Low Complexity Near-Maximum Likelihood Decoding for MIMO Systems

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***Abstract*—In recent literature, an increasing radii algorithm (IRA) is introduced to decode the signals in multiple-input multiple-output (MIMO) systems. It is developed from the idea of sphere decoder (SD) and can achieve near-maximum likelihood (ML) decoding performance with relatively lower complexity than the SD by taking the noise statistics into consideration. In this paper, an improved IRA (IIRA) is proposed to further reduce the complexity. Apart from the noise statistics, channel information is taken into consideration as well. Additionally, a radii update scheme is introduced which enables the search space of the proposed algorithm to be further pruned. As a result, the proposed algorithm achieves a performance close to that of the IRA but with substantial computational savings. The effectiveness of the proposed decoder is verified by simulations.**

***Index Terms*—MIMO, sphere decoder, radii, complexity**

I. Introduction

MIMO communications have attracted a lot of research interests due to its capability of improving channel capacity without extra bandwidth, and a number of decoding algorithms have been proposed in the literature. Among them, the maximum likelihood decoder (MLD) is optimal in terms of decoding performance. Under the assumption that the noise is Gaussian distributed, which is usually the case in cellular systems, the ML decoding is equivalent to solving the integer least square (LS) problem. It requires a search over the whole m-dimensional constellation space which is known to be NP-complete [1]. Here m denotes the number of inputs in the MIMO system. The decoding complexity increases exponentially with m and linearly with the constellation size, thus limiting its application in real systems.

Sphere decoder (SD) is one of the methods developed to reduce the ML decoding complexity by reducing the search space [1], [2], [3], [4]. In the SD, the search space is constructed as a hyper-sphere centered at the received signal with adjustable radius. Among all the lattice points being found in the search space of the SD, the one with the minimum distance to the center is the optimal solution. This decoder guarantees to find the exact ML solution with expected complexity of $\mathcal{O}(m^3)$.

Although the SD could significantly reduce the complexity without performance loss from the MLD, its complexity is still rather high for practical application, especially under low signal-noise-ratio (SNR) region and/or in high-order MIMO systems. Different attempts have been tried to further reduce the complexity by shrinking the search space. Update of sphere radius is one of them, which was proposed in [5], [6]. Though the solution found remains optimal, the complexity reduction is quite limited. Tree pruning is another intuitive approach proposed in [7], [8]. Unfortunately, it necessitates performance degradation as a trade-off. Recently, ordering is introduced to the tree pruning approach [9], [10]. Though reduction on the complexity could be achieved at lower performance degradation, the performance gap to ML is still not insignificant.

In [11], an increasing radii algorithm (IRA), which is another modification of the SD, was proposed to achieve near ML decoding. It considers the decoding problem from a statistical point of view. By taking the noise statistics into consideration, the search space is pruned to a subset that contains the ML solution with high probability. It offers substantial computational savings over the SD while maintaining the performance fairly close to the MLD.

In this paper, an improved decoding algorithm based on the idea of IRA is proposed. It takes not only the noise statistics but also the channel information into account when pruning the search space. In addition, an update scheme of radii is introduced to further shrink the search space once a lattice point is found. After analyzing the performance and complexity, it is observed that the proposed decoder could achieve similar performance to the IRA with substantial complexity reduction. In other words, the proposed decoder offers near-ML decoding performance with low complexity. This is verified by computer simulations.

The paper is organized as follows. Section II gives a brief introduction on the system model, sphere decoder, as well as the IRA. Section III presents the proposed decoding algorithm and its performance analysis. Simulation results are shown in Section IV, while conclusions are drawn in Section V.

