International Diversification with Factor Funds

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1. Introduction

Classic studies such as Grubel (1968) and Levy and Sarnat (1970) establish a strong case for international diversification using country market indices. In recent years, as international capital markets have become more integrated, stock market correlations have risen, diminishing the potential gains from the country market index-based diversification strategy.[1] Against this backdrop, we propose a new international portfolio diversification strategy involving size (SMB), book-to-market (HML), and momentum (MOM) factor mimicking funds.[2] We examine the extent to which the additional efficiency gains can be achieved by holding factor funds in addition to country market indices. In addition, we discuss various factor fund investment strategies and the associated efficiency gains achievable under investment constraints and market frictions.

The proposed factor diversification strategy is motivated by the empirical success of international asset pricing models incorporating SMB, HML, and MOM factors.[3] In particular, Fama and French (1993, 1996, and 1998) show that a three-factor asset pricing model yields a substantial empirical success in describing stock returns and hypothesize that SMB and HML factors can represent two state-variable mimicking portfolios in ICAPM or additional risk factors in APT (Ross (1976)). Carhart (1997) and others further provide evidence that the MOM factor has additional explanatory power for stock returns beyond Fama-French factors do. Although the interpretation of these factors remains a contentious issue,[4] Vassalou (2003) and Petkova (2006) show that SMB and HML factors contain significant information about changes in future GDP growth, term premium, default premium, and dividend yield in the U.S., lending support to the risk-based explanation of Fama-French factors. Liew and Vassalou (2000) further provide international evidence that SMB and HML but not MOM contain significant information about future GDP growth for the 10 countries they study.

A direct portfolio implication from these factor asset pricing models is that investors would hold factor mimicking funds to enhance portfolio efficiency. However, in choosing among various versions of

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[1] Longin and Solnik (1995), Solnik and Roulet (2000), and Goetzmann, Li, and Rouwenhorst (2005) provide evidence on the rising trend in international stock market correlations. Bekaert, Hodrick, and Zhang (2009) find evidence that return correlation has been rising in European markets, though not for other regions.

[2] We use the SMB, HML, and MOM factor (mimicking) funds to refer to zero investment portfolios which take long positions in, respectively, large-cap, value, and winner stocks, and short positions in small-cap, growth, and loser stocks.

[3] The link between asset pricing theories and investors' portfolio choice problems has been well established by existing literature. See, e.g., Solnik (1974), Huberman and Kandel (1987), De Roon and Nijman (2001), and De Roon, Nijman, and Werker (2003). In particular, it has been shown that for a single- or multi-index model, a statistically insignificant regression intercept (Jansen's alpha) would imply that the benchmark index portfolio(s) is mean-variance efficient with respect to the testing assets. We offer more detailed discussion in Section 3.1.

[4] There exist alternative views to the risk-based explanation of Fama-French factors. For example, Lakonishok, Shleifer, and Vishny (1994) favor a mispricing-based explanation of the value effect. Daniel and Titman (1997) suggest that stocks' characteristics, rather than risks, are priced in the cross section of average returns. A recent paper by Lewellen, Nagel, and Shanken (2009) points out the econometric problems in existing empirical asset pricing tests and argues that results from these tests can be misleading.

factor funds, the issue of whether these factors are global or country-specific becomes important. Presumably, if factors are mainly driven by common global forces, there ought to be no discernable difference for a U.S. investor to engage in either a pure U.S. factor diversification strategy, i.e., holding only the U.S. domestic (SMB, HML, and MOM) factor funds and the U.S. market index, or a global factor diversification strategy, i.e., holding global factor funds and the global market index.[5] Likewise, a global factor diversification strategy should also perform similarly to a local factor diversification strategy, which involves holding factor funds and market indices from individual countries. On the contrary, if factors contain substantial country-specific components, a local version of factor funds would allow the investor to expand her investment opportunity set significantly beyond what can be achieved by the U.S. or global factor funds alone.

So far, the issue of whether SMB, HML, and MOM are global or local is still unsettled in the asset pricing literature. On the one hand, Griffin (2002) argues that a local version of the Fama-French three-factor model describes international stock returns better than the global version of the model. Griffin, Ji, and Martin (2003) also argue that the momentum factor contains a significant country-specific component. On the other hand, Hou, Karolyi, and Kho (2007) advocate a model of global factors, which they show can describe the average returns of a wide range of international stock portfolios. By directly investigating the relative efficiency gains/losses from the global versus local versions of factor funds, our results not only bear direct portfolio implications but also shed new light on the recent debate in the asset pricing literature.

In our empirical analysis, we examine the aforementioned portfolio allocation problem from the perspective of a U.S. (or dollar-based) investor who currently holds only country market indices but desires to augment her investment with factor funds, domestically or internationally. We include ten developed countries in our study with a geographical balance: Two are from North America, four from Europe, and four from Asia. Our sample period spans January 1981 through December 2008. There is a broad understanding that investors would not have faced major barriers to international investments in these countries during this period. In addition, the growing variety of investment products available in these markets makes the factor investment approach proposed in this paper increasingly accessible to retail investors.[6,7]

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[5] A global SMB (HML, MOM) factor fund refers to a country market value-weighted average of SMB (HML, MOM) funds from individual countries in our sample. The global market index is defined analogously.

[6] Our informal conversations with institutional fund managers confirm that factor funds are indeed feasible in most developed markets. Many hedge funds and some mutual funds are known to actively take long and/or short positions. Particularly, long/short strategies, such as 130/30 (i.e., 130% long and 30% short positions) and 120/20, have gained increasing popularity among mutual fund managers (*Wall Street Journal*, June 6, 2007, p. C11). Currently there also exist several style funds accessible to retail investors. According to the CRSP and International Thomson Reuter fund databases, there are 493 (277) large- (small-) cap and 292 (492) value (growth) mutual funds in the U.S., and

Overall, our results show that investors benefit significantly from investing in factor funds. Domestically, the Sharpe ratio for the optimal portfolio comprising the U.S. market index and the U.S. SMB, HML, and MOM factor funds is 0.33, triple the Sharpe ratio (0.10) for the U.S. market index alone and more than double the Sharpe ratio (0.14) for the optimal portfolio comprising the ten country market indices during our sample period. Furthermore, the domestic factor diversification strategy works well in countries outside the U.S. For the nine non-U.S. countries in our sample, the domestic factor diversification strategy on average yields a Sharpe ratio of 0.22 compared to 0.08 of their respective domestic country market index.

Furthermore, if the investor expands her portfolio to include local factor funds, the optimal portfolio yields a Sharpe ratio of 0.69 during our sample period, which exceeds the Sharpe ratios of 0.14, 0.33, and 0.44 obtained from the strategies using, respectively, the ten country market indices, U.S. domestic factor funds, and global factor funds. The evidence strongly rejects the intersection hypothesis that the efficient frontier spanned by the ten country market indices plus the three factor funds from each of the ten sample countries intersects the one spanned by the assets employed in the competing diversification strategies.[8] These results are robust to various constraints on portfolio weights.

Among the three classes of factor funds, the HML factor funds receive the most significant weights in the optimal portfolios, regardless of the factor diversification strategy – domestic, global, or local. In particular, when we consider each class of factors separately for the augmented diversification, the optimal portfolio of ten country market indices plus the local HML (SMB, MOM) funds yields a Sharpe ratio of 0.48 (0.34, 0.30) during our sample period. We provide some evidence as to why HML offers the most significant diversification benefits among the three classes of funds.

Specifically, we examine the link between factor funds and news on important economic variables, including GDP growth, consumption growth, dividend yield, and term premium, for our 10 sample countries following the methodology employed by Vassalou (2003), which involves constructing economic mimicking portfolios (EMPs). We find international evidence that factor funds, especially SMB and HML, are significantly related to news on these economic variables. Further, GDP growth appears to be the most important economic news for the purpose of international portfolio investment because an

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142 (233) large- (small-) cap and 101 (450) value (growth) funds in the other 9 countries in our sample as of 2008. There are also around 200 ETFs featuring large/small/value/growth styles in these 10 countries. Furthermore, momentum investing has garnered increasing interest among fund managers, which is evident in a recent *Wall Street Journal* article, titled 'momentum funds for small investors' (July 14, 2009, p. C9). For example, AQR offers Momentum Fund, Small Cap Momentum Fund, and International Momentum Fund.

[7] Bailey, Kumar, and Ng (2008) examine foreign investment behavior of U.S. individual investors. They show that about 49 percent of U.S. investors who trade domestic equities also trade foreign equities. Interestingly, around 38 percent of investors invest abroad via international mutual funds.

[8] As we discuss in Section 3.1, the "intersection hypothesis" posits that the efficient frontier spanned by the benchmark assets and the one augmented by the additional assets intersect at the tangency point. A failure to reject the hypothesis would suggest that the benchmark assets are mean-variance efficient with respect to the new assets.

investment strategy involving the EMPs of GDP growth yields the highest Sharpe ratio among all the economic variables we examine. We conjecture that the relatively strong association of HML with GDP growth may contribute to the outperformance of the HML funds compared to other factor funds. Lastly, our result suggests that factor funds may contain information beyond news in these economic variables, as indicated by the higher Sharpe ratio of holding local factor funds as opposed to EMPs.

We also perform out-of-sample tests on our key findings. Using a ten-year rolling window, we show that local factor funds are not spanned by the benchmark assets out-of-sample. This holds for the case of international diversification across the ten countries as well as for domestic diversification within each country. In addition, we compare the local factor investment strategy with some simple trading rules which are of particular interest to practitioners. We show that the former still outperform the latter both in- and out-of-sample and for different subperiods.[9]

We further conduct several additional analyses. Specifically, we impose (additional) transaction costs on the factor funds but not on the market indices. The local factor strategy continues to deliver the most significant gains among all diversification strategies. We find that the added gains from the augmented optimal portfolio with local factor funds remain significant at the 5% level as long as the transaction costs do not exceed 4% to 6% per annum over our sample period, depending on the benchmark portfolio chosen. In addition, the optimal international portfolio continues to include factor funds until the transaction costs exceed 13% per annum. Further, we consider the extreme case in which short sales are not allowed. In this case, investors may only take long positions on the long and short components of the factor funds. Under this constraint, the augmented optimal portfolio is dominated by value funds. Comparing the international country market index diversification strategy with the strategy involving the components of the local factors, the intersection hypothesis is again soundly rejected. In addition, we show that over time the benefits from the local factor diversification strategy have not declined compared to the benchmark diversification strategies.

Our study relates to the literature of international portfolio diversification beyond developed market indices. A large body of the recent literature has been devoted to the search for asset classes that have relatively low correlations with the broad-based developed country market indices and/or among themselves. The existing approaches proposed include, among others, an international diversification approach with either emerging market stocks (Harvey (1995) and Bekaert and Urias (1996)) or developed markets' small-cap stocks (Eun, Huang, and Lai (2008)), and an industry-based diversification approach (Roll (1992); Cavaglia, Brightman, and Aked (2000); and Carrieri, Errunza, and Sarkissian (2004)).

