Financial Institutions and The Wealth of Nations: Tales of Development*

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Abstract

Interactions between economic development and financial development are studied by looking at the roles of financial institutions in selecting R&D projects (including for both imitation and innovation). Financial development is regarded as the evolution of the financing regimes. The effectiveness of R&D selection mechanisms depends on the institutions and the development stages of an economy. At higher development stages a financing regime with ex post selection capacity is more effective for innovation. However, this regime requires more decentralized decision-making, which in turn depend on contract enforcement. A financing regime with more centralized decision-making is less affected by contract enforcement but has no ex post selection capacity. Depending on the legal institutions, economies in equilibrium chose regimes that lead to different steady-state development levels. The financing regime of an economy also affects development dynamics through a 'convergence effect' and a 'growth inertia effect.' A backward economy with a financing regime with centralized decision-making may catch up rapidly when the convergence effect and the growth inertia effect are in the same direction. However, this regime leads to large development cycles at later development stages. Empirical implications are discussed.

Key Words: development, transition, financial institutions, R&D

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1 Introduction

It has been documented that almost all successful development in history has involved intertwined institutional and technological changes. Moreover, such development is always associated with an economy's catching up to the more developed economies in terms of wealth and technology. Most prominent examples include the continental European economies in the 19th century, Japan after the Meiji Restoration and after World War II, and Korea after World War II.[1] Gerschenkron and Cameron, in particular, have independently observed that the banking systems in continental Europe played an essential role in its catching up in the nineteenth century (Gerschenkron, 1962; Cameron, 1967). Schumpeter (1936) ascertained the relationship between financial institutions and development. He argued that banks play important roles in selecting projects that ultimately affect technological change and economic development.

There is a growing literature that has made great progress in exploring and testing the relationship between institutions and economic development (e.g., King and Levine, 1993; La Porta et al., 1998; Engerman and Sokoloff, 2000; Acemoglu et al., 2002). However, many gaps still remain and many important questions are still being debated. What are the institutional mechanisms that help or hinder technological change and economic development? How are these mechanisms chosen in the development process?

This paper is an attempt to address these questions with a focus on the financial institutions. We develop an endogenous growth model in which financing mechanisms, development levels, R&D activities, and economic growth are endogenized jointly. Financial development is regarded as an evolution of the financing regimes, together with the economy's development level. In our model, R&D is broadly defined to consist of all activities that improve knowledge about technology, including imitation, innovation, and invention.[2] Furthermore, exogenously given legal conditions and an endogenously determined development level affect the choice of financial institution jointly, which in turn determines the R&D selection mechanism and efficiency. As a result, economies develop along different paths. Our theory has implications for how to measure financial development, how to explain existing observations, and what new empirical evidence should be collected. Figure 1 summarizes the basic structure of our model.

[1] There is a substantial literature to support this claim. Due to space limitations we do not quote them here.

[2] Because our definition of R&D, the usual R&D statistical measurements cover only part of the R&D in our model.

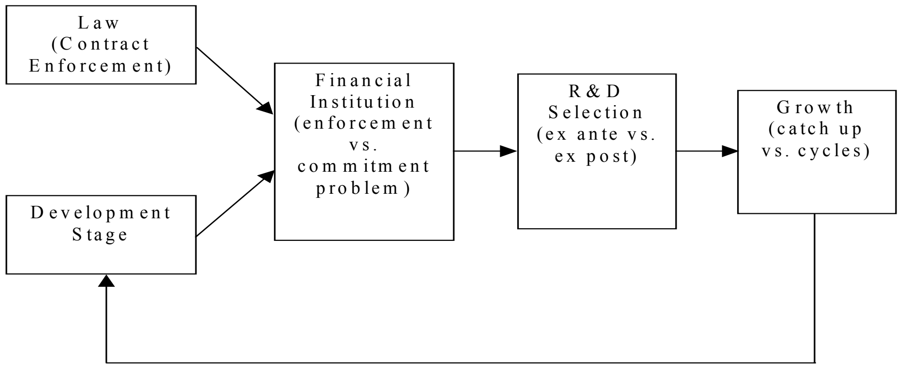

Figure 1: A model of endogenized institution, R&D, development level, and growth

In our model we analyze the impacts of project selection mechanisms associated with different financial institutions on development. We also explore how the development level determines the choice of financial institution. Project selection mechanisms are related to the incentives provided by the financial institutions to entrepreneurs for R&D. These incentives include 'carrots' to reward entrepreneurs and 'sticks' to prevent cheating. Our model focuses on the latter because, in our view, these are particularly important in dealing with the following important features of R&D: a) the uncertainty of R&D projects for innovation/invention can be very high such that essential knowledge for a project is only known ex post; whereas the uncertainty for imitation is low since reasonably accurate information can be collected ex ante; b) individuals with R&D ideas usually do not have the resources to finance projects so they need outside investment; c) entrepreneurs have informational advantages over projects that they work with, and with these advantages they may benefit by cheating on the worth of the projects.

Cheating can be deterred if it is punished whenever it is revealed. Moreover, an effective deterrence is a better R&D selection mechanism for innovation. However, such punishments can be enforced only when they are consistent with the financiers' ex post calculations. But the commitment to ex post punishments depends on the financing mechanisms. Two types of financial institutions are studied: regime s with more centralized decision-making,[3] reflecting the conglomerates' internal financing, 'relation-based fi-

[3] When financing decisions are concentrated in the hands of one financier we regard the corresponding financial institution as a single-financier financing regime, i.e., regime s.

nancing'[4] or a centralized finance system in reality; and regime m with more decentralized decision-making, reflecting venture capital or syndicated financing in reality. The institutions of regime s have no commitment for ex post punishment, i.e., they are associated with a soft budget constraint problem (SBC);[5] whereas regime m is committed to punishing cheating ex post, i.e., it is associated with hard budget constraints.[6]

An alternative approach to deal with the R&D-related incentive problem in our model is to select R&D projects ex ante. Associated with the above-mentioned R&D feature a), the effectiveness of pre-screening R&D depends on the information available ex ante. The more novel the project, the less information available to make ex ante judgments; whereas it is much easier to evaluate projects that have marginal novelty, such as those involving technology imitation. Thus a relatively backward economy will benefit from imitation which reduces the problems of cheating in R&D when projects are selected ex ante. We model the degree of backwardness of an economy as the distance from that economy to that of the world frontiers.

Regime m institutions are more efficient in innovation; whereas under certain conditions regime s institutions can be more efficient in technology imitation although they are less efficient in innovation. We predict a conditional convergence such that in equilibrium, economies with stronger legal institutions chose a regime m that leads to higher steady-state development levels, whereas those with weaker legal institutions chose a regime s. Since ex ante R&D selection is less effective in solving incentive problems when the development level is higher and imitation opportunities diminish, this leads to lower steady-state development levels for regime s.

Another major contribution of our work is to analyze the catch-up dynamics by decomposing the impacts of institutions on the development dynamics into a 'convergence effect' and a 'growth inertia effect.' The magnitude and the direction of the two effects govern the development dynamics of an economy. The key factor that determines the magnitude of each effect is what we discover from the model: the 'inertia factor' of the economy. The 'inertia factor,' a measure of the ability to reserve the momentum of

[4] The term is borrowed from Rajan and Zingales (2003).

[5] An observable indication of the existence of a substantial SBC problem in an economy is a large amount of non performing loans (NPL), such as those in transition economies and in Japan during the last decade. The NPL/GDP ratio in Japan was 15.3% in 2001, far higher than any other developed economy.

[6] For the contractual foundations of the commitment problems associated with centralized and decentralized financing regimes, see Dewatripont and Maskin (1995); for the contractual foundations of the commitment problems associated with different financing regimes in market economies, see Huang and Xu (1998, 2003).

growth performance, is determined by institutions. Moreover, it is empirically observable as the auto-correlation coefficient of the growth rate. At a catching-up stage, the convergence effect and the growth inertia of an economy are in the same direction. Thus, a backward economy with a higher 'inertia factor' will catch up faster. However, when an economy's development level is close to its steady state, a higher 'growth inertia' may make the economy prone to growth cycles.

Among other factors, the 'inertia factor' of an economy is affected by how much ex ante R&D selection is used in the economy, which in turn is determined by the financing regime. In general, the 'inertia factor' under regime m is smaller than that under regime s.

Together with the results of how financing regimes determine their steady state, our theory predicts that the institutions of regime s lead to a fast catch up when an economy is at an earlier development stage; however, it is likely to fall into growth cycles around the relatively low steady-state development levels. In contrast, although the institutions of regime m may have higher steady-state development levels associated with more stable catch-up patterns, their catch-up speed may vary depending on the legal institutions.

These predictions shed light on why economies associated with some financial institutions, such as centralized financing or 'relationship financing,' catch up quickly at earlier stages but encounter serious problems later even when investments in R&D are high. Our theory is consistent with some observed development patterns, including the rise and fall of centralized economies.

The structure of the paper is as follows. Section 2 presents some motivating observations and discusses the related literature. Section 3 sets up an endogenous growth model, focusing on the role of financial intermediation on R&D project selection. Section 4 describes equilibrium financing regimes, the R&D level, the steady-state growth rate, and the steady-state development level. Section 5 explores the catch-up dynamics by which an economy converges with or diverges from its steady-state path. Section 6 briefly provides suggestive empirical evidence; moreover, policy implications of the theory are demonstrated through simulations. Finally, section 7 offers some conclusions.

2 Catching-up Patterns and the Related Literature

In this section we present some observations motivating our theory. We first briefly compare the development paths in the last half-century between some West European economies, where the financial institutions were relatively closer to regime m in our model, and some Central and Eastern European (CEE) economies that were under centralized financial systems prior to the 1990s, thus representing an extreme case of regime s in our model. It is well recognized that a centralized financial system, where all national financial resources are concentrated in state banks, creates the so-called 'soft-budget constraint' syndrome that is one of the most serious problems in centralized economies (Kornai, 1979; Dewatripont and Maskin, 1995; for recent surveys, see Maskin and Xu, 2001; Kornai, Maskin, and Roland, 2003). However, the rise of the centralized financing regimes is puzzling, i.e., they appeared to catch up quickly at earlier development stages, given that the SBC is inefficient. Moreover, the fall of the centralized financing regimes is also puzzling, i.e., they experienced a reversed catch-up pattern at later development stages, given that the negative experience was associated with their heavy investments in R&D (both in monetary and in human capital terms). Our model provides an explanation for the rise and fall of the SBC economies together with their R&D activities.

Presenting the growth rate differences with those of the world frontiers by the vertical axis, Figure 7 shows the development paths of four Western European economies (Austria, Belgium, France, and Italy) for the period from 1950 to 2000 based on a five-year average (data source: Madison, 2003).[7] A regular catching-up pattern consistent with the predictions of our model (Fig. 5) seems to emerge such that all these economies had catch ups before the 1980s; thereafter, their catch ups ended with small growth cycles when their development levels were close to that of the frontiers (from 70% to 81% of the U.S. level).

In comparison, Figure 8 illustrates the catch-up patterns of some CEE economies (Hungary, Poland, Romania, and the USSR), which appear to be

[7] The development path of each economy is plotted in a development level growth rate space so that the development level and growth rate of each economy can be compared with those of the world frontiers, which are proxied by those of the U.S., given that the data come from the latter half of the twentieth century. The development level relative to that of the world frontiers (hereafter referred to as the development level) is measured by the ratio of the per capita GDP of this economy to that of the U.S. It is presented by the horizontal axis.

consistent with the predictions of our model (Fig. 6). Although associated with the SBC problem, all the CEE economies underwent fairly rapid catch ups in their earlier stages (before 1975), together with the rapid adoption of new technologies.[8] However, the catch ups all ended with large growth cycles when their development levels reached one-third that of the U.S. level.[9] The reversed catch-up trend since the 1980s seems to confirm our prediction that these economies will decline on a large scale after overshooting.

With respect to the mechanism for the fall of the centralized economies, we predict an increase in R&D expenditure when the development of regime s begins to decline. Our explanation focuses on the failure of the financing regime to deal with R&D. A corresponding fact is that when the catch-up reversal occurred, the R&D (both in the civilian and military sectors) in these economies was among the highest in the world (Tables 1-3). Specifically, beginning in 1975 R&D intensity in the USSR was the highest in the world both in monetary terms and in terms of human capital inputs: compared with the U.S., the R&D intensity and the number of scientists employed in R&D in the USSR in 1975 was about 47% and 63% higher respectively (Table 1 and 3).[10]

Moreover, our theory seems consistent with much of the existing evidence in the recent literature. Rajan and Zingales (1998, RZ hereafter) make major progress in confirming the causality between financial and economic development after the pioneering work by King and Levine (1993) which established positive correlations between the two developments. Interpreting regime m/s in our model as external/internal financing[11] and relating

[8] For example, except for in synthetic fibers, the USSR adopted major new technologies that were introduced in the 1940s and 1950s (oxygen steel making, continuous steel casting, synthetic fibers, polyolefins, HVAC [300 kv and over], nuclear power, NC machine tools) around the same time as the UK, FRG, and Japan (Bergson, 1989, Table 10, p.124).

[9] The data reflecting the collapse of the centralized economies over the last decade are the last two points on the curves. The basic development pattern of these economies will not change qualitatively if these data are excluded. The only reason for including the data for the period from 1990 to 2000 is to present the data in the same way as those for the West European economies.

[10] For all countries under comparison, the R&D includes both the civilian and military sectors. Great care has been taken to rely on the most prominent experts in the field for the source of our data: we use Madison for the GDP/GNP data; and Bergson and Hanson for the R&D data.

[11] Indeed, regime m in our model reflects such external financing as venture capital, syndicated loans, or other 'market-based institutions' that are able to solve the commitment problem; and regime s reflects internal financing within large conglomerates, financial institutions with strong government intervention, or 'relation-based institutions' that are unable to solve commitment problems.

our model to RZ's work in broad terms, our model predicts that external financing is essential for the growth of industries with technologies close to the frontiers of knowledge.[12] In contrast, internal financing may be more efficient for industries with technologies far from the frontiers of knowledge. The data in RZ show that industries involving more 'new technologies,' such as pharmaceuticals, electronics, and computers, are more dependent on external financing for growth. In contrast, industries involving 'traditional technologies,' such as iron and steel, auto vehicles, and machinery, are much less dependent on external financing for growth (the industries least dependent on external financing in RZ are tobacco, pottery, and leather) (RZ, 1998, Table 1). Indeed, in the U.S. an overwhelming proportion of the most innovative R&D projects in 'new technologies' were financed by syndicated venture capital (Gompers and Lerner, 2001), whereas most innovations involving 'traditional technologies' in all developed economies were created by in-house R&D.[13]

A high (or low) bank concentration in an economy may make the financing in that economy closer to regime s (or m). If we interpret the financing regimes in this way, our theory implies that a higher bank concentration may be beneficial for catching-up economies, whereas a lower bank concentration may be more efficient for industries at technological frontiers in developed economies. Carlin and Mayer (2003) found that bank concentration is negatively correlated with growth in developed OECD economies: a lower bank concentration is associated with higher R&D shares and faster growth of externally financed industries. However, for countries at earlier stages of development, the converse result is found, such that a high bank concentration is associated with faster growth (Carlin and Mayer, 2003, Tables 6 and 8). Moreover, our results about how contract enforcement affects development are consistent with empirical evidence that shows that economies with stronger legal institutions have better financial development and better economic development (La Porta et al., 1998).

Using data from 72 countries for the 1978-2000 period, a discovery by Demetriades and Law (2004) also fits well with our model. They found

[12] A fundamental feature of the frontiers of knowledge in our model is the degree of uncertainty in innovation. This can be very different from merely being the most advanced in the field. For example, inventing a new drug requires the frontiers of knowledge in biology and involves high uncertainties, whereas innovations to improve the quality of steel may not involve very high uncertainties.

[13] The financing regimes in the U.S. before the mid-nineteenth century (the early catching-up stage) may further illustrate this point. At that time New England banks effectively facilitated development. Instead of being commercial banks, they were financial arms of kinship networks that lended mainly to insiders (Lamoreaux, 1986).

that financial development has significant effects on growth; the effects are stronger for economies with better institutions; moreover, for underdeveloped economies, the quality of the institutions has a dominant impact on growth. To link their discovery to our model, we need to mention the following features of their data: a) their institutional quality data are essentially related to the contract enforcement in our model; b) most underdeveloped countries sampled in their study have financial institutions closer to regime m in our model.

Our theory is developed based on two strands of literature. The first is the R&D-based growth literature (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). The second is the literature on the commitment problems of financial institutions (or the theory of soft budget constraints) (Kornai, 1979; Dewatripont and Maskin, 1995; Huang and Xu, 2003), which provides the contractual foundation for our growth model.

Qian and Xu (1998) develop the idea that financing regimes are used as different R&D selection mechanisms dealing with different types of R&D projects. However, the implications for growth or development are only mentioned are not modeled. In an endogenous growth model Huang and Xu (1999) study how financial institutions affect growth through their R&D selection mechanisms. Although this paper notes the implications of R&D selection mechanisms for catch-up's economies and for economies at other development stages, the discussions are very brief and there is no full-fledged model of these issues. Moreover, since the HX model takes the financial institution as exogenous and the analysis is restricted to steady states, it is unable to make most of the predictions generated by our model.

