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TECHNICAL NOTE

Numerical simulation of pressure-controlled cavity expansion process in clay at constant volumetric expansion rate

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The cavity expansion theory is being applied to describe many geotechnical engineering processes. Many of these processes are controlled by the cavity volumetric expansion rate in practice. As soil is not an elastic material, the injection pressure is a non-linear function of the cavity volume, rendering it very difficult to produce the appropriate time history of injection pressure so that the controlled cavity volumetric expansion rate can be simulated. In this technical note, an algorithm to develop the time history of injection pressure using appropriate soil properties to generate the required cavity volumetric expansion rate in clay is presented. The simulation and experimental results of a series of laboratory-scale compaction grouting experiments are presented as an example to demonstrate the validity of the approach.

KEYWORDS: clays; compaction; grouting; laboratory tests; numerical modelling; pore pressures

La théorie de l'expansion de cavités est appliquée pour décrire de nombreux procédés techniques, un grand nombre desquels sont contrôlés, en pratique, par le taux d'expansion volumétrique des cavités. Le sol n'étant pas une matière élastique, la pression d'injection est une fonction non linéaire du volume de la cavité, en rendant ainsi très difficile la production d'un historique approprié de la pression d'injection, permettant une simulation du taux d'expansion volumétrique des cavités contrôlé. Dans la présente communication technique, on présente un algorithme permettant de développer l'historique de la pression d'injection, en utilisant des propriétés appropriées du sol pour produire le taux d'expansion volumétrique des cavités requis dans l'argile. La simulation et les résultats expérimentaux d'une série d'expériences de scellement par tassement en laboratoire sont présentés à titre d'exemples pour démontrer la validité de cette approche.

INTRODUCTION

The cavity expansion theory is applied to describe the behaviour of geomaterials in many geotechnical engineering processes with varying degrees of success (Silvestri *et al.*, 2005). Depending on the boundary conditions of the expanding cavity, cavity expansion problems can be broadly classified into two categories: (a) pressure-controlled cavity expansion; and (b) displacement-controlled cavity expansion. During a pressure-controlled cavity expansion process, spatial distribution of pressure within the cavity is uniform and is a function of time only. On the contrary, the cavity remains spherical or cylindrical throughout the displacement-controlled cavity expansion process (Au *et al.*, 2006b). Depending on the nature of geotechnical engineering processes, the processes can be idealised as pressure-controlled or displacement-controlled cavity expansion processes. Most existing closed-form solutions are for displacement-controlled cavity expansion problems in an infinite, homogeneous and isotropic continuum under either undrained (no volume change) or drained (no excess pore-water pressure) isotropic stress state (Yu & Houlsby, 1991, 1995; Cao *et al.*, 2002; Mantaras & Schnaid, 2002), although advances are being made to accommodate anisotropic finite continuum under an anisotropic stress state.

As the behaviour of soil during a cavity expansion process can be very complex due to the non-linearity of soil, complexity of boundary conditions and simultaneous occurrence of several processes, idealised closed-form solutions cannot always capture the behaviour of soil during and after the process. Numerical simulation may provide an alternative avenue to develop a better understanding of the associated processes and the changes in soil properties resulting from these processes.

Numerical simulation of pressure-controlled cavity expansion in soil is quite straightforward if the time history of injection pressure is known. However, many pressure-controlled cavity expansion processes in practice are controlled by the cavity volumetric expansion rate, for example, compaction grouting. As soil is not an elastic material, the injection pressure is a non-linear function of the cavity volume. It is thus necessary to develop the time history of injection pressure to generate the required cavity volumetric expansion rate as input for the simulation. An algorithm developed to meet this end is presented in this technical note. It should be noted that the shape of the expanding cavity is not assumed a priori to be cylindrical or spherical as in the analyses of displacement-controlled cavity expansion processes. The simulation and experimental results of a series of laboratory-scale pressure-controlled cavity expansion tests are also presented as an illustration.