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[9] Specifically, we examine a trading rule in which an investor invests in country market indices as well as local factor funds but restricting the portfolio weights to be the same across countries. Further, we examine a trading rule in which an investor maintains a 50-50% allocation between HML and MOM funds, but do not invest in SMB funds in part due to empirical evidence on the diminishing size premium. The detailed results are discussed in Section 4.2.

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Emerging market stocks and developed markets' small-cap stocks are shown to be less integrated with the world market, thereby having relatively low correlations with developed market indices. Industry diversification can be more effective than a country market index-based strategy if stock returns are driven more by industry rather than country factors. The existing evidence for the relative importance of country vs. industry factors in stock returns, however, is mixed.[10] Although each of these strategies may yield international diversification benefits, the choice of asset classes can be arbitrary, and hence the resulting portfolios may not be mean-variance efficient. Unlike the previous studies mentioned above, our approach is motivated by the empirical success of factor pricing models in explaining the cross-section of stock returns and the direct link between asset pricing theories/tests and investors' portfolio choice problem. We thus contribute to the literature by proposing a parsimonious set of funds that can systematically enhance the mean-variance efficiency of international, as well as domestic, investments.

The rest of the paper is organized as follows. Section 2 describes the data and sample statistics. Section 3 discusses the test methodology and presents the main results on the efficiency gains from international allocation strategies with local factor funds. In Section 4, we conduct out-of-sample tests and a series of robustness checks that include the effects of transaction costs, short-sales constraints, and other portfolio constraints on the gains from factor diversification strategies. The time trends of the gains are also examined. Section 5 discusses the link between factor funds and news on important economic variables. Section 6 provides concluding remarks.

2. Data and Sample Statistics

Our data include monthly stock returns, year-end book-to-market ratios for exchange-listed companies, and monthly stock market indices from ten major developed countries – Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the United Kingdom, and the United States – from January 1981 to December 2008.[11] We limit our analysis to this sample period because book value information from Datastream, our main data source for international stocks, is generally not available prior to 1981. In addition, there is a broad understanding among researchers that investors

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[10] Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998) argue that country factors outweigh industry factors in explaining stock returns, while Roll (1992) argues otherwise. Cavaglia, Brightman, and Aked (2000) and Baca, Garbe, and Weiss (2000) document a rising importance of industry factors relative to country factors, while Bekaert, Hodrick, and Zhang (2009) argue that the phenomenon is short-lived and largely confined to the 1990s. Examining the issue from a portfolio perspective, Eiling, Gerard, and de Roon (2006) and Gerard, Hillion, de Roon, and Eiling (2006) show that the two strategies generate indistinguishable Sharpe ratios. Hou, Karolyi, and Kho (2007) document in their asset pricing tests that their proposed factors (i.e., global market, momentum. and cash flow/price factors) can explain the average returns of industry-based portfolios from the 49 countries they study, implying no efficiency gains from adding industry portfolios to these global factor funds.

[11] We do not extend our sample period to prior to the 1980s partly because formal investment barriers were still in place in some developed countries during that period and partly because the book value data were not available for a wide range of stocks from Datastream until the end of 1980.

would not have faced major barriers to international investments in these developed markets during this sample period. We use the NYSE/AMEX/Nasdaq composite index from CRSP to proxy for the U.S. market index and employ MSCI stock market indices from Datastream to proxy for the market indices of the remaining countries. The U.S. T-Bill rate, which proxies for the risk-free interest rate in our portfolio analysis, also comes from CRSP. Our firm-level data for the U.S. are from CRSP and those for international companies are from Datastream. The U.S. data include all common stocks traded on NYSE/AMEX/Nasdaq, while the international data include all local firms from each of the nine countries for which Datastream provides the necessary data during our sample period. For each country, we consider both active and inactive stock files to avoid survivorship bias.[12] Both the factor and market index returns are computed in U.S. dollar terms. For simplicity, we assume the investor is not concerned with foreign exchange risk and hence does not hold currency forwards. In fact, the added foreign exchange exposure from holding factor funds is minimal due to the long/short nature of the funds.

In forming local SMB and HML factor funds, we follow the method of Fama and French (1998).[13] Specifically, to form SMB factor funds, we rank all our sample firms in each country in descending order based on their market capitalization at the end of each year. We then form a "large-cap fund" with the top 30% of the stocks and a "small-cap fund" with the bottom 30% of the stocks. We use the relative market value of each stock to determine its weight in the fund. We then calculate the monthly (value-weighted) returns for each fund in terms of U.S. dollars. The SMB factor fund involves taking a short position in the large-cap fund and a long position in the small-cap fund. We calculate the return of the SMB factor fund by subtracting the return of the large-cap fund from that of the small-cap fund. HML factor funds are constructed in a similar manner. That is, we rank all stocks in each country in descending order by their book-to-market ratios at the end of each year. We then form a "value fund" with the top 30% of the stocks and a "growth fund" with the bottom 30% of the stocks. The HML factor fund involves taking a long position in the value fund and a short position in the growth fund. The return of the HML fund is computed as the return of the value fund minus the return of the growth fund.

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[12] For international data obtained from Datastream, we carefully screen and exclude non-common stocks such as preferred stocks, REITs, closed-end funds, warrants, etc. We also exclude those firms that are incorporated outside their home countries as well as those indicated by Datastream as duplicates. To filter out the recording errors in Datastream, we set monthly holding-period returns greater than 300% as missing values. In addition, in view of Datastream's practice to set the return index to a constant once a stock ceases trading, we also treat those constant values as missing values in the inactive file.

[13] In particular, Fama and French (1998, pages 1977-1981 and 1994-1996) form international value, growth, small, and big portfolios at the end of each calendar year, with returns calculated for the following year. They also form the U.S. portfolios at the end of each calendar year, but the portfolio returns are calculated from July to June of the next year due to accounting considerations. We construct our local SMB and HML factors in the same way as Fama and French (1998). For the U.S. factors, we use the data posted on Kenneth R. French's website.

To form the MOM factor fund, at the end of June and December each year we rank all the stocks in each country in descending order by their cumulative returns over the prior two to 12 months.[14] We then form a value-weighted "winner fund" with the top 30% of the stocks and "loser fund" with the bottom 30% of the stocks. The MOM factor fund involves taking a long position in the winner fund and a short position in the loser fund. The return of the MOM factor fund is computed as the return of the winner fund minus the return of the loser fund.

The SMB and HML factors are updated once a year, while the MOM factor is updated every six months. We generate separate time series of monthly returns, in U.S. dollar terms, on the SMB, HML, and MOM factor funds for each sample country over the 1981-2008 period. In later tests, we also consider a "global" version of factor funds as alternative benchmark portfolios. Employing a similar approach used by Fama and French (1998) in constructing the global HML factor for their 13 sample countries, we form the global market index and the global SMB, HML, and MOM factors for our ten sample countries. We use the relative stock market value of each country to determine its weight in the global portfolio. Our estimation of a country's stock market value is taken from the monthly market capitalization value of Datastream's global country market index. We do not include the global cash flow/price factor suggested by Hou et al. (2007) in our analysis partly because they show that the HML factor is a close contender of the cash/price factor in their empirical asset pricing tests and that the two factors are highly correlated, and partly because the cash flow information for international stocks is only available from Worldscope, whose coverage is tilted toward larger-cap stocks.[15]

Table 1 provides summary statistics for each country market index (MKT) and the SMB, HML, and MOM factor funds. Specifically, the table reports the average number of stocks (N) used to construct each factor fund as well as the monthly percentage mean return (μ) and percentage standard deviation (σ_μ) of the market indices and factor funds. The Sharpe ratios (SHP) for the market indices and factor funds are also reported. In calculating the Sharpe ratio for the market indices, we use the U.S. T-Bill rate averaged over 1981-2008, which is 0.447% per month, to proxy for the risk-free rate. The Sharpe ratio for each zero-investment factor fund is calculated as the ratio of excess return to standard deviation. The correlation with the U.S. market index ($\rho_{\text{US MKT}}$) of each country market index and factor fund, as well as the correlation between the U.S. factor funds and those abroad ($\rho_{\text{US SMB}}$, $\rho_{\text{US HML}}$, and $\rho_{\text{US MOM}}$) are also

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[14] In forming the MOM factor, we follow the same approach used in Carhart (1997), except that Carhart rebalances the factor every month while we rebalance it every six months to mitigate the transaction costs involved in frequent rebalancing. A six-month rebalancing period is also employed by Griffin et al. (2003) and is consistent with prevailing industry practice for momentum strategies. In addition, we use value-weighted instead of equal-weighted winner and loser funds in constructing the MOM factor.

[15] Hou et al. obtain the cash flow data from Worldscope. We choose to use the Datastream data for the current study because Worldscope tracks mainly larger-cap stocks. The resulting "large-cap bias" is likely to weaken the SMB factor whose property we would like to examine. The Datastream data, on the other hand, provides a comprehensive coverage of publicly listed international stocks.

reported. We also calculate the cross-country correlation matrices for market indices and factor funds, but to conserve space we do not report them in the table.

The table shows that the number of stocks included in factor funds varies greatly across countries, reflecting the different sizes of the stock markets in our sample. The U.S. factor funds include the most stocks, and the funds corresponding to the Netherlands and Singapore include the least. Further, in each country, the number of stocks included in the HML funds is typically smaller than that in the SMB and MOM funds. This is because book value data are less readily available than return and market capitalization data. As a result, Datastream provides book value data for only those stocks with such information available. There are also fewer stocks in MOM funds than in SMB funds because stocks without returns for the previous two to 12 months are excluded from the former but not from the latter.

The mean returns of the SMB, HML, and MOM factor funds are positive in most countries, implying that these factors command premiums during our sample period. The only exceptions are negative size premiums in Germany, the Netherlands, and the U.S.[16] The last column of the table shows that the international average mean and standard deviation of monthly returns are 0.95% and 6.32%, respectively, for the market index, with a Sharpe ratio of 0.08. By comparison, the Sharpe ratio is the highest for the HML factor (0.16), followed by the MOM factor (0.11) and then the SMB factor (0.06).

Further, the country market index (MKT) is significantly positively correlated with the U.S. market index at the 1% level for all countries, with an international average of 0.58. In contrast, the majority of SMB, HML, and MOM factor funds have low correlations with the U.S. market index, with an international average of -0.15 (SMB), -0.06 (HML), and -0.06 (MOM), respectively. Intra-factor correlations are generally higher for the MOM funds than for the SMB and HML funds. For example, the correlation between U.S. factor funds and those abroad is on average 0.28 for the MOM funds, followed by 0.11 and 0.08 for the HML and SMB funds.

