Commission Sharing and Search Agents*

Zhiyun (Frances) Xu
University of Hong Kong

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Abstract

When a principal hires an agent to do searching, she needs to motivate the agent to pay effort as well as to deliver a suitable result. Since different principals have different taste and the suitability of the search result is not entirely determined by the agents' effort, there is an opportunity for agents to cooperate among themselves and use commission sharing to match search results to principals. This paper studies how such fee-sharing arrangement affects the agents' incentive when exerting effort and principals' incentive when offering contracts. I show that principals would offer contracts with lower piece-rates and the agents would exert lower effort in searching when such arrangement is possible. However, efficiency may increase because the search results would be better matched to the principals.

1 Introduction

Often agents are hired to perform the function of searching. After exerting efforts, the agents may find that the result of the search does not match the principal's taste perfectly. When different principals have different taste, an outcome bad for one principal may be good for another, so opportunities exist for agents to trade among themselves based on the realized values of the result of their efforts to different principals. Therefore, principals have

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to simultaneously motivate the agents to exert effort and also to cooperate among themselves to better match the principals' taste.

Such principal and agent relationship exists in the real estate and rental markets. Brokers pay effort to look for potential buyers for the seller, but in the process they may find buyers that are not entirely suitable to his contracted seller. To solve this problem, brokers cooperate with each other through the MLS ("Multiple Listing Servic"), a cooperative arrangement among brokers in a particular area to share their property listings with each other. Through the MLS, the brokers can deliver the right buyers to the right sellers and share the commission if a seller eventually makes a deal with a buyer that is found by a broker other than the one that is directly hired by the seller.

Such cooperative commission sharing arrangement may also be found desirable among other agents who are hired to do searching, such as third party recruiters, also called headhunters. For instance, Denise DeMan Williams, president and chief executive of Branch International, a US-based headhunting firm, argued in Executive Recruiter News, an industry newsletter, that acting in the client's best interest demanded that if the retained firm could not complete the assignment, it should partner with a firm that could finish the job on a fee-sharing basis. Guidebooks for recruiter also have sections about how to write fee-sharing contract[1]. Various websites provide sample fee sharing agreements for recruiters to use.

In these situations, the suitability of a project is subject to some uncertainty that is out of the agent's control. The principal contracts with the agent on the outcome of the search project, but leaves the agent the freedom to trade with other agents. This freedom obviously increases the efficiency in terms of matching search results to principals. However, this raises interesting questions such as how the possibility of inter-agent trading affects the incentive of the agents when exerting efforts, as well as the incentive of the principals when offering contracts.

This paper studies a two-principal and two-agent model where each principal can contract with only one agent by offering him a linear contract. Agents first exert effort and then consider whether or not to swap the outcome with the other agent through monetary transfers. The fee-sharing promise is determined through Nash Bargaining. Then the principals pay according to

[1]See, for example, "The Recruiter's Edge: Comprehensive Recruiter Training System" published by AuthorHouse.

the project received and then the agents pay each other according to the fee sharing agreement they reached.

We show that when inter-agent trading is possible, on the unique symmetric equilibrium, the principals offer a lower-powered incentive contract (i.e., with a lower piece-rate) than when inter-agent trading is not possible. These are socially sub-optimal piece-rates in the sense that they are too low to make agents exert the efficient effort in searching.

The intuition is that the principals face a free-rider problem when motivating the agents to do efficient inter-agent trading. Notice that one principal only internalizes one agent's payoff through the individual rationality constraint, even though this principal's piece-rate affects both agents' incentives to trade. When a principal unilaterally decreases her piece-rate from the first-best, the loss to the surplus from trading is shared by both agents, and thus by the other principal as well. In other words, the principals piece-rate also affect the shares of the two principals. A slightly lower piece-rate from the first-best (given that the other principal uses the first-best contract) does not change the total pie much, but gives the principal a bigger share of the pie. This creates an incentive for both principals to shirk in motivating inter-agent trading. The result is that the equilibrium piece-rate is lower than the first-best level. When the cost function is quadratic, the principals are always better-off overall if the agents can swap search outcomes. This is because the loss from lower effort is more than compensated by the gain from better matched outcomes.

This paper highlights several effects of the commission rate. A higher commission rate not only makes one's own agent work harder, it also transfers surplus to the other agent (and ultimately to the other principal) because trading surplus are shared between the agents before they are extracted away by principals. Also, heterogenous commission rates will cause inefficiency in agents' trading decisions because they make that agents as a whole put non-equal weights on the welfare of the different principals.

This paper relates to the literature on multi-task agency, where the principal wants to motivate the agent to do two or more tasks. Multi-task problem was studied by Holmstrom and Milgrom (1991), where they looked at the tension between allocating risks and rewarding productive work. Itoh (1991) studied a multi-task problem with multiple agents, where the principal wants to motivate the agents to exert effort on their own tasks and also to help other agents. Unlike previous multi-task problems, in this paper, the tension between two tasks arises from the fact that there are two principals. In other words, if the two principals can cooperate then both tasks can be achieved with first-best outcomes.