EXPERIMENTAL SET-UP

The laboratory experimental set-up is shown in Fig. 1. A detailed description of the apparatus is given by Au *et al.* (2006a). The modified consolidometer is 100 mm in internal diameter and 280 mm high. Pressure-controlled cavities were expanded in normally consolidated 100 mm high E-grade kaolin clay specimens by precisely controlled injection of epoxy resin or water, using a GDS pressure/volume controller, into a specially designed latex balloon through a

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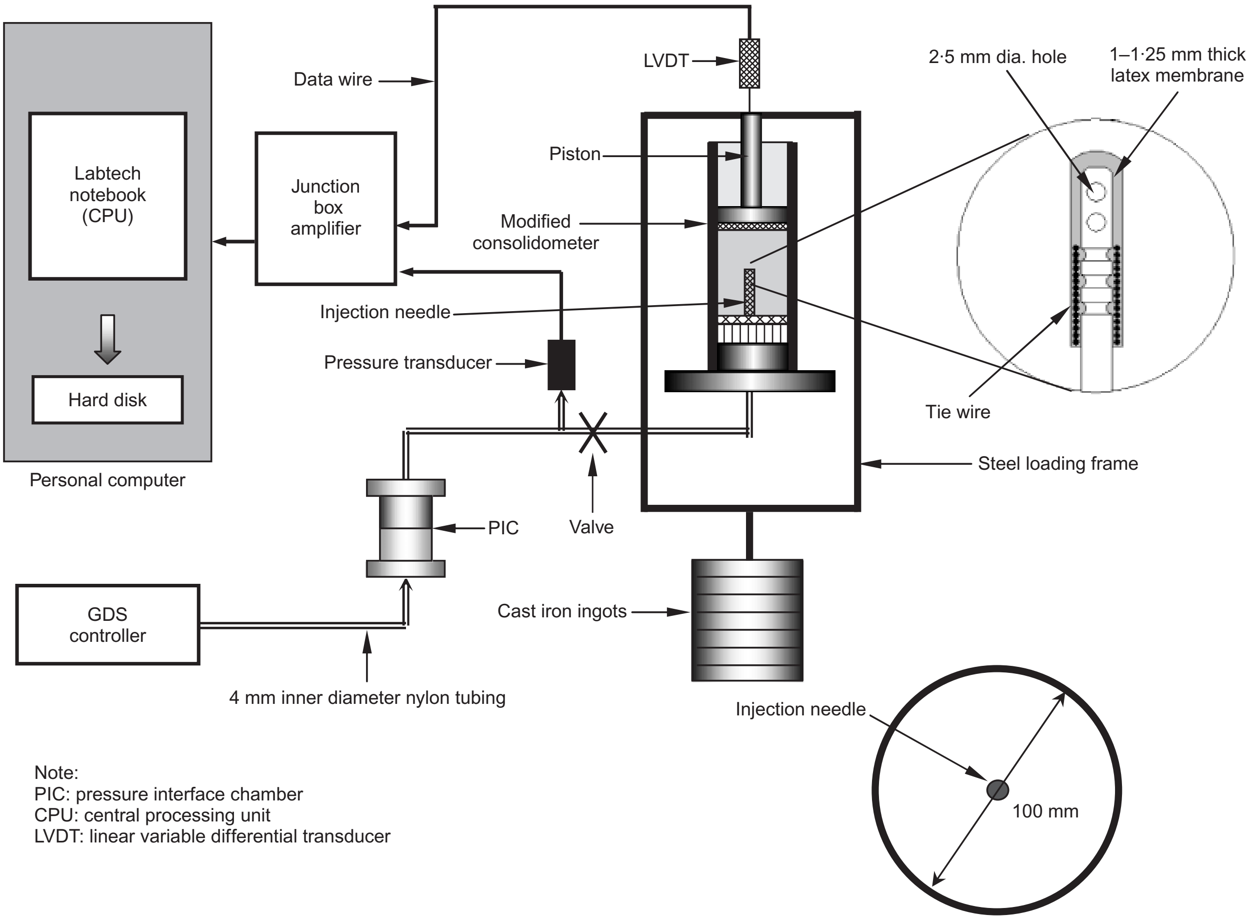

Fig. 1. Schematic diagram of experimental set-up of grout injection

multiple-hole injection needle embedded in the kaolin specimen to simulate an ideal subsurface compaction grouting process (i.e. no bleeding or permeation of grout). The vertical effective stress exerted on the specimen was maintained at 140 kPa, simulating a typical overburden during compaction grouting. The injection volume, vertical displacement of the top surface of the clay specimen and injection pressure were measured by the GDS, a linear variable differential transducer (LVDT) and a Druck PDCR pressure transducer, respectively, during the cavity expansion process. Post-cavity expansion settlement was measured continuously by the LVDT to evaluate the consolidation process after the procedure.

