Commission Sharing among Agents*

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Abstract

When a principal hires an agent to create a result, she would like to motivate the agent to pay effort to increase the probability of a good fit with her own characteristics. However, an outcome that does not fit this principal may fit another, so her agent might have an incentive to divert a result to another principal through that principal's contracted agent. This is commonly achieved in practice through a fee-sharing arrangement among the agents. This paper studies how such result-diverting and fee-sharing arrangements affect the agents' incentive to exert efforts and the principals' incentive when offering contracts. We show that, under fee-sharing arrangement, a contract signed between a principal and her agent is able to influence the future transfers among the agents when they bargain, so each principal has incentive to lower her commission (the reward for a good fit) to reduce the outflows of surplus to other principals. Also, the ability of the commission to motivate effort in an agent decreases when fee-sharing is allowed. These effects lower the symmetric equilibrium effort level compared to the benchmarks where fee-sharing is not possible. As a result, efficiency is improved as the agents' efforts would have been wastefully high in the absence of fee-sharing. However, the symmetric equilibrium commission level can be higher depending on the cost function.

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1 Introduction

Principals are typically different, so they need their agents to exert effort to get them a result that fits their individual characteristics. A result that is bad for one principal may be good for another, so opportunities exist for an agent to divert a result to other principals if that can give the agent a higher payment, and at the same time for an agent who cannot create a good fit to turn to other agents for results that can be delivered to his own principal.

An example of such interaction exists among third party recruiters, also called headhunters. They routinely write fee-sharing agreements with other headhunters to finish a job. Other agents, such as talent agents, can make use of the same arrangement to match performance opportunities to artists. Real estate agents also adopt commission-sharing and exchange information on their search results through Multiple Listing Services (MLS).

In these situations, the suitability of a result to a principal is subject to some uncertainty. For example, after exerting search effort for a potential job candidate, an agent might find one that is more suitable for a different position. At the same time, agents typically have good access to information such as other agents' identity and their search results and thus have the freedom to get together with other agents to reach a mutually beneficial arrangement. This freedom obviously brings some efficiency gain in terms of matching results to principals. However, its impact on the equilibrium effort levels and equilibrium contract offers is not immediately clear without careful analysis.

From a more theoretic point of view, it is also interesting to study a multi-principal multi-agent model where agents can interact with each other and principals are restricted to contract only with one agent. We will elaborate on the latter assumption more in terms of how it differs from the existing literature.

More specifically, this paper studies a two-principal and two-agent model, which is easily extendable to n-principal-and-n-agent, where each principal can contract with only one agent by offering him a contract, written to be contingent on the result received. Agents first exert effort, which affects the probability of whether the result is a good fit or not. Then, observing the outcomes, the agents consider whether to deliver the result to his own principal, or to deliver it to the other agent. The fee-sharing arrangement is determined through Nash Bargaining with equal bargaining power between the agent that diverts and the agent that receives.[1]

Contracts, offered by the principals and accepted by the agents, indirectly control the

[1]The equilibrium outcome will look like as if there is an industry convention of splitting the commission half-half and they are just adhering to the convention.

flows of transfers in-between the agents by controlling the agents' total surplus to be shared through bargaining. Principals actually directly care about these inflows and outflows of transfers, because she internalizes her agent's payoff through a contract that makes the participation constraint binding for her agent, but she does not internalizes the other agents' payoff as there is no contract with them. Therefore, the principal would want to use her contract to increase the inflow of surplus and decrease the outflow of surplus. There is a limit to how much a principal can increase the inflow and the incentive for reducing the outflow is more interesting because there the principal also faces tradeoffs, explained below. First, the desire to reduce outflow of surplus makes a principal want to pay less for a good fit. We will call the effect of the contract on her payoff the "leakage effect". This effect was absent in the no fee-sharing benchmarks, as there the principal does not care how much she pays for a good fit as long as it motivates the right effort level because she extracts the payment back. When fee-sharing is allowed, in the event of receiving a result from agent 2, principal 1's payment for a good fit (beyond what principal 2 pays for a bad fit) gets shared between two agents. Since principal 1 does not contract with agent 2, so she cannot extract back the surplus from agent 2, so the part of the commission that goes to agent 2 ultimately goes to the other principal.

Second, the principal has an incentive to increase the payment for a good fit to motivate her own agent to pay effort (as long as the payment is less than the incremental value of a good fit). We will see that since the principal internalizes the cost of the effort of her own agent, this effect alone would call for a sell-out contract. We will call this the "self-motivating effect".

Third, the principal has an incentive to increase the payment for a good fit to de-motivate the other agent to pay effort. If the agent pays less effort, there is a higher chance that the other agent's result is useful to this principal. Since the principal does not internalize the other agent's cost of effort, this effect alone calls for a higher payment as long as the leaked out surplus is still less than the incremental value of a good fit. We will call this the "cross-motivating effect".

The second and third effect of a contract is through its effect on agents' effort. They form a trade-off with the first effect. How the equilibrium contracts and efforts of a model with result-diverting and fee-sharing would compare to the case of no fee-sharing depends on how the above three effects play out. Except for when the cost of effort is too low or too high (where the equilibrium effort level hits the boundary constraints), the equilibrium effort level is lower than the benchmarks with no fee-sharing allowed. The reason is that the ability of a given level of reward for a good fit to motivate effort is weaker when there can be fee-sharing. To motivate the same level of effort as in the no fee-sharing benchmarks, the reward for a good fit has to higher. However, the principal will never want to offer such a high level of reward because at that high level of reward, both the leakage and the self-motivating effects are negative, while the cross-motivating effect is zero.

At the same time, because the ability of a given level of reward for a good fit to motivate effort is weaker when there can be fee-sharing, the equilibrium reward for a good fit might have to be *higher*, just to motivate that *lower* level of effort.

We also show that the welfare is improved when fee-sharing is allowed. The improvement not only comes from better matching of project results to principals, but also from lower level of socially-wasteful efforts.

This model also highlights the role of the restriction that a principal cannot contract with all agents whose action affects this principal's payoff. This is the reason why sell-out contracts are not efficient. If each principal can offer two contracts, one to each agent. We show that there is always a symmetric equilibrium that is first-best, with properly structured "sell-out" contracts that sell out to all agents.[2]

Literatures The paper is related to the common agency models in the sense that one agent's effort affect multiple principals' payoff. Common Agency models however assumes that each principal can offer contracts to every agent.[3] In this paper, since a principal only contract with one agent and thus does not internalize the other agent/principal's utility, the first-best outcome cannot be achieved even though the collusive outcome would be first-best. The fact that efficiency can be restored in this model if each principal can offer two contracts, one to each agent, echoes the result in Segal (1999), which showed that, when a principal can offer multiple bilateral public offers, if there is no externality on agents' reservation utilities, the equilibrium is efficient.

There is also a literature that focuses on agents that are hired to do search. Lewis and Ottaviani (2008) studied a general single-principal and single-agent model where the agent can gain private benefits over the cause of searching, and their focus was on the interaction of the agent's incentives to exert search effort with agent's incentives to report the private information the agent acquires during the search process. In comparison, this paper looks at the agents' incentives to exert targeted search effort when their payoff is linked to other agents and thus other principals.

There is also a literature on referral, another form of interaction among agents. It has been studied in a non-principal-agent setting in Garicano and Santos (2004), which focused on matching opportunities with agents' talent, rather than on matching known results with principals' characteristics.

[2]In other words, there exists an optimal mechanism to implement the efficient outcome.

[3]Bernheim and Whinston (1986), Bernheim and Whinston (1985).

The rest of the paper is organized as follows. Section 2 presents the model. Next, section 3 presents the analysis of the three benchmarks. Section 4 shows the main model and contrasts its results with the three benchmarks. Section 5 discusses the assumptions, and extends the model in different direction, including a n-principal-n-agent model. Section 7 concludes.

2 Model

There are two principals, 1 and 2. They only differ in just one dimension, so we can say without loss of generality that an outcome that is a good fit for one principal is a bad fit for the other.

Principal 1 can contract with agent 1; and principal 2 can contract with agent 2. The principal-agent relationship is assumed to be fixed. We will refer to an agent as him and a principal as her.[4]

We assume the value of a bad fit to a principal to be 0, and that of a good fit normalized to be $d \geq 0$. Agents can exert effort to increase the probability of a good fit to his own principal. Specifically, if agent exerts effort e_1, then the probability of getting a good fit for principal 1 is $\frac{1}{2} + e_1$. We will refer to such effort as "targeted effort". A principal can enjoy more than one piece of results, while the agents can only create one piece of result each.

