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Preface

It is our great pleasure to welcome you to the 5th International Conference on Optimization and Control with Applications (OCA 2012), held in Beijing, China on December 4-8, 2012. As a continuation of the OCA series, the goal of the 5th OCA is to provide an international forum for scientists, engineers, researchers, and practitioners to exchange ideas and approaches, to present research findings and state-of-the-art solutions, to share experiences on potentials and limits, and to open new avenues of research and developments, on all issues and topics related to optimization and control.

The OCA 2012 proceedings include 125 peer-reviewed abstracts and brief papers. The proceedings can help young scientists, postgraduate students and practitioners to conduct further investigations in the development and applications of control and optimization with their applications.

We wish to acknowledge the financial support from all organizing institutions and sponsors: Capital Normal University, Beijing, China University of Petroleum, Beijing, Chongqing University, Chongqing Normal University, Chongqing Science & Technology Commission, Chongqing, Curtin University, Hunan University, Shanghai University, and the National Nature Science Foundation of China (No. 11171079, 11071257, 11210301064). In particular, we express our appreciation to China University of Petroleum, Beijing, which provides us such a wonderful venue to meet together. We are also grateful to all friends, colleagues, and conference participants, especially, Dr. Changjun Yu and Dr. Bin Li, who have provided professional general and technical assistance to the conference. Last but not least, we would like to express our acknowledgement to our plenary and invited speakers, the members of the Organizing Committee and International Program Committee, the reviewers and the technical session chairs for their support and contributions.

Finally, welcome you to OCA 2012 again and hope you have an enjoyable and fruitful stay in Beijing.

Honglei Xu, Xinmin Yang and Yi Zhang

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1 ON THE BOUNDARY VALUE PROBLEM FOR BLOOD FLOW IN A SMALL VESSEL WITH FLUID-STRUCTURE INTERACTION

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Abstract: Coronary artery disease or coronary heart disease is the leading cause of human death in the developed industrialized nations. It involves the progressive narrowing of the arteries that supply blood to heart muscle. When arteries become severely narrowed, the heart muscle cannot get sufficient blood or oxygen it needs. This can lead to a heart attack. In order to understand the pathogenesis of coronary artery diseases, various in-vivo and vitro experiments have been conducted using animal models. Due to the difficulty in determining the critical flow conditions for both in-vivo and vitro experiments, the exact mechanism involved is not well understood. On the other hand, mathematical modeling and numerical simulation can lead to better understanding of the phenomena involved in vascular diseases. In this paper, we propose a mathematical model of blood flow in a small vessel with fluid structure interaction. The unsteady state flow of blood cell and plasma through the coronary artery and the deformation of the arterial wall in a cardiac cycle are investigated in this study. Numerical simulations based on the finite volume method are carried out for the flow field, pressure field, internal wall shear stress and the wall deformation in a cardiac cycle.

Key words: Coronary heart disease; Fluid-Solid flow; Fluid-Structure Interaction; Non-Newtonian Fluid; Finite Volume Method.

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2 POSITIVE SOLUTIONS FOR $2p$-ORDER AND $2q$-ORDER SYSTEMS OF SINGULAR SEMIPOSITONE BOUNDARY VALUE PROBLEMS

Lishan Liu*[a], Chengjuan Yin[a] and Zhaocai Hao[a]
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Abstract: In this paper, we study the existence of positive solutions for $2p$-order and $2q$-order systems of singular semipositone boundary value problems with Sturm-Liouville boundary conditions

$$\begin{cases} (-1)^p u^{(2p)}(t) = f(t, u, -u'', \cdots, (-1)^{(p-1)} u^{(2p-2)}, v, -v'', \cdots, (-1)^{(q-1)} v^{(2q-2)}), \\ (-1)^q v^{(2q)}(t) = g(t, u, -u'', \cdots, (-1)^{(p-1)} u^{(2p-2)}, v, -v'', \cdots, (-1)^{(q-1)} v^{(2q-2)}), \\ a_i u^{(2i)}(0) - b_i u^{(2i+1)}(0) = 0, \quad 0 \le i \le p-1, \qquad 0 < t < 1, \\ c_i u^{(2i)}(1) + d_i u^{(2i+1)}(1) = 0, \quad 0 \le i \le p-1, \\ \alpha_j v^{(2j)}(0) - \beta_j v^{(2j+1)}(0) = 0, \quad 0 \le j \le q-1, \\ \gamma_j v^{(2j)}(1) + \delta_j v^{(2j+1)}(1) = 0, \quad 0 \le j \le q-1, \end{cases}$$

where $f, g : (0,1) \times (\mathbb{R}^+)^p \times (\mathbb{R}^+)^q \to \mathbb{R}$ are continuous; f, g may be singular at $t = 0$ and/or $t = 1$ and may take negative values, in which $\mathbb{R}^+ = [0, +\infty)$, $p, q \in \mathbb{N}^+$, $a_\eta \ge 0, b_\eta \ge 0, c_\eta \ge 0, a_\eta \ge 0, \rho_\eta = a_\eta c_\eta + a_\eta d_\eta + b_\eta c_\eta > 0, 0 \le \eta \le p-1$; and $\alpha_\theta \ge 0, \beta_\theta \ge 0, \gamma_\theta \ge 0, \delta_\theta \ge 0, \tilde{\rho}_\theta = \alpha_\theta\gamma_\theta + \alpha_\theta\delta_\theta + \beta_\theta\gamma_\theta > 0, 0 \le \theta \le q-1$. By using the fixed point index theorem, some new existence results are established, and an example is given to demonstrate the application of our main results.

Key words: Higher-order semiposition systems; Different orders; Positive solutions; Cone; Fixed point index.

3 ANALYTICAL SOLUTION OF TIME PERIODIC ELECTROOSMOTIC FLOW WITH SLIP BOUNDARY

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Abstract: Recent research confirms that slip of a fluid on solid surface occurs at small scale.Slip causes the change of interior material deformation and velocity profile and stress field. Electroosmosis is one of the major electrokinetic phenomena in which ionized liquid flows with respect to a changed surface in the presence of an external electric field.Time periodic electroosmotic flow is driven by an alternating electric field which is an important electrokinetic effect that can be utilized for particle manipulation and separation, for example,flow pumping and mixing enhancement.Althongh exact and numerical solutions to various flow problems of electroosmotic flows under the no-slip condition have been obtained. Exact solutions for problems under slip boundary conditions have seldom been addressed. In this paper,we will derive an exact solution of the time periodic electroosmotic flow in two-dimensional straight channels under slip boundary conditions.

Key words: Slip boundary condition; Newtonian fluid; Navier-stokes equations; Fluid flow; Time periodic electroosmotic flows.

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4 OPTIMAL INVESTMENT UNDER OPERATIONAL FLEXIBILITY AND RISK AVERSION

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Abstract: Fluctuating global economic conditions require proper responsive strategies to ensure the effectiveness of investment decisions. When market uncertainty increases and decision makers are risk averse, the discretion to modify or suspend investment becomes much more important. This paper addresses the problem of investment under uncertainty assuming that the decision maker is risk averse. We construct an optimal investment model driven by a jump diffusion stochastic differential equation. Our analysis shows that risk aversion reduces the likelihood of investment but this can be mitigated by incorporating operational flexibility of embedded suspension and resumption options. We also illustrate the impact of risk aversion on the optimal decision threshold and the optimal suspension and resumption thresholds under complete operational flexibility.

Key words: Real options; Jump diffusion stochastic differential equation; Optimal investment under uncertainty; Operational flexibility; Risk aversion

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5 ANALYSIS AND CONTROL OF MICROFLOWS UNDER VARIOUS GEOMETRY CONDITIONS

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Abstract: Study and control of fluids through micro-channels are important to the application of a wide range of biological and engineering micro-devices and systems, such as biochemical lab-on-the-chip systems, micro-electromechanical systems, fuel cell devices, drug delivery systems, biological sensing and energy conversion devices. In this talk, we first present the underlying boundary value problem for transient microflows under dynamic condition. Then some analytical and numerical results will be given for various geometry conditions. Finally, an investigation is given to demonstrate the flow behaviour and its control through the system parameters.

