Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

On improvements of the Rozanova's inequality

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Abstract

In the present paper, we establish some new Rozanova's type integral inequalities involving higher-order partial derivatives. The results in special cases yield some of the interrelated results on Rozanova's inequality and provide new estimates on inequalities of this type.

MS (2000) Subject Classifiication: 26D15.

Keywords: Opial's inequality, Hölder's inequality, Rozanova's inequality

1 Introduction

In the year 1960, Opial [1] established the following integral inequality:

Theorem A *Suppose* $f \in C^1[0, h]$ *satisfies* $f(0) = f(h) = 0$ *and* $f(x) > 0$ *for all* $x \in (0, h)$. *Then*

$$\int_0^h |f(x)f'(x)|\,dx \le \frac{h}{4}\int_0^h (f'(x))^2 dx. \tag{1.1}$$

The first Opial's type inequality was established by Willett [2] as follows:

Theorem B *Let* $x(t)$ *be absolutely continuous in* $[0, a]$, *and* $x(0) = 0$. *Then*

$$\int_0^a |x(t)x'(t)|dt \le \frac{a}{2}\int_0^a |x'(t)|^2 dt. \tag{1.2}$$

A non-trivial generalization of Theorem B was established by Hua [3] as follows:

Theorem C *Let* $x(t)$ *be absolutely continuous in* $[0, a]$, *and* $x(0) = 0$. *Futher, let* l *be a positive integer. Then*

$$\int_0^a |x(t)x'(t)|dt \le \frac{a^l}{l+1}\int_0^a |x'(t)|^{l+1} dt. \tag{1.3}$$

A sharper inequality was established by Godunova [4] as follows:

Theorem D *Let* $f(t)$ *be convex and increasing functions on* $[0, \infty)$ *with* $f(0) = 0$. *Further, let* $x(t)$ *be absolutely continuous on* $[0, \tau]$, *and* $x(\alpha) = 0$. *Then, following inequality holds*

$$\int_\alpha^\tau f'(|x(t)|)|x'(t)|dt \le f\left(\int_\alpha^\tau |x'(t)|dt\right). \tag{1.4}$$

Rozanova [5] proved an extension of inequality (1.4) is embodied in the following:

Theorem F *Let* $f(t)$ *and* $g(t)$ *be convex and increasing functions on* $[0, \infty)$ *with* $f(0) = 0$, *and let* $p(t) \ge 0$, $p'(t) > 0$, $t \in [\alpha, a]$ *with* $p(\alpha) = 0$. *Further, let* $x(t)$ *be absolutely* *continuous on* $[\alpha, a)$, *and* $x(\alpha) = 0$. *Then, following inequality holds*

$$\int_{\alpha}^{a} p'(t) \cdot g\left(\frac{|x'(t)|}{p'(t)}\right) \cdot \left[f'\left(p(t) \cdot g\left(\frac{|x(t)|}{p(t)}\right)\right)\right] dt \le f\left(\int_{\alpha}^{a} p'(t) \cdot g\left(\frac{|x'(t)|}{p'(t)}\right) dt\right). \qquad (1.5)$$

The inequality (1.5) will be called as Rozanova's inequality in the paper.

Opial's inequality and its generalizations, extensions and discretizations play a fundamental role in establishing the existence and uniqueness of initial and boundary value problems for ordinary and partial differential equations as well as difference equations [6-13]. For Opial-type integral inequalities involving high-order partial derivatives, see [14,15]. For an extensive survey on these inequalities, see [16].

The first aim of the present paper is to establish the following Opial-type inequality involving higher-order partial derivatives, which is an extension of the Rozanova's inequality (1.5).

