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ORIGINAL ARTICLE

Curriculum traditions in Berlin and Hong Kong: a comparative case study of the implemented mathematics curriculum

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Abstract Many studies (such as Pepin in Learners and pedagogy, Sage Publications, London, 1999; Kaiser in ZDM 34(6):241–257, 2002; Park and Leung in Mathematics education in different cultural traditions: a comparative study of East Asia and the West. The 13th ICMI Study, pp. 227–238, Springer, New York, 2006) have revealed that there is a strong dependence on cultural traditions in mathematics teaching in different countries. Education in Germany is influenced by the Central and North European Didaktik tradition (Westbury in Teaching as a reflective practice: the German Didaktik tradition, L. Erlbaum Associates, Mahwah, pp. 15–39, 2000), while that in East Asia is influenced by Confucian heritage culture. However, there have not been studies investigating the relationships between these two cultural traditions and their influences on teaching and learning. This study aims at filling this gap in knowledge. Some commonalities in the aims and beliefs in the underlying philosophies in education in traditional China and Germany were found and are presented in this paper. Specifically, the relationship between cultural traditions and the implemented mathematics curriculum was investigated, using Berlin and Hong Kong as examples. It was found that culture affects the implemented curriculum in a complicated way and that other factors such as the intended curriculum and textbooks may also influence the implemented curriculum.

Keywords Didaktik tradition · Confucian heritage culture · Implemented curriculum

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1 Introduction

Many studies (such as Pepin 1999; Kaiser 2002; Park and Leung 2006) have revealed that there is a strong dependence on cultural traditions in mathematics teaching in different countries. Education in Germany is influenced by the Central and North European Didaktik tradition (Westbury 2000), while that in East Asia is influenced by the Confucian heritage culture (CHC). However, there have not been studies investigating the relationships between these two cultural traditions and their influences on teaching and learning. This study aims at filling this gap in knowledge. In this paper, the relationship between cultural traditions and the implemented mathematics curriculum will be investigated. The implemented curriculum is what teachers actually teach in classrooms and includes all classroom activities. Berlin and Hong Kong will be taken as examples, rather than representative of the two cultural traditions, to illustrate the relationship.

Culture may refer to ideas, methods of thinking, values, beliefs, customs and traditions (Leung et al. 2006). Valsiner (1989), for example, suggested that culture has been understood as "something that is, in its essence, shared in a qualitatively similar manner by all (or almost all) members of the given 'culture' (as a population, society, or an ethnic group)" (p. 503). Education in a particular social environment is influenced in many ways by the culture of such environment and hence differs across countries or regions with different cultural backgrounds. Such difference is particularly significant between the West and the East (Leung 2006). Nonetheless, some commonalities in the aims and beliefs in the underlying philosophies in education in traditional China and Germany were found from the literature and will be presented in this paper. Further, given a similar humanistic approach in education, will the

implemented curricula in Berlin and Hong Kong be similar? If yes, in what ways are they similar? If no, in addition to the cultural traditions what factors also influence the implemented curriculum? Our research questions are: What are the similarities and differences in the implemented curriculum in Berlin and Hong Kong? How are they related to the underlying cultural traditions?

In Sect. 2 of this paper, cultural traditions in Germany and China are depicted and discussed. The methodology of this study is described in Sect. 3, followed by the results in Sect. 4. In Sect. 5 there is discussion and a conclusion.

2 The educational traditions in Germany and Hong Kong

2.1 Germany

Didaktik originates from the Greek *didaskein*, which meant "to teach, to be a teacher, to educate". In modern German, *Didaktik* is generally defined as the art or study of teaching. Since the sixteenth century, Didaktik has been the most important tool for planning, enacting and thinking about teaching in most of northern and central Europe (Westbury 1998). Indeed, it is impossible to understand German schooling without appreciating the role and impact of Didaktik. The Bohemian bishop Jan Amos Komensky (also known as Comenius), 1592–1670, wrote the book *Didactica Magna* (Comenius 1638), which suggested education for all. For example, in Chapter 8 and Chapter 9, he suggested that the young of both sexes must be educated. He also suggested some principles of teaching and learning (Chapters 16–24). The rise of mass schooling, sparked by the Lutheran school reformation in the sixteenth century, brought about a situation in which Didaktik became the common approach for planning lessons and legitimizing schooling in central and northern Europe (Hopmann and Riquarts 2000).

Bildung is an important concept in Didaktik. Hudson et al. (1999) conceived Bildung as "an (intermediate) actual state in the process of personality development" and also as "an ideal norm". According to Howson et al. (1981), Bildung comprises learning as universal as possible with strong emphasis on humanities—philosophy, history, literature, art and music—and also with an emphasis on mathematics and sciences. The ideal was the completely cultivated, fully educated human being. Bildung was not a process ending at the end of one's studies, but was just the base laid in youth to be enlarged and enriched throughout life. Bildung is an attitude and a path as much as an accomplishment (Keitel 2006). In general, the concept of Bildung incorporates encyclopedic rationalism as well as humanist moralism (Pepin 1999).

2.2 Hong Kong

The population of Hong Kong is predominantly ethnic Chinese. Although some people have adopted Western lifestyles, the majority still adhere to traditional Chinese values in various aspects of living. These include an emphasis on interpersonal relationships, courtesy, "face" and trust in verbal agreements (Leung 1999). The Chinese are also known to place high emphasis on education. This can be explained by the Confucian view of education. The aim of education is not the pursuit of knowledge for knowledge's sake, but the development of the character of the learner (Leung 1999). Lee (1996) pointed out that the Chinese emphasis on education "rests upon the Confucian presumption that everyone is educable" (p. 28). Confucius acknowledged that there are individual differences in intelligence, but he believed that "differences in intelligence…do not inhibit one's educability".