Recent work by Acemoglu, Aghion, and Zilibotti (2003) studies the selection of managers related to the growth of firms using an investment-based strategy and an innovation-based strategy with an emphasis on competition. Their model creates multi-equilibria and a development trap in that context. However, they do not focus on the relationship between financial development and economic development, and they do not derive large growth development cycles. Accordingly, their model does not make predictions related to the observations that we discuss here (e.g., Fig. 7 and 8; RZ, 1998; Carlin and Mayer, 2003, etc.).

With respect to our contribution to growth cycles, Matsuyama (1999, 2001) studies possible growth paths cycling between a non-innovative (competitive) phase and an innovative (monopolistic) phase due to the interaction between innovation and the accumulation of capital. In our model we have imitation vs. innovation, and the growth cycles are determined by the financial institutions.

3 The Model

Our model focuses on the impacts of institutional solutions to incentive problems in R&D on long run growth. In our theory, the Romer model (1990) is the benchmark model whereby if there is no capital requirement or information asymmetry in R&D, our model becomes a discrete-time version of the Romer model. In the following, we start from the benchmark model, and then add institutional features to the model.

3.1 Production

In our model, consumers are risk neutral and infinitely lived. The representative consumer's maximization problem is,

$$\begin{array}{l} \max \sum_{\tau=t}^{\infty} \frac{c_\tau}{(1+\rho)^{\tau-t}} \\ s.t.: b_{\tau+1} = w_\tau + b_\tau \left(1 + r_\tau\right) - c_\tau, \end{array} \tag{1}$$

where c_τ is consumption, and w_τ is labor income, b_τ is the holding of bond with interest rate r_τ, ρ is the discount rate; in equilibrium, $r_\tau = \rho$.

Production in this economy is standard which consists of a final good sector and an intermediate good sector. The final good sector is perfectly competitive and it has a Cobb-Douglas technology with intermediate inputs x_{it} and labor input L_{1t}, such that the output is

$$Y_t = L_{1t}^{1-\alpha} \sum_{i=1}^{A_t} x_{i_t}^{\alpha}, \ 0 < \alpha < 1. \tag{2}$$

The firm's program is

$$\max_{x_{it}, L_{1t}} \left(L_{1t}^{1-\alpha} \sum_{i=1}^{A_t} x_{i_t}^{\alpha} - (1+\rho)\, p_{it} x_{it} - L_{1t} w_t \right),$$

where p_{it} is the price of intermediate good x_{it}, and w_t is the cost of labor at period t. The final good producers pay the intermediate goods producers at the beginning of each period to get the inputs, and sell their own products and pay their workers at the end of each period. The optimal demands for intermediate goods and labor are:

$$x_{it} = \alpha \frac{Y_t}{(1+\rho)\, p_{it}} \frac{x_{it}^{\alpha}}{\sum_{i=1}^{A_t} x_{i_t}^{\alpha}} \tag{3}$$

and

$$L_{1t} = \frac{(1-\alpha)\,Y_t}{w_t} \tag{4}$$

The producer of intermediate good i is a monopolist with the following profit maximization program:

$$\begin{aligned} \max_{p_{it},x_{it}} \pi_{it} \quad &= \quad \max_{p_{it},x_{it}} \{p_{it}x_{it} - x_{it}\} \\ s.t. \quad &: \quad \alpha L_{1t}^{1-\alpha} x_{it}^{\alpha-1} = (1+\rho)\, p_{it} \end{aligned} \tag{5}$$

And its solution is

$$p_{it} = \frac{1}{\alpha}. \tag{6}$$

The solutions for all intermediate goods producers are symmetric, so the subscript i can be dropped. Then we have,

$$x_t = \frac{\alpha^2 Y_t}{A_t\,(1+\rho)} = \alpha^{\frac{2}{1-\alpha}}\,(1+\rho)^{\frac{-1}{1-\alpha}}\,L_{1t} \tag{7}$$

$$p_t = \frac{1}{\alpha} \tag{8}$$

$$\pi_t = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\,(1+\rho)^{\frac{-1}{1-\alpha}}\,L_{1t} \tag{9}$$

$$w_t = \frac{(1-\alpha)\,Y_t}{L_{1t}} \tag{10}$$

and

$$\frac{Y_t}{A_t} = \alpha^{\frac{2\alpha}{1-\alpha}}\,(1+\rho)^{\frac{-\alpha}{1-\alpha}}\,L_{1t} \tag{11}$$

and

$$L_{1t} = \frac{(1-\alpha)\,Y_t}{w_t} \tag{12}$$

Define $\bar{w} \triangleq \frac{w_t}{A_t}$, the ratio of wage-knowledge stock, we have

$$\bar{w} = \frac{w_t}{A_t} = \frac{(1-\alpha)\,Y_t}{L_{1t}A_t} = (1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}\,(1+\rho)^{\frac{-\alpha}{1-\alpha}}.$$

$$w_t = A_t\bar{w} \text{ for } L_{1t} > 0$$

and w_t is undefined when $L_{1t} = 0$.

Then, the profit for every intermediate firm in each period is,

$$\pi_t = \frac{\alpha \bar{w}}{1+\rho} L_{1t}. \tag{13}$$

Denoting the steady state L_{1t} as L_1, the steady state x_t and π_t are

$$x = \alpha^{\frac{2}{1-\alpha}} (1+\rho)^{\frac{-1}{1-\alpha}} L_1$$

and

$$\pi = \frac{\alpha \bar{w}}{1+\rho} L_1. \tag{14}$$

The number of new intermediate products introduced in period $t+1$ as a result of R&D activities at t is determined by the productivity efficiency of R&D sector δ, labor input in R&D sector L_{2t},[14] and the knowledge stock A_t, i.e.,

$$A_{t+1} - A_t = \delta L_2 A_t, \tag{15}$$

where, L_{2t}, the labor input in the R&D sector, is determined by the labor market clearing condition

$$L_{2t} = L - L_{1t}. \tag{16}$$

3.2 Financial Intermediation and R&D

In our model, we assume that R&D is uncertain and is subject to incentive problems, whereas all other productions are certain. A major function of financial intermediaries is to solve the entrepreneurs' incentive problems when they finance uncertain R&D projects.

The incentives to be provided involve 'carrots' to reward entrepreneurs and 'sticks' to prevent cheating. The latter issue is particularly serious and difficult for R&D due to the following features of R&D: a) individuals with R&D ideas usually have no wealth to finance projects such that they need others' wealth as investment; b) R&D projects can be highly uncertain such that ex ante financiers may not be able to know which project is worth doing. Although ex ante it seems to be obvious that cheating can be deterred by severe punishment once it is revealed ex post, such punishments will be enforced only when the ex post punishment is consistent with financiers' ex post benefits. That is, only when ex post punishment is time consistent for

[14] Here, a crucial departure from the standard Romer model arises. As will be elaborated upon the next subsection, each R&D project needs capital inputs to complement one unit of labor input.

the financiers can the incentive problem be solved properly and this depends on the financial institutions.

To deal with the R&D incentive problem in a growth model, in our economy there are always some consumers who can generate innovative ideas during every period following an independent identical stochastic distribution. The consumers with innovative ideas will become entrepreneurs if their ideas are financed. The i.i.d. assumption implies that no entrepreneur can automatically continue to be an entrepreneur during the next period.

We assume that there are two possible types of every project proposed by an entrepreneur: a good type and a bad type. The returns of the two types of projects are the same – the present value of profits, $\delta A_t \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau$ (notice that $\delta L_2 A_t$ is the number of new R&D outcomes introduced in period $t+1$ and each R&D project uses one unit of labor). But the costs of the two types of projects differ. For a project being carried out in period t, a good type takes two stages to complete, requiring (capital) investments I_{1t} and I_{2t}; and it is profitable. However, a bad type takes three stages to complete requiring (capital) investments I_{1t}, I_{2t} and I_{3t} and it is unprofitable. Moreover, we also assume that all early stage investments to a bad project I_{1t} and I_{2t} are sunk, such that a bad project's liquidation value at stage 2 is zero. The magnitude of each investment is given by $I_{it} = I_i A_t$, where I_i is constant, for $i = 1, 2$ and 3.

With respect to information, when a project is proposed by an entrepreneur, no one (including the entrepreneur) knows the type of each project, although it is a common knowledge that the probability that it is a good (bad) type is q ($\bar{q} \triangleq 1-q$). Facing unknown-type R&D projects and potential losses associated with financing bad type projects, financiers may do better to select projects ex post by eliminating bad ones once the projects' types become known, i.e., if the financiers commit not to make last stage investment I_{3t}. However, this ex post selection mechanism may not be implemented if investing I_{3t} can make a bad project ex post profitable, unless investing I_{3t} makes an ex post loss due to some additional transaction costs.

To study R&D selection mechanisms, we model two alternative financing regimes: a multi-financier financing regime (regime m, with a dispersed claim structure) and a single-financier financing regime (regime s, with a concentrated claim structure). We suppose that a transaction cost

$$F_t = FA_t$$

is incurred whenever a project is to be re-financed by multi-financiers at stage 2, where $F \in \Re_+$. F_t may be justified as a negotiation cost due to the conflict of interests and asymmetric information among the co-financiers under the regime m. When a project is financed by regime s, this transaction cost does not appear.[15] In the following we summarize our major assumptions in an intuitive way; formal expressions of these assumptions can be found in Appendix A.

A-1 Financing a bad project is ex ante unprofitable.

A-2 Given that earlier investments are sunk, financing a bad project at its last stage is ex post profitable.[16]

A-3 With the transaction cost F_t, financing a bad project at its last stage is ex post unprofitable.[17]

To describe the incentive problems in financing R&D, we illustrate the stages of R&D financing in period t as follows:

Stage 0 Financiers choose the financial institution – regime s or m. Potential entrepreneurs propose R&D projects to financiers under a chosen regime when no one knows the types of proposed projects. Based on ex ante selection, which is to be analyzed in the next subsection, financiers make take-it-or-leave-it contract offers to the proposers of the chosen project. If a contract is signed, the project proposer becomes an entrepreneur and the financier(s) invest I_{1t} units of money into the project during stage 1. Stage 1 takes no time and requires no labor input.[18]

Stage 1 An entrepreneur learns the type of the project proposed. However, the financier(s) still does (do) not know the type of the project and stage 2 of R&D is launched (unless a project is stopped by the entrepreneur), which requires I_{2t} units of capital and one unit of labor

[15] This cost F_t can be regarded as a reduced form of the endogenized renegotiation costs in Huang and Xu (1998, 2003). Moreover, there is a literature on different reasons why costs will be higher when there are more parties involved (e.g., Dewatripont and Maskin, 1995; Bolton and Scharfstein, 1996; Hart and Moore, 1995).

[16] This assumption is a variation of similar assumptions made by Dewatripont and Maskin (1995); Qian and Xu (1998).

[17] This assumption is a reduced form of the results in Huang and Xu (1998, 2003).

[18] Relaxing this simplification assumption will not change the model qualitatively. A justification for the assumption is the following. Suppose testing a proposed idea is quick, then the time spent on it can be ignored. Moreover, suppose the number of people working on it can be very small, then compared with the later stage, it is small enough to be ignored.

inputs. If the entrepreneur stops the project, he gets a low private benefit $b_1 > 0$.

Stage 2 All good projects are completed, thus the types of the projects become common knowledge. For a good project, all the financial returns go to the financier(s) and the entrepreneur gets a high private benefit of b_g. All bad projects are incomplete, thus they have no return and their liquidation values are zero. The financier(s) decide either to continue, or to liquidate them. If a project is liquidated the financier(s) get(s) zero return and the entrepreneur loses, i.e., the private benefit is $b_2 < b_1$. If it is to be reorganized, I_{3t} will be invested. To simplify the model, we assume no further labor input is required to continue.

Stage 3 Bad projects are completed. All the financial returns go to the financier(s) and the entrepreneur gets a moderate private benefit, $b_b \in (b_1, b_g)$.

Given assumption A-2, financiers under regime s will continue to invest in bad projects at stage 2. Anticipating this, entrepreneurs with bad projects will lie at stage 1 to benefit from continuing bad projects. However, following A-3, financiers under regime m will liquidate bad projects at stage 2. Facing the credible threat of liquidation of bad projects at stage 2, entrepreneurs endowed with bad projects will stop bad projects "voluntarily" at stage 1 to avoid heavier losses later. These results are summarized in the following.[19]

Lemma 1 *: Under regime s all bad projects will not be stopped, i.e., there is a pooling equilibrium. Under regime m all bad projects will be liquidated at date 1, i.e., there is a separating equilibrium.*

Proof. See Appendix A.

The above lemma shows that through its commitment to liquidate bad projects at date 2, the decentralized nature of regime m provides incentives for entrepreneurs to honestly disclose information about the quality of the projects. In contrast, financial systems where key decisions are made by a single agent (regime s) do not have a commitment to liquidate bad projects ex post. Without a commitment entrepreneurs under this regime will hide bad news about their projects. Examples of regime m, or a dispersed claim structure, are syndicated venture capital financing and decentralized financial markets such as equity markets; whereas examples of regime s include

[19] There are other possible contractual foundations that we can use to derive the Lemma1, such as internal influence activities within regime s (Milgrom, 1988).

large firms' internal financing (e.g. conglomerates) or in-house R&D; a centralized economy is an extreme example (see Dewatripont and Maskin, 1995; Qian and Xu, 1998).

Regime m involves multi-party contracts. Thus, law enforcement such as contract enforcement, accounting standards, etc. will affect its operation. To capture this, we suppose that when a project is to be financed at stage 0, regime m will involve a transaction cost σFA_t, $\sigma \in [0,1]$ when a project is started to align the interests of investors and entrepreneurs. In short, we call σ an institutional cost. This institutional cost σ reflects the degree of imperfection of the legal infrastructure with respect to the costs involved in multi-party contracts. In an economy with perfect law enforcement, $\sigma = 0$; but in an economy with imperfect legal institutions, $\sigma > 0$; moreover, the more imperfect the legal institutions the higher σ.[20]

With respect to the 'waste' caused by bad projects in the two regimes, in the benchmark case of regime m (i.e. $\sigma = 0$), the present value of expected 'waste' for each complete project due to liquidating a bad project is $(1-q)\frac{I_1}{q}$. In comparison, under regime s the present value of the expected 'waste' due to extra costs involved in the final stage of financing is $(1-q)\frac{I_3}{1+\rho}$. In this paper we make an assumption that regime s has higher 'waste' than regime m. Formally we have the following.

A-4: $q\frac{I_3}{1+\rho} > I_1$ (benchmark regime m waste less than regime s).

Furthermore, we assume that to produce intermediate goods from a completed R&D project takes no time. Although it is a simplification assumption, a plausible example is that of producing a software package in a large scale when the code is there. Finally, we assume that the law of large numbers applies whenever appropriate. Therefore, we use mathematical expectations to replace the average of samples throughout the rest of the paper.

3.3 Pre-screening and Development Level

At its best, ex post selection is at the expense of I_1; at its worst, it does not exist (in regime s). As an alternative R&D selection mechanism, in our model financiers also select projects ex ante, which we call "pre-screening". This also captures important features of technology imitation. We assume that the effectiveness of ex ante project selection depends on the quality of ex

[20] This transaction cost σ or σFA_t can be seen as a reduced form of endogenized law enforcement cost in Xu and Pistor (2002). The assumption that regime s does not incur cost σ is not only a simplification but also captures the idea that a regime s, such as conglomerates, is an institutional substitute when the market is less developed.

ante information on R&D projects, which is determined by how far an economy is from the technology frontier. Supposedly, imitation-featured R&D projects (e.g., reverse engineering, etc.) are less uncertain and it is easier to make ex ante judgments about the projects; then a backward economy may rely more on ex ante selection, which allows for trying new technologies at a lower cost.[21]

Formally, we suppose that by investing κ_t, a prior signal can be collected about the quality of a project before starting it. The precision of the signal, i.e., the probability that a signal is correct, is θ, where $\theta \in \left[\frac{1}{2}, 1\right]$. We also suppose that the pre-screening cost increases in the development level and in the pre-screening precision. That is, we assume $\kappa_t = \kappa(a_t, \theta) A_t$, where $a_t \triangleq \frac{A_t}{A_{ft}}$ is the relative development level of an economy and A_{ft} is the world frontier of knowledge stock. A_{ft} grows at a constant rate g_f, which is to be determined endogenously. We make the following assumption about the properties of the pre-screening cost function κ.

A-5: $\kappa = \lambda(a_t)\psi(\theta)$ where, $\psi\left(\frac{1}{2}\right) = 0$, $\psi(1) = \infty$, $\psi'(\cdot) > 0, \psi''(\cdot) > 0$;[22] $\lambda(0) = 0$, $\lambda'(\cdot) > 0$, $\lambda(\cdot) \leq \frac{2\bar{q}I_1}{\psi'\left(\frac{1}{2}\right)}$.[23]

4 Equilibrium

4.1 Endogenous Financing Regime

We model a continuum of economies in the world with $\sigma \in [0, \infty)$. For any period t at stage 0, after receiving R&D proposals from entrepreneurs, financiers choose the optimal financing regime and pre-screening precision $\{\zeta, \theta_\zeta\}$ to maximize the expected net present value (NPV) of the projects they finance, where ζ is the regime variable, $\zeta \in \{m, s\}$, and θ_ζ is the pre-screening precision under a chosen regime ζ. In the following, we analyze the two financing regimes, and then we look at how at the equilibrium financing structure and the equilibrium pre-screening precision.