Interestingly, the inter-factor correlations are generally low, which are -0.11, -0.01, and -0.05, respectively, for the MKT-SMB, MKT-HML, and MKT-MOM pairs, and 0.05, -0.01, and -0.08, respectively, for the SMB-HML, SMB-MOM, and HML-MOM pairs. Overall, the statistics reported above suggest that the correlation structures of SMB, HML, and MOM factors are substantially different from that of market factors. In particular, global components do not seem as important in the returns of factor funds as in those of country market indices. Among the three classes of factor funds, MOM funds appear to be more closely related across countries.

3. Gains from Factor Fund Diversification

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[16] Griffin (2002) also finds a negative size premium in the U.S. over his sample period, 1981-1995.

In this section we formally assess the gains from international diversification augmented with factor funds. In our main test, we consider an investor whose investment opportunity set includes the ten country market indices as the "base assets" and the local factor funds from each of the ten countries as the "additional assets." We first briefly describe the methodology and then present the empirical results.

3.1. Regression-based tests for the intersection hypothesis

Consider K base assets and N additional assets. Investors are not able to achieve further diversification gains by holding the N additional assets if the efficient frontier constructed using the base assets intersects the one spanned by the base assets and the additional assets at the tangency point for a given risk-free rate (the intersection hypothesis). Huberman and Kandel (1987) propose a regression-based approach to test the intersection hypothesis for mean-variance investors. De Roon, Nijman, and Werker (2001) extend the Huberman-Kandel method to allow for various market frictions such as transaction costs and no-short-sales constraints. We adopt the same regression-based intersection test methods to test whether the efficient set spanned by the benchmark market index portfolios intersects the one spanned by the portfolios augmented with factor funds.[17]

Let $R_{B,t+1}$ denote a K-dimensional vector of returns for the K base assets, and R_{t+1} for N additional assets. In the absence of market frictions, Huberman and Kandel (1987) show that the intersection hypothesis implies that

$$R_{t+1} = A + BR_{B,t+1} + e_{t+1}, \tag{1}$$

where $\dfrac{A}{R_f} + B1_K - 1_N = 0$. Since the additional assets are zero-net-investment funds in our study, for our purpose it is useful to rewrite equation (1) in excess return form:

$$r_{t+1} = \alpha + \beta(r_{B,t+1}) + \varepsilon_{t+1}, \tag{2}$$

where r_{t+1} is a N-dimensional excess return vector, $r_{B,t+1}$ is the excess return vector on the base assets. The intersection hypothesis is then equivalent to the condition that the intercepts are insignificantly different from zero, i.e., $\alpha = 0$. Kan and Zhou (2008) also provide evidence that regression based spanning tests are better than those based on the stochastic discount factor approach. We conduct standard Wald tests and report the corresponding p-values. Following Kan and Zhou (2008), we also report p-values from GMM versions of Wald tests, so that we don't rely on the normality assumption and can control for conditional heteroskedasticity (both in the cross-section and time-series) in estimating the covariance matrices.

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[17] For a detailed discussion on the tests of the mean-variance intersection and spanning, see the survey paper by de Roon and Nijman (2001) and the references therein.

If short sales are not allowed, de Roon, Nijman, and Werker (2001) show that the intersection condition implies that:

$$r_{t+1} = \alpha + \beta(r_{B,t+1}^{NS}) + \varepsilon_{t+1} \ , \tag{3}$$

where $r_{B,t+1}^{NS}$ is a subset of $r_{B,t+1}$ for which the no-short-sales constraints are not binding, and the constraint becomes $\alpha \leq 0$.

De Roon, Nijman, and Werker (2001) propose a Wald test to test the above inequality constraint, where the coefficient α in equation (3) is estimated without imposing any constraint, and the covariance matrix $\text{var}(\alpha)$ is estimated subject to the constraint $\alpha = 0$. Following Kodde and Palm (1986), de Roon et al. show that the test statistic $\xi = \min_{\alpha \leq 0} (\widehat{\alpha} - \alpha)' \text{var}(\widehat{\alpha})^{-1} (\widehat{\alpha} - \alpha)$ is asymptotically distributed as a mixture of χ^2 distributions under the null hypothesis of intersection, where $\widehat{\alpha}$ is the estimated value of α. Gourieroux, Holly, and Monfort (1982) propose a numerical simulation to determine the p-value of ξ. We follow the same approach for our empirical tests of the intersection hypothesis.

To allow for transaction costs, we follow de Roon, Nijman, and Werker (2001) and Luttmer (1996) and treat long and short positions separately. Let c denote the proportional transaction costs. For the base assets, we define a $2K$-dimensional excess return vector $r_{B,t+1}^{2K}$, where $r_{B,t+1,i}^{2K} = r_{B,t+1,i} - c$, and $r_{B,t+1,i+K}^{2K} = r_{B,t+1,i} + c$, $i = 1,...,K$. We can then incorporate transaction costs by imposing no-short-sales constraints on the first K assets and no-buying constraints on the next K assets. Similarly, we can expand the N additional assets into $2N$ assets and denote $r_{t+1,i}^{long} = r_{t+1,i} - c$, and $r_{t+1,i}^{short} = r_{t+1,i+N} + c$, $i = 1,...,N$.[18] Testing the intersection hypothesis in the presence of transaction costs can then be treated in the same way as under no-short-sales constraints.

3.2. Domestic factor fund diversification

We now turn to the empirical results. Before we present the details corresponding to local factor fund diversification, we first examine the factor diversification strategy using U.S. domestic factor funds to illustrate the effectiveness of the "factor funds" approach in general. We consider a U.S. investor who holds the U.S. market index but wishes to further diversify using domestic factor funds SMB, HML, and MOM. Note that due to the zero-net-investment feature of factor funds, in the optimal portfolio the market index always has a portfolio weight of 100% and there is no restriction on factor funds' weights. Table 2 reports the results. The benchmark portfolio, which includes only the U.S. market index, has a Sharpe ratio of 0.10. Adding SMB, HML, and MOM factor funds significantly improves the mean-

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[18] In our empirical analysis, we regard a factor fund as a single asset and treat the factor fund (asset) itself as a long position and the reverse as a short position.

variance efficiency of the portfolio, with a Sharpe ratio of 0.33. The interaction hypothesis is rejected at the 1% level.

We also examine the domestic factor diversification strategy from the perspective of investors residing in each of the nine other countries. We use stock returns and the risk-free rate in the local currency from each individual country. The domestic factor diversification strategy provides significant benefits for the majority of the countries in our sample. Across the nine countries, the increase in the Sharpe ratio ranges from 0.06 for the Netherlands to 0.27 for Canada. It is remarkable that the Sharpe ratio on average increases from 0.08 to 0.22 across countries.

3.3. Portfolio diversification with local factor funds

We next formally investigate the diversification benefits with local factor funds. As discussed previously, given the alternative views on the effectiveness of global versus local factors in explaining international stock returns, it is interesting to compare the benefits of global versus local factor diversification strategies. For completeness, we also examine the domestic factor diversification strategies as an alternative benchmark. We consider five portfolios. The benchmark portfolio (i.e., portfolio 1) comprises the ten country market indices. We then augment the benchmark portfolio with local SMB factor funds in portfolio 2, HML factor funds in portfolio 3, and MOM factor funds in portfolio 4. Portfolio 5 includes all three classes of local factor funds as well as the market indices.

Table 3 reports the performance measures for each of the five portfolios. When investors limit their portfolio choice to country market indices (MKT), the optimal portfolio has a monthly mean return of 1.27% and standard deviation of 5.84%. The Sharpe ratio of this portfolio is 0.14, which numerically is higher than the Sharpe ratio of the U.S. market index. However, the test for the intersection hypothesis yields a *p*-value of 0.91 based on the GMM version of the Wald test. Thus, we cannot reject the null hypothesis that the efficient set spanned by the ten country market indices intersects the U.S. market index.[19]

When SMB, HML, or MOM factor funds are added to the base assets, it is apparent that all three classes of factor funds contribute to the improvement of portfolio efficiency. The addition of each class of local factor funds to the base assets separately increases the Sharpe ratio from 0.14 for the benchmark optimal portfolio to 0.48 for the augmented portfolio with the HML factor, 0.34 with the SMB factor, and 0.30 with the MOM factor funds. Clearly, adding the HML factor funds achieves the greatest improvement in portfolio efficiency.

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[19] We also conduct direct tests on the differences in Sharpe ratios between the benchmark and augmented assets, based on the GMM analysis. The results are consistent with those of the Wald tests and available upon request.

When we add all three factor classes to the base assets, the resulting optimal portfolio has a Sharpe ratio of 0.69, which is more than four times that of the benchmark optimal portfolio (0.14).[20] The Sharpe ratio is also higher than that of the all-inclusive U.S. factor portfolio (i.e., the optimal portfolio for the U.S. market index augmented by the three domestic factor funds), 0.33. Moreover, it is higher than that of the all-inclusive global factor portfolio (i.e., the optimal portfolio for the global market index augmented by the three global factor funds), 0.44.[21] We reject the intersection hypothesis at the 1% level for both cases. The above results show that local factor funds provide both economically and statistically significant efficiency gains beyond what domestic and global factor funds can achieve.[22, 23] In Figure 1, we plot the mean-variance efficient frontier spanned by the ten country market indices, as well as those spanned by country market indices plus factor funds. The figure clearly shows that the local factor investment strategy offers the most efficient risk-return trade-off to investors.

Overall, the empirical results presented in this section lend strong support to the hypothesis that international diversification augmented with factor funds substantially improves the efficiency of international investment. Further, the local version of the factor funds outperforms its global counterpart.[24, 25]

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[20] The result is robust to alternative rebalancing frequencies for the MOM factor–the resulting Sharpe ratio is 0.72, 0.70, 0.69, and 0.64, respectively, for monthly, quarterly, semiannual, and annual rebalancing. In all cases, the intersection hypothesis is rejected at the 1% level.

[21] In unreported results, we find that the Sharpe ratio for the optimal portfolio of the global market index augmented by, separately, the global SMB, HML, and MOM factor funds is, respectively, 0.14, 0.36, and 0.17. The all-inclusive global factor portfolio has a Sharpe ratio of 0.44. The details of the results are available upon request.

[22] When we conduct the intersection test with the global factor portfolio as the benchmark, we include in our augmented portfolio the assets in the local factor portfolio, as well as those in the benchmark portfolio. The resulting augmented portfolio generates a Sharpe ratio almost identical to the one generated by the local factor portfolio alone. This is understandable because we construct each global asset as a linear combination of local assets. For consistency, in the tables we simply refer to the augmented portfolio as the local factor portfolio, i.e., 10 country market indices plus local factor funds.

