When Capital Follows Profitability: Non-linear Residual Income Dynamics

GARY C. BIDDLE biddle@ust.hk
Department of Accounting, Hong Kong University of Science & Technology
Clear Water Bay, Kowloon, Hong Kong

PETER CHEN acpchen@ust.hk
Department of Accounting, Hong Kong University of Science & Technology
Clear Water Bay, Kowloon, Hong Kong

GUOCHANG ZHANG acgzhang@ust.hk
Department of Accounting, Hong Kong University of Science & Technology
Clear Water Bay, Kowloon, Hong Kong

Abstract. Economic reasoning suggests that capital follows profitability. This study introduces into residual income valuation "capital follows profitability" investment dynamics whereby capital investments are guided by the profitability of underlying investment opportunities. These investment dynamics predict convex versus linear relations between future and current residual income, with slope and convexity dependent on investment opportunity. We test these predictions against the linear information dynamics (LID) proposed by Ohlson (1995) and Feltham and Ohlson (1996), with supportive results. These findings point the way to further development of links between firm value and the economics of value creation.

Keywords: Residual Income, Valuation, Linear Information Dynamics, Investment Dynamics, Non-linear Residual Income Dynamics

JEL Classification Codes: M41, G12, G31

Contacts:	*Email*	*Telephone*	*Fax*
Gary C. Biddle	biddle@ust.hk	(852) 2358-7552	(852) 2358-1693
Peter Chen	acpchen@ust.hk	(852) 2358-7572	(852) 2358-1693
Guochang Zhang	acgzhang@ust.hk	(852) 2358-7569	(852) 2358-1693

(A previous version of this paper was entitled "Capital Investment and Nonlinear Residual Income Dynamics.")

Version date 15 November 2000

When Capital Follows Profitability: Non-linear Residual Income Dynamics

The residual income model (RIM) expresses the market value of equity as current book equity, which is observable, plus discounted expected future residual income (RI), which is not.[1] To provide a link between current observables and expected future RI, Ohlson (1995) and Feltham and Ohlson (1996) propose that future RI is related to current RI via a linear stochastic process. Subsequent tests of these "linear information dynamics" (LID) have yielded mixed results. Dechow, Hutton, and Sloan (1999), for example, find support in mean reverting residual income, but are puzzled that LID does not improve on a simple capitalization of period-ahead earnings in explaining equity values. Myers (1999) examines four formulations of LID to find that none outperforms book equity alone, inferring that "current theoretical models of the information dynamics are incomplete and/or RI may be a nonstationary time series" (page 3). Beaver (1999, page 41) calls for "more aggressive exploration of the accounting and economic factors" influencing RI and contexts in which LID might be violated, sentiments echoed by Verrecchia (1998), Lee (1999), and Lo and Lys (1999).[2]

---

[1] See Preinreich (1938), Ohlson (1995), and Fetham and Ohlson (1996). Others contributing to the early development of the residual income model include Canning (1929), Edey (1957), Edwards and Bell (1961), Solomons (1965), Anthony (1973), Kay (1976), and Peasnell (1981, 1982).

[2] Recent studies documenting related non-linearities include Barth, Beaver, and Landsman (1998), Berger, Ofek, and Swary (1996), Burgstahler and Dichev (1997), Easton (1999), Hayn (1995), and Subramanyam and Wild (1996).

In this study we introduce into residual income valuation "capital investment dynamics" that allow net capital investment to be informed by current profitability, and thereby influence future profits, as economic "capital follows profitability" intuition would suggest. Relations between current profitability, capital investment, and future profits are modeled in terms of characteristics of firms' underlying investment opportunity sets. These more endogenous investment dynamics predict convex relations between future and current RI, and between unrecorded goodwill (market value added) and current RI, with slope and convexity dependent on investment opportunity. These predictions contrast with Ohlson's (1995) and Feltham and Ohlson's (1996) LID, which predict linear relations.[3]

Empirical tests based on 83,826 firm-year observations from 1981-98 support multiple predictions of the proposed investment dynamics versus LID. First, we find one- (and three-) year ahead growth in operating assets (net investment) to be positively related to current RI, consistent with capital following profitability. This contrasts with LID where capital growth is either unspecified (Ohlson (1995)) or capital grows at a constant rate that is independent of current RI (Feltham and Ohlson (1995)). Second, piecewise regressions of future on current RI reveal slopes increasing in current RI (except for very high residual income), confirming convexity versus the linearity predicted by LID. Third, slopes and convexity of future RI with respect to current RI increase in investment opportunity (measured by growth in operating assets). Fourth, slopes decrease and convexity increases with divestment opportunity, also as predicted, except for high residual income divesting firms (which suggests that other factors are at work for firms

[3] Because "other information" is defined in LID to be information not reflected in RI, this treatment precludes a role for accounting profitability (as reflected in RI) to inform investment decisions, and thus influence future RI, as allowed by the investment dynamics modeled here.

with these characteristics). LID do not offer these predictions regarding the effects of investment / divestment opportunities.

A fifth finding consistent with the proposed investment dynamics is that unrecorded goodwill is distinctly convex in current RI for both investing and divesting firms. LID would predict linear relations. Sixth, tests for the effects of investment opportunities on the slopes and convexity of unrecorded goodwill are supported for investing firms; only for divesting firms are these predictions not supported. Multiple robustness checks controlling for variable definitions, time periods, capital costs, firm size, and industry membership yield qualitatively identical findings. Taken together, these findings support the proposed investment dynamics over LID.

The model and findings of this study introduce into residual income valuation economic relations between investment opportunities, profitability, and the investment decisions that underlie the creation of value. They provide a framework for further development of RI valuation, including the determinants and predictors of investment opportunities and capital flows. Other recent studies that also incorporate contingent investment decisions include G. Zhang (2000), Chen and Zhang (2000), and Yee (2000).[4]

This study also may help to explain why empirical tests of LID have found limited support (Dechow, Hutton, and Sloan (1999), Hand and Landsman (1999), Myers (1999)). The proposed investment dynamics suggest that LID will hold only in specific contexts, and that in

[4] G. Zhang (2000) and Chen and Zhang (2000) consider the effects of contingent investments on the properties of the valuation function where firms have both growth and adaptation options; Yee (2000) considers adaptation options only (Burgstahler and Dichev (1997)). Unlike the present study, these studies do not consider the properties of residual income dynamics.

general, non-linearity should be recognized in the relation between future and current residual income. A further implication is that linear valuation models may not describe empirical data for firms experiencing changes in operating scales (via expansion or restructuring / downsizing).[5]

As background and motivation, Section 1 discusses the residual income model, Ohlson's (1995) LID, and prior findings. Section 2 develops the proposed investment dynamics and their testable implications. Section 3 describes data collection and characteristics. Section 4 presents the tests and empirical results. Section 5 provides a summary.

1. The Residual Income Valuation Model and Linear Information Dynamics

The residual income valuation model (RIM) expresses the market value of equity as current book equity plus discounted expected residual income to equity holders. Residual income (RI) is defined as profit earned on equity capital above the cost of equity capital. It reflects intuitively that to create wealth, capital must earn more than its cost, as long recognized by economists and accountants.[6]

RIM derives from the discounted dividends valuation model and two additional assumptions (Preinreich (1938), Ohlson (1995)). The discounted dividends model in Equation (1) expresses equity value (V_t) as the present value of expected dividends, where dividends (D_t)

[5] See G. Zhang (2000) for a discussion of non-linear regression model specifications in a valuation context.

[6] See Hamilton (1777) and Marshall (1890). Coined "residual income" by General Electric in the 1950s, this concept has been called *excess earnings* by Canning (1929) and Preinreich (1938); *super-profits* by Edey (1957); *excess realizable profit* by Edwards and Bell (1961); *excess income* by Kay (1976); *abnormal earnings* by Peasnell (1981, 1982) and Ohlson (1995); and *economic value added* (EVA®) by Stern Stewart and Company (Stewart (1991)).

are defined as cash flows to equity holders (cash dividends plus net share repurchases), r is the cost of equity capital (assumed constant for simplicity), and E is an expectations operator:

$$V_t = \sum_{\tau=1}^{\infty} E_t \left[\frac{\tilde{D}_{t+\tau}}{(1+r)^{\tau}} \right]. \tag{1}$$

The first additional assumption is that book equity (B_t) follows a "clean surplus relation" that only earnings (X_t) and (net) dividends modify book equity, a condition approximately satisfied in the financial accounting principles of most jurisdictions:[7]

$$B_t = B_{t-1} + X_t - D_t. \tag{2}$$

The second assumption is a regularity condition that the present value of book equity converges to zero as τ goes to infinity (Prienreich (1938)):

$$E_t \left[\frac{\tilde{B}_{t+\tau}}{(1+r)^{\tau}} \right] \xrightarrow[\tau \to \infty]{} 0. \tag{3}$$

RI expressed as $X_t^a \equiv X_t - rB_{t-1}$ (for "abnormal earnings" following Ohlson's (1995) notation) implies $X_t = X_t^a + rB_{t-1}$.[8] Substituting $X_t^a + rB_{t-1}$ for X_t in equation (2), solving equation (2) for D_t, substituting the result into (1), and applying regularity condition (3) yields the RIM, an isomorphic expression for equity market value expressed in terms of current book equity and discounted expected residual income (Prienreich (1938)):[9]

---

[7] For recent evidence on conformity of US GAAP with clean surplus, see Lo and Lys (1999).

[8] Adopting Ohlson's (1995) notation, we use RI and X_t^a interchangeably as abbreviations for residual income.

[9] For a discussion of the finite horizon case, see Penman (1997, 2001).

$$V_t = B_t + \sum_{\tau=1}^{\infty} E_t\left[\frac{\tilde{X}_{t+\tau} - r\tilde{B}_{t+\tau-1}}{(1+r)^{\tau}}\right] = B_t + \sum_{\tau=1}^{\infty} E_t\left[\frac{\tilde{X}^a_{t+\tau}}{(1+r)^{\tau}}\right]. \tag{4}$$

Two features of (4) should be noted. First, only B_t is observable at time t; r is assumed fixed, and all other right-side variables are forecasts. Second, the clean surplus relation (2) underlying (4) is a weak constraint that is satisfied for any accumulation B_t of X_t and D_t; it imposes no restriction otherwise on the relations between B_t, X_t, and D_t. As a result, the measures described in (4) need bear little resemblance to traditional concepts of book equity, earnings, and dividends. Thus, while RIM (4) provides a translation of the discounted dividends model (1) into what appear to be accounting measures, additional structure is required to approach descriptive validity and practical utility.

To provide a link between current observables and residual income forecasts, Ohlson (1995) proposes the following linear information dynamics (LID):

$$\tilde{X}^a_{t+1} = \omega X^a_t + \nu_t + \tilde{\varepsilon}_{1,t+1} \tag{5}$$

$$\tilde{\nu}_{t+1} = \gamma\nu_t + \tilde{\varepsilon}_{2,t+1} \tag{6}$$

where $0 < \omega, \gamma < 1$ are fixed and known parameters defined "loosely" by a firm's economic environment and accounting principles, ε_1 and ε_2 are zero-mean disturbance terms, and ν_t is "information other than abnormal earnings" that relates to residual income with a one-period delay (Ohlson (1995), page 668).[10] Thus, both residual income X^a and "other information" ν are assumed to follow linear stochastic (first-order auto-regressive, AR1) processes.[11]

---

[10] The parameters ω and γ are bounded above by 1 to provide stationarity.

[11] It is tempting to suggest that the auto-regressive specification for RI is supported by prior empirical evidence on the time-series properties of earnings (Ball and Watts (1972), Watts and Leftwich (1977)). But applying the clean surplus relation, RI is seen to depend on both

Combining linearity assumptions (5) and (6) with (4) yields a linear expression relating current equity market value to currently observable book equity, residual income, and "other information" ν :

$$V_t = B_t + \alpha_1 X_t^a + \alpha_2 v_t \tag{7}$$

$$\text{where } \alpha_1 = \frac{\omega}{1+r-\omega}, \qquad \alpha_2 = \frac{1+r}{(1+r-\omega)(1+r-\gamma)}. \tag{8}$$

Expression (7) is appealing as it provides a parsimonious linear expression linking equity market values with observable accounting (book equity and earnings) and "other information." Taking the first difference of (7) and dividing by book equity yields a corresponding linear expression relating current equity returns with scaled earnings levels and earnings changes.

A number of recent studies have used (7) to motivate empirical specifications (Easton and Harris (1991), Collins, Maydew, and Weiss (1997)). Other studies provide empirical evidence that calls (7) into question. For example, Dechow, Hutton, and Sloan (1999) find that various empirical implementations of (7) perform no better in explaining contemporaneous stock prices than simply capitalizing analysts' earnings forecast in perpetuity. Myers (1999) examines four formulations of LID to find that they underperform book value alone in estimating equity values, and that they significantly underestimate the market expectation of future residual income. Hand and Landsman (1999), in their words, "find robust evidence that dividends are materially positively priced, sharply contrasting with the negative relation predicted by dividend displacement" in Ohlson (1995). Independent of RIM, Burgstahler and Dichev (1997) document a non-linear relation between equity market value and net income (versus RI), a result they

---

earnings and dividends, $X_t^a \equiv X_t - rB_{t-1} = X_t - r(B_{t-2} + X_{t-1} - D_{t-1})$. Dividends defined as cash flows to equity holders (to include share buybacks and equity investments) make it less clear that RI will be auto-regressive, even if this condition holds for earnings.

ascribe to the effects of an adaptation option applicable to lower ranges of profitability. Other studies documenting related non-linearities include Barth, Beaver, and Landsman (1998), Berger, Ofek, and Swary (1996), Easton (1999), Hayn (1995), and Subramanyam and Wild (1996).

Ultimately, the linear form of (7) hinges critically on LID assumptions (5) and (6), which warrant closer examination. Of particular interest is their treatment of capital investments and of dividends (defined as cash flows to equity holders).[12] Here it is revealing to expand (5) using the residual income definition $X^a_{t+1} \equiv X_{t+1} - rB_t$ and the clean surplus relation (2):

$$\tilde{X}^a_{t+1} = \tilde{X}_{t+1} - r(B_{t-1} + X_t - D_t) = \omega X^a_t + \nu_t + \tilde{\varepsilon}_{1,t+1}. \tag{9}$$

In Ohlson (1995), dividends (and capital investment) are not modeled explicitly beyond (1), (2), and (9).[13] Since Ohlson ((1995), page 666) assumes reasonably that dividends have no effect on same-period earnings ($\partial X_t/\partial D_t = 0$), D_t must decrease earnings X_{t+1} at rate r in order for the equality (9) to hold. This implies $\partial E_t(\tilde{X}_{t+1})/\partial D_t = -r$ (Ohlson (1995), property P1, page 672).

[12] In Ohlson (1995), in the absence of financial assets and liabilities, dividends defined as cash flows to equity holders, minus earnings, equal net capital divestment when positive, and net capital investment when negative, as follows from the clean surplus relation (see also Section 2 below).

[13] This explains why Ohlson ((1995), page 669) does not predict earnings beyond one period ahead. Earnings two periods ahead depend on dividends (and thus capital) one period ahead, which are unspecified. Relatedly, LID equation (5) models RI without reference to the level of capital employed; in an extreme case where all capital is paid out as dividends, the generation of future residual income would nonetheless continue according to the pre-specified AR1 process under LID. Our alternative investment dynamics developed below model RI more intuitively as a function of capital employed. Under Ohlson's (1995) assumptions, and ignoring "other information," simple capitalization of price-to-book or price-earnings ratios should perform as well as (7) for valuing market equity (Lee (1999), page 420).