[21] This approach captures the Gerschenkron effect of the 'advantage of backwardness' (1962).

[22] One example which satisfies these conditions is $\psi(\theta) = \frac{\theta - \frac{1}{2}}{1-\theta}$.

[23] When this upper bound is respected, the equilibrium level θ is an interior solution, i.e., some pre-creening is desirable regardless of the country characteristics and the stage of development.

For a project financed under regime s, the expected NPV is

$$
\begin{aligned}
E_{NPVs} &= q\theta_s \left(\frac{\Delta A_{t+1}}{L_2} \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - I_{1t} - I_{2t} - \frac{w_t}{1+\rho} \right) + \\
& \quad \bar{q}\bar{\theta}_s \left(\frac{\Delta A_{t+1}}{L_2} \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - I_{1t} - I_{2t} - \frac{I_{3t}}{1+\rho} - \frac{w_t}{1+\rho} \right) - A_t \lambda \psi(\theta_s) \\
&= \overline{q\theta_s} \left(\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - \bar{C}_s \right) A_t, \quad \text{if } L_{1t} > 0, \qquad (17)
\end{aligned}
$$

[24]where $\overline{q\theta_s} \triangleq q\theta_s + \bar{q}\bar{\theta}_s$ and

$$
\bar{C}_s \triangleq \frac{\lambda \psi(\theta_s)}{\overline{q\theta_s}} + I_1 + I_2 + \frac{\bar{w}}{1+\rho} + \frac{\bar{q}\bar{\theta}_s}{\overline{q\theta_s}} \frac{I_3}{1+\rho} \qquad (18)
$$

is the expected cost of completion of one project. (Note: if $L_{1t} = 0$, then $w_t \neq \bar{w} A_t$, i.e., the wage rate is not determined by the final good sector.)

Similarly, for a project financed under regime m, the expected NPV is

$$
\begin{aligned}
& E_{NPVm} = q\theta_m \left(\frac{\Delta A_{t+1}}{L_2} \sum_{\tau=t}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - I_{1t} - I_{2t} - \frac{w_t}{1+\rho} - \sigma F_t \right) \\
& -\bar{q}\bar{\theta}_m (I_{1t} + \sigma F_t) - A_t \lambda \psi(\theta_m) \\
= \; & q\theta_m \left(\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - \bar{C}_m \right) A_t, \quad \text{if } L_{1t} > 0, \qquad (19)
\end{aligned}
$$

[25]where

$$
\bar{C}_m \triangleq \frac{\lambda \psi(\theta_m)}{q\theta_m} + \frac{q\theta_m + \bar{q}\bar{\theta}_m}{q\theta_m} (\sigma F + I_1) + I_2 + \frac{\bar{w}}{1+\rho} \qquad (20)
$$

is the expected cost of completion of one project. In a competitive capital market only the most efficient financing regime survives. The following result reflects this intuition.

Proposition 2 *With free-entry in the capital market, the equilibrium financing regime minimizes the expected capital cost of a completed project, i.e., at equilibrium a regime is chosen such that* $(\zeta^*, \theta^*) = \arg\min \left\{ \min_{\theta_s} \bar{C}_s, \min_{\theta_m} \bar{C}_m \right\}$.

[24] For the corner solution that $L_{1t} = 0$ the condition is $E_{NPVs} = \overline{q\theta_s} \left(\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - \bar{C}_s + \left(\frac{w_t}{A_t} - \bar{w} \right) \right) A_t$.

[25] For the corner solution that $L_{1t} = 0$ the condition is $E_{NPVm} = q\theta_m \left(\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau - \bar{C}_m + \left(\frac{w_t}{A_t} - \bar{w} \right) \right) A_t$

Proof. The financiers choose the optimal selection regime (ζ, θ) to maximize the expected NPV of the projects they finance, i.e., to solve the following program:

$$\max_{\zeta,\theta} E_{NPV} = \max \left\{ \max_{\theta_s} E_{NPVs}, \max_{\theta_m} E_{NPVm} \right\}. \tag{21}$$

Given free entry into the capital market, a break-even condition ensues, i.e., $\max_{\zeta,\theta} E_{NPV} = 0$. (Otherwise, an efficient outside financier would find it profitable to enter, or an inside financier would find it profitable to quit.) Using eq. (17) and (19), it is easy to verify that

$$\begin{aligned} & \delta \sum_{\tau=t}^{\infty} (1+\rho)^{-(\tau-t)} \pi_{\tau} \\ = \quad & \min \left\{ \min_{\theta_s} \bar{C}_s, \min_{\theta_m} \bar{C}_m \right\} \quad \text{for } L_{1t} > 0, \end{aligned}$$

[26]and $\arg\min \left\{ \min_{\theta_s} \bar{C}_s, \min_{\theta_m} \bar{C}_m \right\}$ is the solution to the program (21). ■

We define the minimum expected cost of completing a project under regime s as $\bar{C}_s^* \triangleq \min_{\theta_s} \bar{C}_s$, the minimum cost under regime m as $\bar{C}_m^* \triangleq \min_{\theta_m} \bar{C}_m$, and the cost at equilibrium as $\bar{C}^* \triangleq \min \left\{ \bar{C}_s^*, \bar{C}_m^* \right\}$. Then applying Proposition 2, we have the following result.

Proposition 3 *If $\bar{C}_s^* < \bar{C}_m^*$, then the equilibrium financing regime is regime s; if $\bar{C}_s^* > \bar{C}_m^*$, then the equilibrium financing regime is regime m.*

We denote the optimal pre-screening precision under the two regimes as θ_s^* and θ_m^* respectively, and define the optimal average pre-screening costs under the two regimes as $K_s^* \triangleq \frac{\lambda\psi(\theta_s^*)}{q\theta_s^* + \bar{q}\bar{\theta}_s^*}$ and $K_m^* \triangleq \frac{\lambda\psi(\theta_m^*)}{q\theta_m^*}$ respectively. The following Lemma shows comparative statics with respect to the institutional cost σ. (For other comparative statics of the model, please see Lemma 25 and 26 in Appendix B.)

Lemma 4 *$\frac{d\bar{C}_s^*}{d\sigma} = 0$, $\frac{d\theta_s^*}{d\sigma} = 0$ and $\frac{dK_s^*}{d\sigma} = 0$; $\frac{d\bar{C}_m^*}{d\sigma} > 0$ and $\frac{\partial\theta_m^*}{\partial\sigma} > 0$ and $\frac{dK_m^*}{d\sigma} > 0$ for $\lambda > 0$. Moreover, if $\sigma = 0$, then $\bar{C}_s^* > \bar{C}_m^*$ for $\lambda > 0$.*

Proof. See Appendix B.

[26]For the corner solution that $L_{1t} = 0$ the condition is $\delta \sum_{\tau=t}^{\infty} (1+\rho)^{-(\tau-t)} \pi_{\tau} = \min \left\{ \min_{\theta_s} \bar{C}_s, \min_{\theta_m} \bar{C}_m \right\} + \left(\frac{w_t}{A_t} - \bar{w} \right)$

This lemma suggests that ceteris paribus under regime m, with a high σ the financiers will spend more on pre-screening and achieve a higher precision, and the expected capital cost of R&D will be higher. In contrast, under the regime s, the change of institutional cost σ has no impact on pre-screening. The last part of the lemma establishes the benchmark case when there is no institutional cost.

Based on the above discussions, the following result demonstrates the determinants of regime choice.

Proposition 5 *For $\lambda(a) > 0$, there exists a threshold $\tilde{\sigma}(\lambda) > 0$ such that regime s is chosen if and only if $\sigma > \tilde{\sigma}(\lambda)$.*

Proof. See Appendix B.

The following Figure 2 illustrates Proposition 5. In the figure, a $\tilde{\sigma}(\lambda)$ curve partitions the $(\lambda(a), \sigma)$ space into two regions: in the upper region regime s prevails; in the lower region, regime m rules. It shows that the choice of financing regime is jointly determined by the exogenous 'institutional cost' σ and the relative development level a through $\lambda(a)$. For a given development level a, or $\lambda(a)$, when the institutional cost is high in comparison with the threshold value $\tilde{\sigma}$, at equilibrium regime s will be chosen; but if the institutional cost is lower than $\tilde{\sigma}$, regime m will be chosen.

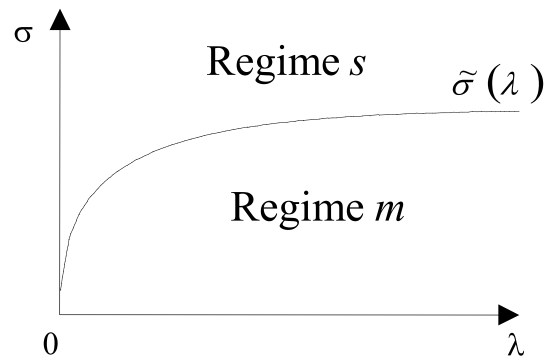

Figure 2: Endogenous financial regimes

4.2 Growth

In order to establish the foundation for examining how growth and financing regimes interact, we first establish the laws of motion, and then characterize the steady state of the system. A complete characterization of the dynamics of the growth paths is in Section 5.

4.2.1 Growth Equations

Let $g_t \triangleq \frac{A_{t+1}-A_t}{A_t}$ be the rate of growth (of knowledge stock) in period t. We suppose that there is free entry into the R&D sector and the capital market. Focusing on interior solutions that $0 < g_t < \delta L$, the number of completed projects in each period is determined by the following arbitrage condition (in equilibrium the expected cost of capital is the same as the asset value of a completed project):[27]

$$\underbrace{\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau}_{\text{PV of profit from completing a project}} = \overbrace{\bar{C}^*(\lambda_t)}^{\text{costs of completing a project}} \tag{22}$$

where π_τ is the expected profit per product in period τ.[28]

From the difference in the arbitrage conditions for period t and period $t+1$ we have

$$\frac{1}{1+\rho}\delta\pi_{t+1} = \bar{C}^*(\lambda_t) - \frac{1}{1+\rho}\bar{C}^*(\lambda_{t+1}). \tag{23}$$

The left-hand side of eq. (23) is the present value of the next period per project dividends to the investors; the right-hand side of eq. (23) is the difference of the present values between the current and next period costs of capital, or between the current and next period per project asset values.

Given that $\lambda_t = \lambda(a_t)$ and $a_{t+1} = a_t\frac{1+g_t}{1+g_f}$, and by combining eq. (13), (15), and (23), we have the following system of difference equations, which characterizes the dynamics of the economy: on the one hand, the current relative development level, a_t, affects the way of financing, which in turn affects the R&D cost and future growth rate, g_{t+1}; on the other hand, the growth rate g_t, affects the future development level, a_{t+1}.

$$\begin{cases} a_{t+1} = a_t\frac{1+g_t}{1+g_f} \\ g_{t+1} = \frac{1+\rho}{\alpha\bar{w}}\bar{C}^*\left(\lambda\left(a_t\frac{1+g_t}{1+g_f}\right)\right) - \frac{(1+\rho)^2}{\alpha\bar{w}}\bar{C}^*(\lambda(a_t)) + \delta L. \end{cases} \tag{24}$$

[27] The conditions for corner solutions are, $\delta\sum_{\tau=t+1}^{\infty}(1+\rho)^{-(\tau-t)}\pi_\tau \le \bar{C}^*(\lambda_t)$ for $g_t = 0$ and $\delta\sum_{\tau=t+1}^{\infty}(1+\rho)^{-(\tau-t)}\pi_\tau = \bar{C}^*(\lambda_t) + \left(\frac{w_t}{A_t} - \bar{w}\right) \ge \bar{C}^*(\lambda_t)$ for $g_t = \delta L$.

[28] In this economy, everyone complying with eq. (22) is a Nash equilibrium. This is because the best one can do in this economy is to gain zero-profit, which can be achieved by complying with eq. (22). Any deviation from the strategy implied by eq. (22) is not profitable given that all other players follow it.

4.2.2 Steady State Growth under Different Regimes

To facilitate our analysis, we define the benchmark economy as the case that there is no institutional cost, i.e., $\sigma = 0$, and that the knowledge stock is at the world frontier, A_{ft}. Moreover, we define the development level of this economy as the benchmark level, i.e., $a_t = 1$; and the benchmark knowledge stock A_{ft} grows at a constant growth rate g_f,

$$A_{ft} = A_{f0} \left(1 + g_f\right)^t .$$

By definition, in a steady state, $g_{t+1} = g_t$ and $a_{t+1} = a_t = \bar{a} > 0$, where $\bar{a}$ is time-invariant. Applying these definitions to eq. (24), we have $g_t = g_f$ and

$$\delta L = \frac{\rho\left(1+\rho\right)}{\alpha \bar{w}} \bar{C}^* \left(\lambda\left(\bar{a}\right)\right) + g_f. \tag{25}$$

To summarize we have,

Lemma 6 *The point $(\bar{a}, g_f)$ is a steady state (rest point) of (a_t, g_t), where $\bar{a}$ is defined as the solution to $\bar{C}^* \left(\lambda\left(\bar{a}\right)\right) = \frac{\alpha\bar{w}\left(\delta L - g_f\right)}{\rho(1+\rho)}$.*

In the remainder of the paper, we will focus on *this* steady state and call it the steady state, although there exists another steady state, which is trivial and unstable.[29]

In steady state, all economies' R&D capital costs are equalized to that of the frontier economy (the benchmark), which is a constant.

Lemma 7 *In the steady state, $g_t = g_f$; $\bar{C}^*_\zeta = \bar{C}^*_f$ where $\zeta = m, s$.*

Proof. By substituting eq. (25) into (24) we obtain

$$g_{t+1} = \frac{1+\rho}{\alpha\bar{w}} \left(\bar{C}^* \left(\lambda\left(a_t \frac{1+g_t}{1+g_f}\right)\right) - \bar{C}^* \left(\lambda\left(a_t\right)\right)\right) + \frac{\rho\left(1+\rho\right)}{\alpha\bar{w}} \left(\bar{C}^* \left(\lambda\left(\bar{a}\right)\right) - \bar{C}^* \left(\lambda\left(a_t\right)\right)\right) + g_f. \tag{26}$$

Then it is obvious that in the steady state $(g_{t+1} = g_t = g_f)$ we must have

$$\bar{C}^* \left(\lambda\left(a_t\right)\right) = \bar{C}^* \left(\lambda\left(\bar{a}\right)\right).$$

Noticing that $\bar{C}^* \left(\lambda\left(\bar{a}\right)\right) = \frac{\alpha\bar{w}\left(\delta L - g_f\right)}{\rho(1+\rho)}$, which is constant regardless of σ, and denoting $\bar{C}^* \left(\lambda\left(\bar{a}\right)\right)$ by $\bar{C}^*_f$ and $\bar{C}^* \left(\lambda\left(a_t\right)\right)$ by $\bar{C}^*_\zeta$. ■

[29] The trivial steady state is $a_{t+1} = a_t = 0$ and $g_{t+1} = g_t = \frac{\rho(1+\rho)}{\alpha\bar{w}} \left(\bar{C}^* \left(f\left(\bar{a}\right)\right) - \bar{C}^* \left(f\left(0\right)\right)\right) + g_f$. This steady state is unstable. For the proof of the result, see Lemma 28 in Appendix B.

The intuition behind this Lemma is that the fixed effects (σ's) associated with the differences among the different economies are compensated for by the adjustment in the relative development level and the way of financing. Based on this Lemma, we have the following proposition. The intuition of this result is related to the cost minimization of the financing regimes (Proposition 2).

Proposition 8 *In steady state a financing regime is chosen as if in each economy there were a social planner who has the objective of maximizing the economy's steady state development level* $\bar{a}$.

Proof. See Appendix B.

In the following we apply Lemma 7 to characterize the optimal regime selection in a steady state. For an economy under regime m, using eq (20) in the steady state, we have

$$\bar{C}_m^* = \frac{\lambda\left(a_{ss}\right)\psi\left(\theta_m^*\right)}{q\theta_m^*} + \frac{q\theta_m^* + \bar{q}\bar{\theta}_m^*}{q\theta_m^*}\left(\sigma_{ss}F + I_1\right) + I_2 + \frac{\bar{w}}{1+\rho} = \bar{C}_m^*\big|_{\sigma=0,a=1}, \tag{27}$$

where θ_m^* is the optimal pre-screening precision in the regime, which depends on λ and σ; since $\lambda = \lambda\left(\bar{a}\right)$ in a steady state, θ_m^* is an implicit function of $(\bar{a}, \sigma)$. We define the relationship between the steady-state development level a_{ss} and the institutional cost σ_{ss} under regime m as a set:

$$\Omega_{ss} \triangleq \left\{\left(a_{ss}, \sigma_{ss}\right) : \left(a_{ss}, \sigma_{ss}\right) \Rightarrow \bar{C}_m^* = \bar{C}_f^*, \text{ i.e., eq. (27)}\right\}.$$

It is obvious that σ_{ss} is an implicit function of a_{ss} and it satisfies the following property.

