Downlink Resource Auction in a Tree Topology Structured Wireless Mesh Network

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Abstract—We analyze the problem of downlink resource allocation in a non-cooperative multi-level tree topology structured wireless mesh network in which a selfish mesh router (MR) may refuse to relay other MRs' traffic so as to improve its own performance at the cost of overall system performance. Based on game theory, we propose an auction framework, where the parent MR serves as the auctioneer while its children MRs act as bidders and compete for time-slots. We derive a payment function from radio resource used for relaying traffic instead of money, so as to simplify the implementation and avoid the possible security problems from monetary payment. We prove the existence and uniqueness of Nash Equilibrium and propose a stochastic best response updating algorithm to allow the bids to iteratively converge to NE in a practical distributed fashion. Simulation results show the proposed auction algorithm greatly outperforms traditional algorithms in non-cooperative environments.

Keywords-Wireless mesh networks, resource allocation, auction algorithms, game theory.

I. INTRODUCTION

With the capability of providing universal Internet access services at affordable monetary and infrastructure costs, infrastructure wireless mesh network (WMN) has become one of the key topologies for future wireless networking [1], [18]. In a typical WMN, mesh routers (MRs) form the backbone of the network to provide wireless access for mesh clients (MCs). A special MR, i.e., the gateway (GW), is installed between the mesh backbone and the Internet to provide broadband out-bound access. In WMNs, wireless resource is usually scheduled in a centralized manner [4], that is, the GW serves as a cluster head and determines to resource allocated to each MRs. This centralized approach tend to organize all MRs in a tree structure rooted at the GW. Consequently, to provide broadband Internet services to a remote area, the MRs close to the GW shall work as relay nodes transmitting traffic between the Internet and the MRs far away from the GW.

Among contemporary wireless technologies, IEEE 802.16 standard has been regarded as a promising solution for WMNs due to its capacity of combating channel fading and supporting high data rate. The research and deployment of IEEE 802.16-based WMNs have gained enormous popularity recently [14]. However, one of the key enabling features

behind such an infrastructure WMN is that a MR should not only serve the MCs in its own local cell, but also relay traffic for outer level MRs. Unfortunately, this kind of cooperation among MRs is not always practical in reality, especially when these MRs are owned by different profit-maximizing entities. In such a non-cooperative environment, a MR will rationally exhibit selfish behaviors driven by self-interests in that the MR may report a bogus channel information or valuation so as to improve its own benefit [11], or refuse to relay other MRs' traffic because relaying traffic will inevitably consume its own resource and consequently degrade its own performance. This selfish problem is further complicated in a multi-level tree topology WMN environment, where the centralized scheduler at the GW can only impose limited loose control over the distributed remote MRs. Though these non-cooperative behaviors could improve the performance for the selfish MR itself, they may lead to inefficient or unfair resource utilization from the system perspective.

Several recent work has addressed the problem of resource allocation in WMNs. In [4], the authors studied the opportunistic scheduling in an IEEE 802.16j based WiMAX relay networks so as to serve the mobiles in a fair manner and improve the throughput and increase range of WMN effectively. But they didn't take the selfish behavior of mesh routers into account. In [5], the problem of joint routing and scheduling in 802.16 based mesh networks was investigated and an ILP optimization model was proposed to determine a minimum schedule period and maximize the spectrum spatial reuse. While in [3], a joint power-frequency-time resource allocation algorithm was designed to optimize the system performance of a clustered WMN. But these two studies were both based on global optimization, and the competition among different MRs was not considered. In [14], a radio resource management framework for scheduling and admission control in an IEEE 802.16-based WMN was proposed to maximize transmission rate as well as satisfy QoS requirements under power constraint in a wireless MR. It just provided a study of the performance of an individual router; overall system performance was not analyzed. In [17], the throughput maximization problems in a WMN for both cooperative and non-cooperative scenarios were analyzed, and a linear pricing scheme was proposed to combat

the selfish behavior of non-cooperative MRs. However, the strategy to relay traffic was not considered. In [7], two auction-based approaches were proposed to help relay nodes to determine how to allocate transmission power among users as well as improve fairness and efficiency performances. In their model a relay node itself does not have local users to serve, which is different with our model where a relay MR should serve both local and relay connections.

