given by Eq. (8). By assuming a gravastar of mass $M = 4 \times 10^6 M_\odot$ and radius $R = 1.4 \times 10^{12}$ cm, we obtain a surface redshift of $z_{grav} = 1.55$. The corresponding value of the redshift for a neutron star with mass $M = 2M_\odot$ and radius $R = 10^6$ cm is $z_{NS} = 0.56$. Therefore the radiation coming from the surface of a gravastar may be highly redshifted (in standard general relativity the redshift obeys the constrain $z \leq 2$).

Type I X-ray bursts show strong spectroscopic evidence for a rapid expansion of the radius of the X-ray photosphere. The luminosities of these bursts reach the Eddington critical luminosity at which the outward radiation force balances gravity, causing the expansion layers of the star. The touchdown luminosity of radius-

expansion bursts from a given source remain constant between bursts to within a few percent, giving empirical verification to the theoretical expectation that the emerging luminosity is approximately equal to the Eddington critical luminosity. The Eddington luminosity at infinity of a gravastar is given by [48]

$$L_E^\infty = \frac{4\pi m_0 R^2}{\sigma} e^{-\lambda} \frac{de^{\nu/2}}{dr} \Big|_{r=R}, \quad (44)$$

where m_0 is the mass of the particle and σ is the interaction cross section. For the gravastar we obtain

$$L_E^\infty = \frac{4\pi m_0 M}{\sigma} \sqrt{1 - \frac{2M}{r}} \Big|_{r=R}. \quad (45)$$

The ratio of the Eddington luminosities at infinity for a gravastar with mass of $4 \times 10^6 M_\odot$ and radius 1.4×10^{12} cm and a neutron star with mass of 2 solar masses and radius of 10 km is $L_{Egrav}^\infty / L_{ENS}^\infty = 1.22 \times 10^6$. Finally, we consider the apparent surface area during burst cooling. Observations of the cooling tails of multiple type I bursts from a single source have shown that the apparent surface area of the emitting region, defined as $S^\infty = 4\pi D^2 F_{c,\infty} / \sigma_{SB} T_{c,\infty}^4$, where $F_{c,\infty}$ is the measured flux of the source during the cooling tail of the burst, $T_{c,\infty}$ is the measured color temperature of the burst spectrum, D is the distance to the source and σ_{SB} is the Stefan-Boltzmann constant, remains approximately constant during each burst, and between bursts from the same source. The color temperature on the surface of the compact object $T_{c,h}$ is related to the color temperature measured at infinity by $T_{c,h} = T_{c,\infty} e^{-\nu(R)/2}$ [48]. By introducing the color correction factor $f_c = T_c / T_{eff}$, where T_{eff} is the effective temperature at the surface, we obtain

$$S^\infty = 4\pi \frac{R^2}{f_c^4} [z(R) + 1]^2. \quad (46)$$

Since the radius of the gravastar as well as its surface redshift may be very large quantities, the apparent area of the emitting region as measured at infinity may be also very large. Hence all the astrophysical quantities related to the observable properties of the X-ray bursts, originating at the surface of the gravastar can be calculated, and have finite values on the surface of the gravastar and at infinity.

It was argued in [29] that any neutron star, composed by matter described by a more or less general equation of state, should experience thermonuclear type I bursts at appropriate mass accretion rates. The question asked in [29] is whether an ‘‘abnormal’’ surface may allow such a behavior. The gravastars may have such a zero velocity, particle trapping, abnormal surface. The presence of a material surface located at the ‘‘event horizon’’ implies that energy can be radiated, once matter collides with that surface. Thus, gravastar models, characterized by high mass, normal matter crusts/surfaces and type

I thermonuclear bursts can be theoretically constructed. Moreover, some of the so-called SXT's (soft X-ray transients), having a relatively low mass function (e.g. SXT A0620-00, with a mass function $f(M) \geq 3M_{\odot}$ [29]), but still exceeding the equilibrium limit of $3M_{\odot}$, or very massive neutron stars showing the presence of a crust, may in fact be gravastars.

