Taxes, Leverage, and Stimuli of Investment under Uncertainty

Kit Pong WONG *
University of Hong Kong

January 2012

Abstract

This paper examines the effect of leverage on the effectiveness of a self-financed tax-subsidy program offered by a government in stimulating a firm's investment. We show that the firm, be it levered or unlevered, has an incentive to hasten its investment because of the agency conflicts arising from the commitment made by the government on the terms of the tax-subsidy program. We further show that the levered firm has a countervailing incentive to defer its investment due to the deadweight loss when bankruptcy occurs, which would be absent should the firm be unlevered. The former incentive is likely to be dominated by the latter incentive, in particular when the corporate income tax rate is sufficiently high and the bankruptcy cost is sufficiently low so that the firm relies heavily on debt. In this case, the tax-subsidy program induces the levered firm to defer, not hasten, its investment. Finally, we show that the levered firm is made worse off with than without the program because of the presence of agency and bankruptcy costs.

JEL classification: G31; G33; H25

Keywords: Capital structure; Investment timing; Real options; Tax-subsidy programs

* *Correspondence to:* Kit Pong Wong, School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 2859-1044, fax: (852) 2548-1152, e-mail: kpwong@econ.hku.hk (K. P. Wong).

Taxes, Leverage, and Stimuli of Investment under Uncertainty

Abstract

This paper examines the effect of leverage on the effectiveness of a self-financed tax-subsidy program offered by a government in stimulating a firm's investment. We show that the firm, be it levered or unlevered, has an incentive to hasten its investment because of the agency conflicts arising from the commitment made by the government on the terms of the tax-subsidy program. We further show that the levered firm has a countervailing incentive to defer its investment due to the deadweight loss when bankruptcy occurs, which would be absent should the firm be unlevered. The former incentive is likely to be dominated by the latter incentive, in particular when the corporate income tax rate is sufficiently high and the bankruptcy cost is sufficiently low so that the firm relies heavily on debt. In this case, the tax-subsidy program induces the levered firm to defer, not hasten, its investment. Finally, we show that the levered firm is made worse off with than without the program because of the presence of agency and bankruptcy costs.

JEL classification: G31; G33; H25

Keywords: Capital structure; Investment timing; Real options; Tax-subsidy programs

1. Introduction

Economists have long argued that changes in tax policy have significant impacts on corporate investment (see, e.g., Cummins et al., 1996; Goukasian and Sarkar, 2006; Wong, 2011). Using a canonical real options model of McDonald and Siegel (1986) and Dixit and Pindyck (1994), Pennings (2000) shows that a government can implement a self-financed tax-subsidy program to hasten a firm's undertaking of a project. Specifically, the program consists of a lump-sum subsidy to the firm's irreversible investment cost, and a proportional tax on the stochastic earnings generated by the project. The government commits to these terms prior to the commencement of the project, which is endogenously chosen by the firm. The terms of the program are devised in a way that the lump-sum subsidy paid to the firm is exactly covered by the present value of all subsequent taxes collected from the firm. The government has rational expectations so that the tax-subsidy program has zero expected cost in equilibrium.

When the tax-subsidy program is utilized by the firm, it is as if the firm sells a fraction of its equity (on a before-tax basis) to the government at a predetermined price set equal to the lump-sum subsidy, where the fraction is simply the constant corporate income tax rate. Due to the commitment made by the government on the terms of the program, there are agency conflicts between the firm and the government. Succinctly, the firm has an incentive to make early investment so as to reduce the value of the equity stake that goes to the government. Fully anticipating this incentive problem, the government cuts down the lump-sum subsidy to ensure that the tax-subsidy program has zero expected cost in equilibrium. The firm as such is induced to hasten its investment, making the tax-subsidy program an effective investment stimulus. Indeed, the higher the corporate income tax rate, the faster the speed at which the project is undertaken. In the limit when the corporate income tax rate goes to unity, the firm follows the naive net present value rule by completely ignoring the opportunity cost arising from killing the investment option upon the commencement of the project. Since all agency costs are ultimately borne by the firm, the firm is made worse off with than without the tax-subsidy program (see also Pennings, 2005; Maoz, 2011).

While Pennings (2000) focuses on the case of all-equity financing, firms rely on debt to different extents in reality. The extant literature has shown that corporate investment and financing decisions are de facto interrelated with each other when market imperfections such as corporate taxes, bankruptcy costs, and agency conflicts prevail (see, e.g., Dotan and Ravid, 1985; Dammon and Senbet, 1988; Mauer and Triantis, 1994; Childs et al., 2005; Wong; 2010). It is thus of great interest to examine the effectiveness of the self-financed tax-subsidy program as an investment stimulus when a firm makes its investment and financing decisions simultaneously. To this end, we extend the real options model of Pennings (2000) by allowing the firm to choose its optimal capital structure that trades off the interest tax shield benefit against the bankruptcy cost of debt as in Leland (1994) and Goldstein et al. (2001).

Similar to the benchmark case wherein the firm is restricted to be all-equity financed, we show that the commitment made by the government on the terms of the tax-subsidy program gives rise to the agency conflicts in that the firm has an incentive to hasten its investment so as to take advantage of the government from over-pricing its equity. We refer to this as the mispricing incentive. Debt financing creates a countervailing incentive due to the deadweight loss when bankruptcy occurs, which would be absent should the firm be unlevered. The levered firm has to take this loss into account because debt holders correctly price the debt contract issued by the firm at the time when the project is undertaken. To reduce the deadweight loss, the levered firm has an incentive to postpone its investment until the firm becomes safer. We refer to this as the bankruptcy-avoidance incentive. We show that the bankruptcy-avoidance incentive is likely to dominate the mispricing incentive so that the tax-subsidy program may in fact induce the levered firm to defer, not hasten, its investment. This is particularly true when the firm borrows a lot, which is the case when the corporate income tax rate is sufficiently high and the bankruptcy cost is sufficiently low. Debt financing as such has the detrimental effect on making the tax-subsidy program ineffective in stimulating investment. Finally, we show that the levered firm is made worse off with than without the program because of the presence of agency and bankruptcy costs.