We assume the effort choice set is $[0, \frac{1}{2}]$. The two agents have the same cost function of effort, equaling to $C(e)$. Suppose $C'(0) = 0$. Assume $C'(e) > 0, C''(e) > 0$ and $C'''(e) \geq 0$ for any $e \in [0, \frac{1}{2}]$.

After exerting effort and after observing the realization of result, an agent can choose from one of the three actions:

1. Divert the result to the other agent, who will in turn deliver it to the other principal.
2. Deliver it to his own principal.
3. Hide it and not give it to anyone.

[4]This fixed pairing is without loss of generality under the assumption that each principal can only hire one agent (or hiring more than one agent is prohibitively costly). Research is under way for endogenizing such a choice by a principal to limit the number of contracting agents without relying on exogenous contracting cost.

We will denote the action set by $A = \{\text{Divert, Deliver, Hide}\}$.

Tie breaking assumptions Assume that when indifferent, an agent will choose not to hide a result. When indifferent, an agent will choose to divert a result.

Note that potentially each principal can receive and derive benefit from more than one result.

2.1 Information and Contracts

The agents' effort choices and the action in the set A are not observable to the principals. The two agents have complete information between themselves when Nash bargaining.

Both principals can offer a contract to her own agent and the payment can be contingent on the verifiable value of the result the principal receives. We restrict the contract to be non-discriminatory over all good fits, i.e., two good fits will be rewarded just twice as much as one good fit.[5]

This implies that the contract can be written to be contingent on three different realizations: a good fit, a bad fit or no result. Therefore, we will characterize a contract by three elements. First, a fixed fee $F \in (-\infty, +\infty)$, to be paid regardless of what happens. Second, an additional payment t, with $t \in (-\infty, +\infty)$, to be paid when a bad fit is delivered to the principal. Third, an additional payment $s + t$ for a good fit with $s \in (-\infty, +\infty)$. So, principal 1 offers (F_1, t_1, s_1) and principal 2 offers (F_2, t_2, s_2). The fixed part of the contract is always set to make the agents' rationality constraints to be binding, so we can characterize all contracts with only t_i and s_i for $i = 1, 2$. A useful notation is $k_i \equiv s_i/d$, which can be thought of as the piece-rate for the extra benefit of a good fit.

2.2 Timing

The timing of the game is as follows.

1. Principals 1 and 2 simultaneously make take-it-or-leave-it contract offers to their respective agents.

[5] The assumption of allowing multiple units per principal and that the contract have to non-discriminatory greatly simplifies the analysis, and rules out the strange effect that a higher reward for a good fit from principal 1 can motivate agent 2 to pay higher effort through a indirect effect from agent 1's effort level. We will explain this in more detail in the appendix.

2. Agents 1 and 2 simultaneously decide whether to accept the offers or not.
3. Agents 1 and 2 simultaneously exert efforts.
4. Project values are realized.
5. Agents decide whether or not to divert their results.
6. A diverter and a receiver Nash Bargain to determine the transfer between them (with full information).
7. Agents 1 and 2 deliver results to their respective principals and receive the promised payments from their principals.

2.3 Equilibrium concept

All players are risk-neutral and they maximize their expected payoff. We assume there is no discounting and normalize the outside options of the agents to be zero. Principals' strategies are simply the contracts they offer. The agents' behavioral strategies are, 1) the acceptance decision given the two contracts offered, 2) effort choices given what contracts are accepted, and 3) diverting or not given the realized results.

Despite that there is incomplete information in this model, since the principals only act once at the beginning of the game by offering contracts, their beliefs will not play a role. Here, like in any pure screening model, the set of Subgame perfect Nash equilibrium is identical to the sets of strategy profiles in weak perfect Bayesian equilibrium or sequential equilibria, so we will use the concept of Subgame Perfect Nash Equilibrium.

2.4 Benchmarks

We will consider the following benchmarks for isolating the effects of fee-sharing arrangement and for drawing welfare implications:

- **No diverting (and no fee-sharing) benchmark.** This is where we exogenously shut-down all interactions among the agents.
- **Allowing diverting but no fee-sharing benchmark.** This is where we allow the agent to divert the result to the other agent, but does not allow them to make any transfers to each other.

- **First-best benchmark.** This is where we solve for socially optimal pair of efforts, i.e., the first-best efforts, and show the pair of contracts that would implement the first-best efforts (allowing the agents to be strategic and allowing diverting and fee-sharing), which we will call the first-best contracts.

We will adopt the following notations for the symmetric equilibrium outcomes for the three benchmarks and the main model:

	Effort	Contract	Welfare
No diverting benchmark	$\hat{e}$	$\hat{k}$	$\hat{W}$
Allowing diverting but no fee sharing benchmark	$\check{e}$	$\check{k}$	$\check{W}$
First-best benchmark	e^*	k^*	W^*
Main model (allowing diverting and fee sharing)	$\dot{e}$	$\dot{k}$	$\dot{W}$

3 Benchmarks

3.1 No diverting benchmark

There is no connection between two principal-agent pairs, so we can just study one pair in isolation. The analysis is very standard: a sell-out contract is optimal for the principal. Any $t_i \geq 0$ and $s_i = d$ (coupled with appropriate F_i that extracts all surplus) will implement the optimal effort for each principal. Here t_i will work just as F_i as a fixed fee for the agent. Other contracts with $s_i + t_i = d$ and $t_i < 0$ (coupled with appropriate F_i that extracts all surplus) will also work, where agents hide the result if it is bad and gets paid d if the outcome is good. No matter the sign of t_i, the basic structure is the same: a fixed fee that extracts surplus and the agent gets paid d more only if he delivers a good fit.

Lemma 1. *In the no-trading benchmark, the equilibrium outcome is the following.*

Both agents exert effort effort level $\hat{e}$, where $\hat{e}$ is determined by the following equation if $d < C'(\frac{1}{2})$, otherwise, it is $\frac{1}{2}$:

$$d - C'(\hat{e}) = 0$$

Let $\Pi = (\frac{1}{2} + \hat{e})d - C(\hat{e})$.

Both principals offer two sell-out contracts: the agent is guaranteed an amount $F = -\Pi < 0$, and if and only if he delivers a good fit, he will be paid an additional $\hat{s} = d$, equivalent to a piece-rate of $\hat{k} = 1$.

Both agents accept the contract and they always deliver a result to their own principals.

The expression $d - C'(\hat{e}) = 0$ is simply the agent's first order condition when the solution is interior.

3.2 Allowing diverting but no fee sharing benchmark

Notice that all symmetric equilibria in the No-diverting Benchmark are also equilibria here. Since agents will not get paid if they divert, they will divert according to our tie-breaking assumption. (Even when the tie-breaking assumptions are different, the equilibrium effort will be the same, while efficiency pay change.)

Moreover, these are the only equilibria, because no matter what the other principal offer, each principal can always implement $\hat{e}$ and extracts all the surplus from her agent. Also, a principal cannot do better than that.

Therefore, this benchmark gives exactly the same equilibrium efforts and contract structure as the previous one.

Lemma 2. *Allowing diverting but no fee-sharing benchmark gives the same equilibrium efforts and contract structure as the no diverting benchmark, i.e.,* $\check{s} = d$ *(equivalent to a piece-rate of* $\check{k} = 1$*) and* $\check{e}$ *is determined by the following equation if* $d < C'(\frac{1}{2})$*, otherwise, it is* $\frac{1}{2}$*:*

$$d - C'(\check{e}) = 0$$

Remark: This shows that what really matters for the effort levels and piece-rate in the agents' interaction is the fee-sharing arrangements. Since the no-diverting and the allowing diverting but no fee-sharing benchmarks have the same equilibrium effort levels and piece-rates, we will sometimes refer to them together as **no fee-sharing benchmarks**.

3.3 First-best benchmark

Since whenever a result is created, it is going to be good for some principal, the efficient effort level is $e^* = 0$.[6] This shows that in the previous two benchmarks, efforts are overly-high.

[6] If agent 1 gets a good fit and agent 2 gets a bad fit, then both agents give the results to principal 1 who enjoys two pieces of good fits, while principal 2 gets nothing.

To implement the efficient level of effort and efficient diverting decision, the contracts must have only a fixed part. (If a contract rewards good fit over bad fit, then effort will be positive. If a contract punishes good fit over bad fit, the diverting decision will be inefficient. Therefore, the contract must pay the same no matter a good or a bad fit is delivered. This implies $s = 0$ and $t \geq 0$. Then diverting will not give an agent any payment, so to make sure that the agent will divert, $t = 0$.