Key words: Boundary value problem; Microflows; Analytical solutions; Flow behaviour; Control.

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6 ESTIMATION OF PARAMETERS IN THE INTEREST RATE MODELS

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Abstract: Stochastic differential equation (SDE) is a very important mathematical tool to describe complex systems in which noise plays an important role. SDEs have been widely used to study various nonlinear systems in biology, engineering, finance and economics, as well as physical sciences. Since a SDE can generate unlimited number of trajectories, it is a difficult problem to estimate model parameters based on experimental observations which may represent only one trajectory of the stochastic model. Although substantial research efforts have been made to develop effective methods, it is still a challenge to estimate parameters in SDE models from observations with large variations. In this work, we use the Bayesian inference and Markov Chain Monte Carlo method to estimate unknown parameters in the interest rate models.

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7 A SWARM OPTIMIZATION ALGORITHM WITH INDIVIDUAL SEARCH STRATEGY

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Abstract: For most swarm optimization algorithms such as Genetic Algorithms, Particle Swarm Optimization focused on the search ability of the group, and not pay more attention to the individual search. In this paper, we proposed a swarm optimization with individual search strategy which combines the swarm search with individual search. This optimization algorithm holds the ability of global and local searching ability. Theoretically proved that the algorithm is convergent, and the numerical results show that this algorithm has better global convergence, but also has faster convergence speed and higher accuracy than other intelligent algorithms.

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8 DUALITY THEOREMS FOR CONTINUOUS-TIME GENERALIZED LINEAR PROGRAMMING PROBLEMS

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Abstract: Duality theorems for continuous-time generalized linear programming problems are established. The method of proof is constructive, indicating that optimal solutions of approximating linear programs converge to the optimal solution of the continuous-time generalized linear programming problem. Hence it also provides a possible solution technique for the continuous-time generalized linear programming problems.

Key words: Continuous-time linear programming problems; Continuous-time generalized linear programming problems; Duality theorem.

1 INTRODUCTION

This paper will concern a class of max-min optimal control problem with linear state constraints. Such a problem is called the *continuous-time generalized linear programming problem* (in short, the problem (CGLP)) and defined as follows. Let $L^\infty([0,T],\mathbb{R}^p)$ and $C([0,T],\mathbb{R}^p)$ be the space of all measurable and essentially bounded functions and the space of all continuous functions from a time space $[0,T]$ into the p-dimensional Euclidean space $\mathbb{R}^p$, respectively. The problem (CGLP) is formulated as follows:

$$\begin{aligned} \text{(CGLP)} \quad \text{maximize} \quad & \min_{k=1,\cdots,r} \int_0^T \mathbf{f}_k(t)^\top \mathbf{x}(t)dt \\ \text{subject to} \quad & B\mathbf{x}(t) \le \mathbf{g}(t) + \int_0^t K\mathbf{x}(s)ds \text{ for all } t\in[0,T] \\ & \mathbf{x}\in L^\infty([0,T],\mathbb{R}^q_+), \end{aligned} \tag{1.1}$$

where B and K are $p\times q$ matrices; $\mathbf{f}_k \in C([0,T],\mathbb{R}^q)$, $\mathbf{g}\in C([0,T],\mathbb{R}^p_+)$, $\mathbb{R}^p_+ = \{(x_1,\cdots,x_p)^\top : x_i \ge 0 \text{ for } i=1,\cdots,p\}$ and the superscript "$^\top$" denotes the transpose operation of matrices. We see that the problem (CGFP) is feasible with the trivial feasible solution $\mathbf{x}(t)=\mathbf{0}$ for all $t\in[0,T]$. Throughout this paper, we also assume that $B=[B_{ij}]_{p\times q}$ and $K=[K_{ij}]_{p\times q}$ are $p\times q$ constant matrices with

- $K_{ij}\ge 0$ for all $i=1,\cdots,p$ and $j=1,\cdots,q$;
- $B_{ij}\ge 0$ and $\sum_{i=1}^p B_{ij} > 0$ for all $i=1,\cdots,p$ and $j=1,\cdots,q$.

The problem (CGLP) is a natural generalization of the continuous-time linear programming problem (in short, the problem (CLP)), which was originated from the "bottleneck problem" proposed by Bellman

(1957). The theory of the problem (CLP) has received considerable attention for a long time. The published work concerned with the problem (CLP) can be loosely divided into two areas, those concerned either with the duality theory for such problems or with methods for their solutions. For the study of duality theory, the literature stems from a paper of Tyndall (1965). Tyndall (1965) considered a class of the problem (CLP) with constant matrices B and K and gave conditions under which the problem (CLP) and its dual (DCLP) have the same objective value. This result was later extended by Levinson (1966), Kumar (1966) and Tyndall (1967), and applied to problems with nonlinear objective functions by Hanson (1967), Hanson (1968), Hanson and Mond (1968) and Wen and Wu (2012). These extensions involved weakening the regularity conditions imposed on the problem data. By virtue of the abstract mathematical programming approach, Grinold (1969) and Grinold (1970) extended the duality theorem through easing the algebraic conditions needed for duality. For the other versions of duality theorem, one can consult the literature review in Anderson and Nash (1994). In this paper, following the methodology of Tyndall (1965), we shall derive a dual problem of the problem (CGLP) and discuss the relations between the problem (CGLP) and its dual problem.

2 MAIN RESULTS

In this study, we shall define the dual problem of primal problem (CGLP) as follows:

$$
\begin{aligned}
\text{(DCGLP)} \quad \text{minimize} \quad & \int_0^T (\mathbf{g}(t))^\top \mathbf{y}(t)dt \\
\text{subject to} \quad & B^\top \mathbf{y}(t) - \int_t^T K^\top \mathbf{y}(s)ds \geq F(t)\boldsymbol{\beta} \text{ for } t \in [0,T] \qquad (2.1) \\
& \mathbf{1}_r^\top \boldsymbol{\beta} = 1 \qquad (2.2) \\
& \mathbf{y}(t) \in L^\infty([0,T], \mathbb{R}_+^p) \text{ and } \boldsymbol{\beta} \in \mathbb{R}_+^r,
\end{aligned}
$$

where $\mathbf{1}_r = (1, \cdots, 1)^\top \in \mathbb{R}^r$ and $F(t) = (\mathbf{f}_1(t), \cdots, \mathbf{f}_r(t))$ is a $q \times r$ matrix. For the remainder of this paper, given any optimization problem (P), we denote by $V(\text{P})$ the optimal objective value of problem (P) ; that is, $V(\text{P})$ will be obtained by taking the supremum or infimum. The main results of this paper are shown below.

Theorem 2.1 (**Weak Duality between (CGLP) and (DCGLP)**) *Considering the primal-dual pair problems* (CGLP) *and* (DCGLP), *for any feasible solutions* $\mathbf{x}(t)$ *and* $(\mathbf{y}(t), \boldsymbol{\beta})$ *of problems* (CGLP) *and* (DCGLP), *respectively, we have*

$$\min_{k=1,\cdots,r} \int_0^T \mathbf{f}_k(t)^\top \mathbf{x}(t)dt \leq \int_0^T (\mathbf{g}(t))^\top \mathbf{y}(t)dt;$$

that is, $V(\text{DCGLP}) \geq V(\text{CGLP})$.

Theorem 2.2 (**Strong Duality between (CGLP) and (DCGLP)**) *There exist optimal solutions* $\mathbf{x}^{(*)}(t)$ *and* $(\mathbf{y}^{(*)}(t), \boldsymbol{\beta}^{(*)})$ *to problems* (CGLP) *and* (DCGLP), *respectively, such that*

$$\min_{k=1,\cdots,r} \int_0^T \mathbf{f}_k(t)^\top \mathbf{x}^{(*)}(t)dt = \int_0^T (\mathbf{g}(t))^\top \mathbf{y}^{(*,\lambda)}(t)dt; \qquad (2.3)$$

that is, $V(\text{DCGLP}) = V(\text{CGLP})$.

