DYNAMIC GAME OF OFFENDING AND LAW ENFORCEMENT: A STOCHASTIC EXTENSION

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Abstract

This note provides an extension of the Fend-Feichtinger-Tragler dynamic game of offending and law enforcement to a stochastic framework. This allows the analysis to reflect actual crime statistics which displays randomness in its distribution. Stochastic paths of crimes are derived. The asymptotic stationary distribution of crime records is also obtained.

Keywords: Economics of Crime; Law Enforcement; Stochastic Differential Game.

1. Introduction

Fent *et al* (2002) analyze a differential game describing the interactions between a potential offender and the law enforcement agency. However, actual crime statistics displays randomness in its distribution as shown in the Appendix. To generate this effect we extend the Fend-Feichtinger-Tragler game in Fent *et al* (2002) to a stochastic differential game. Stochastic dynamics are derived. This note is organized as follows. In Section 2, an intertemporal game played between an offender and a law enforcer with stochastic dynamics is set up. Analysis on the outcome of the game is performed and the solution crime evolution path is derived. Section 4 considers the case when the time horizon approaches infinity. The stationary distribution of crime records is obtained. Concluding remarks are given in Section 5.

2. A Stochastic Fend-Feichtinger-Tragler Game of Law Enforcement

Following Fent *et al* (2002), we consider the behaviour of two rational players. The first player is a (group of) potential offender who tries to maximize profit gained from illegal activities, while the second player is a law enforcement agency that tries to maximize public welfare. The offender's decision variable is $u_1(t)$, the rate of offence, and the agency chooses its rate of crime investigation $u_2(t)$.

As argued in Fent *et al* (2002), it may be less plausible why the authority should concentrate its activities -- and therefore its spending on investigation, prosecution, and execution of sentences -- on one particular offender. The state $x(t)$ represents the offender's experience in committing crimes or her record of prior criminal offences. The state variable has two possible interpretations. First, it can be considered as the record of prior crimes. Following Greenwood *et al* (1994), it is assumed that the increase of this record only depends on the criminal activity and the number of convictions, but not on the punishment. A decay term $-\delta x$ describes those cases in which former crimes are only considered for a limited period. The second interpretation is to regard x as the offender's level of experience. The experience increases in proportion to the intensity u_1. However, the offender also forgets, and the experience decays with time due to changes of law and technology. Thus, the value of experience will be reduced by a rate δ.

To introduce randomness we use the stochastic differential equation

$$dx(t) = (u_1 - \delta x)dt + \varepsilon x dz(t) \tag{2.1}$$

to describes the dynamics of the state x. In particular, where ε is a constant and $z(s)$ is a Wiener process and the initial state x_0. Equation (2.1) shows that the decay rate δ is subject to stochastic shocks.

The utility the offender obtains from criminal activities consists of revenues minus costs. The revenues of the offender is

$$R(u_1) = \gamma u_1 . \tag{2.2}$$

The offender's costs consist of two terms, S and C. The term S represents the costs connected with the sentence. These costs depend on the decision variables u_1 and u_2 and the state x:

$$S(x,u_1,u_2) = \pi(u_1,u_2) + \sigma(x) = u_1 u_2 + \varphi x . \tag{2.3}$$

In particular, the punishment an offender will suffer is a function of his own offending intensity, the rate of crime investigation, and the criminal record. The offence level u_1 influences the probability of being convicted and the level of punishment, the investigation activities u_2 affect the probability of being convicted and prosecuted, and the criminal record x influences the level of punishment. With the formulation in (2.3), the function $S(x, u_1, u_2)$ takes positive values in case of a criminal record $x > 0$ even if $u_1 = 0$. This represents, for example, the disadvantages one might experience in the labour market after having been convicted.

The second cost term C represents costs that are not related to incarceration or conviction. These costs are increasing in the criminal intensity the following cost-function is adopted.

$$C(u_1) = \psi u_1^2 . \tag{2.4}$$

At terminal time T, there is a salvage value function $Q_1(x(T))$ which assesses the value of the state x at the end of the planning period. It represents the damage or harm (or utility) caused to the offender by having the criminal record $x(T)$. It is assumed that

$$Q_1(x(T)) = -\eta x(T) .$$

If the criminal record $x(T)$ has a negative impact, η must be positive. In the case where the offender does not consider a criminal record to be something bad at all, the parameter η can be equal to or less than zero.

