A Spatial Portfolio Theory of Household Location Choice

by

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Abstract: Classical residential location choice models were constructed as uncertainty-free. Using the expected utility theorem, urban researchers have dealt with different types of uncertainties, such as uncertain income, uncertain housing price, uncertain transportation cost, etc. This paper, however, considers uncertain traveling frequencies in 2-workplace setting, a novel theory on the emergence of a new centre between two existing CBDs can then be formulated. It can be regarded as a spatial portfolio theory as the theory predicts that household location choice would strike a balance between commuting cost savings (return) and variance of the savings (risk). Empirical evidence on the housing transaction price gradient changes in Hong Kong supports the theory.

JEL Classification: R21 – Housing Demand

Keywords: multiple workplaces, housing demand, price gradient, commuting distance, traveling frequency

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1. INTRODUCTION

The whole structure of urban economics is underpinned by the concept of central business districts (CBDs). A CBD is often defined as the peak of the inverted cone of negative price gradients (Lösch, 1938, 1940), i.e. the centre of higher land value than its peripherals. Theories and empirical studies explaining the existence of a CBD almost always rest on commuting costs, job availability, social factors and demographics. Furthermore, household location choice studies often found downward sloping housing price gradients originated from the CBD. However, similar to Darwin's (1894) theory of evolution, the theories of CBD formation are good at explaining, but not good at predicting where a new CBD would be formed. We have many examples of failure attempts of artificially forming a centre by building infrastructures, but counterexamples of natural evolution of new business centre spontaneously. This paper aims to put forward a novel spatial portfolio theory of household location choice to both explain and predict why centres are evolved at locations between existing centres. It is based on the assumption of multiple workplaces with uncertain frequency of commuting.

This model is coined as a spatial portfolio theory because it is analogous to Markowitz's (1952) Portfolio Theory and shared the same background of the writing motivation. Markowitz (1952) contended that "investor does consider expected return a desirable thing and variance of return an undesirable thing", which predicts diversification behavior of the investor. Analogously, housing buyer regards commuting cost savings a desirable thing and variance of commuting cost savings an undesirable thing. Traditional analyses of household location choice assumed zero variance of commuting cost savings, thus predicting a downward price gradient from the CBD. However, in case of multiple workplaces with uncertain commuting frequency, i.e. variance of commuting cost savings is not negligible, the best strategy is not to select household location close to a CBD, but in-between CBDs, i.e. a portfolio. This theory predicts the formation of new CBD in-between existing CBDs, which agrees closely with intuition and casual observations. Empirical evidence to this theory is separately discussed.

The arrangement of this paper is as follows: Section 2 provides a literature review on household location choice. Section 3 discusses the spatial portfolio theory of household location choice. Section 4 presents some casual observations and Section 5 is a concluding section.

2. LITERATURE REVIEW

Since von Thunen's (1826) isolated state model, and then Alonso's (1964) bid-rent curve, a declining unit land rent against distance from the CBD is predicted. The determinants of the location of a CBD are often explained by the competition of accessibility, or put in Fujita's (1989) words in his standard urban economic (SUE) framework, a price gradient surface should reflect commuting costs, income and demographics. These factors in a polycentric city setting have been modeled by Papageorgiou (1971), Papageorgiou and Casetti (1971), Hartwick and Hartwick (1974), Amson (1976), White (1976, 1988), Odland (1978), Ramanos (1979), Ogawa and Fujita (1980), Griffith (1981a, b), Fujita and Ogawa (1982), Gordon and Wong (1986), Peiser (1987), Wieand (1987), Richardson et al. (1990), Sasaki (1990), Helsley and Sullivan (1991), Gordon and Richardson (1996), Sasaki and Mun (1996), Yinger (1992), Sivitanidou (1996, 1997), and Anas et al. (1998).

Summing up, previous studies on urban spatial structure was based on the following n-1-1 assumptions: (1) n known centres (monocentric when n=1, bi-centric when n=2 and polycentric when n>2); (2) every household travels to a single workplace at a certain period of time (1-workplace); and (3) the factors affecting location decisions are deterministic (1=certain; 0=uncertain). Numerous attempts have been conducted to relax these assumptions, which Yiu and Tam (2004) provided a comprehensive review of these studies.

