A NEW FAMILY OF LINEAR DISPERSION CODE FOR FAST SPHERE DECODING

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ABSTRACT

In this paper, a new family of Linear Dispersion Codes (LDCs) that can be decoded using a fast Sphere Decoding (SD) algorithm in MIMO systems is proposed. The basic principle of this structure is to make the LDC to have as many as possible the rows orthogonal in the dispersion matrices. Monte Carlo simulation results show that the optimum LDCs with this orthogonal structure have nearly identical bit-error-rate (BER) performances as other optimal LDCs. We develop a simplified Sphere Decoding (SD) algorithm that can significantly reduce the decoding complexity in decoding the new LDCs with proposed orthogonal structure. Simulation results show that the complexity reduction is more significant for MIMO system transmitting higher level modulation. For 2×4 MIMO systems transmitting 4 64QAM and 256QAM symbols in a block length of 4, the reductions are about 71-83% and 76-88%, respectively.

***Index Terms*—**MIMO, Sphere decoding, orthogonal, complexity

1. INTRODUCTION

Linear Dispersion Code (LDC) [1] is well-known for its advantages in providing full-ergodic capacity to Multiple-Input Multiple-Output (MIMO) communication systems. However, the detection complexity of LDCs has always been a problem in the design and implementation of high speed MIMO systems. The tremendous complexity of the Maximum Likelihood (ML) decoding process makes the implementation of high speed Space-time block codes (STBCs) impractical. Linear decoding algorithms like Minimum-Mean-Square Error (MMSE) [2] and Zero-Forcing (ZF) [3] algorithms are much less complicated, but the achievable BER performances are not very satisfactory.

Alamouti [4] proposed a remarkable space-time (ST) code in 1998 for MIMO systems with two transmit antennas which was later on extended to orthogonal space-time block codes (OSTBCs) [5]. The OSTBCs and Alamouti's code attracted much attention because they allow the uses of very simple decoding algorithms to achieve the same BER performances as with ML decoding. However, the main disadvantage of the OSTBCs is that they cannot achieve the full-transmission rates for MIMO systems with more than two transmit antennas [6].

In [7], Sphere Decoding (SD) was proposed to substantially reduce the complexity of ML decoding, yet having the same BER performance. However, as Jalden *et al.* pointed out in [8], for a fixed signal-to-noise ratio (SNR), the complexity of SD increases exponentially with the number of symbols jointly decoded. When data rate is high, SD is still too complicated for practical use. Improvements in different aspects of SD have been proposed. In [9], a so-called Babai Point method was used to set the initial searched point in SD. In [10], a Schnorr Euchner (SE) enumeration method was proposed to refine the search strategy of SD. In [11], Paredes et al. reduced the complexity of SD by reducing the number of search levels in the tree search process. By combining the advantages of OSTBCs and SD, they constructed a family of fast decodable full-rate, full diversity codes for a 2×2 MIMO system. Biglieri *et al*. extended this concept to a 4×2 MIMO system and developed a family of quasi-orthogonal structured codes [12].

In this paper, we propose a new family of fast-decodable full-diversity LDCs for MIMO systems and the LDCs can be designed for arbitrary number of transmit and receive antennas. To reduce the decoding complexity, we also develop a simplified SD for the codes to achieve the same BER performance as those using conventional SD or ML decoding, but with much less decoding complexity.

The rest of this paper is organized as follows. The system model used for the study is defined in Section 2. In Section 3, our proposed new family of LDCs together with the corresponding simplified SD algorithm is explained. Monte Carlo simulation results and discussions are given in Section 4. Section 5 concludes this paper.

2. SYSTEM MODEL

The system model used for the study is an $N_t \times N_r$ MIMO system with N_t transmit antennas and N_r receive antennas, over a quasi-static Rayleigh fading channel. The $N_r \times T$ received signal matrix **R** is given by:

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$$\mathbf{R} = \mathbf{HC} + \mathbf{W} \quad (1)$$

where the entries of $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ represent the channel coefficients which are assumed to be perfectly known at the receiver but not at the transmitter, $\mathbf{C} \in \mathbb{C}^{N_t \times T}$ is the codeword matrix with block length T, and $\mathbf{W} \in \mathbb{C}^{N_r \times T}$ represents the complex additive white Gaussian noise (AWGN) matrix with elements being independently and identically distributed (iid) and following the normal distribution $N_C(0,N_0)$.