Lemma 9 $\sigma_{ss}\left(a_{ss}\right)$ *decreases in* a_{ss}*; particularly,* $\sigma_{ss} > 0$ *when* $a_{ss} = 0$*; and* $\sigma_{ss} = 0$ *when* $a_{ss} = 1$.

Proof. Using eq. (27) and the envelope theorem, then

$$\frac{d\sigma_{ss}}{da_{ss}} = -\frac{\psi\left(\theta_m^*\right)\lambda'\left(a_{ss}\right)}{\left(q\theta_m^* + \bar{q}\bar{\theta}_m^*\right)F} < 0, \tag{28}$$

which implies a negative, one-to-one mapping, hence, a functional relationship between a_{ss} and σ_{ss}. ■

Similarly, applying Lemma 7 we have the following result.

Lemma 10 (a^*, g_f) *is the steady state for any economy under regime s.*

Proof. Applying Lemma 7 to eq. (18) we have

$$\bar{C}_s^* = \frac{\lambda(a^*)\,\psi(\theta_s^*)}{q\theta_s^* + \bar{q}\bar{\theta}_s^*} + I_1 + I_2 + \frac{\bar{w}}{1+\rho} + \frac{\bar{q}\bar{\theta}_s^*}{q\theta_s^* + \bar{q}\bar{\theta}_s^*}\frac{I_3}{1+\rho} = \bar{C}_m^*\big|_{\sigma=0,a=1}, \quad (29)$$

where θ_s^* is optimal pre-screening precision under regime s and a^* is the unique solution to eq. (29). ■

To facilitate our analysis on determination of financing regimes in the steady state, corresponding to a^* we denote $\sigma^* = \sigma_{ss}(a^*)$. By definition, $(a^*, \sigma^*) \in \Omega_{ss}$. The following result demonstrates how financing regimes are chosen at the steady state.

Proposition 11 *With endogenized financing regimes, in the steady state, if $\sigma \geq \sigma^*$, then regime s is chosen and $\bar{a} = a^*$; if $\sigma < a^*$, then regime m is chosen and $\bar{a} > \sigma^*$ where $(\bar{a}, \sigma) \in \Omega_{ss}$, with $\frac{d\bar{a}}{d\sigma} < 0$, particularly, $\bar{a} = 1$ as $\sigma = 0$.*

Proof. From eqs.(27 and 29), (a^*, σ^*) is a solution to the condition $\bar{C}_s^* = \bar{C}_m^*$. Applying Lemma 4 ($\frac{d\bar{C}_s^*}{d\sigma} = 0$ and $\frac{d\bar{C}_m^*}{d\sigma} > 0$) and Proposition 2, if $\sigma \geq \sigma^*$, then regime s is chosen and $\bar{a} = a^*$; if $\sigma < a^*$, then regime m is chosen. Consequently, from Lemma 9 we have $\frac{d\bar{a}}{d\sigma} < 0$ hence, $\bar{a} > a^*$. ■

The following Figure 3 illustrates endogenized financing regimes in the steady state (Proposition 11). The σ_c curve partitions the (a, σ) space into a regime m region and a regime s region, whereby the two different regimes have comparative advantages in minimizing R&D costs respectively (see Proposition 5). The bold σ_{ss} curve and the connecting vertical line are the sets of the steady state for economies with $\sigma < \sigma^*$ and $\sigma > \sigma^*$ respectively. Instead of a universal convergence, economies converge to two 'clubs': steady state economies with $\sigma < \sigma^*$ go to the regime m club and economies with $\sigma > \sigma^*$ go to the regime s club. Related to this institutional divide, economies also differ in their steady state development levels: countries in the regime m club are wealthier than those in the regime s club.

The two regimes put different weights in ex ante R&D project selection.

Lemma 12 *In the steady state $K_s^* > K_m^*$ if $I_3 > \breve{I}_3$ where $\breve{I}_3$ is a finite threshold.*

Proof. See Appendix B. ■

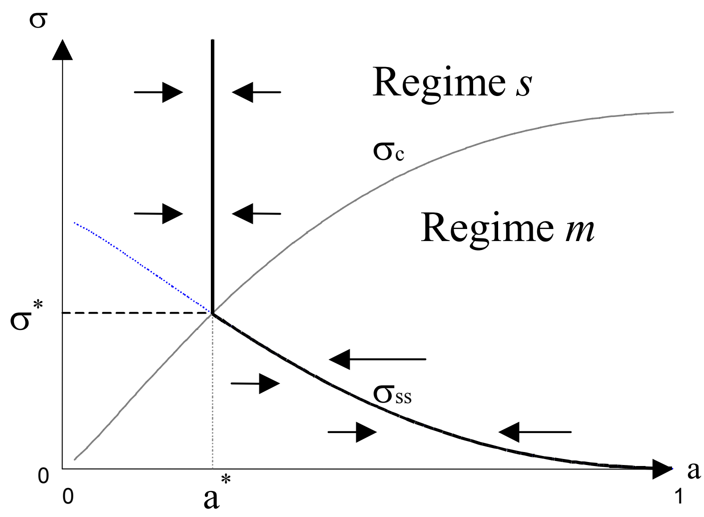

Figure 3: Endogenous financial regimes and their steady states (The arrows indicate the convergence effects)

Lemma 12 says that regime s spends more on pre-screening than regime m in the steady state. This is because regime s does not have ex post screening capacity and pre-screening serves as a *substitute*. In the remainder of the paper, we will focus on the case of $I_3 > \breve{I}_3$. This condition will not be mentioned unless we note otherwise. Given regime s selects projects only ex ante, it is more demanding than regime m in pre-screening. As a result, when R&D is more uncertain, regime s selects a smaller portfolio of R&D projects to begin with.

Proposition 13 *In the steady state, regime s imposes higher standards than regime m in pre-screening, i.e., $\theta_s^* > \theta_m^*$. Moreover, when projects are more uncertain ($q < \frac{1}{2}$) regime s has a lower acceptance rate in pre-screening than regime m, i.e., $q\theta_s^* + \bar{q}\bar{\theta}_s^* < q\theta_m^* + \bar{q}\bar{\theta}_m^*$.*

Proof. See Appendix B. ■

Another major difference between the two regimes is in ex post project selection. This difference becomes more significant when the development level of an economy becomes higher such that the regime m relies more on ex post selection.

Proposition 14 *The project termination rate under regime m increases with a, i.e.,* $\frac{\partial}{\partial a}\frac{\bar{q}\bar{\theta}_m^*}{q\theta_m^*+\bar{q}\bar{\theta}_m^*} > 0$*; whereas the rate under regime s is a constant* 0.

Proof. See Appendix B. ■

When the development level is low, with imitation opportunities relying on pre-screening, regime s can do well. However, when the development level becomes higher such that imitation opportunities diminish, regime m's ex post screening mechanisms become more effective. This explains why regime s has lower steady-state development levels than regime m.

5 Catching-up Dynamics and Cycles

The catching up process is modelled as transition dynamics starting from a below-steady-state development level towards the steady-state level. Under different financing mechanisms, some economies may converge to their steady state faster than others; and the growth of some economies may be cyclical (unstable).[30]

5.1 Convergence and Stability

The linearized growth equation (24) around their steady state $(\bar{a}, g_f)$ is that

$$\begin{pmatrix} a_{t+1}-\bar{a} \\ g_{t+1}-g_f \end{pmatrix} \approx \begin{pmatrix} 1 & \frac{\bar{a}}{1+g_f} \\ -\frac{\rho\mathfrak{B}}{\frac{\bar{a}}{1+g_f}} & \mathfrak{B} \end{pmatrix} \begin{pmatrix} a_t-\bar{a} \\ g_t-g_f \end{pmatrix}, \tag{30}$$

where $\bar{C}^*(\lambda(\bar{a})) = \frac{\alpha\bar{w}(\delta L-g_f)}{\rho(1+\rho)}$

$$\mathfrak{B} \triangleq \frac{\delta L-g_f}{\rho(1+g_f)}\frac{\partial\bar{C}^*}{\partial\bar{a}}\frac{\bar{a}}{\bar{C}^*}. \tag{31}$$

Eq. (30) decomposes the cause of growth, $(g_{t+1}-g_f)$, into two effects: the convergence effect, $-\frac{\rho\mathfrak{B}}{\frac{\bar{a}}{1+g_f}}(a_t-\bar{a})$; and the growth inertia effect, $\mathfrak{B}(g_t-g_f)$.

[30] The combination of a discrete-time setup and a flat capital supply differentiates our model significantly from most techonological diffusion-based growth models (e.g., Barro and Sala-i-Martin, 1995, Chapter 8) in the dynamics of the system. Flat capital supply can also arise in many situations, such as in a small economy with a free access to international capital market where interest is almost exogenous.

The common factor in the two effects in the system (30) is $\mathfrak{B}$,which is a measure of the ability to reserve the momentum of growth performance. We call it the inertia factor. Moreover, $\mathfrak{B}$ is observable as the auto-correlation coefficient of the growth rates. In the following we will first focus on impacts of $\mathfrak{B}$ on the dynamic system. Then in Section5.2 we will discuss on how $\mathfrak{B}$ is determined, the interaction between financing regimes and dynamics of the system.

$\mathfrak{B}$ determines the magnitude of the convergence effect in the system (30). When the current development level a_t is below $\bar{a}$, an economy with a higher $\mathfrak{B}$ tends to invest more on R&D than other economies; whereas when a_t is above $\bar{a}$, then an economy with a higher $\mathfrak{B}$ tends to reduce R&D more than other economies. Moreover, $\mathfrak{B}$ also determines the magnitude of the growth inertia. An economy with a higher $\mathfrak{B}$ may expect higher future R&D capital costs (associated with more rapid exhaustion of opportunities for imitation), hence a higher future valuation of current projects (capital gain). Thus, when a higher $\mathfrak{B}$ economy has a high $(g_t - g_f)$, it tends to invest more on R&D, which drives an even faster growth in the future.

The combination of the above convergence effect and inertia effect determines the speed of catching up and the stability of an economy. In a catching-up phase (i.e., $a_t < \bar{a}$ and $g_t > g_f$), the two effects work in the same direction; therefore a higher $\mathfrak{B}$ implies a higher speed of catching up. However, when there is an overshooting (i.e., $a_t > \bar{a}$ and $g_t > g_f$), the two effects work in opposite directions and, importantly, the inertia effect dominates in a divergence direction. Thus, the inertia effect ultimately determines the stability of the system.

Proposition 15 *If $\mathfrak{B} < \frac{1}{1+\rho}$, the steady state $(\bar{a}, g_f)$ is asymptotically stable (it is a sink); if $\mathfrak{B} \in \left(\phi, \frac{1}{1+\rho}\right)$, where $\phi = 1 + 2\rho - 2\sqrt{(\rho + \rho^2)}$, the economy spirals toward $(\bar{a}, g_f)$. If $\mathfrak{B} > \frac{1}{1+\rho}$, the steady state $(\bar{a}, g_f)$ is unstable (it is a source).*[31]

Proof. The stability of the steady state $(\bar{a}, g_f)$ depends on the eigenvalues of the matrix

$$\begin{pmatrix} 1 & \frac{\bar{a}}{1+g_f} \\ -\frac{\rho\mathfrak{B}}{\frac{\bar{a}}{1+g_f}} & \mathfrak{B} \end{pmatrix},$$

[31] When $\mathfrak{B} > \frac{1}{1+\rho}$, the economy may spiral toward limit cycles. In some of the numerical examples in this and the next section, we simulate the limit cycles.

which are:

$$r_1 = \frac{1}{2}(\mathfrak{B}+1) + \frac{1}{2}\sqrt{\eta}, \quad r_2 = \frac{1}{2}(\mathfrak{B}+1) - \frac{1}{2}\sqrt{\eta}.$$

where $\eta \triangleq \mathfrak{B}^2 - 2\mathfrak{B} + 1 - 4\rho\mathfrak{B}$. If $\mathfrak{B} < \frac{1}{1+\rho}$ then $|r_1| < 1$ and $|r_2| < 1$, $(\bar{a}, g_f)$ is asymptotically stable; if $\mathfrak{B} \in \left(\phi, \frac{1}{1+\rho}\right)$, the two eigen values are complex with the norm being smaller than unity, $(\bar{a}, g_f)$ is a cyclical attractor. If $\mathfrak{B} > \frac{1}{1+\rho}$, then $|r_1| > 1$ and $|r_2| > 1$, $(\bar{a}, g_f)$ is unstable. ■

The essence of the above result is that when $\mathfrak{B}$ is small, the inertia effect is weak, and an economy will converge to its steady-state level without over-shooting; and when $\mathfrak{B}$ is in the medium range, the inertia effect is strong enough to generate overshooting and contracting cycles, but not strong enough to cause sustained cycles, which will occur when $\mathfrak{B}$ is even larger. Next, we analyze how an economy's $\mathfrak{B}$ affects the economy's convergence speed by solving the growth path. We start from the asymptotically stable case, i.e., $\mathfrak{B} \in (0, \phi)$. In this case, the two real eigenvalues of the system are

$$r_2 = \frac{1}{2}(\mathfrak{B}+1) - \frac{1}{2}\sqrt{\eta} < r_1 = \frac{1}{2}(\mathfrak{B}+1) + \frac{1}{2}\sqrt{\eta} < 1$$

and $\frac{\partial r_1}{\partial \mathfrak{B}} < 0$. The associated eigenvectors are

$$\mathbf{v}_1 = \begin{pmatrix} \frac{\bar{a}\left(\frac{1}{2}\mathfrak{B} - \frac{1}{2} - \frac{1}{2}\sqrt{\eta}\right)}{\rho\mathfrak{B}\left(1+g_f\right)} \\ 1 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} \frac{\bar{a}\left(\frac{1}{2}\mathfrak{B} - \frac{1}{2} + \frac{1}{2}\sqrt{\eta}\right)}{\rho\mathfrak{B}\left(1+g_f\right)} \\ 1 \end{pmatrix}.$$

The solution confirms that when $\mathfrak{B}$ is small, a higher $\mathfrak{B}$ leads to a faster convergence toward the steady-state development level $\bar{a}$.

Proposition 16 *If $\mathfrak{B} \in (0, \phi)$ and $a_0 < \bar{a}$ and $g_0 = g_f$, then when t is sufficiently large, a_t increases with $\mathfrak{B}$, i.e., $\frac{\partial a_t}{\partial \mathfrak{B}} > 0$, for $a_t < \bar{a}$.*

Proof. See Appendix B.

Next, we analyze the cases where the value of $\mathfrak{B}$ is at the medium level and the growth path is cyclical, i.e., $\mathfrak{B} \in (\phi, \overline{\phi})$, where $\overline{\phi} \triangleq 1 + 2\rho + 2\sqrt{(\rho + \rho^2)}$. Within this range of $\mathfrak{B}$ values, the two eigen values are complex

$$r_1 = \sqrt{\mathfrak{B}(1+\rho)}e^{i\cos\omega}, \ r_2 = \sqrt{\mathfrak{B}(1+\rho)}e^{-i\cos\omega}$$

and their associated eigenvectors are

$$\mathbf{v}_1 = \begin{pmatrix} \frac{\bar{a}\sqrt{\rho\mathfrak{B}}}{\rho\mathfrak{B}\left(1+g_f\right)}e^{-i\varphi} \\ 1 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} \frac{\bar{a}\sqrt{\rho\mathfrak{B}}}{\rho\mathfrak{B}\left(1+g_f\right)}e^{i\varphi} \\ 1 \end{pmatrix},$$

where $\omega \triangleq \arccos\left(\frac{\frac{1}{2}+\frac{1}{2}\mathfrak{B}}{\sqrt{\mathfrak{B}(1+\rho)}}\right)$ is the angular velocity, $\varphi \triangleq \arccos\left(\frac{\frac{1}{2}\mathfrak{B}-\frac{1}{2}}{\sqrt{\rho\mathfrak{B}}}\right)$ and the norm is $|r_1| = \sqrt{\mathfrak{B}(1+\rho)}$. Some properties of ω and φ are the following:

$$\varphi \in (0, \pi) \text{ and } \frac{\partial \varphi}{\partial \mathfrak{B}} < 0; \tag{32}$$

A useful approximate relation between ω and φ is given by

$$\omega \approx \left(\arccos \frac{1}{\sqrt{(1+\rho)}}\right) \sin \varphi, \tag{33}$$

the derivation of which is provided in Appendix B. Solving the growth path we find that with a medium value of $\mathfrak{B}$, although the growth path is cyclical, a higher $\mathfrak{B}$ still leads to a faster catch up toward the steady-state position $\bar{a}$.

Proposition 17 *If $\mathfrak{B} \in \left(\phi, \overline{\phi}\right)$ and $a_0 < \bar{a}$ and $g_0 = g_f$, then the catching-up speed increases in $\mathfrak{B}$.*

Proof. See Appendix B.

The essence of Propositions 15 to 17 is that economies with a larger inertia factor $\mathfrak{B}$ have a stronger convergence effect and growth inertia; therefore, they tend to catch up faster. But they are also more likely to overshoot their steady-state targets. Relating these findings to the property of B, we have the following empirically testable predictions.