There is also a literature that focuses on agents that are hired to do search. Lewis and Ottaviani (2008) studies a general single-principal and single-agent model where the agent can gain private benefits over the cause of searching, and their focus is on the interaction of the agent's incentives to exert search effort with agent's incentives to report the private information the agent 1cquires during the search process. Other papers have modeled search agents specifically for the real estate market, but have not looked at the fee sharing arrangements in a principal-agent framework. Several papers look at the conflict between the effort (search) dimension and the informational (suggest a reservation price) dimension [2] In comparison, this paper looks at the tension between the dimension of effort (search) and the dimension of cooperating with other agents (share commission).

The paper is also related to the common agency models in the sense that each agent's action affect both principals. Bernheim and Whinston (1986) showed that whenever collusive behavior among the principals would implement the first-best action at the first-best level of cost, noncooperative equilibrium is fully efficient. However, since here a principal only contract with one agent 1nd thus does not internalize the other agent's utility, the first best outcome cannot be achieved even though the collusive outcome would be first-best. Multiple-principals and multiple-agents settings have been studied without considering directly the interaction among the agents. Attar et. al. (2008) evaluates revelation principle in a general multiple-principals and multiple-agents setting.

There is a literature on referral, another form of interaction among agents. It has been studied in a non-principal-agent setting in Garicano and Santos (2004), which focused on matching opportunities with agents' talent, rather than on matching outcomes with principals' tastes.

The paper is organized as follows. Section 2 presents the model. Next, section 3 shows the benchmark when the agents are unable to trade with each other. Section 4 shows the main model and its results. Section 5 discusses the assumptions and robustness of the results. Section 6 concludes.

[2]See Geltner, Kluger and Miller (1991) and Arnold (1992).

2 Model

There are two principals 1 and 2. They each have an agent 1 and 2. The principal-agent relationship is assumed to be fixed, meaning that principal 1 can only contract with agent 1 and principal 2 and only contract with agent 2. We will refer to an agent as him and a principal as her.

An agent can exert effort to find an outcome. Denote the effort by $e_i \geq 0$, with $i = 1, 2$. Agents incur a cost of $C(e_i)$. Assume C is infinitely differentiable and $C(0) = 0$, $C'(0) = 0$, $C'(e) > 0$ for all $e > 0$, $C''(e) > 0$ for all e.

Principal 1 and principal 2 have differentiated taste. Whether or not agents exerts effort, the taste element in the outcome is a random draw. More specifically, agent 1 gets a realization x_1 and agent 2 gets a realization x_2, both of which is independent random draws from a distribution on $[-\frac{1}{2}, \frac{1}{2}]$ and with a continuously differentiable density that is symmetric around 0.

The project from agent i is worth $v+e_i-x_it$ to principal 1 and $v+e_i+x_it$ to principal 2, where $t > 0$ and $i = 1, 2$.

To make the problem continuous and avoid consider the case of negative project value, we have assumed the agent can create quality v with no effort and he always does so. We also assume $v > \frac{1}{2}t$, so that there is enough base value to the project such that a project will always have some value to any of the two principals. We assume the agents have the same outside option if they do not get hired, and normalize it to be 0. Denote the socially efficient level of effort by e^*, i.e., e^* is defined by $1 = C'(e^*)$.

Both principals can offer a linear contract to her own agent. In particular, principal 1 offers (F_1, k_1) and principal 2 offers (F_2, k_2) with $k_1 \geq 0$ and $k_2 \geq 0$ denoting the piece-rate. Denote the value of the project that principal 1 *receives* to be v_1 and that of the project that principal 2 *receives* to be v_2. The contracts oblige principal 1 to pay $k_1(v_1 - v) + F_1$ and principal 2 to pay $k_2(v_2 - v) + F_2$. We restricted the contracts to be contingent only on the realized value received by a principal. In other words, we assume that only the realized value of the projects v_1 and v_2 are verifiable and contractible by both the principal who receives it and the agent who delivers it. The effort of the agents are not verifiable or contractible. We will omit the base project value v as it does not affect the analysis.

The timing of the game is as follows.

1. Principals 1 and 2 simultaneously make take-it-or-leave-it linear con-

tract offers to their respective agents.

2. Agents 1 and 2 simultaneously decide whether to accept the offers or not.

3. Agents 1 and 2 simultaneously exert efforts.

4. Project values are realized.

5. Agents 1 and 2 decide whether to swap projects or not and Nash-bargain (with equal bargaining power) to determine the transfer between them.

6. Agents 1 and 2 hand projects to their respective principals and receive the promised payments from their principals.

7. Agents 1 and 2 pay each other transfers as promised. (The timing of this one step can be right after Nash Bargaining.)

All players are risk-neutral and they maximized their expected payoff. We consider symmetric subgame perfect equilibrium.

3 No-trading Benchmark

When the two agents cannot trade with each other[3] , the two principal-agent pairs are not related in this game, so we can just look at one pair. The analysis is very standard. Since the principal can extract all the surplus from the agent through the fixed part of the linear contract, the principal is effectively maximizing the total expected surplus of the principal-agent pair. WLOG, we consider the pair 1-1. Principal 1 solves:

$$\begin{aligned} \max_{k_1} \quad & e_1 - E[x_1]t - C(e_1) = e_1 - C(e_1) \\ s.t. \quad & e_1 = argmax_e\{k_1(e - E[x_1]t) - C(e)\} = argmax_e\{k_1 e - C(e)\} \end{aligned}$$

[3]If one generalizes the game such that principals first choose to whether or not to ban trading (maybe through banning the communications between agents) and trading can only occur if both principles allow it, then this benchmark of no trading will exist as an equilibrium, because given that one principal bans trading, the other weakly prefers to ban trading as well.