In this paper, we design a novel auction-based resource allocation scheme. In this scheme, the whole WMN is divided into multiple auction sub-markets. Each sub-market comprises of a parent MR and its children MRs. The parent MR serves as the auctioneer, who has traffic for its children and grandchildren MRs, while the children MRs act as bidders and compete with each other for the time-slots from their parent. Within this model, we propose a relay auction (RA) algorithm based on a concave utility function, where a MR's bid is a scalar directly associated with its reported valuation, and the payment is proportional to this bid. Though in some auction algorithms [7], [19], the payments are expressed in terms of money, they cannot be easily implemented in practice and may suffer from security problems. In this paper, one of our major contributions is that we associate the payment with the time-slots used by a MR to relay others' traffic. Specifically, to win time-slots allocated by the auctioneer, a bidder must "pay" some wireless resources, i.e., time-slots, and use them to forward its children's traffic. We prove the existence and uniqueness of Nash equilibrium (NE) [15] for the bidding profile. We also present a stochastic bid update algorithm to achieve NE in a practical situation and prove its convergence. The simulation show that the proposed RA algorithm greatly outperforms traditional algorithms in non-cooperative environments, while having the capability of balancing the efficiency and fairness by adjusting the payment function parameter.

The rest of this paper is organized as follows. We present the system model and auction framework in Section II. In Section III, our novel relay auction approach is proposed. Section IV gives the simulation results. We conclude in Section V.

II. SYSTEM MODEL

In this paper, we consider an IEEE 802.16-based WMN with tree topology and focus on downlink resource allocation. Such a hierarchical network structure is highly practical and can be used to model a WMN in a rural area [4]. We assume the tree structure has been already determined using link metrics such as Expected Transmission Time (ETT) [16]. The proposed WMN model is represented by a tree graph $G = (V, E)$. Here, V is a set of MRs in the WMN, and E is a set of edges between every parent and child MR, i.e., the wireless links on the routing tree. There are $|V| = N + 1$ MRs in the tree, where N is the number of

MRs other than the GW. The tree's maximal level is L, and the root GW is at level 0. Specifically, a MR $i \in V$ is at level l if it is l hops away from the GW, and the edge $e(i)$ between node i and its parent MR P_i is also at level l.

We assume every cell centered on a MR operates under IEEE 802.16 OFDM/TDMA-TDD mode, and each adjacent cell uses a different frequency band. Except the MRs at the farmost level L, every MR has both local MC connections served in its local cell and relay connections for its children MRs. The MAC frame is composed of downlink and uplink subframes. Each downlink subframe is composed of T_d time-slots, each of which is used for transmission of packets corresponding to one connection. For uplink and downlink transmission using OFDM, each MR uses M subchannels with total bandwidth of B MHz. Without loss of generality, we show in Fig. 1 a part of tree graph which consists of the MR i at level l, its parent P_i at level $l-1$, its siblings $j \in S_i$ with the same parent, and its children MRs C_i at level $l+1$. S_i is the set of i's siblings, N_i is the number of MRs in S_i. For the sake of ease, we also let $i \in S_i$. For MR i, a_i^L and a_i^R are the average number of local packets and relay packets that can be successfully transmitted to its local cell and its children MRs per time-slot, respectively. They can be measured according to channel and traffic conditions.
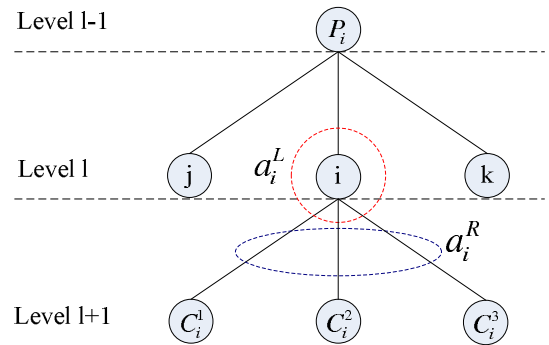

Figure 1: Tree model.