With regard to the features and values in education of the East Asian society, as highlighted by Leung (2001), the traditional view is that the body of knowledge should be "transferred" from teachers to students. On the premise of this main focus, the East Asian society stresses the importance of reflection and understanding in learning, but also registers a degree of emphasis on memorization or repetitive learning. Referring to Marton (1997), repetitive learning is "continuous practice with increasing variation" which leads to deep understanding. However, because of such emphasis, the learning process in the East Asian society is sometimes just regarded as learning by rote (committing to memory without understanding). Taking the view that understanding is a continuous process or a continuum, and considering the general belief in the East Asian society that the learning process inevitably interacts with repeated practice, memorizing and understanding, Leung (2001) nonetheless argued that this was too simplistic.

Leung (2001) also pointed out that "studying is a serious endeavor" is almost a motif in the Chinese culture. Following this, the East Asian society mostly believes extrinsic motivations to be an acceptable and healthy means to drive students to learn. Also, through hard work, the studying process should lead to contentedness with a deep knowledge of the subject. The important position of such extrinsic motivations in the East Asian society could be attributed to a number of factors. These include the great trust in competition and examination as a fair method of differentiating between the able and the less able (Cheng 1994), the perception that good academic achievement signifies recognition and honor, and society's emphasis on academic achievement for career success.

Leung (2001) suggested that the East Asian society highly treasures learning together in a social setting.

In such a setting, teachers are considered as an essential role model and are expected to be experts or learned figures. It is believed that, when compared with expertise in pedagogy, a good grasp of the subject matter is more important and serves as a prerequisite for invoking appropriate pedagogy. This naturally leads to the typical "direct teaching to the whole class" mode of mathematics teaching commonly found in the East Asian society.

2.3 Discussion of the two traditions

2.3.1 Aims

The aims of both the Didaktik tradition and Confucianism in education are similar. In the Didaktik tradition, it was expected that one would become part of an intellectual or cultural élite through education. In East Asia, the teaching of Confucius expected one to become a proper man or a gentleman. Thus, these two cultures have both stressed the goal of becoming a good member of society.

Gravemeijer and Terwel (2000) suggested that Bildung refers to the ideal of personality formation, and does not only entail simply the transmission of knowledge, but also the development of the knowledge, norms and values associated with "good" citizenship and/or membership of the cultural and intellectual élite. Similarly, the Chinese also espoused humanistic views. The classics *Analects of Confucius* taught the values of social and ritual propriety (禮), righteousness and loyalty. All these values were directed toward the central thought of Confucius—humanity and becoming a "proper man" or "gentleman" (君子).

There is a large range of content that can be chosen for students to learn. The Didaktik tradition suggested identifying those elements which have the potential to bring about the state of Bildung. In other words, the Didaktik tradition informs what knowledge and capabilities should be addressed in order to become educated (Reid 1998). Similarly, the contents chosen in the *Analects of Confucius* have the goal of bringing about five elements, which are called *Wuchang* (五常). They are *Ren* (仁, humanity), *Yi* (義, righteousness), *Li* (禮, ritual), *Zhi* (智, knowledge) and *Xin* (信, integrity). Hence, the two traditions both chose contents to bring about some humanistic values in the students.

2.3.2 The belief that everyone should be taught

The Didaktik tradition and Confucianism shared the same belief, to teach everyone. In Comenius' definition, Didaktik contains three elements, omnes omnia docere, that is, "teaching everything to everyone". The three components, the teacher (teaching), the content (everything) and the learner (everyone), formed the *Didaktik triangle*. This is a tool to structure the field of Didaktik research and theory. Also, von Humboldt (1903), who promoted general education in Germany, wrote, "Every suitable head is able to practice mathematical strength" (p. 282). In ancient China, Confucius said, "I have yet to not instruct even someone who comes with a small bundle of dried meat (gift for the instructor) (凡自行束脩以上者, 吾未嘗無誨焉)" (verse 7 of chapter 7 in the *Analects of Confucius*). This is equivalent to the saying: "Instruct all and reject none (有教無類)." Thus, both Confucianism and the Didaktik tradition have the same notion of teaching everyone. In addition, these two traditions also have belief in the teacher as a moral educator. Pepin (1999) suggested that, under the Didaktik tradition, teachers hold two functions: that of an academic specialist and that of a moral educator. Teachers in East Asia, as mentioned before, are also considered as role models and are expected to be experts or learned figures (Leung 2001).

2.3.3 The process of education

Despite the similarities mentioned above, the processes of education were different in the Didaktik tradition and Confucianism. Based on Comenius' suggestion, lessons should instruct pupils to become independent, and rules and laws should be discovered by themselves (Kaiser 1999). Students in East Asia learned through whole-class instruction. This has been the setting in traditional Chinese classrooms for more than 2,000 years since the time of Confucius. Students have always been expected to listen to and respect the teachers.

2.3.4 Summary

Although the ideas of Confucianism were developed in the East while those of the Didaktik tradition were developed in the West, it was found that they shared some commonalities in aims and beliefs and the main difference was in the process of education.

3 Methodology of the study

It was found, as described in the previous section, that the two cultural traditions, Didaktik tradition and Confucianism, shared many common ideas though some differences were also found. To investigate how these two cultural traditions influenced the implemented curricula in Berlin and Hong Kong, respectively, and examine whether the implemented curricula are similar, it is necessary to investigate the classroom activities. One way of doing this is through video studies. Videotaping provides multiple perspectives in data analysis and interpretation. It also

provides information about classroom processes that do not rely on a teacher's own descriptions.

