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Effects of various shape parameters on packing of aggregate particles

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The shape of the aggregate particles used has significant effects on the properties of the concrete produced. One major effect is on the packing density of the aggregate which determines the amount of cement paste needed to fill the voids between the aggregate particles. In order to study how the various shape parameters of aggregate particles would affect the packing of aggregate, aggregate samples of different rock types from different sources have been analysed for their shape characteristics using a newly developed digital image processing technique and their packing densities measured in accordance with an existing method given in the British Standard. The packing densities of the aggregate samples are correlated to the shape parameters to evaluate the effects of the various shape parameters on packing. From the results of the correlation, it is found that the shape factor and the convexity ratio are the most important shape parameters affecting the packing of an aggregate. Two alternative formulas revealing the combined effects of these two shape parameters on the packing density of aggregate are proposed.

Introduction

The packing of an aggregate for concrete is the degree of how good the solid particles of the aggregate would fill up the volume of the concrete. It is usually measured in terms of 'packing density', which is defined as the ratio of the solid volume of the aggregate particles to the bulk volume occupied by the aggregate, as given by:

$$\text{packing density} = \frac{\text{solid volume of particles}}{\text{bulk volume of aggregate}} \quad (1)$$

From the packing density, the 'voids ratio', that is, the ratio of the volume of voids between the aggregate particles to the bulk volume occupied by the aggregate, may be obtained as:

$$\text{voids ratio} = 1 - \text{packing density} \quad (2)$$

Depending on the size distribution and shape characteristics of the aggregate, the packing density may vary from 55 to 85%, while the corresponding voids ratio may vary from 45 to 15%.

Since the voids between the aggregate particles must be filled up with cement paste, the voids ratio determines the minimum volume of cement paste needed to produce concrete using the aggregate. A higher packing density leads to a smaller voids ratio and thus a smaller amount of cement paste needed. Hence, concrete producers and engineers have been trying to improve the packing of aggregates. From the concrete producers' point of view, the incentive is to reduce the amount of cement needed in order to cut down the cost of production. From the engineers' points of view, the incentive is to reduce the heat of hydration and the drying shrinkage, both of which may cause cracking problems and are roughly proportional to the volume of cement paste in the concrete.

Packing of the aggregate used has a significant effect on the workability or the water requirement of the concrete mix produced. Kaplan[1] has shown that for a given mix proportion, the workability of the concrete mix decreases as the voids ratio increases. On the other hand, Bloem and Gaynor[2] have found that the water requirement of a concrete mix increases more or less linearly with the voids ratio of the aggregate used. These effects may be explained by the fact that the cement paste added must first fill up the voids between the aggregate particles and it is the cement paste in excess of the amount needed to fill the voids that lubricates the concrete mix and gives the mix workability. Another possible reason is that some of the

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shape parameters affecting the packing of the aggregate such as the angularity of the aggregate particles may also be affecting the frictional forces between the particles and hence the workability of the concrete mix.

A related effect of the packing of aggregate is on the pumpability of the concrete mix produced.[3] Generally, for a concrete mix to be pumpable, the voids sizes of the aggregate should be as small as possible and the volumetric content of the cement paste should be greater than the voids content of the aggregate. For a given cement paste content, the pumpability is higher when the voids ratio is smaller and, for a given pumpability, a larger voids ratio would lead to a larger required amount of cement paste.

Packing of an aggregate also has an indirect effect on the strength of the concrete made with the aggregate. If the packing is good, the water requirement will be smaller and a lower water–cement ratio may be used to produce a higher strength concrete. If, however, the packing is not good, the water requirement will be larger and for the same amount of cement used, the water–cement ratio will become higher and the concrete strength lower. Hence, in the case of high-performance concrete where both high strength and high workability are demanded, it is important to select an aggregate with good packing or otherwise the cement paste volume might become undesirably high.

The packing of an aggregate is dependent on the size distribution of the aggregate particles. An aggregate having a good grading, in the sense that it contains particles of a wide range of size with the voids between particles filled by successively smaller particles, would generally have a relatively high packing density. The maximum density theory calls for an ideal grading curve which is parabolic in shape when plotted to a natural scale. However, such an 'ideal' grading curve, though capable of minimising the amount of cement paste needed, produces a harsh concrete mix. In actual practice, the addition of more than enough cement paste to fill the voids of the fine aggregate and more than enough mortar (cement paste + fine aggregate) to fill the voids of the coarse aggregate would produce a more workable concrete mix.[3]

The packing of an aggregate is dependent also on the shape of the aggregate particles but the effect of particle shape is more difficult to comprehend. There are many confusions on how the various shape parameters would affect the packing of aggregate, mainly because up to now many of the shape parameters are not yet clearly defined[4] and there are no commonly accepted methods for their measurement. For instance, in the British Standard BS 812: Sections 105.1 and 105.2,[5] the flakiness/elongation of an aggregate sample are measured indirectly in terms of flakiness/elongation indexes which are respectively defined as percentages by mass of particles classified as flaky/elongated particles. The particles are classified as flaky/elongated particles according to the rather arbitrary assumptions that a particle is flaky if its thickness is less than 0·6 times the sieve size and that a particle is elongated if its length is greater than 1·8 times the sieve size. These measures of flakiness and elongation are far from being satisfactory for the main reason that the degrees of how flaky/elongated the particles actually are have not been taken into account and for some other reasons explained by Kwan *et al.* in a recent study.[6]