For LDCs, the codeword $\mathbf{C}$ can be expressed as [1]:

$$\mathbf{C} = \sum_{i=1}^{N} \mathbf{M}_i \cdot s_i \quad (2)$$

where $\{\mathbf{M}_i\}_{i=1}^{N}$ are the dispersion matrices of the LDC, $\{s_i\}_{i=1}^{N}$ are the transmitted symbols taking values from some complex constellation in a finite set $\mathbb{S}$, and N is the number of symbols in one codeword. All elements in $\mathbf{R}$, $\mathbf{H}$, $\mathbf{C}$ and $\mathbf{W}$ of (1) are complex variables. The transmitted symbols in (1) can be expressed in vector form as $\mathbf{s} = [s_1, s_2, s_3, s_4 ..., s_N]$. Then substituting (2) into (1) and taking vectorization on both sides yields [1]:

$$\bar{\mathbf{r}} = \mathbf{K}\bar{\mathbf{X}}\bar{\mathbf{s}} + \bar{\mathbf{w}} \quad (3)$$

where $\bar{\mathbf{r}} = vec(\mathbf{R}) = \left[r_{1,1}, r_{2,1}, ..., r_{N_r,1}, r_{1,2}, ..., r_{N_r,2}, ..., r_{N_r,T} \right]^T$ with $r_{i,j}$ being the entry in the i^{th} row and j^{th} column of matrix $\mathbf{R}$ and $[\ ]^T$ denoting matrix transposition, $\bar{\mathbf{X}} = [vec(\mathbf{M}_0), vec(\mathbf{M}_1), ..., vec(\mathbf{M}_N)]$, $\bar{\mathbf{s}} = vec(\mathbf{s}) = \mathbf{s}^T = [s_1, ... s_N]^T$ and $\bar{\mathbf{w}} = vec(\mathbf{W})$. $\mathbf{K} = \mathbf{I} \otimes \mathbf{H}$, where $\mathbf{K} \in \mathbb{C}^{N_r T \times N_t T}$, $\mathbf{I}$ is an identity matrix and $\otimes$ denotes the Kronecker product.

Separating the real and imaginary parts of the elements in $\bar{\mathbf{r}}$, $\bar{\mathbf{s}}$ and $\bar{\mathbf{w}}$ of (3) and then vectorizing them give the real-valued expression:

$$\tilde{\mathbf{r}} = \tilde{\mathbf{G}}\tilde{\mathbf{s}} + \tilde{\mathbf{w}} \quad (4)$$

where $\tilde{\mathbf{r}} = \left[\text{Re}(\bar{\mathbf{r}}^T), \text{Im}(\bar{\mathbf{r}}^T) \right]^T$, $\tilde{\mathbf{s}} = \left[\text{Re}(\mathbf{s}), \text{Im}(\mathbf{s}) \right]^T = [\text{Re}(s_1), ..., \text{Re}(s_N), \text{Im}(s_1), ..., \text{Im}(s_N)]^T$, and $\tilde{\mathbf{w}} = [\text{Re}(\bar{\mathbf{w}}^T), \text{Im}(\bar{\mathbf{w}}^T)]^T$, with Re(.) and Im(.) denoting the real and imaginary parts, respectively, of (.). In (4), $\tilde{\mathbf{G}}$ is a $2N_rT \times 2N$ real matrix given by:

$$\tilde{\mathbf{G}} = \begin{bmatrix} \text{Re}(\mathbf{K}\bar{\mathbf{X}}) & -\text{Im}(\mathbf{K}\bar{\mathbf{X}}) \\ \text{Im}(\mathbf{K}\bar{\mathbf{X}}) & \text{Re}(\mathbf{K}\bar{\mathbf{X}}) \end{bmatrix} \quad (5)$$

Here, we adopt an ordering scheme to construct our proposed code structure. First, we arrange the vector $\tilde{\mathbf{s}}$ in (4) to

$$\begin{aligned} \tilde{\tilde{\mathbf{s}}} &= [\text{Re}(s_1), \text{Im}(s_1), \text{Re}(s_2), \text{Im}(s_2), ..., \text{Re}(s_N), \text{Im}(s_N)]^T \\ &= [\tilde{\tilde{s}}_1, \tilde{\tilde{s}}_2, \tilde{\tilde{s}}_3, \tilde{\tilde{s}}_4 ..., \tilde{\tilde{s}}_{2N}]^T \end{aligned} \quad (6)$$