By considering a sequence of successively finer discretization, Theorem 2.2 can be achieved through a sequence of finite dimensional approximations. The provided proof is constructive and shows, in a sense, that the optimal solution to the problem (CGLP) can be approximated. Hence it also provides a possible solution technique for the problem (CGLP). Furthermore, the derived solution technique has several advantages: the discrete solutions can be utilized to construct feasible approximate solutions to the problem (CGLP); we can know how accurate the searched solution is, and there exists easily checked termination criterion. These advantages make it possible to establish a practical algorithm for solving the problem (CGLP). The related results are now in progress.

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9 SOLVING CONTINUOUS-TIME QUADRATIC FRACTIONAL PROGRAMMING PROBLEMS WITH APPROXIMATION

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Abstract: In this article, a hybrid of the parametric method and discretization approach is proposed to solve a class of continuous-time quadratic fractional programming problems (CQFP). Using different step sizes of discretization problems pertaining to (CQFP), we construct a sequence of continuous, convex and strictly decreasing upper and lower bound functions. The zeros of upper and lower bound functions then determine a sequence of intervals shrinking to the optimal value of (CQFP) as the size of discretization getting larger. By utilizing the intervals we can find corresponding approximate solutions to (CQFP). We also establish upper bounds of lengths of these intervals, and thereby we can predetermine the size of discretization such that the accuracy of the corresponding approximate solution can be controlled within the predefined error tolerance. Therefore, this approach leads to a polynomial-time approximation algorithm that solves the problem (CQFP) to any required accuracy.

Key words: Continuous-time linear programming problems; Continuous-time quadratic Fractional programming problems; Infinite-dimensional nonlinear programming problems; Interval-type algorithm; Duality Theory.

1 INTRODUCTION

Given $p,\ q \in \mathbb{N}$. Let $L^\infty([0,T],\mathbb{R}^p)$ be the space of all measurable and essentially bounded functions from a time space $[0,T]$ into the p-dimensional Euclidean space $\mathbb{R}^p$ and let $C([0,T],\mathbb{R}^p)$ be the space of all continuous functions from $[0,T]$ into the $\mathbb{R}^p$. In this article, we shall pay our attention to the *continuous-time quadratic fractional programming problem* (in short, the problem (CQFP)) that is formulated as follows:

$$
\begin{aligned}
\text{(CQFP)}\quad \text{maximize}\quad & \frac{\mu + \displaystyle\int_0^T \left\{1/2\,\mathbf{x}(t)^\top D(t)\,\mathbf{x}(t) + \mathbf{f}(t)^\top \mathbf{x}(t)\right\} dt}{\xi + \displaystyle\int_0^T \left\{1/2\,\mathbf{x}(t)^\top E(t)\,\mathbf{x}(t) + \mathbf{h}(t)^\top \mathbf{x}(t)\right\} dt} \\
\text{subject to}\quad & B\mathbf{x}(t) \le \mathbf{g}(t) + \int_0^t K\mathbf{x}(s)ds \text{ for all } t \in [0,T] \\
& \mathbf{x}(t) \in L^\infty([0,T],\mathbb{R}^q_+),
\end{aligned}
$$

where

- $\mathbf{x}(t)$ is the decision variable, $T > 0$ is a given time horizon, and the superscript "$^\top$" denotes the transpose operation of matrices.

- B and K are $p \times q$ matrices, $\mathbf{g} \in C([0,T], \mathbb{R}^p_+)$ and $\mathbb{R}^p_+ = \{(x_1, \cdots, x_p)^\top : x_i \geq 0 \text{ for } i = 1, \cdots, p\}$.

- $D(t) = [\, d_{ij}(t)\,]_{q\times q}$ is a symmetric negative semi-definite matrix with $d_{ij}(t) \in C([0,T],\mathbb{R})$, $\mathbf{f} \in C([0,T],\mathbb{R}^q)$ and $\mu \in \mathbb{R}_+$; $E(t) = [\, e_{ij}(t)\,]_{q\times q}$ is a symmetric positive semi-definite matrix with $e_{ij}(t) \in C([0,T],\mathbb{R})$, $\mathbf{h} \in C([0,T],\mathbb{R}^q_+)$ and $\xi > 0$.

We also assume that $B = [B_{ij}]_{p\times q}$ and $K = [K_{ij}]_{p\times q}$ are $p \times q$ constant matrices satisfying

- $K_{ij} \geq 0$ for all $i = 1, \cdots, p$ and $j = 1, \cdots, q$;

- $B_{ij} \geq 0$ and $\sum_{i=1}^{p} B_{ij} > 0$ for all $i = 1, \cdots, p$ and $j = 1, \cdots, q$.

The problem (CQFP) is a generalization of the so-called *continuous-time linear programming problem* (in short, the problem (CLP)). The theory of the problem (CLP), which was originated from the "bottleneck problem" proposed by Bellman (1957), has received considerable attention for a long time, for the related results, one can consult the literature review in Anderson and Nash (1994).

The optimization problem in which the objective function appears as a ratio of two real-valued function is known as a fractional programming problem. Due to its significance appearing in the information theory, stochastic programming and decomposition algorithms for large linear systems, the various theoretical and computational issues have received particular attention in the last decades. For more details on this topic, one may refer to Stancu-Minasian (1997). In the literature, a number of optimality principles and duality models for linear and nonlinear fractional programming problems have been extended to some continuous-time fractional programming problems. For a survey of developed results, one can consult Zalmai (1986, 1987, 1990, 1997). Moreover, Stancu-Minasian and Tigan (2000) studied the stochastic continuous-time linear fractional programming problem. Under some positivity conditions, by using the minimum-risk approach, the stochastic continuous-time linear fractional programming problem can be shown to be equivalent to the deterministic continuous-time linear fractional programming problem. However, in these works, the computational issues were not addressed. Recently, Wen and Wu (2011), Wen et al. (2012) and Wen (2012) have developed computational procedures by combining the parametric method and discrete approximation method to solve subclasses of the present problem (CQFP). In this paper, by extending the methodology of Wen (2012), we shall develop a polynomial-time approximation algorithm that solves the problem (CQFP) to any required accuracy.

2 PARAMETRIC CONTINUOUS-TIME QUADRATIC PROGRAMMING PROBLEMS

For convenience, given any optimization problem (P), we denote by $V(\mathrm{P})$ the optimal objective value of (P); that is, $V(\mathrm{P})$ will be obtained by taking the supremum or infimum. Now, we propose an auxiliary problem associated with (CQFP) which will be proposed and formulated as the parametric continuous-time quadratic programming problem. Let us write

$$\Theta^{(\lambda)}(t) = [\, \theta^{(\lambda)}_{ij}(t)\,]_{q\times q} = D(t) - \lambda\, E(t) \tag{2.1}$$

and

$$\mathbf{a}^{(\lambda)}(t) = \mathbf{f}(t) - \lambda\, \mathbf{h}(t). \tag{2.2}$$

Given $\lambda \geq 0$, we consider the following continuous-time quadratic programming problem (in short, the problem (CQP_λ)):

$$\begin{aligned}
(\mathrm{CQP}_\lambda) \quad \text{maximize} \quad & \mu - \lambda\xi + \int_0^T \left\{ 1/2\, \mathbf{x}(t)^\top \Theta^{(\lambda)}(t)\, \mathbf{x}(t) + \mathbf{a}^{(\lambda)}(t)^\top \mathbf{x}(t) \right\} dt \\
\text{subject to} \quad & B\mathbf{x}(t) \leq \mathbf{g}(t) + \int_0^t K\mathbf{x}(s)ds \text{ for all } t \in [0,T] \\
& \mathbf{x}(t) \in L^\infty([0,T], \mathbb{R}^q_+).
\end{aligned}$$

According to Wen et al. (2012), the dual problem (DCQP_λ) can be defined as follows:

$$\begin{aligned}(\mathrm{DCQP}_\lambda)\quad \text{minimize}\quad & \mu-\lambda\xi+\int_0^T\left\{-1/2\,\mathbf{u}(t)^\top\Theta^{(\lambda)}(t)\mathbf{u}(t)+\mathbf{g}(t)^\top\mathbf{w}(t)\right\}dt\\ \text{subject to}\quad & B^\top\mathbf{w}(t)-\int_t^T K^\top\mathbf{w}(s)ds\geq\Theta^{(\lambda)}(t)\mathbf{u}(t)+\mathbf{a}^{(\lambda)}(t)\text{ for }t\in[0,T],\\ & \mathbf{w}(\cdot)\in L^\infty([0,T],\mathbb{R}^p_+)\text{ and }\mathbf{u}(\cdot)\in L^\infty([0,T],\mathbb{R}^q).\end{aligned}$$

Since $\Theta^{(\lambda)}(t)$ is symmetric negative semi-definite for all $\lambda\geq 0$, the weak and strong duality properties can be established as follows.