[23] Brooks and Del Negro (2005) and Bekaert, Hodrick, and Zhang (2009) argue that a significant regional component is present in stock returns, notably in the North America, Europe, and Far East regions. To examine the relative performance of a local vs. regional factor diversification strategy, we replicate the analysis in Table 3 using regional market indices and factor funds as the benchmark portfolio. We construct the stock market index and factor funds for each of the three regions: North America, Europe and Asia. We form each regional asset by taking a country value-weighted average of the local assets in our sample countries. Our results show that the local factor diversification strategy outperforms its regional counterpart. Specifically, the difference in Sharpe ratio between the two all-inclusive optimal portfolios is 0.19 during our sample period. The intersection hypothesis is rejected at the 1% level.

[24] This conclusion may seem contradictory to the findings by Hou et al. (2007). However, Hou et al. do not directly test whether the global market, cash flow/price, and momentum factor model which they propose can explain the returns of country-specific characteristic-based portfolios, i.e., local size, value, growth, and momentum portfolios. Therefore, it is not clear whether the global factor model they propose can outperform a local factor model. Further, although Hou et al. do not include the HML factor in their proposed model, they show that cash flow/price and HML are highly correlated and often yield indistinguishable results in their tests. Lastly, Hou et al. obtain their sample mainly from Worldscope, which is tilted toward larger-cap stocks. This may help explain the poor performance of the SMB factor in their tests.

4. Additional Analyses

In this section we conduct a series of tests to check the robustness of our results, including evaluating the effect of transaction costs, out-of-sample tests, and the effects of imposing no-short-sales constraints and other constraints on the portfolio weights. We also examine the time trends in the gains from various investment strategies. Our analysis is mainly focused on the performance of the local factor diversification strategy relative to the performance of the international country market index, U.S. domestic factor, and global factor diversification strategies.

4.1. The effect of transaction costs

Although the composition of stock market indices and factor funds changes over time, rebalancing is likely to be more extensive for factor funds than for market indices. In addition, because factor funds require shorting stocks, holding factor funds can be more costly than holding market indices.[26] If the costs associated with investing in factor funds are excessive, the gains documented above may not be real. To address this concern, we examine the effect of transaction costs on the potential gains from international diversification augmented with factor funds. To represent the additional costs associated with factor funds, we treat a factor fund as a single asset and impose a percentage cost on the asset. We then apply the methodology discussed in Section 3.1. This percentage cost amounts to a proportional tax on holding factor funds. As in Stulz (1981), this tax is meant to capture broadly whatever additional costs investors may face when they invest in factor funds relative to the country market indices.

Table 4 provides summary results for the test of the intersection hypothesis under additional transaction costs for factor funds. As one may expect, the Sharpe ratios of the augmented optimal international portfolios decline as transaction costs increase. However, the overall message conveyed by the table is that the benefits from augmented diversification remain significant unless the (additional) transaction costs for factor funds become excessive. For international diversification using local factor funds, the gains over the benchmark portfolio of the country market indices are statistically significant at the 5% level, even when the annual transaction costs reach the 5% level. The local factor diversification outperforms the U.S. domestic diversification strategy unless the annual transaction costs exceed 6%. The closest competitor for the local factor diversification strategy is the global factor diversification strategy.

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[25] We focus on the Sharpe ratio as the performance measure. However, in untabulated tests we also examine two alternative performance measures: The Goetzmann, Ingersoll, Spiegel and Welch (2007) manipulation-proof performance measure and a certainty equivalent return measure based on the expected utility of a mean-variance investor (DeMiguel, Garlappi, and Uppal (2009)). We show that under both measures, our main results hold robust. Those results are available upon requests.

[26] Bris, Goetzmann, and Zhu (2007) study short-selling practices around the world during the 1990-2001 period. They document that short-selling was allowed in the ten countries in our sample except Hong Kong. Hong Kong allowed short-selling on 33 stocks in 1994 and relaxed the rule to a wide range of stocks in 1998.

They are statistically indistinguishable when the annual transaction costs exceed 4%. However, the Sharpe ratio of the former is still much higher than that of the latter at that level of transaction costs, suggesting that the difference between the two is still economically significant. Lastly, in an unreported test we find that for the local factor funds to receive zero weight in the optimal portfolio, the transaction costs need to be as high as 13%.

4.2. Out-of-sample-period results

In this subsection, we conduct out-of-sample-period analyses to examine how well the factor fund strategy works ex ante. Following de Roon, Nijman, and Werker (2003), we solve the optimal portfolio weights for the various benchmark portfolios considered earlier once a month using a 10-year rolling window. We then construct the optimal portfolio using these weights and hold the portfolio for one month. We record the monthly returns for these portfolios and test whether these out-of-sample portfolio returns can span the returns of the augmented portfolios. The advantage of this approach is that it allows us to easily incorporate market frictions and conduct formal tests as we do for in-sample analysis.

Table 5 reports the out-of-sample-period results for the international local factor diversification strategy against three benchmark strategies discussed in the earlier sections. In the absence of additional transaction costs for factor funds, we reject the intersection hypothesis that the additional assets are spanned by the benchmark portfolio at the 1% level for all cases. The local factor diversification remains the best strategy for the out-of-sample periods. We also test the intersection hypothesis under additional transaction costs for factor funds for the out-of-sample periods. The local factor fund strategy outperforms the alternative strategies unless the annual transaction costs exceed 4%-5%.

In untabulated results, we also examine the out-of-sample performance for the domestic factor investment strategy for each of the ten sample countries. We find that this strategy outperforms the benchmark country-market-index strategy at the 5% level for all but two countries, i.e., Singapore and the U.K. Over the more recent period, 1995-2008, the domestic factor investment strategy still outperforms its benchmark at the 5% level for 5 countries. In addition, we examine the performance of two simple trading rules which are of particular interest to practitioners: An equal-weighted global factor strategy in which an investor invests in country market indices as well as local factor funds but restricting the portfolio weights to be the same across countries, and a strategy in which an investor maintains a 50-50% allocation between HML and MOM funds, but does invest in SMB funds. We examine two variants of the second trading rule: With and without restricting the portfolio weights to be the same across countries. Overall, our results indicate that the local factor diversification strategy still outperforms these two simple trading rules, both in- and out-of-sample. Interestingly, compared to its value-weighted counterpart, the equal-weighted global factor investment strategy generally outperforms, indicating that the smaller markets in our sample do better than the larger ones. Further, allowing the flexibility of different portfolio

weights across countries can improve the performance of the second trading rule substantially.[27] These results are of particular relevance to practitioners who intend to implement these trading rules.

4.3. Constraints on the portfolio weights

In constructing the optimal portfolios presented in the previous tables, we impose no constraints on portfolio weights, except that the weights of the market indices should add up to one. In reality, some investors may prefer to put some weight constraints on their portfolio, such as allocating their funds internationally with the portfolio weight in each country comparable to the relative size of the country's stock market in the world market portfolio. Similarly, professional money managers often benchmark their global capital allocation to the relative size of each market. This leads one to ask if investors may still benefit from local factor diversification when their portfolio allocation is subject to certain weight constraints.

In this subsection, we impose various constraints on the optimal portfolio weights and check whether the gains from factor diversification remain robust. Specifically, we require that for each country market index and factor fund, the weight must be bounded within the range $[(1-\delta)\text{MPW}, (1+\delta)\text{MPW}]$, where MPW refers to the world market portfolio weight of the country to which the fund belongs and δ, a non-negative fraction, represents the weight constraint. We calculate the world market portfolio weight for each country according to its relative stock market capitalization averaged over our 1981-2008 sample period. In our analysis, we examine the effect of the weight constraint δ ranging from zero to 100%. Thus, the optimal portfolio weight for any fund must lie between zero and twice the world market portfolio weight of the country to which the fund belongs.

In untabulated results, we find that as we relax the weight constraint δ from 0% toward 100%, the Sharpe ratio of the constrained optimal portfolio of the ten country market indices increases only slightly from 0.10 to 0.11. We cannot reject the null hypothesis that the U.S. market index is mean-variance efficient with respect to the ten country market indices in any of the cases. We also examine the results for the augmented assets of the ten country market indices plus the local factor funds. In all cases, the Sharpe ratios of the constrained augmented optimal portfolios remain much higher than those of the benchmark portfolios comprising ten country market indices. For example, when $\delta = 100\%$, the Sharpe ratio for the augmented portfolio with factor funds is 0.50 and that of the benchmark portfolio comprising

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[27] For example, in the post-1995 period the Sharpe ratios of the equal- and value- weighted global factor investment strategies are 0.45 and 0.37. The two variants of the second trading rule yields Sharpe ratios of 0.18 and 0.61, compared to that of 0.88 for the local factor investment strategy over the same period. Separately, we have also examined the performance of the momentum funds and value funds in the post-1998 period. We find that during this period investors still benefit substantially by holding value funds. Adding value funds to the 10 country market indices enhances the portfolio Sharpe ratio from 0.23 to 0.76. The increase is statistically significant at the 1% level. Although the performance of momentum funds is weaker, it still can improve the Sharpe ratio of the benchmark country-market-index portfolio by 0.22, which is marginally significant at the 10% level.

country market indices is 0.11. We reject the null hypothesis that the factor funds are spanned by the ten country indices at the 1% level. Even when the weight constraint is most stringent, i.e., $\delta = 0\%$, the Sharpe ratio of the augmented optimal portfolio (0.37) is much higher than that of the benchmark portfolio (0.10),[28] rejecting the intersection hypothesis. In summary, the results show that the significant diversification gains from local factor funds hold under various portfolio weight constraints.[29]

4.4. Short-sales constraints

In this subsection, we consider the situation in which short sales are not allowed. In this case, one can no longer hold the zero-net-investment factor funds. Given this constraint, we decompose each of the three factor funds into their component funds (i.e., small-cap, large-cap, value, growth, winner, and loser funds) and let investors take only long positions in each of the six component funds. We then repeat the optimal portfolio allocation exercise for both in- and out-of-sample periods.[30] Table 6 reports the results.

When we consider all six local component funds together with the market indices, only two fund types receive positive weights: Value (66.9%) and small-cap (32.8%) funds. Market indices as well as large-cap, growth, winner and loser funds all receive zero weights. Our findings here further highlight the importance of expanding one's investment opportunity set beyond the market indices. Obviously, the value funds are the most important component. The Sharpe ratio for the portfolio is 0.25, almost twice that of the benchmark portfolio consisting of country market indices only (0.13), and also higher than that of the domestic (0.16) and global (0.18) factor fund portfolios for the in-sample period. The local factor fund strategy also outperforms the country market index and U.S. factor investment strategies significantly during the out-of-sample periods, though its performance is only marginally better than that of the global factor investment strategy.

4.5. Time trends in the gains from international diversification with local factor funds

Given the increasing evidence that the gains from international diversification using country market indices have been diminishing over time, it is important to examine whether the same trend is observed for the diversification strategy involving local factor funds. In this section, we specifically examine the difference in the Sharpe ratio between the local factor diversification strategy and the alternative strategies over time. For each diversification strategy, we first compute the Sharpe ratio for the

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[28] When $\delta = 0\%$, the resulting portfolio of country market indices resembles the global market portfolio constructed in Section 2. The only difference is that the country weight used in the former is the time-series average over 1981-2008, while that in the latter varies every month. In both cases, the monthly country weight is estimated as a country's relative stock market capitalization value in Datastream's Global Country Market Indices.