To keep the received signal vector $\tilde{\mathbf{r}}$ in (4) unchanged, we need to arrange the columns of $\tilde{\mathbf{G}}$ correspondingly. That is, for $\tilde{\mathbf{G}} = [\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, ..., \tilde{\mathbf{g}}_{2N}]$, where $\tilde{\mathbf{g}}_i$, for i = 1, 2, …, 2N, denotes the i^{th} column of $\tilde{\mathbf{G}}$, we arrange the columns of $\tilde{\mathbf{G}}$ to give:

$$\tilde{\tilde{\mathbf{G}}} = [\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_{N+1}, \tilde{\mathbf{g}}_2, \tilde{\mathbf{g}}_{N+2}, ..., \tilde{\mathbf{g}}_N, \tilde{\mathbf{g}}_{2N}] \quad (7)$$

With these arrangements, the received signal vector $\tilde{\mathbf{r}}$ in (4) can be re-written as:

$$\tilde{\mathbf{r}} = \tilde{\tilde{\mathbf{G}}}\tilde{\tilde{\mathbf{s}}} + \tilde{\mathbf{w}} \quad (8)$$

3. PROPOSED LDC AND DECODING ALGORITHM

Proposed orthogonal row structure

We propose a new family of LDCs which have the first m dispersion matrices among the N dispersion matrices $\{\mathbf{M}_i\}_{i=1}^{N}$ in (2) satisfying the following condition:

$$\mathbf{M}_i \mathbf{M}_j^H = \mathbf{0} \qquad (i \neq j;\ i, j \leq m) \quad (9)$$

where $\mathbf{0}$ is an $N_t \times N_t$ matrix with all elements being zero and $(.)^H$ denotes the transpose conjugate of a matrix (.). The condition of (9) implies that among the first m dispersion matrices, any row of one dispersion matrix is orthogonal to the rows of any other dispersion matrices. It can be easily proved that with proper scaling, LDCs with the dispersion matrices satisfying (9) will satisfy the full-capacity constraint and power constraint in [1], which are the basic requirements for full-ergodic capacity LDCs.

Simplified SD

Sphere decoding attempts to obtain the solution of [7]:

$$\hat{\mathbf{s}} = \arg \min_{\tilde{\tilde{\mathbf{s}}} \in \mathbb{S}} \left\| \tilde{\mathbf{r}} - \tilde{\tilde{\mathbf{G}}}\tilde{\tilde{\mathbf{s}}} \right\|^2 \quad (10)$$

To do this, we can first conduct the QR decomposition of $\tilde{\tilde{\mathbf{G}}}$:

$$\tilde{\tilde{\mathbf{G}}} = \mathbf{Q}\begin{bmatrix} \mathbf{P} \\ \mathbf{0}_{(2N_rT-2N)\times 2N} \end{bmatrix} \quad (11)$$

where $\mathbf{P} \in \mathbb{R}^{2N\times 2N}$ is an upper triangular matrix, $\mathbf{Q} \in \mathbb{R}^{2N_rT\times 2N_rT}$ is an orthonormal matrix, and $N_rT \geq N$. Then (10) can be written as:

$$\hat{\mathbf{s}} = \arg\min_{\tilde{\tilde{\mathbf{s}}}\in\mathbb{S}} \left\| \mathbf{y} - \mathbf{P}\tilde{\tilde{\mathbf{s}}} \right\|^2 \quad (12)$$

where $\mathbf{y} = \mathbf{Q}_1^T\tilde{\mathbf{r}}$, $\mathbf{Q}_1$ is the first $2N$ orthonormal columns of $\mathbf{Q}$. Based on (12), the SD can now search only the points within a hypersphere centered at the received signal with radius d by solving the following inequality iteratively [7]:

$$\left\| \mathbf{y} - \mathbf{P}\tilde{\tilde{\mathbf{s}}} \right\|^2 \leq d^2 \quad (13)$$

To explain how the orthogonal row structure can be used to simplify the SD process, we first introduce the following Lemma:

Lemma 1: If a LDC has an orthogonal row structure and so satisfies (9), the elements p_{ij}, for i = 1, 2, …, $2m$-1 and j = i+1, i+2, …, $2m$, in the upper triangular matrix $\mathbf{P}$ in (12) are all zeros. The proof is omitted here due to the page limit.