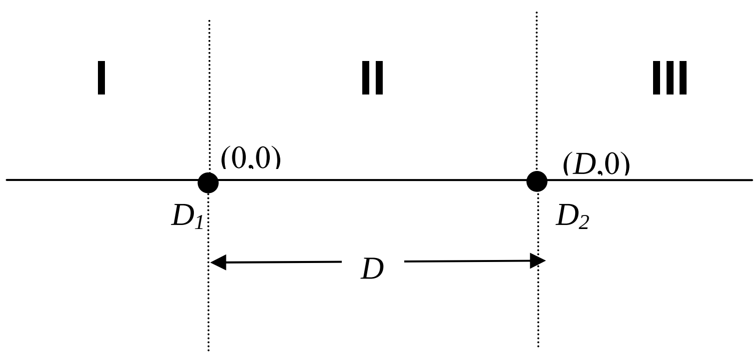
However, the rapid decentralization of economic activity and the globalization trend make the classical negative price gradient irresponsive to the distance from employment centers. Anas et al. (1998) argued that sub-center employment is widely dispersed and households may have idiosyncratic preferences for particular locations. Clapp et al. (2001) also contended that improvements in telecommunications and transportation have aided suburbanization of households. Decentralization and globalization trend does not only result in edge cities (Garreau, 1991), but also bring along multiple-workplace concept. There are at least three reasons of multiple workplaces per household, namely: (1) multiple earners households, (2) job location changes in the future, and, (3) multiple workplaces per earner. Madden (1980), Curran, et al. (1982), Timmermans, et al. (1991), Hotchkiss and White (1993), Kim (1995), Freedman and Kern (1997), van Ommeren, et al. (1998) and White (1999) investigated the first cause: residential location choice in two-earner households. Crane (1995) studied the second: effects of uncertain job location on urban form. Romanos (1977) and Yiu and Tam (2007) and Yiu (2008) probed into the third: household location choice of earners with multiple workplaces. Yet, all the previous studies assumed known commuting frequency to each centre, i.e. risk levels are assumed to be certain.

In view of the nature of housing consumption, it was commonly recognized that households do not know all the factors with certainty when making housing consumption and location decisions. The introduction of uncertainty into the urban location theory then blossomed. They can be categorized into four types of uncertainty, namely (1) uncertainty in income (studied in Andrulis, 1982; DeSalvo and Eeckhondt, 1982; Turnbull et al., 1991 and Turnbull, 1995); (2) uncertainty in housing user cost (studied in Turnbull et al., 1991 and Turnbull, 1995); (3) uncertainty in transportation cost (studied in Papageorgiou and Pines, 1988 and Turnbull, 1995); and (4) uncertainty in quality and neighborhood externalities (studied in Papageorgiou, 1991 and Turnbull, 1991, 1995). Incorporating these uncertainties into the urban location model, however, does not affect the general downward sloping bid-rent curve from the CBD. Furthermore, the uncertainties were theoretically addressed in the urban residential choice model in term of a stochastic variable; yet the origination of the uncertainty was seldom considered within the model, i.e. the stochastic term is appeared *as is*. This study, on the other hand, investigates the effect of uncertainty in traveling frequency to multiple workplaces on the households' location decisions. This uncertainty is the result of the multiple workplaces phenomenon.

3. PORTFOLIO THEORY OF HOUSEHOLD LOCATION CHOICE

This paper develops a two-centre two-workplace with uncertain traveling frequency model (2-2-0 model) of a linear city with two working locations similar to Romanos (1977). Suppose the city is built on a narrow strip with the width equal to one unit; housing unit can be found along the whole city. There are two workplaces, D_1 and D_2 respectively. Each household is required to travel to either one or both of the workplaces in every period; and the mean and variance of the visiting frequencies for the two workplaces are μ_1, σ_1^2 and μ_2, σ_2^2 respectively, which are exogenously determined. Using the working locations as landmarks, the city can be divided into three quadrants: the LHS of D_1, between D_1 and D_2, and the RHS of D_2; they are denoted as quadrants **I**, **II** and **III** respectively as in Figure 1.