It is generally expected that most of the astrophysical objects grow substantially in mass via accretion. Recent observations suggest that around most of the active galactic nuclei (AGN's) or black hole candidates there exist gas clouds surrounding the central far object, and an associated accretion disk, on a variety of scales from a tenth of a parsec to a few hundred parsecs [43]. These clouds are assumed to form a geometrically and optically thick torus (or warped disk), which absorbs most of the ultraviolet radiation and the soft x-rays. The gas exists in either the molecular or the atomic phase. Evidence for the existence of super massive black holes comes from the very long baseline interferometry (VLBI) imaging of molecular H_2O masers in active galaxies, like NGC 4258 [49], and from the astrometric and radial velocity measurements of the fully unconstrained Keplerian orbits for short period stars around the supermassive black hole at the center of our galaxy [50, 51]. The VLBI imaging, produced by Doppler shift measurements assuming Keplerian motion of the masering source, has allowed a quite accurate estimation of the central mass, which has been found to be a $3.6 \times 10^7 M_{\odot}$ super massive dark object, within 0.13 parsecs. Hence, important astrophysical information can be obtained from the observation of the motion of the gas streams in the gravitational field of compact objects.

Therefore the study of the accretion processes by compact objects is a powerful indicator of their physical nature. However, up to now, the observational results have confirmed the predictions of general relativity mainly in a qualitative way. With the present observational precision one cannot distinguish between the different classes of compact/exotic objects that appear in the theoretical framework of general relativity [29]. However, with important technological developments one may allow to image black holes and other compact objects directly [52]. Recent observations at a wavelength of 1.3 mm have set a size of microarcseconds on the intrinsic diameter of SgrA* [53]. This is less than the expected apparent size of the event horizon of the presumed black hole, thus suggesting that the bulk of SgrA* emission may not be centered on the black hole, but arises in the surrounding accretion flow. A model in which Sgr A* is a compact object with a thermally emitting surface was considered in [54]. Given the very low quiescent luminosity of Sgr A* in the near-infrared, the existence of a hard surface, even in the limit in which the radius approaches the horizon, places a severe constraint on the steady mass accretion rate onto the source: $\dot{M} \leq 10^{-12} M_{\odot}/\text{yr}$. This limit is well below the minimum accretion rate needed to power the observed submillimeter luminosity of Sgr

A*: $\dot{M} > 10^{-10} M_{\odot}/\text{yr}$. Thus it follows that Sgr A* does not have a surface, i.e., that it must have an event horizon. This argument could be made more restrictive by an order of magnitude with microarcsecond resolution imaging, e.g., with submillimeter very long baseline interferometry. Submilliarcsecond astrometry and imaging of the black hole Sgr A* at the Galactic Centre may become possible in the near future at infrared and submillimetre wavelengths [55]. The expected images and light curves, including polarization, associated with a compact emission region orbiting the central black hole were computed in [56]. From spot images and light curves of the observed flux and polarization it is possible to extract the black hole mass and spin. At radio wavelengths, disc opacity produces significant departures from the infrared behavior, but there are still generic signatures of the black hole properties. Detailed comparison of these results with future data can be used to test general relativity, and to improve existing models for the accretion flow in the immediate vicinity of the black hole.

With the improvement of the imaging observational techniques, it will also be possible to provide clear observational evidence for the existence of gravastars, and to differentiate them from other types of compact general relativistic objects. Indeed, in this work we have shown that the thermodynamic and electromagnetic properties of the disks (energy flux, temperature distribution and equilibrium radiation spectrum) are different for these two classes of compact objects, consequently giving clear observational signatures. More specifically, comparing the energy flux emerging from the surface of the thin accretion disk around black holes and gravastars of similar masses, it was found that its maximal value is systematically lower for gravastars, independently of the values of the spin parameter or the quadrupole momentum. These effects are confirmed from the analysis of the disk temperatures and disk spectra. In addition to this, it is also shown that the conversion efficiency of the accreting mass into radiation is always smaller than the conversion efficiency for black holes, i.e., gravastars provide a less efficient mechanism for converting mass to radiation than black holes. Thus, these observational signatures may provide the possibility of clearly distinguishing rotating gravastars from Kerr-type black holes.

Acknowledgments

We would like to thank the two anonymous referees for suggestions and comments that helped us to significantly improve the manuscript. The work of TH was supported by the General Research Fund grant number HKU 701808P of the government of the Hong Kong Special Administrative Region. ZK was supported by the Hungarian Scientific Research Fund (OTKA) grant No. 69036.

