Discussions and Closures

Discussion of "A Half-Size Singularity Test Matrix for Fast and Reliable Passivity Assessment of Rational Models"

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A new passivity test for a regular state-space system has recently been proposed in [1] (conference version in [2]) based on a half-size singularity matrix, whose dimension is half that of the conventional Hamiltonian matrix. A computational speedup of about eight is recorded, apparently due to the algorithmic complexity reduction from $O((2n)^3)$ to $O(n^3)$ with n being the order of the state vector. A symmetric or slightly unsymmetric system is assumed in [1]. The discussers would like to remark that the half-size singularity matrix test, in fact, applies *only* to symmetric systems, unlike the Hamiltonian matrix passivity test that is applicable to state-space systems without regard to symmetry considerations. This is, however, inaccurately captured in [1] which describes the singularity matrix test as applicable to symmetric and unsymmetric models, but the Hamiltonian approach applies to symmetric models only (conclusion point 4, Section IX). While the authors' contribution in deriving a faster singularity matrix passivity test for symmetric systems should be commended for, the discussers find it necessary to clarify its assumptions and limitations, and, in particular, its inapplicability to unsymmetric systems. Moreover, the discussers have come up with a more elegant derivation of the singularity test matrix and new results concerning the stronger nature of the passivity condition employed in [1].

A passive system means a dissipative system whose time-domain input $u(t) \in \mathbb{R}^m$ and output $y(t) \in \mathbb{R}^m$ satisfy [3]

$$\int_{-\infty}^{t} y^T(\tau)u(\tau)d\tau \geq 0, \quad \forall t \in \mathbb{R}. \tag{1}$$

For strict passivity, the "$\geq$" sign is replaced with "$>$." Let $U(s)$ and $Y(s)$ denote the Laplace transforms of $u(t)$ and $y(t)$, respectively. A linear time-invariant system with state-space matrices $A \in \mathbb{R}^{n\times n}$, $B, C^T \in \mathbb{R}^{n\times m}$, and $D \in \mathbb{R}^{m\times m}$ is studied, which is assumed minimal, causal, and stable (i.e., all eigenvalues of A are in the open left-half complex plane) with the following compact notation for the Laplace domain transfer function $G(s)$:

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = D + C(sI - A)^{-1}B,$$

so that $Y(s) = G(s)U(s)$. (Strict) passivity of the system is equivalent to $G(s)$ being (strictly) positive real, described by the following theorem (e.g., [3]).

Theorem 1: A rational matrix-valued transfer function $G(s)$ is positive real (strictly positive real) if

1) $G(s)$ is analytic in $\mathbb{C}_+ = \{s \in \mathbb{C} | \mathrm{Re}(s) > 0\}$;
2) $\overline{G(s)} = G(\bar{s}), \forall s \in \mathbb{C}$;
3) $G(s) + G^*(s) \geq 0, \forall s \in \mathbb{C}$ (>0 for strict positive realness) where "$*$" denotes conjugate transpose.

Here, $M \geq 0$ ($M > 0$) means that M is Hermitian, or symmetric for real M, and contains all nonnegative (positive) eigenvalues. This matrix is also called a positive semidefinite (positive definite) matrix. With our assumptions of real and stable state space, conditions 1) and 2) are automatically satisfied, whereas condition 3) becomes equivalent to $G(j\omega) + G^T(-j\omega) \geq 0$ (or >0), $\forall \omega \in \mathbb{R} \cup \infty$.

Define the system

$$\Phi(s) = G(s) + G^T(-s) = \left[\begin{array}{cc|c} A & 0 & B \\ 0 & -A^T & C^T \\ \hline C & -B^T & D + D^T \end{array}\right]. \tag{2}$$

$\Phi(s)$ is easily seen to be Hermitian on $s = j\omega$. Assuming $R = D + D^T > 0$ and, therefore, invertible (a possible trick to deal with a singular $R \geq 0$ is the iterative reduction in [4] but it is not elaborated here), the system is strictly positive real if and only if $\Phi(j\omega) > 0$, $\forall \omega \in \mathbb{R} \cup \infty$ [5]. Since $\Phi(j\infty) = R > 0$, strict positive realness is equivalent to $\Phi(s)$ having no zeros, or $\Phi^{-1}(s)$ having no poles, along $s = j\omega$. From (2)

$$\Phi^{-1}(s) = \left[\begin{array}{c|c} H & \begin{bmatrix} B \\ C^T \end{bmatrix} R^{-1} \\ \hline -R^{-1}[C \quad -B^T] & R^{-1} \end{array}\right] \tag{3}$$

where

$$H = \begin{bmatrix} A & 0 \\ 0 & -A^T \end{bmatrix} - \begin{bmatrix} B \\ C^T \end{bmatrix} R^{-1} [C \quad -B^T]. \tag{4}$$

The conventional Hamiltonian matrix test for passivity [5] computes the eigenvalues of H, and the existence of purely imaginary eigenvalues at $j\omega_i, i = 0, 1, \cdots$, then corresponding to singularity at $\Phi(j\omega_i)$ and, therefore, nonpassivity.