The offender's expected objective functional can be expressed as:

$$J_1 = E\left\{ \int_0^T e^{-rt}[\gamma u_1 - u_1 u_2 - \varphi x - \psi u_1^2]dt - e^{-r_1 T}\eta x(T) \right\}, \tag{2.5}$$

where r is the discount rate.

The expected objective functional of the law enforcement agency is

$$J_2 = E\left\{ -\int_0^T e^{-rt}[D(u_1) + K(x, u_2) + \nu x + L(u_{1,} u_2)]dt \right\} \tag{2.6}$$

where all terms are costs for the state. Since the criminal record and the experience of a previously convicted offender at the terminal time do not influence public welfare there is no salvage value of the state x.

The damage $D(u_1)$ caused by illegal activities increases with the offence rate and it is assumed:

$$D(u_1) = \zeta u_1^2 .$$

The costs of law enforcement, $K(x,u_2)$, increase and have non-decreasing marginal costs. More experienced offender might be more difficult to arrest but, on the other hand, the higher the level of criminal experience already is, the smaller the advantage of one additional unit of experience. The law enforcement cost is assumed to be:

$$K(x,u_2) = \vartheta u_2^2 + \nu x .$$

The term $L(u_1,u_2)$ in the law enforcement agency's objective functional reflects the costs of imposing a certain punishment. For instance, the costs implied by maintaining prisons might be included here. In particular:

$$L(u_1,u_2) = \omega u_1 u_2 .$$

The expected objective functional of the law enforcement agency in (2.6) can then be expressed as:

$$J_2 = E\left\{ -\int_0^T e^{-rt}[\zeta u_1^2 + \vartheta u_2^2 + \nu x + \omega u_1 u_2]dt \right\}. \tag{2.7}$$

3. Analysis

The game (2.1), (2.5) and (2.7) is a stochastic version of the Fend-Feichtinger-Tragler game. Given the presence of stochasticity, a feedback solution has to be sought. A Nash equilibrium solution for this stochastic differential game can be characterized as:

Theorem 3.1. A pair of feedback strategies $\{\phi_1^*(t,x);\ \phi_1^*(t,x)\}$ provides a Nash equilibrium solution to the game (2.1), (2.5) and (2.7) if there exist suitably smooth functions $V^i(t,x):[0,T]\times R \to R$, $i\in\{1,2\}$, satisfying the partial differential equations

$$-V_t^1(t,x)-\frac{1}{2}\sigma^2x^2V_{xx}^1(t,x)=\max_{u_1}\left\{ e^{-rt}[\gamma u_1-u_1\phi_2(t,x)-\varphi x-\psi u_1^2]\right.$$

$$\left. +V_x^1(t,x)(u_1-\delta x) \right\},$$

$$V^1(T,x)=-e^{-rT}\eta x;$$

$$-V_t^2(t,x)-\frac{1}{2}\sigma^2x^2V_{xx}^2(t,x)=\max_{u_2}\left\{ -e^{-rt}[\zeta(\phi_1(t,x))^2+\vartheta u_2^2+\nu x+\omega\phi_1(t,x)u_2]\right.$$

$$\left. +V_x^2(t,x)[\phi_1(t,x)-\delta x] \right\}$$

$$V^2(T,x)=0. \tag{3.1}$$

Proof. Follow the proof of Theorem 2.5.1 in Yeung and Petrosyan (2006). □

Performing the indicated maximization in (3.1) yields the conditions:

$\gamma-\phi_2(t,x)-2\psi\phi_1(t,x)+e^{rt}V_x^1\le 0$,

$u_1[\gamma-\phi_2(t,x)-2\psi\phi_1(t,x)+e^{rt}V_x^1]=0$;

$-2\vartheta\phi_2(t,x)-\omega\phi_1(t,x)\le 0$, $u_2[-2\vartheta\phi_2(t,x)-\omega\phi_1(t,x)]=0$;

$\phi_1(t,x)\ge 0$, $\phi_2(t,x)\ge 0$.