ν_t is defined as information not contained in X_t^a, where $\partial v_t / \partial D_t = 0$. Combining (2), (7), $\partial X_t / \partial D_t = 0$, and $\partial v_t / \partial D_t = 0$ yields a dividend irrelevance property $\partial V_t / \partial D_t = -1$ for dividends defined as equity cash flows. [14]

Putting the above into words, LID expressions (5) and (6) leave unspecified how net dividends (capital investment) may be influenced by other variables in the model such as current profitability. Importantly, they also dictate that a dollar of net dividends reduces period-ahead profits by r. Because a dollar of dividends reduces equity capital by a dollar, and the capital

[14] In Ohlson (1995), dividend irrelevance is obtained by assuming that capital investments (subsequent to an initial period) yield zero NPV, and thus, *net* dividends (defined as equity cash flows) affect equity value dollar-for-dollar (see also next paragraph). This is a more restrictive assumption than invoked by Miller and Modigliani (1961), where dividend irrelevance is defined in terms of direct (cash) dividends and investment decisions are fixed. Our proposed investment dynamics (Section 2) allow for positive and negative NPV investment opportunities in each period. They maintain dividend irrelevance as defined by Miller and Modigliani (1961) since equity value is invariant to direct (cash) dividends; replacement capital for the dividends is obtained from investors to fund the (fixed) investment projects. With positive and negative NPV investment opportunities, but without financial assets and liabilities as in Ohlson (1995), *net* dividends are not irrelevant since, given earnings, they are equivalently net capital investments.

charge by r, the foregone investment return on capital must also be r. This implies that all investments (subsequent to an initial period) are zero net present value (NPV).[15]

While the assumption of zero NPV investments should hold approximately for investments at the margin, and perhaps in the longer run in competitive markets, it does not apply plausibly to investment opportunity sets faced by going concerns in the short run being modeled. Investment opportunity sets will encompass capital scale ranges over which rates of return will differ from r, and thus, so will the averages. As follows from the implicit assumption that investments are zero NPV beyond an initial period, Ohlson (1995) does not develop the possibility that earnings and capital investments (net dividends) are related, within the context of the clean surplus relation (2). While Feltham and Ohlson (1995) allow for non-zero NPV investments beyond an initial period, they assume an exogenous linear investment process that does not explicitly respond to accounting information as modeled here. The investment dynamics we propose next generalize RIM by allowing ongoing non-zero NPV investments and investment decisions that respond to firms' underlying investment opportunities as informed by current profitability.

2. Capital Investment and Non-linear Residual Income Dynamics

Firms create value by directing capital to activities that earn more than the cost of capital. Capital investment decisions are informed by profitability currently, and projected forward. The

---

[15] Ohlson (2000) argues that LID as in Ohlson (1995) allow for non-zero NPV investments. This inference derives ultimately from a non-zero NPV condition in an initial period (when LID are set into motion) since, by construction, LID are independent of incremental capital investments and thus, capital investments subsequent to an initial period must have zero NPV. Therefore, Ohlson's (2000) argument does not apply to incremental capital investments made by going concerns that are the focus of this study.

proposed investment dynamics introduce this economic intuition into the residual income model.[16]

More specifically, we assume that investment decisions are guided by profitability signals and constrained by available investment (divestment) opportunities. Current profitability (measured by the "spread" between the rate of return on capital and the cost of capital) is assumed to be informative regarding future profitability, reflecting persistence in economic rents. Expected future profitability in turn determines investment and hence future capital. Future residual incomes are forecast based on the expected capital and profitability. Thus, and in contrast to Ohlson (1995) and Feltham and Ohlson (1995), we introduce a role for profitability to inform investment decisions that underlie value creation.

To clarify the exposition of the proposed investment dynamics, we first consider a simple setting where accounting profitability reflects economic profitability without bias. We then show that the predictions of the model generalize to a setting where accounting profitability differs from economic profitability due to the effects of conservative accounting (presented in the Appendix). In the context of this generalized view, the initial treatment is a special case where depreciation is unbiased, and consequently, accounting profitability equals the internal rate of return.[17]

---

[16] Consistent with our intuition, Penman (2001, page vii) states: "Financial statements have many uses, but the predominant one is to provide information for investing in businesses."

[17] For a derivation of this equivalence, see G. Zhang (2000). Two other accounting assumptions underlying both the simple and generalized cases are the clean surplus relation and historical cost accounting.

2.1. Investment Dynamics

Following Ohlson (1995), we initially assume a firm with no financial assets and liabilities.[18] In this case, period t ending capital defined as operating assets (OA_t) equals book equity (B_t), and period t net capital investment (I_t) defined as the change in operating assets equals the change in book equity. Via clean surplus relation (2), net capital investment also equals earnings less dividends. Thus, $I_t \equiv OA_t - OA_{t-1}$ which (in the absence of financial assets and obligations) equals $B_t - B_{t-1} = X_t - D_t$. For expositional clarity and to allow investment and divestment decisions to reflect different influences, we separate the case of $I_t \geq 0$ from $I_t < 0$ by denoting $I_t^+ = I_t$ if $I_t \geq 0$, as the amount of non-negative investment, and $I_t^- = -I_t$ if $I_t < 0$ as the amount of negative investment. Henceforth, the term "divestment" refers to a reduction in capital (operating assets) during a period.

Applying this notation, consider a multi-period setting in which capital investment decisions are made each period contingent on perfitability signals (where in this simple setting economic and accounting profitability are equivalent). Profitability in period t (q_t) is defined as "spread," or the difference between the rate of return earned on beginning-of-period t capital (OA_{t-1}) and its cost (r), $q_t \equiv (X_t^a / OA_{t-1}) = (X_t / B_{t-1}) - r$.[19] When applied across a scale of investment opportunity, spread determines the envelope of a firm's investment opportunity set.[20] Importantly, spread also lends itself to prediction since, as an economic rent on capital, it

---

[18] This assumption is relaxed in Section 2.3, where we show that our predictions are unaffected.

[19] This definition of spread is consistent with its use in residual income valuation and incentive applications (Biddle, Bowen, and Wallace (1997, 1999, 2000)).

[20] The investment opportunity envelope defines the rates of return that can be earned on each unit of capital within a firm's available investment opportunity set. For expositional

can be expected to exhibit persistence, but converge gradually to zero due to the effects of competitive economic forces.

Applying this reasoning, we assume that spread in period t is informative about spread in period $t+1$. For tractability, we assume that spread q_t evolves according to the following process:

Assumption 1. Spread is persistent, $\tilde{q}_{t+1} = \theta q_t + \tilde{\varepsilon}_{t+1}$, (10)

where $0 < \theta < 1$ is a parameter representing the speed at which spread (economic rent) decays over time. The lower bound on θ conveys a positive relation between current and future spreads,[21] and the upper bound (as in Ohlson (1995)) is a useful regularity condition. Based on (10), and conditional on date t spread q_t, one dollar of incremental net investment at date t is expected to produce a series of future expected dollar spreads, θq_t, $\theta^2 q_t$, $\theta^3 q_t$, . In present value, this series equals $\frac{\theta}{1+r-\theta} q_t \equiv \Theta q_t$, which represents the expected dollar present value created by one dollar of incremental net capital investment at time t.

We capture the economic intuition that "capital follows profitability" by assuming that the amount of net capital invested at date t depends on the marginal value of investment (Θq_t). We assume that a firm will expand its scale of operations (capital) if the marginal value of investment is positive, and it will reduce its scale if the marginal value of investment is negative. It is envisioned that these changes in capital jointly determined by actions of investors and managers. This "capital follows profitability" assumption is expressed as:

---

simplicity, we model this envelope as a rectangle defined by an average rate of return on capital over the investment opportunity set.

[21] Spread reflects fundamental factors such as the firm's competitive advantages or disadvantages in the marketplace, including customers, location, technology, and management skills. To the extent that such factors tend to endure from one period to next, a positive correlation in (10) is justified. In this study, we focus on spread that is sufficiently persistent to influence investment decisions.

Assumption 2. Capital follows profitability.[22]

2a). If the marginal value of investment (Θq_t) is positive, $I_t^+ = \pi_1 B_{t-1} \Theta q_t$, where $\pi_1 > 0$ is a parameter reflecting the firm's achievable investment opportunity at the end of period *t*.[23]

2b). If the marginal value of investment (Θq_t) is negative, $I_t^- = - \pi_2 B_{t-1} \Theta q_t$, where $\pi_2 > 0$ is a parameter reflecting the firm's achievable divestment opportunity at the end of period *t*.

We do not explicitly impose an upper bound on divestment, as might apply, say, in liquidation, to concentrate our analyses on more representative interior cases. Also, as our focus in this study is

---

[22] Given the equivalence in this initial view between accounting and economic profitability, Assumption 2 is consistent with the NPV rule. In the generalized version of the model presented in the Appendix, capital investments follow economic NPV, and economic profitability can differ from accounting profitability due to the effects of accounting conservatism. In this more general setting it is shown that investing (divesting) firms do not necessarily exhibit positive (negative) current accounting profitability, nor is this condition critical to our predictions.

[23] Since Θq_t represents the expected dollar present value created by one dollar of incremental net capital investment at time *t*, π_1 represents the achievable investment growth expressed as a fraction of existing capital and reflects constraints on the speed at which capital can be invested. Investment I_t, (following conventions used in Ohlson (1995)), is assumed to occur simultaneously with the realization of period *t* spread, so as to adjust capital in period *t+1*, and thus residual income in period *t+1*. A simple linear form is used to enhance expositional clarity. More general relations between current net capital investment (divestment) and the marginal value of investment, such as $I_t^+ = \pi_1 (B_{t-1} \Theta q_t)^\kappa$ and $I_t^- = \pi_2 (-B_{t-1} \Theta q_t)^\lambda$, $\kappa, \lambda > 0$, can be assumed without changing qualitatively the model's predictions.

relationships among current profitability, current investment, and future residual income, we do not introduce an additional dynamic for "other information" as in Ohlson (1995).[24]

2.2. Properties of Residual Income

Utilizing the partition of I_t in Assumption 2, we next examine the properties of future residual income in relation to current residual income.

Case a). The firm makes positive investment.

Based on Assumptions 1 and 2a, the residual income for period $t+1$ is:

$$\tilde{X}^a_{t+1} = B_t q_{t+1} = (B_{t-1} + I^+_t)(\theta q_t + \tilde{\varepsilon}_{t+1}) = \theta X^a_t + \pi_1 B_{t-1} \theta \Theta q_t^2 + \tilde{e}_{t+1} \tag{11}$$

where $\tilde{e}_{t+1} \equiv B_t \tilde{\varepsilon}_{t+1}$ is a mean zero disturbance term. Differentiating $E_t(\tilde{X}^a_{t+1})$ with respect to q_t, or equivalently, with respect to X_t^a given B_{t-1} (assumed positive), yields:

$$\frac{\partial E(\tilde{X}^a_{t+1})}{\partial X^a_t} = \theta + \frac{2\pi_1 \theta \Theta X^a_t}{B_{t-1}} = \theta \left[1 + \frac{2I^+_t}{B_{t-1}} \right] > 0, \tag{12}$$

and

$$\frac{\partial^2 E(\tilde{X}^a_{t+1})}{\partial X^{a^2}_t} = \frac{2\pi_1 \theta \Theta}{B_{t-1}} > 0 . \tag{13}$$

Equations (12) and (13) show that when investment is positive, expected period-ahead residual income is an increasing and convex function of current residual income, given the beginning book value, B_{t-1}. This contrasts with LID, which assume a linear relation between current and period-

[24] In Ohlson (1995), "information other than abnormal earnings" is defined as affecting value independent of current residual income; the relation between this other information and capital investment is not specified and value is unrelated to investment.

ahead residual income. The intuition is that larger residual income in period *t*, for given capital, implies larger spread (an hence, greater marginal value of investment). This larger spread in turn attracts a larger capital investment in period *t* that earns a decayed level of spread in period *t+1*, thereby boosting period *t+1* residual income and producing a steeper slope. Thus, convexity in the residual income dynamic follows from the multiplicative effect of period-ahead spread and capital, both of which are influenced by current spread.[25] Linearity obtains only when the numerator of the second term of (13) is zero, which applies when π_1 and/or θ, and thus net investment, are zero.

For a given level of current RI, investment at date *t* (and hence period-ahead RI) also is influenced by the investment opportunity parameter π_1. To examine the influence of π_1, we differentiate (12) and (13) as follows:

$$\frac{\partial^2 E(\tilde{X}^a_{t+1})}{\partial(X^a_t)\partial\pi_1} = \frac{2\theta\Theta X^a_t}{B_{t-1}} > 0\,, \tag{14}$$

$$\frac{\partial^3 E(\tilde{X}^a_{t+1})}{\partial(X^a_t)^2\partial\pi_1} = \frac{2\theta\Theta}{B_{t-1}} > 0\,. \tag{15}$$

Thus, for a given level of current residual income, the slope and convexity of expected period-ahead residual income increase with investment opportunity.

Case b). The firm makes negative investment.

For the case of an interior level of divestment, the residual income for period *t+1* is:

$$\tilde{X}^a_{t+1} = B_t q_{t+1} = (B_{t-1} - I^-_t)(\theta q_t + \tilde{\varepsilon}_{t+1}) = \theta X^a_t + \pi_2 B_{t-1}\theta\Theta(-q_t)^2 + \tilde{e}_{t+1}\,. \tag{16}$$

Differentiating $E_t(\tilde{X}^a_{t+1})$ with respect to X^a_t yields:

[25] Similar reasoning applies to divestments considered in Case b) below.

$$\frac{\partial E(\tilde{X}^a_{t+1})}{\partial X^a_t} = \theta\left[1 + \frac{2\pi_2 \Theta X^a_t}{B_{t-1}}\right] = \theta\left[1 - \frac{2I^-_t}{B_{t-1}}\right] \tag{17}$$

and

$$\frac{\partial^2 E(\tilde{X}^a_{t+1})}{\partial (X^a_t)^2} = \frac{2\pi_2 \theta\Theta}{B_{t-1}} > 0\,. \tag{18}$$

As in the case of positive investment, the expected residual income next period is a convex function of the current spread. However, expression (17) reveals that period-ahead residual income is not always positively related to current spread. In situations where downsizing is highly sensitive to spread, implying a large π_2, and current spread is very negative, it is possible that $(2I^-_t - B_{t-1}) < 0$, hence causing $E_t(\tilde{X}^a_{t+1})$ to decrease with X^a_t. This corresponds to situations where poor performance causes the firm to drastically reduce its operating scale to avoid large future losses. Note that in the extreme case of complete liquidation, period-ahead residual income reverts to zero. Linearity obtains only when π_2 and/or θ are zero, implying zero net capital divestment.

Parameter π_2 represents the firm's opportunity to divest when faced with negative spread. The effect of divestment opportunity on the residual income dynamic can be examined similarly to the case of positive investment. Differentiating (17) and (18) with respect to π_2 yields:

$$\frac{\partial^2 E(\tilde{X}^a_{t+1})}{\partial (X^a_t)\partial \pi_2} = 2\Theta\theta q_t < 0 \tag{19}$$

and

$$\frac{\partial^3 E(\tilde{X}^a_{t+1})}{\partial (X^a_t)^2 \partial \pi_2} = \frac{2\theta\Theta}{B_{t-1}} > 0\,. \tag{20}$$

Equation (19) shows that the slope of the residual income dynamic becomes flatter as divestment opportunity increases, as the firm is able to limit its operating loss more quickly. Equation (20) shows that as divestment opportunity increases, the residual income dynamic becomes more convex.

2.3. Generalizations of the Model

The above analysis generalizes readily to a firm with financial assets and liabilities. As above, define *OA* as the book value of the firm's operating assets, *FA* as financial assets, *FO* as financial obligations, and *B* as book equity value, with *OA* + *FA* = *FO* + *B* from the balance sheet equality. Define residual operating income as $OX_t^a \equiv OX_t - rOA_{t-1}$ and spread as $q_t \equiv OX_t^a / OA_{t-1}$, where *OX* is operating income. Applying the "capital follows profitability" intuition to operating activities, capital investment (divestment) in a period equals the change in *OA*, and it is immediate that residual *operating* income follows the same dynamics as residual income as described by (11) and (16) above, with OX^a and *OA* taking the places of X^a and *B* respectively.

If it is further assumed (following Miller and Modigliani (1961), Feltham and Ohlson (1995), Nissim and Penman (1999)) that financial activities have zero NPV, then residual income equals residual operating income ($X^a = OX^a$), with spread $q_t = X_t^a / OA_{t-1}$.[26] It follows that the residual income dynamics for this case are as described by (11) and (16), and the qualitative properties are the same as shown by (12)-(15) and (17)-(20).