To implement auction, we first divide the WMN into several sub-markets. A sub-market is made up of a MR, its siblings and their parent. For example, in Fig. 1, the MR i's siblings S_i and their parent P_i form a sub-market Ψ_i, where each MR i acts as bidder and competes for time-slots from the auctioneer P_i through auction. Each sub-market can operate independently and locally. To be noticed, because the outmost MRs at level Y do not have relay connections, they are not organized into auction sub-markets and we assume they get resources through their parents' scheduling.

In our model, we assume if receiving t_i time-slots per frame, MR i's valuation is $V_i^L(a_i^L, t_i)$, thus its valuation function can be fully characterized by the scalar parameter a_i^L. Further, as in [9], we consider to use the following logarithmic function to express MR i's valuation on its

received resource, i.e.,

$$V_i^L(a_i^L, t) = N_i^L \cdot \log(\frac{a_i^L}{N_i^L} \cdot t + 1), \qquad (1)$$

where N_i^L is the number of MCs served by MR i. Thus, $V_i^L(a_i^L, t)$ also represents MR i's local valuation when it serves N_i^L local MCs. Similarly, we let $V_i^R(a_i^R, t)$ be MR i's relay valuation, i.e,

$$V_i^R(a_i^R, t) = N_i^R \cdot \log(\frac{a_i^R}{N_i^R} \cdot t + 1), \qquad (2)$$

where N_i^R is the number of its relay connections. We assume that P_i has already obtained a_i^R during the process of routing tree construction. It can also know N_i^R and N_i^L according to the buffered traffic.

In an auction, a succinct and expressive bidding language related to bidders' valuation is necessary. According to (1), a bidders valuation function on the allocated resource is actually fully characterized by a single scalar parameter a_i^L. Because a_i^L is only related to MR i's local downlink transmission and is not used for WMN tree construction, the auctioneer doesn't know it before auction. Thus, this scalar a_i^L can be utilized as bidding language and submitted to the auctioneer to represent bidder's valuation, then it leads to an efficient and practical bidding process.

Specifically, during the auction, every bidder i calculates the valuation parameter a_i^L and submits a bid $b_i = \mu_i(a_i^{L_1})$ to the auctioneer according to a rationally selfish strategy. After receiving the announced bidding profile $B = \{b_1, b_2, \ldots, b_{N_i}\}$ from its children, the parent P_i knows each bidder's reported valuation, and then calculates the allocation $T = T(B) = \{T_1, T_2, \ldots, T_i, \ldots, T_{N_i}\}$, which represents the number of time-slots allocated to each bidder, as well as the payment $T^R = T^R(B) = \{T_1^R, T_2^R, \ldots, T_i^R, \ldots, T_{N_i}^R\}$ representing the number of time-slots that should be used by MR i to relay its descendant MRs' traffic. Subsequently, the allocation and payment results are transmitted to each bidder. Then, each MR i gets to know $T_i^L = T_i - T_i^R$ and payment T_i^R.

In a typical auction, the utility of a bidder is the value received by this bidder minus the payment assigned by the auctioneer. However, in our model, to provide practical meanings for payment, we associate the payment with the practical radio resource, i.e., time-slots. Thus, when MR i receives T_i time-slots and accept T_i^R time-slots as payment, its utility for serving local connections is its true valuation when getting $T_i^L = T_i - T_i^R$ time-slots, i.e.,

$$\begin{aligned} &U_i^L(a_i^L, a_i^R, T_i, T_i^R) \\ =&V_i^L(a_i^L, T_i - T_i^R) - c_i \cdot (a_i^L \cdot (T_i - T_i^R) + a_i^R \cdot T_i^R), \end{aligned}$$
$$(3)$$

where the second item of (3) is the cost of packet transmission, such as power consumption and so on. c_i is the average cost per packet for MR i. On the other hand, its relay connections can get a net benefit of

$$U_i^R(a_i^R, T_i^R) = V_i^R(a_i^R, T_i^R). \qquad (4)$$

III. RELAY AUCTION

One well known auction is the Vickrey-Clarke-Groves (VCG) auction, which has been regarded as one of the most effective mechanisms to induce truth-revealing strategies [12]. However, the VCG auction does not consider fairness performance. Furthermore, it just applies to the problem where bidders have a quasilinear utility function [2]. From (3), we see the utility function cannot be expressed as the difference between the valuation and payment, implying that they are not quasi-linear. Thus, the VCG method cannot be applied here. We need to propose another new auction approach.