[29] We have also examined the cases where portfolio weights are constrained to between 0 and 1 for the whole sample periods as well as for subperiods. We again find that the local factor diversification strategy outperforms all other competing strategies.

[30] Note that the no-short-sales constraints we impose here are more stringent than those faced by investors in practice. In reality, it is possible for investors to hold factor funds yet not violate no-short-sales constraints as long as the short positions in their factor holdings can be offset by the long positions in their holdings in market indices.

optimal portfolio on a 10-year rolling window, moving forward one month at a time. This procedure yields a time series of 217 Sharpe ratios over our sample period. The estimation periods span from January 1981- December 1990 to January 1999- December 2008. We plot the time-series difference in Sharpe ratio (diff SHP) for various pairs of investment strategies in Figure 2 and report the summary statistics at the bottom of the figure.

As a benchmark case, we compare the optimal portfolio comprising the 10 country market indices with that of the U.S. market index in Panel A. We also estimate the time-trend coefficient of the differential Sharpe ratio of these two portfolios. As expected, a slight downward trend is observed, with an estimated time-trend coefficient of -0.0002 (Newey-West adjusted t-statistic = -0.93). The mean (median) diff SHP is 0.191 (0.180), with a standard deviation of 0.049. Among the 217 estimation periods, the differential Sharpe ratio is statistically significant at the 5% level in only 24 periods.

The picture is different when we compare the local factor diversification strategy with three alternative strategies, namely, the international country market index, the U.S. domestic factor, and the global factor diversification strategies, in Panels B, C, and D. We find that the additional benefits of holding local factor funds remain substantial and are not declining over time, irrespective of the choice of benchmark portfolios or subperiods. When we use the international country market indices as the benchmark, the average increase in Sharpe ratio is 0.692. When the benchmark is the U.S. domestic (global) factor strategy, the corresponding number is 0.634 (0.602). The increase in Sharpe ratio is statistically significant at the 5% level for almost all estimation periods.

5. The Relation between Factor Funds and Economic Fundamentals

We have shown empirically that factor funds enhance the risk-return trade-off of an investor's portfolio. However, from a theoretical perspective whether SMB, HML, and MOM factors proxy for state variables that affect the investment opportunity set as outlined in Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM) is still a contentious issue (see, e.g., Lakonishok, Shleifer, and Vishny (1994), and Daniel and Titman (1997)). Although Vassalou (2003) and Petkova (2006) provide supportive evidence on the risk-based explanation of Fama-French factors in the U.S., international evidence on this issue remains scarce. In this section, we investigate this issue internationally for the 10 sample countries we study.

Specifically, we examine the relation between factor funds and important economic variables, including GDP growth, consumption growth, aggregate dividend yield, and term premium, following the methodology employed by Lamont (2001) and Vassalou (2003), which involves constructing an

economic mimicking portfolio (EMP) to track news related to each economic variable.[31] In particular, we estimate the following regression model:

$$EconVar_{t,t+4} = \alpha + \gamma BaseAssets_{t-1,t} + \psi_1 TERM_{t-2,t-1} + \psi_2 DEFY_{t-2,t-1} + \psi_3 RF_{t-1,t} + \varepsilon_{t,t+4} \qquad (4)$$

EconVar denotes one of the four economic variables we examine. GDP growth and consumption growth are seasonally-adjusted and continuously compounded from quarter t to t+4. Aggregate dividend yield and term premium (TERM) are the average value from quarter t to t+4. The aggregate dividend yield is based on the dividend yield on the Datastream country market index. Term premium is measured by the difference in return between the ten-year government bond and short-term T-bill.

Following Vassalou (2003), we include six equity portfolios and two fixed-income portfolios as the base assets. The six equity portfolios are the excess returns of the component portfolios of SMB, HML, and MOM funds, i.e., large-cap, small-cap, value, growth, winner, and loser funds. The returns on the two fixed-income portfolios are DEF and TERM, where DEF measures the return difference between long-term corporate bonds and long-term government bonds. In addition, we include a constant, RF, TERM, and DEFY as control variables. RF is the 3-month T-bill rate, and DEFY is the yield spread between Moody's BAA and AAA corporate bonds.[32] Since DEF and DEFY are generally not available for countries outside the U.S., we use the U.S. data as a proxy instead.[33]

We estimate Equation (4) for each economic variable by country from Q1 1981 to Q4 2008.[34] With the estimated regression coefficients, the returns of EMPs can be calculated as follows:

$$EMP_{t-1,t} = \gamma BaseAssets_{t-1,t} \qquad (5)$$

We generate separate time-series of quarterly returns on the EMP of each of the four economic variables for each sample country. Table 7 reports the results. Panel A reports the cross-country average return and standard deviation of EMPs for each economic variable, as well as their correlations with the SMB, HML,

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[31] We are not able to include default premiums in our analysis because the data are not available for countries outside the U.S.

[32] We obtain the GDP growth, consumption growth, dividend yield, government bond yields, corporate bond yields and Treasury bill rates data from Datastream. The consumption growth data are only available from Q2 1991 for Germany, from Q2 1994 for Japan, and from Q2 1988 for the Netherlands. The 10-year Treasury bond data are only available from October 1996 for Hong Kong, and from June 1998 for Singapore. We use one-month or 3-month T-bill rates to proxy for short-term T-bill rates if they are available. Otherwise, we use 3-month Eurocurrency rates instead.

[33] TERM is not available prior to October 1996 for Hong Kong and prior to June 1998 for Singapore. Therefore, when the variable is required as one of the independent variables in the regression, we use the U.S. data in place of missing values for these two countries.

[34] To conserve space, we do not report the regression estimates. They are available upon request. In total we estimate 40 regressions. We correct for serial correlation up to three lags and adjust for heteroskedasticity in standard errors. To reduce the effect of the Internet bubble on our estimates, we exclude the period Q1 1999 - Q2 2000 from the estimation. On average, the adjusted R-square for these regressions is 40.61%. The hypothesis that the regression coefficients for the 8 base assets are jointly zero is rejected at the 5% (10%) level for 62.5% (70%) of the cases based on the asymptotic χ^2 (8) test, suggesting that these base assets contain important incremental information about innovations on these economic variables.

and MOM factor funds. The average returns of the EMPs are positive for GDP growth, consumption growth, and term premium but negative for dividend yield. These results are broadly consistent with the findings by Vassalou (2003), Lamont (2001) and Petkova (2006).

To examine whether factor funds are significantly related to news on these economic variables, we run a pooled regression of EMPs on SMB, HML, and MOM factor funds, a constant, and country dummies, for each of the four economic variables. The results, reported in Panel B, show that factor funds, especially SMB and HML, are significantly related to news on economic variables. In particular, consistent with Vassalou (2003), we find a significantly positive association between HML and GDP growth. HML is also significantly positively related to consumption growth but negatively related to dividend yield, consistent with the finding by Petkova (2006). Further, SMB is significantly positively related to consumption growth and term premium, while MOM is negatively related to dividend yield.

To gauge the relative importance of news on these four economic variables for portfolio investment purpose, we compare the performance of investment strategies involving EMPs of individual economic variables. We use EMPs constructed from monthly returns of the six equity base assets for this analysis to increase the number of observations and to ensure that the base assets are available in all sample countries. Panel C reports the results. In Portfolios (1)-(4), we examine the optimal risky portfolio comprising the EMPs of, respectively, GDP growth, consumption growth, term premium, and dividend yield in addition to the 10 country market indices. Their Sharpe ratios are 0.36, 0.29, 0.20, and 0.18, respectively. Therefore, from a portfolio perspective GDP growth appears to be the most important economic news among the four we examine.

Lastly, we compare the performance of investment strategies involving EMPs with those involving factor funds. The Sharpe ratio of the optimal risky portfolio comprising the ten country market indices and the four EMPs from each of the ten sample countries is 0.52, which is lower than the Sharpe ratio (0.69) of the local factor investment strategy. In Portfolios (5)-(7), we augment the benchmark portfolio of EMPs with, respectively, the U.S., global, and local factor funds, to examine whether there are additional gains from holding factor funds over and above the benchmark strategy.

Our results indicate that local factor funds contain important information about fundamental economic variables. However, investors are still better off diversifying with local factor funds than with EMPs of the four economic variables we examine. One possible explanation for this finding is that SMB, HML, and MOM factor funds may contain information not only on these four economic variables but also on other state variables, or they may possibly capture non-risk-based factors. It is, however, beyond the scope of this paper to identify these factors. In summary, factor funds serve as a convenient vehicle for investors to capture information on changing investment opportunities.

6. Concluding Remarks

In this paper we propose a new international diversification strategy using factor funds to systematically enhance the portfolio efficiency. Our approach is motivated by the direct link between factor-based asset pricing theories/tests and the tests of portfolio mean-variance efficiency. As shown by the recent literature, the size, book-to-market, and momentum factors help explain stock returns. The portfolio implication is that investors would hold these factor portfolios to enhance portfolio efficiency. Furthermore, Griffin (2002) and Griffin et al. (2003) suggest that these factors are substantially local, i.e., country-specific, while Hou et al. (2007) advocate a model of global factors. We examine these issues from a portfolio perspective, employing data from ten developed countries.

We show that factor fund diversification strategies yield substantial improvements in portfolio efficiency beyond what can be achieved by the traditional country market index diversification approach. Among the three factor diversification strategies (i.e., domestic, global, and local), local factor diversification provides the largest efficiency gains. We find that the Sharpe ratio of the augmented optimal international portfolio comprising local factor funds and country market indices far exceeds that of the benchmark optimal portfolio comprising country market indices only. The local factor diversification strategy continues to outperform even if we include domestic or global factor funds in the benchmark portfolio. Among the three classes of factor funds, the HML factor funds receive the most significant weights in the optimal portfolio and contribute most to portfolio efficiency.

Our key findings hold for both in-sample and out-of-sample periods, and remain robust to a reasonable range of additional transaction costs for factor funds and to the imposition of no-short-sales constraints. Our findings are also robust to various other constraints on portfolio weights. Furthermore, we show that unlike the case for country market index-based diversification strategies, the gains from local factor diversification remain statistically and economically significant over time.