Theorem 1.1 *Let f and g be convex and increasing functions on* $[0, \infty)$ *with* $f(0) = 0$, *and let* $p(s, t) \ge 0$, $D_1D_2p(s,t) = \frac{\partial^2}{\partial s \partial t}p(s,t)$, $D_1D_2p(s, t) > 0$, $s \in [\alpha, a]$, $t \in [\beta, b]$ *with* $p(s, \beta) = p(\alpha, t) = p(\alpha, \beta) = 0$ *and* $D_1D_2p(s, t)\,|_{t=\tau} = 0$. *Further, let* $x(s, t)$ *be absolutely continuous on* $[\alpha, a) \times [\beta, b]$, *and* $x(s, \beta) = x(\alpha, t) = x(\alpha, \beta) = 0$. *Then following inequality holds*

$$\begin{aligned}\int_{\alpha}^{a}\int_{\beta}^{b} D_1D_2p(s,t) \cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right) \cdot \frac{\partial}{\partial t}\left[f\left(p(s,t) \cdot g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right)\right] dsdt \\ \le f\left(\int_{\alpha}^{a}\int_{\beta}^{b} D_1D_2p(s,t) \cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right) dsdt\right).\end{aligned} \qquad (1.6)$$

We also prove the following Rozanova-type inequality involving higher-order partial derivatives.

Theorem 1.2 *Assume that*

(i) *f, g and* $x(s, t)$ *are as in Theorem 1.1,*
(ii) $p(s, t)$ *is increasing on* $[0, a] \times [0, b]$ *with* $p(s, \beta) = p(\alpha, t) = p(\alpha, \beta) = 0$,
(iii) *h is concave and increasing on* $[0, \infty)$,
(iv) $\varphi(t)$ *is increasing on* $[0, a]$ *with* $\varphi(0) = 0$,
(v) *For* $\gamma(s,t) = \int_0^s \int_0^t D_1D_2p(\sigma,\tau) g\left(\frac{|D_1D_2x(\sigma,\tau)|}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau$,

$$D_1D_2f\left(\gamma(s,t)\right) D_1D_2\gamma(s,t) \cdot \phi\left(\frac{1}{D_1D_2\gamma(s,t)}\right) \le \frac{c_{(a,b)}}{\gamma(a,b)} \cdot \phi'\left(\frac{t}{\gamma(a,b)}\right).$$

Then

$$\begin{aligned}\int_0^a \int_0^b D_1D_2f\left(p(s,t)g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right) \cdot v\left(D_1D_2p(s,t)g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)\right) dsdt \\ \le w\left(\int_0^a \int_0^b D_1D_2p(s,t)g\left(\frac{|x(s,t)|}{D_1D_2p(s,t)}\right) dsdt\right),\end{aligned} \qquad (1.7)$$

where

$$v(z) = zh\left(\phi\left(\frac{1}{z}\right)\right),$$
$$w(z) = c_{(a,b)}h\left(a\phi\left(\frac{b}{z}\right)\right),$$

and

$$c_{(a,b)} = \int_0^a \int_0^b D_1D_2 f\left(y(s,t)\right) D_1D_2 y(s,t) ds dt.$$

2 Main results and proofs

Theorem 2.1 *Let f and g be convex and increasing functions on* $[0, \infty)$ *with* $f(0) = 0$, *and let* $p(s, t) \geq 0$, $D_1D_2p(s,t) = \frac{\partial^2}{\partial s \partial t}p(s,t)$, $D_1D_2p(s, t) > 0$, $s \in [\alpha, a]$, $t \in [\beta, b]$ *with* $p(s, \beta) = p(\alpha, t) = p(\alpha, \beta) = 0$ *and* $D_1D_2p(s, t)\,|_{t=\tau} = 0$. *Further, let* $x(s, t)$ *be absolutely continuous on* $[\alpha, a) \times [\beta, b]$, *and* $x(s, \beta) = x(\alpha, t) = x(\alpha, \beta) = 0$. *Then, following inequality holds*

$$\begin{aligned} &\int_\alpha^a \int_\beta^b D_1D_2p(s,t)\cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)\cdot \frac{\partial}{\partial t}\left[f\left(p(s,t)\cdot g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right)\right] ds dt \\ &\quad \leq f\left(\int_\alpha^a \int_\beta^b D_1D_2p(s,t)\cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right) ds dt\right). \end{aligned} \tag{2.1}$$