The situation with angularity measurement is even worse. Different researchers used different definitions for angularity. Some defined angularity as the number of protrusions without taking into account the actual shape of the protrusions,[7] some measured the angularity in terms of the mean angle of the protrusions,[8] while others measured the angularity as a function of the sharpness of the protrusions and the probabilities of the protrusions being contacted by other bodies.[9] To overcome the difficulties of measuring angularity directly from the geometry of the particle boundaries, the British Standard BS 812: Part 1[10] measures the angularity indirectly in terms of an angularity number, which is defined as 67 minus the packing density of a single-sized fraction of the aggregate expressed as a percentage value (or as the voids ratio expressed as a percentage value minus 33). This is based on the belief that the packing density of an aggregate is determined solely by the angularity of the aggregate. However, while angularity may be the major factor affecting packing, there may be other shape parameters also having significant effects on packing. In fact, Popovics[11] has suggested that the packing density may be dependent also on sphericity and surface texture. It is, therefore, doubtful whether the angularity number measured by the packing density test is an appropriate measure of angularity. In reality, the angularity number is nothing more than a measure of packing density or voids ratio.

Nevertheless, some progress in particle shape measurement has been made in recent years by employing the digital image processing (DIP) technique to capture and analyse the geometry of the particle boundaries.[6–9,12–14] Experiences gained so far have indicated that the DIP technique is suitable for this purpose and is fast, accurate and reliable. With the use of the DIP technique, sophisticated mathematical analysis of the geometry of the particle boundaries can be carried out to characterise the shape of the particles. However, there is the major problem with the DIP technique that only the two-dimensional projection of the particles is captured and measured. In other words, the third dimension, that is, thickness, of the particles is not directly obtainable from the DIP results. Due to this problem:

(*a*) the quantity measurements of the DIP analysis have to be expressed in terms of area fractions rather than mass fractions and are thus more difficult to interpret, as most people are more used to measuring quantity by mass, and (*b*) the flakiness of the particles, which is dependent on the thickness of the particles, cannot be measured using the DIP method.

Nevertheless, the authors[6] have recently resolved these problems by supplementing the DIP results with the weight of the aggregate sample to obtain the mean thickness–breadth ratio of the aggregate particles, based on which the area fractions may be converted to mass fractions and the flakiness of the aggregate particles may be evaluated.

In the present study, in order to investigate how the various shape parameters of the aggregate particles affect the packing of aggregate, 46 coarse aggregate samples of different rock types from different sources are analysed for their particle shape using the DIP technique and their packing densities measured using the packing density test described in the British Standard BS 812: Part 1.[10] After the shape analysis and the packing density tests, the measured shape parameters are correlated to the packing densities of the aggregate samples to evaluate the effects of various shape parameters on packing.

Measurement of packing density

The packing density of an aggregate sample is measured in accordance with the method described in Section 7·5 of British Standard BS 812: Part 1.[10] Since both the size distribution and the particle shape of the aggregate would affect the packing density, it is necessary to remove the effect of size distribution in order to isolate the effect of particle shape. This is done by conducting the packing density test on single-sized fractions of the aggregate. Hence, the first step in the test procedure is to separate the aggregate sample by sieving into different size fractions, each falling within a narrow sieve size range so that each size fraction is essentially a single-sized aggregate. The sieve size ranges are: 20·0–14·0 mm, 14·0–10·0 mm, 10·0–6·3 mm and 6·3–5·0 mm.

The packing density of each size fraction is measured by filling up a steel cylinder with the aggregate particles, subjecting the particles to prescribed tamping and weighing the amount of particles in the cylinder. The cylinder is of size 150 mm diameter × 170 mm height. Its volume is determined by filling it up with water such that no meniscus is present above the rim and weighing the amount of water inside. Having determined the weight of aggregate in the cylinder, the packing density of the aggregate may be evaluated as

$$\text{packing density} = \frac{\text{weight of particles in cylinder}}{\rho \times \text{volume of cylinder}} \qquad (3)$$

where ρ is the density of the aggregate particles.

Measurement of shape parameters by DIP

To conduct image analysis of an aggregate sample, an image of the aggregate particles is first acquired by placing the particles on a sample tray and putting the sample tray underneath a video camera. Details of the image acquisition process and calibration procedures have been reported previously.[6] Having acquired an image of the aggregate particles, DIP is performed to discriminate the aggregate particles from the background. Once the particle boundaries are located, their geometry is analysed to measure the dimensions and shape characteristics of the particles.

Estimating thickness and volume of particles

Since only the two-dimensional projection is captured for image analysis, the thickness and volume of the particles are not directly obtainable from the DIP results. Nevertheless, a method of estimating the thickness and volume of the particles has been developed previously.[6] It is based on the assumption that aggregate particles from the same source should have more or less the same shape characteristics and thus similar thickness to breadth ratio. Using this assumption, the mean thickness and volume of a particle may be estimated as

$$\text{mean thickness} = \lambda \times \text{breadth} \qquad (4)$$

$$\text{volume} = \text{mean thickness} \times \text{area} = \lambda \times \text{breadth} \times \text{area} \qquad (5)$$

in which λ is a parameter dependent on the flakiness of the aggregate. Adding the volume of all particles and multiplying by the density ρ, the equation for the total mass of the aggregate sample M is derived as

$$M = \rho \times \lambda \times \sum_{i=1}^{n} (\text{breadth} \times \text{area}) \qquad (6)$$

where n is the total number of particles. Solving the above equation, λ is determined as

$$\lambda = \frac{M}{\rho \times \sum_{i=1}^{n} (\text{breadth} \times \text{area})} \qquad (7)$$

Substituting the value of λ into equations (4) and (5), the mean thickness and volume of each particle may then be obtained. Being equal to the mean thickness–breadth ratio of the aggregate sample, λ may be taken as a measure of flakiness.