Theorem 2.1 (**Weak Duality between** ($\mathbf{CQP}_\lambda$) **and** ($\mathbf{DCQP}_\lambda$) *Given* $\lambda\geq 0$. *Considering the primal-dual pair problems* (CQP_λ) *and* (DCQP_λ), *for any feasible solutions* $\mathbf{x}^{(0)}(t)$ *and* $(\mathbf{u}^{(0)}(t),\mathbf{w}^{(0)}(t))$ *of problems* (CQP_λ) *and* (DCQP_λ), *respectively, we have*

$$\begin{aligned}&\mu-\lambda\xi+\int_0^T\{1/2\,\mathbf{x}^{(0)}(t)^\top\Theta^{(\lambda)}(t)\mathbf{x}^{(0)}(t)+\mathbf{a}^{(\lambda)}(t)^\top\mathbf{x}^{(0)}(t)\}dt\\ &\leq\quad \mu-\lambda\xi+\int_0^T\{-1/2\,\mathbf{u}^{(0)}(t)^\top\Theta^{(\lambda)}(t)\mathbf{u}^{(0)}(t)+\mathbf{g}(t)^\top\mathbf{w}^{(0)}(t)\}dt;\end{aligned}$$

that is, $V(\mathrm{CQP}_\lambda)\leq V(\mathrm{DCQP}_\lambda)$.

Theorem 2.2 (**Strong Duality between** ($\mathbf{CQP}_\lambda$) **and** ($\mathbf{DCQP}_\lambda$)) *Given* $\lambda\geq 0$. *There exist optimal solutions* $\bar{\mathbf{x}}^{(\lambda)}(t)$ *and* $(\bar{\mathbf{u}}^{(\lambda)}(t),\bar{\mathbf{w}}^{(\lambda)}(t))$ *of the primal-dual pair problems* (CQP_λ) *and* (DCQP_λ), *respectively, such that* $\bar{\mathbf{x}}^{(\lambda)}(t)=\bar{\mathbf{u}}^{(\lambda)}(t)$ *and*

$$\begin{aligned}&\mu-\lambda\xi+\int_0^T\left\{1/2\,\bar{\mathbf{x}}^{(\lambda)}(t)^\top\Theta^{(\lambda)}(t)\,\bar{\mathbf{x}}^{(\lambda)}(t)+\mathbf{a}^{(\lambda)}(t)^\top\bar{\mathbf{x}}^{(\lambda)}(t)\right\}dt\\ &=\quad \mu-\lambda\xi+\int_0^T\left\{-1/2\,\bar{\mathbf{u}}^{(\lambda)}(t)^\top\Theta^{(\lambda)}(t)\,\bar{\mathbf{u}}^{(\lambda)}(t)+\mathbf{g}(t)^\top\bar{\mathbf{w}}^{(\lambda)}(t)\right\}dt;\end{aligned}$$

that is, $V(\mathrm{CQP}_\lambda)=V(\mathrm{DCQP}_\lambda)$.

3 MAIN RESULTS

Using the solvability of the problem (CQP_λ), the relations between (CQFP) and its associated auxiliary problem (CQP_λ) can be realized. To see this, we define a function $\mathcal{F}:\mathbb{R}_+\to\mathbb{R}$ by $\mathcal{F}(\lambda)=V(\mathrm{CQP}_\lambda)$ for all $\lambda\geq 0$. Then we obtain the following results.

Proposition 3.1 *The following statements hold true.*

(i) *The real-valued function* $\mathcal{F}(\lambda)$ *is convex, hence is continuous.*

(ii) *If* $\lambda_1<\lambda_2$, *then* $\mathcal{F}(\lambda_1)>\mathcal{F}(\lambda_2)$; *that is, the real-valued function* $\mathcal{F}(\cdot)$ *is strictly decreasing.*

Proposition 3.2 *The following statements hold true.*

(i) *Given any* $\lambda\geq 0$, *then* $\mathcal{F}(\lambda)>0$ *if and only if* $\lambda<V(\mathrm{CQFP})$. *Equivalently,* $\mathcal{F}(\lambda)\leq 0$ *if and only if* $\lambda\geq V(\mathrm{CQFP})$.

(ii) *Suppose that* $\bar{\mathbf{x}}(t)$ *is an optimal solution of* (CQFP) *with* $V(\mathrm{CQFP})=\lambda^*$. *Then* $\bar{\mathbf{x}}(t)$ *is an optimal solution of* (CQP_{λ^*}) *with* $V(\mathrm{CQP}_{\lambda^*})=0$; *that is* $\mathcal{F}(\lambda^*)=0$.

(iii) *If there exists a* $\lambda^*\geq 0$ *such that* $\mathcal{F}(\lambda^*)=0$, *then the optimal solution* $\bar{\mathbf{x}}^{(\lambda^*)}(t)$ *of the problem* (CQP_{λ^*}) *is also an optimal solution of* (CQFP) *and* $V(\mathrm{CQFP})=\lambda^*$.

Let $\mathbf{1}=(1,1,\cdots,1)^\top\in\mathbb{R}^p$ and

$$\widehat{\rho}=\max_{j=1,\cdots,q}\left\{\frac{\sum_{i=1}^p K_{ij}}{\sum_{i=1}^p B_{ij}},\frac{\max_{t\in[0,T]}f_j(t)}{\sum_{i=1}^p B_{ij}}\right\}\geq 0.$$

We define $\mathbf{w}^\star(t) = \widehat{\rho}\, e^{\widehat{\rho}(T-t)}\mathbf{1}$ for all $t \in [0,T]$ and

$$\eta^\star = \frac{1}{\xi}\left\{\mu + \int_0^T \mathbf{g}(t)^\top \mathbf{w}^\star(t)dt\right\} \geq 0. \tag{3.1}$$

The following result shows the solvability of the problem (CQFP).

Theorem 3.1 *The following statements hold true.*

(i) *There exists a unique* λ^* *in the closed interval* $[\mu/\xi,\ \eta^\star]$ *such that* $\mathcal{F}(\lambda^*) = 0$. *Hence, we have* $\frac{\mu}{\xi} \leq V(\text{CQFP}) \leq \eta^\star$.

(ii) *If* $\bar{\mathbf{x}}^{(\lambda^*)}(t)$ *is an optimal solution of the problem* (CQP_{λ^*})*, then it is also an optimal solution of the problem* (CQFP).

From the above results, it follows that solving the problem (CQFP) is equivalent to determine the unique root of the nonlinear equation $\mathcal{F}(\lambda) = 0$. However, it is notoriously difficult to find the exact solution of every (CQP_λ). Given a λ in the closed interval $[\mu/\xi,\ \eta^\star]$, we shall develop a discrete approximation procedure to find the approximate value of $\mathcal{F}(\lambda)$ and to estimate its error bound. Hence, a hybrid of the parametric method and discretization approach will be proposed to solve the problem (CQFP) with approximation. More precisely, using different step sizes of discretization problems of (CQP_λ) and (DCQP_λ), we shall construct a sequence of continuous, convex and strictly decreasing upper and lower bound functions. The zeros of upper and lower bound functions then determine a sequence of intervals shrinking to the optimal value of (CQFP) as the size of discretization getting larger. Besides, by virtue of the intervals we can find corresponding approximate solutions to (CQFP). We also establish upper bounds for the lengths of these intervals, and thereby we can predetermine the size of discretization such that the accuracy of the corresponding approximate solution can be controlled within the predefined error tolerance. Therefore, this approach leads to a polynomial-time approximation algorithm that solves the problem (CQFP) to any required accuracy.