Singularity Matrix and Its Limitation: In [1], an invertible D is assumed and the condition on the real part of $G(j\omega)$, denoted by

$$\mathrm{Re}\,(G(j\omega)) = \frac{1}{2}\left(G(j\omega) + G(-j\omega)\right) > 0 \tag{5}$$

is used to characterize passivity, which coincides with the correct definition in condition 3) of Theorem 1 only when $G(s)$ is symmetric (i.e., $G(s) = G^T(s)$). Moreover, a singularity matrix S is proposed in [1], namely,

$$S = A(BD^{-1}C - A) \tag{6}$$

for replacing H in (4). It is claimed that a purely imaginary eigenvalue $j\omega_i$ in H is then manifested as a positive real eigenvalue ω_i^2 in S. This is, however, only true for a symmetric $G(s)$. An unsymmetric system is given where this singularity matrix test fails. Consider the system

$$A = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}, \; B = C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; D = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}. \tag{7}$$

In that case, S has a repeated eigenvalue at -2, while the eigenvalues of H are located at ± 2.7788 and $\pm j0.6233$. In other words, the Hamiltonian test detects a nonpassive system but the singularity matrix test, due to the absence of positive real eigenvalues, fails to identify nonpassivity.

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Simple Derivation of S: Let $G(j\omega) = D + C(j\omega I - A)^{-1}B$ be symmetric, noting

$$(j\omega I - A)^{-1} = -A(\omega^2 I + A^2)^{-1} - j\omega(\omega^2 I + A^2)^{-1},$$

The real and imaginary parts of $G(j\omega)$ are, respectively

$$\mathrm{Re}\,(G(j\omega)) = D - CA(\omega^2 I + A^2)^{-1}B \quad (8a)$$

$$\mathrm{Im}\,(G(j\omega)) = -\,\omega C(\omega^2 I + A^2)^{-1}B. \quad (8b)$$

Similar to the flow of ideas from (2)– (4), a simple exposition for the origin of the singularity matrix S in [1] can be obtained. To ensure $\mathrm{Re}(G(j\omega)) > 0, \forall\omega \in \mathbb{R} \cup \infty$, it is required that $D = D^T > 0$ and no zero of $\mathrm{Re}(G(j\omega))$ appear along the positive real ω^2-axis. But the zeroes of $\mathrm{Re}(G(j\omega))$ are given as the poles of its inverse. Recognizing (8a) as a transfer function along $s = \omega^2$, its inverse is

$$\left[\begin{array}{c|c} -A^2 & B \\ \hline -CA & D \end{array}\right]^{-1} = \left[\begin{array}{c|c} -A^2 + BD^{-1}CA & BD^{-1} \\ \hline D^{-1}CA & D^{-1} \end{array}\right]. \quad (9)$$

Subsequently, the zeros of $\mathrm{Re}(G(j\omega))$ (i.e., where $\mathrm{Re}(G(j\omega))$ becomes singular and no longer positive definite), are $\mathrm{eig}((BD^{-1}C - A)A) = \mathrm{eig}(A(BD^{-1}C - A)) = \mathrm{eig}(S)$, where $\mathrm{eig}(\circ)$ denotes the eigenvalues.

$Re(G(j\omega)) > 0$ *Being a Stronger Passivity Condition:* The discussers present the less obvious result that $\mathrm{Re}(G(j\omega)) > 0$ (i.e., symmetric positive definite) actually implies $G(j\omega)$ is symmetric too. To see this, it is noted that a causal and stable physical system must satisfy the Kramers–Krönig relation (see, e.g., [3])

$$\mathrm{Re}\,(G(j\omega))_{(i,j)} = \frac{1}{\pi}\mathrm{pv}\int_{-\infty}^{+\infty} \frac{\mathrm{Im}\,(G(j\omega'))_{(i,j)}}{\omega - \omega'} d\omega' \quad (10a)$$

$$\mathrm{Im}\,(G(j\omega))_{(i,j)} = -\,\frac{1}{\pi}\mathrm{pv}\int_{-\infty}^{+\infty} \frac{\mathrm{Re}\,(G(j\omega'))_{(i,j)}}{\omega - \omega'} d\omega' \quad (10b)$$

where "pv" denotes the principal value while the subscript (i,j) indexes the (i,j)th element in $\mathrm{Re}(G(j\omega))$ and $\mathrm{Im}(G(j\omega))$. [Equation (10), however, is generally not fulfilled with an unstable transfer function.] That is, the real and imaginary parts of a causal and stable transfer function are not independent and, in particular, a symmetric real part constrains the imaginary part to be symmetric too. This results in $\mathrm{Re}(G(j\omega)) > 0 \Rightarrow G(j\omega) + G^*(j\omega) = G(j\omega) + G(-j\omega) = 2\mathrm{Re}(G(j\omega)) > 0$. In other words, $\mathrm{Re}(G(j\omega)) > 0$ actually serves as a stronger condition for passivity.

The authors' comments on the aforementioned items would be greatly appreciated.

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Closure on "A Half-Size Singularity Test Matrix for Fast and Reliable Passivity Assessment of Rational Models"

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We would like to thank the discussers for their clarification and appreciate their elegant derivation. We already discovered that we were incorrect in claiming the half-size test matrix to be applicable to unsymmetrical models [1]. The error was clarified in a subsequent paper [2]. We would like to mention that we have also derived a half-size test matrix for scattering parameter-based models [3], again applicable only to symmetrical models.

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