II. System Model and Existing Decoders

A. System Model

Consider a discrete-time block-fading MIMO system with m transmit and m receive antennae. The received signal vector $\boldsymbol{x} \in \boldsymbol{C}^m$ is given by

$$\boldsymbol{x} = \boldsymbol{H}\tilde{\boldsymbol{s}} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{H} \in \boldsymbol{C}^{m \times m}$ is the channel matrix with independent identically distributed (*i.i.d.*) Gaussian elements of variance

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σ_h^2 and it is assumed to be known at the receiver; $\boldsymbol{n} \in \boldsymbol{C}^m$ is the noise vector, whose elements are also *i.i.d.* Gaussian random variables with zero mean and variance σ_v^2; $\tilde{\boldsymbol{s}} \in \boldsymbol{S}^m$ denotes the transmitted signal vector. Here $\boldsymbol{S}$ represents the signal constellation with average power σ_s^2. Under Gaussian noise, the MLD is equivalent to LS decoder which is to find the lattice point closest to the received signal in m-dimensional constellation space. Mathematically, it is to solve the minimization problem:

$$\min_{\boldsymbol{s}\in\boldsymbol{S}^m} \|\boldsymbol{x}-\boldsymbol{H}\boldsymbol{s}\|^2. \tag{2}$$

Apparently, this decoder requires a search over the whole $\boldsymbol{m}$-dimensional space, *i.e.*, $\boldsymbol{S}^m$ and the complexity increases exponentially with m, thus limits its application in real systems when m is large.

B. Sphere Decoder

Sphere decoder (SD) is one decoder which could find the ML solution without searching over the whole m-dimensional space. Its complexity is significantly reduced since it only searches the lattice points within a hyper-sphere $\boldsymbol{D}_{\text{SD}}$ of radius d centering at the received signal $\boldsymbol{x}$, which is represented as

$$\boldsymbol{D}_{\text{SD}} : \|\boldsymbol{x}-\boldsymbol{H}\boldsymbol{s}\|^2 \leqslant d^2. \tag{3}$$

With proper d, the search space of the SD could be limited to a subset of the ML search space.

Generally, QR factorization is firstly performed on the channel matrix $\boldsymbol{H}$ to simplify the search process. After QR factorization, the channel matrix is written as $\boldsymbol{H}=\boldsymbol{Q}\boldsymbol{R}$ where $\boldsymbol{Q}$ is a unitary matrix with $\boldsymbol{Q}'\boldsymbol{Q}=\boldsymbol{I}$ and $\boldsymbol{R}$ is an upper triangular matrix with positive diagonals. Here the superscript $(\cdot)'$ denotes the conjugate transpose of a matrix. Then (3) could be rewritten as

$$\boldsymbol{D}_{\text{SD}} : \|\boldsymbol{x}-\boldsymbol{Q}\boldsymbol{R}\boldsymbol{s}\|^2 = \|\boldsymbol{Q}'\boldsymbol{x}-\boldsymbol{R}\boldsymbol{s}\|^2 \leqslant d^2. \tag{4}$$

Defining $\boldsymbol{y}=\boldsymbol{Q}'\boldsymbol{x}$, the hyper-sphere is transformed as:

$$\boldsymbol{D}_{\text{SD}} : \|\boldsymbol{y}-\boldsymbol{R}\boldsymbol{s}\|^2 \leqslant d^2. \tag{5}$$

In the SD, the key step is to find all the lattice points within the hyper-sphere $\boldsymbol{D}_{\text{SD}}$. It is equivalent to find the points satisfying the condition of $\|\boldsymbol{y}-\boldsymbol{R}\boldsymbol{s}\|^2 \leqslant d^2$. More specifically, the following conditions should be satisfied simultaneously:

$$\sum_{j=i}^{m}\left(y_j-\sum_{k=j}^{m}R_{jk}s_k\right)^2 \leqslant d^2, i=1,2,\cdots,m. \tag{6}$$

where y_j and s_k are the j th and k th elements of $\boldsymbol{y}$ and $\boldsymbol{s}$, respectively, while R_{jk} denotes the (j,k) th entry of $\boldsymbol{R}$. The decoding process is performed in the following way. It starts from finding the candidates of s_m using the inequality in (6) with $i=m$. For each candidate of s_m, the candidates of s_{m-1} are obtained by solving the inequality in (6) with $i=m-1$. This process continues until the candidates of s_1 are found by solving the inequality in (6) with $i=1$ using the predetermined candidates for $s_2,\cdots,s_m$. Each candidate group $\{s_1\cdots s_m\}$ forms one lattice point found in this hyper-sphere. With all the lattice points found by successively solving (6), the one minimizing $\|\boldsymbol{x}-\boldsymbol{H}\boldsymbol{s}\|^2$ is selected as the ML solution.