Generally, the auctioneer wants to maximize the aggregate utility of it transmitted packets. At the same time, it also need to enforce the bidder to relay other's traffic through payments. Specifically, the more resource a bidder requests, the higher payment it should pay to the auctioneer. Thus, we define the payment as a function increasing with the bidding scalar b_i, and propose a relay auction (RA) algorithm in sub-market, which is formalized as follows.

A. Auctioning Mechanism

1) **Bidding rule:** In a sub-market Ψ_i, bidder i submits a scalar value b_i to its parent P_i, and let $\mathbf{b} = \{b_i : b_{min} \leq b_i \leq b_{max}\}$.

2) **Payment rule:** If bidder i gets T_i time-slots, it must use $T_i^R = f(b_i) \cdot T_i$ time-slots for relaying its children's traffic, where $f(b_i)$ is increasing with b_i, and $0 \leq f(b_i) \leq 1$. Here, we consider a linear payment function: $f(b_i) = \frac{b_i}{A}$, where A is a payment parameter and $A > b_{max}$.

This payment scheme has two implications. First, it can restrict bidder's selfish behavior due to the fact that when given a payment parameter A, larger resource request means more obtained time slots should be used for relay. Thus, a rational bidder must request resource strategically. On the other hand, for a fixed bid, a larger A implies that larger portion of resource can be used for local traffic; while more resource are utilized for relay connection in terms of a smaller A. Therefore, it can adjust the tradeoff between efficiency and fairness in the system, which is further verified through simulations in Section IV.

3) **Allocation rule:** We assume the auctioneer's objective is to maximize the aggregate utilities of all local connections and relay connections in its sub-market,

i.e.,

$$T_* = \{T_1^*, T_2^*, \cdots, T_N^*\}$$
$$= \arg\max_{\{T_i\}} \sum_{i \in S_i} U_i^L(T_i - T_i^R) + U_i^R(T_i^R)$$
$$= \arg\max_{\{T_i\}} \sum_{i \in S_i} (N_i^L \cdot \log(\frac{b_i}{N_i^L} \cdot (T_i - T_i^R) + 1) \quad (5)$$
$$+ N_i^R \cdot \log(\frac{a_i^R}{N_i^R} \cdot T_i^R + 1)$$
$$- c_i \cdot (b_i \cdot (T_i - T_i^R) + a_i^R \cdot T_i^R))$$

subject to

$$(I) \sum_i (T_i^L + T_i^R) = T_d$$
$$(II) \; T_i^R = f(b_i) \cdot T_i.$$

where the constraint (I) implies the time-slots constraint in one frame, and the constraint (II) is the payment function.

B. Nash Equilibrium

When the bidder i and its siblings compete with each other in a sub-market Ψ_i, our proposed relay auction can be formulated as a strategic non-cooperative game with N_i players, i.e., $\Gamma \triangleq [S_i, \{b_i\}_{i \in S_i}, \{U_i^L(\mathbf{B})\}_{i \in S_i}]$, where S_i is the player set, $\{b_1, \cdots, b_{N_i}\}$ is the action profile, and U_i^L is the utility function of player i.

A useful solution concept of such a game is called a Nash equilibrium (NE) [15], which is a bidding profile B^* where no MR wants to deviate unilaterally, i.e.,

$$U_i^L(b_i^*; \mathbf{b}_{-i}^*) \ge U_i^L(b_i; \mathbf{b}_{-i}^*), \quad \forall i \in S_i, \quad \forall b_i \ge 0, \quad (6)$$

where $\mathbf{b}_{-i}$ is the bidding profile of bidder i's opponents. We first establish the existence of the NE.