The unique solution clearly is to set $k_1 = 1$. The resulting effort level is efficient and in this benchmark $F_1 = -(e^* - C(e^*))$.

4 Trading Allowed

Despite that the effort levels are efficient in the no-trading Benchmark, there exists a source of inefficiency there: the projects are not efficiently matched to principals. principal 1 always receives the project created by agent 1 and principal 2 always receives the project created by agent 2 in the Benchmark, while the most efficient pairing between projects and principals depends on the realization of x_1 and x_2. It is clear that it is efficient to trade if and only if $x_1 \geq x_2$, i.e., $x_1 - x_2 \geq 0$.

4.1 Trading incentive given efforts

Because of the nature of Nash Bargaining between the two agents, trading will occur whenever it is subgame-efficient to do so, i.e., efficient given the effort level e_1, e_2, the realized value of x_1 and x_2 and the contracts the agents have accepted. Consider the subgame just after x_1 and x_2 are realized. Therefore, trading happens if and only if[4]:

$$
\begin{aligned}
&[k_1(e_2 - x_2 t) + k_2(e_1 + x_1 t)] - [k_1(e_1 - x_1 t) + k_2(e_2 + x_2 t)] \geq 0 \\
\Leftrightarrow &-(k_1 - k_2)(e_1 - e_2) + (k_1 + k_2)(x_1 - x_2)t \geq 0
\end{aligned}
$$

For any $k_1 + k_2 > 0$, the above trade-condition is equivalent to:

$$
x_1 - x_2 \geq \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)}
$$

Let's contrast this with the efficient condition for trading. Trading is efficient if and only if,

$$
x_1 - x_2 \geq 0
$$

This raises three questions:

[4]We assume the tie-breaking rule that they will trade if indifferent.

- Why $k_1 - k_2 \Rightarrow$ efficient trading?
- Why $e_1 - e_2 \Rightarrow$ efficient trading?
- Why $(k_1 - k_2)(e_1 - e_2) \neq 0$ implies inefficient trading?

To answer these questions, notice that trading can increase efficiency in only two ways.

- Case 1. Benefit both principals. $\Rightarrow$ Agents will do it no matter what contracts they are given (as long as $k_1 \geq 0$ and $k_2 \geq 0$).
- Case 2. Benefit one principal while hurt the other. $\Rightarrow$ Agents may not agree to do the trading.

When $e_1 = e_2$, i.e., when there is no vertical difference in the quality of the outcomes, only Case 1 is possible. In other words, if trading is efficient, then trading benefits both principals, so agents trade.

When $k_1 = k_2$, then agents collectively view the two principals as a social planner would: equally, so through nash bargaining, they will trade whenever trading is efficient (even when it is Case 2). Notice that the efficiency criteria is simply a utilitarian criteria that gives the same weights to both principals' outcomes.

When $e_1 \neq e_2$ and $k_1 \neq k_2$, only Case 2 will happen and the agents collectively give different "weights" to the two principals, therefore, they do not act like a social planner and they do not trade efficiently.

Since trading is efficient if $k_1 = k_2$, the first-best pair of contracts when trading is allowed is simply all contracts that satisfy $k_1 = k_2 = 1$. We will denote the total gain of trade (for all players as a whole) as S:

$$S \equiv 2t \int_0^1 sf(s)ds$$

Given a pair of contracts and a set of realization of x_1, x_2, let's denote the agents' total gain of trade as $g(k_1, k_2, x_1, x_2)$:

$$g(k_1, k_2, x_1, x_2) \equiv -(k_1 - k_2)(e_1 - e_2) + (k_1 + k_2)(x_1 - x_2)t$$

For the sake of simplicity of notation, we will sometimes suppress the arguments of g. By Nash-Bargaining, this gain is equally shared between the two agents, so each gets $\frac{1}{2}g$.

4.2 Incentive to exert efforts

Now we go one step backward in the game. Let $Pr()$ denote the probability of the event in the bracket. Agent 1's problem in choosing effort is:

$$\begin{aligned}
\max_{e_1} \quad & E[k_1(e_1 - x_1 t)|no\,trade]Pr(no\,trade)+ \\
& E[k_1(e_1 - x_1 t) + \frac{1}{2}g|trade]Pr(trade) - C(e_1) = \\
& k_1(e_1 - E[x_1]t) - C(e_1)+ \\
& E\left[\frac{1}{2}g|g \geq 0\right] Pr\left(g \geq 0\right)
\end{aligned}$$

Since the fall-back from breaking up of the Nash-Bargaining is to supply the project to one's own principal, the payoff from a trading is $k_1(e_1 - x_1 t) + \frac{1}{2}g$ for agent 1.

Notice that when $k_1 = k_2$ two things are true. First, the gain from trade, g, does not depend on e_1 (as explained in Subsection 4.1), and second (as a result of the first) the probability of trading does not depend on e_1 either. In other words, as long as $k_1 = k_2$, the trading decision in the subgame will be efficient.

The agent's problem highlights the three effects of a commission rate.