Each principal can extract half of the total surplus, which is d.

Lemma 3. *The first-best outcomes are $e^* = 0$. Given two contracts with only a negative fixed payment $-d$, there is a symmetric equilibrium where the first-best level of welfare is achieved.*

4 Main Model

4.1 Preliminary results

We denote the subgame effort equilibrium by ($\tilde{e}_1(s_1, s_2, t_1, t_2)$ and $\tilde{e}_2(s_1, s_2, t_1, t_2)$) or simply $\tilde{e}_1$ and $\tilde{e}_2$) for convenience.[7] We will also use $1 \to 2$ to denote the event that agent 1 diverts a bad fit to agent 2 (so that a bad fit becomes a good fit), and $1 \leftarrow 2$ to denote a diversion of a bad fit in the other direction.

Because of the following lemma, we will focus on studying principal 1's best response to principal 2 given that principal 2 choose a strategy satisfying $s_2 + t_2 \geq 0$.

Lemma 4. *On any symmetric equilibrium, it must be that agents deliver a good fit to their own principals and $\dot{s} + \dot{t} \geq 0$.*

This lemma is proved in the Appendix.

We first tackle the choice of t_1. The following lemma allows us to convert a problem where a principal can potentially control two parameters s and t into a problem where she essentially picks only one choice variable.

Lemma 5. *Given any strategy of principal 2 with $s_2 + t_2 \geq 0$, there always exists a best response of principal 1 must have $t_1 = 0$ and $s_1 \geq 0$.*

[7]The bounded choice set implies that the subgame effort equilibrium exists and the convexity of $C(\cdot)$ implies that it is unique.

Remark: In general, a principal can choose to reward or punish an agent upon receiving a bad fit relative to receiving no result. Interestingly, increasing the amount promised for a bad fit, while keeping the incremental reward for a good fit (over a bad fit) the same would give the agent more incentive to create a good fit (even though on equilibrium a principal never receives a bad fit). This is because rewarding a bad fit increases the agent's threat point and reduces the net gain for the agent from Nash bargaining when the agent tries to divert away a bad fit. This decreases the attractiveness of the event of getting a bad fit for the agent and thus can motivate the agent to pay effort to create a good fit. However, we show that it is not profitable for the principal to reward a bad fit (over no-result) despite this subtle effect.

Proof. Here we prove for the case of the subgame following s_1, s_2, t_1, t_2 such that $s_1 + t_1 > \max\{t_2, 0\}$ and $s_2 + t_2 > \max\{t_1, 0\}$. For the other cases, refer to the Appendix.

The threat point of agent 1's Nash Bargaining when he gets a bad fit is what he can get if the bargaining fails, which is $\max\{t_1, 0\}$ (disregarding the fixed payment he is guaranteed of). So the role of t_1 depends on whether it is positive or negative.

Case 1. $t_1 \geq 0$. Given the contracts and the other agent's effort choice, agent 1 solves:

$$\begin{aligned} &\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)(s_1 + t_1) + (\frac{1}{2} - e_2)\frac{s_1 + t_1 - \max\{t_2, 0\}}{2} \\ &\qquad\qquad + (\frac{1}{2} - e_1)(\frac{s_2 + t_2 - t_1}{2} + t_1) - C(e_1) \\ \Leftrightarrow &\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)s_1 + (\frac{1}{2} - e_1)\frac{s_2 + t_2 - t_1}{2} - C(e_1) \end{aligned}$$

Therefore, the first order condition for an interior solution of e_1 is:

$$s_1 - \frac{s_2 + t_2 - t_1}{2} = C'(e_1)$$

A higher positive t_1 decreases the payment from agent 2 to agent 1 when $1 \to 2$, so it increases agent 1's incentive to pay effort to create a good fit. Both s_1 and t_1 can increase the effort of agent 1, but the effect from s_1 is stronger. For any interior effort levels, we have:

$$\frac{\partial \tilde{e}_1}{\partial s_1} = \frac{1}{C''(\tilde{e}_1)} > \frac{1}{2C''(\tilde{e}_1)} = \frac{\partial \tilde{e}_1}{\partial t_1} > 0$$

Principal 1's payoff is:

$$U_1 \equiv (\frac{1}{2}+\tilde{e}_1)d + (\frac{1}{2}-\tilde{e}_2)(d - \frac{s_1+t_1-\max\{t_2,0\}}{2}) + (\frac{1}{2}-\tilde{e}_1)\frac{s_2+t_2-t_1}{2} - C(\tilde{e}_1)$$

From the principal 1's payoff, we see that, aside from effects on $\tilde{e}_1$ and $\tilde{e}_2$, a higher s_1 increases the payment leaked to agent 2 when $1 \leftarrow 2$ and a higher t_1 when $t_1 > 0$ increases the payment leaked to agent 2 (with the same strength as s_1) and in addition reduces the payment gained by agent 1 when $1 \rightarrow 2$. Therefore, a cost-benefit analysis suggests that principal should use a more cost effective s_1 to motivate effort instead of t_1. Formally, we can prove by contradiction.

Suppose there is a best response with $t_1 > 0$. Consider an alternative for principal 1 t_1', s_1' with $t_1' = t_1 - \epsilon$ with $\epsilon > 0$ small enough such that t_1' is still above zero, and at the same time Let $s_1' = s_1 + \frac{\epsilon}{2}$. When ϵ is small enough, we are still in the parameter case $s_1 + t_1 > \max\{t_2, 0\}$ and $s_2 + t_2 > \max\{t_1, 0\}$. This alternative makes $s_1' + \frac{t_1'}{2} = s_1 + \frac{t_1}{2}$, so the incentive for agent 1 is the same, i.e. $e_1' = e_1$. However, $s_1' + t_1' < s_1 + t_1$, so $e_2' \leq e_2$. This implies the payoff strictly increases. Therefore, this contradicts the definition of a best response.

Case 2. $t_1 \leq 0$. Given s_1, s_2, t_1, t_2 and e_2, agent 1 solves:

$$\begin{aligned}
&\max_{e_1\in[0,\frac{1}{2}]} \quad (\frac{1}{2}+e_1)(s_1+t_1) + (\frac{1}{2}-e_2)\frac{s_1+t_1-\max\{t_2,0\}}{2} \\
&\qquad +(\frac{1}{2}-e_1)(\frac{s_2+t_2}{2}+0) - C(e_1) \\
\Leftrightarrow &\max_{e_1\in[0,\frac{1}{2}]} \quad (\frac{1}{2}+e_1)(s_1+t_1) + (\frac{1}{2}-e_1)\frac{s_2+t_2}{2} - C(e_1)
\end{aligned}$$

The first order condition for an interior solution is:

$$(s_1+t_1) - \frac{s_2+t_2}{2} = C'(e_1)$$

Notice that when $t_1 < 0$, t_1 has the same motivating effect on e_1 as s_1.

When $t_1 \leq 0$, principal 1's payoff is:

$$U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_2)(d - \frac{s_1 + t_1 - \max\{t_2, 0\}}{2}) + (\frac{1}{2} - \tilde{e}_1)\frac{s_2 + t_2}{2} - C(\tilde{e}_1)$$

Here we see that in both the agents' incentive and the principal's payoff, only the sum of $s_1 + t_1$ matters. Therefore, If there is a best response with $t_1 < 0$, we can find another best response with $t_1 = 0$ by decreasing s_1.

□

Other things to notice about the first order condition is that one's own principal's contingent payments motivate effort and the other principal's contingent payments de-motivate effort.

4.2 Main results

Lemma 5 implies that we only need to consider principal 1's best response in a class of strategy with $t_1 = 0$ against a given strategy of principal 2 with $t_2 = 0$.

Taking derivative of principal 1's payoff with respect to k_1, while setting $t_1 = t_2 = 0$,

$$\frac{\partial U_1}{\partial k_1} = -\frac{1}{2}(\frac{1}{2} - \tilde{e}_2)d + ((1 - \frac{k_2}{2})d - C'(\tilde{e}_1))\frac{\partial \tilde{e}_1}{\partial k_1} + (1 - \frac{k_1}{2})d(-\frac{\partial \tilde{e}_2}{\partial k_1})$$

We can identify three effects influencing the choices of k_1:

Leakage effect First, a higher piece-rate leaks more surplus to the other agent, which ultimately goes to the other principal through the binding participation constraint of the other agent. This gives each principal incentive to lower the piece-rate, represented by the negative term $-\frac{1}{2}(\frac{1}{2} - \tilde{e}_2)d$.