In Appendix A we confirm that the predictions derived above extend to a more general setting where accounting profitability differs from economic profitability due to the effects of accounting conservatism. Specifically, non-linear residual income dynamics are shown to derive from the "capital follows profitability" assumption (applied to economic profitability) irrespective of the degree of accounting conservatism. Importantly, convexity also is seen to obtain for the underlying cash receipts dynamics by the same intuition. In fact, all of the

[26] The equivalence between residual income and residual operating income is shown by Feltham and Ohlson (1995) for the case of risk neutrality, where the cost of debt and equity capital are equal. Feltham and Ohlson (1999) show this equivalence more generally under conditions of risk aversion and stochastic interest rates (footnote 17, page 178).

predictions derived above continue to hold, with one refinement – that the slopes of period-ahead residual income with respect to current residual income should increase with investment opportunity for investing firms *with sufficiently high current profitability*.

These results suggest the following testable hypotheses.

2.4. Testable Hypotheses

2.4.1 Hypothesis About Capital Investment Behavior (a model assumption)

H1. Investment is related positively to current profitability (spread).

2.4.2 Hypotheses About the Relation between Period-Ahead and Current Residual Income[27]

H2. Period-ahead residual income is a convex function of current residual income.

H3a. For firms with positive net capital investment and sufficiently high current profitability, the slope of period-ahead residual income (with respect to current residual income) increases with investment opportunity.

H3b. For firms with positive net capital investment, the convexity of period-ahead residual income (with respect to current residual income) increases with investment opportunity.

[27] We do not separately list the hypothesis that current and period-ahead residual incomes are positively related as it is already well established empirically (Dechow, Hutton, and Sloan (1999), Myers (1999)).

H4a. For firms with negative net capital investment, the slope of period-ahead residual income (with respect to current residual income) decreases with divestment opportunity.

H4b. For firms with negative net capital investment, the convexity of period-ahead residual income (with respect to current residual income) increases with divestment opportunity.

Ohlson's (1995) LID from Equation (5) assume a linear relation between future and current residual income, and make no predictions regarding the effects of investment opportunities.

2.4.3 Hypotheses About the Relation between Unrecorded Goodwill and Current Residual Income

The discussion above focuses on relations between period-ahead and current residual income. Inferences also obtain concerning the relation between equity value and current residual income.[28] We express this relation in terms of "unrecorded goodwill" (Ohlson 1995), or in EVA® terminology, "market value added" (Biddle, Bowen, and Wallace (1999)). From RIM relation (4) we obtain unrecorded goodwill G_t by subtracting book equity from both sides:

$$G_t \equiv V_t - B_t = \sum_{\tau=1}^{\infty} \frac{E_t(\tilde{X}^a_{t+\tau})}{(1+r)^{\tau}}. \tag{21}$$

By inspection, unrecorded goodwill equals discounted expected residual income (EVA®).

[28] The proposed investment dynamics provide additional inferences for relations among other variables, including value, dividends, θ, π, and accounting conservatism ($\gamma - \delta$) (Appendix). To reasonably limit its scope, we focus in this study on implications for relations involving residual income as driven by investment dynamics.

Applying (12) through (15) recursively,[29] we infer that for firms with q_t sufficiently high to induce positive net capital investment: i) G_t is a convex function of spread q_t or, equivalently, a convex function of current residual income, given beginning capital;[30] and ii) the slope dG_t/dX_t and the degree of convexity d^2G_t/dX_t^2 both increase with investment opportunity, given q_t. Similar reasoning applies to firms with negative investments. These extensions provide the following additional hypotheses regarding the relation between unrecorded goodwill and current residual income that parallel those above for period-ahead residual income:

H5. Unrecorded goodwill is a convex function of current residual income.

H6a. For firms with positive net capital investment and sufficiently high current profitability, the slope of unrecorded goodwill (with respect to current residual income) increases with investment opportunity.

H6b. For firms with positive net capital investment, the convexity of unrecorded goodwill (with respect to current residual income) increases with investment opportunity.

H7a. For firms with negative net capital investment, the slope of unrecorded goodwill (with respect to current residual income) decreases with divestment opportunity.

---

[29] This recursive derivation extends the inferences of the proposed investment dynamics to multi-period ahead residual income forecasts and applies regardless of whether or not the firm has financial assets and liabilities, as discussed above.

[30] Similar convexity properties are derived for equity value in G. Zhang (2000).

H7b. For firms with negative net capital investment, the convexity of unrecorded goodwill (with respect to current residual income) increases with divestment opportunity.

From Equation (7) above, it follows that Ohlson's (1995) LID predict a linear relation between unrecorded goodwill and current residual income, and make no predictions regarding the effects of investment opportunities.

3. Data Collection Procedures

To test the above hypotheses, annual data were collected from Compustat for net income for year *t* and *t+1*, book equity for year *t-1* and *t* (for estimating residual income), and operating assets for year *t-1* (as a scaler) and year *t, t+1,* and *t+3* (to provide one- and three-year ahead estimates of capital growth). We also collected share price and shares outstanding for year *t* in order to estimate unrecorded goodwill.[31] To provide comparability with previous studies (Dechow, Hutton, and Sloan (1999), Myers (1999), Hand and Landsman (1999)), we define net income as income before extraordinary items and discontinued operations (rather than as comprehensive income), and we use a 12% cost of equity capital that is assumed constant across firms and time.[32] We control for cross-sectional variation in firm size by scaling residual income in year *t* and *t+1* and unrecorded goodwill in year *t* by operating assets in year *t-1* (to abstract from year *t* events).[33] Investment opportunity is used as a partitioning variable. Investment

---

[31] Additional data collected for robustness checks are described in Section 4.4 below.

[32] Qualitatively similar results are obtained with an assumed cost of equity capital of 9%.

[33] Residual income in year *t* scaled by operating assets at the beginning of year *t* (OA_{t-1}) equals operating spread in year *t* if financial assets/liabilities have zero NPV. Our choice of OA_{t-1} as the scaler follows directly from the theoretical model; it also has the advantage of eliminating the effect of financial leverage on residual income properties.

opportunity is measured *ex post* using the percentage change in operating assets from year *t* to *t+1*, i_{t+1}, and as a robustness check, from year *t* to *t+3*, i_{t+3}. Table 1 provides a listing of variable abbreviations, definitions, and sources, expressed in terms of the notation developed in Section 2 above.

Table 1 about here.

Our sample consists of all firm years on Compustat with available observations for all test variables, with the following adjustments. First, we delete firm years with non-positive book equity or operating assets. This provides 94,472 available observations for study. Second, we divide these observations into subsamples of positive capital investment (64,008 observations) and negative capital investment (30,464 observations) by the change of operating assets in year *t+1*. To abstract from the influence of unrepresentative data, for each subsample we delete firm years in the top and bottom two percent of residual income and the extreme two percent of investment and divestment (4,496 and 2,360 observations, respectively).[34] We further delete observations more than three standard deviations from the mean of residual income (2,566 and 1,224 observations for the two subsamples, respectively). Combined, these deletions reduce the original sample size by 11.3 percent for the overall sample. Our final sample consists of 83,826 firm year observations from 1981 to 1998. Similar sample selection procedures yield 60,781 observations for robustness tests with three-year growth partitions.

---

[34] This trimming procedure is similar to those used in prior studies. For example, Burgstahler and Dichev (1997) trim the top and bottom 3% of the earnings-to-equity ratio. Collins, Pincus, and Xie (1999) trim observations beyond three standard deviations from their means for their dependent and independent variables.

Table 2 about here.

Table 2 presents descriptive statistics, in Panel A for the pooled sample, and in Panel B by year. Consistent with prior studies (Myers (1999)), Panel A indicates a mean and median for scaled residual income (x^a_{t+1} and x^a_t) close to zero and slightly negative with a 12% assumed cost of equity capital. Capital investment (i_{t+1}) averages ten percent of beginning operating assets. Unrecorded goodwill (g_t) exhibits right skewness with a mean of 61 percent of prior operating assets and a median of approximately 19 percent of prior operating assets. Panel B confirms that each of the test variables has a similar distribution through time, with a slight indication of business cycle effects.

4. Empirical Tests and Results

4.1. Tests of Investment Behavior

We test the "capital follows profitability" hypothesis H1 by regressing year-ahead realized investment growth (measured by i_{t+1}) on current residual income (measured by x^a_t):

$$i_{t+1} = \beta_0 + \beta_1 x^a_t + \mu_{t+1},^{35} \tag{22}$$

where β_0 and β_1 are estimated intercept and slope parameters, respectively, and μ_{t+1} is a mean zero random disturbance term. If financial assets and liabilities are assumed to be zero NPV as in Feltham and Ohlson (1995), current residual income x^a_t scaled by prior operating assets (from

[35] Because operating capital growth defined over the year ahead does not include capital invested during the current year *t* in response to current spread, it is biased against the predicted relation.

Section 2.3 above) equals the spread on operating assets, $x_t^a \equiv X_t^a / OA_{t-1} = OX_t^a / OA_{t-1}$. In i_{t+1}, period-ahead capital investment is scaled by current operating assets. Thus, (22) is effectively a regression of percentage operating asset growth over a one-year horizon on current operating spread.

Results in Table 3 reveal that growth rates in operating capital over one-year horizons are positively related to current operating spread in each year, and for the pooled sample, at high levels of statistical significance, consistent with a "capital follows profitability" prediction. This is seen in estimated slope coefficient β_1 and associated *t*-statistic $t_{\beta 1}$.[36] Together with intercept β_0 (also significantly positive for all years), the results indicate, for the pooled sample, one-year ahead growth in operating capital is 13% plus 96% of prior operating spread. We interpret these findings as supportive of hypothesis H1 that "capital follows profitability."

Table 3 about here.

4.2. Tests of Relations between Period-Ahead and Current Residual Income

4.2.1 Tests for Convexity between Period-Ahead and Current Residual Income

We test the convexity hypotheses H2 first by examining linear and piecewise linear regressions of period-ahead residual income x_{t+1}^a on current residual income x_t^a, following the design of Burgstahler and Dichev (1997):

$$x_{t+1}^a = \alpha_0 + \alpha_1 M + \alpha_2 H + \omega_0 x_t^a + \omega_1 M x_t^a + \omega_2 H x_t^a + \varepsilon_{t+1}, \quad (23)$$

[36] Transitory components in earnings, and thus in x_t^a and i_{t+1}, would bias against finding a positive slope. Lower slopes during the late 1980s and early 1990s suggest that investment may be less responsive to profits in recession years.

where $\alpha_j, j = \{0,1,2\}$, are estimated intercepts, $\omega_j, j = \{0,1,2\}$, are estimated slope parameters, M and H are indicator variables for the middle and high thirds of x_t^a, and ε_{t+1} is a random mean zero disturbance term.

Results are presented in Table 4. For comparison, we present first in Panel A results for a linear relation as would be implied by Ohlson (1995):

$$x_{t+1}^a = \alpha + \omega x_t^a + \varepsilon_{t+1} \tag{24}$$

where α and ω are estimated intercept and slope parameters, respectively, and ε_{t+1} is a random mean zero disturbance term. The results in Panel A are comparable to those in other studies that have employed the linear form, indicating a positive and significant relation with a slope of .71 for the pooled sample, with slight variation across years.[37]

Table 4 about here.

Panel B of Table 4 presents results for piecewise linear regressions (23) for the low (implicit), middle (M) and high (H) thirds of current residual income x_t^a. Focusing on the slope coefficients, a comparison of ω_0 and $\omega_0 + \omega_1$ indicates a statistically significant increase in slopes between the low and middle ranges of x_t^a for the pooled sample and for all but four years (as indicated by F-test asterisks), consistent with convexity prediction H2. However, a comparison of $\omega_0 + \omega_1$ with $\omega_0 + \omega_2$ reveals the slopes to decline between the middle and high ranges of x_t^a for the pooled sample and for all years except 1981. Associated F-tests (asterisks) reveal this decline in slopes to be statistically significant for the pooled sample and for 13 of 18 years. The picture that emerges is an upward sloping "S-shaped" curve with steeper slope in the middle

[37] For comparison, Dechow, Hutton, and Sloan (1999) obtain an estimate of 0.62 for ω using 50,133 annual observations from 1976 to 1995 with residual incomes deflated by market value of equity at the end of year *t*.

range. The decline in slopes between the middle and high range of x_{t+1}^a is inconsistent with both H2 and with LID. Possible explanations offered by prior studies include transitory earnings in the extreme ranges of x_t^a and the dissipation of extreme spreads (rents) due to the effects of competition.[38] To differentiate our model, our further tests examine unique predictions of investment dynamics that are provided neither by LID nor by these other explanations. To begin, we examine our model's predictions regarding the effects of investment / divestment opportunities on the slope and convexity of x_{t+1}^a with respect to x_t^a (H3a to H4b).

4.2.2 Tests for Increasing Slope and Convexity in Residual Income for Investing Firms

Following the theoretical development in Section 2, we examine independently results for firms with positive and negative operating capital growth. First we consider firms with positive growth, partitioning the sample into quartiles based on i_{t+1} as an *ex post* proxy for investment opportunity.[39] Results are presented in Table 5. Panel A presents summary statistics for the test variables (including, for convenience, unrecorded goodwill which is examined in the following section). Consistent with the "capital follows profitability" result in Table 3 above, the central

[38] One line of research finds "S-shaped" relations between stock returns and earnings surprises (and levels), with proposed explanations including greater transitory effects, measurement errors, and imprecision in extreme ranges of earnings surprises and levels (Freeman and Tse (1992), Das and Lev (1994), Subramanyan (1996), Beneish and Harvey (1998)). A second related line of research finds evidence of mean reverting scaled profitability (Beaver (1970), Brooks and Buckmaster (1976), Freeman, Ohlson, and Penman (1982), Fama and French (2000)). Because x_{t+1}^a and x_t^a are scaled by period *t-1* operating assets and the cost of capital is fixed, these variables resemble operating ROA if dividends and other changes in book equity are ignored.

[39] As a robustness check, we also examined partitions based on investment (operating capital growth) over 3-year horizons (see Section 4.4 below).

tendencies of residual income (and unrecorded goodwill) increase monotonically with realized investment i_{t+1}.

Table 5 about here.

In Panel B of Table 5, the estimated slope parameters ω are seen to increase monotonically with i_{t+1} for the linear specification (24), as predicted by H3a. In Panel C, we present results for the piece-wise regressions (23) by quartile of i_{t+1}. The piece-wise slopes increase monotonically in i_{t+1} for the middle and high ranges of x_t^a (reading vertically down the columns $\omega_0, \omega_0 + \omega_1, \omega_0 + \omega_2$), but not for ω_0, consistent with H3a which applies to ranges with sufficiently high spread. Results consistent with the convexity prediction H3b are seen in Panel D of Table 5, where incremental slopes for the middle and high ranges of x_t^a increase monotonically in i_{t+1}.

Overall, the results in Table 5 support H3a and H3b in exhibiting slopes and convexity for x_{t+1}^a with respect to x_t^a, respectively, that increase with investment opportunity. As capital growth increases from the lowest quartile (i1) to the highest quartile (i4), and in contrast to the results in Panel B of Table 4, the S-shape relation between x_{t+1}^a and x_t^a evolves towards a J-shape relation more consistent with convexity hypothesis H2. The results are stronger for higher ranges of spread where H3a and H3b, according to the model, are more applicable. This distinct evolution of shapes with investment opportunity (low to high partitions) is illustrated graphically in Figure 1.

Figure 1 about here.

4.2.3 Tests for Decreasing Slope and Increasing Convexity in Residual Income for Divesting Firms

For firms with negative net capital investment, hypothesis H4a predicts that with increasing divestment opportunity, the slope of x_{t+1}^{a} with respect to x_{t}^{a} will decline (in contrast to the positive investment case). H4b predicts that convexity will increase as before. Results are presented in Table 6, where for the divestment case, i1 (i4) denotes lowest (highest) divestment. Panel A presents summary statistics for the test variables (including, for convenience, unrecorded goodwill). Consistent with the "capital follows profitability" finding in Table 3, the central tendencies of residual income decline monotonically with realized divestment i_{t+1}.