- The term $k_1 e_1 - C(e_1)$ shows that k_1 has the normal effect of motivating agents to work harder.
- If $k_1 = k_2$, then the rest of the agent 1's objective function becomes the following.

  $$E[\frac{1}{2}g|g \geq 0]Pr(g \geq 0) = \frac{1}{4}(k_1 + k_2)E[x_1 - x_2|x_1 - x_2 \geq 0]$$

  By the same logic, there is a term in agent 2's objective function that is also

  $$\frac{1}{4}(k_1 + k_2)E[x_1 - x_2|x_1 - x_2 \geq 0]$$

  Therefore, higher commission rate transfers surplus to both agents. This is a crucial effect because as we will see, principal 1 is only able to extract back the part given to agent 1 through individual rationality constraint but not the part given to agent 2.

- When $k_1 \neq k_2$, trading is inefficient because trading happens if and only if $x_1 - x_2 \geq \frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}$ (as explained in Subsection 4.1).

Let f denote the the density of the distribution of $x_1 - x_2$. By definition, f is symmetric around 0 and is positive over $[-1, 1]$.

Lemma 1. *Given any k_1 and k_2, a pure strategy subgame equilibrium exists in the effort game. Let $(\tilde{e}_1(k_1, k_2), \tilde{e}_2(k_1, k_2))$ denote an equilibrium.*

On the equilibrium, the probability of trading is strictly less than one. If an equilibrium is such that the probability of trading is positive, then it is characterized by:

$$k_1 - C'(e_1) - \frac{1}{2}(k_1 - k_2) \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\, ds = 0 \qquad (FOC_1)$$

$$k_2 - C'(e_2) - \frac{1}{2}(k_2 - k_1) \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\, ds = 0 \qquad (FOC_2)$$

Proof. In the Appendix. □

Remark: The proof for existence of equilibrium is essentially an application of results from supermodular game. Notice that trading may not happen in the subgame equilibium.

The term $k_1 - C'(e_1)$ in the FOC condition is the same as that in the no-trading Benchmark. The rest in the FOC condition is the incentive provided by the possibility of trading. Notice that this term disappears if $k_1 = k_2$. In other words, if $k_1 = k_2$, the effort incentive is the same as in the Benchmark: agents exert efforts as if trading is banned. However when $k_1 \neq k_2$, there is "spill over" of incentive and 2) "free-riding" in efforts.

When principal 1 increases k_1 slightly above k_2, it also motivates agent 2 to work harder because now agent 2 gets a share of principal 1's commission through Nash-bargaining if agent 2's project is supplied to principal 1. However, at the same time, when $k_1 > k_2$, there is a free-riding problem. Agent 1 gains from agent 2's effort, so higher effort from agent 2 reduces agent 1's incentive to work. [5]

These "spill-over" and "free-riding" effects causes e_1 to go down when k_1 is increased if we *only* consider the effects coming from trading. However, the

[5] Let $\mathbf{e_1}(e_2)$ denote a's reaction function as implied by Condition FOC_1, then around $(\tilde{e}_1, \tilde{e}_2)$, we have:

effort level is still higher for the agent who is promised a higher piece-rate, as shown in Lemma 3 below. This is because the direct motivating effect from the piece-rate dominates the indirect effects through trading.

The above lemma however does not establish that the subgame equilibrium to be unique.

Lemma 2. *When the cost function is quadratic and when the distribution of $x_1 - x_2$ is uniform, the subgame equilibrium given any non-negative k_1, k_2 is unique.*

Proof. See the Appendix. □

Lemma 3. $k_1 > k_2 \Rightarrow \tilde{e}_1 > \tilde{e}_2$.

Proof. WLOG, let $k_1 > k_2$. There are two possible cases. Either there is positive probability of trading on the equilibrium or there is not.

Case 1, positive probability of trading.

Let $\Delta = k_1 - k_2 > 0$. Let $Prob(\Delta) = \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\,ds$, then FOCs imply that:

$$\begin{aligned} C'(\tilde{e}_1) - C'(\tilde{e}_2) &= (k_2 + \Delta - \frac{1}{2}\Delta Prob(\Delta)) - (k_2 + \frac{1}{2}\Delta Prob(\Delta)) \\ &= \Delta - \Delta Prob(\Delta) \\ &\geq 0 \end{aligned}$$

This implies that $\tilde{e}_1 \geq \tilde{e}_2$ [6]. Next, we prove the inequality by contradiction. Suppose $\tilde{e}_1 = \tilde{e}_2$, then we have:

$$\Delta - \Delta Prob(\Delta) = 0 \Rightarrow Prob(\Delta) = 1$$

$$\frac{d\mathbf{e_1}(e_2)}{de_2} = \frac{\frac{1}{2}\frac{(k_1-k_2)^2}{t(k_1+k_2)} f\left(\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}\right)}{-C''(e_1) + \frac{1}{2}\frac{(k_1-k_2)^2}{t(k_1+k_2)} f\left(\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}\right)} < 0 \text{ when } k_1 \neq k_2$$

The inequality follows from the necessary condition that second order conditions must be satisfied at $(\tilde{e}_1, \tilde{e}_2)$.

[6] We will suppress the arguments of $\tilde{e}_1(k_1, k_2), \tilde{e}_2(k_1, k_2)$ when there is no ambiguity.