One important video study that has been carried out is the Learners' Perspective Study (LPS). LPS aimed to construct rich, detailed portrayals of the practices of individual well-taught 8th grade mathematics classrooms over sequences of ten consecutive lessons. The participating countries/regions of the LPS project were Australia, the Czech Republic, Germany, Hong Kong and Mainland China, Israel, Japan, Korea, the Philippines, Singapore, South Africa, Sweden and the USA. In each of these countries/regions, a sample of three "well-taught" lessons was chosen for data collection. Each lesson was taught by a teacher who was identified by the local mathematics education community as competent. Data generation in the LPS adopted a three-camera approach (teacher camera, student camera, whole class camera). In the post-lesson student interviews, in which a video record was used as stimulus for students to reconstruct classroom events, students were given control over the video playback and were also asked to identify classroom events of personal importance and to comment. The post-lesson student interviews were conducted as individual interviews in all countries except Germany, Israel and South Africa, where student preference for group interviews was sufficiently strong to make that approach essential. Each participating teacher was interviewed at least three times using a similar protocol (Clarke et al. 2006a, b). The teacher and student interviews offered insight into both the teacher's intentions in the enactment of particular lesson events and the significance and meaning that the students associated with that event (Clarke 2006).

This research studies the junior mathematics curriculum in Berlin and Hong Kong. Due to limited resources, the researcher was not able to carry out videotaping in Berlin; therefore, the LPS videos and interviews from both Berlin and Hong Kong were used instead. The data from these showed various elements including lesson structure, lesson proceedings, students' reaction and teachers' perspective. Videotaping provided multiple perspectives in data analysis and interpretation. It also provided information about classroom processes that did not rely on a teacher's own descriptions. The data set of LPS for Berlin and Hong Kong contained a total of 78 videotaped "well-taught" 8th grade mathematics lessons from 6 schools, 3 from Berlin (G1–G3) and 3 from Hong Kong (HK1–HK3), supplemented by 6 teacher interviews and about 100 student interviews. There were 10 lessons in G1, 11 lessons in G2 (with data of one of the lessons missing), 11 lessons in G3, 18 lessons in HK1 and 14 lessons in each of HK2 and HK3. In this study, the LPS-videotaped classroom activities were analyzed according to the framework suggested by Mok and Lopez-Real (2006).

The researchers were aware that the LPS videos were taken more than 10 years ago in 2000. The curriculum changed within this period and so the teaching practice may have also changed. Therefore, classroom observations for one or two lessons for the same schools as those that participated in the LPS were taken during 2009 and 2010 to check for consistency with the LPS videos. The first author observed the classroom activities in Berlin in 2009 and in Hong Kong in 2010, and took field notes to see if there were any major changes in these years (see Sect. 3.2). The observation is not a generalization of the teaching practices. As resources were limited, the researcher could not stay in Berlin for a long period of time. Hence, there are no claims for either the representativeness of the sample or the generalizability of the findings.

In this study, teacher interviews were also administered. In-depth teacher interview is obviously the best way to understand teachers' perceptions of their own situations and understand the reasons for some classroom activities. Teachers in different places have different intentions that affect their teaching of mathematics. Using the words of Westbury (1998), the "most dramatic difference in viewpoint" between the two traditions, the Didaktik and Confucian traditions, is the "views of the teachers". Hence, questions are asked on what teachers think about the curriculum documents and the textbooks, how teachers select the contents to be taught, how they select the methods of teaching, and how they help students with learning difficulty. Once this information is gathered, it can help us to understand the intentions of teachers at greater depth. The teacher interviews are semi-structured ones in order to allow flexibility for the researcher to ask follow-up questions. The nature of the work environment is also of interest.

3.1 Framework for LPS data analysis

LPS itself does not provide a framework for data analysis. Clarke et al. (2006a, b) used four classroom activities (*reviewing previous material*; *presenting the topic and the problems for the day*; *developing the procedures to solve the problem*; and *practicing*) suggested by Stigler and Hiebert (1999) to code the three LPS classes in Berlin. It was found that lessons in all three German schools included classroom activities, such as summary and assignment homework which fell outside the predicted categories. Therefore, the coding suggested by Stigler and Hiebert (1999) may not be suitable in this study. Nonetheless, the coding suggested by Mok and Lopez-Real (2006) covers most, if not all, aspects of the classroom activities. Also, it was developed in Hong Kong and is sensitive to the Hong Kong culture. Therefore, in this study, the coding according to Mok and Lopez-Real (2006) in classroom

organization will be adopted as the framework for detailed analysis. This codes the classroom organization into three types: *Classwork*, *Seatwork: Individual* and *Seatwork: Small group*. *Classwork* is further coded into five exclusive types: *Exploratory*, *Directive*, *Summarization*, *Exercises and Practice*, and *Assigning Homework*.

Exploratory includes the following events (Mok and Lopez-Real 2006, p. 238):

- The focus is on a relatively open or difficult problem which has more than one possible answer.
- The teacher gave a signal for pair or group discussion.
- Whole-class discussion with the following features: inviting more than one student to give answers, and inviting explanations and peer comments.

In some other literature, exploratory teaching has similar meaning to "whole-class discussion" (Pepin 1999).