Figure 1. The city coordinate



For analytical purpose, any particular point along the city is represented by a Euclidian coordinate; by taking D_1 as the origin without loss of generality. Any point along the city can be identified by a coordinate (z,0); where the magnitude of z is the distance between the point and D_1, which is positive in quadrants **II** and **III** and negative in quadrant **I**. Hence the coordinates of D_1 and D_2 are (0,0) and (D,0) where

$z = D$ at D_2. The expected commuting distance $E(d)$ at any point (z,0) weighted with the commuting frequencies, μ_1 and μ_2 is:

$$E(d) = \mu_1 z + \mu_2 (D - z) \tag{1}$$

Assuming fixed unit cost of commuting c, the expected commuting cost $E(c)$ at any point (z,0) is then:

$$E(c) = c[\mu_1 z + \mu_2 (D - z)] \tag{2}$$

The variance of the expected commuting cost is:

$$\begin{aligned}\sigma_c^2 &= Var(c \bullet d) \\ &= c^2 Var(d) \\ &= c^2 \left[z^2 \sigma_1^2 + (D - z)^2 \sigma_2^2 + 2Corr(D_1, D_2) \bullet \sigma_1 \sigma_2 z (D - z) \right]\end{aligned} \tag{3}$$

When the studied period is fixed, then the more you travel to D_1, the less you travel to D_2. It is plausible to assume the correlation between them is -1, i.e. $Corr(D_1, D_2) = -1$. Therefore,

$$\begin{aligned}\sigma_c^2 &= c^2 \left[z^2 \sigma_1^2 + (D - z)^2 \sigma_2^2 - 2Dz\sigma_1\sigma_2 + 2z^2 \sigma_1 \sigma_2 \right] \\ &= c^2 \left[z^2 (\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2) + D^2\sigma_2^2 - 2Dz\sigma_2^2 - 2Dz\sigma_1\sigma_2 \right] \\ &= c^2 \left[z^2 (\sigma_1 + \sigma_2)^2 + D\sigma_2 (D\sigma_2 - 2z\sigma_2 - 2z\sigma_1) \right]\end{aligned} \tag{4}$$

The housing price along the city can be revealed by the classical household's utility maximization problem subject to the budget constraints:

$$\begin{aligned}&\max_{y,h,r} U(x, h, \sigma_c^2) \\ &\text{s.t.}\, Y = x + hR(z) + cE(d) \,\text{and}\, \sigma_c^2 = \sigma_c^2(z)\end{aligned} \tag{5}$$

where x is the non-housing composite goods with price equal to one; h is the quantity of homogeneous housing consumption. The household's utility increases with x, h and decreases with σ_c^2, i.e. U_x, $U_h > 0$ and $U_{\sigma_c^2} < 0$; subscripts are the corresponding partial derivatives. Y is the household's income per period, R(.) is the function of unit housing rent, and c is the fixed unit commuting cost. The Lagrangian of the maximization is:

$$U(x,h,\sigma_c^2)+\lambda[Y-x-hR(z)-cE(d)] \tag{6}$$

The first order conditions are:

$$\begin{aligned}
&U_x-\lambda^*=0\\
&U_h-\lambda^*R(z)=0\\
&U_\sigma(\frac{\partial\sigma_c^2}{\partial z})-\lambda^*hR'(z)-\lambda^*c(\mu_1-\mu_2)=0\\
&\Rightarrow R'(z)=\left[U_\sigma(\frac{\partial\sigma_c^2}{\partial z})-U_x c(\mu_1-\mu_2)\right]\frac{1}{h\cdot U_x}\\
&=-\frac{c(\mu_1-\mu_2)}{h}+\frac{U_\sigma}{h\cdot U_x}\left(\frac{\partial\sigma_c^2}{\partial z}\right)
\end{aligned} \tag{7}$$

Differentiate (4) with respect to z, we get $\frac{\partial\sigma_c^2}{\partial z}=c^2\left[2z\left(\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2\right)-2D\left(\sigma_1\sigma_2+\sigma_2^2\right)\right]$. Consider the sign of the terms inside the bracket, then

$$\begin{aligned}
&\left[2z\left(\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2\right)-2D\left(\sigma_1\sigma_2+\sigma_2^2\right)\right]\geq 0\\
&\Rightarrow\frac{z}{D}\geq\frac{\sigma_2^2+\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2}=\theta
\end{aligned} \tag{8}$$

For the sake of illustration, a simple case of $\sigma_1^2=\sigma_2^2$ is used, i.e. $\theta=\frac{1}{2}$. Therefore,

$$\frac{\partial\sigma_c^2}{\partial z}=c^2\left[2z\left(\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2\right)-2D\left(\sigma_1\sigma_2+\sigma_2^2\right)\right]\left.\begin{matrix}>0\\=0\\<0\end{matrix}\right\}when\frac{z}{D}\left\{\begin{matrix}>1/2\\=1/2\\<1/2\end{matrix}\right. \tag{9}$$

