ESTABLISHMENT OF BASELINE MODELS FOR LONG-SPAN CABLE-STAYED BRIDGES

Jing Zhang and Francis T.K. Au
Department of Civil Engineering, The University of Hong Kong, Hong Kong, People's Republic of China

ABSTRACT

To enable the long-term health monitoring and condition assessment of a long-span cable-stayed bridge, an accurate and effective baseline finite element model is indispensable. With proper calibration, the model provides more realistic prediction of static and dynamic behaviour of structure. This paper describes the development of baseline finite element model of Ting Kau Bridge on the basis of the measured frequencies and initial equilibrium configuration of the bridge. The flexibility method is employed in order to avoid lengthy computations and possible convergence problems in the iterative procedure. The calibrated baseline model gives realistic internal force distributions and more reliable deformed shapes under permanent loading. Then the ability of the baseline model to predict the static and dynamic displacements under moving loads is examined. The baseline model can provide more reliable predictions for the subsequent analyses as the initial deck profile under permanent load has been so calibrated to agree with the design profile with discrepancies not exceeding a specified tolerance. The baseline model is particularly important to fatigue assessment of this kind of bridges.

KEYWORDS

Baseline model calibration, cable-stayed bridges, fatigue assessment, finite element modelling

INTRODUCTION

Cable-stayed bridges have become more and more popular over the past 50 years because of their desirable structural and aesthetic characteristics. With the continual development in design methodology and construction technology, cable-stayed bridges have ushered in a new era of long-span structures with spans over 1,000m, which has previously been occupied exclusively by suspension bridges. Since the 1990's, a few notable long-span cable-stayed bridges have been built in Hong Kong, including the Kap Shui Mun Bridge, Ting Kau Bridge and Stonecutters Bridge, to meet the needs of rapid transportation.

The long-term health monitoring and condition assessment of long-span cable-stayed bridges are important measures to ensure safety during their service lives (Ren and Peng 2005). Increase in span length makes the cable-stayed bridges more flexible and sensitive to dynamic loading. The early identification and localization of any potential damage therefore become more complicated due to the structural complexity. The finite element method is extensively employed to analyse how structures perform when subjected to dynamic excitations such as earthquake, wind and vehicular loading. However, in the development of finite element models, various simplifying assumptions are normally made regarding the material and geometric properties, degrees of freedom in describing the deformations, and the boundary conditions. While conservative simplifying assumptions are acceptable for design purposes, more sophisticated numerical models are often required to reflect the actual structural behaviour. Generally, for monitoring and assessment purposes, the initial model needs to be calibrated properly to make sure that the discrepancies between theoretical predictions and field measurements are within certain acceptable tolerances. Considering these, the development of baseline model is an important milestone to provide a reference for more accurate static and dynamic analyses afterwards.

The baseline finite element model of a bridge is a finite element model so calibrated that the initial geometry of the model under permanent loading agrees with that specified in the as-built drawings within reasonable tolerance. In particular, the net deflection of the bridge deck under permanent loading should be zero or minimal. This can be achieved through proper calibration of the initial cable forces. There are various methods to derive an optimal set of cable forces, including the zero displacement method, optimization method, force equilibrium method and unit force method. Many existing methods for baseline calibration employ the iterative approach by which the final cable forces are determined by systematically updating a set of initial cable forces in each cycle until certain criteria or objective functions are satisfied (Wang *et al.* 1993; Chen *et al.* 2000). The force equilibrium method can easily account for the effect of prestressing and the additional bending moments due to the vertical profile of bridge deck, and therefore it is much more rational as well as simpler than the traditional zero displacement method (Chen *et al.* 2000). The unit force method takes into account all relevant effects for the design of cable-stayed bridges, including construction sequence, second-order behaviour, large displacements, cable sag and time-dependent effects, such as creep, shrinkage and relaxation of prestressing tendons (Janjic *et al.* 2003).