The conditions above give:

$$\phi_1(t,x)=\frac{\gamma+e^{rt}V_x^1}{2\psi} \text{ and } \phi_2(t,x)=0 . \tag{3.2}$$

Note that the authority will decide not to investigate at all.

Substituting $\phi_1(t,x)$ and $\phi_2(t,x)$ into (3.1) yields

$$-V_t^1-\frac{1}{2}\sigma^2x^2V_{xx}^1=e^{-rt}\left[\frac{\gamma(\gamma+e^{rt}V_x^1)}{2\psi}-\varphi x-\frac{(\gamma+e^{rt}V_x^1)^2}{4\psi}\right]+V_x^1\left[\frac{\gamma+e^{rt}V_x^1}{2\psi}-\delta x\right],$$

$$-V_t^2 - \frac{1}{2}\sigma^2 x^2 V_{xx}^2 = -e^{-rt}\left[\zeta\left(\frac{\gamma + e^{rt}V_x^1}{2\psi}\right)^2 + \nu x\right] + V_x^2\left[\frac{\gamma + e^{rt}V_x^1}{2\psi} - \delta x\right],$$

$$V^1(T,x) = -e^{-rT}\eta x,$$

$$V^2(T,x) = 0. \tag{3.3}$$

Proposition 3.1.

The system (3.3) admits a solution

$$V^1(t,x) = e^{-rt}[A_1(t)x + C_1(t)] \text{ and } V^2(t,x) = e^{-rt}[A_2(t)x + C_2(t)], \tag{3.4}$$

where

$A_1(t)$, $C_1(t)$, $A_2(t)$ and $C_2(t)$ satisfy

$$\dot{A}_1(t) = (r+\delta)A_1(t) + \varphi,$$

$$\dot{C}_1(t) = rC_1(t) - \left[\frac{\gamma(\gamma + A_1(t))}{2\psi} - \frac{[\gamma + A_1(t)]^2}{4\psi}\right] - A_1(t)\frac{\gamma + A_1(t)}{2\psi},$$

$$\dot{A}_2(t) = (r+\delta)A_2(t) + \nu,$$

$$\dot{C}_2(t) = rC_2(t) + \zeta\left(\frac{\gamma + A_1(t)}{2\psi}\right)^2 - A_2(t)\frac{\gamma + A_1(t)}{2\psi},$$

$$A_1(T) = -\eta,\ C_1(T) = 0,\ A_2(T) = 0,\ C_2(T) = 0. \tag{3.5}$$

Proof. Substituting the relevant derivatives of $V^1(t,x)$ and $V^2(t,x)$ into (3.3) yields

$$re^{-rt}[A_1(t)x + C_1(t)] - e^{-rt}[\dot{A}_1(t)x + \dot{C}_1(t)]$$
$$= e^{-rt}\left[\frac{\gamma(\gamma + A_1(t))}{2\psi} - \varphi x - \frac{(\gamma + A_1(t))^2}{4\psi}\right] + e^{-rt}A_1(t)\left[\frac{\gamma + A_1(t)}{2\psi} - \delta x\right],$$

$$re^{-rt}[A_2(t)x + C_2(t)] - e^{-rt}[\dot{A}_2(t)x + \dot{C}_2(t)]$$
$$= -e^{-rt}\left[\zeta\left(\frac{\gamma + A_1(t)}{2\psi}\right)^2 + \nu x\right] + e^{-rt}A_2(t)\left[\frac{\gamma + A_1(t)}{2\psi} - \delta x\right].$$

$$re^{-rT}[A_1(T)x + C_1(T)] = -e^{-rT}\eta x,$$

$$re^{-rT}[A_2(T)x + C_2(T)] = 0. \qquad (3.6)$$

For (3.6) to be satisfied, it is required that (3.5) to hold. Hence Proposition 3.1 follows. □

System (3.5) forms a block recursive system of differential equations. $A_1(t)$ and $A_2(t)$ can be solved independent of $C_1(t)$ and $C_2(t)$. In particular:

$$A_1(t) = \left(\frac{\varphi}{r+\delta} - \eta\right)e^{(r+\delta)(t-T)} - \frac{\varphi}{r+\delta},$$

$$A_2(t) = \left(\frac{\nu}{r+\delta}\right)e^{(r+\delta)(t-T)} - 1. \qquad (3.7)$$

Note that $A_2(t)$ is negative for all $t \in [0,T]$ and $A_1(t)$ is negative for all $t \in [0,T]$ if $\eta \le 0$. This means that an additional unit of criminal record (or experience) x is negatively evaluated by both players at any instant of time. Having a negative shadow price, x is a "bad stock". This is clear since x enters negatively in both objectives.