Danielova and Sarkar (2011) have recently examined the effect of leverage on a government's decision to offer a tax cut versus an investment subsidy in order to promote corporate investment. They show that it is generally optimal to adopt a combination of tax cut and investment subsidy from the government's perspective. Using numerical examples, they further show that a suboptimal combination of investment stimuli can result in substantial reduction of benefits for the government. This paper is similar to theirs in that both show that debt financing has significant impacts on the design of tax-subsidy programs as investment stimuli. The interaction between corporate investment and financing decisions has to be taken into account, or else a correct tax policy recommendation cannot be made.

The rest of this paper is organized as follows. Section 2 delineates our continuous-time model of an owner-managed firm that has a perpetual option to invest in a project under uncertainty. The firm has to make investment and financing decisions simultaneously after the government has committed to a self-financed tax-subsidy program. Section 3 derives the values of debt and equity of the firm at the investment instant. Section 4 examines the firm's optimal investment decision in the benchmark case of all-equity financing. Section 5 characterizes the firm's optimal investment and financing decisions, and shows that the tax-subsidy program can be ineffective as an investment stimulus. The final section concludes.

2. The model

Consider a risk-neutral, owner-managed firm that has monopoly access to a perpetual option to invest in a project.[1] The firm operates in continuous time indexed by $t \in [0, \infty)$. The riskless rate of interest is known and constant at $r > 0$ per unit time.

To undertake the project at an endogenously chosen time, $t \geq 0$, the firm has to incur a fixed investment cost, I, at that instant. The project then immediately generates a stream of stochastic earnings before interest and taxes (EBIT), X_t, that evolves over time according to the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \ \mathrm{d}t + \sigma X_t \ \mathrm{d}Z_t, \tag{1}$$

where $\mu < r$ and $\sigma > 0$ are constant parameters, and $\mathrm{d}Z_t$ is the increment of a standard Wiener process under the risk-neutral probability space, $(\Omega, \mathcal{F}, \mathcal{Q})$.[2] Eq. (1) implies that the growth rate of X_t is normally distributed with a mean, μ, and a variance, σ^2, per unit time. The initial value of the state variable, $X_0 > 0$, is known at $t = 0$. We assume throughout the paper that X_0 is sufficiently small such that an immediate exercise of the investment option by the firm is not optimal.

At $t = 0$, the government launches a tax-subsidy program with terms defined by a pair, (δ, τ), to stimulate the firm's investment. Specifically, the government subsidizes the firm by funding a fraction, $\delta \in (0, 1)$, of the fixed investment cost. The lump-sum subsidy, δI, is

[1] The assumption of risk neutrality is innocuous as long as there are arbitrage-free and complete financial markets in which assets can be traded to span the state variable that determines the value of the firm.

[2] The assumption that $\mu < r$ is needed to ensure the value of the firm to be finite.

financed by taxing the project's stream of EBIT at a constant corporate income tax rate, $\tau \in (0, 1)$, subject to full loss-offset provisions. Following Pennings (2000), we assume that the government commits to the terms of the tax-subsidy program, (δ, τ), at $t = 0$, and devises the program in a way that the lump-sum subsidy paid to the firm is equal to the value of all taxes collected from the firm. The government has rational expectations so that the program has zero expected cost in equilibrium.

When the state variable, X_t, is large enough such that the project becomes sufficiently lucrative, the firm finds it optimal to undertake the project by incurring the fixed investment cost, I, and the opportunity cost arising from killing the investment option. Hence, to solve the firm's investment problem is tantamount to finding a threshold value, $X_I > X_0$, such that the firm optimally exercises the investment option at the first instant when X_t reaches X_I from below (see, e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994). We refer to X_I as the investment trigger. Let $T_I = \inf\{t > 0 : X_t = X_I\}$ be the (random) first passage time that X_t reaches X_I from below, starting off at $t = 0$.

At the investment instant, T_I, the firm can issue debt and equity to finance the fixed investment cost, I, net of the lump-sum subsidy, δI, paid by the government. The debt issued by the firm is perpetual in that debt holders receive a constant coupon payment, $C \geq 0$, per unit time until default occurs, where C is the firm's financing decision. The coupon payments to debt holders are tax-deductible so that the interest tax-shield per unit time is τC prior to default and zero thereafter.

Shareholders have limited liability and thus the option to default on their debt obligations. We follow Leland (1994) and Goldstein et al. (2001) to adopt a stock-based definition of default in that shareholders optimally inject equity capital into the firm as long as the firm has positive economic net worth. To solve the optimal default policy for shareholders is tantamount to finding a threshold value, X_D, of the state variable, X_t, such that the value of equity vanishes at the first instant when X_t reaches X_D from above. We refer to X_D as the default trigger. Let $T_D = \inf\{t > T_I : X_t = X_D\}$ be the (random) first passage time that X_t reaches X_D from above, starting off at the investment instant, T_I.