Directive includes the following events (Mok and Lopez-Real 2006, p. 239):

- No comment on the student's answer, no attempt to discuss the answer with the other students, simply stating what should be done (e.g., the conventional notation).
- Emphasis is purely on following a convention.
- Insistence on precise language.
- Repetition of what had been learnt in an earlier lesson or in the earlier part of the lesson at a fast pace, using this as a foundation for establishing further knowledge.
- Insistence on articulation of procedures.
- Clear and directive definition of concept or method after an illustrative example or discussion.
- Teacher plays the role of directing students to work on problems.
- Probing for "expected" answers.
- Directive explanation by teacher.

Summarization includes the following event (Mok and Lopez-Real 2006, p. 239):

- Teacher does summarizing during the lesson, or to conclude the topics or problems discussed.

Exercises and Practice includes:

- In the situation of doing textbook exercises, there can be teacher talking about/explaining the question, and students having seatwork.

Assigning Homework includes:

- Teacher assigns homework or questions for students to do at home.

Although the results in Mok and Lopez-Real confirmed that the schools in Hong Kong spent little time on assigning homework, we stick to their original framework to include *Assigning Homework* as an element. Also, Pepin (1999) found that, in terms of routines, German teachers started nearly every lesson with the correction of homework, with the whole class. Therefore, homework constitutes an important component in the lessons.

Mok and Lopez-Real (2006) defined individual seatwork to be one where "students work on a task individually, without any discussion with other students" (p. 238). Nonetheless, when Mok and Lopez-Real analyzed the data on individual seatwork, they included the following situation: during or after individual seatwork, some students might explain to the one who sat next to him or her. Students might also compare their answers with each other after working. To be consistent with Mok and Lopez-Real's data analysis, we will modify this definition in this study. S*eatwork: Individual* is redefined as the activity when the teachers instruct the students to work on a task individually, no matter whether they explain their answers to one another or not.

The final category *Seatwork: Small group* is an activity that two or more students discuss or do among themselves (Mok and Lopez-Real 2006, p. 238).

3.2 Classroom observation 2009 (Berlin) and 2010 (Hong Kong)

As mentioned above, classroom observations were conducted in Berlin and Hong Kong in June 2009 and January 2010, respectively, to verify whether the classroom organization (*Classwork*, *Seatwork: individual* and *Seatwork: group work*), the five approaches and activities in *Classwork* (*Exploratory*, *Directive*, *Summarization*, *Exercises and Practice*, and *Assigning Homework*) and their salient features (if any) were similar to those in 2000.

All the three teachers in Berlin have teaching licenses. None of them were teaching grade 8 in the academic year 2008–2009. The G1 teacher had been teaching for 21 years by 2009, including 1 year as a teacher trainee. He was teaching three classes (2 grade 12 mathematics and 1 grade 9 physics classes) in the academic year 2008–09. The researcher observed his grade 12 mathematics class on Probability, which lasted for 45 min. At the beginning of the lesson, the teacher started the question and answer session with his students. Although the contents were different, this question and answer session was very similar to what the teacher did in the year 2000. Each time the teacher asked a question, some (around 10), students would raise their hands to answer it. Then he checked the homework with them. After that, the teacher discussed the mathematical rules and laws with them. This was consistent with the pattern in the LPS videos in 2000. Most of the lesson time was on whole-class discussion.

The G2 teacher had been teaching for 32 years by 2009, including 2 years as a teacher trainee. He only taught one mathematics class this year because he was a principal at the same time. His grade 7 mathematics class on symmetry was observed. There were 30 students. This was a double-period lesson, lasting for one-and-a-half hours. At the beginning of the lesson, the teacher checked homework with the students. The teacher asked three groups of students to present their work in front of the class. Then, they had whole-class discussion on the mathematical contents. The teacher gave instructions and students formed groups and worked together. Homework was given at the end of the lesson. Working in groups, presentation in front of the class and whole-class discussion were three unchanged elements in the lessons. Similar to LPS in 2000, students were eager to raise their hands and answered questions which the teacher posed.

The G3 teacher had been teaching for 29 years by 2009, including 2 years as a teacher trainee. She was teaching three mathematics classes this year. All of them were grade 9. The researcher observed a lesson on simultaneous equations, which lasted for 45 min. The lesson was organized in this way: the teacher posed a problem and then a student worked on it in front of the class. As she solved it, she explained to the others what she was doing. She also answered other students' questions. Then students were formed into groups and worked on the problems posed by the teacher. Group work in G3 was also an unchanged element in the past 10 years.

In Hong Kong, the teacher in HK1 was deceased. The teacher in HK2 is a qualified teacher and had been teaching for 18 years. He was teaching four classes (grade 8–grade 11) in the academic year 2009–2010. The researcher observed his grade 8 mathematics class on rate and ratio, which lasted an hour. There were 37 students. The lesson was organized in this way: the teacher first checked homework with students. Then the teacher started his lesson in a lecture style. Most of the time, he gave instructions and told the students explicitly about what steps they should work on. He introduced a new mathematical concept with some examples. After that, he gave some exercises for the students to work on individually. He would check the answers with the whole class. After working on a few more examples with the whole class, he gave homework to the students. It was noted that there was no group work in the lesson observed. In the interview, the teacher was asked if he encouraged group work. He said yes, but he could not facilitate group work in the lesson. This was because there were many students in the class. In short, teachers in Hong Kong did not facilitate group work, while the teachers in Berlin did. The HK3 teacher was working for the Curriculum Development Institute of the Education Bureau and hence he was not teaching in 2010.

4 Results

There were no significant changes observed in the teaching practices in Berlin and Hong Kong in the 10-year period. Results for classroom observations were similar to those in the LPS videos, especially in the structure of the lessons (which will be discussed in Sect. 5.1). In this section, the results on *Classwork*, *Seatwork: individual*, and *Seatwork: group work* will be presented from the LPS study, with supplementary comments from the 2009 and 2010 observations.