Arithmetic and weighted mean values of shape parameters

An aggregate sample consists of many particles. To determine the shape parameter of an aggregate, it is necessary to first measure the shape parameter of each particle and then calculate the mean value of the shape parameters of the particles. There are at least two ways of calculating the mean value. The first is to calculate the mean value as the arithmetic mean of the shape parametres of the particles, with each value of shape parameter derived from a particle given equal weight. However, since larger particles generally have greater effects on the overall performance of the aggregate, a better alternative is to take the mean value as the weighted mean of the shape parameters of the particles, as given by:

$$\text{weighted mean of shape parameter} = \frac{\sum_{i=1}^{n}(\text{volume} \times \text{shape parameter})}{\sum_{i=1}^{n}(\text{volume})} \quad (8)$$

The weighted mean values are used in the present study because they generally give more consistent results and better correlation to each other.

Shape parameters measured

Flakiness ratio. The flakiness ratio is defined as the thickness–breadth ratio. It is also called flatness ratio.[12] Although the thickness is not directly obtainable by DIP, the value of λ is actually a weighted mean value of the mean thickness–breadth ratio of the aggregate sample. Hence, λ is used as the flakiness ratio in this study.

Elongation ratio. The elongation ratio is defined as the breadth–length ratio.[12] It is obtained directly from the DIP results.

Sphericity. Sphericity is usually defined as the ratio of the surface area of a sphere having the same volume as the particle to the actual surface area of the particle.[12] However, since the surface area has to be evaluated by three-dimensional analysis and cannot be determined by the present or any other two-dimensional DIP method, an alternative definition of sphericity proposed by Krumbein [13] is used. It is given by:

$$\text{sphericity} = \sqrt[3]{\frac{\text{thickness} \times \text{breadth}}{\text{length}^2}} \quad (9)$$

Substituting the thickness by $\lambda \times$ breadth, the sphericity can be expressed as:

$$\text{sphericity} = \sqrt[3]{\lambda\left(\frac{\text{breadth}}{\text{length}}\right)^2} \quad (10)$$

which can then be evaluated from the DIP results.

Shape factor. Shape factor is a commonly used index but different researchers adopt different definitions for it to describe different aspects of shape[12,14] In the present study, the definition used follows that adopted by Barksdale *et al.*, [12], as given by

$$\text{shape factor} = \frac{\text{thickness} \times \text{length}}{\text{breadth}^2} \quad (11)$$

Replacing the thickness by $\lambda \times$ breadth, equation (11) becomes:

$$\text{shape factor} = \frac{\lambda \times \text{length}}{\text{breadth}} \quad (12)$$

Convexity ratio. Convexity ratio is a measure of convexity.[13] Due to the difficulty of three-dimensional shape analysis, the convexity ratio is evaluated from the two-dimensional projection of the particle, as shown in Fig. 1. It is defined as:

$$\text{convexity ratio} = \frac{\text{area}}{\text{convex area}} \quad (13)$$

where the convex area is the area of the minimum convex boundary circumscribing the particle.

Fullness ratio. Fullness ratio is another measure of convexity that is also evaluated from the two-dimensional projection of the particle.[14]. It is defined as:

$$\text{fullness ratio} = \sqrt{\frac{\text{area}}{\text{convex area}}} \quad (14)$$

Correlation of packing density to shape parameters

To study the effects of the various shape parameters on the packing of aggregate, three types of rock aggregates from five different sources have been tested for their packing densities and analysed by the DIP method for their shape parameters. They are crushed granitic rock from Hong Kong and mainland China, crushed volcanic aggregate from Hong Kong and mainland China, and gravel from Canada. The maximum particle sizes of the aggregate samples vary from 10 to 40 mm. As the crushed rock aggregates are quite angular, while the gravel aggregates are very rounded, the aggregate

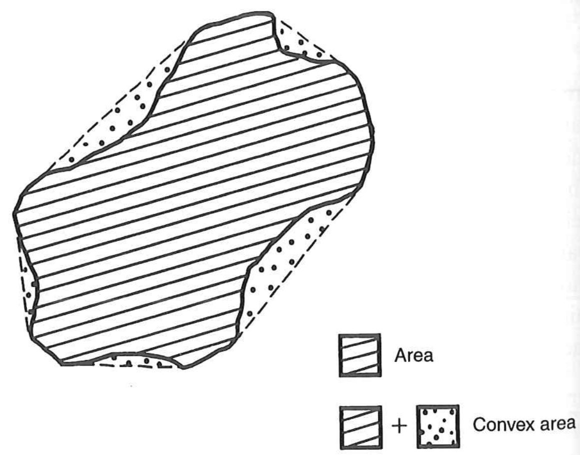

Fig. 1. Definitions of area and convex area

samples studied cover both ends of the angularity–roundness scale. In order to produce more intermediate results for statistical analysis, some of the crushed rock aggregates have been subjected to artificial attrition using the Los Angeles testing machine to change their shape characteristics before measurement of their packing densities and shape parameters.