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10 SINGULAR TRAJECTORIES IN THE CONTRAST PROBLEM IN NUCLEAR MAGNETIC RESONANCE

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Abstract: the analysis of the contrast problem in NMR medical imaging is essentially reduced to the analysis of the so-called singular trajectories of the system modeling the problem, a coupling of two spin 1/2 control systems. They are solutions of a Hamiltonian system and in this article we study such dynamics, restricting to the zero level set of the Hamiltonian. They define a vector field on $B^2 \times B^2$ where B^2 is the Bloch ball of each spin particle. The behaviors of the solutions are discussed in relation with the relaxation parameters of each spin and the trajectories are used to provide a final contrast using bang and singular magnetic control sequences.

Key words: Geometric Optimal Control; Medical Imaging; Exceptional Singular Trajectories.

1 INTRODUCTION

Experimental projects in quantum control using finite-dimensional systems as the control of spin systems in nuclear magnetic resonance (see e.g. (Assemat E. (2010); Gershenzon N. I. (2007); Khaneja N. (2003)) and references therein) are motivating new theoretical studies in the case where the system interacts with its environment. The primary objective of this article is the application of techniques from geometric optimal control theory to the control of the spin dynamics by magnetic fields in nuclear magnetic resonance (NMR). Through interaction with a magnetic field, NMR involves the manipulation of nuclear spins. It has many potential applications extending from the determination of molecular structures (NMR spectroscopy) and quantum computing, where NMR remains one of the most promising road in the construction of a scalable quantum computer, to medical imagery (MRI).

In this article we focus on the *contrast problem* in medical imaging using NMR. The system is modeled by a coupling of two spins particles governed by the *Bloch equation* with distinct relaxation parameters. In this problem the primary goal is to bring the magnetization vector of the first spin to the origin of the Bloch ball (so as to appear dark when imaged) while maximizing the modulus of the magnetization vector of the second spin (to appear light when imaged). The resulting difference in moduli gives the image its contrast. This is the basic question that has to be solved in order to improve the quality and the resolution of medical imaging. Solving the contrast imaging problem can potentially have a profound impact on how medical imaging is done in hospitals. Indeed, by designing magnetic fields to maximize the distance between the two spin we increase the image resolution and therefore improve its quality which improves patient care.

Previous work has treated the time-optimal control of a single spin 1/2 particle (Bonnard B. (2009)). In this setting, a spin particle is modeled by a magnetization vector whose dynamics are given by the Bloch equation. Using a normalization (Lapert M. (2010)), the coordinates are (x, y, z) and the control is $u = (u_x, u_y)$ with $|u| \leq 2\pi$. A substance's parameters are γ and Γ, determined by its relaxation times. In this formulation, the system is written

$$\dot{x} = -\Gamma x + u_y z, \quad \dot{y} = -\Gamma y - u_x z, \quad \dot{z} = \gamma(1-z) + (u_x y - u_y x). \tag{1.1}$$

The uncontrolled system has a globally attractive equilibrium point $N = (0, 0, 1)$. For parameters such that $2\Gamma \geq \gamma \geq 0$, the dynamics are invariant for the unit ball.

This foundation is built upon with the contrast problem, where two such non-interacting spins are controlled by the same magnetic control field. For this, we take a pair of such systems with respective coefficients $\Lambda_1 = (\gamma_1, \Gamma_1)$ and $\Lambda_2 = (\gamma_2, \Gamma_2)$, controlled by the same magnetic field. Each is governed by (1.1), which we denote individually by $\frac{dq_i}{dt} = F_i(q_i, \Lambda_i, u)$. Together, this gives the system $\frac{dq}{dt} = F(q, u)$, where $q = (q_1, q_2)$ and $q_i = (x_i, y_i, z_i)$, $i = 1, 2$.

With this system, the *Mayer-type* optimal control problem is stated as follows. From the initial point $((0, 0, 1), (0, 0, 1))$, find a control u defined on $[0, T]$ with $q_1(T) = 0$ that maximizes $|q_2(T)|$ (which is the contrast since $q_1 = 0$) or, identically, minimizes $c(q(T)) = -|q_2(T)|^2$. A subcase of this problem that is treated in this work is the *single-input case* where $u_y = 0$, restricting the state to $x_1 = x_2 = 0$.

To summarize, we have a system of the form $\dot{q} = F(q, u)$, $q(0) = q_0$, $q \in \mathbb{R}^4$ with the terminal condition of the form $\psi(q(T)) = 0$, while minimizing $\phi(q(T))$. Fixing the level set to $\phi(q) = m$, this with $\psi(q(T)) = 0$ leads us to introduce a family of manifolds denoted M_m. We denote by $A(q(0), T) = \bigcup_{u \in \mathcal{U}} q(T, q_0, u)$ the union of terminal points of trajectories emanating at time $t = 0$ from q_0 for each admissible control $u(\cdot) \in \mathcal{U}$, $u(\cdot) \in L^\infty[0, T] \cap \{u : |u| \leq u_{\max}\}$, such that $q(\cdot, q_0, u)$ is defined on the whole interval. Clearly an optimal control u^* must be chosen such that the terminal point $q^*(T)$ belongs to the boundary of $A(q_0, T)$ and satisfies the transversality condition of the maximum principle. Hence, this viewpoint allows us to re-state the problem as a time-optimal control problem which leads to the following optimality conditions.

Proposition 1.1 *Define the Hamiltonian $H(q, p, u) = \langle p, F(q, u) \rangle$. By the maximum principle (Pontryagin L. S. (1962)), an optimal control has to satisfy the following necessary optimality conditions: (i) $\dot{q} = \frac{\partial H}{\partial p}(q, p, u)$ $\quad \dot{p} = -\frac{\partial H}{\partial q}(q, p, u)$; (ii) $H(q, p, u) = \max_{|v| \leq M} H(q, p, v)$; (iii) $\psi(q(T)) = 0$; and (iv) $p(T) = p_0 \frac{\partial \phi}{\partial q} + {}^t\delta \frac{\partial \psi}{\partial q}$, where ${}^t\delta \in \mathbb{R}^k$ and $p_0 \leq 0$.*

A pair (q, p) which satisfies the maximum principle, in the sense just stated, is called an *extremal.* The maximum principle provides only necessary conditions, hence to complete the analysis one must classify the behaviors of extremals of order zero near the switching surface to analyze the possible connections between singular arcs of order zero.

A direct application to the contrast imaging problem gives $q_1(T) = 0$ and splitting $p = (p_1, p_2)$, $p_2(T) = -2p_0 q_2(T)$, $p_0 \leq 0$, since $\phi(q) = -|q_2|^2$ in the contrast problem. In the nontrivial case $p_0 \neq 0$, one can normalize $p_0 = -\frac{1}{2}$.

Remark 1.1 *The optimal control problem can be mainly reduced to the analysis of the so-called singular trajectories since the optimal solution is a concatenation of a sequence of bang-singular arcs.*

The motivation of this article is to pursue the analysis of the singular flow with a special emphasis on the exceptional extremals. The motivations are the following.
(1) Find the precise BS sequence in connection with the relaxation parameters.
(2) Relate the relaxation parameters to the feedback invariants of the system coded by the singular flow.
(3) Analyze the attractivity properties of the north pole for the singular flow in relation with the relaxations parameters. This is related to the experimental aspect of the problem, since in this setting the experiment is repeated several times, requiring that the system relax to the equilibrium between each run. This return time is much longer than the time to produce the desired contrast. The time-optimal solutions are among singular trajectories, again demonstrating the significance of these extremals.
(4) The optimality status is coded in the singular flow using the concept of conjugate points.