-
- [1] R. Doran, F. S. N. Lobo and P. Crawford, *Found. Phys.* **38**, 160187 (2008).
- [2] P. O. Mazur and E. Mottola, *Proc. Natl. Acad. Sci.* **111**, 9545 (2004).
- [3] M. Visser and D. L. Wiltshire, *Class. Quantum Grav.* **21**, 1135 (2004).
- [4] B. M. N. Carter, *Class. Quantum Grav.* **22**, 4551 (2005).
- [5] P. Rocha, R. Chan, M. F. A. da Silva and A. Wang, *JCAP* **0811**, 010 (2008).
- [6] C. Cattoen, T. Faber and M. Visser, *Class. Quantum Grav.* **22**, 4189 (2005).
- [7] A. DeBenedictis, D. Horvat, S. Ilijic, S. Kloster and K. S. Viswanathan, *Class. Quant. Grav.* **23**, 2303 (2006).
- [8] F. S. N. Lobo and A. V. B. Arellano, *Class. Quantum Grav.* **24**, 1069 (2007).
- [9] N. Bilic, G. B. Tupper and R. D. Viollier, *JCAP* **0602**, 013 (2006).
- [10] G. Chapline, “Dark energy stars,” [arXiv:astro-ph/0503200].
- [11] F. S. N. Lobo, *Class. Quantum Grav.* **23**, 1525 (2006).
- [12] A. DeBenedictis, R. Garattini and F. S. N. Lobo, *Phys. Rev. D* **78**, 104003 (2008).
- [13] O. Bertolami and J. Paramos, *Phys. Rev. D* **72**, 123512 (2005).
- [14] F. S. N. Lobo, *Phys. Rev. D* **75** 024023 (2007).
- [15] A. E. Broderick and R. Narayan, *Class. Quantum Grav.* **24**, 659 (2007).
- [16] C. B. M. H. Chirenti and L. Rezzolla, *Class. Quantum Grav.* **24**, 4191 (2007).
- [17] D. Horvat and S. Ilijic, *Class. Quantum Grav.* **24**, 5637 (2007).
- [18] V. Cardoso, P. Pani, M. Cadoni and M. Cavaglia, *Phys. Rev. D* **77**, 124044 (2008).
- [19] C. B. M. H. Chirenti and L. Rezzolla, *Phys. Rev. D* **78**, 084011 (2008).
- [20] D. Horvat, S. Ilijic and A. Marunovic, *Class. Quantum Grav.* **26**, 025003 (2009).
- [21] B. V. Turimov, B. J. Ahmedov and A. A. Abdujabbarov, *MPLA*, in press, arXiv:0902.0217 (2009).
- [22] I. D. Novikov and K. S. Thorne, in *Black Holes*, ed. C. DeWitt and B. DeWitt, New York: Gordon and Breach (1973).
- [23] N. I. Shakura and R. A. Sunyaev, *Astron. Astrophys.* **24**, 33 (1973).
- [24] D. N. Page and K. S. Thorne, *Astrophys. J.* **191**, 499 (1974).
- [25] K. S. Thorne, *Astrophys. J.* **191**, 507 (1974).
- [26] T. Harko, Z. Kovács and F. S. N. Lobo, *Phys. Rev. D* **78**, 084005 (2008); T. Harko, Z. Kovács and F. S. N. Lobo, *Phys. Rev. D* **79**, 064001 (2009).
- [27] S. Bhattacharyya, A. V. Thampan and I. Bombaci, *Astron. Astrophys.* **372**, 925 (2001).
- [28] D. Torres, *Nucl. Phys. B* **626**, 377 (2002).
- [29] Y. F. Yuan, R. Narayan and M. J. Rees *Astrophys. J.* **606**, 1112 (2004).
- [30] F. S. Guzman, *Phys. Rev. D* **73**, 021501 (2006).
- [31] C. S. J. Pun, Z. Kovács and T. Harko, *Phys. Rev. D* **78**, 084015 (2008).
- [32] Z. Kovács, K. S. Cheng and T. Harko, *Astron. Astrophys.*, **500**, 621 (2009).
- [33] C. S. J. Pun, Z. Kovács and T. Harko, *Phys. Rev. D* **78**, 024043 (2008).