Theorem 1. *In Ψ_i, there exists a bidding profile $B^* = \{b_1^*, \cdots, b_{N_i}^*\}$ to achieve Nash equilibrium in the relay auction.*

Proof: A Nash equilibrium B^* exists in the game if the following two conditions are satisfied for all $i \in S_i$:
  1) the bidding strategy b_i is a nonempty, convex and compact subset of some Euclidean space $\Re^N$; and
  2) $U_i^L(\mathbf{B})$ is continuous in $\mathbf{B}$ and quasi-concave in b_i

The first condition is easily satisfied because we have $b_i \in [b_{min}, b_{max}]$. For utility function U_i^L, because $\frac{\partial^2 U_i^L}{\partial b_i^2} < 0$, it is strictly concave on b_i. As a concave function is also quasi-concave, the second condition is then satisfied. Thus, the existence of NE is proven. ∎

We then construct MR i's best response function (for fixed $\mathbf{b}_{-i}$) as:

$$\beta_i(\mathbf{b}_{-i}) = \{b_i \mid b_i = \arg\max_{\widetilde{b}_i \ge 0, i \in S_i} U_i^L(\widetilde{b}_i; \mathbf{b}_{-i})\}. \quad (7)$$

Then, similar to the proof in [6], we show the uniqueness of NE in the following.

Theorem 2. *In Ψ_i, there exists a unique NE $B^* = \{b_1^*, \cdots, b_{N_i}^*\}$ in the relay auction.*

Proof: We assume that NE is not unique, and there exists at least another one $\tilde{B} = \{\tilde{b}_1, \cdots, \tilde{b}_{N_1}\}$, and $B^* \ne \tilde{B}$. According to the definitions of NE and the best response function, if given b_{-i}, b_i^* and $\tilde{b}_i$ must satisfy:

$$b_i^* = max_{b_i^*} U_i^L(b_i^*, b_{-i}), \quad (8)$$
$$\tilde{b}_i = max_{\tilde{b}_i} U_i^L(\tilde{b}_i, b_{-i}). \quad (9)$$

However, because U_i^L is concave and differentiable, for a fixed b_{-i}, there is a unique best strategy b_i that satisfies $b_i = max_{b_i} U_i^L(b_i, b_{-i})$. These contradict the assumption that $B^* \ne \tilde{B}$. Thus, $B^* = \tilde{B}$, and there is a unique NE in this auction sub-market. ∎

C. Iterative Best Response Update Algorithm to Achieve NE

In our model, it is difficult to get the close-form $\beta_i(\mathbf{b}_{-i})$ and $U_i^L(b_i, \mathbf{b}_{-i})$. However, because the nonlinear constrained optimization problem defined by (5) can be efficiently solved by numerical methods, and there exists a unique NE in the auction, and MR i's utility function U_i^L in (3) is differentiable and concave related to b_i, we can still design an stochastic bid updating algorithm to achieve NE based on each bidder's local information and the limited feedback received from the auctioneer. Although it is still hard to obtain the exact expression of $U_i^L(b_i, \mathbf{b}_{-i})$, we can regard the optimization problem to find the best response in (7) as a stochastic approximation problem [13], and then get recursive solutions for the best response through the stochastic gradient form of Robbin-Monro algorithm [13] and the finite difference (FD) method. The key idea is to let the auctioneer gives the bidders some additional information besides the auction results so that the bidder can use it to estimate the gradient of its utility by FD method and then update its bid according to Robbin-Monro algorithm.

Specifically, at time t, when the auctioneer P_i receives $\mathbf{b}$ and calculates the allocation $T(\mathbf{b})$ and payment $T^R(\mathbf{b})$, it also calculates $T_i^{L+} = T_i^L(\mathbf{b} + c_t \cdot \mathbf{e}_i)$ and $T_i^{L-} = T_i^L(\mathbf{b} - c_t \cdot \mathbf{e}_i)$. Here, $c_t \to 0$ is a FD interval and $\mathbf{e}_i$ be the standard unit vector in the ith coordinate direction. Then they are all sent to bidder i. With this knowledge, MR i can estimate its utility gradient $\nabla_{b_i(t)} U_i(b_i(t), b_{-i}(t))$ by FD method as follows:

$$\nabla_{b_i(t)} U_i(\mathbf{b}) = \frac{U_i^L(\mathbf{b} + c_t \cdot \mathbf{e}_i) - U_i^L(\mathbf{b} - c_t \cdot \mathbf{e}_i)}{2c_t}$$
$$= \frac{U_i^L(T_i^{L+}) - U_i^L(T_i^{L-})}{2c_t}. \quad (10)$$