NUMERICAL MODEL

The two-dimensional axisymmetric numerical model developed is shown in Fig. 2. Eight-node quadrilateral full integration consolidation elements with four integration points for pore-pressure calculations were adopted for the clay specimen, and eight-node quadrilateral full integration elements (without pore-pressure calculations) were adopted for the piston base. There are 1571 elements in the mesh of the specimen. The size of element increases with the radial distance from the cavity centre, where most rapid variations of various parameters with distance would occur. The vertical cylindrical boundary and the bottom boundary were modelled as roller boundaries. Drainage boundaries were provided at the top and bottom of the specimen.

The modified Cam-clay model implemented in Abaqus

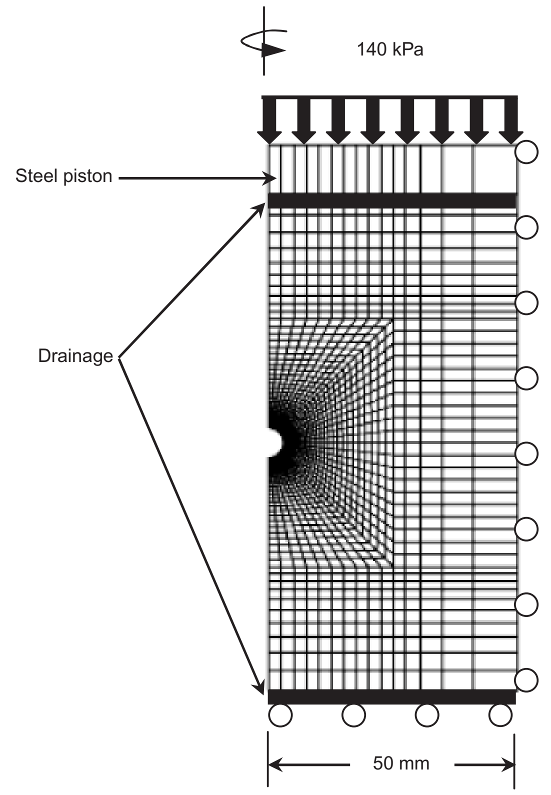

Fig. 2. Numerical model

was used in the analyses, as it can capture the plastic deformation behaviour of the clay around the expanding cavity. As the clay around the expanding cavity would undergo large deformation, geometric non-linearity and updated Lagrangian formulation were adopted in the analyses. Details of the model are given by Abaqus (2004).

Material properties

The Cam-clay parameters of E-grade kaolin used for the series of experiments obtained from triaxial tests are: $\kappa = 0{\cdot}03$, $\lambda = 0{\cdot}13$, $M = 1{\cdot}05$, $\Gamma = 2{\cdot}65$, $\upsilon = 0{\cdot}2$ and $k = 2 \times 10^{-9}$ m/s. A detailed description of the material is given by Elmes (1985).

Initial geostatic conditions

The self-weight of the clay specimen was neglected. The initial stress was set at 140 kPa and the corresponding void ratio was taken as the initial void ratio. The coefficient of at-rest lateral pressure K_0 was estimated by the empirical relationship (Schmidt, 1966)

$$K_0 = [1 - \sin(1{\cdot}2 \times \varphi'_{\text{crit}})] \times \text{OCR}^{\sin(1{\cdot}2\times\varphi'_{\text{crit}})} \tag{1}$$

where φ'_{crit} is the the effective critical state angle of shearing resistance of clay (Muir Wood, 1990); and OCR is the overconsolidation ratio.

The initial outside radius of the latex balloon of 3·25 mm was used as the initial cavity radius. The nodal reaction forces around the cavity boundaries at equilibrium under geostatic conditions were first calculated assuming there was no cavity. The nodal reaction forces so obtained were then applied at the nodes on the cavity boundaries to maintain the initial cavity shape. The initial cavity pressure was calculated. The clay elements within the cavity were then removed to create the cavity.

Development of the time history of injection pressure

Taking the initial injection pressure p_0 to be zero at t_0, the injection pressure p_α at t_α was given by

$$\begin{gathered} p_t = p_{\alpha-1} + \frac{\Delta p_\alpha}{\Delta t_\alpha} \times (t - t_{\alpha-1}) \\ t_{\alpha-1} \leqslant t \leqslant t_\alpha \quad \text{and} \quad \alpha = 1, 2, 3, \ldots \end{gathered} \tag{2}$$

where $\Delta t_\alpha = (t_\alpha - t_{\alpha-1})$; and $\Delta p_\alpha = (p_\alpha - p_{\alpha-1})$ = incremental injection pressure over the time increment Δt_α.