C. Increasing Radii Algorithm

The increasing radii algorithm (IRA) proposed in [11] is a modification of the SD. In the IRA, the search space is statistically pruned to a subset that contains the ML solution with high probability, thereby further reducing the decoder's complexity. Here the search space $\boldsymbol{D}_{\text{IRA}}$ is defined as

$$\boldsymbol{D}_{\text{IRA}} : \sum_{j=i}^{m}\left(y_j-\sum_{k=j}^{m}R_{jk}s_k\right)^2 \leqslant r_{i,\text{IRA}}^2, i=1,2,\cdots,m. \tag{7}$$

where a set of radii $r_{i,\text{IRA}}$ is designed by taking the statistics of noise into consideration, given by

$$r_{i,\text{IRA}}^2 = [\delta\log m+(m-i+1)]\cdot\sigma_v^2, i=1,2,\cdots,m. \tag{8}$$

In (8), δ is an adjustable parameter to control the probability of excluding the ML solution in $\boldsymbol{D}_{\text{IRA}}$ and is provided in a lookup table in [11]. Once the search space is defined as (7), the decoding process of the IRA is similar to the SD, except that the universal sphere-radius d in (6) is replaced by a set of radii $r_{i,\text{IRA}}$ in (7).

III. Improved Increasing Radii Algorithm

In the IRA, only the statistics of noise is taken into consideration for the radii design. In this paper, an improved algorithm which also takes channel information into account to the radii design and involves a radii update scheme is proposed. It can reduce the complexity of decoding with unnoticeable performance degradation from the IRA, since extra information about the channel is exploited. In the following, the proposed algorithm will be introduced and its performance will be analyzed.

A. The Proposed Algorithm

Since the proposed algorithm expliots channel information for radii design, the statistics of the channel matrix will be discussed first. As mentioned in Section II.A., elements in the channel matrix $\boldsymbol{H}$ are Gaussian *i.i.d.*. After QR factorization, the diagonals of $\boldsymbol{R}$ are also independent. From [9], we further know that square of the normalized diagonal element R_{ii}^2/σ_h^2 follows Gamma distribution as $\left(R_{ii}^2/\sigma_h^2\right) \sim G(m-i+1)$. Here $G(k)$ stands for the standard Gamma distribution Gamma(k,θ) with $\theta=1$. According to the properties of Gamma distribution, $\mathrm{E}\left[(R_{ii}^2/\sigma_h^2)\right]=(m-i+1)$. Due to the independence among diagonal elements R_{ii}, the summations $\sum_{k=i}^{m}\left(R_{kk}^2/\sigma_h^2\right)$ and $\sum_{k=1}^{i-1}\left(R_{kk}^2/\sigma_h^2\right)$ also follow the independent Gamma distributions $\sum_{k=i}^{m}\left(R_{kk}^2/\sigma_h^2\right) \sim G\left(\frac{1}{2}(m-i+1)(m-i+2)\right)$, $\sum_{k=1}^{i-1}\left(R_{kk}^2/\sigma_h^2\right) \sim G\left(\frac{1}{2}(i-1)(2m-i+2)\right)$, respectively.

It follows that

$$\mathrm{E}\left[\sum_{k=i}^{m}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)\right]=\sum_{k=i}^{m}\mathrm{E}\left[\frac{R_{kk}^2}{\sigma_h^2}\right]=\frac{1}{2}(m-i+1)(m-i+2), \tag{9}$$

$$\mathrm{E}\left[\sum_{k=1}^{i-1}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)\right]=\sum_{k=1}^{i-1}\mathrm{E}\left[\frac{R_{kk}^2}{\sigma_h^2}\right]=\frac{1}{2}(i-1)(2m-i+2). \tag{10}$$

for $i=1,2,\cdots,m$. Defining

$$p(i)=\frac{\sum_{k=i}^{m}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)}{\sum_{k=1}^{m}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)}=\frac{\sum_{k=i}^{m}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)}{\sum_{k=1}^{i-1}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)+\sum_{k=i}^{m}\left(\frac{R_{kk}^2}{\sigma_h^2}\right)} \tag{11}$$