Corollary 18 *Ceteris paribus, economies with larger coefficients of autocorrelation in the growth rate tend to catch up faster, but they are more likely to experience growth cycles.*

The magnitude of the inertia factor $\mathfrak{B}$ is determined by R&D selection mechanisms used by financing regimes. In the next part we analyze the growth dynamics of economies under regimes m and s.

5.2 Catching up and Stability Properties of Financing Regimes

From eq. (31) a key factor which determines $\mathfrak{B}$ is $\frac{\partial \bar{C}^*}{\partial \bar{a}} \frac{\bar{a}}{\bar{C}^*}$, the steady-state elasticity of R&D capital costs with respect to the development level. It turns out the elasticity is affected by the R&D selection mechanism used by the financing regime.

Lemma 19 *Ceteris paribus, the R&D capital cost in both regimes,* $\bar{C}^* (\lambda (a_t))$ *increases in* a_t*, i.e.,* $\frac{\partial \bar{C}^*(\lambda(a_t))}{\partial a_t} = \frac{K^*_\zeta(\lambda(a_t))}{\lambda(a_t)} \lambda'(a_t) > 0,$ *where* $\zeta = s, m.$

Proof. See Appendix B.

Using Lemma 19 and eq. (31) we obtain inertia factor $\mathfrak{B}$ under different financing regimes

$$\mathfrak{B} = \begin{cases} \mathfrak{B}_s \triangleq \frac{\delta L - g_f}{\rho\left(1+g_f\right)} \frac{\lambda'(\bar{a})\bar{a}}{\lambda(\bar{a})} K^*_s (\lambda(\bar{a})) \text{ under regime } s, \\ \mathfrak{B}_m \triangleq \frac{\delta L - g_f}{\rho\left(1+g_f\right)} \frac{\lambda'(\bar{a})\bar{a}}{\lambda(\bar{a})} K^*_m (\lambda(\bar{a})) \text{ under regime } m. \end{cases} \quad (34)$$

To compare the dynamics of the two regimes, we study two economies that are identical in every aspect except for a difference in financing regimes. That is, we look at $\sigma = \sigma^*$ where the two regimes have identical steady state a^*; and they start from the same initial values (a_0, g_0) where $a_0 < a^*$. Then, Proposition 20 implies that $\mathfrak{B}_s > \mathfrak{B}_m$.

Proposition 20 $\mathfrak{B}_s > \mathfrak{B}_m$ *at* (a^*, σ^*)*.*

Proof. By Proposition 12, if $I_3 > \breve{I}_3$ then $K^*_s > K^*_m$ at (a^*, σ^*); consequently, $\mathfrak{B}_s > \mathfrak{B}_m$ at (a^*, σ^*). ■

Given growth inertia factors $\mathfrak{B}_s$ and $\mathfrak{B}_m$ are observable as growth rate auto-correlations, Proposition 20 not only makes testable predictions but also, combined with some previous results, our model further predicts that regime s (as compared to regime m) is more likely to fluctuate around its steady state, which has a lower development level than regime m, although it may catch up faster.

Proposition 21 *For an economy with* $\sigma = \sigma^*$ *and starting from the initial point* (a_0, g_0) *where* $a_0 < a^*$ *and* $g_0 = g_f$*,*
(1) the growth path under regime s *is more likely to be cyclical than that under regime* m*.*
(2) it converges to its steady state faster under regime s *than under regime* m *if* $\mathfrak{B}_s \leq \phi$*;*
(3) it reaches the level of a^* *earlier under regime* s *than under regime* m *if* $\mathfrak{B}_s \in \left(\phi, \overline{\phi}\right)$*.*

Proof. See Appendix B.

As has been shown, regime s gives greater advantages to more backward economies, in particular economies with higher σ values. Nevertheless, some low σ backward economies may benefit from adopting regime s at their early stage of catching up as well. The following proposition characterizes an optimal regime selection at different development stages. At an early stage of development, regime s is more efficient and catches up faster. However, as an economy approaches its steady-state development level, the financing regime should be changed to regime m.

Proposition 22 *For an economy with $\sigma \in (0, \sigma^*]$, there exists a threshold value $\underline{a}$ such that if $a_0 < \underline{a}$, the optimal financing regime is regime s when $a_t < \underline{a}$ and regime m when $a_t \geq \underline{a}$.*

Proof. See Appendix B.

Regime s is optimal at low development levels because it saves contract enforcement costs and relies more on ex ante R&D selection, which is more effective at a low development level. When the development level is low, the growth path of an economy under regime s is independent from σ (see eq. 24) such that an economy with $\sigma < \sigma^*$ can grow like an economy with $\sigma \geq \sigma^*$. However, an economy with low σ can do better by switching to regime m when it is close to its steady-state development level.

Simulation results (Example 29) are shown in Appendix C to illustrate the above theoretical results.

6 Empirical and Policy Implications

A basic prediction of our theory on financing regimes (Proposition 20) is that regime s has higher growth inertia $\mathfrak{B}$ than regime m, where $\mathfrak{B}$ is measured as the growth rate auto-correlations. Although systematic tests of this prediction will be conducted during the next step of our research, some preliminary observations seem to suggest that this prediction is consistent with the data. Figure 4 in the following plots a lagged growth rate vs. the growth rate for the CEE economies (as proxies for regime s) and the West European (WE) economies (as proxies for regime m) for the period from 1950 to 2000.[32] The figure suggests that after controlling for the convergence effect, the average $\mathfrak{B}$ of the CEE economies is higher than that of the WE economies: the slope for the CEE economies is positive, whereas the slope for the WE economies is negative.

[32] To be more precise, the horizontal axis of the figure is the lagged 5-year-average growth rate, and the vertical axis is the 5-year-average growth rate (net off the convergence effect, the adjustment of the U.S. 5-year-average growth rate, and the regression constant term).

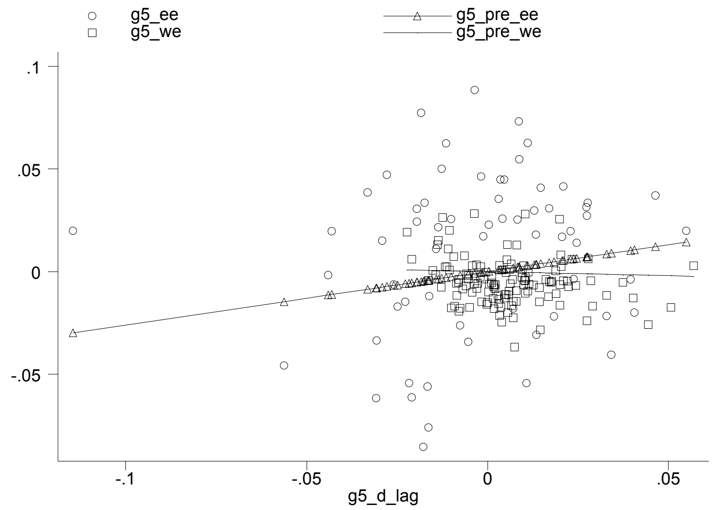

Figure 4: Growth rate auto-correlations: CEE vs WE economies

Based on Proposition 20, our model (e.g., Proposition 21, etc.) predicts that regime s is more likely to fluctuate around the steady-state development level, which is lower than that under regime m.[33] To demonstrate the empirical implications of these predictions we provide the following simulations.[34] Using similar coordinates as those in Fig. 7 and 8 the vertical axis is the difference between an economy's growth rate and the world frontiers, $g_t - g_f$; and the horizontal axis is the ratio between the economy's development level and that of the world frontiers.

Fig. 5 is a simulation of regime m, which catches up first and then converges to its steady-state development level with moderate overshooting. Fig. 6 is a simulation of regime s for an economy with the same parameter values as those for regime m. It demonstrates substantially larger growth cycles around a lower steady-state development level. Although we leave formal tests of the predictions of our model for future work, the simulated growth patterns of the two financing regimes seem consistent with the observed growth patterns of the two financing systems in Figures 7 and 8.

[33] Our model also predicts that the steady-state level of regime m is determined by institutional factor σ such that an economy with a higher σ should have a lower steady-state development level.

[34] All the parameter values of the simulation are presented in Example 30 of Appendix C.

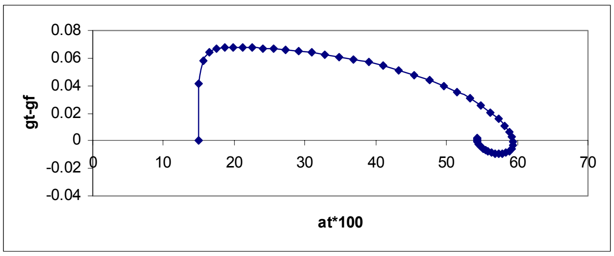

Figure 5: A catch-up pattern of a regime m

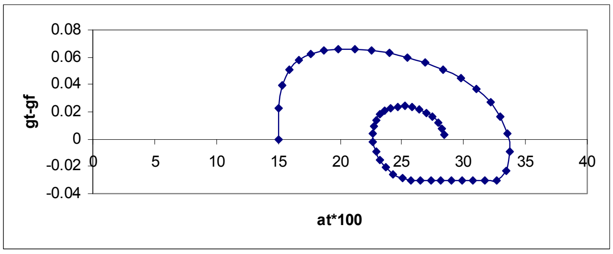

Figure 6: A catch-up pattern of a regime s

With simultaneously endogenized financing and development, our model has rich policy implications. However, a complete exploration of the policy implications of the model is beyond the scope of this paper. Here we briefly illustrate some of the implications.

In reality, the choice (or change) of financing regime may be affected by political institutions, legal restrictions, etc. For example, for a regime s, where the economy's development level has caught up to a level that is close to its steady state, it is optimal to switch to regime m at this time. However, this may not be implemented since some stakeholders who benefit from regime s may have strong incentives to block a regime change. As a result, the economy may decline after an overshooting, which may trigger an economic/political crisis later. Therefore, a change of financial institution may occur as a consequence of a political regime change. Related to this, political regime changes (e.g., the collapse of centralized economies) or outside pressures (e.g., reform conditions imposed by international institutions like the IMF) may result in a different timing of regime change which can be much worse than than optimal. Our model derives some useful policy implications from the above-mentioned scenario.

Example 23 . *Consider an economy starting from $a_0 < a^* < \bar{a}$ and $g_0 = g_f$. The following examples show the impacts of deviations from the second-best financing regime on the consequent development of the economy.*

1. *Optimal regime change: at first, regime s is chosen optimally, which delivers a high speed of catching up. Then the regime is changed at the optimal timing to regime m, which guarantees a stable convergence to the steady-state development level and growth rate (Panel a and b of Fig. 11 in Appendix C). After the regime change, the development level keeps increasing toward the steady-state level, while there is a moderate initial drop in the growth rate to adjust to the convergence to the steady-state growth rate. Regarding in-house R&D as regime s and venture-capital-financed R&D as regime m, this may shed some light on the impact of the rise of venture capital in the U.S. economy since the 1970s.*

2. *Late regime change: A reform takes place at the end of a positive overshooting (Panel c and d of Fig. 11 in Appendix C). Immediately after the regime change, there is a sharp drop in the growth rate which makes the development level decline as well. Then there is a recovery of growth and the development level. Interpreting a centralized financial system as regime s, this may shed some light on the impact of failing to reform the financial system on time, as well as on the sharp decline of the transition economies after the change in financial systems.*

3. *Very late regime change: the regime change takes place at the end of a negative overshooting (Panel e and f of Fig. 11 in Appendix C). Although the economy suffered low growth during the negative overshooting, immediately after the regime change there was a second sharp drop in the growth rate. This may reflect the experience of some of the transition economies.*

7 Conclusion

Our model suggests that under certain conditions related to legal institutions, some financial systems may be better than others in allowing an economy to catch up faster. The same underlying force, however, may also result in large growth cycles around lower steady-state development levels if financial reform lags behind growth needs. Therefore, the timing of financial system reforms (e.g., financial liberalization) can play a critical role. Moreover, our model implies that changes in the financial regime of an economy should be conditional on the legal institutions in that economy. When legal institutions are very weak, the promotion of financial liberalization may impair the economy. This can shed some light on the problems of the former centralized economies and their transition.

Our theory takes the decades-long debates on financial development and economic development,[35] on alternative financial institutions one step further. It also provides alternative explanations for some important recent discoveries, such as the 'irrelevance' of financial structures (Beck and Levine, 2002).

Furthermore, our theory has implications for the convergence/divergence debate (e.g., Barro and Sala-I-Martin, 1995; Quah, 1996; Barro, 1997). We predict that when convergence will occur is conditional on the financial institutions and legal conditions.

Finally, our model focuses on one important aspect of the mechanisms of financial institutions - R&D project financing/selection. We are fully aware that financial institutions have other important features, such as affecting capital investment in general, and affecting risk sharing between households and firms (Allen and Gale, 2000), etc. To make our point clear with a simple model, we abstract from many other mechanisms of financial institutions. These factors will be incorporated into our model in the future.

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[35] The debates on finance and development involve the following different major views/observations. Cameron et al. (1967), Rajan and Zingales (1998), and Carlin and Mayer (2003) argue that some specific financial institutions are more helpful than others for economic development. Gerschenkron (1962) argues that the development stage may determine the choice of financial institutions (backward countries might benefit from banking institutions for catching up). In contrast, Robinson (1952) believes financial development follows economic growth. Lucas (1988) argues that financial development is not important and it has been overemphasized.

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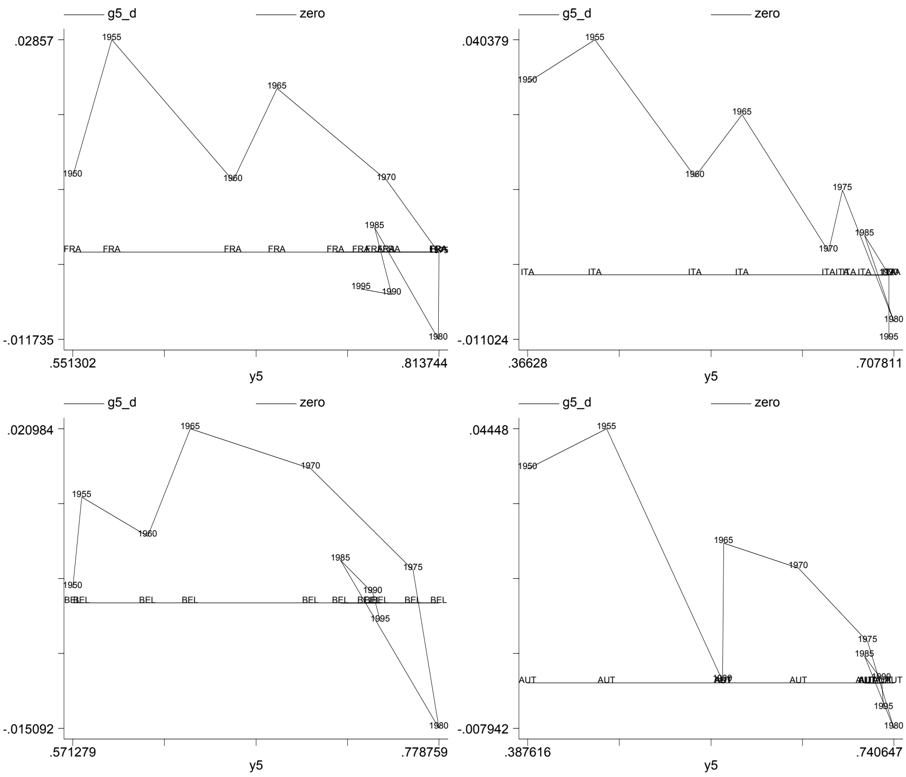

Figure 7: Catching-up patterns of France, Italy, Belgium, and Austria

Table 1. Total R&D Expenditure, 1967-85 (Percentage of GNP)

	US	Japan	WE	USSR	Czech	Hungary
1967	3.07	1.58	1.78	3.0	$2.9^{(a)}$	$1.8^{(a)}$
1975	2.38	2.01	1.81	3.5	$3.1^{(a)}$	$2.8^{(a)}$
1982	2.69	2.47	2.04	3.6		
1985	$2.68^{(b)}$	$2.80^{(b)}$		$5.0^{(c)}$		

Note: All the data in the table include both military and civilian sectors.
Source: Hanson and Pavitt (1987), Table 2, p.53.
$^{(a)}$ Poznanski (1985), Table 1, p.53.
$^{(b)}$ National Science Board (1989), Table 4-19, p.96.