At the default instant, T_D, the firm is immediately liquidated and absolute priority is enforced. Following Danielova and Sarkar (2011), we assume that the liquidation value of the firm at T_D is given by

$$(1-b)\mathrm{E}_{\mathcal{Q}}^{X_D}\left[\int_{T_D}^{\infty} e^{-r(t-T_D)}X_t \,\mathrm{d}t\right] = (1-b)\left(\frac{X_D}{r-\mu}\right), \tag{2}$$

where $\mathrm{E}_{\mathcal{Q}}^{X_D}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_D, and $b \in (0,1)$ is a parameter gauging the severity of bankruptcy costs. Since absolute priority is enforced, shareholders get nothing and debt holders receive the liquidation value upon default at T_D.

We describe the firm's investment and financing decisions by a pair, (X_I, C), that specifies the investment trigger, X_I, and the coupon payment, C. To obtain the solution to the model, we use backward induction and proceed in three steps. In the first step, we derive the value of debt, $D(X_I, C)$, and that of equity, $E(X_I, C)$, at the investment instant, T_I, taking the terms of the tax-subsidy program, (δ, τ), and the firm's investment and financing decisions, (X_I, C), as given. In the second step, we derive the firm's optimal investment and financing decisions, (X_I^L, C^L), that maximize the ex-ante value of equity prior to the debt issuance, taking the terms of the tax-subsidy program, (δ, τ), as given. Specifically, at the investment instant, T_I, the firm issues perpetual debt to raise $D(X_I, C)$ from debt holders. The difference, $(1-\delta)I - D(X_I, C)$, is raised from (paid to if negative) shareholders whose claim right after the debt issuance is worth $E(X_I, C)$. The ex-ante value of equity is therefore given by $E(X_I, C) - [(1-\delta)I - D(X_I, C)] = V(X_I, C) - (1-\delta)I$, where $V(X_I, C) = D(X_I, C) + E(X_I, C)$ is the value of the firm at T_I. Hence, maximizing the ex-ante value of equity is equivalent to maximizing the net present value of the project, taking the effect of leverage into account. In the final step, we solve for the rational expectations equilibrium in which the government breaks even with the tax-subsidy program, and the firm's optimal investment and financing decisions are consistent with the terms of the tax-subsidy program.

3. Valuation of corporate securities

Taking the terms of the tax-subsidy program, (δ, τ), and the firm's investment and financing decisions, (X_I, C), as given, the values of debt and equity at the investment instant, T_I, are given by

$$D(X_I, C) = \mathrm{E}_{\mathcal{Q}}^{X_I}\left\{ \int_{T_I}^{T_D(C)} e^{-r(t-T_I)} C \ \mathrm{d}t + e^{-r[T_D(C)-T_I]}(1-b)\left[\frac{X_D(C)}{r-\mu}\right]\right\}, \tag{3}$$

and

$$E(X_I, C) = \mathrm{E}_{\mathcal{Q}}^{X_I}\left[\int_{T_I}^{T_D(C)} e^{-r(t-T_I)}(1-\tau)(X_t - C) \ \mathrm{d}t\right], \tag{4}$$

respectively, where $T_D(C)$ is the default instant at which the state variable, X_t, reaches the default trigger, $X_D(C)$, from above, $\mathrm{E}_{\mathcal{Q}}^{X_I}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_I, and the liquidation value is given by Eq. (2).

It is well-known (see, e.g., Karatzas and Shreve, 1988; Dixit and Pindyck, 1994) that

$$\mathrm{E}_{\mathcal{Q}}^{X_I}\left\{e^{-r[T_D(C)-T_I]}\right\} = \left[\frac{X_D(C)}{X_I}\right]^{\alpha}, \tag{5}$$

if $X_I > X_D(C)$, where $\alpha = \mu/\sigma^2 - 1/2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} > 0$. Using the strong Markov property of Ito diffusions and Eq. (5), we can write Eqs. (3) and (4) as

$$D(X_I, C) = \frac{C}{r}\left\{1 - \left[\frac{X_D(C)}{X_I}\right]^{\alpha}\right\} + (1-b)\left[\frac{X_D(C)}{r-\mu}\right]\left[\frac{X_D(C)}{X_I}\right]^{\alpha}, \tag{6}$$

and

$$E(X_I, C) = (1-\tau)\left(\frac{X_I}{r-\mu} - \frac{C}{r}\right) + (1-\tau)\left[\frac{C}{r} - \frac{X_D(C)}{r-\mu}\right]\left[\frac{X_D(C)}{X_I}\right]^{\alpha}, \tag{7}$$

respectively. Differentiating Eq. (7) with respect to $X_D(C)$ and solving the first-order condition yields the optimal default trigger:

$$X_D(C) = (r-\mu)\left(\frac{\alpha}{\alpha+1}\right)\frac{C}{r}. \tag{8}$$

It is evident from Eq. (8) that $0 < X_D(C) < C$, i.e., the firm is insolvent on a flow basis at the default instant (see also Leland, 1994; Goldstein et al., 2001; Wong, 2010).

The value of the levered firm, $V(X_I, C)$, at the investment instant, T_I, is equal to the sum of the value of debt, $D(X_I, C)$, and the value of equity, $E(X_I, C)$, at that time. Using Eqs. (6) and (7), we have

$$V(X_I, C) = \frac{X_I}{r-\mu} - \tau\left\{\frac{X_I}{r-\mu} - \frac{C}{r} + \left[\frac{C}{r} - \frac{X_D(C)}{r-\mu}\right]\left[\frac{X_D(C)}{X_I}\right]^{\alpha}\right\}$$

$$-b\left[\frac{X_D(C)}{r-\mu}\right]\left[\frac{X_D(C)}{X_I}\right]^{\alpha}. \tag{9}$$

The second term on the right-hand side of Eq. (9) is the value of all taxes, net of interest tax-shields, prior to default, and the last term is the value of bankruptcy costs paid upon default, all discounted back to T_I.