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Table 1. Summary Statistics for Market Indices and Factor Funds

The table reports the monthly percentage mean return (μ), percentage standard deviation (σ_μ), correlation with the U.S. market index ($\rho_{US\ MKT}$), and correlation with U.S. factor funds ($\rho_{US\ SMB}$, $\rho_{US\ HML}$, and $\rho_{US\ MOM}$) for each country market index (MKT) and the size (SMB), book-to-market (HML), and momentum (MOM) factors from the ten sample countries we study. We also report the Sharpe ratio (SHP) for the market index and the SMB, HML, and MOM factors. The Sharpe ratio for the factors is calculated as the return to the standard deviation for each zero-investment factor fund. The number of securities (N) used to construct each SMB, HML, and MOM factor is also reported. All the returns are in terms of U.S. dollars. SMB, HML, and MOM denote, respectively, the return on the small-cap fund less the return on the large-cap fund, the return on the value fund less the return of the growth fund, and the return on the winner fund less the return on the loser fund. We form small-cap and large-cap (value and growth) funds based on the market capitalization of individual stocks (book-to-market ratio) at the end of each calendar year and returns are calculated for the following year; we form winner and loser funds in June and December each year based on individual stocks' cumulative returns during the previous two to 12 months and returns are calculated for the subsequent six months. The small-cap (large-cap) fund represents the 30% of stocks with the smallest (largest) market capitalization in the country; the value (growth) fund represents the 30% of stocks with the highest (lowest) book-to-market ratio stocks in the country; and the winner (loser) fund represents the 30% of stocks with the highest (lowest) cumulative returns in the country over the previous two to 12 months. The risk-free rate is proxied by the U.S. T-bill rate averaged over the sample period, which is 0.447% per month. * and ** denote that the respective statistic is significantly different from zero at the 5% and 1% level. The significance test for the Sharpe ratio is conducted using the GMM estimator. The ten sample countries included in our analysis are Australia (AU), Canada (CN), France (FR), Germany (GR), Hong Kong (HK), Japan (JP), the Netherlands (NL), Singapore (SG), the U.K., and the U.S. Monthly data from January 1981 to December 2008 are employed for the analysis.

Country	AU	CN	FR	GR	HK	JP	NL	SG	UK	US	Average
					MKT						
μ	0.93 **	0.84 **	1.05 **	1.06 **	1.16 *	0.74 *	1.13 **	0.78	0.89 **	0.87 **	0.95 **
σ_μ	6.68	5.63	6.16	6.50	8.74	6.61	5.49	7.60	5.30	4.47	6.32
SHP	0.07	0.07	0.10	0.09	0.08	0.04	0.13 *	0.04	0.08	0.10	0.08
$\rho_{US\ MKT}$	0.55 **	0.79 **	0.59 **	0.57 **	0.48 **	0.37 **	0.69 **	0.57 **	0.64 **	1.00 **	0.58 **
					SMB						
N	1125	1150	1021	769	550	2708	320	264	3693	5192	1679
μ	0.04	0.98 **	0.29	-0.04	0.52	0.78 **	-0.19	0.43	0.05	-0.11	0.27 *
σ_μ	5.49	4.61	3.34	3.86	8.25	5.51	3.84	6.69	2.89	3.18	4.77
SHP	0.01	0.21 **	0.09	-0.01	0.06	0.14 **	-0.05	0.06	0.02	-0.04	0.06 *
$\rho_{US\ MKT}$	-0.06	-0.10 **	-0.23	-0.27	-0.08	-0.12 **	-0.54	0.04	-0.29	0.15	-0.15 *
$\rho_{US\ SMB}$	0.01	0.24 **	0.09	0.07	0.09	-0.07	0.01	0.10	0.14 **	1.00 **	0.08 *
					HML						
N	584	533	538	532	390	2292	136	195	1438	4476	1111
μ	0.70 **	0.49 *	0.81 **	0.74 **	0.58	1.04 **	0.66 **	0.99 **	0.38 *	0.59 **	0.70 **
σ_μ	3.79	4.31	4.25	4.30	5.72	4.64	4.75	5.89	3.59	3.06	4.43
SHP	0.18 **	0.11	0.19 **	0.17 **	0.10	0.22 **	0.14 **	0.17 **	0.11	0.19 **	0.16 **
$\rho_{US\ MKT}$	-0.15 **	-0.15 **	0.07	0.02	-0.01	-0.17 **	0.02	0.12 *	0.05	-0.43 **	-0.06
$\rho_{US\ HML}$	0.17 **	0.21 **	0.18 **	0.15 **	0.05	0.16 **	-0.03	-0.04	0.08	1.00 **	0.11 **
					MOM						
N	1041	1096	949	713	480	2060	309	243	3564	5134	1559
μ	0.86 **	0.97 **	0.52 **	0.68 *	0.53	0.13	0.40	0.21	0.43 *	0.56 **	0.53 **
σ_μ	4.58	5.19	3.73	5.86	6.42	6.05	4.80	6.71	3.42	3.36	5.01
SHP	0.19 **	0.19 **	0.14 *	0.12 *	0.08	0.02	0.08	0.03	0.13 *	0.17 **	0.11 **
$\rho_{US\ MKT}$	-0.02	-0.07	-0.10	-0.18 **	-0.04	-0.04	-0.07	-0.12 *	0.09	-0.07	-0.06 *
$\rho_{US\ MOM}$	0.21 **	0.54 **	0.41 **	0.32 **	0.11 *	0.15 **	0.32 **	0.05	0.38 **	1.00 **	0.28 **

Table 2. Domestic Diversification with Factor Funds

The table examines the potential gains from domestic diversification augmented with factor funds for Australia (AU), Canada (CN), France (FR), Germany (GR), Hong Kong (HK), Japan (JP), the Netherlands (NL), Singapore (SG), the U.K. (UK), and the U.S. (US). We report the composition of the optimal augmented portfolio comprising the domestic country market index plus SMB, HML, and MOM factor funds. We evaluate the performance of each domestic optimal risky portfolio. The risk-free rate is proxied by the domestic T-bill rate (or its close substitute) averaged over the sample period. Returns for the T-bill rate, market index, and factor funds are all measured in local currency units. For each portfolio, we report the percentage mean return, percentage standard deviation, and the Sharpe ratio, as well as the p-value of the Wald test and the p-value calculated using the GMM estimator. The last column of the table reports the average statistics across the nine countries. The null hypothesis of the intersection test is that the mean-variance efficient frontier spanned by the augmented portfolio intersects that spanned by the benchmark portfolio at the tangency point. The benchmark portfolio is the domestic country market index. Monthly data from January 1981 to December 2008 are employed for the analysis.

Countries	Australia (AU)	Canada (CN)	France (FR)	Germany (GR)	Hong Kong (HK)	Japan (JP)	Netherlands (NL)	Singapore (SG)	U.K. (UK)	U.S. (US)	Average
Augmented Portfolio: Individual Country Market Index, and SMB, HML, and MOM Factor Funds											
Portfolio Return (%)	3.66	3.07	3.62	5.76	3.32	2.91	2.93	3.70	4.16	2.53	3.56
Portfolio Std dev. (%)	10.55	7.76	16.32	29.65	14.28	16.64	11.74	20.58	20.04	6.33	15.39
Portfolio Sharpe Ratio	0.28	0.32	0.19	0.18	0.20	0.16	0.21	0.17	0.18	0.33	0.22
Benchmark Portfolio: Individual Country Market Index											
Portfolio Return (%)	0.99	0.80	1.04	0.94	1.26	0.44	1.29	0.64	1.02	0.87	0.93
Portfolio Std dev. (%)	5.17	4.75	5.82	6.13	8.52	5.64	5.62	7.16	4.69	4.47	5.80
Portfolio Sharpe Ratio	0.05	0.05	0.09	0.08	0.10	0.04	0.15	0.06	0.08	0.10	0.08
Mean-Variance Intersection Tests: Augmented vs. Benchmark Portfolio											
Δ Sharpe Ratio	0.23	0.27	0.10	0.10	0.10	0.12	0.06	0.11	0.09	0.23	0.14
p-Wald	0.00	0.00	0.03	0.04	0.02	0.04	0.07	0.04	0.05	0.00	
p-GMM	0.00	0.00	0.03	0.02	0.03	0.10	0.07	0.05	0.04	0.00	

Table 3. International Diversification with Local Factor Funds

The table examines the potential gains from international diversification augmented with local factor funds. The local factor funds refer to the SMB, HML, and MOM factor funds from each of the ten sample countries we study, which are Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. We examine the optimal risky portfolio for five sets of assets: (1) country market indices, (2) country market indices plus SMB factor funds, (3) country market indices plus HML factor funds, (4) country market indices plus MOM factor funds, and (5) country market indices plus SMB, HML, and MOM factor funds. The risk-free rate is proxied by the U.S. T-bill rate averaged over the sample period, or 0.447% per month. We also compare each augmented portfolio with a benchmark portfolio. The augmented portfolio refers to portfolios (1)-(5). The benchmark portfolio is described below each corresponding augmented portfolio. The p-value of the Wald test and the p-value calculated using the GMM estimator are both reported. The null hypothesis of the intersection test is that the mean-variance efficient frontier spanned by the augmented portfolio intersects that spanned by the benchmark portfolio at the tangency point. Monthly data from January 1981 to December 2008 are employed for the analysis.

	Optimal Risky Portfolios				
Portfolios	*1*	*2*	*3*	*4*	*5*
Assets	*10 Country MKT*	*10 Country MKT and SMB*	*10 Country MKT and HML*	*10 Country MKT and MOM*	*10 Country MKT, SMB, HML, and MOM*
Portfolio Return (%)	1.27	3.31	5.15	3.50	7.55
Portfolio Std dev. (%)	5.84	8.39	9.88	10.05	10.30
Portfolio Sharpe Ratio	0.14	0.34	0.48	0.30	0.69
Mean-Variance Intersection Tests					
Benchmark Portfolio		10 Country MKT	10 Country MKT	10 Country MKT	10 Country MKT
Δ Sharpe Ratio		0.20	0.34	0.16	0.55
p-Wald		0.00	0.00	0.01	0.00
p-GMM		0.00	0.00	0.01	0.00
Benchmark Portfolio	US MKT	US MKT and SMB	US MKT and HML	US MKT and MOM	US MKT, SMB, HML, and MOM
Δ Sharpe Ratio	0.04	0.23	0.20	0.10	0.36
p-Wald	0.94	0.01	0.00	0.51	0.00
p-GMM	0.91	0.03	0.00	0.36	0.00
Benchmark Portfolio	Global MKT	Global MKT and SMB	Global MKT and HML	Global MKT and MOM	Global MKT, SMB, HML, and MOM
Δ Sharpe Ratio	0.06	0.20	0.12	0.13	0.25
p-Wald	0.85	0.03	0.08	0.33	0.00
p-GMM	0.81	0.05	0.02	0.09	0.00

Table 4. International Diversification with Local Factor Funds:
The Effects of Transaction Costs

The table reports the results of three sets of mean-variance intersection tests with various levels of (additional) transaction costs imposed on SMB, HML, and MOM factor funds, but not on the country market indices. The null hypothesis of the mean-variance intersection tests is that the mean-variance efficient frontier spanned by the augmented set of risky assets intersects that spanned by the benchmark set of risky assets at the tangency point. The risk-free rate is proxied by the U.S. T-bill rate averaged over the sample period, which is 0.447% per month. The augmented assets and benchmark assets are described in the second and third row of the table. The annualized transaction costs imposed on SMB, HML, and MOM factor funds range from 0% to 10% of the fund value. For each optimal augmented portfolio, we report its Sharpe ratio, the differential Sharpe ratio (ΔSHP) between the optimal augmented and benchmark portfolio, the p-value of the Wald test, as well as the p-value calculated using the GMM estimator. The null hypothesis of the intersection test is that the mean-variance efficient frontier spanned by the augmented portfolio intersects that spanned by the benchmark portfolio at the tangency point. We construct the global market index and global factor funds, respectively, as the value-weighted average of local country market indices and local factor funds from the ten sample countries. We use the relative stock market value of each country to determine its weight in the global market index and global factor funds. The ten sample countries included in our analysis are Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. Monthly data from January 1981 to December 2008 are employed for the analysis.