However, because of the symmetry of the density function f around 0, we know that $Prob(\Delta) = \int_0^1 f(s)\, ds = \frac{1}{2}$ when $\tilde{e}_1 = \tilde{e}_2$. Therefore, it is a contradiction, and we know that $\tilde{e}_1 \neq \tilde{e}_2$.

Case 2, zero probability of trading.

This implies that $\tilde{e}_1 = (C')^{-1}(k_1)$ and $\tilde{e}_2 = (C')^{-1}(k_2)$. Therefore, $\tilde{e}_1 > \tilde{e}_2$. □

4.3 Principal's choices of piece-rates

The equilibrium fixed rates are implied by the agents' individual rationality constraints, so an equilibrium is characterized by a pair of piece-rates, which we denote by $\tilde{k}_1$ and $\tilde{k}_2$. Recall that S denotes the total gain of trading under efficient trading, i.e., when agents trade if and only if $x_1 - x_2 \geq 0$.

Proposition 1. *(Necessary Condition)*

If a pure strategy symmetric equilibrium exists such that $\tilde{k}_1 = \tilde{k}_2 = k$, we have the following results.

First, we must have $k < 1$. In other words, the equilibrium piece-rates must be lower than those in the no-trade Benchmark and the first-best.

Second, k is unique and is decreasing in S.

For $S < \frac{2}{C''(0)}$, k is uniquely determined by the necessary condition:

$$\frac{1-k}{C''((C')^{-1}(k))} = t \int_0^1 sf(s)ds = \frac{S}{2}$$

For $S \geq \frac{2}{C''(0)}$, $k = 0$.

Proof. See the Appendix. □

Remark:

In the Benchmark, principal 1 fully internalizes the effect of her piece-rate: even though higher piece-rate gives her agent 1 higher cut of the result of his effort, she extracts all of the agent's cut back through her fixed rate. Now when there is trading, higher piece-rate by principal 1 increases the total gain from trading for the agents, but that gain is shared by agent 1 and 2, so the value accrued to agents from principal 1's sacrifice (in the sense of a higher piece-rate) cannot be fully recovered through principal 1's fixed rate. There is a "leakage". This gives principal 1 an incentive to reduce her piece-rate. In other words, decreasing k_1 helps principal 1 to grab a bigger share of the trading surplus. To see that, given any symmetric contracts:

Principal 1 gets

$$e_1 - C(e_1) + \frac{(1-k_1) + \frac{1}{2}(k_1+k_2)}{2} S$$

Principal 2 gets

$$e_2 - C(e_2) + \frac{(1-k_2) + \frac{1}{2}(k_1+k_2)}{2} S$$

The effects of decreasing k_1 on e_1, e_2, and S are second order given $k_1 = k_2 = 1$ as the initial condition, while the effect on the sharing of S is first order. Therefore, Principal 1 has a unilateral incentive to reduce k_1 from $k_1 = 1$.

The intuition for the comparative statics result is that higher S increases the incentive to lower the piece-rate because the principal wants to capture a larger share of the gain of trading. Several things can increase S: higher level of heterogeneity in principal's taste, such as higher t; and higher chance of getting extreme characteristic projects, such as an f that is heavier on the two ends. When $S = 0$, we go back to the standard case where agents do not trade and the only equilibrium involves $k = 1$.

Sufficiency of the equilibrium has been proved for uniform f distribution and quadratic cost function, and the search for more general sufficient condition for the existence of the equilibrium is under progress.

Proposition 2. *(Welfare) If the cost function is quadratic then allowing trading increases social surplus for any parameters.*

Proof. Let $W(S)$ denote the total social surplus on the symmetric equilibrium when trading is allowed and let W denote the total social surplus when trading is not allowed. We will show $W(S) > W$ for any $S \geq 0$.

From the benchmark analysis, we already know that $W = 2(e^* - C(e^*))$.

$$W(S) = 2(\tilde{e} - C(\tilde{e})) + S$$

Quadratic cost function implies that $C''(e)$ is a positive constant, which we will simply denote by C''. This implies that $e^* = \frac{1}{C''}$. To see this, we know that e^* satisfies the FOC:

$$1 - C'(e^*) = 1 - C''e^* = 0$$

Therefore, we have $\frac{2}{C''} > 2(e^* - C(e^*))$. We will look at two cases:
Case 1. For $S < \frac{2}{C''}$, by Proposition 1,

$$\frac{dW}{dS} = 2\frac{1 - C'}{C''}\left(-\frac{C'' + \frac{S}{2}C'''}{(C'')^2}\right) + 1 = -2\frac{1 - C'}{(C'')^2} + 1$$

$$\frac{d^2W}{d^2S} = -\frac{2}{(C'')^2} < 0$$

Therefore, the social surplus is strictly concave over $S \in [0, \frac{2}{C''}]$. Note $W(0) = W$ and recall that $W(S) = S$ when $S = \frac{2}{C''}$, which implies $W(\frac{2}{C''}) = \frac{2}{C''} > W$. Therefore, $W(S) \neq W$ for $S \in [0, \frac{2}{C''}]$, with the inequality strict for $S > 0$.

Case 2. For $S \geq \frac{2}{C''}$, we have $W(S) = S > W$.

□

Remark: Trading increases the efficiency gain from matching the projects to principals, but reduces the surplus from stage of exerting efforts because agents are under-incentivized. However, when the cost function is quadratic, the first effect dominates.