4. Benchmark case of all-equity financing

As a benchmark, we restrict the firm to finance the project solely with equity in this section. This is the case analyzed by Pennings (2000) (see also Pennings, 2005; Yu et al., 2007; Maoz, 2011).

Taking the terms of the tax-subsidy program, (δ, τ), as given, the value of the unlevered firm at $t = 0$ isgiven by

$$F^U(X_0) = \max_{X_I > X_0} \mathrm{E}_{\mathcal{Q}}^{X_0}\left[\int_{T_I}^{\infty} e^{-rt}(1-\tau)X_t \,\mathrm{d}t - e^{-rT_I}(1-\delta)I\right], \tag{10}$$

where T_I is the investment instant, and $\mathrm{E}_{\mathcal{Q}}^{X_0}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_0. It is well-known (see, e.g., Karatzas and Shreve 1988; Dixit and Pindyck 1994) that

$$\mathrm{E}_{\mathcal{Q}}^{X_0}\left(e^{-rT_I}\right) = \left(\frac{X_0}{X_I}\right)^{\beta}, \tag{11}$$

if $X_I > X_0$, where $\beta = 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} > 1$. Using the strong Markov property of Ito diffusions and Eq. (11), we can write Eq. (10) as

$$F^U(X_0) = \max_{X_I > X_0} \left[(1-\tau)\left(\frac{X_I}{r-\mu}\right) - (1-\delta)I\right]\left(\frac{X_0}{X_I}\right)^{\beta}. \tag{12}$$

The first-order conditions for the optimization problem on the right-hand side of Eq. (12) are given by

$$(1-\tau)\left(\frac{X_I^U}{r-\mu}\right) = \left(\frac{\beta}{\beta-1}\right)(1-\delta)I, \tag{13}$$

where X_I^U is the optimal investment trigger of the unlevered firm. Solving Eq. (13) yields

$$X_I^U = (r-\mu)\left(\frac{1-\delta}{1-\tau}\right)\left(\frac{\beta}{\beta-1}\right)I. \tag{14}$$

The expression, $\beta/(\beta-1) > 1$, is referred to as the option value multiple (Abel et al., 1996). It measures the wedge between the value of the project at the investment instant, $(1-\tau)X_I^U/(r-\mu)$, and the fixed investment cost, $(1-\delta)I$, which captures the opportunity cost arising from killing the investment option when the project is undertaken, as is evident from Eq. (14).

The government devises the terms of the tax-subsidy program, (δ, τ), in a way that the lump-sum subsidy paid to the firm is equal to the value of all taxes collected from the firm:

$$\delta I = \mathrm{E}_{\mathcal{Q}}^{X_I^U}\left[\int_{T_I^U}^{\infty} e^{-r(t-T_I^U)}\tau X_t \, \mathrm{d}t\right] = \frac{\tau X_I^U}{r-\mu}, \tag{15}$$

where T_I^U is the investment instant at which the state variable, X_t, reaches the investment trigger, X_I^U, from above, and $\mathrm{E}_{\mathcal{Q}}^{X_I^U}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_I^U. For a fixed corporate income tax rate, $\tau \in (0,1)$, the rational expectations equilibrium is the one that solves Eqs. (14) and (15) simultaneously, which yields $\delta = \beta\tau/(\beta - 1 + \tau)$ and

$$X_I^U = (r-\mu)\left(\frac{\beta}{\beta-1+\tau}\right)I. \tag{16}$$

Substituting Eq. (15) into Eq. (12) yields the value of the unlevered firm at $t = 0$ in the rational expectations equilibrium:

$$F^U(X_0) = \left(\frac{X_I^U}{r-\mu} - I\right)\left(\frac{X_0}{X_I^U}\right)^{\beta}, \tag{17}$$

where X_I^U is given by Eq. (16).

If there are neither taxes nor subsidies, i.e., $\tau = \delta = 0$, Eq. (14) reduces to

$$X_I^0 = (r-\mu)\left(\frac{\beta}{\beta-1}\right)I, \tag{18}$$

which is the investment trigger given in the real options literature (see, e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994). The value of the firm at $t = 0$ is given by

$$F^0(X_0) = \left(\frac{X_I^0}{r-\mu} - I\right)\left(\frac{X_0}{X_I^0}\right)^{\beta}, \tag{19}$$

where X_I^0 is given by Eq. (18). It is evident from Eqs. (16) and (18) that $X_I^U < X_I^0$ for all $\tau \in (0,1)$. The break-even tax-subsidy program as such speeds up investment by lowering the investment trigger.[3] Since X_I^0 is the optimal investment trigger when $\tau = \delta = 0$, comparing Eqs. (17) and (19) immediately implies that $F^U(X_0) < F^0(X_0)$ for all $\tau \in (0,1)$. The unlevered firm as such is made worse off with than without the break-even tax-subsidy program. We thus establish the following proposition.

Proposition 1. *All break-even tax-subsidy programs induce the unlevered firm to lower its investment trigger, and reduce the value of the unlevered firm, as compared to those without the break-even tax-subsidy program, i.e.,* $X_I^U < X_I^0$ *and* $F^U(X_0) < F^0(X_0)$ *for all* $\tau \in (0,1)$.