It is common for algorithms based on the iterative approach to incorporate objective functions so that iteration can be terminated once the objectives are satisfied. However if too many of these objective functions are imposed on the algorithm or if they are set to be too stringent, the algorithm may fail to converge. The flexibility method is therefore employed here to calibrate the cable forces and to make sure that the bridge deck profile of the finite element model is at the specified level.

First, the initial finite element model is established based on the geometry shown on the design drawings to obtain the initial equilibrium geometry under the permanent load and the trial initial cable forces. Then the flexibility method is applied to adjust the cable forces in order to control the equilibrium configuration. With the completion of calibration, the frequencies of the baseline model should agree with those calculated from the data gathered by the Wind and Structural Health Monitoring System (WASHMS) installed on the bridge. More than 200 sensors of different types have been installed on the Ting Kau Bridge by the Highways Department of the Hong Kong Government to monitor the structural health and performance.

The main objective of instrumentation and health monitoring is to detect and evaluate any symptoms of operational anomalies and/or deterioration or damage that may induce adverse effects on service or safety reliability through the processing and analysis of data collected from transducers and sensors (Wong 2004). Over the past two decades, a variety of methods have been proposed for detecting the presence, location and severity of structural damage (Elkordy *et al.* 1994; Doebling *et al.* 1998; Park *et al.* 2001; Lam *et al.* 2006; Ni *et al.* 2008). In addition, the fatigue effect caused by moving vehicular loads is one of the concerns. Therefore moving loads are applied on the initial and baseline models before and after calibration respectively to simulate the excitations of vehicles. Possible effects on the prediction of fatigue life will be examined.

INITIAL FINITE ELEMENT MODEL BEFORE CALIBRATION

Ting Kau Bridge is a cable-stayed bridge with a total length of 1,177 m forming part of Route 3 of the highway network in Hong Kong. As shown in Figure 1, there are three reinforced concrete towers, namely the Ting Kau Tower, Main Tower and Tsing Yi Tower, and 384 longitudinal main cables to support the bridge decks. The two decks are connected by a series of connecting cross girders provided at regular intervals. At each end of the bridge deck, each main girder is connected to a mechanical rocker bearing that is tied to the abutment at the Tsing Yi End or the pier at the Ting Kau End. The main girders are numbered from 1 to 4 from the west to the east. The finite element model is constructed using the commercial package ANSYS Multiphysics 10. Four types of elements are used in the model, namely BEAM4, BEAM44, LINK8 and LINK11 to build the initial three-dimensional model in Figure 2. At the base of each tower, all degrees of freedom are assumed to be fixed.

Figure 3 shows the vertical displacements of the four main girders of the initial finite element model under permanent loading, which can be interpreted as the discrepancies between the deck profile of the initial model and the correct deck profile. Note that the discrepancies are quite substantial and the maximum discrepancies around Section B as shown in Figure 1 all exceed 1.3m. In view of the possible adverse effects of the discrepancies on the accuracies of results, the initial finite element model needs to be further calibrated.