According to the properties of Gamma distribution, $p(i)$ follows Beta distribution as $p(i)\sim\mathrm{Beta}(\alpha,\beta)$ where $\alpha=\frac{1}{2}(m-i+1)(m-i+2)$ and $\beta=\frac{1}{2}(i-1)(2m-i+2)$ [12]. Its expectation is given by

$$\mathrm{E}\left[p(i)\right]=\frac{(m-i+1)(m-i+2)}{m(m+1)}, i=1,2,\cdots,m. \tag{12}$$

With this channel statistics, now we propose an algorithm called improved IRA (IIRA), where the search space is chosen similarly to that of the IRA, but the initial radii are chosen differently as follows:

$$\boldsymbol{D}_{\mathrm{IIRA}}^{0}:\sum_{j=i}^{m}\left(y_j-\sum_{k=j}^{m}R_{jk}s_k\right)^2\leqslant r_{i,\mathrm{IIRA},0}^2, \tag{13}$$

$$r_{i,\mathrm{IIRA},0}^2=\left[\delta\log m+\frac{(m-i+1)+p(i)\cdot m}{2}\right]\cdot\sigma_v^2, \quad i=1,2,\cdots,m. \tag{14}$$

Obviously, the radii are chosen based not only on the statistics of noise but also the channel information. It is adaptive to the channel condition related $\boldsymbol{R}$ matrix and the coefficient $p(i)$ could be regarded as a channel adaptive factor.

Since $p(i)$ follows Beta distribution with the expectation of $(m-i+2)(m-i+1)/\left[m(m+1)\right]$, the expectations of the initial radii in the IIRA could be easily proved to be smaller than the respective radii in the IRA as follows:

$$\begin{aligned}\mathrm{E}\left[r_{i,\mathrm{IIRA},0}^2\right]&=\left[\delta\log m+\frac{2m-i+3}{2(m+1)}\cdot(m-i+1)\right]\cdot\sigma_v^2\\&\leqslant\left[\delta\log m+(m-i+1)\right]\cdot\sigma_v^2=r_{i,\mathrm{IRA}}^2,\\&\qquad i=1,2,\cdots,m.\end{aligned} \tag{15}$$

It implies that the search space of the IIRA would be statistically smaller than that of the IRA, thus resulting in a complexity reduction from the IRA.

We now introduce a radii update scheme for the IIRA. Let $\boldsymbol{s}_\lambda$ denote the λ th lattice point found within the search space with initial radii chosen as (14). Once the first candidate $\boldsymbol{s}_1$ is found, the radii are updated as

$$r_{i,\mathrm{IIRA},1}^2=g(1)+\frac{(m-i+1)+p(i)\cdot m}{2}\cdot\sigma_v^2, \quad i=1,2,\cdots,m. \tag{16}$$

where $g(1)=\min\{\delta\log m\cdot\sigma_v^2,\|\boldsymbol{y}-\boldsymbol{R}\boldsymbol{s}_1\|^2\}$. After the λ th lattice point $\boldsymbol{s}_\lambda$ is found, the radii are re-updated as

$$r_{i,\mathrm{IIRA},\lambda}^2=g(\lambda)+\frac{(m-i+1)+p(i)\cdot m}{2}\cdot\sigma_v^2, \quad i=1,2,\cdots,m. \tag{17}$$

where $g(\lambda)=\min\{\|\boldsymbol{y}-\boldsymbol{R}\boldsymbol{s}_\lambda\|^2,g(\lambda-1)\}$. In this radii update scheme, when a lattice point with shorter distance to the received signal is found, the radii would be updated to smaller values based on this distance. Accordingly, the search space of the IIRA would be further pruned. This is different from the IRA, where the search space is constant during the decoding process. Consequently, the complexity could be further reduced by this update scheme.