The intuition for Proposition 1 is as follows. Given the tax-subsidy program with terms, (δ, τ), the government commits to purchasing a fraction, τ, of the firm's equity (on a before-tax basis) at a price set equal to δI, which is fixed at $t = 0$. The firm as such has an incentive

[3] It is well known (see, e.g., Sarkar, 2000; Shackleton and Wojakowski, 2002; Wong, 2007) that the expected time to exercise the investment option (investment timing) is given by $\mathrm{E}_Q^{X_0}(T_I) = \ln(X_I/X_0)/(\mu - \sigma^2/2)$, whenever $\mu > \sigma^2/2$. Hence, the investment trigger and the investment timing are positively related.

to take on the project earlier so as to make the value of the equity stake that goes to the government worth less than δI. The government fully anticipates this incentive and thus lowers the price, δI, to ensure that the tax-subsidy program indeed has zero expected cost in equilibrium. The investment trigger, X_I^U, as such is smaller than the one without the program, X_I^0 (see also Pennings, 2000). Eq. (16) implies that X_I^U decreases with an increase in τ. As τ goes to unity, Eq. (16) reduces to $X_I^U/(r-\mu) = I$ so that the firm follows the naive net present value rule and completely ignores the opportunity cost arising from killing the investment option upon the commencement of the project. The agency cost is defined by $F^0(X_0) - F^U(X_0) > 0$, which is strictly increasing in τ. This cost is ultimately borne by the unlevered firm, thereby making the unlevered firm worse off with than without the break-even tax-subsidy program (see also Pennings, 2005; Maoz, 2011).

5. Investment and leverage

In this section, we resume our original case that the firm can issue perpetual debt to raise $D(X_I, C)$ from debt holders at the investment instant, T_I. The difference, $(1-\delta)I - D(X_I, C)$, is raised from (paid to if negative) shareholders. The ex-ante value of equity prior to the debt issuance is, therefore, given by $E(X_I, C) - [(1-\delta)I - D(X_I, C)] = V(X_I, C) - (1-\delta)I$.

Taking the terms of the tax-subsidy program, (δ, τ), as given, the value of the levered firm at $t = 0$ is given by

$$
\begin{aligned}
F(X_0) &= \max_{X_I > X_0, C \geq 0} \mathrm{E}_{\mathcal{Q}}^{X_0}\left\{ e^{-rT_I}[V(X_I, C) - (1-\delta)I] \right\} \\
&= \max_{X_I > X_0, C \geq 0} [V(X_I, C) - (1-\delta)I]\left(\frac{X_0}{X_I}\right)^{\beta}, \qquad (20)
\end{aligned}
$$

where $V(X_I, C)$ is given by Eq. (9), and the second equality follows from Eq. (11). We solve the optimization problem on the right-hand side of Eq. (20) in two steps. First, we derive the optimal coupon payment, $C(X_I)$, for a fixed value of $X_I > X_0$. Then, we derive the optimal investment trigger, X_I^L, taking the schedule of the optimal coupon payments, $C(X_I)$, as given. The solution to the optimization problem on the right-hand side of Eq. (20) is therefore given by X_I^L and $C^L = C(X_I^L)$.

For a given value of $X_I > X_0$, the first-order condition for the optimization problem on the right-hand side of Eq. (20) is given by

$$\tau - (\tau + b\alpha)\left\{\frac{X_D[C(X_I)]}{X_I}\right\}^{\alpha} = 0, \tag{21}$$

where $C(X_I)$ is the optimal coupon payment. Using Eq. (8), we solve Eq. (21) to yield

$$C(X_I) = r\phi\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{X_I}{r-\mu}\right), \tag{22}$$

where $\phi = \tau^{1/\alpha}/(\tau + b\alpha)^{1/\alpha} \in (0, 1)$. Substituting Eqs. (21) and (22) into the right-hand side of Eq. (20) yields

$$F(X_0) = \max_{X_I > X_0} \left\{[1 - \tau(1-\phi)]\left(\frac{X_I}{r-\mu}\right) - (1-\delta)I\right\}\left(\frac{X_0}{X_I}\right)^{\beta}. \tag{23}$$

Inspection of Eqs. (12) and (23) reveals that the effect of the optimal leverage on firm value, i.e., $F(X_0) - F^U(X_0)$, is equivalent to that of a reduction in the corporate income tax rate from τ to $\tau(1-\phi)$ on the value of the unlevered firm. It is evident from Eq. (21) that the schedule of the optimal coupon payments, $C(X_I)$, is linear in the EBIT at the investment instant, X_I. The values of equity and debt, evaluated at $C(X_I)$, are thus also linear in X_I, as is evident from Eqs. (7) and (6). This is referred to as the scaling property in the real options literature on capital structure (see, e.g., Goldstein et al., 2001; Strebulaev, 2007; Wong, 2010; Liu and Wong, 2011). As is shown on the right-hand side of Eq. (23), the value of the interest tax-shield benefits of debt, net of bankruptcy costs, is equal to $\tau\phi X_I/(r-\mu)$, which is tantamount to the case of a reduction in the corporate income tax rate from τ to $\tau(1-\phi)$ faced by the unlevered firm. Using Eq. (14) with τ replaced by $\tau(1-\phi)$ yields the optimal investment trigger of the levered firm:

$$X_I^L = (r-\mu)\left[\frac{1-\delta}{1-\tau(1-\phi)}\right]\left(\frac{\beta}{\beta-1}\right)I. \tag{24}$$

The government devises the terms of the tax-subsidy program, (δ, τ), in a way that the lump-sum subsidy paid to the firm is equal to the value of all taxes collected from the firm at the investment instant, T_I^L:

$$\delta I = \mathrm{E}_{\mathcal{Q}}^{X_I^L}\left[\int_{T_I^L}^{T_D(C^L)} e^{-r(t-T_I^L)}\tau(X_t - C^L)\ \mathrm{d}t\right] = \left(\frac{\tau}{1-\tau}\right)E(X_I^L, C^L), \tag{25}$$

where $T_D(C^L)$ is the default instant at which the state variable, X_t, reaches the default trigger, $X_D(C^L)$, from above, $\mathrm{E}_{\mathcal{Q}}^{X_I^L}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_I^L, and the second equality follows from Eq. (4).[4] Inspection of Eq. (25) reveals that the break-even tax-subsidy program is equivalent to letting the government pay δI to buy a fraction, τ, of the firm's equity (before tax) at the investment instant, T_I^L. Using Eqs. (7), (8), (21), and (22), we can write Eq. (25) as

$$\delta I = \left(\frac{X_I^L}{r-\mu}\right)[\tau(1-\phi) - b\phi^{\alpha+1}]. \tag{26}$$