The game equilibrium strategies of the offender and law enforcer are respectively

$$\phi_1(t,x) = \frac{1}{2\psi}\left[\gamma + \left(\frac{\varphi}{r+\delta} - \eta\right)e^{(r+\delta)(t-T)} - \frac{\varphi}{r+\delta}\right] \text{ and } \phi_2(t,x) = 0. \qquad (3.8)$$

Substituting the game equilibrium strategies into the state dynamics yields:

$$dx(t) = \left\{\frac{1}{2\psi}\left[\gamma + \left(\frac{\varphi}{r+\delta} - \eta\right)e^{(r+\delta)(t-T)} - \frac{\varphi}{r+\delta}\right] - \delta x\right\}dt + \sigma x dz(t). \qquad (3.9)$$

Equation (3.9) is a linear stochastic differential equation which solution can be expressed as:

$$x(t) = x_0 + \int_0^t \left\{\frac{1}{2\psi}\left[\gamma + \left(\frac{\varphi}{r+\delta} - \eta\right)e^{(r+\delta)(t-T)} - \frac{\varphi}{r+\delta}\right] - \delta x(s)\right\}ds$$

$$+ \int_0^t \sigma x(s)dz(s). \qquad (3.10)$$

The stochastic path (3.10) manages to exhibit random elements in its evolution.

4. Infinite Horizon and Stationary State

Now we consider the case when the terminal T approaches infinity. An infinite horizon version of the game (2.1), (2.5) and (2.7) can be specified as:

$$\max_{u_1} E\left\{ \int_0^\infty e^{-rt}[\gamma u_1 - u_1 u_2 - \varphi x - \psi u_1^2]dt \right\} \text{ and}$$

$$\max_{u_2} E\left\{ -\int_0^\infty e^{-rt}[\zeta u_1^2 + \vartheta u_2^2 + \nu x + \omega u_1 u_2]dt \right\} \tag{4.1}$$

subject to

$$dx(t) = (u_1 - \delta x)dt + \sigma x dz(t) . \tag{4.2}$$

A Nash equilibrium solution for this infinite horizon stochastic differential game can be characterized as:

Theorem 4.1. A pair of feedback strategies $\{\phi_1^*(x)\,;\ \phi_1^*(x)\}$ provides a Nash equilibrium solution to the game (4.1)-(4.2) if there exist suitably smooth functions $V^i(x): R \to R$, $i \in \{1,2\}$, satisfying the partial differential equations

$$rV^1(x) - \frac{1}{2}\sigma^2 x^2 V_{xx}^1(x) = \max_{u_1}\left\{ [\gamma u_1 - u_1\phi_2(x) - \varphi x - \psi u_1^2] \right.$$

$$\left. + V_x^1(x)(u_1 - \delta x) \right\},$$

$$rV_t^2(x) - \frac{1}{2}\sigma^2 x^2 V_{xx}^2(x) = \max_{u_2}\left\{ -[\zeta(\phi_1(x))^2 + \vartheta u_2^2 + \nu x + \omega\phi_1(x)u_2] \right.$$

$$\left. + V_x^2(x)[\phi_1(x) - \delta x] \right\}. \tag{4.3}$$

Proof. Follow the proof of Theorem 2.7.1 in Yeung and Petrosyan (2006). □

Performing the indicated maximization yields the conditions:

$$\gamma - \phi_2(x) - 2\psi\phi_1(x) + V_x^1 \le 0\,,\ u_1[\gamma - \phi_2(x) - 2\psi\phi_1(x) + V_x^1] = 0\,;$$

$$-2\vartheta\phi_2(x) - \omega\phi_1(x) \le 0\,,\ u_2[-2\vartheta\phi_2(x) - \omega\phi_1(x)] = 0\,;$$

$$\phi_1(x) \ge 0\,,\ \phi_2(x) \ge 0\,. \tag{4.4}$$

The conditions in (4.4) yields:

$$\phi_1(x) = \frac{\gamma + V_x^1}{2\psi} \text{ and } \phi_2(x) = 0 . \qquad (4.5)$$