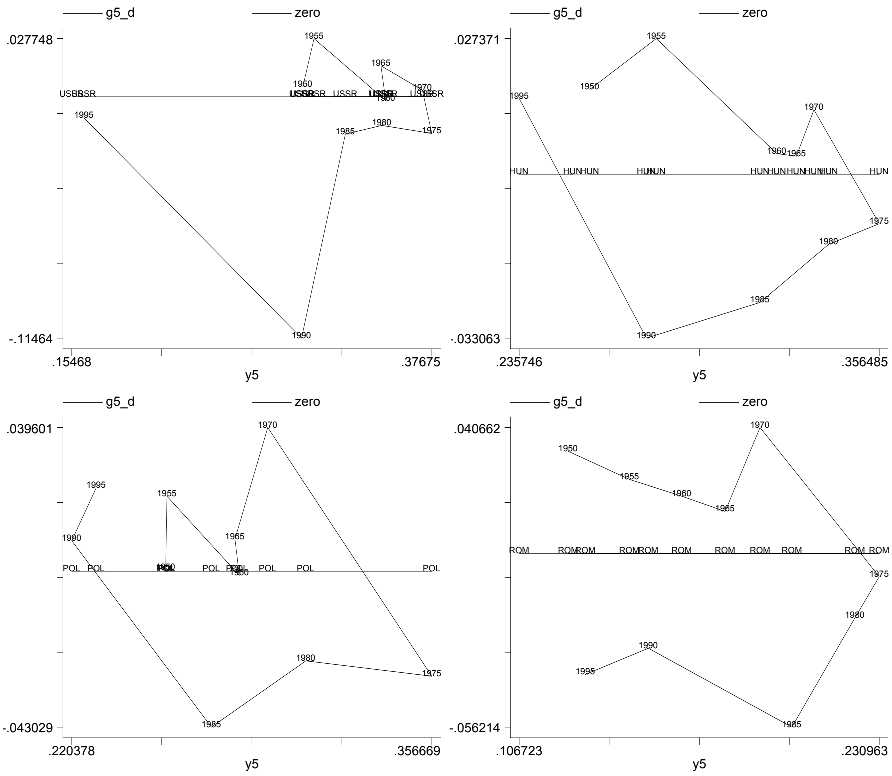

Figure 8: Catching-up pattern of the USSR, Hungary, Poland, and Romania

[c] Linz and Thorton (1988).

Table 2. Civilian Sector R&D Expenditure Comparison (percentage of GNP)

	US	Japan	France	West Germany	UK	USSR
1967	1.2	1.8	1.5	1.6	1.9	1.4

Note: Defense and space sector R&D expenditures excluded for all countries.

Source: Bergson (1989), Table 11, p.125.

Table 3. Scientists and Engineers in R&D, USSR vs. U.S.

	USSR million	U.S. million	USSR per 10,000 workers	U.S. per 10,000 workers
1950	1.25	1.59	14.7	26.2
1960	2.73	3.81	27.5	55.8
1970	6.62	5.47	54.2	66.8
1975	8.74	5.35	66.0	61.5

Note: Scientists and engineers in both countries exclude humanities specialists, social scientists, and psychologists.

Source: Bergson (1989), Table 12, p.126.

A Appendix A

Assumptions

A-1: $I_2 + \frac{I_3}{1+\rho} + \frac{\bar{w}}{1+\rho} > \frac{\delta \alpha L \bar{w}}{\rho}$ (the cost of a bad project at stage 2 is higher than the upper bound of profit).

A-2: $I_1 + I_2 + \frac{\bar{w}}{1+\rho} \geq \frac{I_3}{1+\rho}$ (the cost of a bad project at stage 3 is lower than the cost of a good project).

A-3: $\frac{I_3+F}{1+\rho} > \frac{\delta \alpha L \bar{w}}{\rho}$ (the cost of a bad project at stage 3, conditional on regime m, is more expensive than the upper bound of profit).

To prove Lemma 1, let us first prove the following Lemma.

Lemma 24 *From assumptions A-1, A-2, and A-3, it follows that in any equilibrium with a positive growth rate,*
(1) $I_2 + \frac{I_3}{1+\rho} + \frac{\bar{w}}{1+\rho} > \delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau > \frac{I_3}{1+\rho}$
(2) $I_1 + I_2 + \frac{\bar{w}}{1+\rho} < \delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau$.
(3) $\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau < \frac{I_3+F}{1+\rho}$

Proof. Since $\pi_\tau = \frac{\alpha L_{1t} \bar{w}}{1+\rho}$ and $L_{1t} \leq L$, $\delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau \leq \frac{\delta \alpha L \bar{w}}{\rho}$ holds, i.e., $\frac{\delta \alpha L \bar{w}}{\rho}$ is an upper bound to the expected profit of a project in any equilibrium with a positive growth rate. Consequently, from A-1 and A-2, it follows that $I_2 + \frac{I_3}{1+\rho} + \frac{\bar{w}}{1+\rho} > \frac{\delta \alpha L \bar{w}}{\rho} \geq \delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau > I_1 + I_2 + \frac{\bar{w}}{1+\rho} \geq \frac{I_3}{1+\rho}$, which implies (1). On the other hand, since the cost of a good project, i.e., $I_1 + I_2 + \frac{\bar{w}}{1+\rho}$, is a lower bound to the expected profit of a project in any equilibrium with a positive growth rate, i.e., $I_1 + I_2 + \frac{\bar{w}}{1+\rho} < \delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau$, then (2) holds. Finally by A-3, $\frac{I_3+F}{1+\rho} > \frac{\delta \alpha L \bar{w}}{\rho} \geq \delta \sum_{\tau=t+1}^{\infty} (1+\rho)^{-(\tau-t)} \pi_\tau$ holds, which implies (3). ■

Sketch of proof of Lemma 1: Applying the above result (3), if ex ante some financiers decide to finance a project jointly, an entrepreneur, who discovers that the project is bad at stage 1 will stop it right away to avoid a costly liquidation at stage 2, since for the entrepreneur the private benefits are $b_1 > b_2$. As a result, under multi-financier financing, all bad projects will be terminated at date 1.

However, if a project is financed by one financier, when the financier discovers that the project is a bad one at stage 2, given that earlier investments are sunk, according to the above result (2), continuing the bad project is ex post profitable, the financier will choose to refinance the project. Anticipating this, at stage 1 when an entrepreneur with a bad project realizes the type of the project, the entrepreneur will not report the private information about the type of the project. Thus, the financier will always refinance a bad project.[36] ■

B Appendix B

Proof of Lemma 4

First consider regime s. Since $\bar{C}_s$ is not a function of σ, it follows that $\frac{\partial\theta_s^*}{\partial\sigma}=0$, $\frac{\partial\bar{C}_s^*}{\partial\sigma}=0$ and $\frac{\partial K_s^*}{d\sigma}=0$.

Under regime m, there is an interior optimal precision θ_m^* for $\lambda>0$ (Lemma 25). At the point $\theta_m=\theta_m^*$, the derivative $\frac{\partial\bar{C}_m^*}{\partial\sigma}=\frac{\partial\bar{C}_m}{\partial\sigma}=\frac{q\theta+\bar{q}\bar{\theta}}{q\theta_m}F>0$. Differentiating the equation $\frac{\partial\bar{C}_m^*}{\partial\theta_m}=0$ w.r.t. σ and reorganizing, it follows that $\frac{\partial\theta_m^*}{d\sigma}=\frac{\bar{q}F}{\lambda\psi''(\theta_m^*)\theta_m^*}>0$ and $\frac{\partial K_m^*}{\partial\sigma}=\frac{\partial K_m^*}{\partial\theta}\frac{\partial\theta_m^*}{\partial\sigma}=\frac{\lambda\psi'(\theta_m^*)\theta_m^*-\lambda\psi(\theta_m^*)}{q\theta_m^{*2}}\frac{\bar{q}F}{\lambda\psi''(\theta_m^*)\theta_m^*}>0$.

Now consider the special case: $\sigma=0$. In the following we are going to show the R&D capital cost is lower under regime m, i.e., $\bar{C}_m^*<\bar{C}_s^*$, where $\bar{C}_s^*=K_s^*+\frac{\bar{q}\bar{\theta}_s^*}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{I_3}{1+\rho}+I_1+I_2+\frac{\bar{w}}{1+\rho}$ and $\bar{C}_m^*=K_m^*+\frac{q\theta_m^*+\bar{q}\bar{\theta}_m^*}{q\theta_m^*}I_1+I_2+\frac{\bar{w}}{1+\rho}$. The comparison is not straightforward, and the plan for doing it is to find a suboptimal level of prescreening, K_m, under regime m such that the corresponding R&D capital cost $\bar{C}_m$ is strictly lower than $\bar{C}_s^*$, i.e., $\bar{C}_m^*\le\bar{C}_m<\bar{C}_s^*$.

Let K_m be chosen such that $K_m=K_s^*$, i.e., $\frac{\lambda\psi(\theta_m)}{q\theta_m}=\frac{\lambda\psi(\theta_s^*)}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}$. Then $\bar{C}_m-\bar{C}_s^*=\frac{\bar{q}\bar{\theta}_m}{q\theta_m}I_1-\frac{\bar{q}\bar{\theta}_s^*}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{I_3}{1+\rho}=\frac{\bar{q}\bar{\theta}_m}{q\theta_m}\left(I_1-\frac{q\bar{\theta}_s^*}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{\theta_m}{\bar{\theta}_m}\frac{I_3}{1+\rho}\right)$. It follows from reorganizing $\frac{\lambda\psi(\theta_m)}{q\theta_m}=\frac{\lambda\psi(\theta_s^*)}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}$ that $\psi(\theta_m)\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)=\psi(\theta_s^*)\,q\theta_m$. It is useful to solve for θ_m (approximately) by linearizing the left-hand side of the equation around θ_s^*: $\left(\psi(\theta_s^*)+\psi'(\theta_s^*)(\theta_m-\theta_s^*)\right)\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)\approx\psi(\theta_s^*)\,q\theta_m$, which gives rise to $\theta_m\approx\frac{(\psi'(\theta_s^*)\theta_s^*-\psi(\theta_s^*))\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)}{\psi'(\theta_s^*)\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)-\psi(\theta_s^*)q}$. With this result, we can approximate the term $\frac{q\bar{\theta}_s^*}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{\theta_m}{\bar{\theta}_m}$ and derive that following:

$\frac{q\bar{\theta}_s^*}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{\theta_m}{\bar{\theta}_m}\approx q\frac{\psi'(\theta_s^*)\theta_s^*-\psi(\theta_s^*)}{\psi'(\theta_s^*)\theta_s^*-\psi(\theta_s^*)+(\psi'(\theta_s^*)(1-2\theta_s^*)+2\psi(\theta_s^*))\bar{q}}$.

The term $\frac{\psi'(\theta_s^*)\theta_s^*-\psi(\theta_s^*)}{\psi'(\theta_s^*)\theta_s^*-\psi(\theta_s^*)+(\psi'(\theta_s^*)(1-2\theta_s^*)+2\psi(\theta_s^*))\bar{q}}$ turns out to be bounded by 1 from below, which is equivalent to $\psi(\theta_s^*)-\psi'(\theta_s^*)\left(\theta_s^*-\frac{1}{2}\right)\le 0$. To see this, note that by the mean value theorem, $\psi(\theta_s^*)-\psi\left(\frac{1}{2}\right)=\psi'\left(\tilde{\theta}\right)\left(\theta_s^*-\frac{1}{2}\right)$

[36] The first and second parts of this lemma are variations of Dewatripont and Maskin (1995) and Huang and Xu (2003) respectively, with working assumptions of reduced form transaction cost and A-3.

where $\tilde{\theta} \in \left[\frac{1}{2}, \theta_s^*\right]$. It follows that $\psi'\left(\tilde{\theta}\right) \leq \psi'\left(\theta_s^*\right)$ due to the assumption that $\psi''(\cdot) > 0$ (by A-5). Then $\psi\left(\theta_s^*\right) - \psi\left(\frac{1}{2}\right) = \psi'\left(\tilde{\theta}\right)\left(\theta_s^* - \frac{1}{2}\right) \leq \psi'\left(\theta_s^*\right)\left(\theta_s^* - \frac{1}{2}\right)$, which implies that $\psi\left(\theta_s^*\right) - \psi'\left(\theta_s^*\right)\left(\theta_s^* - \frac{1}{2}\right) \leq 0$. The useful implication of this result is that (conditional on the approximation error being sufficiently small) $\frac{q\bar{\theta}_s^*}{q\theta_s^* + \bar{q}\bar{\theta}_s^*} \frac{\theta_m}{\bar{\theta}_m} > q$, hence,
$\bar{C}_m^* - \bar{C}_s^* \leq \bar{C}_m - \bar{C}_s^* \leq \frac{\bar{q}\bar{\theta}_m}{q\theta_m}\left(I_1 - q\frac{I_3}{1+\rho}\right) < 0$ since $q\frac{I_3}{1+\rho} > I_1$ (by A-4). ■

Proof of Proposition 5

Denote by $\tilde{\sigma}(\lambda)$ the solution to $\bar{C}_s^* = \bar{C}_m^*$ for a given $\lambda > 0$. Lemma 4 suggests that $\bar{C}_s^* > \bar{C}_m^*$ as $\sigma = 0$ and $\frac{\partial \bar{C}_m^*}{\partial \sigma} > 0$. It is also easy to verify that $\bar{C}_m^* \to \infty$ as $\sigma \to \infty$. Then it follows that $\tilde{\sigma}(\lambda)$ exists (by the mean-value theorem) and is unique. Consequently, according to Proposition 3 and Lemma 4, $\bar{C}_s^* < \bar{C}_m^*$ and $\zeta^* = s$ if and only if $\sigma > \tilde{\sigma}(\lambda)$. ■

Proof of Proposition 8

Suppose at equilibrium the steady-state level of relative development is $a = \bar{a}$, but there exists $\tilde{a} > \bar{a}$ which can be achieved by an alternative financing regime, denoted by ζ, in the steady state. Hence, $\bar{C}_\zeta^*(\tilde{a};\sigma) = \bar{C}^*(\bar{a};\sigma) = \bar{C}_f^*$ (by Lemma 7) and $\bar{C}_\zeta^*(\bar{a};\sigma) < \bar{C}^*(\bar{a};\sigma)$ (by Proposition 19). Combining the two conditions gives rise to $\bar{C}_\zeta^*(\bar{a};\sigma) < \bar{C}_\zeta^*(\tilde{a};\sigma)$ and applying Proposition 19 again, we have $\bar{a} < \tilde{a}$, which contradicts the assumption that $\tilde{a} > \bar{a}$. ■

Proof of Lemma 12

For each value of I_3 equation $K_s^* = K_m^*$ define a curve $\sigma_k(a; I_3)$ which divides the (a, σ) space, (see Figure 9) and $\sigma_k(a; I_3)$ increases in I_3 (see Lemma 27). Define $\breve{I}_3 \triangleq \inf\{I_3 : \sigma_k(a; I_3) \geq \sigma_{ss}(a) \text{ for all } a \in (0,1)\}$. Since $\sigma_k(a; I_3)$ increases in I_3, $\sigma_k(a; I_3) \to \infty$ as $I_3 \to \infty$ and $\sigma_{ss}(a)$ is finite, it follows that $\breve{I}_3$ is finite. So $\sigma_k(a; I_3) > \sigma_{ss}(a)$ as $I_3 > \breve{I}_3$ (by definition), which implies that $K_s^* > K_m^*$ along the $\sigma_{ss}(a)$ curve for $a > 0$. ■

Proof of Proposition 13

In the steady state, regime m is associated with $\sigma \leq \sigma^*$ and $\bar{a} \geq a^*$, regime s is associated with $\bar{a} = a^*$; if $I_3 > \breve{I}_3$ then $K_m^* < K_s^*$ (by Proposition 12). Consequently, $\theta_m^* < \theta_s^*$ and $q\theta_m^* < q\theta_s^* + \bar{q}\bar{\theta}_s^*$. Finally, the fact $\frac{\partial\left(q\theta + \bar{q}\bar{\theta}\right)}{\partial \theta} < 0$ as $q < \frac{1}{2}$ implies that $q\theta_m^* + \bar{q}\bar{\theta}_m^* > q\theta_s^* + \bar{q}\bar{\theta}_s^*$ as $q < \frac{1}{2}$. ■

Proof of Proposition 14

The algebraic fact $\frac{\partial}{\partial \theta}\frac{\bar{q}\bar{\theta}}{q\theta + \bar{q}\bar{\theta}} = \frac{-\bar{q}\left(q\theta + \bar{q}\bar{\theta}\right) - \bar{q}\bar{\theta}(q - \bar{q})}{\left(q\theta + \bar{q}\bar{\theta}\right)^2} = \frac{-\bar{q}q}{\left(q\theta + \bar{q}\bar{\theta}\right)^2} < 0$ is to be used, together with Lemma 25 and A-5, to derive the following:
$\frac{\partial}{\partial a}\frac{\bar{q}\bar{\theta}_m^*}{q\theta_m^* + \bar{q}\bar{\theta}_m^*} = \left(\frac{\partial}{\partial \lambda}\frac{\bar{q}\bar{\theta}_m^*}{q\theta_m^* + \bar{q}\bar{\theta}_m^*}\right)\lambda'(a) = \left(\frac{\partial}{\partial \theta_m^*}\frac{\bar{q}\bar{\theta}_m^*}{q\theta_m^* + \bar{q}\bar{\theta}_m^*}\right)\frac{\partial \theta_m^*}{\partial \lambda}\lambda'(a) > 0.$ ■

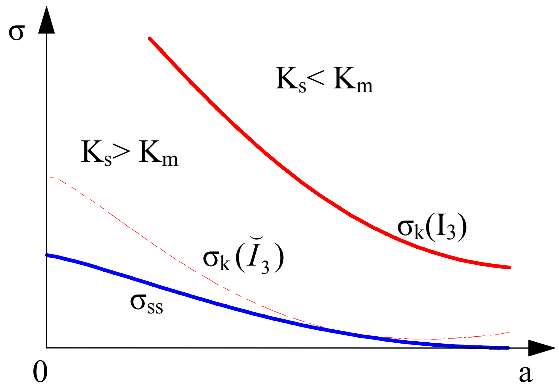

Figure 9: $\sigma_k > \sigma_{ss}$ as $I_3 > \breve{I}_3$

Proof of Proposition 16

We first solve the growth path through the following steps.