Self-motivating effect Second, a higher piece-rate by principal 1 increases effort by agent 1. The positive effect is represented by $((1-\frac{k_2}{2})d - C'(\tilde{e}_1))\frac{\partial \tilde{e}_1}{\partial k_1} = (1-k_1)d\frac{\partial \tilde{e}_1}{\partial k_1}$. However, at $k_1 = 1$, this effect is second order due to the Envelope Theorem.

Cross-motivating effect Third, there is an effect on k_1 through agent 2's effort, $\tilde{e}_2$. When $k_1 < 2$, principal 1 wants a lower $\tilde{e}_2$ to increase the chance of a fit with agent 2' result. This pushes up k_1. When $k_1 > 2$, principal 1 wants to discourage receiving result from agent 2 because too much commission would be leaked to agent 2 in that case. The effect is represented by the term $(1 - \frac{k_1}{2})d(-\frac{\partial \tilde{e}_2}{\partial k_1})$, which is first positive, then negative, as k_1 increases, with a cutoff of $k_1 = 2$.

In the appendix, Lemma 7 (the necessary condition) and Lemma 8 (the sufficient condition) characterize the unique (in terms of effort level) symmetric equilibrium of this model for different parameter cases, based on which we can immediately reach at the comparison with the benchmarks.[8]

Proposition 1. *If cost of effort is high enough such that $C''(0) \geq 6d$, then the first-best level of effort is achieved, i.e., $\dot{e} = e^* = 0 < \check{e} = \hat{e}$.*

If cost of effort is low enough such that $C'(\frac{1}{2}) \leq \frac{3}{5}d$, then the unique equilibrium effort is $\dot{e} = \frac{1}{2}$ and any $\dot{k} > \frac{2}{d}C'(\frac{1}{2})$ and $\dot{t} = 0$ can form an equilibrium.

If cost of effort is in the intermediate range such that $C''(0) < 6d$ and $C'(\frac{1}{2}) > \frac{3}{5}d$, then the unique equilibrium effort is strictly above the first-best level and below that of the no fee-sharing benchmarks, i.e., $e^* < \dot{e} < \check{e} = \hat{e}$. When $\dot{t} = 0$, the piece-rate $\dot{k}$ may or may not be below 1.

Note that at a symmetric pair of contracts with piece-rate k, the subgame equilibrium $\tilde{e}$ (if interior) satisfies $\frac{k}{2}d = C'(\tilde{e})$. Recall $d = C'(\check{e})$ when $\check{e}$ is interior, so to motivate an effort level as high as the one in the no fee-sharing benchmarks, we need a piece-rate as high as 2, at which the cross-motivating effect drops to zero and the other two effects are zero, so the there cannot be an equilibrium with effort level as high as that in the no fee-sharing benchmarks.

However, even when the effort level is interior, the equilibrium piece-rate might be above the sell-out contract. This is because at the piece-rate of the sell-out contract, $k = 1$, there are two effects competing. The leakage effect calls for a lower piece-rate and the cross-motivating effect calls for a higher piece-rate. Lemma 7 shows that the relative strength of the two forces depend on the cost function. For the special case of quadratic function, i.e., $C(e) = ae^2$ with $a > 0$, the condition has a simple form[9]:

[8] The equilibrium contracts are not unique, because given any pair of equilibrium contract $(\dot{k}, \dot{t})$ with $\dot{t} = 0$, we can find another pair $(\dot{k}', \dot{t}')$ by $\dot{k}' = \dot{k} + \frac{x}{d}$ and $\dot{t}' = -x$ for any $x > 0$, which also form an equilibrium. This is because what matters in payoff both on and off the equilibrium is just $s + t$ and $\max\{t, 0\}$ as long as $s + t \geq 0$.

[9] The parameter condition for an interior $\dot{e}$ becomes $a \in (\frac{3}{5}d, 3d)$.

$$\dot{k} \leq 1 \Leftrightarrow a \geq d$$

When cost is very low, even a small piece-rate (when the cross-motivating effect is still very strong and positive), the upper limit of the effort can always be hit. (If we consider a special cost function with $\lim_{e \to \frac{1}{2}} C'(e) = \infty$, then this parameter case will disappear.)

Figure 2 and Figure 1 show the equilibria across the benchmarks and the main model for specific cost function examples. The figures highlight the reasons why equilibrium effort level is lower than the no fee-sharing benchmarks. First, principals want to reduce leakage of surplus to other agents and principals, which makes them offer contract with lower piece-rates than what would be necessary to induce effort level as high as in the no fee-sharing benchmarks ($k = 2$). Second, the ability of a piece-rate to motivate effort becomes lower, as now a bad fit can be turned into a good fit by diverting it to other principals, so an agent works less hard to create a good fit given a level of piece-rate. These two effects cause effort level to be lower than the no fee-sharing benchmarks.

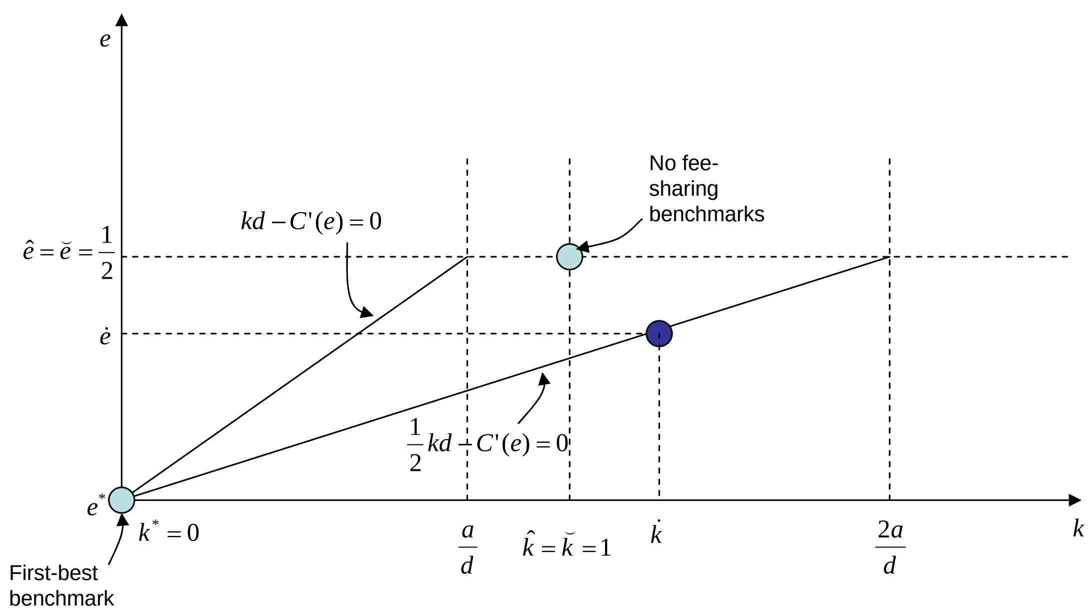

Figure 1: Quadratic cost function $C(e) = ae^2$ with constant $a \in (\frac{3}{5}d, d)$.

Since the matching between results and principal is efficient under first-best benchmark, allowing diverting but no fee-sharing benchmark and the main model, and the main model's equilibrium effort lies in between the first-best and that of the allowing diverting but no fee sharing, the welfare ranking is immediate.

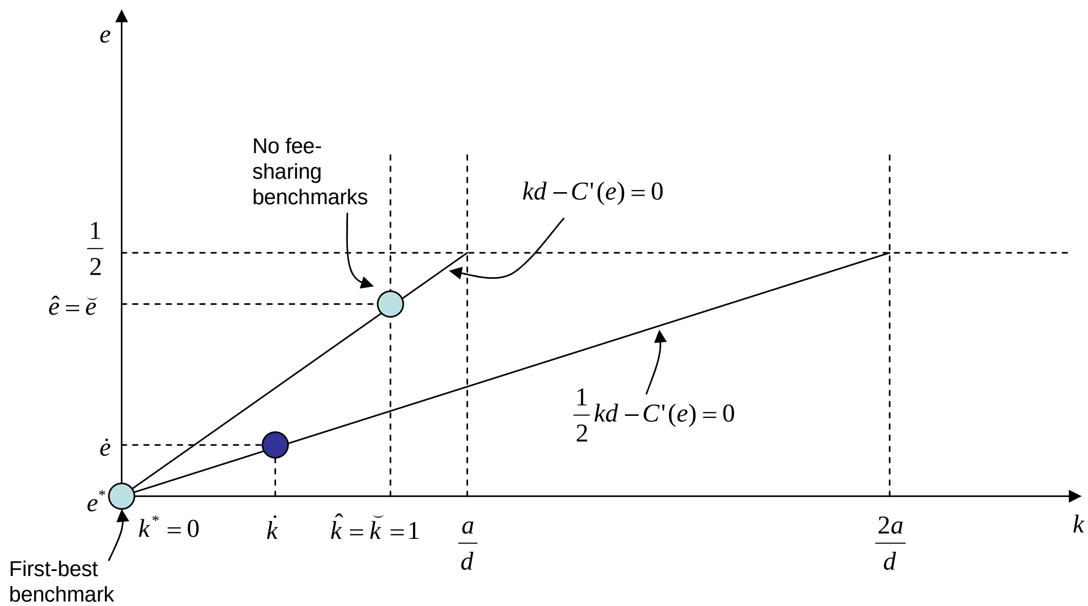

Figure 2: Quadratic cost function $C(e) = ae^2$ with constant $a \in (d, 3d)$.