Table 6 about here.

In Panel B of Table 6, the estimated slope parameters ω for the linear specification (24) are seen to decrease monotonically with i_{t+1} as predicted by H4a. In Panel C, we present results for the piece-wise regressions (23) by quartile of i_{t+1}. Consistent with H4a, the piece-wise slopes decrease monotonically in i_{t+1} for the low range of x_{t}^{a} where it is predicted to operate most strongly (reading vertically down column ω_0). For the middle and high ranges of x_{t}^{a} $(\omega_0 + \omega_1, \omega_0 + \omega_2)$, where it is less applicable, slope changes are inconsistent with H4a. Results consistent with the convexity prediction H4b are seen in Panel D of Table 6, where incremental slopes between the lower and middle ranges of x_{t}^{a} increase monotonically in i_{t+1} where, by theory, H4b most readily applies; they decrease monotonically in i_{t+1} between the middle and high ranges of x_{t}^{a}.

Figure 2 about here.

Figure 2 illustrates the relations observed in Table 6. Immediately apparent are the negative slopes in the extreme right range of x_t^a, which grow increasingly negative with divestment. They indicate that for divesting firms, larger current profitability is associated with larger negative future profitability. This is suggestive of earnings management in that managers may attempt to raise current profits in the face of declining prospects, and/or larger discretionary losses (big baths) with larger divestitures in the period ahead. This range of negative slopes works against the regression tests in Table 6; it is apparent visually that if this range is excluded, the graphs are more consistent with hypotheses 4a and 4b.

Overall, we interpret the results in Tables 5 and 6 and Figures 1 and 2 as supporting the proposed investment dynamics over LID, which predict linear relations between current and period-ahead residual income (spread) and which offer no predictions regarding the effects of investment opportunities.

4.3. Tests of Relations between Unrecorded Goodwill and Current Residual Income

4.3.1 Tests for Convexity between Unrecorded Goodwill and Current Residual Income

We next test predictions of the proposed investment dynamics for unrecorded goodwill (market value added) as described in Section 2.4.3 above. These tests of H5-H7b for unrecorded goodwill parallel those in Section 4.2 above for residual income (H2-H4b). We test the convexity hypothesis H5 by examining linear and piecewise linear regressions of unrecorded goodwill g_t on current residual income x_t^a, once again following the design of Burgstahler and Dichev (1997):

$$g_t = \varphi_0 + \varphi_1 M + \varphi_2 H + \phi_0 x_t^a + \phi_1 M x_t^a + \phi_2 H x_t^a + \xi_t, \qquad (25)$$

where φ_j, $j = \{0,1,2\}$, are estimated intercepts, ϕ_j, $j = \{0,1,2\}$, are estimated slope parameters, M and H are indicator variables for the middle and high thirds of x_t^a, and ξ_t is a random mean zero disturbance term.

Table 7 about here.

Results are presented in Table 7. For comparison, we present first in Panel A results for a linear relation as would be implied by Ohlson (1995):

$$g_t = \varphi + \phi x_t^a + \xi_t \tag{26}$$

where φ and ϕ are estimated intercept and slope parameters, respectively. The results in Panel A for this linear form indicate a positive and statistically significant relation, with some variation across years. For the pooled sample, the results suggest (scaled) unrecorded goodwill on average of .65 plus 1.67 times current operating spread.

Panel B of Table 7 presents results for piecewise linear regression (25) for the low (implicit), middle (M) and high (H) thirds of current residual income x_t^a. Focusing on the slope coefficients, a comparison of ϕ_0 and $\phi_0 + \phi_1$ indicates an increase in slopes between the low and middle ranges of x_t^a for the pooled sample and for all but two years (and with statistical significance indicated for the pooled sample and for five of the increase years, as indicated by F-test asterisks). A comparison of $\phi_0 + \phi_1$ and $\phi_0 + \phi_2$ reveals the slopes to further increase between the middle and high ranges of current operating spread for the pooled sample and for all years (with associated F-tests (asterisks) indicating statistical significance in all cases). These findings support H5, which predicts a convex versus linear relation between unrecorded goodwill and current residual income.

4.3.2 Tests for Increasing Slope and Convexity in Unrecorded Goodwill for Investing Firms

We next test the predictions regarding the effects of investment opportunities on the slope and convexity of unrecorded goodwill g_t with respect to x_t^a (H6a to H7b). As above, and following the theoretical development in Section 2, we examine separately results for firms with positive and negative investment growth. First we consider firms with positive growth, partitioning the sample into quartiles based on i_{t+1} as an *ex post* proxy for investment opportunity. Results are presented in Table 8.

Table 8 about here.

In Panel A of Table 8, the estimated slope parameters ϕ are seen to increase monotonically in i_{t+1} for the linear specification (26), consistent with H6a, except for the highest investment portfolio i4. Panel B presents results for the piece-wise regressions (25) by quartile of i_{t+1}. Hypothesis H6a is supported for the middle and high ranges of x_t^a ($\phi_0 + \phi_1$ and $\phi_0 + \phi_2$), where by theory it should operate most strongly; the piece-wise slopes for the low range of x_t^a (ϕ_0) do not increase monotonically in i_{t+1} where by H6a they should operate less strongly. Correspondingly, the tests in Panel C are consistent with H6b, where incremental slopes are observed to increase monotonically with investment. Overall, we interpret the results in Table 8 as supportive of hypotheses H6a and H6b.

These findings are illustrated graphically in Figure 3. A striking feature in Figure 3 is the convexity (versus linearity) of the relation which, as documented in Table 7, supports the proposed investment dynamics over LID. A second striking feature is the negative slope for smaller values of x_t^a, which is consistent with prior findings by Burghstahler and Dichev (1997). This negative slope is particularly evident for the firms with the highest investment growth, and is suggestive of a role for “other information” in informing valuation and investment decisions above and beyond the role of current profitability.

Figure 3 about here.

4.3.3 Tests for Decreasing Slope and Increasing Convexity in Unrecorded Goodwill Income for Divesting Firms

For firms with negative net capital investment, hypothesis H7a predicts that with increasing divestment opportunity, the slope of g_t with respect to x_t^a will decline (in contrast to the investment case). H7b predicts increasing convexity with divestment. Results are presented in Table 9. Panel A presents results for the linear specification (26), where slopes ϕ are observed to decrease monotonically in realized divestment. Panel B presents results for the piece-wise regressions (25) by quartile of i_{t+1}, where the piece-wise slopes for the low range of x_t^a (ϕ_0) do not decrease monotonically with divestment as predicted by H7a. H7a is supported for the middle range ($\phi_0 + \phi_1$), except for i2, and for the high range of x_t^a ($\phi_0 + \phi_2$), but theory would suggest that it should operate most strongly in the low rather than high range of x_t^a. Contrary to H7b, incremental slopes for the middle and high ranges of x_t^a (ϕ_1 and ϕ_2) do not increase monotonically in divestment. Overall, the results in Table 9 do not support hypotheses H7a and H7b.

Table 9 about here.

These findings are illustrated in Figure 4. As was observed for investing firms in Figure 3, the striking features of Figure 4 are convexity and the negative slopes for lower values of x_t^a. Thus, Table 9 and Figure 4 lend strong support to hypothesis H5, but no support to hypotheses H7a and H7b, suggesting that with respect to H7a and H7b, other factors are operating for divesting firms.

Figure 4 about here.

Summarizing the findings in Tables 3 through 9, Table 3 supports hypothesis H1 that "capital follows profitabilty." Tables 4 and 7 support the convexity predictions of investment dynamics with respect to both residual income (H2) and unrecorded goodwill (H5) over the linear predictions of LID. Tables 5 and 6 support the predictions of investment dynamics that the slopes and convexity of period-ahead residual income (with respect to current residual income) should increase in investment opportunity for investing firms (H3a and H3b, respectively). Tables 5 and 6 also generally support the predictions that slope should decrease and convexity should increase in divesting opportunity for divesting firms (H4a and H4b, respectively). Tables 8 and 9 support the predictions of investment dynamics that the slopes and convexity of unrecorded goodwill (with respect to current residual income) should increase in investment opportunity for investing firms (H6a and H6b, respectively). Corresponding predictions for divesting firms (H7a, H7b) are not supported, suggesting that other factors are operating for these firms. Overall, these tests support multiple predictions of the proposed investment dynamics over LID.

4.4. Robustness Checks

We describe below a series of robustness checks that control for variable definitions, industry membership, and firm size. The results are qualitatively the same as those reported above. All are presented in an Addendum available on request from the authors.

4.4.1 A Further Test of the "Capital Follows Profitability" Hypothesis H1

To further test the hypothesis H1 that "capital follows profitability," we match firms by size (operating assets) within 4-digit SIC classifications and examine whether *differences* between paired firms' period-ahead capital investments (Δi_{t+1} = difference in percentage operating asset growth of paired firms) are related to *differences* between their current profitabilities (Δx_t^a = difference in residual operating spreads of paired firms). Results are as follows:

$$\Delta i_{t+1} = 0.01 + 0.67\Delta x_t^a \quad \text{Adj. } R^2 = \ .06. \tag{27}$$
$$(-5.0) \quad (37.50)$$

This positive significant relation confirms that the "capital follows profitability" hypothesis H1 holds for firms matched by industry and size.

4.4.2 Robustness Checks of Variable Definitions for Capital Investment Growth, Cost of Equity Capital, and Residual Income

To control for random variation in the one-year ahead growth measure and to increase its independence from the test variables, we repeated the analysis for positive investment firms shown in Tables 5 and 8 with investment partitions based on three-year ahead investment growth, i_{t+3}. The results are qualitatively similar to those reported above. To test the sensitivity of the results to the growth partitioning procedure, we partitioned firms by thirds and quintiles and also examined within-partition cutoffs, with qualitatively similar findings. We further examined a 9% (versus 12%) cost of equity capital, equity capital costs adjusted by industry medians, and samples that excluded regulated industries, with similar results.

To assess whether our results were influenced by the definitions employed for residual income (spread), we replicated our analysis in using: (i) residual operating income as defined in Nissim and Penman (1999) and (ii) residual operating income defined as operating income after depreciation with the cost of capital charged on operating assets. These results are qualitatively similar to those reported in Tables 1-9.

4.4.3 Robustness Checks for Industry Effects

To address the possibility that the convexity findings for the residual income dynamic might be influenced by the pooling of firms from different industries, we conducted our analysis at the industry level for the twenty 3-digit SIC classifications with more than 600 firm-year observations. The results by industry are qualitatively similar to those reported for Table 4 (Panel B) above.[40]

To further test the predictions of our model, we also examined the Pearson correlations between the incremental piecewise slope coefficients from the residual income regressions estimated by industry j $(\omega_{1,j}, \omega_{2,j})$ and median industry investment opportunity, $i_{t+1,j}$, with the following results: Corr $(\omega_{1,j}, i_{t+1,j})$ = 0.25 (p-value = 0.29) and Corr $(\omega_{2,j}, i_{t+1,j})$ = 0.47 (p-value = 0.03). These results are consistent with the prediction that convexity in the residual income dynamic should increase with investment opportunity, especially in higher ranges of current profitability.

We correspondingly examined by industry the relation between unrecorded goodwill and residual income. The results are qualitatively similar to our previous Table 7 (Panel B). To further test the predictions of our model, we examined the Pearson correlations between the incremental piecewise slope coefficients for the unrecorded goodwill regressions estimated by industry j $(\phi_{1,j}, \phi_{2,j})$ and median industry investment opportunity, $i_{t+1,j}$, with the following results: Corr $(\phi_{1,j}, i_{t+1,j})$ = 0.62 (p-value = 0.00) and Corr $(\phi_{2,j}, i_{t+1,j})$ = 0.54 (p-value = 0.01). Consistent with our model's predictions, convexity in unrecorded goodwill increases with investment opportunity.

[40] Exceptions are utilities and telecoms, which we interpret as consistent with our predictions since our "capital follows profitability" assumption is less applicable in these regulated industries.

4.4.4 Robustness Checks for Firm Size Effects

To address the possibility that the convexity findings for the residual income dynamic might be due to the pooling of firms of different sizes, we conducted our analysis by size quartiles based on operating assets. The results for each quartile are qualitatively similar to the findings in Table 5.

To further test the predictions of our model, we also examined the effects of investment opportunity within size quartiles. Consistent with the predictions of our model and with the empirical findings described above (Table 5, Panel C), the piecewise slopes generally increase with investment opportunity within each firm size quartile, especially for higher spread where this relation is predicted to operate most strongly.

We similarly examined by firm size quartile the relation between unrecorded goodwill and residual income. The results are qualitatively similar to our previous Table 8 (Panel B), with convexity observed for all size quartiles. To test the predictions that convexity should increase with growth opportunity, we further examined the effects of investment opportunity within size quartiles. Consistent with our predictions and with our previous empirical findings (Table 8, Panel B), the piecewise slopes generally increase with investment opportunity within each firm size quartile, and especially for higher spread where this relation is predicted to operate most strongly.

Combined, these additional findings indicate that the empirical tests of the investment dynamics presented in Sections 4.1 to 4.3 above are robust with respect to alternative variable definitions and to factors associated with firm size and industry membership, including accounting method differences related to size and industry. Importantly, these findings generally confirm a set of properties predicted by investment dynamics that are not predicted by LID.

5. Summary and Discussion

This study introduces into the residual income valuation "capital follows profitability" investment dynamics whereby capital investment is guided by profitability. To the extent current profitability conveys inferences regarding a firm's investment opportunities, it can serve to inform capital investment decisions. By this reasoning, future residual income is related both to current profitability and to capital investment informed by current profitability. These investment dynamics predict non-linear relations between future and current residual income that are distinct from linear information dynamics (LID) proposed in Ohlson (1995) and Feltham and Ohlson (1995, 1996).

Specifically, our proposed investment dynamics predict that future residual income will generally be a convex rather than a linear function of current residual income. They further predict that the slope and convexity should increase with investment opportunity, and that the slope should decrease and convexity should increase with divestment opportunity. Similar properties extend to unrecorded goodwill (market value added) with respect to current residual income. These properties hold regardless of whether accounting is conservative.

We test empirically these predictions of the proposed investment dynamics with confirming results. First, we find that future capital growth is positively related to current profitability, consistent with economic reasoning that "capital follows profitability." This contrasts with LID, where capital growth is either unspecified (Ohlson (1995)) or capital grows at a constant rate that is independent of current RI (Feltham and Ohlson (1995)). Second, our results confirm convexity, versus linearity assumed by LID, in the residual income dynamic (except for very high residual income). Third, slopes and convexity of future RI with respect to current RI increase in investment opportunity (measured by growth in operating assets). Fourth, slopes decrease and convexity increases with divestment opportunity, also as predicted, except for high residual income divesting firms (which suggests that other factors are at work for firms with these characteristics). LID do not offer these predictions regarding the effects of investment / divestment opportunities.

A fifth finding consistent with the proposed investment dynamics is that unrecorded goodwill is distinctly convex in current RI for both investing and divesting firms. LID would predict linear relations. Sixth, tests for the effects of investment opportunities on the slopes and convexity of unrecorded goodwill are supported for investing firms; only for divesting firms are these predictions not supported. A series of robustness checks indicate that these results are insensitive to alternative variable definitions, time periods, capital costs, and to the influences of factors related to firm size and industry membership, including accounting differences related to firm size and industry. Altogether, these findings support the proposed investment dynamics over LID.

The findings and approach of this study hold two main implications. First and foremost, they point the way to further development of valuation theory incorporating the economics of value creation. Possible directions include the examination of more general investment dynamics, their economic determinants, the role of "other information," and their linkages to firm value. Second, they suggest that for empirical applications, non-linear rather than linear forms may be more appropriate, especially for firms that are expected to undergo significant changes in operating scale (expansion or restructuring / downsizing) as "capital follows profitability."