It further apply Robbin-Monro algorithm to update the best response by

$$b_i(t+1) = b_i(t) + a_t \cdot \nabla_{b_i(t)} U_i(\mathbf{b}), \quad (11)$$

where the gain sequence $a_t > 0, a_t \to 0, \Sigma_{t=o}^{\infty}(a_t) = \infty$. Then bidder i submits bid $b_i(t+1)$ again at time $t+1$. We let the bids update asynchronously, e.g., the auctioneer P_i only calculates bidder i's additional information at time $\{i, i + N_i, i + 2N_i, \cdots\}$. Then the NE can be achieved gradually. The stochastic best response update algorithms for the auctioneer and the bidders are formalized as follows in Algorithm 1, and Algorithm 2, respectively.

Algorithm 1 The stochastic best response update algorithm for the auctioneer

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1: In sub-market Ψ_i, at time t, the auctioneer P_i receives bidding profile **b** from all children $i \in S_i$;
2: Calculates T_i and T_i^R for every bidder $i \in S_i$;
3: $j = (t \mod N_i) + 1$;
4: Calculates T_j^{L+}, T_j^{L-}
5: Sends T_i and T_i^R to every bidder $i \in S_i$; and sends T_j^{L+}, T_j^{L-} and c_t to bidder j.

Algorithm 2 The stochastic best response update algorithm for bidders

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1: Initialization: At time $t = 0$, every bidder $i \in S_i$ randomly submits a bid $b_i(0) \in [b_{min}, b_{max}]$;
2: **while** $t = 1, 2, \cdots$ **do**
3:    **if** $i \neq (t \mod N_i) + 1$ **then**
4:       $b_i(t+1) = b_i(t)$;
5:    **else**
6:       Bidder i receives $T_i, T_i^R, T_i^{L+}, T_i^{L-}$ and c_t;
7:       Bidder i approximates the gradient of its utility by (10) and update its bid according to Robbin-Monro algorithm by (11);
8:    **end if**
9:    Submit $b_i(t+1)$;
10: **end while**

Theorem 3. *For each bidder $i \in \Psi_i$, under any initial condition $\mathbf{b}(0)$, the sequence $\{\mathbf{b}(t)\}$ converges weakly (i.e., in probability) to NE B^*.*

Proof: We first interpolate the iterates $\mathbf{b}(t)$ into a continuous time interpolation process $\mathbf{b}^{\epsilon_t}(t) = \{b_i^{\epsilon_t}(t)\}$ with interpolation intervals ϵ_t, and define $b_i^{\epsilon}(t)$ as follows:

$$\bar{b}_i^{\epsilon_t}(t) = \begin{cases} b_i(0) & t < 0 \\ b_i(n) & n\epsilon_t - \epsilon_i \leq t \leq n\epsilon_i, \, n = 1, 2, \cdots \end{cases}$$
(12)

Thus, as shown in [13], when $\epsilon_t \to 0$, $b_i^{\epsilon_t}(t)$ converges in trajectory to $\bar{b}_i(t)$. Here, $\bar{b}_i(t)$ is the solution of the ordinary differential equation (ODE) function

$$\frac{d\bar{b}_i(t)}{dt} = \nabla_{\bar{b}_i(t)} U_i(\bar{b}_i(t), b_{-i}(t)).$$
(13)

Because $U_i(b_i(t), b_{-i}(t))$ is bounded and quasi-concave with respect to b_i, the ODE in (13) is asymptotically Liapunov stable. Consequently, as $\epsilon_t \to 0$, the sequence $b_i(t)$ converges weakly to an optimizer of its utility function, i.e.,Nash equilibrium b_i^*. ∎