For a fixed corporate income tax rate, $\tau \in (0,1)$, the rational expectations equilibrium is the one that solves Eqs. (24) and (26) simultaneously, which yields $\delta = \beta[\tau(1-\phi) - b\phi^{\alpha+1}]/[\beta - 1 + \tau(1-\phi) - b\beta\phi^{\alpha+1}]$ and[5]

$$X_I^L = (r-\mu)\left\{\frac{\beta}{\beta - 1 + \tau(1-\phi) - b\beta\phi^{\alpha+1}}\right\}I. \tag{27}$$

Substituting Eq. (26) into Eq. (23) yields the value of the levered firm at $t = 0$ in the rational expectations equilibrium:

$$F(X_0) = \left[(1 - b\phi^{\alpha+1})\left(\frac{X_I^L}{r-\mu}\right) - I\right]\left(\frac{X_0}{X_I^L}\right)^{\beta}, \tag{28}$$

where X_I^L is given by Eq. (27).

Denote the function, $H(\tau) = \tau(1-\phi) - b\beta\phi^{\alpha+1}$. Inspection of Eqs. (18) and (27) reveals that $X_I^L < (>)\ X_I^0$ if $H(\tau) > (<)\ 0$ for all $\tau \in (0,1)$. In contrast to the benchmark case of all-equity financing, we show in the following proposition that the break-even tax-subsidy

[4] The government's tax revenue is $\tau(X_t - C^L)$ per unit time until bankruptcy occurs. After bankruptcy, the government gets nothing since the firm's project is liquidated.

[5] When $\phi = 0$, which is the case when the firm is unlevered, Eq. (27) reduces to Eq. (16).

program may in fact induce the levered firm to raise, not lower, its investment trigger above X_I^0.

Proposition 2. *Let b^* be the unique solution to $(1+b^*\alpha)^{1/\alpha+1} - b^*(\beta+\alpha) - 1 = 0$. If the bankruptcy cost parameter, b, is less than $\min(b^*, 1)$, there exists a unique corporate income tax rate, $\tau^* \in (0,1)$, that solves $H(\tau^*) = 0$ such that a break-even tax-subsidy program with $\tau < (>)\ \tau^*$ induces the levered firm to choose its investment trigger below (above) the one without the break-even tax-subsidy program, i.e., $X_I^L < (>)\ X_I^0$ for all $\tau < (>)\ \tau^*$. If $b \geq \min(b^*, 1)$, all break-even tax-subsidy programs induce the levered firm to choose its investment trigger below the one without the break-even tax-subsidy program, i.e., $X_I^L < X_I^0$ for all $\tau \in (0,1)$.*

Proof. See Appendix A. □

It is easily verified that $b^* > (<)\ 1$ if $\beta > (<)\ (1+\alpha)[(1+\alpha)^{1/\alpha} - 1]$. To see that either condition can hold, suppose that $\mu = 0$ so that $\beta = 1 + \alpha$. In this case, the condition reduces to $(1+\alpha)^{1/\alpha} < (>)\ 2$, which holds if $\alpha > (<)\ 1$, i.e., if $r > (<)\ \sigma^2$ given that $\mu = 0$. When the firm can choose its capital structure optimally, Proposition 2 shows that a break-even tax-subsidy program may in fact induce the levered firm to raise, not lower, its investment trigger above X_I^0. For example, if the project has zero growth, $\mu = 0$ and there is not much uncertainty, $\sigma^2 < r$, introducing a break-even tax-subsidy problem with a high enough corporate income tax rate, $\tau > \tau^*$, defers the levered firm's investment for all $b \in (0,1)$.

To see the intuition for Proposition 2, suppose that the government naively believes that the firm would choose the investment trigger, X_I^0, given by Eq. (18). The government as such chooses δ that solves Eq. (26) with X_I^L replaced by X_I^0 and C^L replaced by $C^0 = C(X_I^0)$. Taking the terms of the tax-subsidy program, (δ, τ), as given, we can write

Eq. (20) as

$$
\begin{aligned}
F(X_0) &= \max_{X_I > X_0} \{V[X_I, C(X_I)] - (1-\delta)I\} \left(\frac{X_0}{X_I}\right)^\beta \\
&= \max_{X_I > X_0} \left(\frac{X_I}{r-\mu} - I\right)\left(\frac{X_0}{X_I}\right)^\beta \\
&\qquad + \left(\frac{\tau}{1-\tau}\right)\{E(X_I^0, C^0) - E[X_I, C(X_I)]\}\left(\frac{X_0}{X_I}\right)^\beta \\
&\qquad - b\left\{\frac{X_D[C(X_I)]}{r-\mu}\right\}\left\{\frac{X_D[C(X_I)]}{X_I}\right\}^\alpha \left(\frac{X_0}{X_I}\right)^\beta, \qquad (29)
\end{aligned}
$$