Substituting $\phi_1(x)$ and $\phi_2(x)$ into (4.3) yields

$$rV^1 - \frac{1}{2}\sigma^2 x^2 V_{xx}^1 = \left[\frac{\gamma(\gamma + V_x^1)}{2\psi} - \varphi x - \frac{(\gamma + V_x^1)^2}{4\psi}\right] + V_x^1\left[\frac{\gamma + V_x^1}{2\psi} - \delta x\right],$$

$$rV_t^2 - \frac{1}{2}\sigma^2 x^2 V_{xx}^2 = -\left[\zeta\left(\frac{\gamma + V_x^1}{2\psi}\right)^2 + \nu x\right] + V_x^2\left[\frac{\gamma + V_x^1}{2\psi} - \delta x\right]. \qquad (4.6)$$

Proposition 4.1.

The system (4.6) admits a solution

$$V^1(x) = [A_1 x + C_1] \text{ and } V^2(x) = [A_2 x + C_2], \qquad (4.7)$$

where

$$A_1 = \frac{-\varphi}{r + \delta},$$

$$C_1 = \left[\frac{\gamma(\gamma + A_1)}{2\psi r} - \frac{(\gamma + A_1)^2}{4\psi r}\right] + A_1\frac{\gamma + A_1}{2\psi r},$$

$$A_2 = \frac{-\nu}{r + \delta},$$

$$C_2 = A_2\frac{\gamma + A_1}{2\psi r} - \frac{\zeta}{r}\left(\frac{\gamma + A_1}{2\psi}\right)^2 . \qquad (4.8)$$

Proof. Follow the proof of Proposition 3.1. □

A Nash equilibrium is given by

$$\phi_1(x) = \frac{1}{2\psi}\left[\gamma - \frac{\varphi}{r + \delta}\right] \text{ and } \phi_2(x) = 0 . \qquad (4.9)$$

Substituting the game equilibrium strategies into the state dynamics (4.2) yields:

$$dx(t) = \left[\frac{1}{2\psi}\left(\gamma - \frac{\varphi}{r + \delta}\right) - \delta x\right]dt + \sigma x(t)dz(t) . \qquad (4.10)$$

Equation (4.10) is a linear stochastic differential equation with constant coefficients which solution can be expressed as:

$$x(t) = x_0 + \int_0^t [\frac{1}{2\psi}\left(\gamma - \frac{\varphi}{r+\delta}\right) - \delta\, x(s)]ds + \int_0^t \sigma x(s)dz(s)\,. \qquad (4.11)$$

The stochastic path (4.11) manages to exhibit random elements in its evolution.

The stochastic system (4.10) will generate a stochastic process governed by a joint transition probability density function. This transition probability density function characterizes the possible realizations of $x(t)$.

Let $\mu[t, x \,|\, 0, x_0]$ denote the transition density function of the vector of x at time t, given the initial values x_0 at time 0. The function μ must satisfy the Fokker-Planck 'forward' equation (see Soong 1973 and Yeung (2009)):

$$\frac{\partial \mu}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{1}{2\psi}\left(\gamma - \frac{\varphi}{r+\delta}\right) - \delta\, x \right]\mu + \frac{1}{2}\frac{\partial^2}{\partial x^2}\sigma^2 x^2 \mu\,. \qquad (4.12)$$

A stationary state of the system (4.10) will be characterized by a process which has invariant probability density over time. It implies that the process $x(t)$ approaches a steady state as the transition time t approaches infinity. Specifically

$$\underset{t\to\infty}{Lim}\frac{\partial \mu}{\partial t} = 0\,.$$

Therefore, the stationary distribution of x will be represented by a time invariant density function $\upsilon(x)$, which in turn will be independent of time t and the initial values of x_0. In particular, the stationary Fokker-Planck equation can be expressed as:

$$0 = -\frac{\partial}{\partial x}\left[\frac{1}{2\psi}\left(\gamma - \frac{\varphi}{r+\delta}\right) - \delta\, x \right]\upsilon + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2}\upsilon\,.$$