Corollary 1. *The welfare comparison is:*

$$W^* \geq \dot{W} \geq \breve{W} > \hat{W}$$

Since all surplus are shared equally by the two principals, the principals' welfare increases when the fee-sharing is allowed. Even though each agent works less hard to create a good fit, the chance of getting a good fit is higher for the principal under fee-sharing. To see that, without fee-sharing, each principal has probability $(\frac{1}{2} + \hat{e})$ of getting a good fit. With fee-sharing, each principal has probability $(\frac{1}{2} + \dot{e})$ of getting a good fit from her own agent and probability $(\frac{1}{2} - \dot{e})$ of getting a good fit from the other agent. These two events can happen at the same time, at which this principal receives two good fits.

5 Discussion

5.1 The restriction of one agent to contract with

We want to highlight the following assumption.

Assumption: each principal can only contract with one agent.

This assumption is important for the comparison with first-best. To see that we can modify the model such that each principal simultaneously offer one contract to each agent and the contract offer also specify that it is void unless the agent also accept the contract offered by the other principal (such as in Bernheim and Whinston (1985)).

We can denote a contract offered to agent 1 by principal 1 as k_1, F_1, one offered to agent 2 by principal 1 as $\tilde{k}_2, \tilde{F}_2$, one offered to agent 1 by principal 2 as $\tilde{k}_1, \tilde{F}_1$, and one offered to agent 2 by principal 2 as k_2, F_2. Recall that e^* denotes the first-best effort level.

Proposition 2. *There exists a first-best symmetric equilibrium, where each principal pays piece-rate $k_1 = k_2 = \frac{1}{2}$ to her own agent and piece-rate $\tilde{k}_1 = \tilde{k}_2 = \frac{1}{2}$ to the other agent. The equilibrium effort level is $e^* = 0$.*

The proof is in the Appendix.

Here principal 1 not only pays agent 1 more when she gets a good fit, she also pays agent 2 more, not knowing who created the good fit. First-best is achieved because the principal internalizes both agents' payoff. Notice that $k_1 + \hat{k}_2 = 1$, so each principal "sells out" the project, not to one agent, but to two agents. Then, when each principal is maximizing by picking piece-rate, she is maximizing the total payoff of four players.

Despite the desirability of such an equilibrium, there are many real world applications where principals do not have the ability to offer contracts to many agents. Just think of the case of headhunters, it is prohibitively expensive for a company to contract with all headhunters who can potentially exert effort to find the company a good candidate.

5.2 Un-targeted effort

The paper modeled targeted effort, efforts that is useful to only one principal. Similar analysis can be applied to efforts that are "un-targeted", i.e., efforts that are valuable to both principals. One example of such effort is if effort just increases the vertical value of the outcome. The "leakage" problem is still there and it depresses the piece-rates and efforts compared to the no-trading benchmark. Since allowing trading or not does not change the value of such "un-targeted" effort, the no-trading benchmark level effort is the same as the first-best level of effort. Therefore introducing trading pushes the equilibrium level of effort away from the first-best.

6 N principals and n agents

The model can be easily extended to n principals and n agents with $n > 1$. One can imagine n principals differ in one dimension and they are located on a circular city. The model is symmetric, so what we describe for agent 1 applies to all other agents with the appropriate substitution of index.

If agent 1 exerts effort e_1, then his result has probability $(\frac{1}{n} + e_1)$ to fit principal 1 and probability $(\frac{1}{n} - \frac{e_1}{n-1})$ to fit any other principals. Principal 1 can choose effort from $[0, \frac{n-1}{n}]$. The rest of the model does not change.

Agent 1's problem, given contracts $(k_1, k_2, \ldots, k_n)$, becomes:

$$\max_{e_1} \quad (\frac{1}{n} + e_1)k_1 d + (\frac{n-1}{n} - \frac{\Sigma_2^n e_i}{n-1})\frac{k_1}{2}d + (\frac{1}{n} - \frac{e_1}{n-1})\frac{\Sigma_2^n k_i}{2}d - C(e_1)$$

The effect of a piece-rate on the subgame equilibrium effort level is:

$$\frac{\partial \tilde{e}_1}{\partial k_1} = \frac{d}{C''(e_1)}$$
$$\frac{\partial \tilde{e}_i}{\partial k_1} = -\frac{1}{n-1}\frac{d}{2C''(e_1)} \text{ for all } i \neq 1$$

Therefore, the principal 1 maximizes the following payoff:

$$U_1 = (\frac{1}{n} + \tilde{e}_1)d + (\frac{n-1}{n} - \frac{\Sigma_2^n \tilde{e}_i}{n-1})(d - \frac{k_1}{2}d) + (\frac{1}{n} - \frac{\tilde{e}_1}{n-1})\frac{\Sigma_2^n k_i}{2}d - C(\tilde{e}_1)$$

$$\frac{\partial U_1}{\partial k_1} = -\frac{1}{2}(\frac{n-1}{n} - \frac{\Sigma_2^n \tilde{e}_i}{n-1})d + (1-k_1)d\frac{\partial \tilde{e}_1}{\partial k_1} + (1 - \frac{k_1}{2})d(-\Sigma_2^n \frac{\partial \tilde{e}_i}{\partial k_1})$$

At a symmetric equilibrium, the first order derivative is:

$$f(k) \equiv \left.\frac{U_1}{k_1}\right|_{k_1=k_2=\ldots=k_n=k} = -\frac{1}{2}(\frac{n}{n-1} - e)d + (d - 2C'(e))\frac{d}{C''(e)} + (d - C'(e))\frac{d}{2C''(e)}$$

with $\frac{kd}{2} = C'(e)$.

Here we see that the only impact of looking at n instead of 2 is that the leakage effect is stronger here. The more other principals and agents there are, the more likely it is for the principal to receive a result from outside. Since equilibrium $\dot{e}$ and $\dot{k}$ is determined through $f(k) = 0$ for intermediate level of cost, higher n gives lower $\dot{e}$ and $\dot{k}$. However, the relative comparison across the benchmarks does not chance. When n goes up, the higher leakage effect does not interact with effort, so the analysis for general n is an almost word-to-word replica of the analysis for the case of $n = 2$.[10]

Proposition 3. *If cost of effort is high enough such that $C''(0) \geq \frac{3dn}{n-1}$, then the first-best level of effort is achieved, i.e., $\dot{e} = e^* = 0 < \check{e} = \hat{e}$.*

If cost of effort is low enough such that $C'(\frac{n-1}{n}) \leq \frac{3}{5}d$, then the unique equilibrium effort is $\dot{e} = \frac{n}{n-1}$ and any pair of $\dot{k} > \frac{2}{d}C'(\frac{n}{n-1})$ can form an equilibrium.

If cost of effort is in the intermediate range such that $C''(0) < \frac{3dn}{n-1}$ and $C'(\frac{n-1}{n}) > \frac{3}{5}d$, then the unique equilibrium effort is strictly above the first-best level and below that of the no fee-sharing benchmarks, i.e., $e^* < \dot{e} < \check{e} = \hat{e}$. The piece-rate $\dot{k}$ may or may not be below 1.

The welfare comparison is the same as in the case of $n = 2$.

6.1 Bargaining with unequal bargaining power

The agents do not have to have equal bargaining power. As long as the contributor of the result gets strictly positive bargaining power, the results of the model go through. To keep the model symmetric, suppose that the agent who divert a result gets r share of the inter-agent surplus and the agent who receives it and delivers it to his principal gets $1 - r$, with $r \in (0, 1]$.