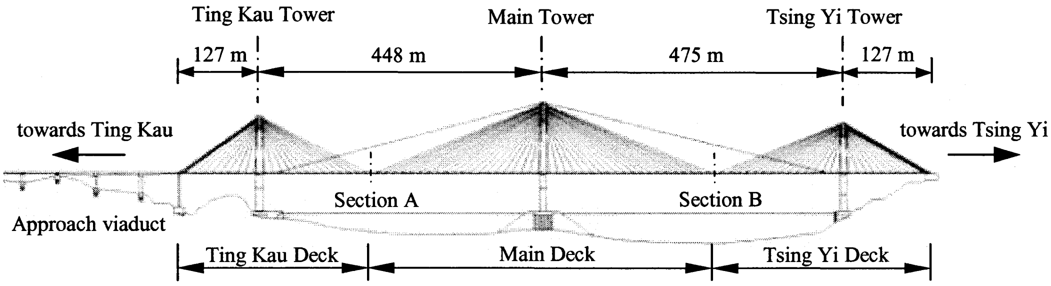


Figure 1. Schematic elevation of Ting Kau Bridge

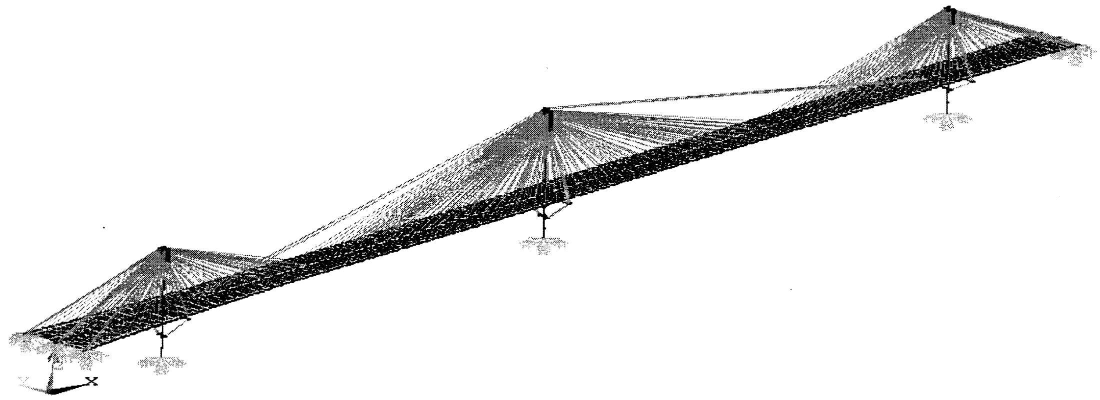


Figure 2. The initial finite element model before calibration

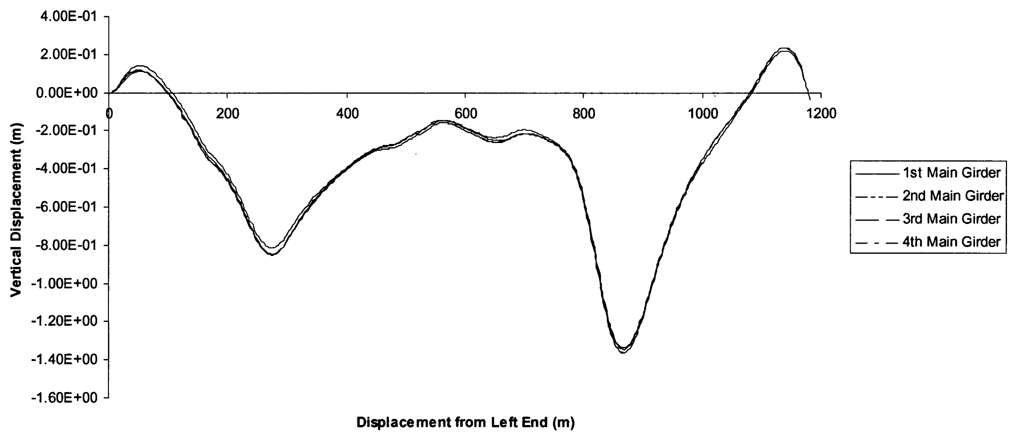


Figure 3. Vertical displacements of main girders of initial finite element model before calibration under permanent loading

THE FLEXIBILITY METHOD FOR CALIBRATION

The flexibility method is employed here to calibrate the cable forces to make sure that the bridge deck profile and tower geometry of the calibrated finite element model are at the specified levels and positions. Various control points are also established to control displacement of the bridge towers and avoid mathematical errors effectively while implementing the flexibility method.