Packing density as single-variable functions of individual shape parameters

The packing density is plotted against the various shape parameters in Fig. 2. To show the overall trend of correlation, best-fit lines are drawn on the graphs along with the data points plotted. As can be seen from the different degrees of scattering of the data points from the best-fit lines, the various shape parameters have different correlation with the packing density. The values of the correlation coefficient r of the correlation between the packing density and each of the shape parameters are listed in Table 1. It is seen that among the six shape parameters measured, the flakiness ratio, shape factor and convexity ratio, which respectively give correlation coefficients of 0·873, 0·859 and 0·828 when correlated to the packing density, seem to have

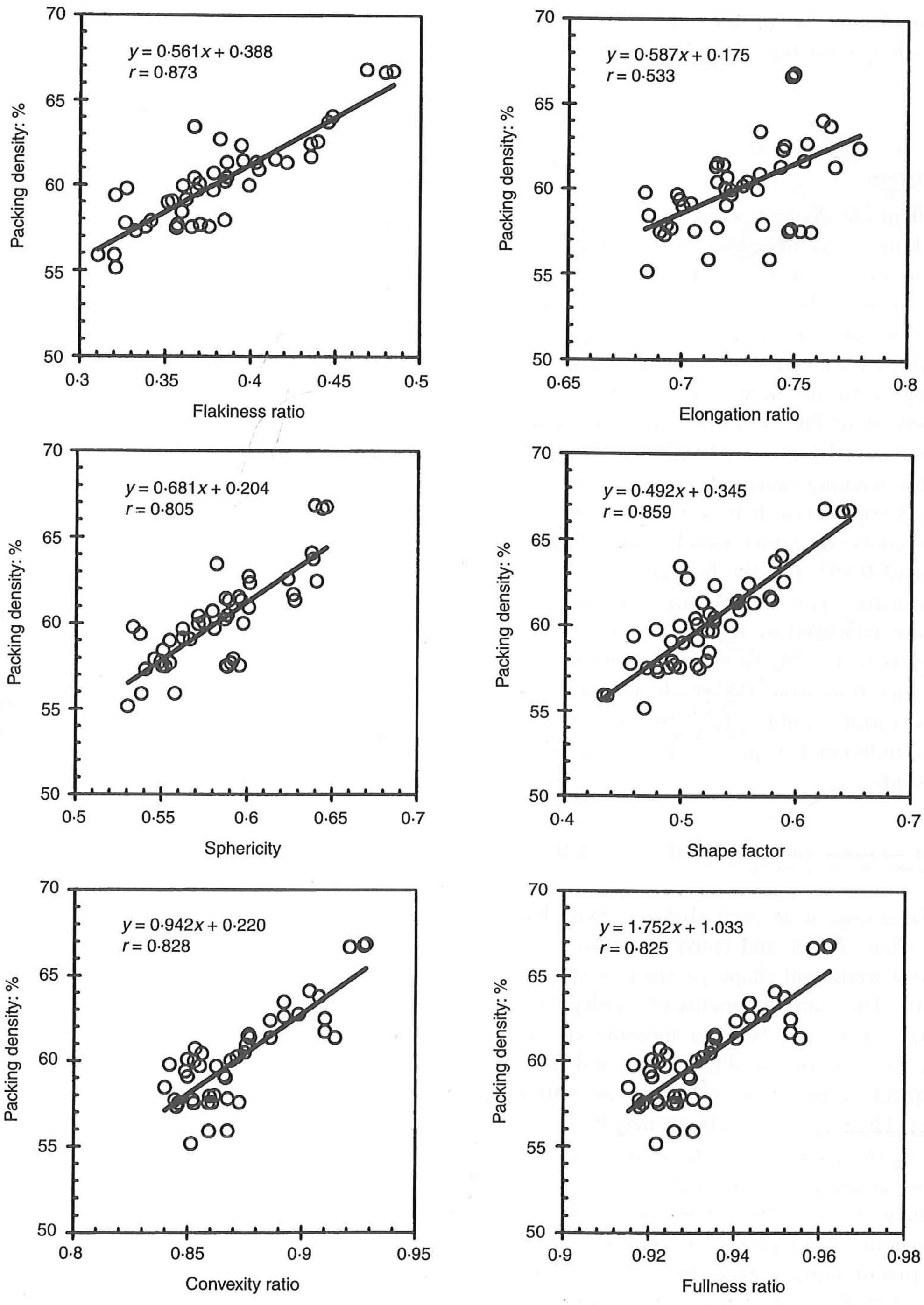

Fig. 2. Correlation of packing density to individual shape parameters

Table 1. Correlation of packing density to various shape parameters

Shape parameter measured by DIP	Correlation coefficient, r
Flakiness ratio, λ	0·873
Elongation ratio	0·533
Sphericity	0·805
Shape factor	0·859
Convexity ratio	0·828
Fullness ratio	0·825

the greatest effects on the packing of an aggregate. Relatively, the elongation ratio has the smallest effect on packing.