This differential equation will immediately lead to the result (see Liu (1969) and Soong (1973)):

$$\upsilon(x) = m\exp\left[\int^x \{\frac{1}{4\sigma^2\tau^2\psi}\left(\gamma - \frac{\varphi}{r+\delta}\right) - \frac{\delta}{2\sigma^2\tau}\}d\tau \right],$$

where m is the normalization factor such that $\int_{-\infty}^{\infty} \upsilon(x)dx = 1$.

The stationary distribution function $\upsilon(x)$ characterizes the probability that the state equal x in a steady state.

5. Concluding Remarks

This note provides a stochastic version of the Fend-Feichtinger-Tragler dynamic game of offending and law enforcement. Stochastic paths of crimes are derived. This allows the analysis to reflect actual crime statistics which displays randomness in its distribution.

Appendix:

United States Crime Index Rates Per 100,000 Inhabitants

Year	Population	Total	Violent	Property	Murder	Forcible Rape	Robbery	Aggravated assault	Burglary	Larceny-Theft	Vehicle Theft
1960	179,323,175	1,887.2	160.9	1,726.3	5.1	9.6	60.1	86.1	508.6	1,034.7	183.0
1961	182,992,000	1,906.1	158.1	1,747.9	4.8	9.4	58.3	85.7	518.9	1,045.4	183.6
1962	185,771,000	2,019.8	162.3	1,857.5	4.6	9.4	59.7	88.6	535.2	1,124.8	197.4
1963	188,483,000	2,180.3	168.2	2,012.1	4.6	9.4	61.8	92.4	576.4	1,219.1	216.6
1964	191,141,000	2,388.1	190.6	2,197.5	4.9	11.2	68.2	106.2	634.7	1,315.5	247.4
1965	193,526,000	2,449.0	200.2	2,248.8	5.1	12.1	71.7	111.3	662.7	1,329.3	256.8
1966	195,576,000	2,670.8	220.0	2,450.9	5.6	13.2	80.8	120.3	721.0	1,442.9	286.9
1967	197,457,000	2,989.7	253.2	2,736.5	6.2	14.0	102.8	130.2	826.6	1,575.8	334.1
1968	199,399,000	3,370.2	298.4	3,071.8	6.9	15.9	131.8	143.8	932.3	1,746.6	393.0
1969	201,385,000	3,680.0	328.7	3,351.3	7.3	18.5	148.4	154.5	984.1	1,930.9	436.2
1970	203,235,298	3,984.5	363.5	3,621.0	7.9	18.7	172.1	164.8	1,084.9	2,079.3	456.8
1971	206,212,000	4,164.7	396.0	3,768.8	8.6	20.5	188.0	178.8	1,163.5	2,145.5	459.8
1972	208,230,000	3,961.4	401.0	3,560.4	9.0	22.5	180.7	188.8	1,140.8	1,993.6	426.1