5 Discussion

This paper assumes that fee sharing is determined though Nash Bargaining. In reality however, often it is already a industry consensus that fee should always be shared half and half between two "cooperating" agents. This alternative will not change the qualitative results. The main driving force still remains the same in the sense that trading surplus will be shared between agents and thus there is a free-riding problem between the principals on contributing to the public good of "trading". However, trading cannot be efficient in any equilibrium. To see that, given that any fee has to be shared half and half, trading only happens if both agents find it better to get the half of the total fee after swapping projects:

$$\frac{1}{2}k_1(e_2 - x_2t) + \frac{1}{2}k_2(e_1 + x_1t) \geq \max\{k_1(e_1 - x_1t), k_2(e_2 + x_2t)\}$$

$$\Leftrightarrow x_1 \geq \frac{(k_1 - k_2)e_1}{t(e_1 + e_2)} \text{ and } x_2 \leq \frac{(k_1 - k_2)e_2}{t(e_1 + e_2)}$$

This means that on a symmetric equilibrium (if exists), the agents trades only when $x_1 \geq 0$ and $x_2 \leq 0$, while complete trading efficiency calls for trading whenever $x_1 - x_2 \geq 0$.

This paper also assumes that uncertainty only lies in the horizontal aspect of the search results. That is, there is no uncertainty in the vertical quality of the search results. If the vertical quality is also uncertain, then it creates a competition between the principals as they can offer higher piece-rate to influence the direction of the inter-agent trading so that they can get the better of the two results. This will be a countervailing force that pushes up the equilibrium piece-rate. That implies this consideration will pressure a home seller not to lower her commission rate below other sellers', not because it will reduce cooperation from other agents, but because she would not want bad buyers be sent her way.

6 Conclusion

This paper shows that when inter-agent trading is possible, on the unique symmetric equilibrium, the principals offer a lower-powered incentive contract (i.e., with a lower piece-rate) than when inter-agent trading is not possible. These are socially sub-optimal piece-rates in the sense that they are too low to make agents exert the efficient effort in searching. Here the principals need to motivate the agents to pay effort in searching and also to cooperate with other agents to deliver the most suitable result to the principals.

Motivating effort calls for paying the agents a high piece-rate. However, principals face negative externality when motivating the agents to do efficient inter-agent trading. Note that the principals internalize the agents' payoff through the individual rationality constraints. When a principal unilaterally decreases her piece-rate, the loss to the surplus from trading is born by both agents, and thus by the other principal as well. This creates an incentive for both principals to shirk in motivating efficient inter-agent trading. The result is that the equilibrium piece-rate is lower than the efficient level. When the cost function is quadratic, the principals are always better-off overall if the agents can swap search outcomes. This is because the loss from lower effort is more than compensated by the gain from better matched outcomes.

7 Appendix

Proof of Lemma 1.

Proof. Step 1. Proving equilibrium exists.

A game is a supermodular game if the strategy set is bounded, the payoff is upper-semi continuous and the payoff has increasing difference between strategies.

To be able to use results from supermodular game, we need to prove that the choice set for efforts is effectively bounded for both players.

Fix any $k_1 > k_2 \geq 0$.

Fix $\forall e_2 \geq 0$. Define two functions $\underline{e}$ and $\bar{e}$ as follows:

$$\frac{(k_1 - k_2)(\underline{e}(e_2) - e_2)}{t(k_1 + k_2)} = -1 \quad \frac{(k_1 - k_2)(\bar{e}(e_2) - e_2)}{t(k_1 + k_2)} = 1$$

$$k_1 - k_2 > 0 \Rightarrow \underline{e}(e_2) < e_2 < \bar{e}(e_2)$$

Note that the payoff to agent 1 depends on which segment e_1 is in. For $e_1 \leq \underline{e}(e_2)$, the probability of trading is 1 and thus the payoff is $u_1(e_1, e_2) = k_1 e_1 - C(e_1) - \frac{1}{2}(k_1 - k_2)(e_1 - e_2) = \frac{1}{2}(k_1 + k_2)e_1 - C(e_1) + \frac{1}{2}(k_1 - k_2)e_2 \geq k_1 e_1 - C(e_1) + \frac{1}{2}t(k_1 + k_2)$. For $e_1 \geq \bar{e}(e_2)$, the probability of trading is 0 and thus the payoff is just $u_1(e_1, e_2) = k_1 e_1 - C(e_1)$.

Case 1. The argmax $(e_1)(e_2) \in [0, \underline{e}(e_2))$, then we have $\mathbf{e_1}(e_2) = e^*(\frac{1}{2}(k_1 + k_2)) < e^*(k_1)$.

Case 2. If $\mathbf{e_1}(e_2) > \bar{e}(e_2)$, then we have $(e_1) = e^*(k_1)$.

Case 3. If $\mathbf{e_1}(e_2) \in [\underline{e}(e_2), \bar{e}(e_2)]$, then it solves the following problem:

$$\begin{aligned}\max_{e_1} \quad & k_1 e_1 - C(e_1) + \\ & \frac{1}{2}[(k_1+k_2)t \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} sf(s)\,ds - \\ & (k_1-k_2)(e_1-e_2)\int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\,ds] \\ & s.t. \quad -1 \geq \frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)} \leq 1\end{aligned}$$

The first order derivative of the objective function is:

$$k_1 - C'(e_1) - \frac{1}{2}(k_1-k_2)\int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\,ds$$

For $e_1 \in [e^*(k_1), \bar{e}(e_2))$, the derivative is strictly negative. Therefore again, $\mathbf{e_1}(e_2) \leq e^*(k_1)$.