Packing density as single-variable functions of products of shape parameters

Since more than one shape parameter has significant effects on packing, it is necessary to investigate the combined effects of the various shape parameters. One simple way is to treat the packing density as single-variable functions of the products of two shape parameters and plot the packing density against the products arising from different combinations of the shape parameters as in Fig. 3. Table 2 lists the values of the correlation coefficient r obtained for the correlation between the packing density and the products of shape parameters considered. It is seen that the product (shape factor × convexity ratio), which gives a correlation coefficient of 0·893, has the best correlation with the packing density. This correlation coefficient is higher than those tabulated in Table 1 and thus, if the packing density is to be estimated as a single-variable function, its estimation from the product (shape factor × convexity ratio) would give better results than from any of the individual shape parameters considered in the present study.

Packing density as multi-variable functions of shape parameters

From the foregoing, it is seen that the two shape parameters — shape factor and convexity ratio — are together the most important shape parameters affecting packing density. They are geometrically independent shape parameters, with one being a measure of form and the other a measure of roundness (form and roundness are independent measures of shape as will be explained later). Their combined effects may be investigated by treating the packing density as multi-variable functions of the shape factor and the convexity ratio. However, to maintain simplicity, only functions of the simplest form should be employed. Two different functions, a linear function and a power function, have been considered for the packing density. They are given respectively by the following equations:

$$\text{packing density} = \text{a} + \text{b (shape factor)} + \text{c (convexity ratio)} \qquad (15)$$

$$\text{packing density} = \text{a (shape factor)}^{\text{b}}\text{(convexity ratio)}^{\text{c}} \qquad (16)$$

in which a, b and c are unknown coefficients to be determined. By curve-fitting of each of the above equations with the experimental data, the corresponding values of the unknown coefficients a, b and c, and the correlation coefficient r may be determined. The results of the curve-fitting are tabulated in Table 3. As indicated by the values of correlation coefficients obtained of 0·903 and 0·899 for equations (15) and (16) respectively, both equations yield very good correlation with the actual measured values of packing density. Either of them may be used for estimating the packing density.

Proposed formulas for packing density

Substituting the corresponding values of a, b and c obtained by the correlation analysis into equations (15) and (16), two alternative formulas for the packing density are derived as follows:

$$\text{packing density} = 0{\cdot}022 + 0{\cdot}311 \cdot \text{(shape factor)} + 0{\cdot}478 \cdot \text{(convexity ratio)} \qquad (17)$$

$$\text{packing density} = 0{\cdot}789 \cdot \text{(shape factor)}^{0{\cdot}271}\,\text{(convexity ratio)}^{0{\cdot}699} \qquad (18)$$

The actual experimental values of the packing density are plotted against the predicted values by equations (17) and (18) in Fig. 4, from which it can be seen that both these equations agree quite closely with the experimental results. Based on these two equations, the relationship between the packing density and the two shape parameters—shape factor and convexity ratio—are plotted in Fig. 5.

Discussion

According to Barret,[4] the shape of a rock particle can be characterized in terms of form (overall shape), roundness (large-scale smoothness) and surface texture (small-scale smoothness). Form, roundness and surface texture are independent properties of shape, although there may be natural correlation between them because of the common physical factors affecting them. Form is normally measured in terms of form factors which are defined as the ratios of the three major dimensions: length, breadth and thickness. Form factors and derivatives of form factors such as sphericity and shape factor are measures of form and cannot be used for measuring roundness. Roundness is not a measure of sphericity,