Appendix

This appendix presents a more general model of residual income dynamic based on fundamental economic events (cash investments and receipts) and conservative accounting policy. In this more general setting, investments follow economic profitability, but accounting is conservative in measuring the operations. The objective is to show that the predictions of the simple version of the model in Section 2 are not driven by unbiased accounting assumed there. Our modeling approach is similar to G. Zhang (2000) which extends Feltham and Ohlson (1996) with contingent investment decisions.

At the beginning of period *t*, i.e., date *t-1*, the firm has a given amount of asset stock, as_{t-1}. This stock produces cash receipts in period t, cr_t, as follows,

$$\tilde{cr}_t = \tilde{k}_t as_{t-1}, \tag{A1}$$

where $\tilde{k}_t$ is a stochastic productivity parameter.

The stock of existing assets diminishes at a rate 1-γ per period, but new investment can be made to replenish the stock. The asset stock at date *t*, after cash investment ci_t, equals:

$$as_t = \gamma as_{t-1} + ci_t. \tag{A2}$$

Based on (A1) and (A2), value created by each unit of assets in period *t*, net of capital charge, is:

$$q_t = [cr_t - (1-\gamma)as_{t-1}]/ as_{t-1} - (R-1) = k_t + \gamma - R, \tag{A3}$$

where R equals one plus the riskless rate of return per period. Thus, q_t measures the economic rent earned per unit of assets in period *t*; referred to as spread.[41]

We assume that over time spread evolves as follows:

Assumption i. $\tilde{q}_{t+1} = \theta q_t + \tilde{e}_{t+1}$, (A4)

where $0<\theta\leq 1$ is a persistence parameter and $\tilde{e}_{t+1}$ is a zero-mean disturbance term that cannot be predicted before date *t+1*.[42] Assumption i implies that productivity parameter evolves as:

$$\tilde{k}_{t+1} = \theta k_t + (1-\theta)(R-\gamma) + \tilde{e}_{t+1}. \quad \text{(A5)}$$

Based on (A1), (A2) and (A4), the net present value (NPV) of an incremental dollar of investment at date *t* can be derived as $\theta(k_t + \gamma - R)/(R - \theta\gamma)$; this is the marginal value of investment.

The "capital follows profitability" intuition is captured by the following investment behavior.

---

[41] It can be shown that spread as defined here reconciles with its definition in Section 2.1 under unbiased depreciation, and given the other modeling assumptions.

[42] The framework specified here is more general than that of Feltham and Ohlson (1996). To demonstrate this, consider the special case of θ=1. Then, (A1), (A2), (A3), and (A4) together imply the following cash flow dynamic, $c\tilde{r}_{t+1} = \gamma cr_t + k_t ci_t + as_t \tilde{e}_{t+1}$. This dynamic is consistent with that in Feltham and Ohlson (1996) except that here the marginal impact on cash receipts of cash investment (k_t) changes over time, just as the marginal impact of assets in place. In Feltham and Ohlson, cash receipts in period *t+1* are specified in terms of previous-period cash receipts and new investment, but their link to underlying assets available is not explicitly specified.

Assumption ii.

iia). If the marginal value of investment is positive ($\theta(k_t+\gamma-R)/(R-\theta\gamma)>0$), the firm invests to increase the asset stock, with

$$ci_t = (1-\gamma)\, as_{t-1} + \pi_1\, as_{t-1}\, [\theta(k_t+\gamma-R)/(R-\theta\gamma)]; \quad \text{(A6)}$$

iib). If the marginal value of investment is negative ($\theta(k_t+\gamma-R)/(R-\theta\gamma)<0$), the firm "divests" to reduce the asset stock, with

$$ci_t = (1-\gamma)\, as_{t-1} + \pi_2\, as_{t-1}\, [\theta(k_t+\gamma-R)/(R-\theta\gamma)], \quad \text{(A7)}$$

where π_1 (π_2) is a parameter that represents the firm's achievable investment (divestment) opportunity.[43]

Based on Assumptions i and ii, the expected period-ahead cash receipts are derived as:

$$\begin{aligned} E_t(cr_{t+1}) = {} & \frac{\pi_1\theta^2}{(R-\theta\gamma)as_{t-1}}\, cr_t^2 + [\frac{\pi_1\theta(1-2\theta)(R-\gamma)}{R-\theta\gamma} + \theta\,]cr_t \\ & + (1-\theta)(R-\gamma)[1-\frac{\pi_1\theta(R-\gamma)}{R-\theta\gamma}]as_{t-1}. \end{aligned} \quad \text{(A8)}$$

The period-ahead cash receipts depend on the amount of asset stock, consisting of cash investment and existing assets, and expected period-ahead spread. Current spread is informative about future spread, and therefore it serves as a signal to guide capital investment. Non-linearity

[43] As above, we do not explicitly impose an upper bound on divestment, as might apply say in liquidation, to concentrate our model and empirical analyses on more representative interior cases.

in (A8) arises because period-ahead cash receipts are a multiplicative product of period-ahead asset stock and spread, with both components dependent on current spread.

To derive the residual income dynamic, we impose three accounting rules: historical cost valuation, the clean surplus relation, and a conservative depreciation policy. With historical cost valuation, the book value of assets at the initial point (date 0), B_0, equals ci_0. The depreciation policy satisfies the clean surplus relation,

$$B_{t+1} = B_t - dep_{t+1} + ci_{t+1}. \tag{A9}$$

Furthermore, depreciation is assumed to be conservative (biased) and has the following form:

$$dep_t = (1\text{-}\delta)\, B_{t-1}, \text{ where } \delta \leq \gamma. \tag{A10}$$

It follows that accounting earnings in period t are:

$$X_t = cr_t - dep_t = k_t as_{t-1} - (1-\delta)B_{t-1}, \tag{A11}$$

the period t residual income is:

$$X_t^a = X_t - (R-1)B_{t-1} = k_t as_{t-1} - (R-\delta)B_{t-1}, \tag{A12}$$

and the expected period $t+1$ residual income is:

$$\begin{aligned} E_t[X_{t+1}^a] &= X_{t+1} - (R-1)B_t = k_{t+1}as_t - (R-\delta)B_t \\ &= [\theta k_t + (1-\theta)(k_t-\gamma)](\gamma as_{t-1} + ci_t) - (R-\delta)B_t. \end{aligned} \tag{A13}$$

Case a). The firm makes positive investment.

For the case of positive investment, substituting the expression for ci_t in (A6) into (A13) and simplifying, we obtain:

$$E_t[X^a_{t+1}] = \frac{\pi_1\theta^2}{(R-\theta\gamma)as_{t-1}}(X^a_t)^2 + [\theta + \frac{2\pi_1\theta^2(R-\delta)B_{t-1}}{(R-\theta\gamma)as_{t-1}} - \frac{2\pi_1\theta^2(R-\gamma)}{(R-\theta\gamma)} - \frac{\pi_1\theta(\gamma-\delta)}{R-\theta\gamma}]X^a_t + M \tag{A14}$$

where

$$M \equiv \frac{\pi_1\theta^2(R-\delta)^2B^2_{t-1}}{(R-\theta\gamma)as_{t-1}} - \{\frac{2\pi_1\theta^2(R-\gamma)(R-\delta)}{(R-\theta\gamma)} + \frac{\pi_1\theta(\gamma-\delta)(R-\delta)}{(R-\theta\gamma)} + (R-\delta)\delta - \theta(R-\delta)\}B_{t-1} + \{[(1-\theta)(R-\gamma)-(R-\delta)(1-\gamma)] - [(1-\theta)(R-\gamma)-(R-\delta)]\frac{\pi_1\theta(R-\gamma)}{R-\theta\gamma}\}as_{t-1}$$

is an expression that arises from conservative depreciation and is unrelated to period t residual income.

It can be shown that under unbiased depreciation (δ=γ), $B_{t-1} = as_{t-1}$ and $M=0$, in which case the residual income dynamic simplifies to:

$$\tilde{X}^a_{t+1} = \frac{\pi_1\theta^2}{(R-\theta\gamma)B_{t-1}}(X^a_t)^2 + \theta X^a_t + \tilde{\varepsilon}_{t+1}, \tag{A15}$$

which reconciles with dynamic (11) in the simple version of the model (Section 2).

The non-linearity in (A14), and in (A15), inherits from the non-linearity of the cash flow dynamic (A8). It arises because current profitability conveys information about future profitability, and this information guides investment decisions and hence affects future asset stock. Comparing (A14) and (A15) reveals that biased accounting policy affects the slope and the constant terms of the residual income dynamic, but it does not affect the non-linearity.[44]

Note that the linear information dynamics employed in Ohlson (1995) and Feltham and Ohlson (1995, 1996) can be viewed as special cases. If investment follows an exogenous process that does not respond to profitability, as assumed in those previous studies, the quadratic term in (A14) or (A15) vanishes and the dynamic reduces to an AR1 process.

To examine the behavior of the residual income dynamic, we differentiate (A14) with respect to X_t^a and then simplify the expression,

$$\frac{\partial E_t[X_{t+1}^a]}{\partial X_t^a} = \frac{2\pi_1\theta^2}{(R-\theta\gamma)as_{t-1}}[k_t - R + \gamma]as_{t-1} + \theta - \frac{\pi_1\theta}{R-\theta\gamma}(\gamma - \delta). \qquad \text{(A16)}$$

The spread persistence parameter (θ) causes the slope to be positive (the second term in A16); positive NPV investment (k_t-R+γ>0) further enhances the positive link, making the slope steeper (the first term in A16). On the other hand, conservative depreciation (γ>θ) creates an offsetting effect on the slope. To the extent that the effect of accounting conservatism is sufficiently small relative to the effect of underlying economic information conveyed by accounting data, a positive slope between period-ahead and current residual income will obtain, especially (from above) for firms with high profitability. Thus, it becomes an empirical question as to which effect dominates.

---

[44] See G. Zhang (2000) for a more detailed discussion of the impact of accounting conservatism on accounting measures such as book value of equity, earnings and book rate of return. While X. Zhang (2000) also addresses the role of accounting conservatism in valuation, he limits the analysis to asymptotic properties in the limiting case and does not consider properties with respect to contemporaneous variables.

The second-order derivative of (A14) with respect to X_t^a is:

$$\frac{\partial^2 E_t[X_{t+1}^a]}{\partial (X_t^a)^2} = \frac{2\pi_1\theta^2}{(R-\theta\gamma)as_{t-1}}. \tag{A17}$$

Thus, the period-ahead residual income is a convex function of current residual income regardless of the degree of accounting conservatism.

The firm's investment opportunity also affects the slope and convexity of the residual income dynamic. Differentiating (A16) and (A17) with respect to π_1 yields:

$$\frac{\partial^2 E_t[X_{t+1}^a]}{\partial X_t^a \partial \pi_1} = \frac{2\theta^2}{(R-\theta\gamma)}[k_t - R + \gamma] - \frac{\theta}{R-\theta\gamma}(\gamma - \delta) \tag{A18}$$

and

$$\frac{\partial^3 E_t[X_{t+1}^a]}{\partial (X_t^a)^2 \partial \pi_1} = \frac{2\theta^2}{(R-\theta\gamma)as_{t-1}} > 0. \tag{A19}$$

From (A18), the effect of investment opportunity on the slope depends on the firm's (economic) profitability (k_t-R+γ) and accounting conservatism (γ-δ); the former creates a positive effect while the latter a negative effect. Thus, for firms with sufficiently high (economic) profitability, the former effect is expected to dominate, which causes the slope to increase with investment opportunity. On the other hand, for firms that are marginally profitable, the effect of conservative depreciation may dominate. From (A19), the degree of convexity of the residual income dynamic always increases with investment opportunity.

Case b). The firm makes negative investment.

For divesting firms, the residual income dynamic has the same mathematical expression as (A14) except that parameter π_1 is replaced by parameter π_2, which represents the achievable divestment opportunity, or the extent to which a firm can remove capital from negative NPV activities.

The properties of the residual income dynamic are derived in the same way as for the positive investment case above. For clarity, the first-order derivative is rewritten as:

$$\frac{\partial E_t[X_{t+1}^a]}{\partial X_t^a} = \theta[1 - \frac{2(as_t - as_{t+1})}{as_{t-1}}] - \frac{\pi_1\theta}{R - \theta\gamma}(\gamma - \delta). \tag{A20}$$

As above, the slope of the residual income dynamic depends on both the extent of divestment and accounting conservatism. If the firm divests a small portion of its assets, the expression in the square brackets in (A20) is positive and is likely to outweigh the negative effect of accounting conservatism. On the other hand, if the firm divests a large portion of its assets, the expression in the square brackets can become sufficiently small or even negative, so that (A20) is negative. The second-order derivative has the same expression as (A17). Thus, as in the case of positive investment, the residual income dynamic is convex.

To examine the effect of divestment opportunity, we have:

$$\frac{\partial^2 E_t[X_{t+1}^a]}{\partial X_t^a \partial \pi_1} = \frac{2\theta^2}{(R - \theta\gamma)}[k_t - R + \gamma] - \frac{\theta}{R - \theta\gamma}(\gamma - \delta) < 0, \tag{A21}$$

and

$$\frac{\partial^3 E_t[X_{t+1}^a]}{\partial (X_t^a)^2 \partial \pi_1} = \frac{2\theta^2}{(R - \theta\gamma) as_{t-1}} > 0. \tag{A22}$$

Therefore, the slope decreases and convexity increases with divestment opportunity.

To summarize, this more general model of residual income dynamics generates the same set of testable hypotheses as in the simple version of the model in Section 2 except for a refinement to hypotheses H3a and H6a. The new version of H3a is: For firms with positive investment *and sufficiently high profitability*, the slope of the residual income dynamic increases with investment opportunity. Extending from residual income to unrecorded goodwill, we make the same modification to H6a. This modification is consistent with our empirical results as illustrated in Tables 5 and 8.

Acknowledgments

Helpful comments were provided by an anonymous reviewer, Peter Easton, Stephen Penman, participants at the *Review of Accounting Studies Conference on Financial Reporting and Equity Valuation*, and our colleagues and workshop participants at Hong Kong University of Science & Technology. Financial support was provided by Hong Kong Research Grants Council Direct Allocation Grants.

References

Anthony, Robert. (1973). "Accounting for the Cost of Equity." *Harvard Business Review* 51, 88-102.

Ball, Ray and Ross Watts. (1972). "Some Time Series Properties of Accounting Income." *Journal of Finance* (June), 663-681.

Barth, William, William Beaver, and Wayne Landsman. (1998). "Relative Valuation Roles of Equity Book Value and Net Income as a Function of Financial Health." *Journal of Accounting and Economics* 25 (February), 1-34.

Beaver, William. (1970). "The Time Series Behavior of Earnings." *Journal of Accounting Research*, Supplement 8, 62-89.

Beaver, William. (1999). "Comments on 'An Empirical Assessment of the Residual Income Valuation Model.'" *Journal of Accounting and Economics* 26, 35-42.

Beneish, Messod and Campbell Harvey. (1998). "Measurement Error and Nonlinearity in the Earnings-Return Relation." *Review of Quantitative Finance and Accounting* 11, 219-247.

Berger, P., E. Ofek, and I. Swary. (1996). "Investor Valuation of the Abandonments Option." *Journal of Financial Economics* 42 (October), 257-287.

Biddle, Gary, Robert Bowen, and James Wallace. (1997). "Does EVA Beat Earnings? Evidence on Associations with Stock Returns and Firm Values." *Journal of Accounting and Economics* 24, 301-336.

Biddle, Gary, Robert Bowen, and James Wallace. (1999). "Evidence on EVA®." *Journal of Applied Corporate Finance* 12, 8-18.

Biddle, Gary, Robert Bowen, and James Wallace. (2000). "EVA® and Financial Analysis." Working paper, Hong Kong University of Science and Technology.

Brooks, L. and D. Buckmaster. (1976). "Further Evidence on the Time Series Properties of Accounting Income." *Journal of Finance* 31, 1359-1373.

Burgstahler, David and Ilia Dichev. (1997). "Earnings, Adaptation, and Equity Value." *The Accounting Review* 73, 187-215.

Canning, John. (1929). *The Economics of Accountancy.* New York: Ronald Press.