Tests	1				2				3			
Augmented	*10 Country* *MKT, SMB, HML, and MOM*				*10 Country* *MKT, SMB, HML, and MOM*				*10 Country* *MKT, SMB, HML, and MOM*			
Benchmark	*10 Country MKT*				*US* *MKT, SMB, HML, and MOM*				*Global* *MKT, SMB, HML, and MOM*			
Transaction Costs	*Sharpe Ratio*	*ΔSHP*	*p-Wald*	*p-GMM*	*Sharpe Ratio*	*ΔSHP*	*p-Wald*	*p-GMM*	*Sharpe Ratio*	*ΔSHP*	*p-Wald*	*p-GMM*
0.0%	0.69	0.55	0.00	0.00	0.69	0.36	0.00	0.00	0.69	0.25	0.00	0.00
1.0%	0.60	0.46	0.00	0.00	0.60	0.31	0.00	0.00	0.60	0.23	0.00	0.00
2.0%	0.51	0.37	0.00	0.00	0.51	0.26	0.00	0.00	0.51	0.19	0.00	0.00
3.0%	0.46	0.33	0.00	0.00	0.46	0.24	0.00	0.00	0.46	0.19	0.01	0.01
4.0%	0.40	0.27	0.00	0.00	0.40	0.22	0.00	0.00	0.40	0.18	0.03	0.03
5.0%	0.31	0.18	0.00	0.02	0.31	0.16	0.00	0.00	0.31	0.13	0.06	0.07
6.0%	0.29	0.16	0.06	0.19	0.29	0.16	0.02	0.03	0.29	0.14	0.13	0.13
7.0%	0.25	0.11	0.42	0.54	0.25	0.14	0.06	0.10	0.25	0.13	0.17	0.24
8.0%	0.21	0.08	0.82	0.91	0.21	0.11	0.15	0.23	0.21	0.11	0.27	0.30
9.0%	0.18	0.05	0.98	0.98	0.18	0.09	0.30	0.45	0.18	0.10	0.28	0.34
10.0%	0.16	0.03	1.00	1.00	0.16	0.06	0.41	0.62	0.16	0.08	0.29	0.41

Table 5. International Diversification with Local Factor Funds:
Out-of-Sample Analysis

The table reports the out-of-sample test results for three sets of mean-variance intersection tests with various levels of (additional) transaction costs imposed on SMB, HML, and MOM factors, but not on country market indices. The null hypothesis of the mean-variance intersection tests is that the mean-variance efficient frontier spanned by the augmented set of risky assets intersects that spanned by the benchmark set of risky assets at the tangency point, where the risk-free interest rate is proxied by the U.S. T-bill rate averaged over the sample period. The augmented assets and benchmark assets are described in the second and third row of the table. The annualized transaction costs imposed on SMB, HML, and MOM factor funds range from 0% to 10% of fund value. The p-value of the Wald test and the p-value calculated using the GMM estimator of the mean-variance intersection test are both reported. We construct the global market index and global factor funds, respectively, as the value-weighted average of local country market indices and local factor funds from the ten sample countries. We use the relative stock market value of each country to determine its weight in the global market index and global factor funds. The ten sample countries included in our analysis are Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. Monthly data from January 1981 to December 2008 are employed for the analysis.

Tests	1		2		3	
Augmented	10 Country MKT, SMB, HML, and MOM		10 Country MKT, SMB, HML, and MOM		10 Country MKT, SMB, HML, and MOM	
Benchmark	10 Country MKT		US MKT, SMB, HML, and MOM		Global MKT, SMB, HML, and MOM	
Transaction Costs	p-Wald	p-GMM	p-Wald	p-GMM	p-Wald	p-GMM
0.0%	0.00	0.00	0.00	0.00	0.00	0.00
1.0%	0.00	0.00	0.00	0.00	0.01	0.00
2.0%	0.00	0.00	0.00	0.00	0.01	0.01
3.0%	0.00	0.00	0.00	0.01	0.01	0.02
4.0%	0.00	0.00	0.01	0.03	0.02	0.04
5.0%	0.01	0.04	0.02	0.08	0.03	0.05
6.0%	0.03	0.12	0.08	0.18	0.13	0.14
7.0%	0.11	0.33	0.20	0.35	0.14	0.16
8.0%	0.24	0.58	0.34	0.55	0.14	0.21
9.0%	0.46	0.83	0.55	0.76	0.19	0.30
10.0%	0.71	0.93	0.76	0.93	0.33	0.43

Table 6. International Diversification with Components of Local Factor Funds:
The Effects of Short-Sales Constraints

The table reports the in-sample and out-of-sample performance of internationally diversified portfolios with local factor funds from the ten sample countries, with the no-short-sales constraint imposed on all assets. Specifically, we consider an internationally diversified portfolio comprising the country market index and factor funds from the ten sample countries. We use the U.S. T-bill rate, averaged over the sample period, to proxy for the risk-free interest rate. We report the percentage mean return, percentage standard deviation, and the Sharpe ratio for the optimal risky portfolio. In addition, we compare each portfolio with its benchmark portfolio, which is described in the table. We report the in-sample and out-of-sample p-value of the Wald test and the p-value calculated using the GMM estimator. The null hypothesis for the intersection tests is that the mean-variance efficient frontier spanned by the augmented set of risky assets intersects that spanned by the benchmark set of risky assets at the tangency point. The in-sample Shape ratio difference (Δ Sharpe Ratio) between the augmented and benchmark portfolios is also reported. The ten sample countries included in our analysis are Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. Monthly data from January 1981 to December 2008 are employed for the analysis.

	Optimal Risky Portfolio		
Assets	*10 Country MKT and Six Component (Small, Large, Value, Growth, Winner, and Loser) Funds*		
Portfolio Return (%)	1.62		
Portfolio Std dev. (%)	4.64		
Portfolio Sharpe Ratio	0.25		
	Mean-Variance Intersection Tests		
Benchmark Portfolio	10 Country MKT	US Market and Six Component Funds	Global Market and Six Component Funds
In-Sample Test			
Δ Sharpe Ratio	0.12	0.09	0.07
p-Wald	0.00	0.02	0.24
p-GMM	0.00	0.06	0.42
Out-of-Sample Test			
p-Wald	0.00	0.00	0.01
p-GMM	0.00	0.02	0.08

We construct economic mimicking portfolios (EMPs) for GDP growth (gGDP), consumption growth (gConsump), aggregate dividend yield (DY), and term premium (TERM) in each sample country using the methodology described in Section 5. gGDP and gConsump are continuously compounded over a year, while DY and TERM are the average value over the same period. The aggregate dividend yield is from Datastream country market index. Term premium represents the difference in return between the ten-year government bond and short-term T-bill. Panel A reports the cross-country average of mean and standard deviation of the EMPs estimated at the quarterly frequency, as well as their average correlations with local SMB, HML, and MOM funds. The numbers in parentheses are the t-statistics for the mean returns of EMPs. In Panel B, we run pooled regressions of the EMPs on local factor funds at the quarterly frequency. The estimated coefficients, white-adjusted t-statistics (in parentheses), and the adjusted-R^2 of the regressions are reported. In Panel C, we examine the international diversification potentials of EMPs and compare them with those using factor funds. We use EMPs constructed using monthly returns of equity base assets for this analysis. We report the returns, standard deviations, and Sharpe ratios for portfolios (1)-(7) in the upper panel, and the comparison between these portfolios and their respective benchmarks in the lower panel. The ten sample countries included in our analysis are Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. The sample period is from January 1981 to December 2008.

Panel A: Mean and Standard Deviation of Economic Mimicking Portfolios and their Correlations with SMB, HML, and MOM

		Standard	Correlation with		
Mimicking Portfolios for	*Mean*	*Deviation*	*SMB*	*HML*	*MOM*
GDP Growth	0.53	1.18	0.15	0.15	-0.15
	(3.43)				
Consumption Growth	0.75	1.05	0.14	0.10	-0.08
	(4.81)				
Dividend Yield	-0.16	0.28	0.04	-0.09	-0.06
	(-6.40)				
Term Premium	1.43	1.18	0.10	0.01	0.02
	(13.01)				

Panel B: Regressions of Economic Mimicking Portfolios on SMB, HML, and MOM

Mimicking Portfolios for	*Constant*	*SMB*	*HML*	*MOM*	*adj-R^2*
GDP Growth	-0.003	0.008	0.018	0.001	0.539
	(-5.14)	(1.46)	(3.01)	(0.11)	
Consumption Growth	-0.001	0.012	0.017	0.005	0.725
	(-1.75)	(2.03)	(2.78)	(0.81)	
Dividend Yield	0.003	-0.001	-0.005	-0.004	0.596
	(8.36)	(-0.37)	(-3.33)	(-3.00)	
Term Premium	0.007	0.010	0.000	0.005	0.220
	(15.98)	(2.96)	(0.06)	(1.26)	

Panel C. International Investment with Economic Mimicking Portfolios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	10 Country Market Indices plus				*Benchmark Assets plus the Following*		
Assets Included in the	*EMPs for*				*Version of Factor Funds*		
Optimal Risky Portfolio	*gGDP*	*gConsump*	*DY*	*TERM*	*U.S.*	*Global*	*Local*
Portfolio Return (%)	1.97	1.83	1.41	1.26	2.45	2.93	3.52
Portfolio Std dev. (%)	4.26	4.73	5.23	4.13	3.25	3.64	3.68
Portfolio Sharpe Ratio	0.36	0.29	0.18	0.20	0.62	0.68	0.83
Mean-Variance Intersection Tests							
Benchmark Portfolio	10 Country Market Indices				10 Country Market Indices plus EMPs of gGDP, gConsump, DY, and TERM		
Δ Sharpe Ratio	0.22	0.15	0.04	0.06	0.10	0.16	0.31
p-Wald	0.00	0.00	0.00	0.00	0.00	0.00	0.00
p-GMM	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 1. Mean-Variance Efficient Frontiers of Alternative Investment Strategies