4.1 Classwork

In this section, the results of the five approaches and activities in *Classwork* (*Exploratory*, *Directive*, *Summarization*, *Exercises and Practice* and *Assigning Homework*) will be presented.

4.1.1 Exploratory

4.1.1.1 Berlin In our analysis of the LPS videos, it was found that the three schools in Berlin, G1, G2 and G3, spent 59.2, 78.6 and 70.7 % on exploratory. This is quite substantial. All of the schools spent more than half of the lesson time on exploratory. In most of the cases, the teachers asked questions, and then students raised their hands. The teachers chose one of them. Students discussed among themselves and answered the questions. They found new features of the mathematics contents and later developed advanced procedures (e.g., devising a general solution for an equation in the form $\frac{a}{x+b} = \frac{c}{x+d}$ with domain $Q\backslash\{-b; -d\}$).

The students were very eager to participate in the classes. In both the LPS videos and the classroom observation, hand raising was found to be intense. Around one-third of students in the LPS videos, and also in the classroom observations in 2009, raised their hands to answer each question the teacher posed. All who wanted to answer questions kept raising their hands, and even those who were never called upon also did so frequently. Students in Berlin would also pay attention to how the others answered the questions.

4.1.1.2 Hong Kong In contrast, the three schools in Hong Kong, HK1, HK2 and HK3, spent 7.4, 3.7 and 0 % on exploratory, respectively. In HK2 and HK3, exploratory teaching was not common. Instead, the teachers preferred to use directive teaching (see next section).

In the lesson in which the teacher in HK1 adopted exploratory teaching, he used the same techniques—asking questions and guiding students to find the solutions—as the ones in Berlin. He taught the problem-solving method of

trial and error. The questions he proposed were of high mathematical maturity as they concerned the extremum for the solution of a problem. He gave a signal for pair or group discussion and invited more than one student to give answers. However, students in Hong Kong were observed as not being active in class discussion. They were not eager or were even reluctant to raise their hands in response to the teachers' questions. Most of the time, teachers had to call students and explicitly require them to answer the questions openly or come out to work on the board. Wong (2004) identified that in the CHC classroom, students seldom interrupted the flow of teaching. It was also found in this study that if a particular student answered the teacher's questions too often, other students laughed at him or her (student interview in LPS in HK1, lesson 11). It was also revealed that some teachers in Hong Kong did not favor students raising their hands in classrooms. The teacher feared that questions might slow down or even distort the teaching schedules. This can be found in the teacher interview in LPS in HK2:

Teacher: This girl at the far side asks many questions…sometimes she would answer… she would ask something that you are about to talk about.

Interviewer: Umm, so in fact is it good? If she asks such questions how will you feel?

Teacher: Sometimes it's good, but sometimes it's troublesome. For instance…I'm going to talk about those three things…but when I'm talking about the first she would raise her hand and ask you about the third problem. …Very often I would have to stop her and tell her to wait until I've finished all the talking before asking. Sometimes she can be quite disruptive for my teaching.

When the Hong Kong students worked at the front, they usually did not have the idea in mind of sharing with the whole class as their German counterparts did. They simply wrote down the solution on the board, without pointing out the methodology or reasoning behind it. There was no exchange between the students at the front and their classmates in the seats. Hence, working at the front rather became a unilateral process of demonstrating the solutions to the teachers.

4.1.2 Directive

4.1.2.1 Berlin For G1, directive teaching made up only 2.2 % of the lesson time. For G2, directive teaching was almost absent. In lesson 9, the teacher spent only 1 min to remind students that they had to factorize the algebraic fractions to get a common denominator first. Directive teaching made up only 0.2 % of the lesson time. For G3, there was no directive teaching in the lesson. So, we could see that directive teaching was not frequent in the three schools in Berlin.

4.1.2.2 Hong Kong Most of the lessons in Hong Kong are directive; 46.4, 57.0 and 36.2 % of the lesson time are directive in HK1, HK2 and HK3, respectively. Teachers played the role of directing students to work on problems and gave clear direction on procedures. Each and every step in working out the solutions was explicitly set out, and students were asked to follow these steps. This means that students were left with little opportunity to think through the problems and develop their own approaches by themselves. For example, the teachers in HK1 and HK2 set the standard steps for solving a system of simultaneous linear equations. Students should solve the problem following the standard steps. Both teachers suggested formal procedures for solving simultaneous equations or word problems. Such formal or systematic ways of solving problems heavily dominated the lessons in Hong Kong. For example, in lesson 5 in HK3, the teacher explained what simultaneous equations were and then told the students how to solve them by graphical methods. The teacher said, "I want you to follow the standard procedure to draw the graph." He then specified how large the paper should be in order to plot the graph. Sometimes (e.g., lesson 13), the teacher just instructed for several minutes without asking whether the students understood. Thus, the time that the three teachers spent on directive was substantial. They all spent more than one-third of the lesson time on directive teaching. They presumed that all students would listen carefully.

4.1.3 Summarization

Summarization is done by the teacher during the lesson, or at the conclusion of the topics or problems discussed. The three teachers in Hong Kong had almost no summary of their lessons. HK1, HK2 and HK3 only spent 0.3, 0.2 and 0.1 %, respectively, of the lesson time on summarization (Mok and Lopez-Real 2006). The researcher found similar results in Berlin from the LPS data. G1, G2 and G3 spent 0.9, 0.9 and 0 % of time, respectively, on summary.