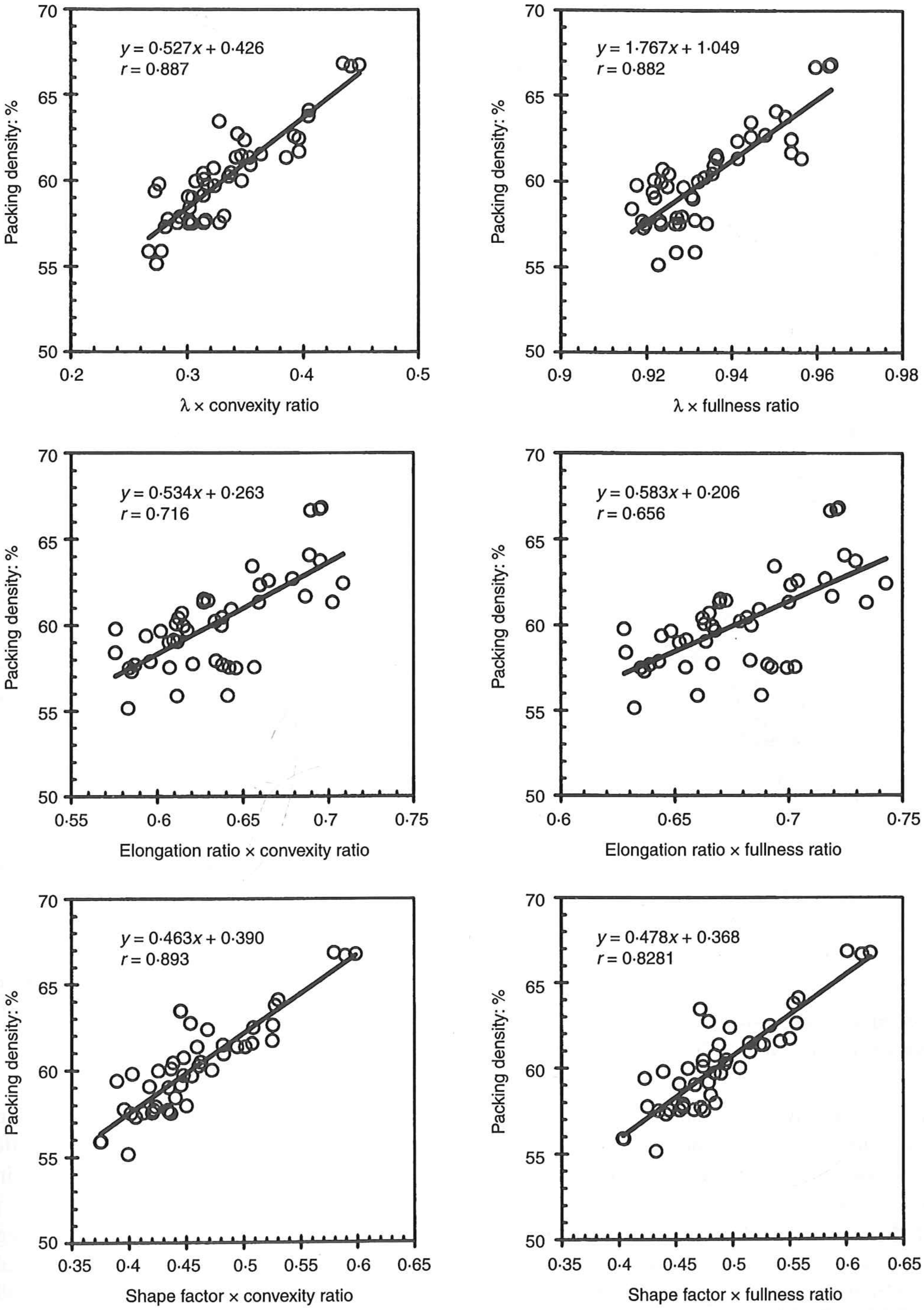

Fig. 3. Correlation of packing density to products of shape parameters

Table 2. Correlation of packing density to products of shape parameters

Product of shape parameters	Correlation coefficient, r
$\lambda \times$ convexity ratio	0·887
$\lambda \times$ fullness ratio	0·882
Elongation ratio × convexity ratio	0·716
Elongation ratio × fullness ratio	0·656
Shape factor × convexity ratio	0·893
Shape factor × fullness ratio	0·881

Table 3. Correlation of packing density to multi-variable functions of shape parameters

Multi-variable function of shape parameters	Values of unknown coefficients			Correlation coefficient, r
	a	b	c	
Equation (15)	0·022	0·311	0·478	0·903
Equation (16)	0·789	0·271	0·699	0·899

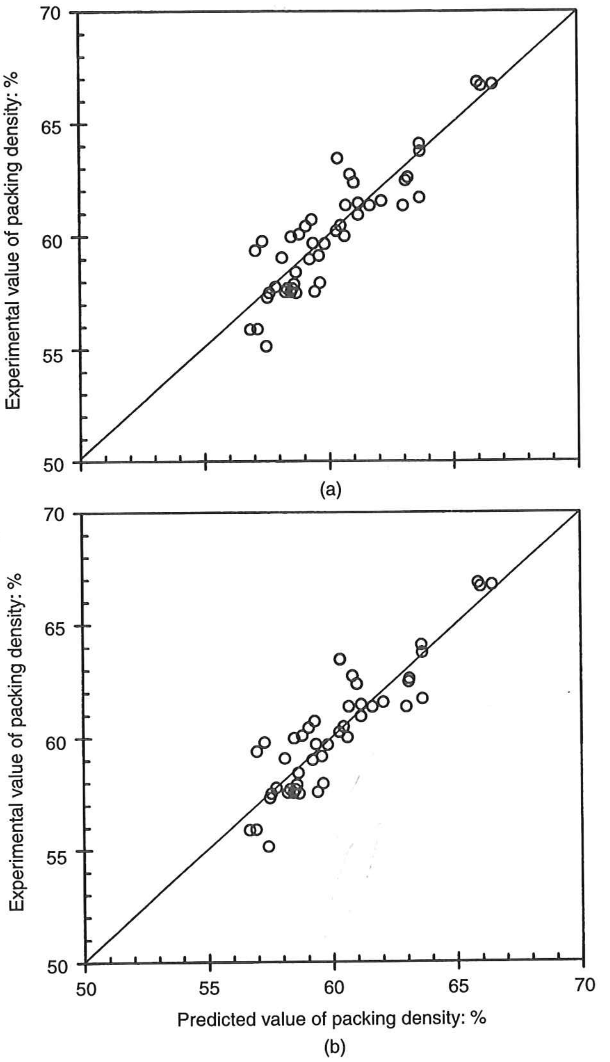

Fig. 4. Experimental plotted against predicted values of packing density for: (a) equations (17); and (b) equation (18)