This proves that $\mathbf{e_1}(e_2) \leq e^*(k_1)$.

We do the same analysis for agent 2's problem. The above defined function $\bar{e}$ and $\underline{e}$ implies:

$$\frac{(k_1-k_2)(e_1-\underline{e}(e_1))}{t(k_1+k_2)} = 1 \quad \frac{(k_1-k_2)(e_1-\bar{e}(e_1))}{t(k_1+k_2)} = -1$$

The payoff to agent 2 also depends on which segment e_2 is in. For $e_2 \leq \underline{e}(e_1)$, the probability of trading is 0, thus the payoff is just $u_2(e_1,e_2) = k_2e_2 - C(e_2)$. For $e_2 \geq \bar{e}(e_1)$, the probability of trading is 1, so $u_2(e_1,e_2) = k_2e_2 - C(e_2) - \frac{1}{2}(k_1-k_2)(e_1-e_2) = \frac{1}{2}(k_1+k_2)e_2 - C(e_2) - \frac{1}{2}(k_1-k_2)e_1 \leq k_2e_2 - C(e_2) - \frac{1}{2}t(k_1+k_2)$.

Case 1. The argmax $\mathbf{e_2}(e_1) \in (\bar{e}(e_1), +\infty)$, then trading happens with probability 1, which implies $\mathbf{e_2}(e_1) = e^*(\frac{1}{2}(k_1+k_2))$.

Case 2. If $\mathbf{e_2}(e_1) < \underline{e}(e_1)$, then trading happens with probability 0, which implies that $\mathbf{e_2}(e_1) = e^*(k_2)$.

Case 3. $\mathbf{e_2}(e_1) \in [\underline{e}(e_1), \bar{e}(e_1)]$.

Notice that the following inequality:

$$\bar{e}(e^*(k_1)) > e^*(k_1) > e^*(\frac{1}{2}(k_1+k_2)) > e^*(k_2)$$

Therefore, when e_1 is bounded by $e^*(k_1)$, agent 2's best response is bounded above by $\bar{e}(e^*(k_1))$.

By symmetry of the setup, the case for $k_1 < k_2$ will yield the same result. Therefore, given any non-negative k_1, k_2, we can find two bounded choices set for e_1 and e_2 that are without loss of generality.

Note that the payoff functions are continuous in e_1 and e_2.

Now look at the cross derivative of agent 1's payoff when $\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)} \in (-1,1)$ (elsewhere the cross derivative trivially equals zero):

$$\frac{\partial^2 u_1}{\partial(-e_2)\partial(e_1)} = \frac{1}{2}\frac{(k_1-k_2)^2}{t(k_1 k_2)} f\left(\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}\right) \geq 0$$

Therefore, u_1 has increasing difference in e_1 and $-e_2$. Similarly, for u_2.

Now we can conclude that the subgame given k_1 and k_2 is a supermodular game. Therefore, we can apply the result that a pure-strategy equilibrium exists.

Step 2. Prove uniqueness of equilibrium.

We show that on equilibrium, probability of trading is less than 1. [to be filled in]

□

Proof of Lemma 2

Proof. There are only two cases: either the probability of trading is zero, or it is one.

Step 1. We show that there are at most only one equilibrium with trading probability of zero. Let $(\tilde{e}_1, \tilde{e}_2)$ be an equilibrium. WLOG, suppose $k_1 > k_2$. Then we must have $e^*(k_1) \geq \bar{e}(\tilde{e}_2)$ because otherwise agent 1's best response is not in a range that makes trading probability zero. It also implies that $\tilde{e}_1 = e^*(k_1)$. Similarly, we have $\tilde{e}_2 = e^*(k_2)$. Therefore, there can at most be only one equilibrium that has trading probability equals zero.

Step 2. We show that if an equilibrium with trading probability zero exists then there does not exist any equilibrium with positive trading probability. WOLG, suppose $k_1 > k_2$. Step 1 already implies that if an equilibrium with trading probability zero exists, then $e^*(k_1) \geq \bar{e}(e^*(k_2))$ and

$e^*(k_2) \leq \underline{e}(e^*(k_1))$. Suppose another equilibrium with positive trading probability exists. Then $\bar{e}(\tilde{e}_2) > \bar{e}(e^*(k_2)) \Rightarrow \tilde{e}_2 > e^*(k_2)$ because otherwise agent 1's best response is one that makes trading probability zero. However, this forms a contradiction, because agent 2's payoff is decreasing over $[e^*(k_2), \bar{e}(\tilde{e}_1)]$.

Step 3. We show that there exists at most one equilibrium that has strictly positive trading probability using quadratic cost function and uniform distribution of $x_1 - x_2$.

Quadratic cost function implies that $C'(e)$ to e is a linear function. Let it be $C'(e) = pe + q$ where $p \neq 0$.

Then we can solve for the equilibrium efforts through the following equations:

$$
\begin{aligned}
k_1 - pe_1 - q &= \frac{1}{4}(k_1 - k_2)(1 - \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)}) \\
k_2 - pe_2 - q &= -\frac{1}{4}(k_1 - k_2)(1 - \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)})
\end{aligned}
$$

Since both equations are linear, there are at most one solution.