The flexibility method is a mathematical approach that involves developing and solving a system of equations that relate the cable forces to the displacements of certain control points using the concept of compatibility. The highly indeterminate structure is first reduced to the simpler primary structure by replacing the cables by their respective internal forces. Therefore for the present case of Ting Kau Bridge, the primary structure consists of the bridge deck supported on rocker bearings at the ends and the free standing bridge towers. The control points are conveniently chosen as the cable anchorages at the deck as well as each tower top. The flexibility method can be written in terms of compatibility as

$$[f_{ij}]\{T_j\}+\{\Delta_i\}=0 \tag{1}$$

where $[f_{ij}]$ is the flexibility matrix, $\{T_j\}$ is the cable force vector that contains the tensile forces in all cables, $\{\Delta_i\}$ is the initial displacement vector containing the displacements at all control points under permanent loading only, and subscripts i and j denote the i^{th} control point and j^{th} cable considered in the calibration respectively. The initial cable forces satisfying the baseline scheme can therefore be determined as

$$\{T_j\}=-[f_{ij}]^{-1}\{\Delta_i\} \tag{2}$$

However in actual practice, certain simplifications may be required in application of the flexibility approach. For example, a group of closely spaced cables may be replaced by one hypothetical cable. Then an iteration procedure may be required to further fine-tune the cable forces to make sure that the equilibrium configuration of the bridge under permanent loading is reasonably close to the specified geometry.

The calibration process of cable forces using the flexibility method can be described as follows:
(a) Build up the initial finite element model based on design drawings of the bridge.
(b) Determine the initial displacements of the primary structure that is essentially the bridge with cables replaced by their respective cable forces.
(c) Obtain the flexibility matrix of the primary structure, which reflects its responses to the cable forces.
(d) Calculate the initial cable forces in order to satisfy the compatibility conditions.
(e) If necessary, carry out further fine-tuning of the cable forces with an iterative procedure.

BASELINE FINITE ELELEMENT MODEL

After calibration, the displacements or discrepancies at deck control points of the baseline finite element model are less than the specified tolerance of 5mm as shown in Figure 4. Actually the maximum deviation from the design deck profile does not exceed 2mm. In order to evaluate the effects of baseline calibration, the internal forces of the deck including bending moments, shear forces and axial forces are obtained and compared to those of the initial model without baseline calibration in Figure 5. In general, baseline calibration does not radically alter the distribution of internal forces in the bridge deck. It is observed that for most part of the bridge deck, the variations of bending moment and shear force tend to be reduced. However, close to the Ting Kau End, there are more fluctuations because of adjustments of the group of cables at this end.