Chen, Peter and Guochang Zhang. (2000). "Operating Decisions and the Incremental Value Impact of Segment Accounting Information: A Theoretical Analysis." Working paper, Hong Kong University of Science and Technology.

Collins, Daniel, Edward Maydew, and Ira Weiss. (1997). "Changes in the Value-Relevance of Earnings and Book Values over the Past Forty Years." *Journal of Accounting and Economics* 24, 39-67.

Collins, Daniel, Morton Pincus, and Hong Xie. (1999). "Equity Valuation and Negative Earnings: the Role of Book Value of Equity. *The Accounting Review* 74, 29-61.

Das, Samnath and Baruch Lev. (1994). "Nonlinearity in the Return-Earnings Relation: Tests of Alternative Specifications and Explanations." *Contemporary Accounting Research* 11, 353-379.

Dechow, Patricia, Amy Hutton, and Richard Sloan. (1999). "An Empirical Assessment of the Residual Income Valuation Model." *Journal of Accounting and Economics* 26, 1-34.

Easton, Peter. (1999). "Security Returns and the Value Relevance of Accounting Data." *Accounting Horizons* 13 (December), 399-412.

Easton, Peter and Trevor Harris. (1991). "Earnings as an Explanatory Variable for Returns." *Journal of Accounting Research* 29, 19-36.

Edey, Harold. (1957). "Business Valuation, Goodwill and the Super-Profit Method." *Accountancy*, January/February.

Edwards, Edgar and Philip Bell. (1961). *The Theory and Measurement of Business Income.* Berkeley: University of California Press.

Fama, Eugene and Kenneth French. (2000). "Forecasting Profitability and Earnings." *Journal of Business* 73, 161-175.

Feltham, Gerald and James Ohlson. (1995). "Valuation and Clean Surplus Accounting for Operating and Financial Activities." *Contemporary Accounting Research* 11, 689-731.

Feltham, Gerald and James Ohlson. (1996). "Uncertainly Resolution and the Theory of Depreciation Measurement." *Journal of Accounting Research* 34, 209-234.

Feltham, Gerald and James Ohlson. (1999). "Residual Income Valuation with Risk and Stochastic Interest Rates." *The Accounting Review* 74, 165-183.

Freeman, Robert, James Ohlson, and Stephen Penman. (1982). "Book Rate-of-Return and Prediction of Earnings Changes: An Empirical Investigation." *Journal of Accounting Research* 2, 639-653.

Freeman, Robert and Senyo Tse. (1992). "A Nonlinear Model of Security Price Response to Accounting Earnings." *Journal of Accounting Research* 30, 185-209.

Hamilton, Robert. (1777). *An Introduction to Merchandize.* Edinburgh.

Hand, John and Wayne Landsman. (1999). "The Pricing of Dividends in Equity Valuation." Working Paper, University of North Carolina at Chapel Hill.

Hayne, Carla. (1999). "The Information Content of Losses." *Journal of Accounting and Economics* 20 (September), 125-153.

Kay, John. (1976). "Accountants, Too, Could be Happy in a Golden Age: The Accountant's Rate of Profit and the Internal Rate of Return." *Oxford Economic Papers* 28, 447-460.

Lee, Charles. (1999). "Accounting-Based Valuation: Impact on Business Practices and Research." *Accounting Horizons* 13, 413-426.

Lo, Kin and Thomas Lys. (1999). "The Ohlson Model: Contribution to Valuation Theory, Limitations, and Empirical Applications." Working Paper, University of British Columbia. Forthcoming, *Journal of Accounting, Auditing and Finance*.

Marshall, Alfred. (1890). *Principles of Economics.* New York: MacMillan Press.

Miller, Morton and France Modigliani. (1961), "Dividend Policy, Growth and the Valuation of Shares," *Journal of Business* 34, 411-433.

Myers, James. (1999). "Implementing Residual Income Valuation with Linear Information Dynamics. *The Accounting Review* 74, 1-23.

Nissim, Donald and Stephen Penman. (1999). "Ratio Analysis and Equity Valuation." Working paper, Columbia University.

Ohlson, James. (1995). "Earnings, Book Value, and Dividends in Equity Valuation." *Contemporary Accounting Research* 11, 661-687.

Ohlson, James. (2000). "Positive (Zero) NPV Projects and the Behavior of Residual Earnings." Working Paper, New York University.

Peasnell, Kenneth. (1981). "On Capital Budgeting and Income Measurement." *Abacus* 17, 52-67.

Peasnell, Kenneth. (1982). "Some Formal Connections between Economic Values and Yields and Accounting Numbers." *Journal of Business, Finance and Accounting* 9, 361-381.

Penman, Stephen. (1997). "A Synthesis of Equity Valuation Techniques and the Terminal Value Calculation for the Dividend Discount Model." *Review of Accounting Studies* 2, 303-323.

Penman, Stephen. (2001). *Financial Statement Analysis and Security Valuation*. New York: McGraw-Hill Book Company.

Preinreich, G. (1938). "Annual Survey of Economic Theory: The Theory of Depreciation." *Econometrica* 6, 219-241.

Solomons, David. (1965). *Division Performance: Measurement and Control.* New York: Financial Executives Research Foundation.

Stewart, G. Bennett III. (1991). *The Quest For Value.* New York: Harper Business.

Subramanyam, K.R. (1996). "Uncertain Precision and Price Reactions to Information." *The Accounting Review* 71, 207-220.

Subramanyam, K.R. and John Wild. (1993). "The Going Concern Assumption and the Informativeness of Earnings." *Contemporary Accounting Research* 13 (Spring), 251-273.

Verrecchia, Robert. (1998). "Discussion of Accrual Accounting and Equity Valuation." *Journal of Accounting Research*, Supplement 36, 113-115.

Watts, Ross and Richard Leftwich. (1977). "The Time-Series Properties of Annual Accounting Earnings." *Journal of Accounting Research* 15, 253-279.

Yee, Kenton. (2000). "Opportunities Knocking: Residual Income Valuation of the Adaptive Firm." Forthcoming, *Journal of Accounting, Auditing, and Finance*.

Zhang, Guochang. (2000). "Accounting Information, Capital Investment Decisions, and Equity Valuation: Theory and Empirical Implications." Forthcoming, *Journal of Accounting Research*.

Zhang, Xiao-Jun. (2000). "Conservative Accounting and Equity Valuation." *Journal of Accounting and Economics* 29 (February), 125-149.

Table 1. Variable definitions and data sources.

Variable	Notation	Definition and Compustat data items used in calculation
Book Equity	B_t	Book value of common equity (#60) at the end of year t.
Operating Assets	OA_t	Book value of operating assets, defined as total assets (#6) minus short-term investments (#193) at the end of year t.
Net Income	X_t	Net income (#18) in year t.
Market Value of Common Equity	V_t	Market value of common equity, defined as the number of common shares outstanding (#25) multiplied by share price (#199) at the end of year t.
Scaled Residual Income in Year *t+1*	x^a_{t+1}	$= (X_{t+1} - rB_t)/OA_{t-1}$, where *r* is cost of capital (=12%).
Scaled Residual Income in Year *t*	x^a_t	$= (X_t - rB_{t-1})/OA_{t-1}$, where r is cost of capital (=12%).
Capital Investment in Year *t+1*	i_{t+1}	$= (OA_{t+1} - OA_t)/OA_t$.
Scaled Unrecorded Goodwill	g_t	$= (V_t - B_t)/OA_{t-1}$.

Table 2. Descriptive statistics.

Panel A: Descriptive statistics for the pooled sample[a]

Variable	Notation[b]	Mean	Median	Std dev	Min.	1st Quartile	3rd Quartile	Max.
Scaled Residual Income in Year *t+1*	x^a_{t+1}	-0.04	-0.01	0.12	-0.78	-0.06	0.02	0.30
Scaled Residual Income in Year *t*	x^a_t	-0.03	-0.00	0.11	-0.62	-0.05	0.02	0.25
Capital Investment in Year *t+1*	i_{t+1}	0.10	0.06	0.29	-0.68	-0.03	0.18	1.97
Scaled Unrecorded Goodwill at Year *t*	g_t	0.61	0.19	1.19	-3.22	-0.01	0.75	9.99

[a] The sample consists of 83,826 firm-year observations for the period from 1981 to 1998.
[b] See Table 1 for definitions of variables.

Table 2. Continued.

Panel B: Annual distributional statistics

Year (t+1)	Obs.	x^a_{t+1} Mean	x^a_{t+1} Med	x^a_{t+1} Std dev	x^a_t Mean	x^a_t Med	x^a_t Std dev	i_{t+1} Mean	i_{t+1} Med	i_{t+1} Std dev	g_t Mean	g_t Med	g_t Std dev
81	3,867	0.00	0.00	0.09	0.01	0.01	0.07	0.13	0.09	0.26	0.42	0.01	1.15
82	4,186	-0.03	-0.01	0.10	0.00	0.00	0.08	0.07	0.04	0.24	0.36	0.00	1.08
83	4,241	-0.02	0.00	0.10	-0.02	-0.01	0.09	0.12	0.07	0.28	0.40	0.03	1.07
84	4,251	-0.03	0.00	0.12	-0.02	0.00	0.10	0.11	0.08	0.26	0.66	0.20	1.27
85	4,366	-0.04	-0.01	0.13	-0.02	0.00	0.11	0.09	0.06	0.29	0.47	0.13	0.97
86	4,268	-0.05	-0.01	0.13	-0.03	-0.01	0.11	0.10	0.06	0.31	0.58	0.19	1.12
87	4,331	-0.04	-0.01	0.12	-0.04	-0.01	0.11	0.10	0.06	0.29	0.62	0.24	1.13
88	4,539	-0.04	-0.01	0.13	-0.03	-0.01	0.11	0.09	0.06	0.28	0.52	0.16	1.08
89	4,445	-0.05	-0.01	0.13	-0.03	-0.01	0.11	0.08	0.05	0.28	0.46	0.17	0.94
90	4,429	-0.05	-0.02	0.13	-0.04	-0.01	0.11	0.05	0.03	0.25	0.53	0.18	1.09
91	4,411	-0.05	-0.02	0.12	-0.04	-0.01	0.11	0.04	0.02	0.25	0.39	0.07	0.99
92	4,426	-0.04	-0.01	0.11	-0.04	-0.01	0.10	0.07	0.04	0.25	0.60	0.16	1.27
93	4,657	-0.04	-0.01	0.12	-0.03	-0.01	0.10	0.11	0.06	0.29	0.68	0.24	1.24
94	4,844	-0.04	-0.01	0.13	-0.03	-0.01	0.11	0.12	0.08	0.28	0.83	0.37	1.32
95	5,528	-0.03	0.00	0.12	-0.02	0.00	0.11	0.13	0.08	0.29	0.64	0.23	1.11
96	5,820	-0.03	0.00	0.13	-0.02	0.00	0.10	0.14	0.08	0.32	0.75	0.27	1.29
97	5,933	-0.04	0.00	0.14	-0.03	0.00	0.11	0.15	0.08	0.33	0.85	0.35	1.40
98	5,284	-0.05	-0.01	0.14	-0.03	0.00	0.12	0.14	0.08	0.33	0.93	0.44	1.39

Table 3. The relation between current residual income and future capital investment[a].

Year (t+1)	Obs.	β_0	$t_{\beta 0}$	β_1	$t_{\beta 1}$	Adj. R^2
81	3,867	0.12	31.03	1.32	24.29	0.13
82	4,186	0.07	21.12	1.07	25.00	0.13
83	4,241	0.14	34.34	1.07	23.99	0.12
84	4,251	0.13	34.10	0.99	25.95	0.14
85	4,366	0.12	29.00	1.08	29.14	0.16
86	4,268	0.14	29.51	1.08	26.67	0.14
87	4,331	0.14	31.75	0.95	25.49	0.13
88	4,539	0.12	30.69	0.94	26.85	0.14
89	4,445	0.11	26.55	0.88	25.51	0.13
90	4,429	0.08	22.50	0.88	27.74	0.15
91	4,411	0.07	19.39	0.82	25.49	0.13
92	4,426	0.09	25.12	0.85	23.58	0.11
93	4,657	0.14	33.80	0.99	26.40	0.13
94	4,844	0.15	37.87	0.93	27.90	0.14
95	5,528	0.15	41.23	0.93	27.05	0.12
96	5,820	0.16	39.62	0.84	21.76	0.08
97	5,933	0.17	41.57	1.02	28.05	0.12
98	5,284	0.17	39.03	0.98	27.44	0.12
81-98 pooled	83,826	0.13	135.79	0.96	109.00	0.12

[a] This table reports results from the following linear regression: $i_{t+1} = \beta_0 + \beta_1 x_t^a + \mu_{t+1}$.

Table 4. Linear versus piece-wise linear regression of future residual income on current residual income.

Panel A: Linear regression of residual income of year *t+1* on residual income of year *t*[a]

Year (t+1)	Obs.	α	t_α	ω	t_ω	Adj. R^2
81	3,867	-0.00	-3.85	0.80	52.33	0.41
82	4,186	-0.03	-19.42	0.71	42.79	0.30
83	4,241	-0.00	-4.69	0.73	52.12	0.39
84	4,251	-0.01	-9.11	0.75	52.78	0.39
85	4,366	-0.03	-17.65	0.75	54.14	0.40
86	4,268	-0.02	-14.64	0.68	47.66	0.35
87	4,331	-0.02	-10.10	0.64	48.22	0.35
88	4,539	-0.01	-9.55	0.75	56.11	0.41
89	4,445	-0.02	-14.14	0.69	50.90	0.37
90	4,429	-0.02	-14.60	0.70	52.20	0.38
91	4,411	-0.02	-16.41	0.64	46.51	0.33
92	4,426	-0.02	-11.96	0.62	42.61	0.29
93	4,657	-0.02	-10.40	0.71	51.69	0.36
94	4,844	-0.01	-8.020	0.76	58.46	0.41
95	5,528	-0.02	-11.32	0.69	56.52	0.37
96	5,820	-0.01	-10.55	0.73	55.16	0.34
97	5,933	-0.02	-12.57	0.79	63.29	0.40
98	5,284	-0.03	-16.15	0.72	56.58	0.38
81-98 pooled	83,826	-0.02	-50.40	0.71	223.09	0.37

[a] This panel reports results from the following linear regression: $x^a_{t+1} = \alpha + \omega x^a_t + \varepsilon_{t+1}$.

Table 4. Continued.