Proof Let $y(s,t) = \int_\alpha^s \int_\beta^t \left|D_1D_2x(\sigma,\tau)\right| d\sigma d\tau$ so that $D_1D_2y(s, t) = |D_1D_2x(s, t)|$ and $y(s, t) \geq |x(s, t)|$. Thus, from Jensen's integral inequality, we obtain

$$\begin{aligned} g\left(\frac{|x(s,t)|}{p(s,t)}\right) &\leq g\left(\frac{y(s,t)}{p(s,t)}\right) \leq g\left(\frac{\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\frac{|D_1D_2x(\sigma,\tau)|}{D_1D_2p(\sigma,\tau)} d\sigma d\tau}{\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau) d\sigma d\tau}\right) \\ &\leq \frac{1}{p(s,t)} \int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau) g\left(\frac{|D_1D_2x(\sigma,\tau)|}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau. \end{aligned} \tag{2.2}$$

By using the inequality (2.2), we have

$$\begin{aligned} &\int_\alpha^a \int_\beta^b D_1D_2p(s,t)\cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)\cdot \frac{\partial}{\partial t}\left[f\left(p(s,t)\cdot g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right)\right] ds dt \\ &\leq \int_\alpha^a \int_\beta^b D_1D_2p(s,t)\cdot g\left(\frac{D_1D_2y(s,t)}{D_1D_2p(s,t)}\right)\cdot \frac{\partial}{\partial t}\left[f\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau\right)\right] ds dt. \end{aligned} \tag{2.3}$$

On the other hand

$$\begin{aligned} &\frac{\partial^2}{\partial s \partial t}\left[f\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau\right)\right] \\ &= \frac{\partial}{\partial s}\left\{\frac{\partial}{\partial t}\left[f\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau\right)\right]\cdot \int_\alpha^s p_{\sigma t}(\sigma,t)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,t)}\right) d\sigma\right\} \\ &= \left\{\frac{\partial^2}{\partial s \partial t}\left[f\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau\right)\right]\right\}\cdot \int_\alpha^s D_1D_2p(\sigma,t)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{p_{\sigma t}(\sigma,t)}\right) d\sigma \\ &\quad \times \int_\beta^t p_{s\tau}(s,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(s,\tau)}\right) d\tau + D_1D_2p(s,t)\cdot g\left(\frac{D_1D_2y(s,t)}{D_1D_2p(s,t)}\right) \\ &\quad \times \frac{\partial}{\partial t}\left[f\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau\right)\right] \\ &= D_1D_2p(s,t)\cdot g\left(\frac{D_1D_2y(s,t)}{D_1D_2p(s,t)}\right)\cdot \frac{\partial f}{\partial t}\left[\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right) d\sigma d\tau\right)\right]. \end{aligned} \tag{2.4}$$

From (2.3) and (2.4), we have

$$\begin{aligned}
&\int_\alpha^a \int_\beta^b D_1D_2p(s,t)\cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)\cdot \frac{\partial}{\partial t}\left[f\left(p(s,t)\cdot g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right)\right]dsdt\\
&\le \int_\alpha^a \int_\beta^b \frac{\partial^2}{\partial s\partial t}\left[f\left(\int_\alpha^s \int_\beta^t D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right)d\sigma d\tau\right)\right]dsdt\\
&= f\left(\int_\alpha^a \int_\beta^b D_1D_2p(\sigma,\tau)\cdot g\left(\frac{D_1D_2y(\sigma,\tau)}{D_1D_2p(\sigma,\tau)}\right)d\sigma d\tau\right)\\
&= f\left(\int_\alpha^a \int_\beta^b D_1D_2p(s,t)\cdot g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)dsdt\right).
\end{aligned}$$

This completes the proof.