To achieve the no fee-sharing benchmark level of effort with a symmetric pair of contract, we must have $k = \frac{1}{1-r}$.

At a symmetric equilibrium, the first order derivative is:

[10]It is arguable that when the number of principals and agents change, the value of a fit d might change as well. Since the magnitude of d does not play a role in the analysis, any assumption regarding d can be easily accommodated by the model.

$$f(k) \equiv \frac{U_1}{k_1}\bigg|_{k_1=k_2=k} = -r(\frac{1}{2} - e)d + (1-k)\frac{d^2}{C''(e)} + (1-rk)\frac{rd^2}{C''(e)}$$

with $(1-r)kd = C'(e)$. This shows that the cross-motivating effect turns negative at $k = \frac{1}{r}$.

There are two cases. Case 1. $r \geq \frac{1}{2}$. This case is equivalent to $\frac{1}{1-r} \geq \frac{1}{r}$. In order to motivate the same level of effort as in the no fee-sharing benchmark when the effort level is interior, the the piece-rate k has to be equal to $\frac{1}{1-r}$, at which level all three effects: leakage, self-motivating and cross-motivating effects are non-positive.

Case 2. $r < \frac{1}{2}$. This implies $\frac{1}{r} > \frac{1}{1-r} > \frac{1}{1+r}$. We will see that when $k > \frac{1}{1+r}$, the negative self-motivating effect is always stronger than the positive cross-motivating effect. This is because $k > \frac{1}{1+r} \Leftrightarrow -r(1-rk) < -(1-k)$. Since $k = \frac{1}{1-r}$ satisfies $k > \frac{1}{1+r}$. This sum of three effects is negative at $k = \frac{1}{1-r}$.

Therefore, the symmetric equilibrium effort level is lower (weakly) than that in the no fee-sharing benchmark regardless of the bargaining powers.

7 Conclusion

In this paper, we show that, when fee-arrangement and result-diverting are allowed among agents, who can exert effort to increase the probability of a fit, not only the efficiency increases because of gain from better matching between results and principals, but also from less wasteful targeted efforts. Lower equilibrium effort is due to two reasons. First, principals want to reduce leakage of surplus to other agents and principals, which makes them offer contract with lower piece-rates than what would be necessary to induce effort level as high as in the no fee-sharing benchmarks. Second, the ability of a piece-rate to motivate effort becomes lower, as now a bad fit can be turned into a good fit by diverting it to other principals, so an agent works less hard to create a good fit given a level of piece-rate. These two effects causes effort level to be lower than the no fee-sharing benchmarks. The equilibrium effort level does not reach the first-best level except when the cost of effort is high enough. This comparison with the first-best is due to our assumption that each principal is restricted to contracting with only one agent. Without this assumption, the equilibrium would be fully efficient.

8 Appendix

Proof for Lemma 4.

Proof. We need to rule out two cases: 1) agent diverts a good fit or 2) agent hides a good fit.

Case 1. agent diverts a good fit.

Subcase 1. Agent diverts a good fit and delivers a bad fit. This implies that $s \leq 0$ and $t \geq 0$. Then following the contracts, both agents pay zero effort and the payoff for principal 1 is:

$$\frac{1}{2}\frac{t_2 - (s_1 + t_1)}{2} - \frac{1}{2}t_1 - (s_2 + t_2)2 \leq \frac{1}{2}\frac{t_2}{2}$$

Principal 1 is strictly better off by offering $s_1 = d$ and $t_1 = -\infty$.

All the other cases are refuted in the same spirit. For any symmetric equilibrium where agents either divert their good fits or hide their good fits, their efforts must be zero. So all the payoff for a principal is purely from transfers and each of them have incentive to unilaterally reduce their transfer to the other principals.

□

The remaining part of proof for Lemma 5.

Proof. Suppose (s_1, t_1) is a best response to $(t_2, s_2$.

(i) Consider the subgame following s_1, s_2, t_1, t_2 such that $0 \leq s_1 + t_1 < t_2$ and $s_2 + t_2 > \max\{t_1, 0\}$. This implies that the agent 2 will not divert a bad fit and agent 1 will divert a bad fit.

Case 1. $t_1 \geq 0$. Given the contracts and the other agent's effort choice, agent 1 solves:

$$\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)(s_1 + t_1) + (\frac{1}{2} - e_1)(\frac{s_2 + t_2 - t_1}{2} + t_1) - C(e_1)$$
$$\Leftrightarrow \max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)s_1 + (\frac{1}{2} - e_1)\frac{s_2 + t_2 - t_1}{2} - C(e_1)$$

Therefore, the first order condition for an interior solution of e_1 is:

$$s_1 - \frac{s_2 + t_2 - t_1}{2} = C'(e_1)$$

Principal 1's payoff is:

$$U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_1)\frac{s_2 + t_2 - t_1}{2} - C(\tilde{e}_1)$$

Suppose $t_1 > 0$. If $\tilde{e}_1 < \frac{1}{2}$, then $(\frac{1}{2} - \tilde{e}_1) > 0$, we can again decrease t_1 by ϵ and increase s_1 by $\frac{\epsilon}{2}$ to improve principal 1's payoff to reach a contradiction. If $\tilde{e}_1 = \frac{1}{2}$, then the principal 1's payoff is simply $(\frac{1}{2} + \tilde{e}_1)d - C(\tilde{e}_1)$. We can replace (s_1, t_1) with $t_1' = 0$ and $s_1' > C'(\frac{1}{2})$.

Case 2. $t_1 \leq 0$.

Agent 1's incentive:

$$(s_1 + t_1) - \frac{s_2 + t_2}{2} = C'(e_1)$$

Principal 1's payoff:

$$U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_1)\frac{s_2 + t_2}{2} - C(\tilde{e}_1)$$

Therefore, only $s_1 + t_1$ matters, so if $t_1 < 0$, we can always replace it with $t_1' = 0$ and decreasing s_1.

(ii) Consider the subgame following s_1, s_2, t_1, t_2 such that $s_1+t_1 > \max\{t_2, 0\}$ and $s_2+t_2 < \max\{t_1, 0\}$. This implies that the agent 1 will not divert a bad fit, and agent 2 will divert a bad fit.

Case 1. $t_1 \geq 0$. Given the contracts and the other agent's effort choice, agent 1 solves:

$$
\begin{aligned}
&\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)(s_1 + t_1) + (\frac{1}{2} - e_2)\frac{s_1 + t_1 - \max\{t_2, 0\}}{2} + (\frac{1}{2} - e_1)t_1 - C(e_1) \\
\Leftrightarrow &\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)s_1 + (\frac{1}{2} - e_2)\frac{s_1 + t_1 - \max\{t_2, 0\}}{2} - C(e_1)
\end{aligned}
$$

Agent 1's incentive:

$$s_1 = C'(e_1)$$

Principal 1's payoff:

$$U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_2)(d - \frac{s_1 + t_1 - \max\{t_2, 0\}}{2}) - C(\tilde{e}_1)$$

If $\frac{1}{2} - \tilde{1}_2 > 0$ then it is in principal 1's interest to reduce t_1. Otherwise, one can replace a positive t_1 with zero.

Case 2. $t_1 \leq 0$. Agent 1 solves:

$$
\begin{aligned}
&\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)(s_1 + t_1) + (\frac{1}{2} - e_2)\frac{s_1 + t_1 - \max\{t_2, 0\}}{2} - C(e_1) \\
\Leftrightarrow &\max_{e_1 \in [0, \frac{1}{2}]} \quad (\frac{1}{2} + e_1)s_1 + (\frac{1}{2} - e_2)\frac{s_1 + t_1 - \max\{t_2, 0\}}{2} - C(e_1)
\end{aligned}
$$

Agent 1's incentive:

$$s_1 + t_1 = C'(e_1)$$

Principal 1's payoff:

$$U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_2)(d - \frac{s_1 + t_1 - \max\{t_2, 0\}}{2}) - C(\tilde{e}_1)$$

So only s_1+t_1 matters. Therefore, one can replace a negative t_1 with zero and decreasing s_1 accordingly.

(iii) Consider the subgame following s_1, s_2, t_1, t_2 such that $0 \leq s_1 + t_1 < \max\{t_2, 0\}$ and $s_2 + t_2 < \max\{t_1, 0\}$. This implies that the agent 1 will not divert a bad fit, and agent 2 will not divert a bad fit.

Agent 1's incentive:

$$s_1 = C'(e_1)$$

Principal 1's payoff is:

$$U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d - C(\tilde{e}_1)$$

Therefore, there is always another best response with $t_1' = 0$.