where we have used Eqs. (9) and (26). The first term on the right-hand side of Eq. (29) is the value of the firm at $t = 0$ in the absence of the tax-subsidy program, which is maximized at $X_I = X_I^0$. The second term captures the mispricing of the fraction, τ, of the before-tax equity stake that is purchased by the government. This gives rise to an incentive that induces the firm to set the investment trigger below X_I^0 to take advantage of the government from the mispricing gain, which we have shown in the case of the unlevered firm in Section 4. We refer to this as the mispricing incentive. The final term captures the loss in value due to bankruptcy, which would be absent should the firm be all-equity financed. To reduce this loss, the levered firm has an incentive to raise the investment trigger. We refer to this as the bankruptcy-avoidance incentive. This incentive is profound only when the firm borrows a lot, which would be the case should the corporate income tax rate, τ, be sufficiently high and the bankruptcy cost parameter, b, be sufficiently low. Specifically, when $\tau > \tau^*$ and $b < b^*$, the bankruptcy-avoidance incentive dominates the mispricing incentive, thereby inducing the levered firm to choose its investment trigger that is higher than the one without the tax-subsidy program, i.e., $X_I^L > X_I^0$. In this case, debt financing has the detrimental effect on making the tax-subsidy program completely ineffective in stimulating investment.

Since $b\phi^{\alpha+1} > 0$ for all $\tau > 0$, it follows from Eq. (28) that

$$
F(X_0) < \left(\frac{X_I^L}{r-\mu} - I\right)\left(\frac{X_0}{X_I^L}\right)^\beta < \left(\frac{X_I^0}{r-\mu} - I\right)\left(\frac{X_0}{X_I^0}\right)^\beta, \qquad (30)
$$

where the second inequality follows from the fact that X_I^0 is the optimal investment trigger when $\tau = \delta = 0$. Eqs. (19) and (30) imply that $F(X_0) < F^0(X_0)$ for all $\tau \in (0, 1)$, thereby invoking the following proposition.

Proposition 3. *The levered firm is always made worse off with than without the break-even tax-subsidy program, i.e.,* $F(X_0) < F^0(X_0)$ *for all* $\tau \in (0, 1)$.

When the break-even tax-subsidy program is in place, the levered firm faces two sources of costs. First, the interest tax shield benefits induce the firm to borrow, which gives rise to the bankruptcy cost. Second, the incentive problem of the firm to take advantage of the government creates the agency cost that is ultimately borne by the firm. The levered firm as such is made worse off with than without the break-even tax-subsidy program.[6]

6. Conclusion

In this paper, we examine the effect of leverage on the effectiveness of a self-financed tax-subsidy program offered by a government in stimulating a firm's investment. To this end, we incorporate the static trade-off model of capital structure (Leland, 1994; Goldstein et al., 2001) into the real options model of Pennings (2000). One the one hand, the firm, be it levered or unlevered, has an incentive to hasten its investment because of the agency conflicts arising from the commitment made by the government on the terms of the tax-subsidy program (the mispricing incentive). On the other hand, the levered firm has a countervailing incentive to defer its investment due to the deadweight loss when bankruptcy occurs (the bankruptcy-avoidance incentive), which would be absent should the firm be unlevered. We show that the bankruptcy-avoidance incentive can dominate the mispricing incentive, in particular when the corporate income tax rate is sufficiently high and the bankruptcy cost is sufficiently low so that the firm relies heavily on debt. In this case, the

[6] Since the levered firm is free to choose to be all-equity financed, the value of the levered firm must be strictly higher than that of the unlevered firm.

tax-subsidy program induces the levered firm to defer, not hasten, its investment. Debt financing as such has the detrimental effect on reducing the effectiveness of the tax-subsidy program as an investment stimulus. Finally, we show that the levered firm is made worse off with than without the program because of the presence of agency and bankruptcy costs.

Appendix A

Differentiating $H(\tau)$ twice with respect to τ yields

$$H'(\tau) = 1 - \phi - \frac{b\phi}{\tau + b\alpha} - \frac{b^2\beta(\alpha+1)\phi}{(\tau+b\alpha)^2}, \tag{A.1}$$

and

$$H''(\tau) = -\frac{b^2(\alpha+1)\phi[\tau(1-2\beta) + b(\alpha+\beta)]}{\tau(\tau+b\alpha)^3}, \tag{A.2}$$

where we have used the fact that $\phi^\alpha = \tau/(\tau + b\alpha)$ and $\partial\phi/\partial\tau = b\phi/\tau(\tau+b\alpha)$. Eq. (A.2) implies that $H(\tau)$ is concave (convex) for all $\tau < (>)\ b(\alpha+\beta)/(2\beta-1)$. From Eq. (A.1), we have $H'(0) = 1$. Since $H(0) = 0$, we have $H(\tau) > 0$ when τ is sufficiently close to zero. Evaluating $H(\tau)$ at $\tau = 1$ yields

$$H(1) = 1 - \frac{1 + b(\alpha+\beta)}{(1+b\alpha)^{1/\alpha+1}}. \tag{A.3}$$

Differentiating Eq. (A.3) with respect to b yields

$$\frac{\partial H(1)}{\partial b} = \frac{b(\alpha+\beta) + 1 - \beta}{(1+b\alpha)^{1/\alpha+2}}. \tag{A.4}$$

Eq. (A.4) implies that $H(1)$ is decreasing (increasing) in b for all $b < (>)\ (\beta-1)/(\alpha+\beta)$. When b approaches zero, we have $H(1)$ approaches zero from Eq. (A.3). Hence, there must exist a unique value, $b^* > 0$, such that $H(1) < (>)\ 0$ for all $b < (>)\ b^*$, where b^* solves $(1 + b^*\alpha)^{1/\alpha+1} - b^*(\beta+\alpha) - 1 = 0$.