We plot the mean-variance efficient frontiers for four investment strategies: (1) country market investment strategy, which involves only country market indices from the 10 countries we study, (2) U.S. domestic factor diversification strategy, which involves the U.S. market index augmented by the three U.S. factor funds, (3) global factor diversification strategy, which involves the global market index augmented by the three global factor funds, and (4) local factor diversification strategy, which involves the market index and factor funds from each of the ten sample countries we study. The dotted line in the graph connects the risk-free rate to the tangency portfolio. The risk-free rate is proxied by the U.S. T-bill rate averaged over the sample period, or 0.447% per month. The round, diamond-shaped, square, and triangular dots in the graph denote the mean return-standard deviation locations of, respectively, the country market indices, SMB, HML, and MOM funds. The ten sample countries included in our analysis are Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. Monthly data from January 1981 to December 2008 are employed for the analysis.
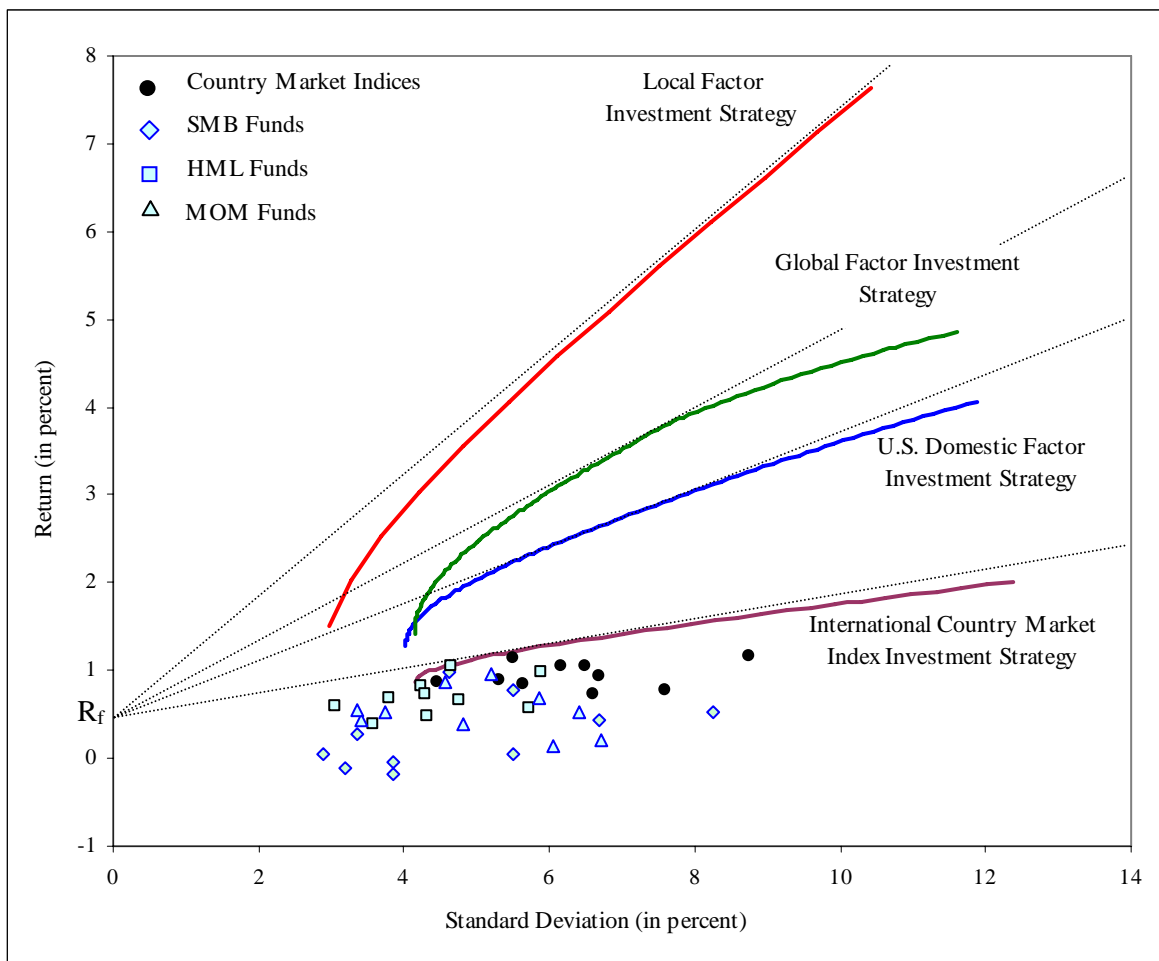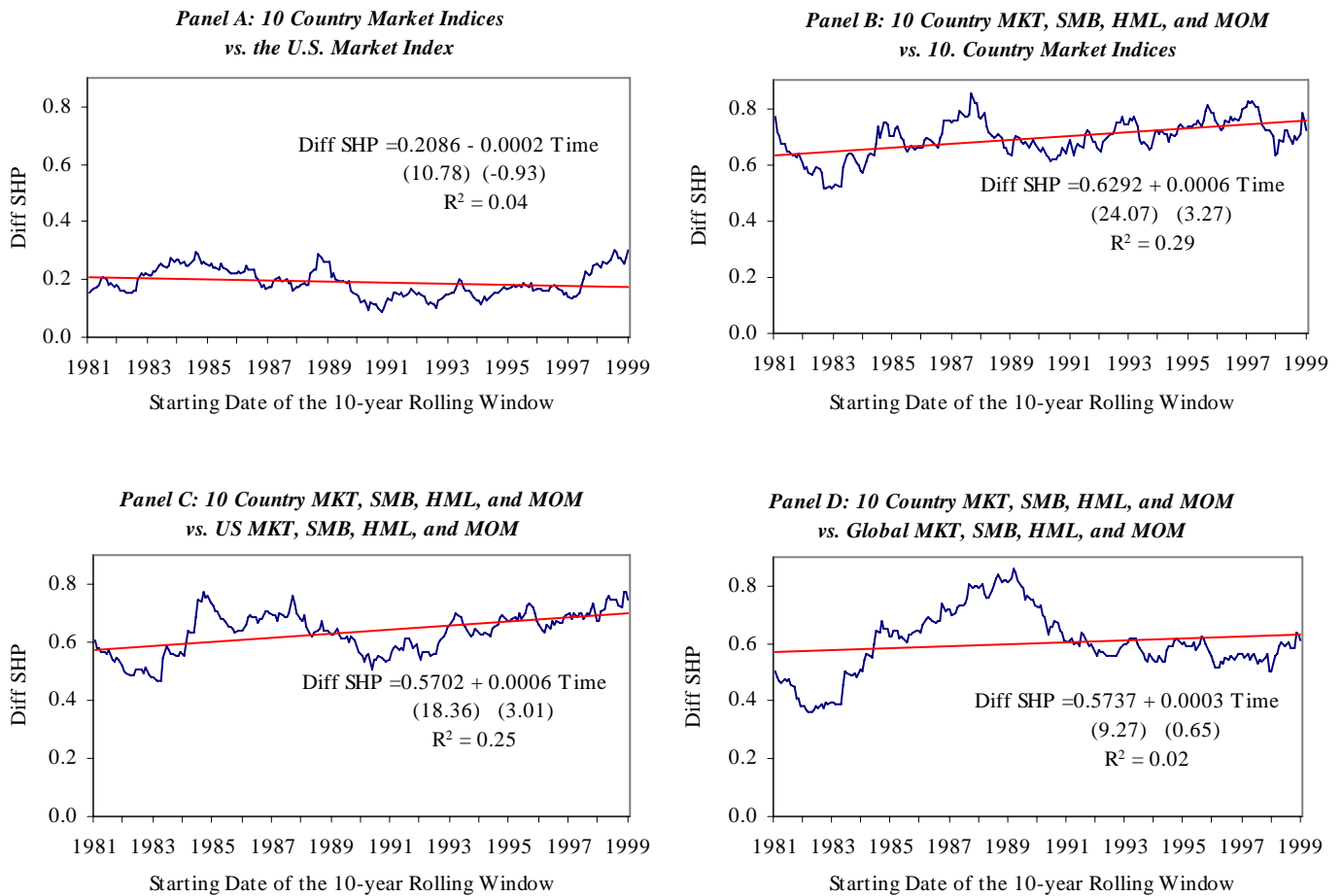
Figure 2. Time Trends in the differential Sharpe Ratios of Alternative Investment Strategies

Panels A-D of the figure plot the differential Sharpe ratio (diff SHP) between the optimal augmented and benchmark portfolio. In Panel A, the augmented portfolio comprises the 10 country market indices in our sample and the benchmark portfolio comprises the U.S. market index only. In Panels B-D, the augmented portfolio is the 10 country market indices plus local factor funds, and the benchmark portfolios are, respectively, the 10 country market indices, U.S. market index plus U.S. factor funds, and global market index plus global factor funds. The ten sample countries we study include Australia, Canada, France, Germany, Hong Kong, Japan, the Netherlands, Singapore, the U.K., and the U.S. For each set of assets, benchmark or augmented, we solve for the optimal portfolio and compute the Sharpe ratio of the portfolio on a 10-year rolling window, moving forward one month at a time. This procedure involves 217 overlapping 10-year estimation periods, where the first (last) period spans from January 1981 (1999) to December 1990 (2008). The regression equation describing the time trend of the differential Sharpe ratio is reported in each panel, where the number in parentheses denotes the Newey-West heteroskedasticity and serial correlation adjusted t-statistic. The summary statistics (mean, median, standard deviation, and number of observations N) for Diff SHP are reported at the bottom of this figure. We also test whether Diff SHP is significantly different from zero using the Wald test. Nsig refers to the number of periods for which Diff SHP is significant.

Panel A: 10 Country Market Indices vs. the U.S. Market Index

Diff SHP =0.2086 - 0.0002 Time
(10.78) (-0.93)
$R^2 = 0.04$

Panel B: 10 Country MKT, SMB, HML, and MOM vs. 10. Country Market Indices

Diff SHP =0.6292 + 0.0006 Time
(24.07) (3.27)
$R^2 = 0.29$

Panel C: 10 Country MKT, SMB, HML, and MOM vs. US MKT, SMB, HML, and MOM

Diff SHP =0.5702 + 0.0006 Time
(18.36) (3.01)
$R^2 = 0.25$

Panel D: 10 Country MKT, SMB, HML, and MOM vs. Global MKT, SMB, HML, and MOM

Diff SHP =0.5737 + 0.0003 Time
(9.27) (0.65)
$R^2 = 0.02$

Statistics for Diff SHP	Panel			
	A	B	C	D
Mean	0.191	0.692	0.634	0.602
Median	0.180	0.696	0.642	0.597
Std	0.049	0.067	0.073	0.113
N	217	217	217	217
Nsig	24	217	217	216