By Lemma 4, we have exhausted all cases. □

Lemma 6. *On any symmetric equilibrium, $\dot{s} \leq 2d$.*

Proof. Suppose $s > 2d$, then when agent 2 sends over a result that is a good fit for principal 1, principal suffers a net loss because $d - \frac{s}{2} < 0$. The payoff of principal 1 is $(\frac{1}{2} + e_1)d + (\frac{1}{2} - e_2)(d - \frac{s_1}{2}) + (\frac{1}{2} - e_1)\frac{s_2}{2} - C(e_1) = d - C(e)$. Consider a deviation principal 1 keeps the same t_1, but reduces t_1 to $t_1' = 2d$. Then the deviation profit is $(\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_1)\frac{s_2}{2} - C(\tilde{e}_1) > d - C(\tilde{e}_1) \geq d - C(e)$, which is a strictly positive deviation. □

Lemma 7. *(Necessary condition) If there exists a symmetric equilibrium in the main model, $(\dot{e}, \dot{k}, \dot{t}$ with $\dot{t} = 0$, then the following is true.*

1. *If $C''(0) \geq 6d$, then $\dot{e} = 0$, $\dot{k} = 0$ and $\dot{(t)} = 0$.*

2. *If* $C'(\frac{1}{2}) \leq \frac{3}{5}d$*, then* $\dot{e} = \frac{1}{2}$ *and* $\dot{k} \geq \frac{2}{d}C'(\frac{1}{2})$*.*

3. *If* $C''(0) < 6d$ *and* $C'(\frac{1}{2}) > \frac{3}{5}d$*, then* $\dot{e} \in (0, \check{e})$ *and is also unique, and* $\dot{k} = \frac{2}{d}C'(\dot{e})$*,* $\dot{t} = 0$*. In particular, if the following condition is satisfied, then* $\dot{k} \leq 1$ *and otherwise,* $\dot{k} \in (1, \frac{6}{5})$*:*

$$\frac{1}{2} - e \geq \frac{C'(e)}{C''(e)} \quad \textit{with } C'(e) = \frac{d}{2}$$

Proof.

$$\max_{k_1,t_1} \quad U_1 \equiv (\frac{1}{2} + \tilde{e}_1)d + (\frac{1}{2} - \tilde{e}_2)(d - \frac{k_1 d + t_1 - t_2}{2}) + (\frac{1}{2} - \tilde{e}_1)\frac{k_2 d + t_2 - t_1}{2} - C(\tilde{e}_1)$$

Denote by $f(k)$ the first order derivative of a principal's payoff with respect to k_1 with $k \in (0, \frac{2}{d}C'(\frac{1}{2})$. Denote the symmetric effort level corresponding to k by e. That is,

$$f(k) \equiv \left.\frac{U_1}{k_1}\right|_{k_1=k_2=k, t_1=t_2=t} = -\frac{1}{2}(\frac{1}{2} - e)d + (d - 2C'(e))\frac{d}{C''(e)} + (d - C'(e))\frac{d}{2C''(e)}$$

with $\frac{kd}{e} = C'(e)$. We use $U_1(k)$ to denote $U_1|_{k_1=k_2=k}$.

First, there cannot be equilibrium with $\dot{k}$ so big such that $d - C'(\dot{e}) < 0$ because it implies $f(\dot{k}) < 0$ and both principal will be better off to decrease the piece-rate by a little bit. Therefore, equilibrium $\dot{k} < 2$, i.e. $d - C'(e) > 0$. This immediately implies $\dot{e} < \check{e} = \hat{e}$.

Second, we will show that $U_1(k)$ is strictly concave in k for all $k \leq \frac{6}{5}$.

$$\begin{aligned}
f'(k) =& \frac{d^2}{4C''(e)} - [d - 2C'(e)]\frac{d}{(C''(e))^2}C'''(e)\frac{\partial e}{\partial d} - 2d\frac{\partial e}{\partial d} \\
& - \frac{d^2}{4C''(e)} - (d - C'(e))\frac{d}{2(C''(e))^2}C'''(e)\frac{\partial e}{\partial d} \\
\sim & - [d - 2C'(e)]\frac{C'''(e)}{(C''(e))^2} - 2 - (d - C'(e))\frac{C'''(e)}{2(C''(e))^2} \\
=& (-\frac{3}{2}d + \frac{5}{4}kd)\frac{C'''(e)}{(C''(e))^2} - 2
\end{aligned}$$

This implies for any $k \leq \frac{6}{5}$, we have $f'(k) \sim -2 < 0$. This means that $U_1(k)$ is strictly concave over $[0, \frac{6}{5}]$.

Third, we show that $U_1(k)$ is decreasing over $k > \frac{6}{5}$.

$$f(\frac{6}{5}) = -\frac{1}{2}(\frac{1}{2} - e)d - \frac{1}{5}d\frac{d}{C''(e)} + \frac{1}{5}d\frac{d}{2C''(e)} = -\frac{1}{2}(\frac{1}{2} - e)d < 0$$

Notice that $k \geq 1$ implies that $(d - 2C'(e))\frac{d}{C''(e)} < 0$. Also, $(d - C'(e))\frac{d}{2C''(e)}$ is decreasing in k (as $-\frac{d^2}{4C''(e)} - (d - C'(e))\frac{d}{2(C''(e))^2}C'''(e)\frac{\partial e}{\partial d} < 0$). Therefore, for all $k < \frac{6}{5}$, we have $f(k) < 0$. This implies that $\dot{k} < \frac{6}{5}$.

Also, for any $k \geq \frac{2}{d}C'(\frac{1}{2})$, the effort level is constant at $\frac{1}{2}$ and the principal's payoff is the same as when $k = \frac{2}{d}C'(\frac{1}{2})$.

When $f(0) \leq 0$, we have $f(k) < 0$ for all $k \in (0, \frac{6}{5})$, therefore, the equilibrium $\dot{k} = 0$ and the corresponding effort level is $\dot{e} = 0$. The parameter case equivalent to $f(0) \leq 0$ is $C''(0) \geq 6d$.

When $f(\frac{2}{d}C'(\frac{1}{2}) \geq 0$ (which by itself implies $\frac{2}{d}C'(\frac{1}{2}) < \frac{6}{5}$), we have $f(k) > 0$ for all $k \leq \frac{2}{d}C'(\frac{1}{2})$, therefore, the equilibrium $\dot{e} = \frac{1}{2}$ and the corresponding piece-rate is $\dot{k} \geq \frac{2}{d}C'(\frac{1}{2})$. This parameter case is equivalent to $C'(\frac{1}{2}) \leq \frac{3}{5}d$.

Note that the above two parameter case is disjoint because:

$$\begin{aligned} C''(0) \geq 6d \Rightarrow & C''(e) \geq 6d \text{ for all } e \in (0, \frac{1}{2}) \\ \Rightarrow & C'(\frac{1}{2}) = \int_0^{\frac{1}{2}} C'(e)de \geq 6d\frac{1}{2} = 3d > \frac{3}{5}d \end{aligned}$$

When $C''(0) < 6d$ and $C'(\frac{1}{2}) > \frac{3}{5}d$, i.e., when the cost is in the intermediate range, $k = 0$ or $k = \frac{2}{d}C'(\frac{1}{2})$ cannot appear on a symmetric equilibrium as each principal has profitable local deviation, therefore, we must have $\dot{k} = 0$. Under this parameter case, we have $f(0) > 0$ and $f(\min\{\frac{2}{d}C'(\frac{1}{2}), \frac{6}{5}\}) < 0$ and $f(\cdot)$ is strictly decreasing over $(0, \frac{2}{d}C'(\frac{1}{2}))$. Therefore, there exists a unique $\dot{k} \in (0, \min\{\frac{2}{d}C'(\frac{1}{2}), \frac{6}{5}\})$ with $f(\dot{k}) = 0$.