The convergence of the stochastic bid update algorithm is also verified from Fig. 2, where $a_1^L = 10$, $a_2^L = 9$, and $A = 90$. We can see the bids converge to NE eventually.
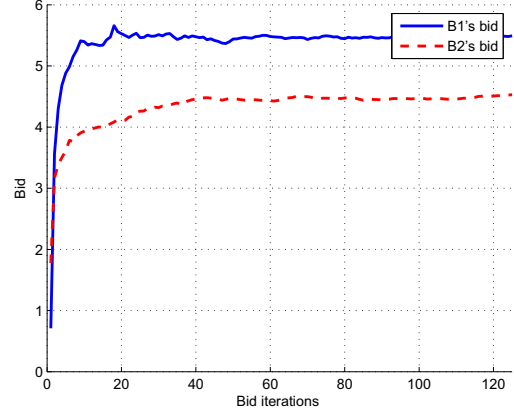

Figure 2: Convergence of bids to NE.

IV. PERFORMANCE RESULTS

In this section, we evaluate the performance of the auction algorithm through simulations. In the simulated tree topolgy WMN, we assume each MR has 64 subcarriers with a total frequency band of 10 MHz. The frequency bands for adjacent MRs are non-coverlapping and frequency reuse is utilized. The DL-subframe length is $2ms$. The wireless channel is modeled as six-path frequency-selective Rayleigh slow fading channel; each path is simulated by Clark's fading model and suffers from different Rayleigh fading with the maximum Doppler frequency of 30 Hz. Because the coherence time $33.3ms$ is much larger than the DL-subframe length, the channel condition can be regarded as static during the iterative bidding period to achieve NE. This is a realistic assumption in a WMN because the MRs are immobile. Furthermore, we assume the average SNR is identical for all MCs in a cell. Like [4], we do not model packet arrival process at the GW and MRs, and assume the GW has infinite backlogged traffic for each MC in WMN and the downlink mesh tree is constructed by using routing algorithms, such as ETT by [16]. We assume each parent MR has at most 3 children, and there are 10 MCs in each cell. Then the traffics generated at GW can be transmitted across the WMN and reach MCs through other MR's relay. b_{min} and b_{max} are 1 and 20, respectively, which also represent the minimum and the maximum successfully transmittable packets per time slots.

We first compare our proposed RA algorithm with two traditional cooperative resource allocation methods, i.e., proportional fairness (PF) [10], and a global optimization method (GLB) which maximizes the aggregate utilities of all MRs based on centralized scheduling, in non-cooperative environments. In this setting, the maximal tree level L is from 2 to 5, and the payment parameter A is 90. Notice that in a cooperative situation which is unrealistic in the operating environment considered in this paper, all MRs report their true valuations to their parents and relay other's traffic according to scheduling results. However, in a non-cooperative environment, a selfish MR may report a bogus valuation parameter to increase its own local benefit without regard to the overall system performance [11] and relay less traffic to others. In our simulations, we assume in each sub-market, there is a MR who always reports a higher valuation in the case of using PF and GLB algorithms. From Fig. 3, we first notice that the average throughput per MR decreases with the size of WMN, due to the downlink transmission capacity limit of the GW. Another reason is that more packets may be dropped due to increased wireless transmission error along the longer path when the network expands. Furthermore, the average throughput per MR of PF and GLB are both much lower than that of RA algorithm due to the fact that less traffics are relayed to remote MRs. While in RA algorithm, a selfish MR may not benefit from bidding higher value because this will also incur higher payment. Consequently, the system performance is maintained in NE state during auctions.
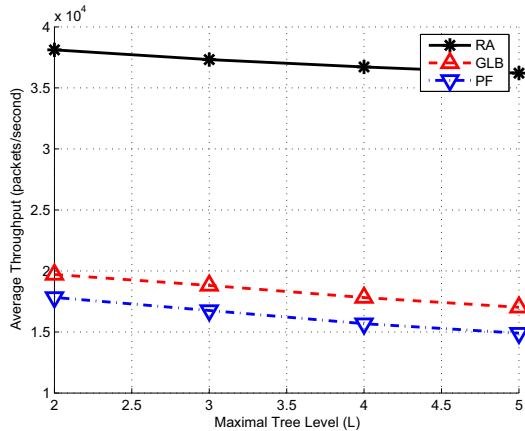

Figure 3: Average throughput per MR with the change of network size in non-cooperative situations.