If $\beta \geq (1+\alpha)[(1+\alpha)^{1/\alpha} - 1]$, then $b^* \geq 1$. In this case, we have $H(1) < 0$ for all $b \in (0, 1)$. It follows from $H(0) > 0$ and the shape of $H(\tau)$, i.e., first concave then convex, that there must exist a unique corporate income tax rate, $\tau^* \in (0,1)$, at which $H(\tau^*) = 0$ such that $H(\tau) > (<)\ 0$ for all $\tau < (>)\ \tau^*$, thereby implying that $X_I^L < (>)\ X_I^0$ for all $\tau < (>)\ \tau^*$. Totally differentiating $H(\tau^*)$ with respect to b yields

$$\left[\frac{b\beta\phi^*}{(\tau^* + b\alpha)^2} - \left(1 + \frac{b\beta}{\tau^* + b\alpha}\right)\frac{\partial\phi^*}{\partial\tau}\right]\frac{\mathrm{d}\tau^*}{\mathrm{d}b} = \frac{\beta\phi^*\tau^*}{(\tau^* + b\alpha)^2} + \left(1 + \frac{b\beta}{\tau^* + b\alpha}\right)\frac{\partial\phi^*}{\partial b}. \qquad \text{(A.5)}$$

Since $\partial\phi/\partial\tau = b\phi/\tau(\tau + b\alpha)$ and $\partial\phi/\partial b = -\phi/(\tau + b\alpha)$, Eq. (A.5) reduces to $\mathrm{d}\tau^*/\mathrm{d}b = \tau^*/b > 0$.

If $\beta < (1+\alpha)[(1+\alpha)^{1/\alpha} - 1]$, then $b^* < 1$. In this case, we have $H(1) < 0$ for all $b \in (0, b^*)$. It follows that there must exist a unique corporate income tax rate, $\tau^* \in (0,1)$, at which $H(\tau^*) = 0$ such that $H(\tau) > (<)\ 0$ for all $\tau < (>)\ \tau^*$, thereby implying that $X_I^L < (>)\ X_I^0$ for all $\tau < (>)\ \tau^*$. Since $\mathrm{d}\tau^*/\mathrm{d}b = \tau^*/b > 0$, we have τ^* goes to unity as b approaches b^*. By continuity, it must be true that $H(\tau) > 0$, and thereby $X_I^L < X_I^0$, for all $\tau \in (0,1)$ when $b \in (b^*, 1)$.

References

Abel, A. B., Dixit, A. K., Eberly, J. C., Pindyck, R. S., 1996. Options, the value of capital, and investment. Quarterly Journal of Economics 111, 753–777.

Childs, P. D., Mauer, D. C., Ott, S. H., 2005. Interactions of corporate financing and investment decisions: The effects of agency conflicts. Journal of Financial Economics 76, 667–690.

Cummins, J. C., Hassett, K. A., Hubbard, R. G., 1996. Tax reforms and investment: A cross-country comparison. Journal of Public Economics 62, 237–273.

Dammon, R. M., Senbet, L. W., 1988. The effect of taxes and depreciation on corporate investment and financial leverage. Journal of Finance 43, 357–373.

Danielova, A., Sarkar, S., 2011. The effect of leverage on the tax-cut versus investment-subsidy argument. Review of Financial Economics 20, 123–129.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainty. Princeton University Press, Princeton, NJ.

Dotan, A., Ravid, S. A., 1985. On the interaction of real and financial decisions of the firm under uncertainty. Journal of Finance 40, 501–517.

Goldstein, R., Ju, N., Leland, H. E., 2001. An EBIT-based model of dynamic capital structure. Journal of Business 74, 483–512.

Goukasian, L., Sarkar, S., 2006. The effect of tax convexity on corporate investment decisions and tax burdens. Journal of Public Economic Theory 8, 293–320.

Karatzas, I., Shreve, S. E., 1988. Brownian Motion and Stochastic Calculus. Springer-Verlag, Berlin, Germany.

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance 49, 1213–1252.

Liu, Q., Wong, K. P., 2011. Intellectual capital and financing decisions: Evidence from the U.S. patent data. Management Science 57, 1861–1878.

Maoz, Y. D., 2011. Tax, stimuli of investment and firm value. Metroeconomica 62, 171–174.

Mauer, D. C., Triantis, A. J., 1994. Interactions of corporate financing and investment decisions: A dynamic framework. Journal of Finance 49, 1253–1277.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. Quarterly Journal of Economics 101, 707–727.

Pennings, E., 2000. Tax and stimuli of investment under uncertainty. European Economic Review 44, 383–391.

Pennings, E., 2005. How to maximize domestic benefits from foreign investments: The effect of irreversibility and uncertainty. Journal of Economic Dynamics and Control 29, 873–889.

Sarkar, S., 2000. On the investment-uncertainty relationship in a real options model. Journal of Economic Dynamics and Control 24, 219–225.

Shackleton, M. B., Wojakowski, R., 2002. The expected return and exercise time of Merton-style real options. Journal of Business Finance and Accounting 29, 541–555.

Strebulaev, I. A., 2007. Do tests of capital structure theory mean what they say? Journal of Finance 62, 1747–1787.

Wong, K. P., 2007. The effect of uncertainty on investment timing in a real options model. Journal of Economic Dynamics and Control 31, 2152–2167.

Wong, K. P., 2010. On the neutrality of debt in investment intensity. Annals of Finance 6, 335–356.

Wong, K. P., 2011. Progressive taxation and the intensity and timing of investment. Economic Modelling 28, 100–108.

Yu, C.-F., Chang, T.-C., Fan, C.-P., 2007. FDI timing: Entry cost subsidy versus tax rate reduction. Economic Modelling 24, 262–271.