B. Performance Analysis

In the IIRA, the radii are updated as (16) or (17), which involves non-linear operation. It complicates the analysis of the error probability. For simplicity, we start the analysis on the IIRA without update. The effect of radii update will be discussed later.

In [11], the probability that the transmitted signal is not included in the search space, $\epsilon=P(\tilde{\boldsymbol{s}}\notin\boldsymbol{D})$), for the IRA-based decoder is derived as

$$\epsilon=P(\tilde{\boldsymbol{s}}\notin\boldsymbol{D})=\sum_{k=1}^{m}e^{-r_{m-k+1}^2}J_{k-1}. \tag{18}$$

where r_i^2 represent the set of radii, which cover $r_{i,\mathrm{IRA}}^2$ and $r_{i,\mathrm{IIRA}}^2$ as special cases; and

$$J_k=\sum_{l=0}^{k-1}(-1)^{k-l+1}\frac{r_{m-l}^{2(k-l)}}{(k-l)!}J_l, J_0=1. \tag{19}$$

Here J_k is a recursive function of r_i^2. After direct computation on J_k, we observe that J_k could be approximated to a close form as $J_k\approx r_1^{2k}/k!$ and therefore the error probability is approximated to a simpler expression as

$$\epsilon=P(\tilde{\boldsymbol{s}}\notin\boldsymbol{D})\approx\sum_{k=1}^{m}e^{-r_1^2}\frac{r_1^{2k}}{k!}. \tag{20}$$

Recall that in the IRA, $r_{1,\mathrm{IRA}}^2=\left[\delta\log m+m\right]\cdot\sigma_v^2$. On the other hand, it is obvious from the definition of $p(i)$ that $p(1)=1$ and then the initial radius $r_{1,\mathrm{IIRA},0}^2$ is

$$\begin{aligned}r_{1,\mathrm{IIRA},0}^2&=\left[\delta\log m+\frac{m+p(1)\cdot m}{2}\right]\cdot\sigma_v^2\\&=\left[\delta\log m+m\right]\cdot\sigma_v^2=r_{1,\mathrm{IRA}}^2.\end{aligned} \tag{21}$$

Using (20) and (21), it follows that $\epsilon_{\mathrm{IIRA}}\approx\epsilon_{\mathrm{IRA}}$, which means that the probability of the transmitted signal is not included in

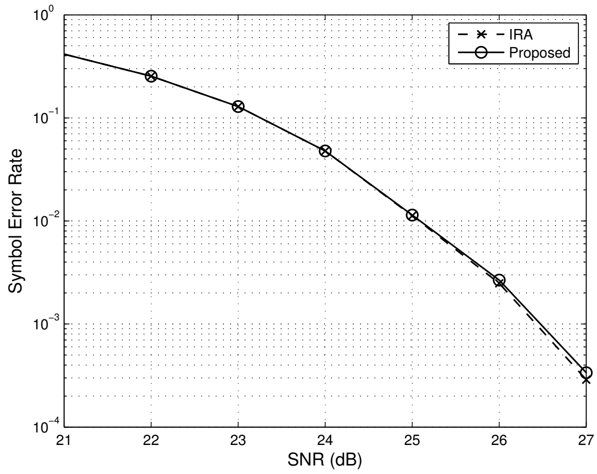

Fig. 1. Symbol Error Rate for 64-QAM 12×12 MIMO System.

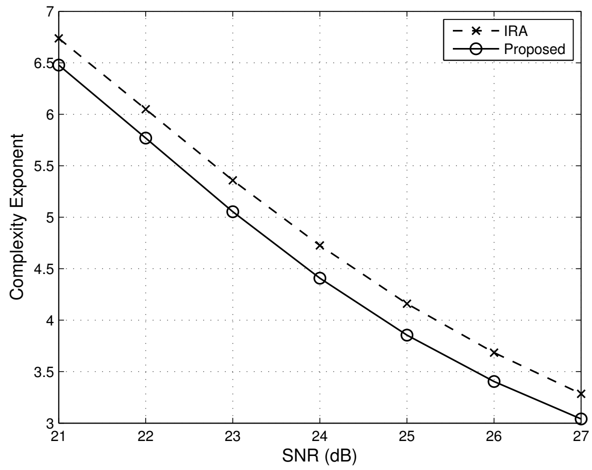

Fig. 2. Complexity Exponent for 64-QAM $\mathbf{12} \times \mathbf{12}$ MIMO.

the search space for both algorithms are roughly the same. Thus we expect that the IIRA without radii update would perform similar to that of the IRA.