Panel B: Piece-wise linear regression of residual income of year *t+1* on residual income of year *t*[b]

Year (t+1)	x_t^a cutoff1	x_t^a cutoff2	α_0	$t_{\alpha 0}$	$\alpha_0+\alpha_1$	$t_{\alpha 1}$	$\alpha_0+\alpha_2$	$t_{\alpha 2}$	ω_0	$t_{\omega 0}$	$\omega_0+\omega_1$	$t_{\omega 1}$	$\omega_0+\omega_2$	$t_{\omega 2}$	Adj.R^2
81	-0.00	0.03	-0.01	-5.66	-0.00	3.26	-0.01	0.40	0.69	26.39	0.66	-0.15	0.93	4.56	0.42
82	-0.01	0.02	-0.04	-12.66	-0.01	5.33	0.00	6.91	0.66	23.28	0.75	0.40	0.40	-4.20	0.31
83	-0.03	0.01	-0.02	-6.64	0.00	4.53	0.00	5.20	0.63	27.85	1.04	2.30*	0.66	0.40*	0.39
84	-0.02	0.02	-0.02	-6.58	0.00	4.08	0.00	4.40	0.71	30.21	1.13	1.95*	0.55	-2.84**	0.40
85	-0.03	0.17	-0.06	-16.47	-0.02	9.02	0.00	10.4	0.59	27.80	1.58	4.44**	0.50	-1.40**	0.42
86	-0.04	0.01	-0.06	-15.25	-0.01	8.98	-0.01	8.36	0.49	21.74	1.29	4.00**	0.72	3.36**	0.36
87	-0.04	0.01	-0.04	-8.99	-0.01	4.66	0.00	5.76	0.54	24.56	1.27	4.12**	0.61	1.18	0.36
88	-0.04	0.01	-0.04	-9.58	-0.00	6.47	0.00	6.03	0.63	28.04	1.55	5.01**	0.71	1.43**	0.42
89	-0.04	0.01	-0.05	-12.39	-0.02	6.35	-0.01	5.84	0.54	24.46	1.32	4.06**	0.74	3.70**	0.38
90	-0.04	0.01	-0.05	-12.21	-0.02	6.80	-0.02	5.41	0.56	25.53	1.35	4.15**	0.77	3.57**	0.39
91	-0.04	0.01	-0.06	-14.06	-0.01	8.13	-0.02	6.97	0.48	21.47	1.38	4.54**	0.66	3.06**	0.34
92	-0.04	0.00	-0.04	-10.52	-0.01	6.62	-0.01	5.38	0.48	19.43	1.14	3.53**	0.63	2.50**	0.30
93	-0.04	0.01	-0.03	-8.33	-0.01	4.27	-0.01	4.32	0.62	26.85	1.09	2.41**	0.70	1.43**	0.37
94	-0.03	0.01	-0.01	-3.39	-0.01	0.98	-0.00	1.58	0.76	36.50	1.04	1.90**	0.63	-2.35**	0.41
95	-0.02	0.01	-0.03	-10.11	-0.01	6.44	-0.01	5.39	0.62	33.29	1.24	2.20**	0.58	-0.68**	0.37
96	-0.02	0.01	-0.02	-5.63	-0.00	2.71	-0.01	2.04	0.72	33.75	1.09	1.40	0.61	-1.93	0.34
97	-0.02	0.01	-0.03	-8.62	-0.00	4.93	-0.01	3.50	0.74	39.03	1.15	1.48	0.67	-1.45	0.41
98	-0.02	0.01	-0.05	-12.54	-0.01	7.52	-0.01	6.05	0.64	32.58	1.39	2.55**	0.56	-1.45**	0.38
81-98	-0.03	0.01	-0.04	-42.28	-0.01	23.9	-0.01	22.3	0.62	121.4	1.25	11.9**	0.64	1.30**	0.38

[b] This panel reports results from the following piece-wise linear regression: $x_{t+1}^a = \alpha_0 + \alpha_1 M + \alpha_2 H + \omega_0 x_t^a + \omega_1 M x_t^a + \omega_2 H x_t^a + \varepsilon_{t+1}$.

Cutoff1 and cutoff2 are within-year cutoff values of x_t^a for the middle and high residual income ranges respectively.

M and *H* are indicator variables for the middle and high residual income ranges defined by x_t^a cutoffs.

** and * denote significance at the 1% and 5% level, respectively, for the change of slope between adjacent ranges.

Table 5. Linear versus piece-wise linear regression of future residual income on current residual income by partitions of positive capital investment in year *t+1*

Panel A: Descriptive statistics of partitions

Partitions by investment		Obs.	i_{t+1}			x^a_{t+1}			x^a_t			g_t		
			Mean	Med	Stddev	Mean	Med	Stddev	Mean	Med	Stddev	Mean	Med	Stddev
Lowest	i1	14,236	0.03	0.03	0.02	-0.02	-0.01	0.06	-0.01	-0.00	0.06	0.36	0.12	0.81
	i2	14,236	0.09	0.09	0.02	-0.00	0.00	0.06	0.00	0.00	0.06	0.51	0.19	0.94
	i3	14,237	0.19	0.18	0.04	0.01	0.01	0.07	0.01	0.01	0.07	0.79	0.34	1.25
Highest	i4	14,237	0.56	0.44	0.33	0.02	0.02	0.10	0.02	0.02	0.08	1.19	0.61	1.62

Panel B: Linear regression of residual income of year *t+1* on residual income of year *t* by partitions[a]

Partitions by investment		α	t_α	ω	t_ω	Adj. R^2
Lowest	i1	-0.01	-25.4	0.48	67.4	0.24
	i2	-0.00	-4.3	0.52	70.8	0.26
	i3	0.01	12.1	0.58	75.6	0.29
Highest	i4	0.01	18.1	0.66	75.8	0.29

[a] This panel reports results from the following linear regression: $x^a_{t+1} = \alpha + \omega x^a_t + \varepsilon_{t+1}$.

Table 5. Continued.

Panel C: Piece-wise linear regression of residual income of year t+1 on residual income of year t by partitions[b]

Partitions by	α_0	$t_{\alpha 0}$	$\alpha_0+\alpha_1$	$t_{\alpha 1}$	$\alpha_0+\alpha_2$	$t_{\alpha 2}$	ω_0	$t_{\omega 0}$	$\omega_0+\omega_1$	$t_{\omega 1}$	$\omega_0+\omega_2$	$t_{\omega 2}$	Adj.R^2
Lowest i1	-0.02	-23.5	-0.00	12.9	0.00	12.8	0.39	32.7	0.61	2.1*	0.27	-4.3**	0.26
i2	-0.02	-13.9	-0.00	9.3	0.01	13.1	0.37	25.9	0.74	3.8**	0.43	2.5**	0.27
i3	-0.01	-8.8	-0.00	6.5	0.01	10.2	0.31	18.7	0.87	4.6**	0.63	13.0*	0.30
Highest i4	-0.01	-3.5	0.00	3.0	0.02	6.7	0.41	21.6	1.01	3.3**	0.75	12.1	0.30

[b] This panel reports results from the following piece-wise linear regression by partitions: $x_{t+1}^a = \alpha_0 + \alpha_1 M + \alpha_2 H + \omega_0 x_t^a + \omega_1 M x_t^a + \omega_2 H x_t^a + \varepsilon_{t+1}$.
M and *H* are indicator variables for the middle and high ranges defined by x_t^a cutoffs in the sample.
**and* denote significance at the 1% and 5% level, respectively, for the change of slope between adjacent ranges.

Panel D: Incremental slope coefficients in the piece-wise linear regression

Partitions by investment	ω_1	ω_2
Lowest i1	0.21	-0.12
i2	0.37	0.06
i3	0.56	0.32
Highest i4	0.60	0.34

Table 6. Linear versus piece-wise linear regression of future residual income on current residual income by partitions of negative capital investment in year *t+1*

Panel A: Descriptive statistics of partitions

Partitions by divestment	Obs.	i_{t+1}			x^a_{t+1}			x^a_t			g_t		
		Mean	Med	Stddev	Mean	Med	Stddev	Mean	Med	Stddev	Mean	Med	Stddev
Lowest i1	6,720	-0.02	-0.02	0.01	-0.05	-0.03	0.09	-0.04	-0.02	0.09	0.34	0.08	0.90
i2	6,720	-0.06	-0.06	0.02	-0.08	-0.05	0.11	-0.06	-0.03	0.11	0.36	0.06	0.99
i3	6,720	-0.14	-0.13	0.03	-0.13	-0.10	0.14	-0.10	-0.06	0.13	0.37	0.05	1.03
Highest i4	6,720	-0.33	-0.30	0.12	-0.23	-0.20	0.19	-0.16	-0.12	0.16	0.50	0.09	1.21

Panel B: Linear regression of residual income of year *t+1* on residual income of year *t* by partitions[a]

Partitions by divestment	α	t_α	ω	t_ω	Adj. R^2
Lowest i1	-0.03	-30.2	0.48	46.0	0.24
i2	-0.05	-36.7	0.46	40.1	0.19
i3	-0.09	-44.3	0.45	37.8	0.18
Highest i4	-0.16	-52.3	0.44	33.8	0.14

[a] This panel reports results from the following linear regression: $x^a_{t+1} = \alpha + \omega x^a_t + \varepsilon_{t+1}$.

Table 6. Continued.

Panel C: Piece-wise linear regression of residual income of year t+1 on residual income of year t by partitions[b]

Partitions by	α_0	$t_{\alpha 0}$	$\alpha_0+\alpha_1$	$t_{\alpha 1}$	$\alpha_0+\alpha_2$	$t_{\alpha 2}$	ω_0	$t_{\omega 0}$	$\omega_0+\omega_1$	$t_{\omega 1}$	$\omega_0+\omega_2$	$t_{\omega 2}$	Adj.R^2
Lowest i1	-0.04	-7.4	-0.03	1.0	-0.02	3.2	0.48	22.5	0.60	1.6	-0.03	-10.3**	0.26
i2	-0.06	-11.9	-0.04	2.8	-0.03	4.5	0.45	20.7	0.68	2.5**	-0.39	-12.8**	0.22
i3	-0.09	-17.0	-0.07	2.8	-0.06	5.0	0.44	20.8	0.78	2.9**	-0.64	-12.5**	0.20
Highest i4	-0.18	-28.3	-0.13	4.4	-0.12	7.3	0.37	18.0	0.83	2.6**	-0.89	-9.3**	0.16

[b] This panel reports results from the following piece-wise linear regression by partitions: $x^a_{t+1} = \alpha_0 + \alpha_1 M + \alpha_2 H + \omega_0 x^a_t + \omega_1 M x^a_t + \omega_2 H x^a_t + \varepsilon_{t+1}$.
M and *H* are indicator variables for the middle and high ranges defined by x^a_t cutoffs in the sample.
**and* denote significance at the 1% and 5% level, respectively, for the change of slope between adjacent ranges.

Panel D: Incremental slope coefficients in the piece-wise linear regression

Partitions by divestment	ω_1	ω_2
Lowest i1	0.12	-0.51
i2	0.23	-0.84
i3	0.35	-1.08
Highest i4	0.45	-1.25

Table 7. Linear versus piece-wise linear regression of unrecorded goodwill on residual income.

Panel A: Linear regression of unrecorded goodwill on residual income[a]

Year (t+1)	Obs.	φ	t_{φ}	ϕ	t_{ϕ}	Adj. R^2
81	3,867	0.39	21.73	4.34	17.33	0.07
82	4,186	0.36	21.83	2.48	11.91	0.03
83	4,241	0.43	25.76	1.55	8.61	0.02
84	4,251	0.70	35.61	2.09	10.60	0.03
85	4,366	0.49	32.44	0.81	5.90	0.00
86	4,268	0.62	35.00	1.24	7.92	0.01
87	4,331	0.67	37.49	1.30	8.42	0.02
88	4,539	0.55	32.97	0.94	6.40	0.01
89	4,445	0.49	33.90	0.95	7.73	0.01
90	4,429	0.57	33.50	1.09	7.49	0.01
91	4,411	0.45	29.31	1.67	12.26	0.03
92	4,426	0.72	37.31	3.48	18.95	0.07
93	4,657	0.76	41.24	2.49	14.71	0.04
94	4,844	0.88	45.58	1.69	10.11	0.02
95	5,528	0.68	45.28	1.90	13.76	0.03
96	5,820	0.80	47.05	2.28	14.03	0.03
97	5,933	0.88	47.99	1.58	9.68	0.02
98	5,284	0.98	50.33	1.66	10.38	0.02
81-98 pooled	83,826	0.65	155.84	1.67	43.20	0.02

[a] This panel reports results from the following linear regression: $g_t = \varphi + \phi x_t^a + \xi_t$.

Table 7. Continued.

Panel B: Piece-wise linear regression of unrecorded goodwill on residual income[b]

Year (t+1)	x_t^a cutoff1	x_t^a cutoff2	φ_0	$t_{\varphi 0}$	$\varphi_0+\varphi_1$	$t_{\varphi 1}$	$\varphi_0+\varphi_2$	$t_{\varphi 2}$	ϕ_0	$t_{\phi 0}$	$\phi_0+\phi_1$	$t_{\phi 1}$	$\phi_0+\phi_2$	$t_{\phi 2}$	Adj.R^2
81	-0.00	0.03	-0.07	-2.28	0.05	0.02	-0.32	-3.90	-3.85	-10.30	5.40	2.98**	19.10	30.00**	0.30
82	-0.01	0.02	-0.14	-4.25	0.03	3.91	-0.31	-3.06	-4.41	-14.37	5.60	3.37**	16.58	32.47**	0.28
83	-0.03	0.01	-0.06	-1.40	0.15	4.18	-0.01	-0.79	-2.78	-10.59	2.63	2.04*	16.51	28.50**	0.22
84	-0.02	0.02	0.15	3.45	0.24	1.78	0.12	-0.49	-2.97	-1037	0.42	1.27	18.01	29.27**	0.26
85	-0.03	0.17	0.04	1.24	0.18	3.47	0.02	-0.37	-2.55	-13.67	1.50	2.06*	14.15	30.85**	0.25
86	-0.04	0.01	0.09	2.20	0.28	3.71	0.14	1.10	-2.42	-10.99	0.55	1.58	17.30	30.17**	0.25
87	-0.04	0.01	0.12	2.78	0.29	3.75	0.29	2.97	-2.42	-10.93	0.44	1.57	16.30	29.32**	0.24
88	-0.04	0.01	-0.02	-0.49	0.22	4.56	0.16	3.38	-2.98	-13.70	0.60	1.70	14.91	30.17**	0.25
89	-0.04	0.01	0.03	0.88	0.17	3.33	0.15	2.55	-2.02	-11.65	-1.60	0.20	12.53	28.75**	0.23
90	-0.04	0.01	-0.01	-0.34	0.22	4.60	0.09	1.95	-2.84	-13.46	0.35	1.85	17.00	34.60**	0.28
91	-0.04	0.01	-0.06	-0.50	-0.01	8.22	0.02	6.96	-2.09	-10.58	0.70	0.74	14.00	31.58**	0.27
92	-0.04	0.00	0.03	0.58	0.33	3.67	0.22	3.35	-1.99	-7.57	0.57	1.24	22.42	38.21**	0.36
93	-0.04	0.01	-0.08	-1.87	0.30	6.93	0.37	7.97	-3.40	-14.42	-1.39	1.05	18.60	37.04**	0.34
94	-0.03	0.01	0.12	3.03	0.39	5.99	0.47	5.39	-3.46	-15.76	1.60	2.35*	18.05	34.29**	0.31
95	-0.02	0.01	0.22	7.07	0.20	-0.30	0.28	1.71	-1.95	-11.03	-3.01	-0.41	16.05	32.03**	0.32
96	-0.02	0.01	0.29	7.65	0.27	-0.63	0.35	1.11	-2.69	-10.09	-3.10	-0.32	17.60	35.43**	0.28
97	-0.02	0.01	0.27	6.73	0.36	1.63	0.49	3.16	-2.98	-13.69	-1.86	0.35	17.81	33.17**	0.26
98	-0.02	0.01	0.49	11.36	0.40	-1.55	0.62	2.26	-2.20	-9.53	3.42	1.68	16.78	29.26**	0.26
81-98	-0.03	0.01	0.09	10.01	0.24	13.00	0.17	6.57	-2.64	-49.1	0.46	5.35**	17.60	134.8**	0.26

[b] This panel reports results from the piece-wise linear regression: $g_t = \varphi_0 + \varphi_1 M + \varphi_2 H + \phi_0 x_t^a + \phi_1 M x_t^a + \phi_2 H x_t^a + \xi_t$.

Cutoff1 and cutoff2 are within-year cutoff values of x_t^a for the middle and high residual income ranges respectively.

M and *H* are indicator variables for the middle and high ranges defined by x_t^a cutoffs.

** and * denote significance at the 1% and 5% level, respectively, for the change of slope between adjacent ranges.

Table 8. Linear versus piece-wise linear regression of unrecorded goodwill on residual income by partitions of positive capital investment in year *t+1*.[a]

Panel A: Linear regression of unrecorded goodwill on residual income by partitions[a]

Partitions by investment	φ	t_{φ}	ϕ	t_{ϕ}	Adj. R^2
Lowest i1	0.39	58.4	2.73	25.9	0.05
i2	0.49	66.5	5.39	44.4	0.12
i3	0.69	71.3	7.65	53.7	0.17
Highest i4	1.09	83.2	6.37	40.5	0.10

[a] This panel reports the following linear versus piece-wise linear regressions by partitions: $g_t = \varphi + \phi x_t^a + \xi_t$.

Table 8. Continued.