Step 1: The general (non-periodical) solution of the linear equation system is $\begin{pmatrix} a_t \\ g_t \end{pmatrix} = \begin{pmatrix} \bar{a} \\ g_f \end{pmatrix} + \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{pmatrix} \begin{pmatrix} r_1^t & 0 \\ 0 & r_2^t \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$.

Step 2: Consider an initial value $\begin{pmatrix} a_0 \\ g_0 \end{pmatrix}$ such that $a_0 < \bar{a}$ and $g_0 = g_f$, which determines that $\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{pmatrix}^{-1} \begin{pmatrix} a_0 - \bar{a} \\ 0 \end{pmatrix}$. It is easy to verify that

$\begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{pmatrix}^{-1} = -\frac{\rho\mathfrak{B}\left(1+g_f\right)}{\bar{a}\sqrt{\eta}} \begin{pmatrix} 1 & -\frac{\bar{a}\left(\frac{1}{2}(\mathfrak{B}-1)+\frac{1}{2}\sqrt{\eta}\right)}{\rho\mathfrak{B}\left(1+g_f\right)} \\ -1 & \frac{\bar{a}\left(\frac{1}{2}(\mathfrak{B}-1)-\frac{1}{2}\sqrt{\eta}\right)}{\rho\mathfrak{B}\left(1+g_f\right)} \end{pmatrix}$, and it follows that

$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = -\frac{\rho\mathfrak{B}\left(1+g_f\right)}{\bar{a}\sqrt{\eta}} \begin{pmatrix} (a_0 - \bar{a}) \\ -(a_0 - \bar{a}) \end{pmatrix}$.

Step 3: The special solution then is

$$\begin{pmatrix} a_t \\ g_t \end{pmatrix} = \begin{pmatrix} \bar{a} \\ g_f \end{pmatrix} - \frac{\rho\mathfrak{B}\left(1+g_f\right)(a_0-\bar{a})}{\bar{a}\sqrt{\eta}} \left(r_1^t\mathbf{v}_1 - r_2^t\mathbf{v}_2\right)$$
$$= \begin{pmatrix} \bar{a} \\ g_f \end{pmatrix} - \frac{\rho\mathfrak{B}\left(1+g_f\right)(a_0-\bar{a})}{\bar{a}\sqrt{\eta}} r_1^t \left(\mathbf{v}_1 - \left(\frac{r_2}{r_1}\right)^t \mathbf{v}_2\right).$$

Since $\left(\frac{r_2}{r_1}\right)^t \approx 0$ as t is sufficiently large, then

$\begin{pmatrix} a_t - \bar{a} \\ g_t - g_f \end{pmatrix} \approx -\frac{\rho\mathfrak{B}\left(1+g_f\right)(a_0-\bar{a})}{\bar{a}\sqrt{\eta}} r_1^t\mathbf{v}_1$ .That is, $\begin{pmatrix} a_t - \bar{a} \\ g_t - g_f \end{pmatrix}$ converges to

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ along the straight line projected by the eigenvector $\mathbf{v}_1$.

In particular, $a_t = \bar{a} - \frac{1}{2}(\bar{a} - a_0) r_1^t \left(\left(\frac{1-\mathfrak{B}}{\sqrt{\eta}} + 1 \right) - \left(\frac{r_2}{r_1} \right)^t \left(\frac{1-\mathfrak{B}}{\sqrt{\eta}} - 1 \right) \right) < \bar{a}$

Approximately, $a_t \approx \bar{a} - \frac{1}{2}(\bar{a} - a_0) r_1^t \left(\frac{1-\mathfrak{B}}{\sqrt{\eta}} + 1 \right)$

since $\left(\frac{r_2}{r_1} \right)^t \approx 0$ as t is sufficiently large.

$\frac{\partial a_t}{\partial \mathfrak{B}} \approx -\frac{1}{2}(\bar{a} - a_0) r_1^{t-1} \left(t \left(\frac{1-\mathfrak{B}}{\sqrt{\eta}} + 1 \right) \underset{(-)}{\frac{\partial r_1}{\partial \mathfrak{B}}} + r_1 \frac{\partial}{\partial \mathfrak{B}} \left(\frac{1-\mathfrak{B}}{\sqrt{\eta}} \right) \right) > 0$

as t is sufficiently large. ■

Derivation of the approximate relation between ω and φ

Recall the definitions of ω and φ : $\omega \triangleq \arccos \left(\frac{\frac{1}{2}+\frac{1}{2}\mathfrak{B}}{\sqrt{\mathfrak{B}(1+\rho)}} \right)$ and $\varphi \triangleq \arccos \left(\frac{\frac{1}{2}\mathfrak{B}-\frac{1}{2}}{\sqrt{\rho \mathfrak{B}}} \right)$

Solve $\sqrt{\mathfrak{B}}$ as a function of φ as follows, $\sqrt{\mathfrak{B}} = \sqrt{\rho}(\cos\varphi) + \sqrt{(\rho(\cos\varphi) + 1)}$therefore,

$\omega = \arccos \frac{1+\rho\cos^2\varphi + \sqrt{\rho}(\cos\varphi)\sqrt{(\rho\cos^2\varphi+1)}}{\left(\sqrt{\rho}\cos\varphi + \sqrt{(\rho\cos^2\varphi+1)}\right)\sqrt{(1+\rho)}}$

Denote $\cos\varphi$ by Φ so that : $\omega = \arccos \frac{1+\rho\Phi^2+\sqrt{\rho}\Phi\sqrt{(\rho\Phi^2+1)}}{\left(\sqrt{\rho}\Phi+\sqrt{(\rho\Phi^2+1)}\right)\sqrt{(1+\rho)}}$

$= \frac{\arccos \frac{1+\rho\Phi^2+\sqrt{\rho}\Phi\sqrt{(\rho\Phi^2+1)}}{\left(\sqrt{\rho}\Phi+\sqrt{(\rho\Phi^2+1)}\right)\sqrt{(1+\rho)}}}{\sqrt{1-\Phi^2}} \sin\varphi$

Linearizing $\frac{\arccos \frac{1+\rho\Phi^2+\sqrt{\rho}\Phi\sqrt{(\rho\Phi^2+1)}}{\left(\sqrt{\rho}\Phi+\sqrt{(\rho\Phi^2+1)}\right)\sqrt{(1+\rho)}}}{\sqrt{1-\Phi^2}}$ around $\Phi = 0$

$\frac{\arccos \frac{1+\rho\Phi^2+\sqrt{\rho}\Phi\sqrt{(\rho\Phi^2+1)}}{\left(\sqrt{\rho}\Phi+\sqrt{(\rho\Phi^2+1)}\right)\sqrt{(1+\rho)}}}{\sqrt{1-\Phi^2}} \approx \arccos \frac{1}{\sqrt{(1+\rho)}} + 0\Phi$

$\omega \approx \left(\arccos \frac{1}{\sqrt{(1+\rho)}} \right) \sin\varphi$

Proof of Proposition 17

Step 1: The general (periodical) solution of the linear equation system is

$\begin{pmatrix} a_t \\ g_t \end{pmatrix} = \begin{pmatrix} \bar{a} \\ g_f \end{pmatrix} + \left(\sqrt{\mathfrak{B}(1+\rho)} \right)^t \begin{pmatrix} \frac{\bar{a}\sqrt{\rho\mathfrak{B}}}{\rho\mathfrak{B}(1+g_f)} e^{-i\varphi} & \frac{\bar{a}\sqrt{\rho\mathfrak{B}}}{\rho\mathfrak{B}(1+g_f)} e^{i\varphi} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{it\omega} & 0 \\ 0 & e^{-it\omega} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}.$

Step 2: Consider an initial value $\begin{pmatrix} a_0 \\ g_0 \end{pmatrix}$ such that $a_0 < \bar{a}$ and $g_0 = g_f$, then we have

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \frac{\bar{a}\sqrt{\rho\mathfrak{B}}}{\rho\mathfrak{B}\left(1+g_f\right)} e^{-i\varphi} & \frac{\bar{a}\sqrt{\rho\mathfrak{B}}}{\rho\mathfrak{B}\left(1+g_f\right)} e^{i\varphi} \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} a_0 - \bar{a} \\ 0 \end{pmatrix} = \frac{\rho\mathfrak{B}\left(1+g_f\right)(a_0-\bar{a})}{2\bar{a}\sqrt{\rho\mathfrak{B}}\sin\varphi} \begin{pmatrix} i \\ -i \end{pmatrix}.$$

Step 3: the special solution is

$$\begin{pmatrix} a_t \\ g_t \end{pmatrix} = \begin{pmatrix} \bar{a} + \left(\sqrt{\mathfrak{B}(1+\rho)}\right)^t \frac{(\bar{a}-a_0)}{\sin\varphi} \sin(t\omega - \varphi) \\ g_f + \left(\sqrt{\mathfrak{B}(1+\rho)}\right)^t \frac{(\bar{a}-a_0)}{\sin\varphi} \frac{\sqrt{\rho\mathfrak{B}}\left(1+g_f\right)}{\bar{a}} \sin(t\omega) \end{pmatrix}.$$ The trajectory is a clockwise spiral around the steady state. The time for the trajectory to pass $a_t = \bar{a}$ from the left for the first time is the solution to the equation:

$t\omega - \varphi = 0 \Rightarrow t = \tau \triangleq \frac{\varphi}{\omega}$, the smaller τ, the faster the speed of catching up. The following proposition reports the result for medium-range values of $\mathfrak{B}$ that an economy with a larger $\mathfrak{B}$ has a faster catching up. Using the approximation: $\omega \approx \left(\arccos \frac{1}{\sqrt{(1+\rho)}}\right) \sin\varphi$, it follows that $\frac{\partial\tau}{\partial\mathfrak{B}} = \frac{\partial}{\partial\mathfrak{B}} \frac{\varphi}{\omega} \approx \frac{\partial}{\partial\mathfrak{B}} \frac{\varphi}{\left(\arccos \frac{1}{\sqrt{(1+\rho)}}\right)\sin\varphi}$

$$= \frac{1}{\left(\arccos \frac{1}{\sqrt{(1+\rho)}}\right)} \underset{(+)}{\left(\frac{\partial}{\partial\mathfrak{B}} \frac{\varphi}{\sin\varphi}\right)} \underset{(-)}{\frac{\partial\varphi}{\partial\mathfrak{B}}} < 0 \text{ as } \mathfrak{B} \in \left(\phi, \overline{\phi}\right). \blacksquare$$

Proof of Proposition 19

In Lemma 25 we establish that $\frac{\partial\bar{C}^*_\zeta}{\partial\lambda} > 0$ for $\zeta = s, m$. Noticing that $K^*_s \triangleq \frac{\lambda\psi(\theta^*_s)}{q\theta^*_s + \bar{q}\bar{\theta}^*_s}$, $K^*_m \triangleq \frac{\lambda\psi(\theta^*_m)}{q\theta^*_m}$, $\lambda = \lambda(a_t)$ and $\bar{C}^* = \min\left(\bar{C}^*_s, \bar{C}^*_m\right)$, it follows that under regime s, $\frac{\partial\bar{C}^*(\lambda(a_t))}{\partial a_t} = \frac{\partial\bar{C}^*_s}{\partial\lambda}\lambda'(a_t) = \frac{K^*_s}{\lambda(a_t)}\lambda'(a_t) > 0$; under regime m, $\frac{\partial\bar{C}^*(\lambda(a_t))}{\partial a_t} = \frac{d\bar{C}^*_m}{d\lambda}\lambda'(a_t) = \frac{K^*_m}{\lambda(a_t)}\lambda'(a_t) > 0$. $\blacksquare$

Proof of Proposition 21

When $I_3 > \breve{I}_3$, for an economy with $\sigma = \sigma^*$, $\mathfrak{B}_s > \mathfrak{B}_m$ in the steady state (Lemma 12).

If $\mathfrak{B}_s \leq \phi$ then $\mathfrak{B}_m < \phi$. By Proposition 16 $\frac{\partial a_t}{\partial\mathfrak{B}} > 0$ as $\mathfrak{B} < \phi$ for sufficiently large t, which implies that $a_{t,s} > a_{t,m}$, i.e., the speed to converge to the steady state is faster under regime s, where $a_{t,s}$ and $a_{t,h}$ are the values of a_t under regime s and regime m.

If $\mathfrak{B}_s \in \left(\phi, \overline{\phi}\right)$ then $\mathfrak{B}_m < \overline{\phi}$. By Proposition 17 $\frac{\partial\tau}{\partial\mathfrak{B}} < 0$ as $\mathfrak{B} \in \left(\phi, \overline{\phi}\right)$; therefore $\tau_s < \tau_m$ if τ_m is finite, where τ_s and τ_m are the values of τ under regime s and regime m respectively. As a result, $a_{t,s}$ reaches a^* earlier than $a_{t,m}$.

It is possible that $\mathfrak{B}_s > \phi > \mathfrak{B}_m$ but not the reverse. Consequently, by Proposition 15 the growth rate is more likely to be cyclical under regime s than under regime m. $\blacksquare$

Proof of Proposition 22

Define $\underline{a}$ such that $\sigma_c(\underline{a}) = \sigma \leq \sigma^*$ and $\underline{a} \leq a^*$. By Proposition 5 regime s is chosen when $a_0 \leq a_t < \underline{a}$ and regime m is chosen when $\underline{a} \leq a_t < \bar{a}$ where $\bar{a} \geq a^*$ by Proposition 11. ■

Lemma 25 *For $\lambda > 0$ there is a unique interior solution, $\theta_\zeta^* \in (1/2, 1)$, to the minimization problem: $\min_{\theta_\zeta} \bar{C}_\zeta$ for $\zeta = s, m$, with the following comparative statics: $\frac{\partial \theta_\zeta^*}{\partial \lambda} < 0$, $\frac{\partial \bar{C}_\zeta^*}{\partial \lambda} > 0$ and $\frac{\partial^2 \bar{C}_\zeta^*}{\partial \lambda^2} < 0$.*

Proof. First consider the case: $\zeta = m$. $\bar{C}_m = \frac{\lambda\psi(\theta_m)}{q\theta_m} + \frac{q\theta_m + \bar{q}\bar{\theta}_m}{q\theta_m}(\sigma F + I_1) + I_2 + \frac{\bar{w}}{1+\rho}$ is twice continuously differentiable in θ_m. At the point $\theta_m = 1/2$, $\bar{C}_m$ is finite; the derivative $\frac{\partial \bar{C}_m}{\partial \theta_m} = 2\frac{\lambda\psi'\left(\frac{1}{2}\right)}{q} - 4\frac{\bar{q}}{q}(I_1 + \sigma F)$ is negative since $\lambda \leq \frac{2\bar{q}I_1}{\psi'\left(\frac{1}{2}\right)}$ (by assumption A-5). If the solution to $\frac{\partial \bar{C}_m}{\partial \theta_m} = 0$ does not exist, it must be the case that $\frac{\partial \bar{C}_m}{\partial \theta_m} < 0$ for all $\theta_m \leq [1/2, 1]$ and hence, $\bar{C}_m$ is smaller at $\theta_m = 1$ than $\theta_m = 1/2$; but this contradicts the fact that $\bar{C}_m$ is positive infinity at $\theta_m = 1$. So we conclude that an interior solution to $\frac{\partial \bar{C}_m}{\partial \theta_m} = 0$ must exist; let it be denoted by θ_m^*. At the point $\theta_m = \theta_m^*$, using the equation implied by $\frac{\partial \bar{C}_m}{\partial \theta_m} = 0$, we can write down the second order derivative in the simple form: $\frac{\partial^2 \bar{C}_m^*}{\partial \theta_m^2} = \frac{\lambda\psi''(\theta_m^*)\theta_m^{*2}}{q\theta_m^{*3}}$, which is positive. Thereby, we infer that the interior critical point (solution to $\frac{\partial \bar{C}_m}{\partial \theta_m} = 0$) is unique (otherwise, at least one of the solutions is not a local minimum, which contradicts the fact that $\frac{\partial^2 \bar{C}_m}{\partial \theta_m^2} > 0$ at each of the solutions) and is the global minimum.