Experiments focus on biological substances such as water, blood, cerebrospinal fluid, cerebral matter, and fat. In this paper, we focus on the contrast between deoxygenated and oxygenated blood. For this pair of substances, the parameters are $\Lambda_{\text{do}} = (\gamma_1 = 1/(32.3 \cdot 1.35), \Gamma_1 = 1/(32.3 \cdot 0.05), \gamma_2 = 1/(32.3 \cdot 1.35), \Gamma_2 = 1/(32.3 \cdot 0.2))$.

2 THE SINGLE SPIN 1/2 CASE

An important analysis concerns the saturation problem for a single spin 1/2 particle. The system is described by (1.1) and the problem is to drive in minimum time the magnetization vector from the north pole $N = (0, 0, 1)$ to the center 0 of the Bloch ball. It is analyzed in detail in (Lapert M. (2010)) and we present the main result. Due to the symmetry of revolution of the problem we can restrict the system to $x = 0$ by $u_y \equiv 0$ and the system is written as the single input, two-dimensional system $\dot{q} = F_0(q)+uF_1(q)$ where $q = (y, z)$, $|u| \leq 1$.

Applying the maximum principle, optimal solutions can be found by concatenating bang arcs, for which $|u| = 1$, and singular arcs for which the control is given by $H_1 = 0$, where $H = H_0 + uH_1$, $H_i = \langle p, F_i \rangle$ while (q, p) is the solution of the Hamiltonian dynamics $\dot{q} = \frac{\partial H}{\partial p}$, $\dot{p} = -\frac{\partial H}{\partial q}$. Differentiating $H_1 = 0$ one gets that the singular trajectories will form two lines: the vertical axis of revolution $y = 0$ and a horizontal line given by $z = \gamma/(\gamma - \Gamma)$ for which the singular control is given by $u_s = \gamma(2\Gamma - \gamma)/(2(\gamma - \Gamma)y)$ and $u_s \to \infty$ as $y \to 0$ if $\gamma \neq 0$. For the applications, the interesting parameter cases satisfy $\Gamma > 3\gamma/2$ and the singular line $z = \gamma/(2(\gamma - \Gamma))$ is such that $0 > z > -1$.

The main result is then the following.

Proposition 2.1 *The optimal solution in the saturation problem is of the form bang-singular-bang-singular.*

3 THE EXCEPTIONAL SINGULAR TRAJECTORIES

Consider a smooth control system of the form $\dot{x} = F_0(x)+uF_1(x)$, $x \in X$, and let $H(x, p, u) = \langle p, F_0(x)+uF_1(x)\rangle = H_0 + uH_1$ be the Hamiltonian lift.

Definition 3.1 *For a system (F_0, F_1), an extremal pair (x, p) is called* singular *on $[0, T]$ if it satisfies almost everywhere the equations $\dot{x} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial x}, \frac{\partial H}{\partial u} = 0$.*

Proposition 3.1 *On $T^*X \setminus \{\{\{F_1, F_0\}, F_1\} = 0\}$, singular extremals are the smooth solutions of $\dot{x} = \frac{\partial H_s}{\partial p}, \dot{p} = -\frac{\partial H_s}{\partial x}$ starting at $t = 0$ from $\Sigma' : H_1 = \{H_1, H_0\} = 0$ where $H_s = \langle p, F_0 + u_sF_1\rangle$, $u_s(x, p) = -\frac{\{\{H_1,H_0\},H_0\}}{\{\{H_1,H_0\},H_1\}}$.*

3.1 Application to the contrast problem

The system under study is $\dot{q} = F_0(q)+uF_1(q)$, $q = (y_1, z_1, y_2, z_2) \in B^2 \times B^2$ where B^2 is the unit (Bloch) ball with $F_0 = \sum_{i=1}^{2} -\Gamma_i y_i \frac{\partial}{\partial y_i} + \gamma_i(1 - z_i)\frac{\partial}{\partial z_i}$ and $F_1 = \sum_{i=1}^{2} -z_i \frac{\partial}{\partial y_i} + y_i \frac{\partial}{\partial z_i}$.

Observe that because of the two constraints $H_1 = \{H_1, H_0\} = 0$ and the linearity with respect to p the equations describing the singular flow reduce to $\frac{dq}{dt} = F_0(q) - \frac{\mathcal{D}'(q,\lambda)}{\mathcal{D}(q,\lambda)}F_1$ where $\mathcal{D} = \{\{H_1, H_0\}, H_1\}$ and $\mathcal{D}' = \{\{H_1, H_0\}, H_0\}$ and λ is a one-dimensional time-dependent parameter whose evolution is described by the adjoint equation for p.

If the transfer time is not fixed, then according to the maximum principle, this leads to the additional constraint $H_s = H_0 + u_s(q, p)H_1 = 0$. Such singular extremals contained in the zero level are called *exceptional*.

Introducing $D = \det(F_0, F_1, [F_1, F_0], [[F_1, F_0], F_1])$ and $D' = \det(F_0, F_1, [F_1, F_0], [[F_1, F_0], F_0])$, the singular control in the exceptional case is given by $-D'(q)/D(q)$ and the exceptional trajectories are solutions of a vector field X^s on the state space $B^2 \times B^2$ defined by $\frac{dq}{dt} = F_0(q) - \frac{D'(q)}{D(q)}F_1$.

The remainder of this article will be restricted to the exceptional case, associated to the analysis of the contrast problem with no fixed transfer time. One consideration being its relation with the vector field X^s and the singular set $D(q) = 0$ on the state space where the singular control explodes. According to our previous analysis for a single spin, this leads to saturation of the singular control u_s near the surface $D = 0$ and the bridge phenomenon to generate optimal sequences, concatenations of bang-singular arcs, with several singular surfaces, a connection between two consecutive singular arcs being realized by a bang arc which is tangent to the switching surface $\Sigma : H_1 = 0$ at both extremities.

An important tool to analyze the problem is the introduction of the feedback classification of the system, showing in particular the dependence with respect to the relaxation parameters. This is presented in the next section.

4 FEEDBACK CLASSIFICATION AND THE EXCEPTIONAL TRAJECTORIES

Definition 4.1 *Let E and F be two $\mathbb{R}$-vector spaces and let G be a group acting linearly on E and F. A homomorphism $\chi : G \to \mathbb{R} \setminus \{0\}$ is called a* character*. Let χ be a character. A* semi-invariant *of weight χ is a map $\lambda : E \to \mathbb{R}$ such that for all $g \in G$ and all $x \in E$, $\lambda(g.x) = \chi(g)\lambda(x)$; it is an* invariant *if $\chi = 1$. A map $\lambda : E \to F$ is a* semi-covariant *of weight χ if for all $g \in G$ and for all $x \in E$, $\lambda(g.x) = \chi(g)g.\lambda(x)$; it is called a covariant if $\chi = 1$.*

Definition 4.2 *Let $\mathcal{C} = (F_0, F_1)$ be the set of smooth systems on $X \simeq \mathbb{R}^n$. Let $(F_0, F_1), (F_0', F_1')$ be two elements of $\mathcal{C}$. They are called feedback equivalent if there exists a smooth diffeomorphism φ of $\mathbb{R}^n$ and a feedback $u = \alpha(x)+\beta(x)v$, α, β smooth and β invertible such that $F_0' = \varphi*(F_0+F_1\alpha)$ and $F_1' = \varphi*(F_1\beta)$, where $\varphi * Z$ denotes the image of the vector field defined by $\varphi * Z = \frac{\partial\varphi}{\partial x}^{-1} Z \circ \varphi$. The set $G = (\varphi, \alpha, \beta)$ endowed with the underlying multiplication rule is called the feedback group.*

Definition 4.3 *Let $(F_0, F_1) \in \mathcal{C}$ and let λ_1 be the map which associates the constrained Hamiltonian vector field $(\vec{H}'_s, \Sigma')$ whose solutions are singular trajectories. We define the action of $(\varphi, \alpha, \beta) \in G$ on $(\vec{H}'_s, \Sigma')$ by the action induced by the symplectic change of coordinates (Mathieu transformation) $\vec{\varphi} : x = \varphi(X), p = p\frac{\partial\varphi}{\partial X}^{-1}$, in particular the feedback acts trivially.*

We have the following.