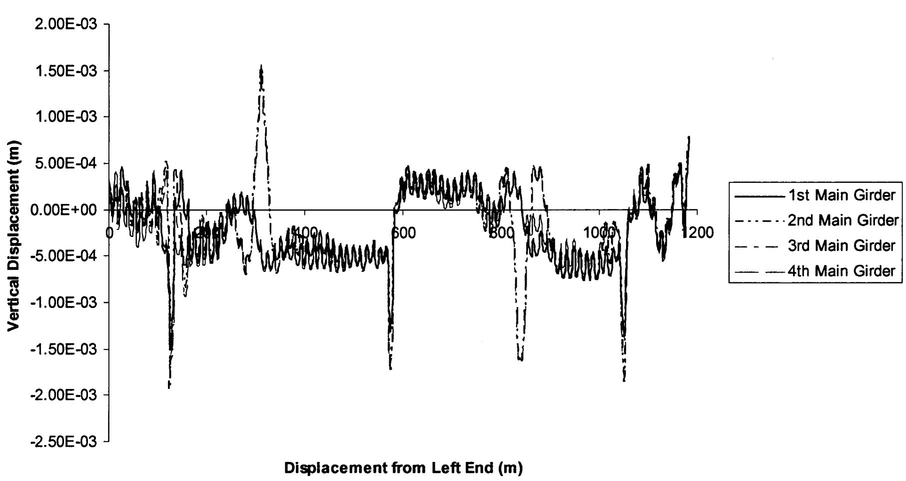
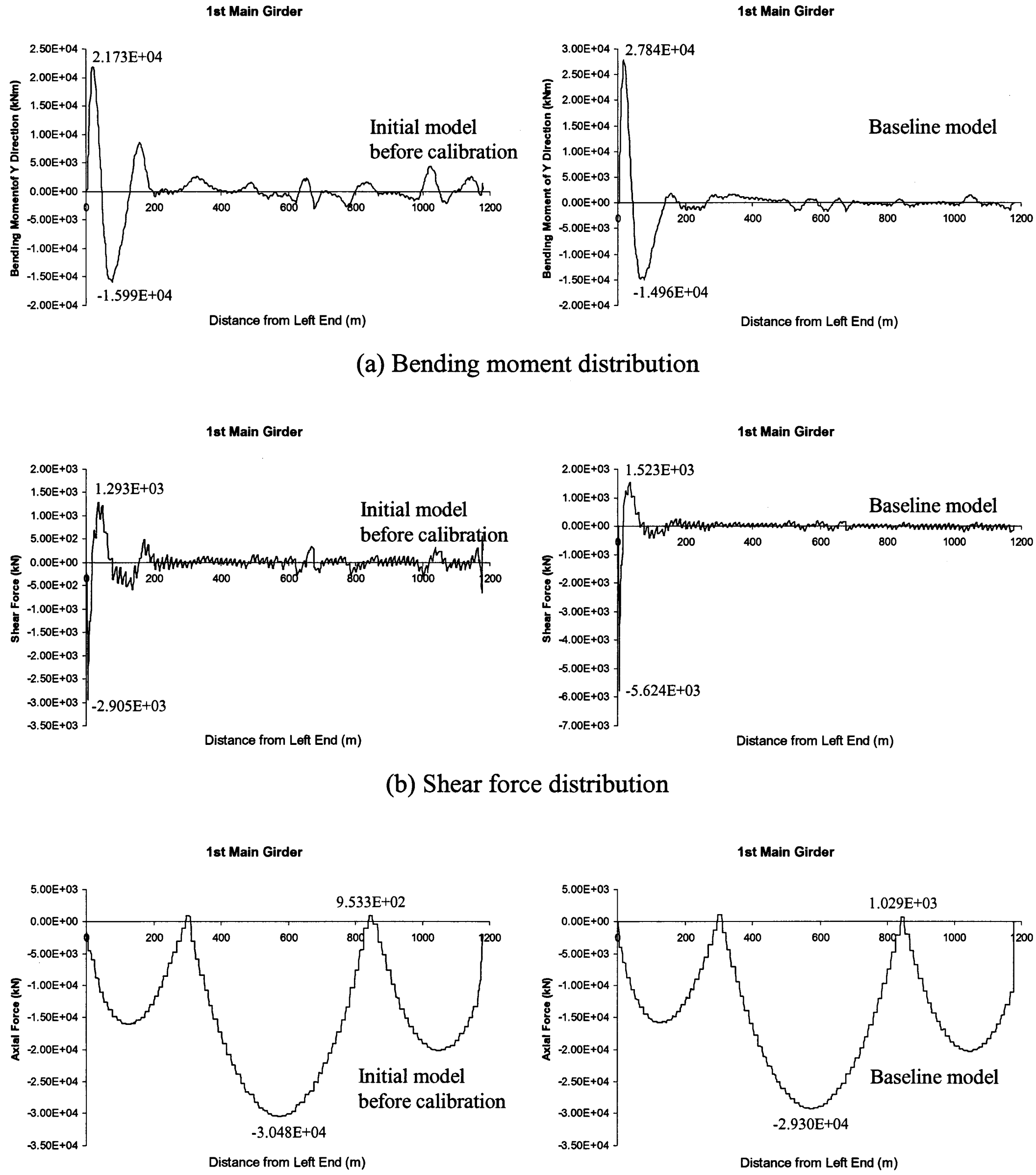


Figure 4. Vertical displacements of main girders of baseline finite element model under permanent loading



(a) Bending moment distribution

(b) Shear force distribution

(c) Axial force distribution

Figure 5. Internal forces in bridge deck of the initial model before calibration and the baseline model

MODAL VERIFICATION

The first 20 natural frequencies obtained by the baseline model are compared with those derived from the WASHMS measurements in Table 1. It shows that, except for the 2nd vertical bending mode, the discrepancies of natural frequencies are less than 5.5%, indicating reasonable agreement.