Now we focus attention to the point $\theta = \theta_m^*$. Differentiating the equation $\frac{\partial \bar{C}_m^*}{\partial \theta} = 0$ w.r.t. λ and reorganizing, it follows that $\frac{\partial \theta_m^*}{\partial \lambda} = \frac{\psi(\theta_m^*) - \psi'(\theta_m^*)\theta_m^*}{\lambda\psi''(\theta_m^*)\theta_m^*} < 0$. Now it is straightforward to derive the following comparative statics: $\frac{\partial \bar{C}_m^*}{\partial \lambda} = \frac{\partial \bar{C}_m}{\partial \lambda} + \frac{\partial \bar{C}_m}{\partial \theta_m}\frac{d\theta_m^*}{d\lambda} = \frac{\partial \bar{C}_m}{\partial \lambda} = \frac{\psi(\theta_m^*)}{q\theta_m^*} > 0$ and $\frac{\partial^2 \bar{C}_m^*}{\partial \lambda^2} = \frac{\partial}{\partial \lambda}\left(\frac{\psi(\theta_m^*)}{q\theta_m^*}\right) = \frac{d}{d\theta_m^*}\left(\frac{\psi(\theta_m^*)}{q\theta_m^*}\right)\frac{\partial \theta_m^*}{\partial \lambda} < 0$.

The same argument goes through for the case: $\zeta = s$. Some particular points we need to make are that $\lambda < \frac{2q\bar{q}I_3}{\psi'\left(\frac{1}{2}\right)(1+\rho)}$ is satisfied as $(1+\rho)I_3 > \frac{I_1}{q}$ (by A-4)) and $\lambda \leq \frac{2\bar{q}I_1}{\psi'\left(\frac{1}{2}\right)}$ (by A-5); that $\frac{\partial \bar{C}_s^*}{\partial \lambda} = \frac{\partial \bar{C}_s}{\partial \lambda} = \frac{\psi(\theta_s^*)}{q\theta_s^* + \bar{q}\bar{\theta}_s^*}$. ■

Lemma 26 *$\frac{\partial \theta_s^*}{\partial I_1} = 0$, $\frac{\partial \theta_s^*}{\partial I_3} > 0$, $\frac{\partial K_s^*}{\partial I_1} = 0$, $\frac{\partial K_s^*}{\partial I_3} > 0$ and $\frac{\partial \bar{C}_s^*}{\partial I_3} > 0$ as $\lambda > 0$; $\frac{\partial \theta_m^*}{\partial I_1} > 0$, $\frac{\partial \theta_m^*}{\partial I_3} = 0$, $\frac{\partial K_m^*}{\partial I_1} > 0$, $\frac{\partial K_m^*}{\partial I_3} = 0$ as $\lambda > 0$.*

Proof. For the case of regime s, the starting point for deriving the set of comparative statics is the FOC: $\frac{\partial \bar{C}_s}{\partial \theta} = 0$, or more precisely, $\frac{\lambda\psi'(\theta_s^*)\left(q\theta_s^* + \bar{q}\bar{\theta}_s^*\right) - \lambda\psi(\theta_s^*)(q-\bar{q})}{q\bar{q}} =$

$\frac{I_3}{1+\rho}$. Differentiating the both sides of the equation w.r.t. I_1 and I_3 respectively and reorganizing, it follows that
$\frac{\lambda\psi''(\theta_s^*)\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)}{q\bar{q}}\frac{\partial\theta_s^*}{\partial I_1}=0\Rightarrow\frac{\partial\theta_s^*}{\partial I_1}=0;$
$\frac{\lambda\psi''(\theta_s^*)\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)}{q\bar{q}}\frac{\partial\theta_s^*}{\partial I_3}=\frac{1}{1+\rho}\Rightarrow\frac{\partial\theta_s^*}{\partial I_3}=\frac{q\bar{q}}{(1+\rho)\lambda\psi''(\theta_s^*)\left(q\theta_s^*+\bar{q}\bar{\theta}_s^*\right)}>0.$
Also $\frac{\partial K_s^*}{\partial I_1}=\frac{\partial}{\partial I_1}\frac{\lambda\psi(\theta_s^*)}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}=\frac{\partial}{\partial\theta_s^*}\frac{\lambda\psi(\theta_s^*)}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{\partial\theta_s^*}{\partial I_1}=0,$
$\frac{\partial K_s^*}{\partial I_3}=\frac{\partial}{\partial I_3}\frac{\lambda\psi(\theta_s^*)}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}=\frac{\partial}{\partial\theta_s^*}\frac{\lambda\psi(\theta_s^*)}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{\partial\theta_s^*}{\partial I_3}>0$ and $\frac{d\bar{C}_s^*}{dI_3}=\frac{\partial\bar{C}_s^*}{\partial I_3}=\frac{\bar{q}\bar{\theta}_s^*}{q\theta_s^*+\bar{q}\bar{\theta}_s^*}\frac{1}{1+\rho}>0$

For the case of regime m, the FOC, $\frac{\partial\bar{C}_m^*}{\partial\theta}=0$ implies that $\frac{\lambda\psi'(\theta_m^*)\theta_m^*-\lambda\psi(\theta_m^*)}{\bar{q}}=I_1+\sigma F.$
Differentiating the both sides of the equation w.r.t. I_1 and I_3 respectively and reorganizing, it follows that
$\frac{\lambda\psi''(\theta_m^*)\theta_m^*}{\bar{q}}\frac{\partial\theta_m^*}{\partial I_1}=1\Rightarrow\frac{\partial\theta_m^*}{\partial I_1}=\frac{\bar{q}}{\lambda\psi''(\theta_m^*)\theta_m^*}>0$ and
$\frac{\lambda\psi''(\theta_m^*)\theta_m^*}{\bar{q}}\frac{\partial_m^*}{\partial I_3}=0\Rightarrow\frac{\partial\theta_m^*}{\partial I_3}=0.$ Consequently,
$\frac{\partial K_m^*}{\partial I_1}=\frac{\partial}{\partial I_1}\frac{\lambda\psi(\theta_m^*)}{q\theta_m^*}=\frac{\partial}{\partial\theta_m^*}\frac{\lambda\psi(\theta_m^*)}{q\theta_m^*}\frac{\partial\theta_m^*}{\partial I_1}>0$
and $\frac{\partial K_m^*}{\partial I_3}=\frac{\partial}{\partial I_3}\frac{\lambda\psi(\theta_m^*)}{q\theta_m^*}=\frac{\partial}{\partial\theta_m^*}\frac{\lambda\psi(\theta_m^*)}{q\theta_m^*}\frac{\partial\theta_m^*}{\partial I_3}=0.$ ■

Lemma 27 $\frac{\partial\sigma_k}{\partial I_3}>0$ *and* $\frac{\partial\sigma_{ss}}{\partial I_3}=0$.

Proof. Differentiating the equation $K_m^*=K_s^*$ w.r.t. to σ and I_3 and reorganizing, it results in $\frac{\partial\sigma_k}{\partial I_3}=\frac{\frac{\partial K_s^*}{\partial I_3}}{\frac{\partial K_m^*}{\partial\sigma}}>0$ since $\frac{\partial K_m^*}{\partial\sigma}>0$ and $\frac{\partial K_s^*}{\partial I_3}>0$ (by Lemma 4 and 26). Also, since I_3 does affect regime m at all, it is obvious that $\frac{\partial\sigma_{ss}}{\partial I_3}=0$. ■

Lemma 28 *The steady state* $(0,\tilde{g})$ *where* $\tilde{g}\triangleq\frac{\rho(1+\rho)}{\alpha\bar{w}}\left(\bar{C}^*\left(\lambda\left(\bar{a}\right)\right)-\bar{C}^*\left(\lambda\left(0\right)\right)\right)+g_f$*, is unstable.*

Proof. Linearizing the growth equation (24) around $(0,\tilde{g})$ results in
$$\begin{pmatrix} a_{t+1} \\ g_{t+1}-\tilde{g}\end{pmatrix}\approx\begin{pmatrix}\mathfrak{B}_1 & 0\\ -\mathfrak{B}_2 & 0\end{pmatrix}\begin{pmatrix} a_t \\ g_t-\left(\frac{\rho(1+\rho)}{\alpha\bar{w}}\left(\bar{C}^*\left(\lambda\left(\bar{a}\right)\right)-\bar{C}^*\left(\lambda\left(0\right)\right)\right)+g_f\right)\end{pmatrix},$$
where $\mathfrak{B}_1=1+\frac{\frac{\rho(1+\rho)}{\alpha\bar{w}}\left(\bar{C}^*(\lambda(\bar{a}))-\bar{C}^*(\lambda(0))\right)}{1+g_f}$ and $\mathfrak{B}_2=\frac{\rho(1+\rho)}{\alpha\bar{w}}\frac{\partial\bar{C}^*(\lambda(0))}{\partial\lambda}\left(\lambda'\left(0\right)\right).$
The transition matrix $\begin{pmatrix}\mathfrak{B}_1 & 0\\ -\mathfrak{B}_2 & 0\end{pmatrix}$ has eigenvalues: $\mathfrak{B}_1, 0$. Since $\mathfrak{B}_1>1$, the steady state $(0,\tilde{g})$ is unstable. ■

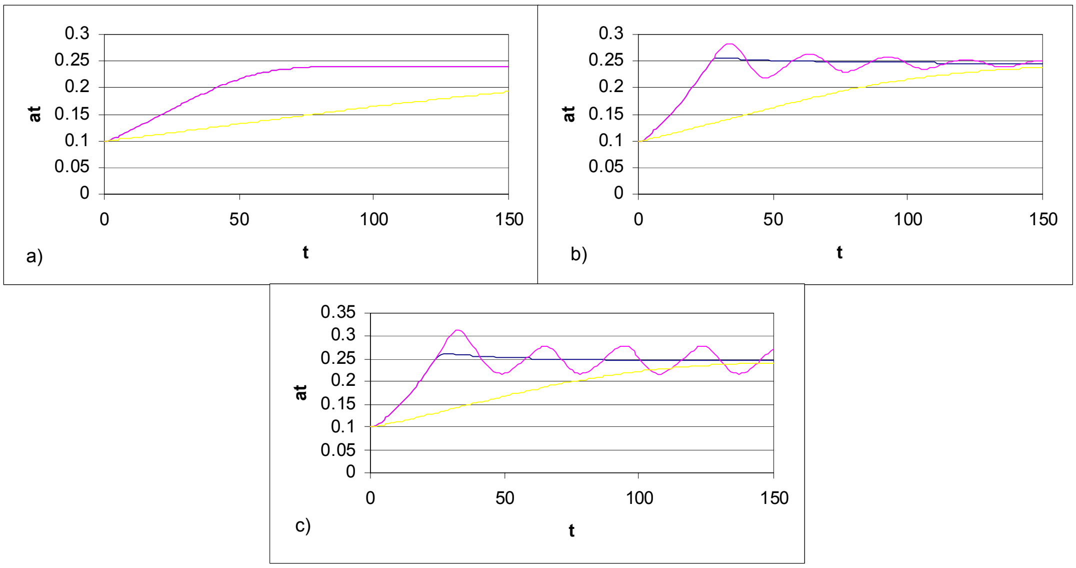

Figure 10:

C Appendix C

Example 29 *See panels a, b, and c of Fig. 10, which correspond to the following three cases:*

a) $\mathfrak{B}_m(\sigma^*) < \mathfrak{B}_s < \phi$*: both regimes have a monotonic catch up; regime* s *is faster in catching up and is the optimal regime.*

b) $\mathfrak{B}_m(\sigma^*)\,\phi < \mathfrak{B}_s < \frac{1}{1+\rho}$*: regime* s *catches up faster than regime* m*, with spirals converging to the steady state; the optimal regime starts with regime* s *and switches to regime* m *before the cycles start.*

c) $\mathfrak{B}_m < \phi < \frac{1}{1+\rho} < \mathfrak{B}_s < \overline{\phi}$*: regime* s *catches up faster at first and converges to limit cycles. The optimal regime starts with regime* s *and switches to regime* m *before the cycles start.*

The following are the common parameter values and derived values for $\frac{1}{1+\rho}$, ϕ and $\overline{\phi}$ for all the above simulations.

α	I_1	I_2	I_3	$\lambda(a)$	g_f	ρ	$\frac{1}{1+\rho}$	ϕ	$\overline{\phi}$
0.3	0.1	0.9	2	$\gamma_0 a^{5/2}$	0.03	0.05	0.952	0.642	1.558

The following are the values of parameters q, γ_0 and δL and the calculated corresponding values of the variables for the above three simulated cases.

	q	γ_0	δL	a^*	σ^*	$B_m(\sigma^*)$	B_s
a)	0.4	0.06	0.398	0.241	.0996	0.3009	0.611
b)	0.3	0.07	0.4125	0.246	.144	0.3756	0.9255
c)	0.29	0.071	0.4415	0.246	.150	0.400	0.968

Example 23 All the simulations share the same following parameters:

α	I_1	I_2	I_3	$\lambda(a)$	g_f	ρ	q	δL
0.3	0.1	0.9	2	$0.071a^{5/2}$	0.03	0.05	0.29	0.415

The corresponding variable values are as follows:

a^*	σ^*	$B_m(\sigma^*)$	B_s
0.246	0.150	0.400	0.968

Interpretation of panels a-f of Figure 11

a) Development levels under an optimal regime.

b) Growth rates under an optimal regime.

c) Development level with a regime that changes after the 1st overshooting.

d) Growth rate with a regime that changes after the 1st overshooting.

e) Development level with a regime that changes at the bottom of a_t.

f) Growth rates with a regime that changes at the end of negative overshooting.

Example 30 *The key feature of the growth pattern is the possible clockwise spiral, which is the joint product of the convergence effect and the growth inertia effect. In the absence of the growth inertia effect, the growth path tends to converge to the steady state (a^*, g_f) monotonically; but the growth inertia effect tends to prevent g_t from converging to g_f as a_t approaches a^* and causes possible overshooting.*
All the simulations share the same following parameters:

α	I_1	I_2	I_3	$\lambda(a)$	g_f	ρ	q	δL
0.3	0.3	0.9	1.7	$0.18a^{5/2}$	0.03	0.05	0.4	0.485

The corresponding variable values are as follows:

σ	$\bar{a}$	$\mathfrak{B}_m$	σ^*	a^*	$\mathfrak{B}_s$	a_0	g_0
0.05	0.55	0.68	0.169	0.257	0.94	0.15	0.03

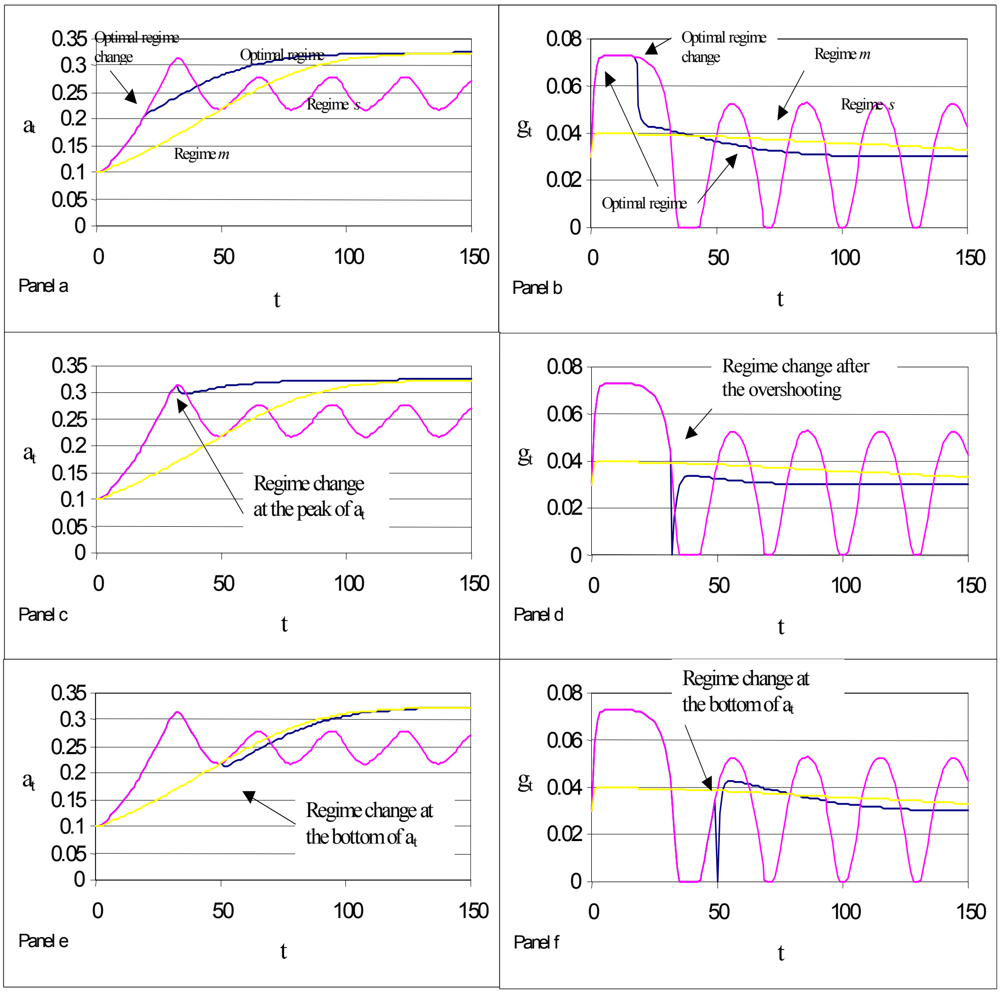

Figure 11: Regime change and development