According to *Lemma 1*, if a LDC with an orthogonal row structure is used, there will be a cluster of zeros in the upper triangular matrix $\mathbf{P}$ in (12). In (13), there are a total of $2N$ inequalities, corresponding to the $2N$ rows in the matrix equation. To solve the inequality iteratively, the SD starts from the bottom row of the matrix and works upwards from the $2N^{\text{th}}$ row. When the SD has completed the $(2m+1)^{\text{th}}$ row, the values of $\tilde{\tilde{s}}_{2N}$, $\tilde{\tilde{s}}_{2N-1}$, …, $\tilde{\tilde{s}}_{2m+1}$ will have been determined. Then benefiting from the cluster of zeros in $\mathbf{P}$, we do not need to use the values of $\tilde{\tilde{s}}_{i+1}$, $\tilde{\tilde{s}}_{i+2}$, …, $\tilde{\tilde{s}}_{2m}$ to determine the values of $\{\tilde{\tilde{s}}_i\}$ for i = 1 to $2m$. Instead, we can simply determine the value of $\tilde{\tilde{s}}_i$ by hard decoding:

$$\tilde{\tilde{s}}_i = \ulcorner (y_i - \sum_{k=2m+1}^{2N} p_{i,k}\tilde{\tilde{s}}_k)/p_{i,i} \lrcorner \quad (14)$$

where $\ulcorner a \lrcorner$ denotes the possible value of $\tilde{\tilde{s}}_i$ closest to a. It should be noted that, compared to the complexity of tree search, the complexity of hard decoding can be neglected, and our proposed code structure can reduce the tree search by $2m$ levels in SD without causing any degradation in BER performance.

4. SIMULATION RESULTS

Studies of this new family of LDCs have been carried out using two 2×4 MIMO systems transmitting four 64QAM and 256QAM symbols in a block length of 4 (i.e. N_t = 2, N_r = 4, T = 4, N = 4) over a block-fading channel. In the construction of these LDCs with orthogonal row structure, we made the first two dispersion matrices to satisfy the orthogonal condition of (9), i.e. m=2. Then we used random search with the Rank & Determinant criterion to obtain the optimal LDC. For the LDCs without the orthogonal row structure, we also used random search with the Rank & Determinant criterion to obtain the optimal LDC. To assess the BER performances of these optimum LDCs in the 2×4 MIMO system, Monte Carlo simulation was used and the results are shown in Figs. 1 and 2 for the 64QAM and 256QAM signals, respectively. It can be seen that the BER performances of the optimum LDCs with and without the orthogonal row structure are about the same.

According to [8], the complexity of SD can be measured by the number of nodes visited in the tree search process. Monte Carlo simulation was therefore used to examine the average numbers of visited nodes to evaluate the complexities of the conventional SD and simplified SD for decoding the same optimum LDC with the orthogonal row structure. The results on the complexities for the 2×4 MIMO systems transmitting four 64QAM and 256QAM symbols in a block length of 4 are shown in Figs. 3 and 4, respectively. In the 64QAM system, the simplified SD reduces the complexity by 71-83%; while in the 256QAM system, it reduces the complexity by 76-88%. Reduction is more significant for signals with higher-level modulation. The reason is that for higher-level modulation, there are more branches in each node and so even more nodes in the lower levels. So for the same number of search levels reduction, more branches reduction and hence complexity reduction will be achieved for higher-level modulation.