EFFECTS OF MOVING LOAD

The deck of Ting Kau Bridge consists of two separate carriageway decks with a width of 18.8m each. Each carriageway deck structure consists of a reinforced concrete slab supported by two longitudinal steel girders along the deck edges and steel cross girders at 4.5m intervals. To evaluate the effects of baseline calibration on the accuracy of responses due to highway loading, a set of moving loads can be applied on both the initial and baseline models to analyze their responses. The corresponding static and dynamic analyses can then be carried out and compared.

Table 1. Summary of modal frequencies obtained from WASHMS and baseline model

Mode	WASHMS (Au *et al.* 2007) (Hz)	Baseline Model (Hz)	Difference from WASHMS (%)
1st vertical bending	0.162	0.169	4.32
1st transverse bending	0.239	0.226	-5.44
2nd transverse bending	0.256	0.249	-2.73
3rd transverse bending	0.276	0.280	1.45
4th transverse bending	0.305	0.311	1.97
2nd vertical bending	0.328	0.286	-12.80
3rd vertical bending	0.338	0.348	2.96
1st torsion	0.359	-	-
4th vertical bending	-	0.374	-
5th vertical bending	0.411	-	-
2nd torsion	0.432	-	-
3rd torsion	0.478	0.490	2.51
6th vertical bending	0.485	0.468	-3.51
4th torsion	0.499	0.521	4.41
5th torsion	0.544	-	-
7th vertical bending	0.568	0.560	-1.41
6th torsion	0.573	0.573	0
8th vertical bending	0.592	0.589	-0.51
7th torsion	0.625	0.617	-1.28
9th vertical bending	0.649	-	-

In the hypothetical case considered, a concentrated load of 320kN moves at a constant speed 50km/h along the 1st main girder. Note that this hypothetical case tends to overestimate the responses as the live load should actually be applied somewhere between two adjacent main girders. In particular, when the load moves across an element, the corresponding equivalent loads comprising two forces and two bending moments are applied to the two end nodes of the element. Section A as shown in Figure 1 has been chosen for more detailed analysis. The stress histories at the bottom flange of the main girder at Section A based on the static responses of the initial and baseline models are shown in Figure 6, while the corresponding statistics of stresses are shown in Figure 7. It is observed that, even based on the results of static analyses, the initial model without proper baseline calibration tends to overestimate the mean stress level while the stress fluctuations remain roughly the same. The corresponding transient dynamic analyses are then carried out. The stress histories at the bottom flange of the main girder at Section A of the initial and baseline models are shown in Figure 8, while the corresponding statistics of stresses are shown in Figure 9. As expected, some additional stress fluctuations are observed as compared to those obtained from the static analyses. As observed before, the initial model without proper baseline calibration greatly overestimates the mean stress level with the stress fluctuations largely unaffected, which is also reflected in the statistics of stresses. The stress levels in general agree well with those of the corresponding static results.