Without loss of generalization, assume $\mu_1>\mu_2$, the first term of Equation (7) shows that the unit housing rent is decreasing with the commuting distance; which is analogous to the prediction of Alonso's land market model. Indeed, his land market model can be regarded as a special case of ours with the second term is equal to zero. However, when the uncertainty of commuting frequency is taken into consideration, the sign of the rent gradient is not necessarily negative. There are three possible cases:

Case 1: *z/D < 1/2*

By Equation (7), the sign of the rent gradient is indeterminate, but depends on the negative marginal utility of risk, U_σ. Intuitively, households may prefer to live further from D_1 when their absolute value of marginal utility of risk is great.

Case 2: *z/D > 1/2*

By Equation (7), the sign of the price gradient is negative on the assumption of $\mu_1 > \mu_2$. The intuition is that housing price is getting higher towards the location with higher visiting frequency, *ceteris paribus*.

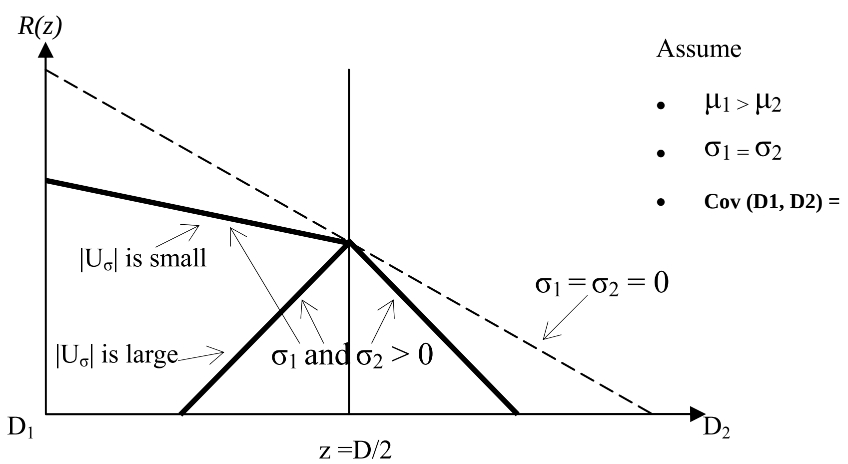
Case 3: *z/D = 1/2*

By Equation (7), the sign of the price gradient is negative as the second term on the marginal utility of risk is zero. The intuition is that households living in the middle of the two workplaces are indifference to the commuting frequency of the two places.

Graphically, the price gradient predicted by the theoretical model can be depicted as in Figure 2. The most interesting implication of this model is the upward sloping of price gradient, i.e. the emergence of a new centre between the two existing centres.