When the injection pressure $p_{\alpha-1}$ at $t_{\alpha-1}$ was known, trial incremental nodal reaction forces were applied at the nodes of cavity boundaries to simulate an incremental injection pressure $\Delta p_{\alpha,i}$ over the time increment Δt_α. A good initial estimate of $\Delta p_{\alpha,i}$ was obtained by extrapolating $\Delta p_{\alpha-1}$ by

$$\Delta p_{\alpha,i} = \Delta p_{\alpha-1} \times \frac{\Delta t_\alpha}{\Delta t_{\alpha-1}} \tag{3}$$

The rate of pressure increase within the time increment Δt_α was therefore $\Delta p_{\alpha,i}/\Delta t_\alpha$. The resulting displacements of the nodes at t_α were calculated. Afterwards, the total cavity volume at t_α was obtained by three-dimensional trapezoidal integration. The cavity volume increase was obtained by subtracting the initial cavity volume from the total cavity volume at t_α. The simulated cavity displacements have already taken into account the expansion of the cavity induced by the injection pressure and any consolidation that might have taken place from t_0 to t_α.

When the simulated cavity volume increase is larger (smaller) than the injected grout volume, smaller (larger) trial incremental nodal forces equivalent to an incremental injection pressure $\Delta p_{\alpha.j}$ were then applied, until the simulated cavity volume increase was smaller (larger) than the injected grout volume. A trial incremental injection pressure $\Delta p_{\alpha,k}$ obtained by interpolating between $\Delta p_{\alpha,i}$ and $\Delta p_{\alpha.j}$ was then used in the simulation. Depending on whether the simulated cavity volume increase induced by $\Delta p_{\alpha,k}$ was larger or smaller than the injected grout volume, another incremental injection pressure was obtained by interpolating between $\Delta p_{\alpha,k}$ and $\Delta p_{\alpha,j}$ or $\Delta p_{\alpha,i}$ and $\Delta p_{\alpha,k}$ for the simulation. The iteration process was continued until the target cavity volume increase at t_α was obtained.

The process was continued until the final cavity volume was reached. Afterwards, the injection pressure was adjusted similarly to maintain the final cavity volume. The time history of injection pressure after completion of grout injection is very useful for the understanding of the post-injection consolidation behaviour of soil. The phenomenon has a profound impact on the long-term effectiveness of compaction grouting as a compensation grouting technique (Au *et al.*, 2007).

RESULTS AND DISCUSSION

Available experimental measurements were: (a) injection volume, (b) injection pressure and (c) heave or settlement of the clay specimen. These measurements were thus compared with simulation results to evaluate the algorithm.

Injection pressure

The injection pressure plotted against time curves for three experiments of volumetric injection rates of 41·67, 83·33 and 500 mm^3/s are presented in Fig. 3. Other experimental parameters were practically identical and the final injection volume was 5000 mm^3. The durations of grout injection were thus 120, 60 and 10 s, respectively.

The significant effects of volumetric injection rate on injection pressure can be observed from the experimental data. A slower volumetric cavity expansion rate generates a lower rate of injection pressure increase. However, the ultimate injection pressure increases with decrease in cavity volumetric expansion rate. The feature was successfully captured by the algorithm as shown in Fig. 3.

The curves of injection pressure plotted against cavity volume are presented in Fig. 4. Both experimental and simulation results indicate that the injection pressure at the same cavity volume increases with decrease in volumetric injection rate. A slower injection rate allows more time for the excess pore-water pressure generated by the expanding