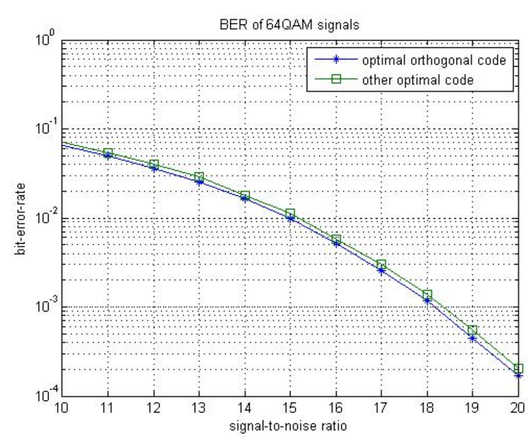

Fig. 1 BER of optimal LDC using 64QAM modulation

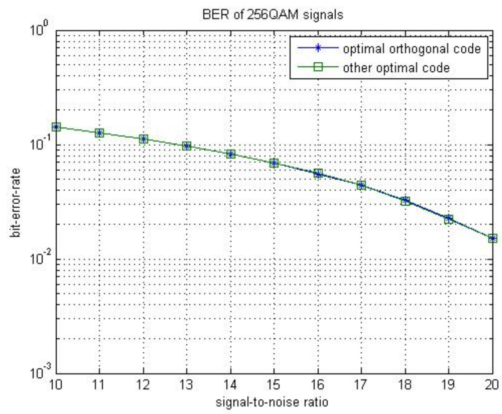

Fig. 2 BER of optimal LDC using 256QAM modulation

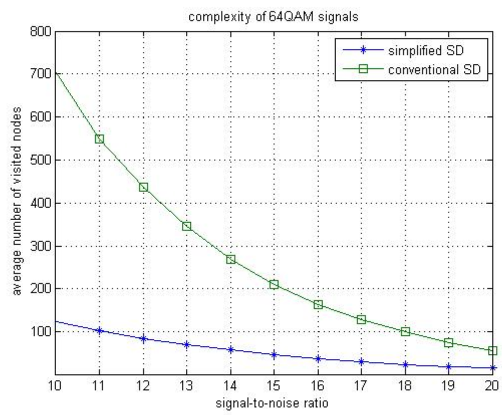

Fig. 3 Complexity of conventional SD and simplified SD for 64QAM

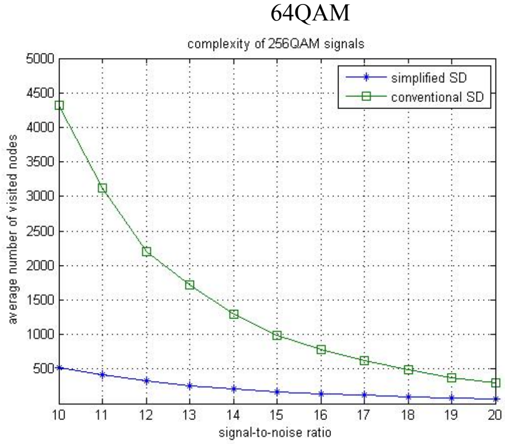

Fig. 4 Complexity of conventional SD and simplified SD for 256QAM

5. CONCLUSIONS

In this paper the design of a new family of fast-decodable full diversity LDC with an orthogonal row structure in the dispersion matrices has been presented. Monte Carlo computer simulation results have shown that the optimal LDCs with and without our orthogonal row structure have nearly identical BER performances. However, the complexity of SD for LDCs with our orthogonal row structure can be significantly reduced by using a simplified SD algorithm. The reduction is more significant for LDCs using higher level modulations.

6. REFERENCES

[1] Heath, R., and A. Paulraj, ‘‘Linear Dispersion Codes for MIMO Systems Based on Frame Theory’’, *IEEE Trans. Sig. Proc.*, vol. 50, no. 10, pp. 2429–2441, October, 2002.

[2] Tarokh, V., et al., ‘‘Space-Time Codes for High Data Rate Wireless Communication: Performance Criteria in the Presence of Channel Estimation Errors, Mobility, and Multiple Paths,’’ *IEEE Trans. Commun.*, vol. 47, no. 2, pp. 199–207, February, 1999.

[3] G. D. Golden, G. J. Foschini, R. A. Valenzuela and P. W. Wolniansky, “Detection algorithm and initial laboratory results using the V-BLAST space-time communication architecture”, *Electronics Letters*, vol. 35, no. 1, pp. 14–15, January, 1999.

[4] Alamouti, S. M., ‘‘A Simple Transmit Diversity Technique for Wireless Communications’’, *IEEE Journal Select. Areas Commun*., vol. 16, no. 8, pp. 1451–1458, October, 1998.

[5] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. on Information Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[6] O. Tirkkonen and A. Hottinen, “Square-matrix embeddable space–time block codes for complex signal constellations”, *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 384–395, February 2002.

[7] U. Fincke and M. Pohst, “Improved methods for calculating vectors of short length in lattice, including a complexity analysis”, *Math. Comput*., vol. 44, no. 170, pp. 463–471, April, 1985.

[8] Jalden J. and Ottersten B., “On the complexity of sphere decoding in digital communications”, *IEEE Trans. Sig*., vol. 53, no. 4, pp. 1474-1484, April, 2005.

[9] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, “Closest point search in lattices”, *IEEE Trans. Inf. Theory*, vol. 48, no. 8, pp. 2201–2214, August 2002.

[10] C. P. Schnorr and M. Euchner, “Lattice basis reduction: Improved practical algorithms and solving subset sum problems,” *Math. Programming*, vol. 66, pp. 181–191, 1994.

[11] J. Paredes, A.B. Gershman, and M. G. Alkhanari, “A 2×2 space-time code with non-vanishing determinants and fast maximum likelihood decoding”, *Proc. ICASSP*., vol. 2, pp. 877-880, April, 2007.

[12] Ezio Biglieri, Yi Hong, Emanuele Viterbo, “A Fast-Decodable, Quasi-Orthogonal Space-Time Block Code for 4x2 MIMO”, *Annual Allerton Conference on Communication, Control and Computing, Illinois*, Sep. 26-28, 2007.