Figure 2 Theoretical Price Gradient in a Bicentric Bi-Workplace Model

with Uncertainty of Commuting Frequency



4. EMPIRICAL EVIDENCE

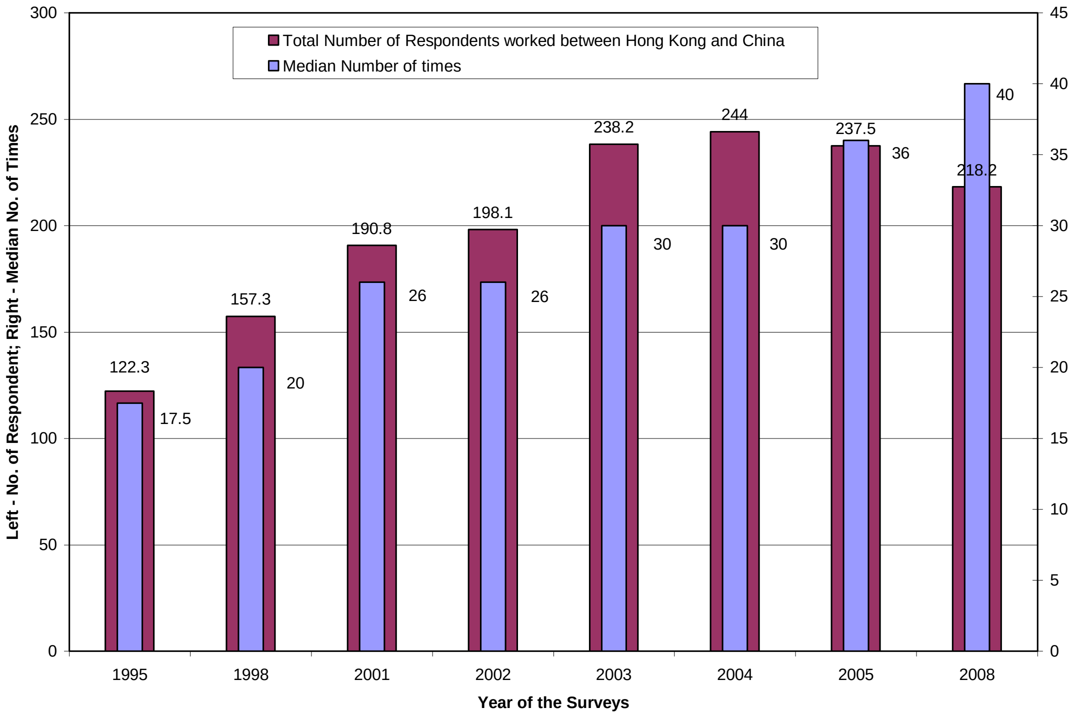
As a more robust empirical study would be separately discussed, the following aims to provide some descriptive empirical evidence to the analytical results of the above theoretical cases; that is, to study the price gradient change between two workplaces due to the uncertainty of visiting frequency to the two workplaces.

An empirical evidence on the emergence of an inverted V-shaped housing price gradient between two workplaces is found in Hong Kong – Shen Zhen, where there is a railway moving commuters between the two CBDs across the boundary between mainland China and Hong Kong. Many companies set up their headquarters at commercial centres (in Hong Kong), and their factories at industrial centres (in Shen Zhen). Such an arrangement results in more frequent travel between the headquarters and the factories.

Shen Zhen is one of the fastest growing cities of China in the past decades. The deep-rooted economic linkage between Hong Kong and Shen Zhen has begun since the 1980's; when there was a rapid migration of manufacturing industry from Hong Kong to the southern part of China. Entrepreneurs resettled their major production lines in Shen Zhen or other neighboring areas while retaining the head offices in Hong Kong. Since then, the managerial and technical staffs of these firms are required to travel to both workplaces frequently. Besides, the reform of market economy in Mainland China attracted a lot of Hong Kong investors to set up new businesses in Shen Zhen, which also results in an increasing trend of frequent business travelers between Hong Kong and Shen Zhen. The Census and Statistics Department of Hong Kong has performed a few household surveys on the topic of *Hong Kong residents working in China* during the past decade on an irregular basis. Figure 3 summarized three important changes in the 1990's.

Figure 3. Number of People and Traveling Frequency between Hong Kong and China

(Source: Census and Statistics Department, 1996-2008)



Notes: The surveys were considering the numbers of Hong Kong residents who had worked in the mainland of China during the 12 months before enumeration by number of times having traveled to work in the mainland of China during the 12 months before enumeration.