4.1.4 Exercises and practice

4.1.4.1 Berlin The three teachers in G1, G2 and G3 spent 36, 16.9 and 25.5 %, respectively, of the time on exercises and practice. When the students finished the assigned exercises, the teacher would check their results in the class. When the teacher asked the students for the results for each step, they raised their hands and gave the answers. The teacher then wrote the answers step by step on the

blackboard. At the same time, the teacher reminded students of some of the important points or common mistakes. He also asked for alternative solutions (G1 lessons 1 and 2). Sometimes, the teacher asked students to write their results on the board (G1 lesson 4; G2 lessons 2, 4, 8, 10 and 11; G3 lesson 9 and 11).

4.1.4.2 Hong Kong The three teachers in schools HK1, HK2 and HK3 spent 44.9, 35.9 and 59.9 %, respectively, of the time on exercises and practice (Mok and Lopez-Real 2006). Usually, the teacher asked a few students to come to the blackboard. The rest of the students were expected to watch and correct errors as the students worked out the problems on the blackboard. The teacher carefully monitored the step-by-step procedures of the students, often asking questions and correcting errors.

4.1.5 Assigning homework

From the LPS data, teachers in Berlin assigned homework to students at the end of almost every lesson. The teachers in G1, G2 and G3 spent 1, 3 and 1 %, respectively, of the time in assigning homework. The frequency of setting homework was 5 out of 10 lessons in G1, 8 out of 10 lessons in G2, and 8 out of 11 lessons in G3. The teachers in HK1, HK2 and HK3 spent 0.9, 3.2 and 3.8 %, respectively, of the time in assigning homework (Mok and Lopez-Real 2006). The frequency of setting homework was 7 out of 18 lessons in HK1, 8 out of 14 lessons in HK2, and 8 out of 14 lessons in HK3. The teachers in both cities did not spend much time in assigning homework.

4.1.6 Summary

The above results are summarized in Table 1. "Others" in the table include greeting, attendance checking and the instances where the teachers gave information on what would be done in the lesson. They also talked about some things outside the content of the lesson. These included, for example, the hand-in of the consent forms for the LPS and that there would be videotaping in the coming week.

Table 1 Percentages of time the teachers in Berlin and Hong Kong spent on different types of *Classwork*

Classwork	Berlin			Hong Kong		
	G1	G2	G3	HK1	HK2	HK3
Exploratory	59.2	78.6	70.7	7.4	3.7	0
Directive	2.2	0.2	0	46.4	57.0	36.2
Summarization	0.9	0.9	0	0.3	0.2	0.1
Exercises and practice	36	16.9	25.5	44.9	35.9	59.9
Assigning homework	1	3	1	0.9	3.2	3.8
Others	0.7	0.4	2.8	0.1	0	0

4.2 Seatwork: individual

The three schools in Berlin, G1, G2 and G3, spent 33,[1] 18 and 13 % on individual seatwork, respectively. The teacher wrote the tasks on the board, distributed worksheets or made use of the tasks in the textbooks. Students then worked on the tasks. At the same time, the teacher walked around the class and gave support to the students (this happened in G1 lessons 1, 2, 3, 4, 5, 8, 9 and 10; G2 lessons 2, 3, 7, 8 and 9; G3 lessons 1, 2, 3, 4, 6, 7 and 8). Sometimes, students might explain to the student who sat next to him or her. They might also compare their answers with each other after working them out.

The three schools in Hong Kong spent relatively more time on individual seatwork: 30.5, 21.9 and 39.9 % for HK1, HK2 and HK3, respectively. The three teachers also walked around the classrooms to give guidance to students when they were working (all lessons in HK1; all lessons except lesson 14 in HK2; all lessons in HK3). Similar to the students in Berlin, students in Hong Kong might also explain their work to others.

4.3 Seatwork: small group

It was found in this study that group work was not observed in any of the Hong Kong lessons. In contrast, group work was observed in many lessons in Berlin. In most LPS lessons, groups of two to three students were formed. "Typically, views are exchanged and mistakes are gone over by each other during the checking," a student said (G2 lesson 1 student interview line 409 and videos). Ideas were shared among members in groups, usually in the sense of the capable students helping the less capable ones. Sometimes, students questioned others' ideas and then they would defend their ideas with evidence.

In G1, group work made up 14 % of the lesson time. In G2, 29 % of the lesson time was spent on group work, which happened in seven out of the ten lessons. The teacher divided the students into groups of four to six. Each group was assigned to solve a problem. Sometimes, all groups worked on the same problem (e.g., lesson 5). The students first worked individually and then discussed their solutions. The teacher assisted them at the same time. After that, the teacher asked a representative from each group to present their results on the blackboard. In G3, group work

[1] This included a test which students did individually.

made up 19 % of the lesson time. Most of the time (lesson 1, 2, 4, 8 and 9), the teacher distributed worksheets to the students so that they could work in groups. At the same time, the teacher walked around and assisted them. Sometimes, the teacher assigned different tasks to different groups and then students had to present their results.

Group work facilitates students to learn through peers. In the interviews in LPS, one German student said that it was important to explain his/her thoughts and discuss problems with members in groups. Another said he/she could learn something through explaining to others, in accordance with the common wisdom that teaching is the best way to learn. In fact, many students felt that it was more comfortable to ask members of their groups. "I wanted him (another student) to explain it to me," a student said in the interview in lesson 5 in G2 (line 310). Also, explaining to classmates could help them grasp the critical points and remember important details. In a student interview in lesson 3 in G1, a student said, "Because I can explain it then I remember it again myself" (line 560). They also believed that they might even learn something more when they were explaining it. In a student interview in lesson 6 in G1, a student said, "After all, he (refers to a student) can explain things to those who have difficulties and then he can learn something when he's explaining it" (line 345). They also said that they had much fun working in groups. "Because it's so much more fun" (line 376), a student said in the student interview in lesson 6 in G1. Another student shared the same view (G1 lesson 3 student interview line 517).