Remark 2.2 Let $x(s, t)$ reduce to $s(t)$, and with suitable modifications in the proof of Theorem 2.1, then (2.1) becomes inequality (1.5) stated in Section 1.

Remark 2.3 Taking for $g(x) = x$ in (2.1), then (2.1) becomes the following inequality.

$$\int_\alpha^a \int_\beta^b \left|D_1D_2x(s,t)\right|\cdot \frac{\partial}{\partial t}(f(\left|x(s,t)\right|))dsdt \le f\left(\int_\alpha^a \int_\beta^b \left|D_1D_2x(s,t)\right|dsdt\right). \quad (2.5)$$

Let $x(s, t)$ reduce to $s(t)$, and with suitable modifications, then (2.5) becomes inequality (1.4) stated in Section 1.

Remark 2.4 For $f(t) = t^{l+1}$, $l \ge 0$, the inequality (2.5) reduces to

$$\int_\alpha^a \int_\beta^b \left|x(s,t)\right|^l \frac{\partial}{\partial t}(\left|x(s,t)\right|)dsdt \le \frac{1}{l+1}\left(\int_\alpha^a \int_\beta^b \left|D_1D_2x(s,t)\right|dsdt\right)^{l+1}. \quad (2.6)$$

In the right side of (2.6), by Hölder inequality with indices $l + 1$ and $(l + 1)l$, gives

$$\int_\alpha^a \int_\beta^b \left|x(s,t)\right|^l \frac{\partial}{\partial t}(\left|x(s,t)\right|)dsdt \le \frac{[(a-\alpha)(b-\beta)]^l}{l+1}\int_\alpha^a \int_\beta^b \left|D_1D_2x(s,t)\right|^{l+1}dsdt. \quad (2.7)$$

Let $x(s, t)$ reduce to $s(t)$ and $\alpha = \beta = 0$, then (2.7) becomes Hua's inequality (1.3) stated in Section 1.

Theorem 2.5 *Assume that*

(i) *f, g and* $x(s, t)$ *are as in Theorem 2.1,*
(ii) $p(s, t)$ *is increasing on* $[0, a] \times [0, b]$ *with* $p(s, \beta) = p(\alpha, t) = p(\alpha, \beta) = 0$,
(iii) *h is concave and increasing on* $[0, \infty)$,
(iv) $\varphi(t)$ *is increasing on* $[0, a]$ *with* $\varphi(0) = 0$,
(v) *For* $y(s,t) = \int_0^s \int_0^t D_1D_2p(\sigma,\tau)g\left(\frac{|D_1D_2x(\sigma,\tau)|}{D_1D_2p(\sigma,\tau)}\right)d\sigma d\tau$,

$$D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)\cdot\phi\left(\frac{1}{D_1D_2y(s,t)}\right) \le \frac{c_{(a,b)}}{y(a,b)}\cdot\phi'\left(\frac{t}{y(a,b)}\right). \quad (2.8)$$

Then

$$\int_0^a \int_0^b D_1D_2f\left(p(s,t)g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right)\cdot v\left(D_1D_2p(s,t)g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)\right)dsdt \\ \le w\left(\int_0^a \int_0^b D_1D_2p(s,t)g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)dsdt\right), \tag{2.9}$$

where

$$v(z) = zh\left(\phi\left(\frac{1}{z}\right)\right), \tag{2.10}$$

$$w(z) = c_{(a,b)}h\left(a\phi\left(\frac{b}{z}\right)\right). \tag{2.11}$$

and

$$c_{(a,b)} = \int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)dsdt.$$

Proof From (2.2), we easily obtain

$$p(s,t)g\left(\frac{|x(s,t)|}{p(s,t)}\right) \le \int_0^s \int_0^t D_1D_2p(\sigma,\tau)g\left(\frac{|D_1D_2x(\sigma,\tau)|}{D_1D_2p(\sigma,\tau)}\right)d\sigma d\tau = y(s,t). \tag{2.12}$$