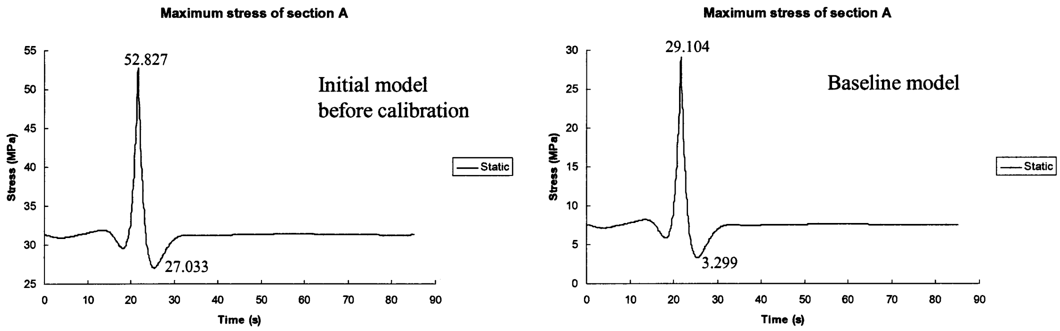


Figure 6. Stress history at Section A based on static response of moving load 320kN

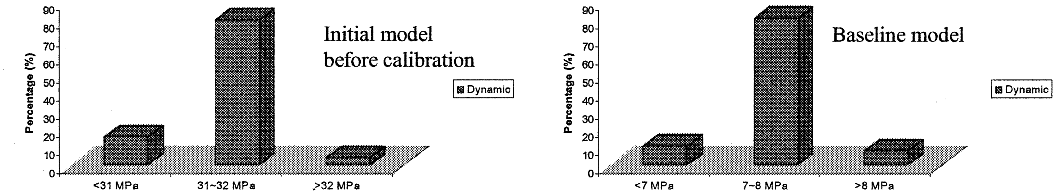


Figure 7. Statistics of stresses at Section A based on static response of moving load 320kN

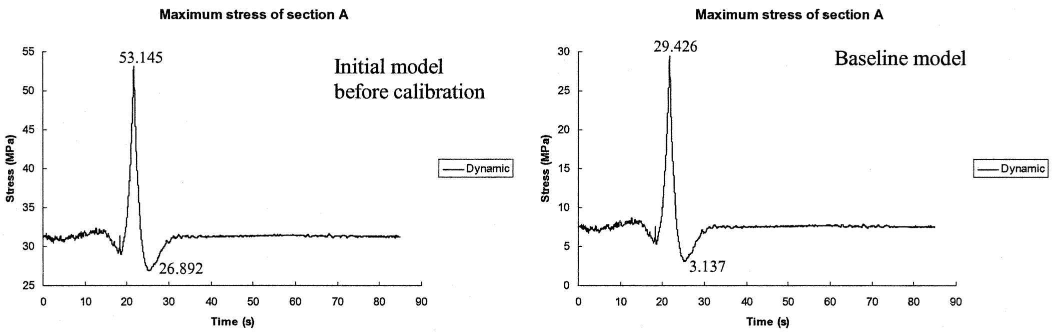


Figure 8. Stress history at Section A based on dynamic response of moving load 320kN

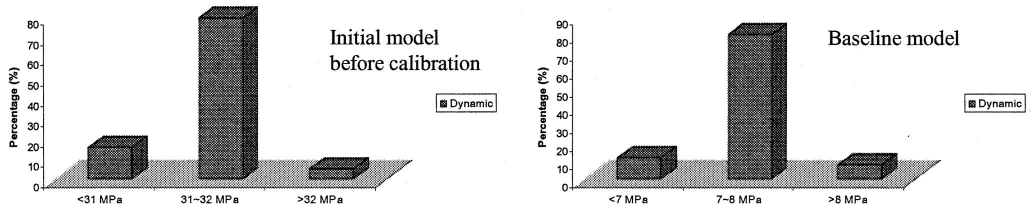


Figure 9. Statistics of stresses at Section A based on dynamic response of moving load 320kN

The fatigue life of a bridge component is basically related to the stress fluctuations experienced by the component as caused by various loading including the highway traffic. Although dynamic analysis can predict the transient stress fluctuations, which are beyond the capability of static analysis, both can come up with similar findings relevant to fatigue analysis. The prevalent stress or the steady stress after the passage of the moving load is overestimated by the initial model without baseline calibration. The maximum tensile stresses are overestimated by the initial model. Such overestimation of stresses is largely caused by the geometric nonlinearity or the "P-delta effect" associated with the deck axial force acting in conjunction with the discrepancy of deck profile.