5 Discussion

In this section, the similarities and the differences in the implemented curriculum in Berlin and Hong Kong will be presented. The relationship between cultural traditions and the implemented curriculum will also be discussed.

5.1 Similarities

In Berlin, most of the lessons were organized as follows: the teacher first reviewed what the students had previously learned. Then he presented a situation or a concept on the board, which would be expanded through a series of question–response sequences, and led a discussion to arrive at some general principle. The teacher and the students then explored the topic. The students then practiced individually or formed groups. They would present their work afterward. When a student presented a problem, he or she always explained the work. The teacher and the rest of the class constantly interacted with the student at the board. The pattern is consistent with the results found by Kaiser (2002, p. 249):

> The introduction of new mathematical concepts is usually done by class discussion, in which the whole learning group participates under the guidance of the teacher. There exist various kinds of teacher guidance. A characteristic of the course of a lesson is that the newly introduced concepts or methods are formulated in detail by phrases and notes on the blackboard, which then is followed by exercises.

In Hong Kong, at the beginning of the lessons, the teacher would check the previously assigned homework with students. Then he would instruct the students what to do. Exercises were given to students, and the students then worked individually. The characteristic of mathematics teaching in Hong Kong is that it is subject based. The lessons are organized in a subject-scheduled order. For example, the HK1 teacher started the topic of simultaneous equations by motivating the students to solve the classic example "to find the number of chickens and rabbits, given the numbers of heads and legs." The other teachers first defined what simultaneous equations were. Then they would teach how to solve the equations. These included graphical methods, substitution and elimination. The teachers also required students to remember the exact procedures and algorithms. They would finally move to applications of the topics. In general, the lessons started from general concepts and rules, and then proceeded to conclusions and applications.

To sum up, the structure in a mathematics lesson in Berlin included the following components:

1. Reviewing previous materials or checking homework with the students.
2. Presenting the topic and the problems for the day.
3. Developing the procedures to solve the problem by *the whole class*.
4. Practicing *in groups*.
5. Assigning homework.

The mathematics lessons in Hong Kong shared a similar structure. The lesson components included:

1. Checking homework with the students.
2. Presenting the new content followed by some examples.
3. Developing the procedures to solve the problem *by the teacher*.
4. Practicing *individually*.
5. Assigning homework.

It can be concluded from the above that the structure of the mathematics lessons in Berlin and Hong Kong was similar. The classroom setting in both cities was teacher dominated. In Berlin, new mathematical concepts and methods were almost exclusively introduced by means of a teacher-guided discussion (Kaiser 2002) (this approach will

be discussed in Sect. 5.2). In Hong Kong, students learned the new mathematical ideas in whole-class instruction. As mentioned before, this setting has been in traditional Chinese classrooms for more than 2,000 years since the time of Confucius.

Another similarity in the classroom is that students in Berlin and Hong Kong spent quite a lot of time on practice (Table 1). After being introduced to a certain topic, they had to do exercises from worksheets or textbooks. The three German schools spent much time (a third in G1, a sixth in G2 and a quarter in G3) on practicing. In Hong Kong, there is an old Chinese idiom "practice makes perfect", which may reflect the phenomenon that students in Hong Kong did a lot of exercises. In the Chinese tradition, learning and practicing are inseparable. According to Leung (1992), "mathematics in Hong Kong was treated more as a set of skills and techniques to be mastered through ample practice. Motivation for learning seemed to be mainly extrinsic, doing well in examinations was the goal. The competitive and pragmatic spirit of the Hong Kong society might also have contributed to this stress by teachers and students on mastery of skills and techniques in preparation for examinations." This may be a reason for the high percentages of classwork time on exercises and practice.

5.2 Differences

5.2.1 Exploratory and directive teaching

Despite the similarities identified above, there were some differences in the lessons. Referring to Table 1, the teachers in Berlin spent more than half of their teaching time on exploratory activities, while the teachers in Hong Kong spent more than a third of their teaching time on directive activities. In exploratory teaching, the students in Berlin were given a task and the teachers guided them to find solutions. The teachers asked questions and gave them clues. The students had to think and give their answers. The teachers tended to ask questions to elicit information from students while they were developing concepts together. The whole class then developed new ideas. As mentioned before, new mathematical concepts and methods are almost exclusively introduced by means of a teacher-guided discussion (Kaiser 2002). The exploratory method used in Berlin is called "das fragend-entwickelnde Unterrichtsgespräch" or "der fragend-entwickelnde Unterricht". (The word "fragend" means inquiry or interrogatory; "entwickelnde" means developing; "Unterricht" means class and "Gespräch" means conversation.) The name of the method gives a clue to what the method means. Discussion with individual contributions from students formed a basis of class teaching (Howson 1995).

At the same time, students were willing to raise their hands and answer the questions from the teachers. Although the classroom setting was still a teacher-dominated one, students could express their own ideas and points of view. Students were encouraged to think. This tradition can be traced back to the 1600s. The educator Jan Amos Komensky (Comenius 1592–1670), the Bohemian bishop, wrote that lessons should instruct pupils to become independent, and rules and laws should be discovered by themselves (Kaiser 1999). Hence, exploratory teaching with whole-class collaboration dominated the classroom in Berlin.