From (2.8), (2.10-2.12) and Jensen's inequality(for concave function), hence

$$\begin{aligned}
&\int_0^a \int_0^b D_1D_2f\left(p(s,t)g\left(\frac{|x(s,t)|}{p(s,t)}\right)\right)\cdot v\left(D_1D_2p(s,t)g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)\right)dsdt \\
&\le \int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)\cdot v\left(D_1D_2y(s,t)\right)dsdt \\
&= \int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)\cdot h\left(\phi\left(\frac{1}{D_1D_2y(s,t)}\right)\right)dsdt \\
&= \frac{\int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)\cdot h\left(\phi\left(\frac{1}{D_1D_2y(s,t)}\right)\right)dsdt}{\int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)dsdt} \\
&\quad\times \int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)dsdt \\
&\le h\left(\frac{\int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)\cdot \phi\left(\frac{1}{D_1D_2y(s,t)}\right)dsdt}{\int_0^a \int_0^b D_1D_2f\left(y(s,t)\right)D_1D_2y(s,t)dsdt}\right)\cdot c_{(a,b)} \\
&\le h\left(\frac{\int_0^a \int_0^b \frac{c_{(a,b)}}{y(a,b)}\cdot \phi'\left(\frac{t}{y(a,b)}\right)dsdt}{c_{(a,b)}}\right)\cdot c_{(a,b)} \\
&= h\left(\frac{1}{y(a,b)}\int_0^a \left(y(a,b)\phi\left(\frac{t}{y(a,b)}\right)\Big|_{t=0}^{t=b}\right)ds\right)\cdot c_{(a,b)} \\
&= h\left(a\phi\left(\frac{b}{y(a,b)}\right)\right)\cdot c_{(a,b)} \\
&= w\left(\int_0^a \int_0^b D_1D_2p(s,t)g\left(\frac{|D_1D_2x(s,t)|}{D_1D_2p(s,t)}\right)dsdt\right).
\end{aligned}$$

This completes the proof.

Remark 2.6 Let $x(s, t)$ reduce to $s(t)$, and with suitable modifications in the proof of Theorem 2.5, then (2.9) becomes the following inequality:

$$\int_0^a f'\left(p(t)g\left(\frac{|x(t)|}{p(t)}\right)\right)\cdot v\left(p'(t)g\left(\frac{|x'(t)|}{p'(t)}\right)\right)dt \le w\left(\int_0^a p'(t)g\left(\frac{|x'(t)|}{p'(t)}\right)dt\right). \quad (2.13)$$

The inequality has been obtained by Rozanova in [17]. For $x(t) = x_1(t), x_1'(t) > 0, x_1(0) = 0, x(a) = b, g(t) = t, f(t) = \phi(t) = t^2$ and $h(t) = \sqrt{1+t}$, the inequality (2.13) reduces to Polya's inequality (see [17]).

Remark 2.7 Taking for $g(x) = x$ in (2.9), then (2.9) becomes the following interesting inequality.

$$\int_0^a\int_0^b D_1D_2f(|x(s,t)|)\cdot v(|D_1D_2x(s,t)|)dsdt \le w\left(\int_0^a\int_0^b |D_1D_2x(s,t)|\,dsdt\right).$$

Acknowledgements
The authors express their deep gratitude to the referees for their many very valuable suggestions and comments. The research of Chang-Jian Zhao was supported by National Natural Science Foundation of China (10971205), and the research of Wing-Sum Cheung was partially supported by a HKU URC grant.

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Authors' contributions
C-JZ and W-SC jointly contributed to the main results Theorems 2.1 and 2.5. Both authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 14 March 2011 Accepted: 18 August 2011 Published: 18 August 2011

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doi:10.1186/1029-242X-2011-33
Cite this article as: Zhao and Cheung: **On improvements of the Rozanova's inequality.** *Journal of Inequalities and Applications* 2011 **2011**:33.