Panel B: Piece-wise linear regression of unrecorded goodwill on residual income by partitions[b]

Partitions by investment	φ_0	$t_{\varphi 0}$	$\varphi_0+\varphi_1$	$t_{\varphi 1}$	$\varphi_0+\varphi_2$	$t_{\varphi 2}$	ϕ_0	$t_{\phi 0}$	$\phi_0+\phi_1$	$t_{\phi 1}$	$\phi_0+\phi_2$	$t_{\phi 2}$	Adj.R^2
Lowest i1	0.06	4.5	0.15	5.4	0.19	4.9	-2.69	-16.8	7.81	7.9**	12.52	41.4**	0.23
i2	0.16	9.8	0.17	0.4	0.16	-0.2	-2.23	-10.5	11.58	9.5**	14.88	47.7**	0.30
i3	0.29	12.4	0.25	-1.2	0.23	-1.6	-2.42	-8.6	12.17	7.1**	17.17	47.1**	0.33
Highest i4	0.47	14.7	0.43	-0.9	0.44	-0.6	-4.60	-14.7	13.74	6.0**	17.43	46.8	0.27

[b] This panel reports results from the following piece-wise linear regression by partitions: $g_t = \varphi_0 + \varphi_1 M + \varphi_2 H + \phi_0 x_t^a + \phi_1 M x_t^a + \phi_2 H x_t^a + \xi_t$.

M and *H* are indicator variables for the middle and high ranges defined by x_t^a cutoffs in the sample.

**and* denote significance at the 1% and 5% level, respectively, for the change of slope between adjacent ranges.

Panel C: Incremental slope coefficients in the piece-wise linear regression

Partitions by investment	ϕ_1	ϕ_2
Lowest i1	10.50	15.21
i2	13.81	17.11
i3	14.59	19.59
Highest i4	18.34	22.03

Table 9. Linear versus piece-wise linear regression of unrecorded goodwill on residual income by partitions of negative capital investment in year *t+1*.[a]

Panel A: Linear regression of unrecorded goodwill on residual income by partitions[a]

Partitions by divestment	φ	t_{φ}	ϕ	t_{ϕ}	Adj. R^2
Lowest i1	0.30	24.9	-0.87	-7.2	0.01
i2	0.27	19.2	-1.36	-12.2	0.02
i3	0.23	14.8	-1.44	-15.2	0.03
Highest i4	0.20	10.0	-1.88	-21.5	0.06

[a] This panel reports the following linear versus piece-wise linear regressions by partitions: $g_t = \varphi + \phi x_t^a + \xi_t$.

Table 9. Continued.

Panel B: Piece-wise linear regression of unrecorded goodwill on residual income by partitions[b]

Partitions by divestment	φ_0	$t_{\varphi 0}$	$\varphi_0+\varphi_1$	$t_{\varphi 1}$	$\varphi_0+\varphi_2$	$t_{\varphi 2}$	ϕ_0	$t_{\phi 0}$	$\phi_0+\phi_1$	$t_{\phi 1}$	$\phi_0+\phi_2$	$t_{\phi 2}$	Adj.R^2
Lowest i1	-0.14	-2.7	0.18	4.8	0.21	6.4	-3.78	-16.4	1.14	5.9**	11.44	28.4**	0.14
i2	-0.16	-3.4	0.09	3.7	0.24	7.7	-3.59	-17.8	-1.01	3.0**	10.35	22.7**	0.12
i3	-0.26	-6.3	0.17	6.5	0.26	10.6	-3.48	-21.5	0.98	4.7**	10.66	21.3**	0.13
Highest i4	-0.15	-3.7	0.17	4.0	0.30	7.8	-3.04	-22.1	-0.07	2.5**	9.14	13.7**	0.11

[b] This panel reports results from the following piece-wise linear regression by partitions: $g_t = \varphi_0 + \varphi_1 M + \varphi_2 H + \phi_0 x_t^a + \phi_1 M x_t^a + \phi_2 H x_t^a + \xi_t$.
M and *H* are indicator variables for the middle and high ranges defined by x_t^a cutoffs in the sample.
**and * denote significance at the 1% and 5% level, respectively, for the change of slope between adjacent ranges.

Panel C: Incremental slope coefficients in the piece-wise linear regression

Partitions by divestment	ϕ_1	ϕ_2
Lowest i1	4.91	15.22
i2	2.58	13.94
i3	4.40	14.14
Highest i4	2.97	12.18

Figure 1. A plot of residual income of year t+1 versus residual income of year t by partitions of positive capital investment in year t+1 (N=56,946)

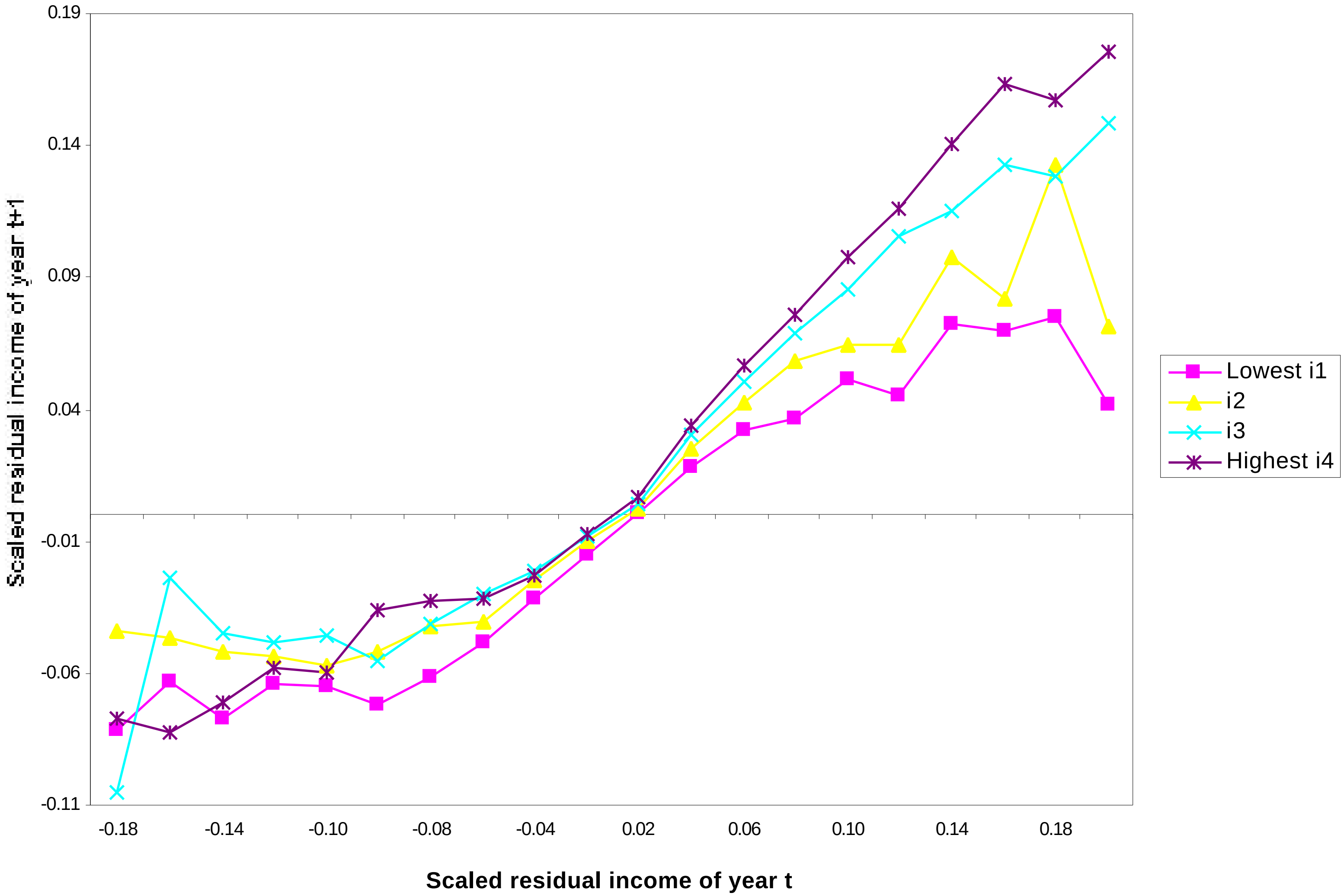

Figure 2. A plot of residual income of year t+1 versus residual income of year t by partitions of negative capital investment in year t+1 (N=26,880)

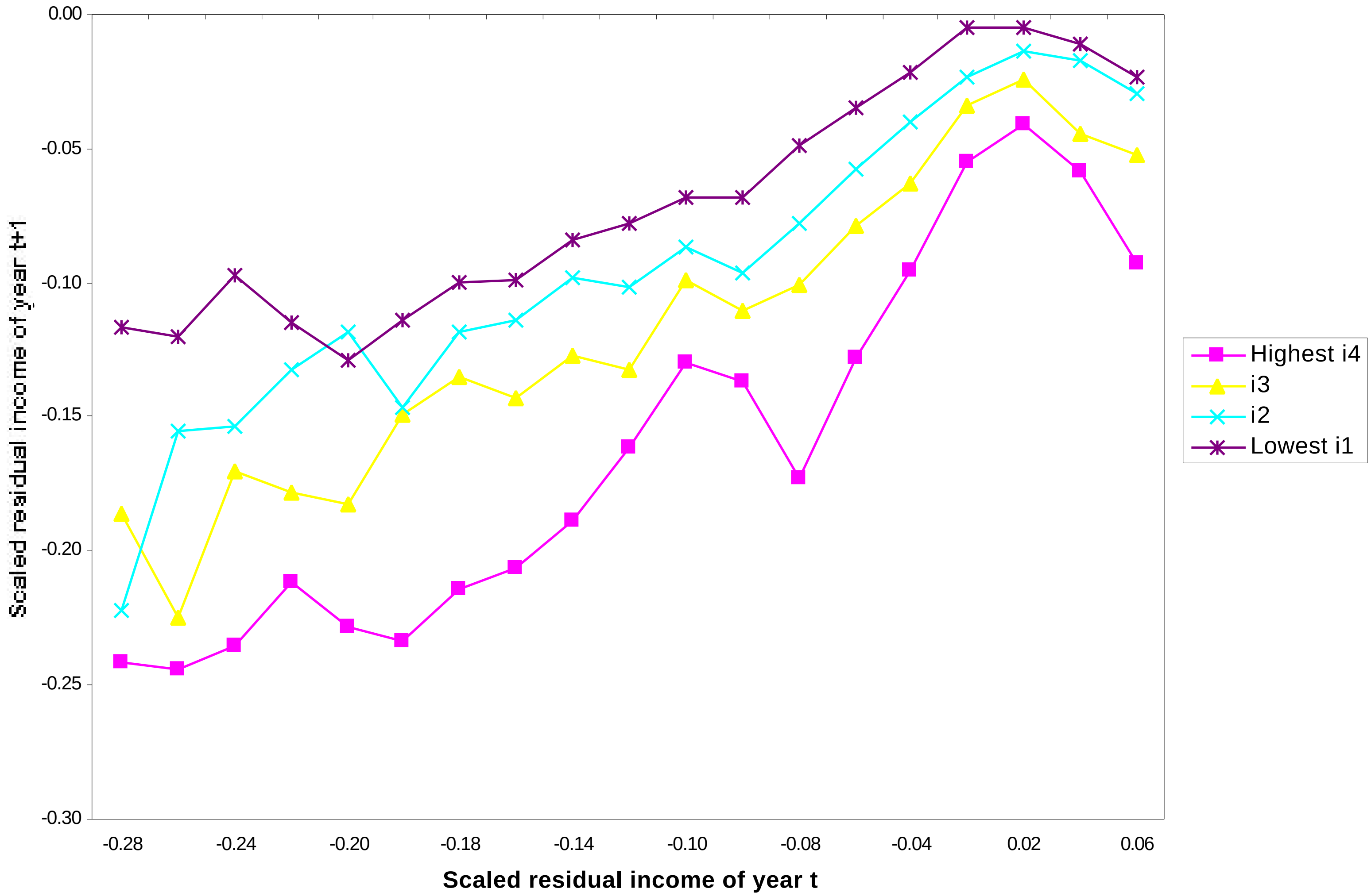

Figure 3. A plot of unrecorded goodwill versus residual income of year t by partitions of positive capital investment in year t+1 (N=56,946)

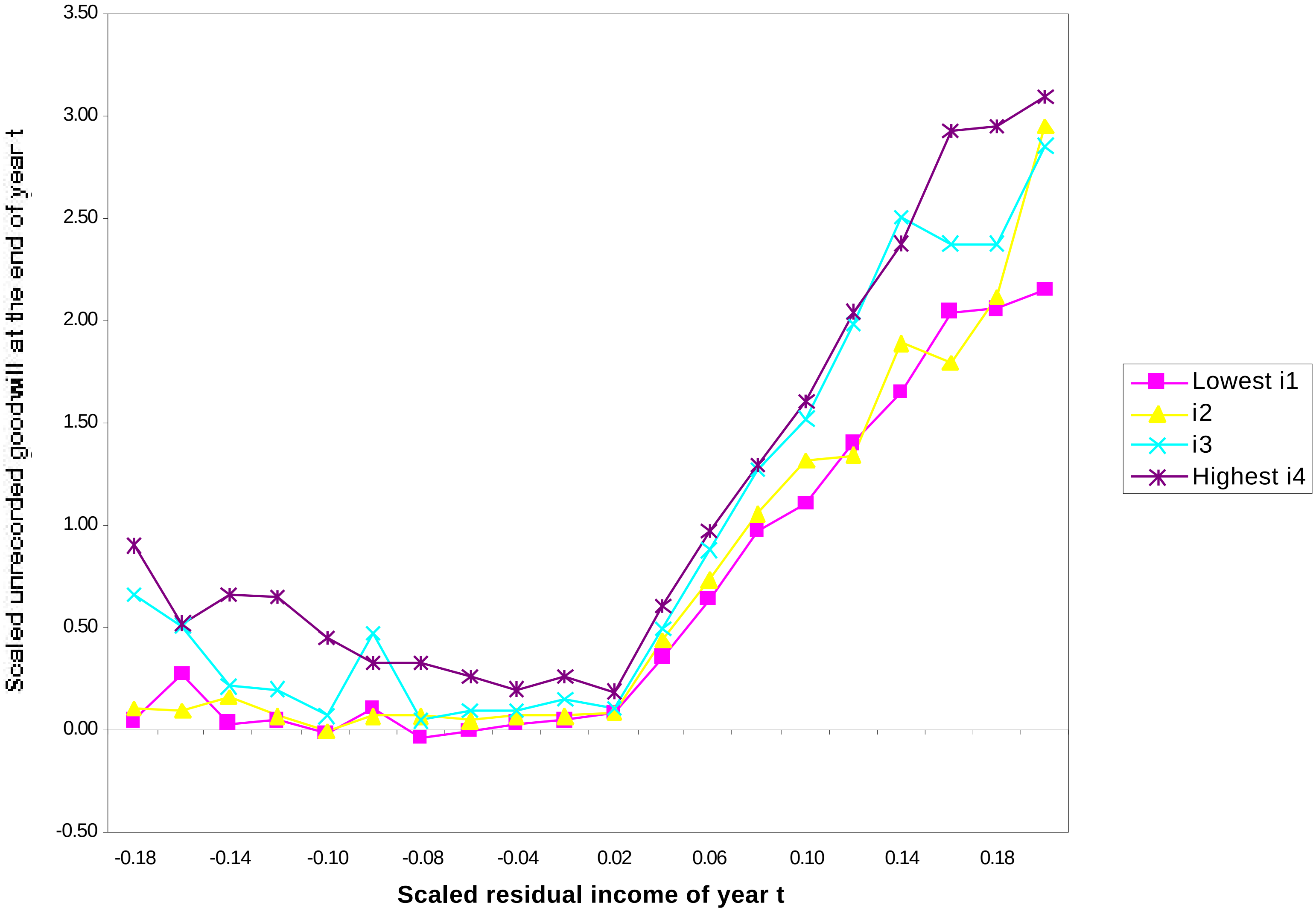

Figure 4. A plot of unrecorded goodwill versus residual income of year t by partitions of negative capital investment in year t+1 (N=26,880)