Now the effect of radii update on the decoder performance is discussed as follows. In fact, after each radii update, the search space would be accordingly updated to a smaller one. As a result, some points within the previous search space would be excluded. Let $\hat{\boldsymbol{s}}$ denote the lattice point that has been found to perform the latest update of $g(\lambda)$ as $g(\lambda) = \|\boldsymbol{y} - \boldsymbol{R}\hat{\boldsymbol{s}}\|^2$ and the radii $r^2_{i,\text{IIRA},\lambda}$ are updated as (17). Define the search space corresponding to these radii as $\boldsymbol{D}^{\lambda}_{\text{IIRA}}$. For any lattice point $\bar{\boldsymbol{s}}$ being excluded from the search space in this update, *i.e.*, $\bar{\boldsymbol{s}} \in \boldsymbol{D}^{\lambda-1}_{\text{IIRA}}$ but $\bar{\boldsymbol{s}} \notin \boldsymbol{D}^{\lambda}_{\text{IIRA}}$, at least one of the inequality in (13) should be violated, *i.e.*:

$$\exists 1 \leqslant a \leqslant m, \sum_{j=a}^{m} \left(y_i - \sum_{k=j}^{m} R_{jk} \bar{s}_k \right)^2 > r^2_{a,\text{IIRA},\lambda}. \quad (22)$$

Since

$$\begin{aligned} \|\boldsymbol{y} - \boldsymbol{R}\bar{\boldsymbol{s}}\|^2 &= \sum_{j=1}^{m} \left(y_j - \sum_{k=j}^{m} R_{jk} \bar{s}_k \right)^2 \\ &\geqslant \sum_{j=a}^{m} \left(y_i - \sum_{k=j}^{m} R_{jk} \bar{s}_k \right)^2, \end{aligned} \quad (23)$$

and

$$\begin{aligned} r^2_{a,\text{IIRA},\lambda} &= g(\lambda) + \frac{(m-a+1) + p(a) \cdot m}{2} \cdot \sigma_v^2 \\ &= \|\boldsymbol{y} - \boldsymbol{R}\hat{\boldsymbol{s}}\|^2 + \frac{(m-a+1) + p(a) \cdot m}{2} \cdot \sigma_v^2, \end{aligned} \quad (24)$$

it follows that

$$\|\boldsymbol{y} - \boldsymbol{R}\bar{\boldsymbol{s}}\|^2 > r^2_{a,\text{IIRA},\lambda} > \|\boldsymbol{y} - \boldsymbol{R}\hat{\boldsymbol{s}}\|^2. \quad (25)$$

It means that for any points $\bar{\boldsymbol{s}}$ being excluded, its distance to the received signal is larger than that of the previous found point $\hat{\boldsymbol{s}}$. In other words, any excluded point $\bar{\boldsymbol{s}}$ cannot be the ML solution. It indicates that the radii update scheme will not exclude the ML solution from the search space and therefore it will not affect the decoding performance. In summary, we can conclude that the IIRA without radii update offers similar decoding performance as the IRA. In addition, the performance of the IIRA with radii update would also be similar to that of the IRA, *i.e.*, near-ML performance. This will be verified by computer simulations in the following section.