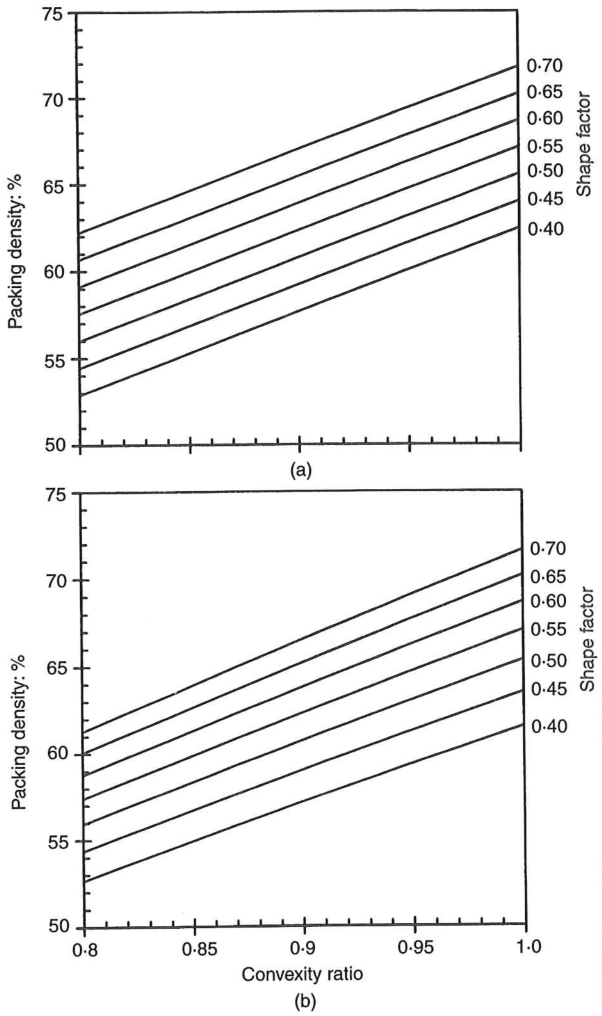

Fig. 5. Charts for packing density as functions of shape factor and convexity ratio, based on: (a) equation (17); and (b) equation (18)

although it is best displayed by a sphere. There are two aspects of roundness, namely the roundness of the corners and the roundness of the outline of the particle. The roundness of the corners is the opposite of the sharpness of the corners and is more important when considering the abrasive and perforation properties of the particles. On the other hand, the roundness of the outline is generally measured in terms of convexity and is more important when considering the interlocking ability and packing density of the particles. Roundness and angularity are opposite to each other. Hence, a measure of roundness is also a measure of angularity. Surface texture is a measure of the smoothness or roughness of the particle surface. It may be measured in terms of the magnitude and sharpness of the small-scale protrusions and indentations on the particle boundary. Since the features of surface texture are one order of magnitude smaller than the features of roundness, it is unlikely that the surface texture would have significant effect on packing density.

Although the research conducted has successfully identified the most important shape parameters affecting packing density and evaluated the effects of these shape parameters, further refinements or improvements are possible, as suggested in the following paragraphs.

Firstly, visual inspection of the aggregate samples revealed that the more flaky particles are also generally more elongated and angular. Perhaps the various physical factors affecting one shape parameter, such as the way that the particles are produced, the weathering and attrition processes that shaped the particles and the hardness of the rock, also affect the other shape parameters in a similar way. Because of the common factors affecting the shape parameters, there is a natural correlation between them. To illustrate such natural correlation, the ranges of shape parameters studied are plotted in Fig. 6. It can be seen from the figure that the aggregates investigated herein have not included aggregate types that are flaky but not elongated nor angular, aggregate types that are elongated but not flaky nor angular, and aggregate types that are angular but not flaky nor elongated. In other words, the ranges of shape