In Hong Kong, teaching is teacher guided. Siu (2009) suggested that the Eastern view of mathematics rather belongs to algorithmic mathematics, which is a view of mathematics as a tool to solve problems. This view has influenced the mathematics classrooms in Hong Kong. Procedures in solving problems were clearly stated by the teachers and the students were supposed to follow them. New mathematical content was introduced by the teachers with a lecture style. Directive whole-class teaching can also be explained by Leung (2001). He suggested that teachers are considered as role models and are expected to be experts or learned figures. It is believed that, when compared with expertise in pedagogy, a good grasp of the subject matter is more important and serves as a prerequisite for invoking appropriate pedagogy. This naturally leads to the typical "direct teaching to the whole class" mode of mathematics teaching commonly found in the East Asian society. Wong (2004) also suggested that students in a CHC classroom were obedient and attentive. They seldom interrupted the flow of the teaching by asking questions, as mentioned before. This can be explained by the Confucian view of education. Everyone should respect their teachers. In the *Classic of Rites* (禮記), one of the five Chinese classics of the Confucian canon, it is mentioned that one should respect the monarch, his father and his teacher, and the teacher is the model or the expert. As students seldom interrupted the lessons, this whole-class instruction would be a very efficient way of teaching. This can help teachers to teach the mathematical content within the limited curriculum time.

There are also be some drawbacks of directive teaching. The teacher in HK3 said in the class, "I'm not asking you to challenge the existence of such a situation. I'm now telling you that it does exist." The desire of the teacher not wanting to be disturbed could also be explained by a practical reason. There was only limited time for teaching and the teacher had to cover the mathematics content within the time available. Another example showed that students were supposed to follow explicitly his instructions. In lesson 5 in HK3, students were required to draw some points on the coordinate plane:

Teacher: Okay? Now, these are important. Draw them. And then mark these points on the graph paper. Negative two, negative one. We have negative two, negative one. Make sure that you are marking it with a cross, with a cross. I don't want to see a dot. I want to see a cross at that point.

One may wonder if there are any differences *mathematically* in plotting dots or crosses on graph paper. It is practically true that the line that connects the points may overwrite the dots. That may be the reason the teacher required students to draw crosses instead of dots, though he did not explain. Students were supposed simply to follow the decisions or the instructions made by the teacher, without knowing the reasons clearly. This may be a drawback of directive teaching.

5.2.2 *Group work*

The second difference is group work. In Berlin, the teachers encouraged the students to work in groups and search for better explanations among themselves. The students worked together to build knowledge for the community. They interacted with each other and shared their solution among themselves. They reached agreement together and explained to each other. After group work, the teachers also provided opportunities for them to present the mathematics in front of the class. They could develop their own problem-solving methods and have chances to apply their mathematical knowledge.

Kaiser (1999) suggested that "the German educational philosophy [is] characterized by the development of two approaches" (p. 149). They are the humanistic-oriented approach and the realistic-oriented approach for the masses. As mentioned in Lui and Leung (2011), the emphasis of the intended curriculum in Berlin was more on the realistic-oriented approach. However, the classrooms were more on the humanistic approach. Their main goal was to help the students to acquire Bildung. Bildung is a state in the process of the acquisition of and the dealing with cultural objects and personality development. In the classrooms, the students in Berlin often worked in groups. A teacher in Berlin suggested that group learning could enhance students' social skills. When working in groups, students had to know how to express their opinions and communicate and negotiate with others. This might foster students' personality development. Thus, the implemented curriculum was more on the humanistic-oriented approach.

In Hong Kong, group work is absent. In the teacher interview in Hong Kong, a teacher said that he could not facilitate group work in the lessons, because there were too many students in the class. Group work was also time-consuming. High productivity and efficiency are always expected in Hong Kong. Another reason is that in traditional Chinese society, students had less chance to present their own work and they were expected to listen to the teachers. Hence, group work was absent in Hong Kong.

5.3 Significance of the findings

It was found in the beginning of this study that there were some commonalities in the aims and beliefs in Didaktik tradition and Confucianism. These two cultures both espouse a humanistic approach in education. They both share the ideologies *educate to become intellectual and cultural élite* (as mentioned in Sect. 2.3.1) and *education for all*. The cultures which look different on the surface actually share some common beliefs. However, they influence the implemented curriculum in different ways. In the previous section, the similarities and differences in the implemented curricula in Berlin and Hong Kong, and their relationships with the cultural traditions, were presented. It was found that the way the teachers presented new mathematical ideas (exploratory vs. directive teaching) and the way students worked in class (individual work vs. group work) were different. This mismatch between the cultural traditions and the implemented curricula was mainly due to the difference in the process of education. While Comenius suggested that lessons should instruct pupils to become independent, and rules and laws should be discovered by themselves (Kaiser 1999), students in East Asia were expected to listen to and respect the teachers. Given a similar goal, the process can always be different.

Cultures affect the implemented curriculum in a complicated or indirect way. In addition, "it is also generally accepted that national cultural traditions of different countries are not an easy area to investigate, because boundaries are usually defined" (Pepin 1999). Germany and China are two big countries. A group of people called Chinese or German may have different values. In terms of beliefs in education, it is generally true that Hong Kong is under the influence of the CHC (Leung 1999), while the German philosopher and educator Humboldt developed many important ideas on Didaktik (Westbury 2000) which influenced the mathematics curriculum in Berlin. Thus, while there may be some social factors or educational factors such as the intended curriculum and textbooks which also influence the implemented curriculum, nonetheless, this study has confirmed again that philosophies have strongly influenced mathematics teaching and learning in classrooms.

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