We then illustrate the throughput and fairness performances of RA in terms of payment parameter, where the maximal tree level L is 2, and compare them with the performances of GLB and PF in cooperative situations. In Fig. 4, we notice that the average throughput per MR of RA algorithm increases with the payment parameter A.

Although GLB and PF generate higher throughput than RA algorithm, the throughput of RA achieves almost 95% of that for GLB with the increase of payment parameter, e.g., $A > 100$, even in non-cooperative environments. This is because the time-slots used for relay will decrease if the payment parameter becomes larger, then more resources are utilized to server local MCs' traffic. Because local MCs are usually closer to the MR and generally experience better transmission conditions than relay connections, the overall system throughput increases when more resources are used for local traffic. We then investigate the fairness performance in terms of Jain fairness index [8]. We can see in Fig. 5 that the Jain fairness index for RA algorithm is dependent on the payment parameter A. RA achieves its highest index value 0.735 when $A = 122$, which is about 95% of PF's fairness index value. Thus, the proposed RA algorithm can be used to strike a proper balance between efficiency and fairness by adjusting payment function parameter. Furthermore, when utilizing RA algorithm in non-cooperative environments, the efficiency and fairness performances are comparable with those obtained by GLB and PF in operative situations.
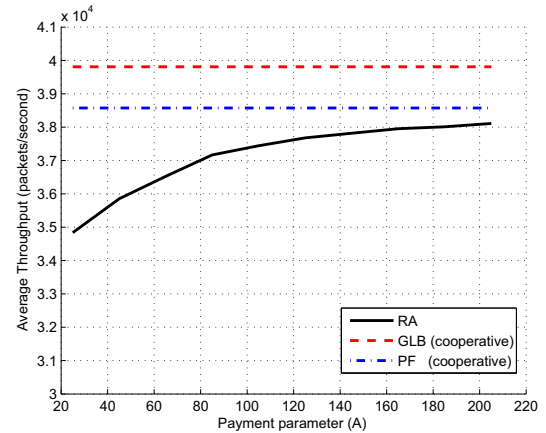

Figure 4: Average throughput per MR with the change of payment parameter.

V. CONCLUSIONS

In this paper, we study the downlink resource allocation problem in a non-cooperative IEEE 802.16 tree topology based wireless mesh network. To motivate selfish MRs to relay other's traffic, we divide the entire network into multiple auction sub-markets and design an auction framework to let rationally selfish MRs compete with their siblings for time-slots from their parent. Based on it, we further propose a novel relay auction algorithm, where the payment is designed as the time-slots used for relaying traffic instead of money, and the allocation policy is to maximize both local and relay connections' utilities in the sub-market. After proving the existence and uniqueness of NE, we also
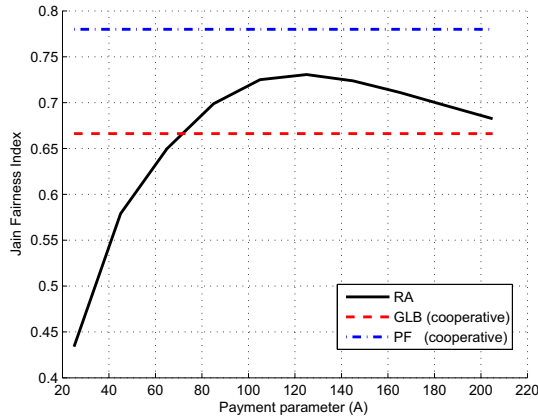
Figure 5: Jain fairness index with the change of payment parameter.

propose a stochastic best response update algorithm to allow bidders to iteratively update their bids based on their local information and the limited feedback from the auctioneer and prove its convergence to NE. We then compare the performances of the RA algorithm with traditional GLB and PF algorithms through simulations, which indicate that the RA algorithm excels GLB and PF in non-cooperative situations. It also can achieve competitive performance in terms of resource allocation efficiency with high payment parameter A, while demonstrating good fairness performance with small payment parameter, so as to strike a proper balance between efficiency and fairness in the network.

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