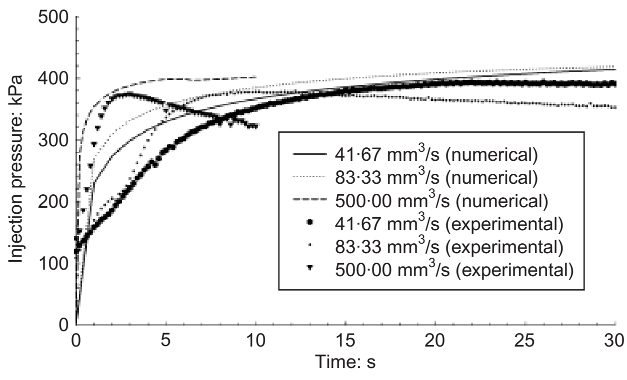

Fig. 3. Injection pressure plotted against time

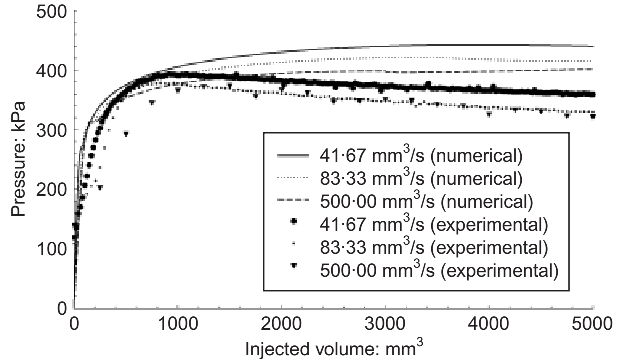

Fig. 4. Injection pressure plotted against injected volume

cavity to dissipate, resulting in an increase of the shear strength of the clay. When the cavity volume reached approximately 100 mm^3, the rate of increase of injection pressure with increase in cavity volume decreased, owing to formation of a plastic zone around the cavity. The peaks in injection pressure observed in experiments cannot be captured by numerical simulations. As the clay is normally consolidated, decrease in injection pressure cannot be described by the modified Cam-clay model. However, formation of microcracks around the cavity, as observed in experiments, facilitated reduction of injection pressure – a feature that cannot be captured by the numerical model. Moreover, the dynamic effect of grout injection cannot be simulated. Such an effect may be more pronounced when the volumetric injection rate is high.

Heave of the clay specimen

Heaves of the clay specimens as a function of time are presented in Fig. 5. The heave induced by subsurface cavity expansion increases with time during injection and decreases afterwards as excess pore-water pressure dissipates. The maximum heave that can be developed decreases with decrease in volumetric injection rate, as a considerable amount of consolidation also takes place during injection. The shape of the consolidation curve after the peak also changes as a function of the amount of consolidation that has taken place. These important heave characteristics were successfully captured by the numerical model. The experimental data and simulation results differ by tenths of a millimetre. However, the rates of consolidation observed in experiments were considerably faster than those observed in numerical simulations, as the formation of microcracks around the cavity provided additional drainage paths of high hydraulic conductivity.

Limitations

There are limitations in the current experimental set-up and numerical simulation scheme. The physical size of the apparatus limits the specimen size and the size of the grout bulb that can be generated within the specimen. The change in stress, pore-water pressure, void ratio or shear strength parameters during the cavity expansion process cannot be measured. Moreover, formation of cracks cannot be observed during the process.

The numerical algorithm requires a trial-and-error approach to determine the pressure increment for each volumetric expansion increment. Therefore, the algorithm can be easily adopted for any volumetric expansion rate. Although the numerical scheme is robust and can be automated, it can be tedious if the tolerance is small. The numerical algorithm cannot simulate the formation of cracks in the soil around the expanding cavity, an inherent limitation of most finite-element programmes.

CONCLUSIONS

The following conclusions can be drawn from the study.

(*a*) A numerical model has been developed to simulate pressure-controlled subsurface cavity expansion at constant volumetric expansion rate. The results of the numerical simulations are in reasonable agreement with available experimental data.
(*b*) The algorithm can be easily adopted for any cavity volumetric expansion rate.
(*c*) The model can be used to develop better insight into the complex processes during pressure-controlled subsurface cavity expansion controlled by cavity volumetric expansion rate.

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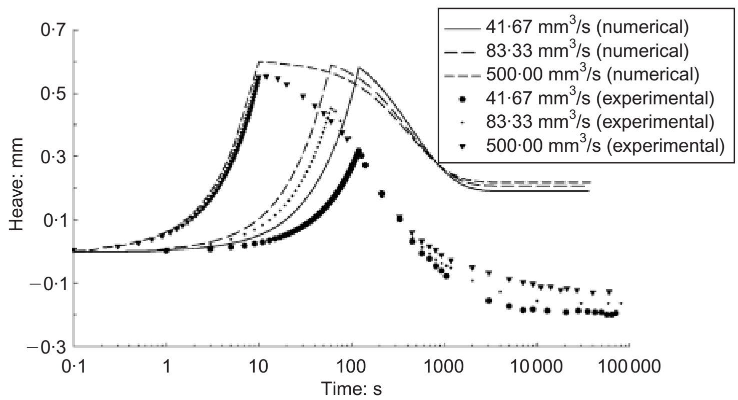

Fig. 5. Heave plotted against time

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