□

Proof of Proposition 1.

Proof. A solves the following problem:

$$
\begin{aligned}
\max_{k_1} \quad U_1(k_1) \equiv & E[(1 - k_1)(\tilde{e}_1(k_1, k_2) - x_1 t)|no\,trade]Pr(no\,trade) \\
& + E[(1 - k_1)(\tilde{e}_2(k_1, k_2) - x_2 t)|trade]Pr(trade) \\
& + E[k_1(\tilde{e}_1(k_1, k_2) - x_1 t)|no\,trade]Pr(no\,trade) \\
& + E[k_1(\tilde{e}_1(k_1, k_2) - x_1 t) + \frac{1}{2}g|trade]Pr(trade) - C(e_1) \\
= & \tilde{e}_1(k_1, k_2) - C(\tilde{e}_1(k_1, k_2)) \\
& - (\tilde{e}_1(k_1, k_2) - \tilde{e}_2(k_1, k_2))[(1 - k_1) + \frac{1}{2}(k_1 - k_2)]Pr(trade) \\
& + [(1 - k_1) + \frac{1}{2}(k_1 + k_2)]E[x_1 - x_2|trade]tPr(trade)
\end{aligned}
$$

Take derivative with respect to k_1 and evaluate the derivative at $k_1 = k_2 = k$, which also implies that $\tilde{e}_1 = \tilde{e}_2 \equiv \tilde{e}$. [7]

$$
\begin{aligned}
\left.\frac{dU_1}{dk_1}\right|_{k_1=k_2=k} &= (1 - C'(\tilde{e}_1))\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k} \\
&-(1-k)\left.\frac{\partial(\tilde{e}_1 - \tilde{e}_2)Pr(trade)}{\partial k_1}\right|_{k_1=k_2=k} \\
&+\left.\frac{\partial E[x_1 - x_2|trade]tPr(trade)}{\partial k_1}\right|_{k_1=k_2=k} \\
&+(-\frac{1}{2})t\int_0^1 sf(s)ds \\
&= (1-k)\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k} - \frac{1}{2}(1-k)\left.\frac{\partial(\tilde{e}_1 - \tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} \\
&-\frac{1}{2}t\int_0^1 sf(s)ds \\
&=\frac{1}{2}(1-k)\left.\frac{\partial(\tilde{e}_1 + \tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} - \frac{1}{2}t\int_0^1 sf(s)ds
\end{aligned}
$$

If $k \geq 1$, then we have and $\frac{1}{2}(1-k)\left.\frac{\partial(\tilde{e}_1+\tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} \leq 0$. Therefore, the above derivative is strictly negative when $k \geq 1$. This shows that principal 1 has unilateral incentive to deviate downward when $k \geq 1$, therefore we must have $k < 1$

From Conditions FOC_1 and FOC_2, we get:

$$
\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k} = \frac{1 - \frac{1}{2}\int_0^1 f(s)ds}{C''(\tilde{e})} = \frac{\frac{3}{4}}{C''(\tilde{e})}
$$

$$
\left.\frac{\partial \tilde{e}_2}{\partial k_1}\right|_{k_1=k_2=k} = \frac{\frac{1}{2}\int_0^1 f(s)ds}{C''(\tilde{e})} = \frac{\frac{1}{4}}{C''(\tilde{e})}
$$

[7] Note that $\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k}$ and $\left.\frac{\partial \tilde{e}_2}{\partial k_1}\right|_{k_1=k_2=k}$ exists. This is because when k_1 and k_2 are close enough, the objective function of agents in the subgame is strictly concave, which implies that the subgame equilibrium of efforts is unique and continuously differentiable in the "parameters" k_1 and k_2.

Since a necessary condition for a symmetric equilibrium where $k > 0$ is that $\left.\frac{dU_1}{dk_1}\right|_{k_1=k_2=k} = 0$, k should satisfy:

$$(1-k)\left.\frac{\partial(\tilde{e}_1+\tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} = t\int_0^1 sf(s)ds \Rightarrow$$
$$\frac{1-k}{C''((C')^{-1}(k))} = t\int_0^1 sf(s)ds$$

Denote the left-hand-side of the necessary condition as $h(k)$, and let $\tilde{e} \equiv \tilde{e}_1(k,k)$. By symmetry of f, we have $S = 2t\int_0^1 sf(s)ds$, so we can rewrite the necessary condition for k as:

$$h(k) = \frac{S}{2}$$

Since $h(1) = 0 < \frac{S}{2}$, we only need to show that $h(k)$ is decreasing in k:

$$\frac{dh(k)}{dk} = -\frac{C''(\tilde{e}) + (1-k)\frac{C'''(\tilde{e})}{C''(\tilde{e})}}{(C''(\tilde{e}))^2} = -\frac{C'' + \frac{S}{2}C'''}{(C'')^2} < 0$$

When $h(0) < \frac{S}{2}$, for any $k < 1$, we have $h(k) < \frac{S}{2}$. This implies that $\left.\frac{dU_1}{dk_1}\right|_{k_1=k_2=k} < 0$ for any $k < 1$, therefore, if a symmetric pure strategy exists it must be $k = 0$. □

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