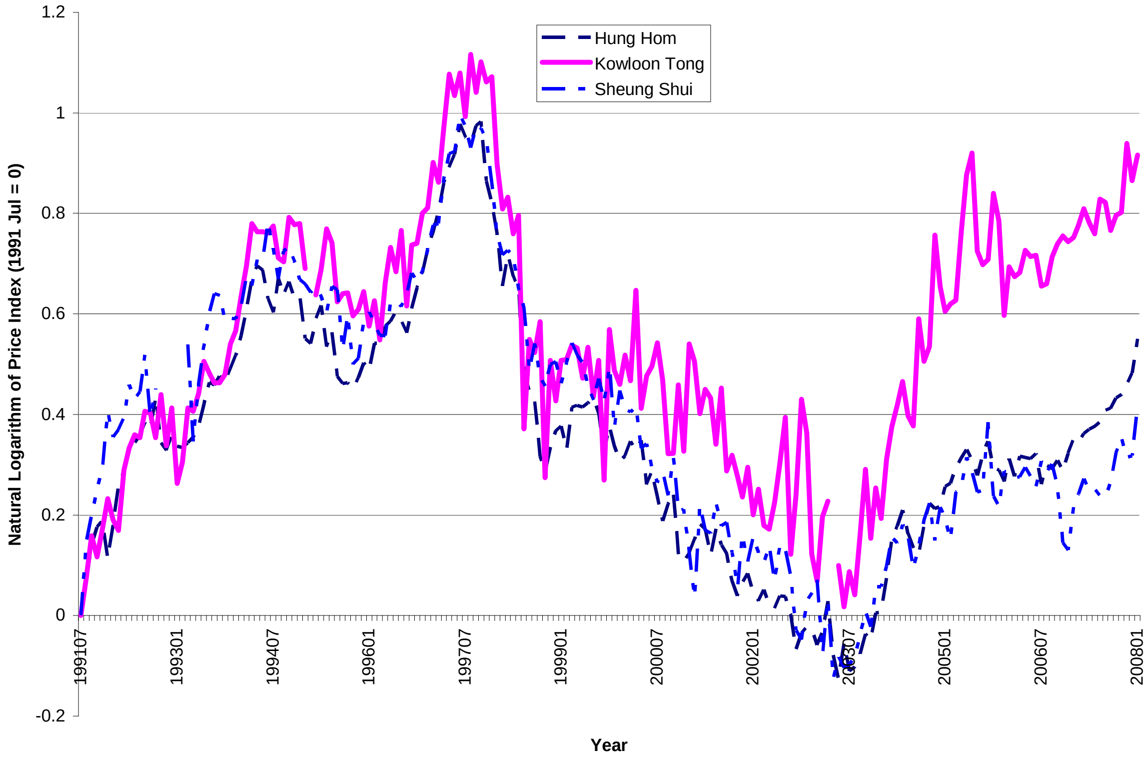
Figure 3 shows that the numbers of Hong Kong residents who work in Mainland China had almost doubled in the period between 1995 and 2008. The number had more than doubled for the median number of travels between Hong Kong and Mainland China in a year. It implies a higher certainty in the requirement of traveling, but a higher uncertainty in the traveling frequency.

The railway is one of the most convenient transportation means between Hong Kong and the southern part of China, it is apposite to assume that the two terminuses of the KCR as the two CBD workplaces, D_1 and D_2 in the multiple workplaces model and the route of the railway as the linear city.

Figure 4 is a plot of the natural logarithm of the price index of the three districts at the two terminuses of the railway (Hung Hom and Sheung Shui) and a station lies in-between

(Kowloon Tong). The data is collected from Chau et al. (2009) based on actual housing transaction prices in Hong Kong. It is clearly shown that the station in-between the terminuses (Kowloon Tong) has a much higher housing price growth than the housing prices in the two terminuses, and the price differences are enlarging in the recent five years. The results support the hypothesis of an emergence of a new housing hub in-between (i.e. an inverted V-shaped price gradient) due to an increasing uncertainty in traveling frequency between the two centres. However, there may have other reasons for the change of housing price in the new centre, a more robust empirical study would be separately carried out to test the hypothesis.

Figure 4 Natural Logarithm of Housing Price Indices of three Stations of the Railway connecting Hong Kong and Shen Zhen, 1991 - 2008



Notes: Hung Hom and Sheung Shui are the two districts where the terminuses of the railway are located at. Kowloon Tong station is located in-between the two terminuses, but closer to Hung Hom side. It has long been a residential district with high quality housing. The natural logarithm of the housing prices in July 1991 of the three districts have been normalized to zero.

8. CONCLUSIONS

This study provides a novel theoretical framework to analyze the choice of residential location with two-centre, two-workplace of uncertain traveling frequency. It has been shown that, under certainty assumption, in a linear bi-centric city, the unit housing price is higher for housing which is closer to the more frequent traveled workplace. However, this paper relaxing the certainty assumption of commuting frequency, derived the conditions for the emergency of a new centre between two existing CBDs. The theory predicts that households' best strategy in location choice would be living in-between the two workplaces when the frequency of traveling is uncertain. The underlying principle is that the housing price gradient between two workplaces will become less negative (or even positive) when the marginal utility of the uncertainty of visiting frequency to the workplaces increases relatively. Casual observations of the housing price gradient change in Hong Kong support the analytical results.

The results can be interpreted as a spatial portfolio theory in analogous with the portfolio theory in finance. Living in the mid point of two workplaces of uncertain traveling frequency is similar to an act of hedging the risk of changing traveling frequency.

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