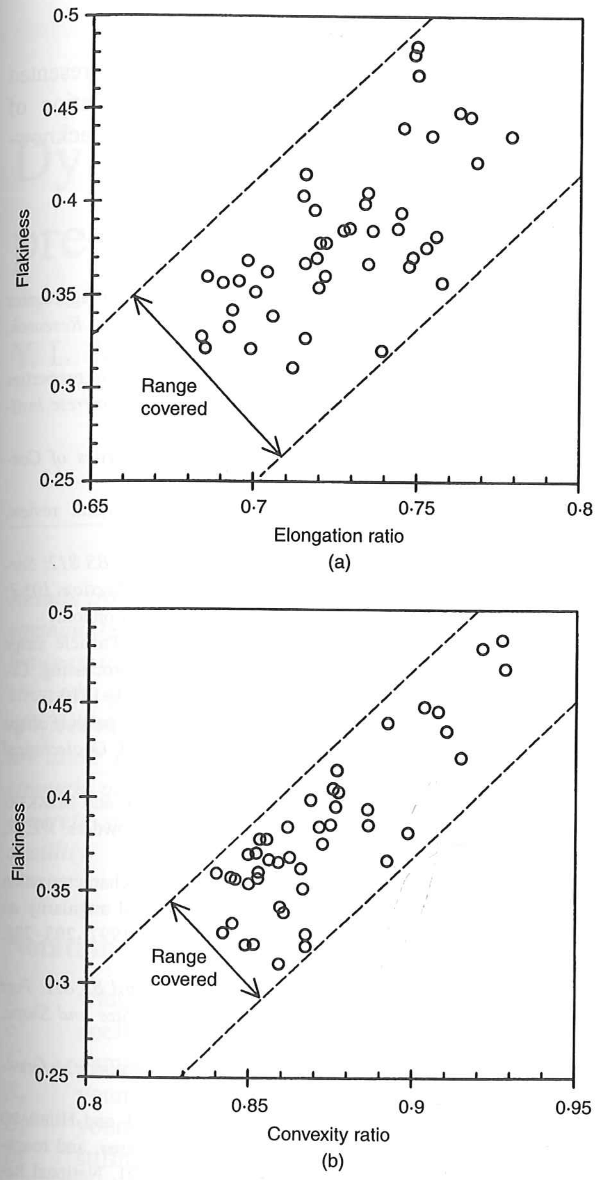

Fig. 6. Ranges of shape parameters covered in present study

parameters studied have not covered some odd shapes that may be uncommon but nonetheless theoretically possible. Care should be taken when applying the proposed formulas to aggregates whose shape parameters fall outside the ranges covered in this study. The authors have not been able to obtain samples of the aforementioned aggregate types but would certainly include them in the research if samples are available.

Secondly, so far the correlation between the packing density and the various shape parameters has been obtained purely by experiment. Theoretical studies are needed to help explain the effects of the various shape parameters. For instance, although the effects of the flakiness ratio and the elongation ratio have been evaluated in the present study, the authors find it difficult to explain why the flakiness ratio should have much greater effects than the elongation ratio. Relatively, the effect of convexity is easier to understand and an explanation is given in the following. The convexity is actually a measure of overall roundness. A rounded particle should not contain too many concave corners and a particle containing many concave corners is not rounded. Since the concave area is equal to convex area minus area, a high area to convex area ratio implies that the amount of concave area is relatively small and the particle is well-rounded, and vice versa. The convexity of the aggregate particles has a direct effect on the packing of the aggregate because the concave areas are generally more difficult to be filled up, especially if the aggregate particles are of similar size, as shown in Fig. 7. Hence, the convexity and fullness ratios should have significant effects on the packing density. This is verified by the high correlation coefficients of 0·828 and 0·825 between the packing density and the two convexity indexes.

Lastly, the authors would like to emphasize the importance of packing in concrete technology and to advocate more research on this topic. Only the packing of aggregate particles has been studied herein. The packing of aggregate particles is itself of importance because it determines the amount of cement paste needed and, for a given amount of cement paste, the workability and pumpability of the concrete mix. However, the packing of the binder particles (cement + pulverized fuel ash + condensed silica fume + other binding materials added) may be of even higher importance.

The first author [15] has shown recently that the workability of a concrete mix can be increased by improving the packing of the binder particles. This can be done by adding ultra-fine binder particles such as condensed silica fume to fill the voids between the larger binder particles. In fact, the amount of water added to a binder paste must be sufficient to fill the voids between the binder particles, or otherwise there will be air trapped inside the binder paste. For this reason, if the binder consists of cement only, in which case the voids ratio is generally quite high, the water–cement ratio must be greater than a certain value to ensure that all voids in the binder paste are filled with water. By the addition of finer binder particles such as pulverized fuel ash and condensed silica fume, the amount of voids can be reduced and a lower water–binder ratio may be used to increase the concrete strength, or for the same amount

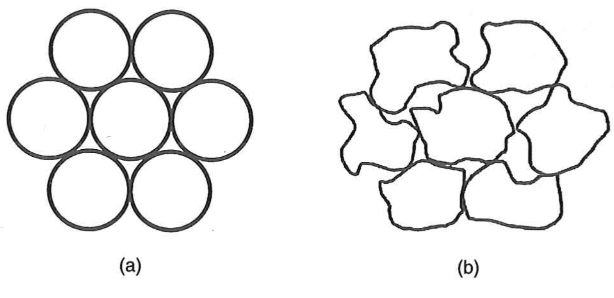

Fig. 7. Effect of convexity on packing of: (a) spherical particles; and (b) particles with concave corners

of water added there will be more water in excess of the amount needed to fill the voids to lubricate the binder paste.

Although the present study is on the packing of aggregate particles only, it is anticipated that some of the conclusions may also be applicable to the packing of binder particles because the only major difference between the aggregate and the binder is just the size. Further research is needed to confirm this anticipation.

Conclusions

The DIP technique has been successfully applied to measure the flakiness ratio, elongation ratio, sphericity, shape factor, convexity ratio and fullness ratio of coarse aggregate particles. Although some of these shape parameters are dependent on the thickness of the particles, which is not directly obtainable by DIP, the simple method of supplementing the DIP results by the weight of the aggregate sample developed previously by the authors can be used to estimate the mean thickness of the particles to allow measurement of shape parameters dependent on thickness.

The aforementioned shape parameters are correlated to the corresponding packing densities of the aggregate samples to study the effects of the various shape parameters on packing. By treating the packing density as single-variable functions of individual shape parameters, single-variable functions of products of shape parameters, and multi-variable functions of shape parameters in turn, and using statistical analysis software to evaluate the corresponding correlation coefficients, it is found that the shape factor and the convexity ratio are the most important shape parameters affecting the packing of aggregate. Two alternative formulas, one being a linear function and the other a power function, are developed for estimating the packing density from the shape factor and the convexity ratio. Charts based on these two formulas that may help to understand better the combined effects of the two shape parameters on the packing of an aggregate are presented.

Further research to refine and improve the correlation between the packing density and the various shape parameters established in the present study is recommended and the importance of packing in concrete technology has been discussed. It is advocated that more research, not only on packing of aggregate particles, but also on packing of binder particles, should be carried out.

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