Theorem 4.1 *The mapping λ_1 is a covariant.*

Now we shall examine this action restricted to the singular flow. Computing, we have the following.

Lemma 4.1 *(1) $D^{F_0+\alpha F_1, \beta F_1} = \beta^4 D^{F_0,F_1}$, (2) $D'^{F_0+\alpha F_1,\beta F_1} = \beta^3(D'^{F_0,F_1}+\alpha D^{F_0,F_1})$, (3) $D^{\varphi * F_0, \varphi * F_1}(x) = \det\left(\frac{\partial\varphi}{\partial x}^{-1}\right) D^{F_0,F_1}(\varphi(x))$, and (4) $D'^{\varphi*F_0,\varphi*F_1}(x) = \det\left(\frac{\partial\varphi}{\partial x}^{-1}\right) D'^{F_0,F_1}(\varphi(x))$.*

From which we deduce the following proposition.

Proposition 4.1 *(1) $\lambda_2 : (F_0, F_1) \to X^s$ is a covariant (the feedback group acting on X^s by a change of coordinates only), (2) $\lambda_3 : (F_0, F_1) \to D$ is a semi-covariant, and (3) $\lambda_4 : (F_0, F_1) \to X_r^s = DF_0 - D'F_1$ is a semi-covariant.*

4.1 Geometric interpretation

Use the action of diffeomorphisms on $(X^s, D = 0)$ to classify the set of systems (F_0, F_1) in relation with the contrast problem. In particular, compute the feedback invariants in relation with the relaxation parameters.

From the geometric point of view this is the classification of the trajectories of X^s, in particular near the north pole which is a singular point. The surface $D = 0$ is the surface where the singular control explodes, except for points where $D' = 0$: the surface $D = D' = 0$ being a feedback invariant. The classification problem is introduced: the set D is a quartic form in dimension four, while $D = D' = 0$ is the intersection of two quartics. A first step is to reduce the problem in the vicinity of the north pole.

5 NUMERICAL SIMULATIONS OF EXCEPTIONAL SINGULAR TRAJECTORIES

In (Bonnard B. (2012)), the authors construct numerical methods to maximize contrast with controls which are concatenations of one or more *bang-singular* arcs (motivated by Remark 1.1). We take a similar approach here: we simulate an extremal of the form *bang-exceptional*, i.e., a bang arc followed by an exceptional singular arc satisfying the problem statement. We denote the *switching time* from the bang to singular arc as t_1, and the final time as T.

5.1 Extremal conditions

As stated in §3, we have $H = \langle F_0+uF_1, p\rangle = 0$ everywhere on an extremal containing an exceptional singular arc. Therefore at the initial time, where $q(0) = (0, 1, 0, 1)$, we have $p(0) \in \text{span}\left\{\big(0, 1, 0, 0\big), \big(0, 0, 0, 1\big), \big(1, 0, -1, 0\big)\right\}$. At the switching time, we have $H_0 = H_1 = \{H_0, H_1\} = 0$, together giving that

$$p(t_1) \in \ker\{F_0, F_1, [F_0, F_1]\}. \tag{5.1}$$

In particular, choosing an $q(t_1)$ along the bang arc at a desired switching time, we can find $p(t_1)$ such that $(q(t_1), p(t_1))$ is a switching point of a possible bang-exceptional extremal. It also must be verified that on the time interval $[0, t_1]$, H_1 is positive.

5.2 Conjugate points

The concept of a conjugate point is related to second-order optimality conditions—a trajectory is locally optimal prior to a conjugate point, and therefore it plays an important role in the study of optimal synthesis.

Definition 5.1 *Let $\hat{H}(q,p) = \langle p, F(q,u)\rangle$ where u is the exceptional singular control. Let $z = (q,p)$ be the reference extremal defined on $[0,T]$. The variational equation $\delta\dot{z} = d\overrightarrow{\hat{H}}(z(t))\delta z$ is called the Jacobi equation. A Jacobi field $J(t)$, a nontrivial solution $\delta z = (\delta q, \delta p)$, is said to be vertical at time t if $\delta q(t) = d\Pi_{z(t)}\delta z(t) = 0$, where Π is the canonical projection $(q,p) \mapsto q$.*

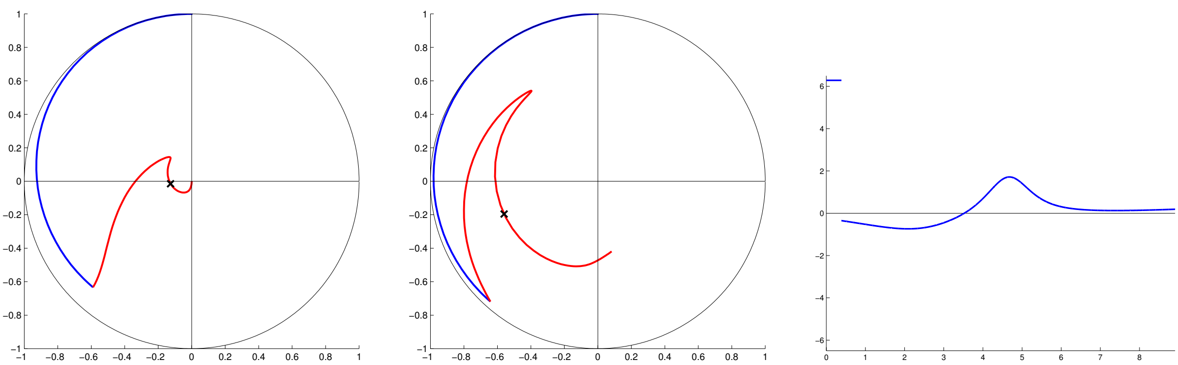
Definition 5.2 *We define the exponential mapping for fixed $q(0) = q_0$ as the mapping $\exp_{q_0} : (t, p_0) \mapsto \Pi(z(t, z_0))$ where $z(\cdot)$ is the solution of $\hat{H}$ with initial condition $z_0 = (q_0, p_0)$, p_0 being normalized by $|p_0| = 1$. A time $t_c > 0$ is said to be geometrically conjugate to zero if the exponential mapping is not of maximal rank $(n-1)$ at $t = t_c$ and the associated point $q(t_c)$ is said to be geometrically conjugate to q_0.*

The conjugate point algorithm is as follows (Bonnard B. (2007)). The reference extremal is a solution of $\frac{dq}{dt} = X^s(q(t))$ and the variational equation reduces to $\frac{d\delta q}{dt} = \frac{\partial X^s}{\partial q}(q(t))\delta q$. To test the existence of a conjugate point, we compute a single Jacobi field $J_0(t)$ whose projection on the q-space is denoted δq_0 and is a solution of the variational equation with initial condition $q(0) = F_1(q(0))$. The conjugate point test is $\delta q_0(t) \in \operatorname{span}\{F_0(q(t)), F_1(q(t))\}$, or equivalently, $\det[\delta q_0(t), F_0(q(t)), F_1(q(t)), \operatorname{ad}^2 F_1.F_0(q(t))] = 0$.

5.3 Numerical simulation

The computation is performed using the COTCOT software package (Bonnard B. (2005)) in the following manner. A bang-exceptional trajectory is characterized by the switching time t_1. From this one-parameter family of trajectories, regions where the first spin passes through a neighborhood of the origin are identified. In such a region, a pair of times such that the trajectory crosses the y_1-axis on opposite sides of the origin, and bisection is used to find t_1 such that $q_1(T) = 0$.

Using this, the extremal producing maximum contrast is given by the switching time $t_1 = 0.3858$, with final time $T = 8.9081$ and contrast $\|q_2(T)\| = 0.4272$. Along this trajectory, a conjugate point is found at $t_c = 4.8776$, and therefore this extremal is not locally optimal in its entirety. This result is illustrated in Figure 5.1, which shows the state trajectory and associated control.