CONCLUSIONS

The present study of cable-stayed bridges shows the importance of baseline calibration in the development of finite element models for health monitoring and condition assessment. The process consists of a few stages, namely (a) initial finite element modelling; (b) cable force adjustment; and (c) fine-tuning of the baseline model in the light of field measurements. For the Ting Kau Bridge being studied, the flexibility method and fine-tuning by iteration have been used in conjunction to achieve the specified tolerance of bridge deck level of 5mm. More reasonable distributions of bending moments and shear forces have been obtained. Reasonably good agreement has been observed between the calculated frequencies and those obtained from WASHMS installed on the bridge. The moving load analyses carried out on the initial and baseline models have confirmed the importance of baseline calibration. The stress histories predicted by the initial model are observed to overestimate the prevalent stresses, which is caused by the geometric nonlinearity associated with the discrepancy of deck profile. The effect on fatigue assessment should be further examined.

ACKNOWLEDGEMENTS

The authors are grateful to the Highways Department of the Hong Kong Government for permission to publish this paper. Any opinions expressed or conclusions reached in the paper are entirely of the authors. The work described in this paper has been partly supported by the Research Grants Council (RGC) of the Hong Kong Special Administrative Region, China (RGC Project No. HKU 7102/08E).

REFERENCES

Au, F.T.K., Jiang, R.J., Leung, C.C.Y., Xie, L.Z., Lee, P.K.K., Wong K.Y. and Chan, W.Y. (2007). "Development and calibration of finite element models for structural health monitoring of Ting Kau Bridge", *Proceedings to the 3rd International Symposium on Environmental Vibrations*, 559-566.

Chen, D.W., Au, F.T.K. Tham, L.G. and Lee, P.K.K. (2000). "Determination of initial cable forces in prestressed concrete cable-stayed bridges for given design deck profiles using the force equilibrium method", *Computers and Structures*, 74(1), 1-9.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1998). "A review of damage identification methods that examine changes in dynamic properties", *Shock and Vibration Digest*, 30(2), 91-105.

Elkordy, M.F., Chang, K.C. and Lee, G.C. (1994). "A structural damage neural network monitoring system", *Microcomputers in Civil Engineering*, 9(2), 83-96.

Janjic, D., Pircher, M. and Pircher, H. (2003). "Optimization of cable tensioning in cable-stayed bridges", *Journal of Bridge Engineering*, 8(3), 131-137.

Lam, H.F., Yuen, K.V. and Beck, J.L. (2006). "Structural health monitoring via measured Ritz vectors utilizing artificial neural networks", *Computer-Aided Civil and Infrastructure Engineering*, 21(4), 232-241.

Ni, Y.Q., Zhou, H.F., Chan, K.C. and Ko, J.M. (2008). "Modal Flexibility Analysis of Cable-stayed Ting Kau Bridge for Damage Identification", *Computer-Aided Civil and Infrastructure Engineering*, 23(3), 223-236.

Park, S., Stubbs, N., Bolton, R., Choi, S. and Sikorsky, C. (2001). "Field verification of the damage index method in a concrete box girder bridge via visual inspection", *Computer-Aided Civil and Infrastructure Engineering*, 16(1), 58-70.

Ren, W.X. and Peng, X.L. (2005). "Baseline finite element modeling of a large span cable-stayed bridge through field ambient vibration tests", *Computers and Structures*, 83(8-9), 536-550.

Wang, P.H., Tseng, T.C. and Yang, C.G. (1993). "Initial shape of cable-stayed bridges", *Computers and Structures*, 46(6), 1095-1106.

Wong, K.Y. (2004). "Instrumentation and health monitoring of cable-supported bridges", *Structural Control and Health Monitoring*, 11(2), 91-124.