IV. Simulation Results

Computer simulations are conducted to investigate the performance and complexity of the IIRA. In the following, two examples are considered: a 64-QAM modulated 12×12 system and a 4-QAM modulated 20×20 system. The performance is measured by symbol-error-rate (SER), while the complexity CP is defined as the number of multiplications required for decoding, and it is presented in terms of complexity exponent C_E, which is defined as $C_E = \log(CP)/\log(m)$. The SNR is defined as SNR $= \sigma_s^2 \cdot \sigma_h^2/m \cdot \sigma_v^2$. In the simulations, the noise power is set as $\sigma_v^2 = 1$, while the variance of the elements in channel matrix varies according to the SNR. For the purpose of comparison, the performance and complexity of the IRA [11] will also be simulated. Notice that in the IIRA, the parameter δ in the radii design is chosen same as that in the IRA. Namely, it is chosen from the table in [11] according to the probability that the transmitted signal is not included in the search space, *i.e.*, ϵ. In detail, the sequence of ϵ is set as $\epsilon = 0.1, 0.01, 0.001$, *etc.*. With ϵ being initially setup as $\epsilon = 0.1$, the corresponding δ is chosen from the table for radii computation. If there is no lattice point being found in this search space, we compute a new set of radii using the value of δ which leads to a probability $\epsilon = 0.01$ and run the decoder algorithm. This continues until the search space is not empty.

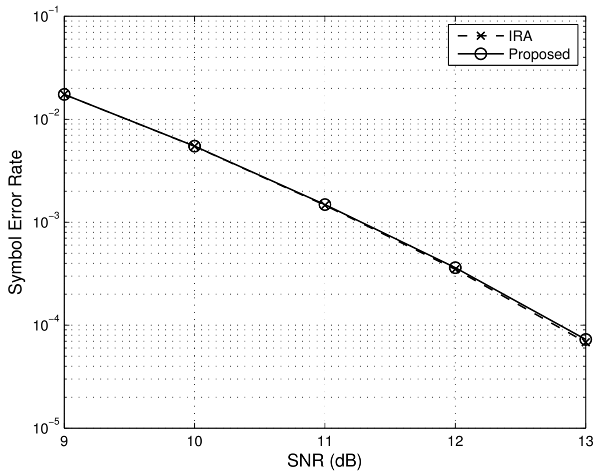

Fig. 3. Symbol Error Rate for 4-QAM 20×20 MIMO System.

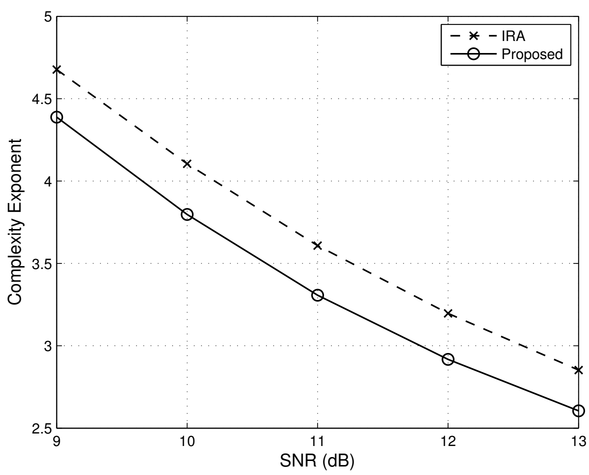

Fig. 4. Complexity Exponent for 4-QAM 20×20 MIMO.

In the example of 64-QAM modulated 12×12 MIMO system, the simulation covers the SNR range from $21 - 27$dB. The SER performance and complexity exponent are shown in Figs.1&2, respectively. It is clear from Fig.1. that the performance of the IIRA is very close to that of the IRA, while as shown in Fig.2, up to 50% of complexity could be saved at $\text{SNR} = 22 - 26$dB when using the IIRA. It demonstrates that the IIRA could significantly reduce the complexity without performance degradation compared with the IRA.

In Fig.3 and Fig.4, the results of the 4-QAM, 20×20 MIMO are shown with SNR in the range of $9 - 13$dB. Similarly, the SER performance of the two decoders is quite close and the maximum complexity reduction of the IIRA is roughly 60% at $\text{SNR} = 10 - 11$dB.

V. Conclusions

In this paper, a low complexity IIRA has been proposed to decode the signals in MIMO systems. It includes an initial channel-adaptive radii design and a radii update scheme. Since both noise statistics and channel information are utilized to prune the search space to a subset of the ML search space, where the ML solution is included with high probability, the IIRA is able to achieve near-ML decoding performance with much lower complexity when compared to the IRA.

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