1973	209,851,000	4,154.4	417.4	3,737.0	9.4	24.5	183.1	200.5	1,222.5	2,071.9	442.6
1974	211,392,000	4,850.4	461.1	4,389.3	9.8	26.2	209.3	215.8	1,437.7	2,489.5	462.2
1975	213,124,000	5,298.5	487.8	4,810.7	9.6	26.3	220.8	231.1	1,532.1	2,804.8	473.7
1976	214,659,000	5,287.3	467.8	4,819.5	8.7	26.6	199.3	233.2	1,448.2	2,921.3	450.0
1977	216,332,000	5,077.6	475.9	4,601.7	8.8	29.4	190.7	247.0	1,419.8	2,729.9	451.9
1978	218,059,000	5,140.4	497.8	4,642.5	9.0	31.0	195.8	262.1	1,434.6	2,747.4	460.5
1979	220,099,000	5,565.5	548.9	5,016.6	9.8	34.7	218.4	286.0	1,511.9	2,999.1	505.6
1980	225,349,264	5,950.0	596.6	5,353.3	10.2	36.8	251.1	298.5	1,684.1	3,167.0	502.2
1981	229,146,000	5,858.2	594.3	5,263.8	9.8	36.0	258.7	289.7	1,649.5	3,139.7	474.7
1982	231,534,000	5,603.7	571.1	5,032.5	9.1	34.0	238.9	289.1	1,488.8	3,084.9	458.9
1983	233,981,000	5,175.0	537.7	4,637.3	8.3	33.7	216.5	279.2	1,337.7	2,869.0	430.8
1984	236,158,000	5,031.3	539.2	4,492.1	7.9	35.7	205.4	290.2	1,263.7	2,791.3	437.1
1985	238,740,000	5,207.1	556.6	4,650.5	8.0	37.1	208.5	302.9	1,287.3	2,901.2	462.0
1986	240,132,887	5,480.4	620.1	4,881.8	8.6	38.1	226.0	347.4	1,349.8	3,022.1	509.8
1987	243,400,000	5,550.0	609.7	4,940.3	8.3	37.4	212.7	351.3	1,329.6	3,081.3	529.5
1988	245,807,000	5,664.2	637.2	5,027.1	8.4	37.6	220.9	370.2	1,309.2	3,134.9	582.9
1989	248,239,000	5,741.0	663.1	5,077.9	8.7	38.1	233.0	383.4	1,276.3	3,171.3	630.4
1990	248,709,873	5,820.3	731.8	5,088.5	9.4	41.2	257.0	424.1	1,235.9	3,194.8	657.8
1991	252,177,000	5,897.8	758.1	5,139.7	9.8	42.3	272.7	433.3	1,252.0	3,228.8	658.9
1992	255,082,000	5,660.2	757.5	4,902.7	9.3	42.8	263.6	441.8	1,168.2	3,103.0	631.5
1993	257,908,000	5,484.4	746.8	4,737.7	9.5	41.1	255.9	440.3	1,099.2	3,032.4	606.1
1994	260,341,000	5,373.5	713.6	4,660.0	9.0	39.3	237.7	427.6	1,042.0	3,026.7	591.3
1995	262,755,000	5,274.9	684.5	4,591.3	8.2	37.1	220.9	418.3	987.1	3,043.8	560.4
1996	265,284,000	5,087.6	636.6	4,451.0	7.4	36.3	201.9	390.9	945.0	2,980.3	525.7
1997	267,637,000	4,927.3	611.0	4,316.3	6.8	35.9	186.1	382.1	919.6	2,891.8	505.7
1998	270,296,000	4,615.5	566.4	4,049.1	6.3	34.4	165.2	360.5	862.0	2,728.1	459.0
1999	272,690,813	4,266.5	523.0	3,743.6	5.7	32.8	150.1	334.3	770.4	2,550.7	422.5

2000	281,421,906	4,124.8	506.5	3,618.3	5.5	32.0	145.0	324.0	728.8	2,477.3	412.2
2001	285,317,559	4,162.6	504.5	3,658.1	5.6	31.8	148.5	318.6	741.8	2,485.7	430.5
2002	287,973,924	4,125.0	494.4	3,630.6	5.6	33.1	146.1	309.5	747.0	2,450.7	432.9
2003	290,690,788	4,067.0	475.8	3,591.2	5.7	32.3	142.5	295.4	741.0	2,416.5	433.7
2004	293,656,842	3,977.3	463.2	3,514.1	5.5	32.4	136.7	288.6	730.3	2,362.3	421.5
2005	296,507,061	3,900.5	469.0	3,431.5	5.6	31.8	140.8	290.8	726.9	2,287.8	416.8
2006	299,398,484	3,808.1	473.6	3,334.5	5.7	30.9	149.4	287.5	729.4	2,206.8	398.4
2007	301,621,157	3,730.4	466.9	3,263.5	5.6	30.0	147.6	283.8	722.5	2,177.8	363.3
2008	304,059,724	3,667.0	454.5	3,212.5	5.4	29.3	145.3	274.6	730.8	2,167.0	314.7

Source: The Disaster Center

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