- [34] G. Darmois, “Les équations de la gravitation einsteinienne,” *Mémorial des sciences mathématiques XXV* (Gauthier-Villars, Paris, France, 1927); W. Israel, *Nuovo Cimento* **44B**, 1 (1966); and corrections in *ibid.* **48B**, 463 (1966).
- [35] J. J. Matese and P. G. Whitman, *Phys. Rev. D* **22** 1270-1275 (1980).
- [36] R. C. Tolman, *Phys. Rev.* **55**, 364 (1939).
- [37] M. R. Finch and J. E. F. Skea, *Class. Quant. Grav.* **6** 467-476 (1989).
- [38] M. K. Mak and T. Harko, *Proc. Roy. Soc. Lond. A* **459**, 393-408 (2003); M. K. Mak and T. Harko, *Annalen Phys.* **11**, 3-13 (2003).
- [39] J. B. Hartle, *Astrophys. J.* **150**, 1005 (1967)
- [40] J. B. Hartle and K. S. Thorne, *Astrophys. J.* **153**, 807 (1968).
- [41] J. P. Luminet, *Astron. Astrophys.* **75**, 228 (1979).
- [42] S. Bhattacharyya, R. Misra and A. V. Thampan, *Astrophys. J.* **550**, 841 (2001).
- [43] C. M. Urry and P. Padovani, *Publ. Astron. Soc. of the Pacific* **107**, 803 (1995).
- [44] R. Shafee, R. Narayan, and J. E. McClintock, *Astrophys. J.* **676**, 549 (2008).
- [45] J. Liu, J. E. McClintock, R. Narayan, S. W. Davis, and J. A. Orosz, *Astrophys. J.* **679**, L37 (2008).
- [46] J. Grindlay, H. Gursky, H. Schnopper, D. R. Parsignault, J. Heise, A. C. Brinkman, and J. Schrijver, *Astrophys. J.* **205**, L127 (1976).
- [47] D. Tournear, E. Raffauf, E. D. Bloom, W. Focke, B. Giebels, G. Godfrey, P. M. Saz Parkinson, K. T. Reilly, K. S. Wood, P. S. Ray, M. T. Wolff, R. M. Bandyopadhyay, M. N. Lovellette, and J. D. Scargle, *Astrophys. J.* **595**, 1058 (2003).
- [48] D. Psaltis, *Phys. Rev.* **D77**, 064006 (2008).
- [49] M. Miyoshi, J. Moran, J. Herrnstein, L. Greenhill, N. Nakai, P. Diamond and M. Inoue, *Nature* **373**, 127 (1995).
- [50] A. M. Ghez, S. Salim, N. N. Weinberg, J. R. Lu, T. Do, J. K. Dunn, K. Matthews, M. Morris, S. Yelda, E. E. Becklin, T. Kremenek, M. Milosavljevic, and J. Naiman, *Astrophys. J.* **689**, 1044 (2008).
- [51] S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins, and T. Ott, *Astrophys. J.* **692**, 1075 (2009).
- [52] H. Falcke, F. Melia, and E. Agol, *Astrophys. J.* **528**, L13 (2000).
- [53] S. Doeleman, J. Weintroub, A. E.E. Rogers, R. Plambeck, R. Freund, R. P.J. Tilanus, P. Friberg, L. M. Ziurys, J. M. Moran, B. Corey, K. H. Young, D. L. Smythe, M. Titus, D. P. Marrone, R. J. Cappallo, D. C.J. Bock, G. C. Bower, R. Chamberlin, G. R. Davis, T. P. Krichbaum, J. Lamb, H. Maness, A. E. Niell, A. Roy, P. Strittmatter, D. Werthimer, A. R. Whitney, and D. Woody, *Nature* **455**, 78 (2008).
- [54] A. E. Broderick and R. Narayan, *Astrophys. J.* **636**, L109 (2006).
- [55] A. E. Broderick and A. Loeb, *Mon. Not. Roy. Astron. Soc.* **367**, 905 (2006).
- [56] A. E. Broderick and A. Loeb, *Mon. Not. Roy. Astron. Soc.* **363**, 353 (2005).