We can further analyze whether the interior $\dot{k}$ is below or above $k = 1$ (i.e., the piece-rate of a sell-out contract). Because $f(\cdot)$ is strictly decreasing, $\dot{k} \leq 1$ is equivalent to

$0 = f(\dot{k}) \geq f(1)$, which is equivalent to the following:

$$\frac{1}{2} - e \geq \frac{C'(e)}{C''(e)} \quad \text{with } C'(e) = \frac{d}{2}$$

By Lemma 5, we know that if $\dot{e} < \frac{1}{2}$, then $\dot{t} = 0$. □

Lemma 8. *(Sufficient condition) All the equilibrium candidates in Lemma 7 are actually equilibria.*

Proof. Define $g_\kappa(k_1)$ by the following:

$$g_\kappa(k_1) = \left.\frac{\partial U_1}{\partial k_1}\right|_{k_2=\kappa}$$

First, we prove sufficiency for the parameter case $C''(0) \geq 6d$. We study the best response of principal 1 given $k_2 = 0$ (which implies that $e_2 = 0$).

$$g_0(k_1) = -\frac{1}{4}d + (1-k_1)\frac{d^2}{C''(e_1)} + (1-\frac{k_1}{2})\frac{d^2}{C''(0)}$$

$$g_0(1) = -\frac{d}{4} + \frac{d^2}{4C''(0)} \leq -\frac{d}{4} + \frac{d^2}{4(6d)} < 0$$

That is, principal 1's best response is a piece-rate strictly less than 1.

$$g_0'(k_1) = -\frac{d^2}{C''(e_1)} - (1-k_1)\frac{d^2}{(C''(e_1))^2}C'''(e_1)\frac{\partial e_1}{\partial k_1} - \frac{d^2}{4C''(0)}$$

Note that when $k < 1$, we have $g_0'(k_1) < 0$, that is, $g_0(\cdot)$ is strictly concave over $[0, 1]$.

$$g_0(0) = -\frac{d}{4} + \frac{3}{2}\frac{d^2}{C''(0)} \leq -\frac{d}{4} + \frac{3}{2}\frac{d^2}{6d} = 0$$

Strict concavity and $g_0(0) \leq 0$ implies that $g_0(k) \leq 0$ for all $k \in [0,1]$. Therefore, principal 1's best response to $k_2 = 0$ is $k_1 = 0$. That is, $\dot{k} = 0$ is an symmetric equilibrium.

Second, we prove sufficiency for the parameter case $C'(\frac{1}{2}) \leq \frac{3}{5}d$. We study principal 1's best response to $k_2 = \frac{2}{d}C'(\frac{1}{2})$. If $k_1 \geq \frac{2}{d}C'(\frac{1}{2})$, then $e_1 = e_2 = \frac{1}{2}$, and if $k_1 < \frac{2}{d}C'(\frac{1}{2})$, then $e_1 < \frac{1}{2}$ and $e_2 = \frac{1}{2}$.

$$g_{\frac{2}{d}C'(\frac{1}{2})}(k_1) = (d - C'(\frac{1}{2}) - C'(e_1))\frac{\partial e_1}{\partial k_1} + (1 - \frac{k_1}{2})d(-\frac{\partial e_2}{\partial k_1})$$

Since $C'(\frac{1}{2}) \leq \frac{3}{5}d$, $k_1 \leq \frac{2}{d}C'(\frac{1}{2})$ implies $k_1 < 2$. The range $k_1 \leq \frac{2}{d}C'(\frac{1}{2})$ also implies that $C'(e_1) \leq C'(\frac{1}{2})$, that is $d - C'(\frac{1}{2}) - C'(e_1) \geq 0$. Therefore, for $k_1 \leq \frac{2}{d}C'(\frac{1}{2})$, $g_{\frac{2}{d}C'(\frac{1}{2})}(k_1) \geq 0$, so $k_1 = \frac{2}{d}C'(\frac{1}{2})$ is a best response, and so does all $k_1 \geq \frac{2}{d}C'(\frac{1}{2})$, which all gives $\dot{e} = \frac{1}{2}$.

Third, we prove sufficiency for the parameter case $C'''(0) < 6d$ and $C'(\frac{1}{2}) > \frac{3}{5}d$.

$$g_\kappa(k_1) = -\frac{1}{2}(\frac{1}{2} - e_2)d + (1 - k_1)d\frac{\partial e_1}{\partial k_1} + (1 - \frac{k_1}{2})d(-\frac{\partial e_2}{\partial k_1})$$

Notice that $g_\kappa(k_1)$ has the property that if $g_\kappa(\tau) < 0$ for some τ, then for all $k_1 > \tau$, $g_\kappa(\tau) < 0$. This implies that a local second order condition is sufficient for global maximization (even though the payoff function may not be strictly concave on the whole range of k_1).

$$g'_\kappa(\kappa_1) = d\frac{\partial e_2}{\partial k_1} - d\frac{\partial e_1}{\partial k_1} + (1 - k_1)\frac{\partial^2 e_2}{\partial k_1^2} + (1 - \frac{k_1}{2})d(-\frac{\partial^2 e_2}{\partial k_1^2})$$

$$g'_\kappa(\kappa_1)|_{k_1=\kappa} = -\frac{3d^2}{2C''(e)} + (-\frac{5}{4} + \frac{7}{8}\kappa)\frac{d^3 C'''(e)}{(C''(e))^2}$$

Since $\kappa < \frac{6}{5}$, we have $g'_\kappa(\kappa_1)|_{k_1=\kappa} < 0$. That is, the local second order condition is satisfied.

□

Proof for Proposition 2.

Proof. The fixed parts of principal 1's contract keeps both agents' rationality constraint binding:

$$-F_1 = \tilde{F}_1 + [(\frac{1}{2} + e_1) + (\frac{1}{2} - e_2)]k_1 d + [(\frac{1}{2} - e_1) + (\frac{1}{2} + e_2)]\tilde{k}_1 d - C(e_1)$$

A similar expression exist for $-\tilde{F}_2$ using agent 2's rationality constraint.

$$-\tilde{F}_2 = F_2 + [(\frac{1}{2} + e_1) + (\frac{1}{2} - e_2)]\tilde{k}_2 d + [(\frac{1}{2} - e_1) + (\frac{1}{2} + e_2)]k_2 d - C(e_2)$$

Therefore, principal 1 maximizes:

$$[(\frac{1}{2} + \tilde{e}_1) + (\frac{1}{2} - \tilde{e}_2)]d + [(\frac{1}{2} - \tilde{e}_1) + (\frac{1}{2} + \tilde{e}_2)](k_2 + \tilde{k}_1)d - C(\tilde{e}_1) - C(\tilde{e}_2)$$

Given that the other principal's strategy is to offer $k_2 + \tilde{k}_1 = 1$, principal 1 maximizes the total surplus of four players:

$$[(\frac{1}{2} + \tilde{e}_1) + (\frac{1}{2} - \tilde{e}_2)]d + [(\frac{1}{2} - \tilde{e}_1) + (\frac{1}{2} + \tilde{e}_2)]d - C(\tilde{e}_1) - C(\tilde{e}_2) = 2d - C(\tilde{e}_1) - C(\tilde{e}_2)$$

Since the piece-rates of principal 1 is no longer in the expression, principal 1 will choose piece-rates so that the effort levels are efficient.

Agent 1's problem is:

$$\max_{e_1} \quad [(\frac{1}{2} + e_1) + (\frac{1}{2} - e_2)]k_1 d + [(\frac{1}{2} - e_1) + (\frac{1}{2} + e_2)]\tilde{k}_1 d - C(e_1)$$

So, to implement efficient effort for agent 1 ($e_1 = 0$), principal 1 just need to set $k_1 = \tilde{k}_1$. Similarly, $\tilde{k}_2 = k_2$. The other constraints are $k_1 + \tilde{k}_2 = 1$ and $k_2 + \tilde{k}_1 = 1$.

This constitutes a continuum of efficient equilibria. In particular, there is a symmetric one with $k_1 = \tilde{k}_1 = \frac{1}{2}$ and $\tilde{k}_2 = k_2 = \frac{1}{2}$. □

Alternative assumptions on the contract and number of units demanded by each principal.

Contracts that can discriminate on the number of units In the paper, we assumed that the contract has to non-discriminatory, i.e., if should pay the same per unit no matter a principal receives two units or one unit. Here we discuss what will happen if the contract can be more general. Suppose a principal does not have to pay twice when receiving two good fits than when receiving one good fit. We can equivalent think of the contract specifying a reward for the first unit of good fit and another reward for the second unit of good fit.

When principal 1 receives two units, she is sure that one of the two units is provided by agent 2. To reduce the amount leaked out to agent 2, she will optimally pay zero (or arbitrarily small amount) for the second unit. Then the reward for the first unit of good fit serves to motivate agent 1's effort. When it is higher, agent 2 may paradoxically want to exert more effort to create a good fit for principal 2. This is because a higher effort by agent 1 reduces the chance that agent 2 can supply the result to agent 1 to earn the reward for the first unit. The relative strength and sign of the three effects are more dependent on the cost functions and thus it is harder to get clear-cut general results.

Each principal can only enjoy one unit The intuition is exactly the same as the one for the "first unit" in the above analysis.

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