(a) Projection of extremal onto (y_1, z_1) plane.

(b) Projection of extremal onto (y_2, z_2) plane.

(c) Value of the control u.

Figure 5.1 Illustration of numerical simulation in §5.3. Subfigures (a) and (b) display the state trajectory, with the initial bang arc in blue, the exceptional singular arc in red, and the conjugate point location marked with a black ×. Subfigure (c) shows the associated control.

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11 OPTIMALITY OF PARTIALLY SINGULAR VARIATIONAL PROBLEMS

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Abstract: Modern mathematical theory of singular variational problems was motivated by the discovery of intermediate thrust arcs of a rocket trajectory in an inverse square law field by D F Lawden in 1962. Independent of this research, P A M Dirac and others in physics have developed since the 1950s a generalized Hamiltonian dynamics for theoretical physics because of singular variational problems in two important fields in theoretical physics. The electromagnetic field and Einsteins general gravitational field are Euler-Lagrange equations of singular variational problems. There is still a need to have better mathematical understanding of an extremal which is nonsingular in some control variables and singular in others. Here some properties of a class of singular variational problems are analysed.

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12 TWO-DIMENSIONAL TIME-MINIMUM SPACE TRANSFERS TO TEMPORARY CAPTURED NEAR EARTH ORBITERS

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Abstract: We present numerical computations of time-minimal transfers within the Earth-Moon system, from the geostationary orbit to temporarily-captured near Earth orbiters. To this end, we use indirect methods in optimal control initialized thanks to previously computed optimal transfers to the Lagrangian point L_1.

Key words: Temporarily-captured near Earth satellites; Time-minimal space transfers; Restricted 3-body problem; Indirect methods in optimal control.

1 INTRODUCTION

This paper deals with numerical computations of space transfers in the Earth-Moon system, from the geostationary orbit to temporarily-captured near Earth orbiters (TCO). Such objects are defined according to two criteria: the planetocentric Keplerian energy and the planetocentric distance (Kary D M. (1996)). The first study about their population characteristics has been carried out recently, providing their steady-state size-frequency and residence-time distribution (Granvik M. (2012)), and the execution of space missions to near Earth TCO would be a major scientific achievement. We present preliminary work regarding the computation of simulated time-minimal transfers to such orbiters, based on indirect methods in optimal control. Such time-minimal transfers are particularly important since the period during which a TCO orbits around the Earth may be very short. As a first step of the analysis, we focus on two-dimensional transfers by modeling the Earth-Moon system using the planar restricted three-body problem. Our methodology relies on initializing a classical shooting method with the time-minimal transfer to the equilibrium point L_1 of the Earth-Moon system (Picot G. (2012)), so as to compute transfers to the planar projections of the TCO going through a small neighborhood of the spatial location of this point with a low velocity. A continuation method is also used to reduce the maximum bound on the engine propulsion. As a result, we obtain a collection of locally time-minimal transfers associated with a wide range of maximal thrusts.

2 GEOMETRIC ANALYSIS AND NUMERICAL METHODS

Spatial trajectories in the Earth-Moon system are frequently modeled as solutions of the restricted three-body problem, where the three bodies are the Earth, Moon, and our spacecraft (**?**). The Earth and Moon are referred to as the primaries, since the mass of the spacecraft is assumed to negligible compared to the masses of the other bodies, and therefore the spacecraft does not affect the motion of the primaries. The primaries are assumed to be revolving circularly around their center of mass, which is chosen as the origin of the coordinate system. This model is reasonable to describe the Earth-Moon system since the eccentricity and inclination of the Moon's orbit around Earth are small (0.0549 and 5.145 to the ecliptic, respectively).

The distance between the primaries, their angular velocities, and the gravitational constant are normalized to 1. So, in the rotating frame, the Earth is located at $(-\mu, 0)$ with mass $1-\mu$, and the Moon is located at $(1-\mu, 0)$ with mass μ, with the parameter $\mu \simeq 0.012153$ being called the reduced mass. As a first approach, we will suppose that the motion of the spacecraft is restrained to the plane defined by the motion of the primaries. This motion is then governed by the equations

$$\ddot{x} - 2\dot{y} - x = \frac{\partial V}{\partial x} + u_1, \ddot{y} + 2\dot{x} - y = \frac{\partial V}{\partial y} + u_2 \tag{2.1}$$

where $-V = \frac{\mu-1}{\varrho_1^3} - \frac{\mu}{\varrho_2^3}$ is the mechanical potential of the problem, ϱ_1 and ϱ_2 are the distances to the primaries and $u = (u_1, u_2)$ is a control term corresponding to the acceleration provided by the the engine. By defining the new variable $q = (x, y, \dot{x}, \dot{y})$, equation (2.3) can be written as a bi-input control system

$$\dot{q} = F_0(q) + F_1(q)u_1 + F_2(q)u_2 \tag{2.2}$$

with

$$F_0(q) = \begin{pmatrix} q_3 \\ q_4 \\ 2q_4 + q_1 - (1-\mu)\frac{q_1+\mu}{((q_1+\mu)^2+q_2^2)^{\frac{3}{2}}} - \mu\frac{q_1-1+\mu}{((q_1-1+\mu)^2+q_2^2)^{\frac{3}{2}}} \\ -2q_3 + q_2 - (1-\mu)\frac{q_2}{((q_1+\mu)^2+q_2^2)^{\frac{3}{2}}} - \mu\frac{q_2}{((q_1-1+\mu)^2+q_2^2)^{\frac{3}{2}}} \end{pmatrix}, \ F_1(q) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \ F_2(q) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

The maximal thrust allowed by the engine is given by $|u|$ which is assumed to be bounded by a parameter ϵ. Our objective is to compute time-minimal numerical transfers from the geostationary orbit $\mathcal{O}_g$ to rendezvous with TCO at specific points on their orbits for values of ϵ representing different thrusts. In mathematical terms, our aim is to solve optimal control problems of the form

$$\begin{cases} \dot{q} = F_0(q) + F_1(q)u_1 + F_2(q)u_2 \\ \min_{u(\cdot)\in B_{\mathbb{R}^2}(0,\epsilon)} \int_{t_0}^{t_f} dt \\ q(0) \in \mathcal{O}_g, \ q(t_f) \in \mathcal{TCO} \end{cases} \tag{2.3}$$

where $\mathcal{TCO}$ is a given trajectory of a TCO and t_f is the transfer time that we want to minimize. Note that a more realistic model would take into account the spacecraft mass variation by considering the equation $\dot{m} = -\delta|u|$. This is not the case in this paper.

Applying the Pontryagin Maximum Principle (Pontryagin L.S. (1962)), it turns out that, in the normal case, every solution $q(t)$ of the optimal control (3.6) is necessarily the projection of an extremal curve $(q(t), p(t))$ solution of the system

$$\dot{q}(t) = \frac{\partial H}{\partial p}, \ \dot{p}(t) = -\frac{\partial H}{\partial q} \tag{2.4}$$

where the pseudo-Hamiltonian function H is defined by $H(q,p,u) = -1 + H_0(p,q) + \epsilon(u_1 H_1(p,q)) + u_2 H_2(p,q))$ with $H_i(p,q) = \langle p, F_i(q)\rangle, i = 0,\ 1,\ 2$ Moreover, we deduce from the maximization condition that, whenever $(H_1, H_2) \neq (0,0)$, the control u is given by $u_i = \frac{H_i}{\sqrt{H_1^2+H_2^2}}$, $i = 1,\ 2$. Substituting in H, yields the expression of the real Hamiltonian function $H_r(z) = -1 + H_0(z) + \epsilon((H_1^2(z) + H_2^2(z))^{\frac{1}{2}})$ which is identically zero on $[0, t_f]$ since the transfer time is not fixed. Defining the switching surface

$\Sigma = \{H_1(p,q) = H_2(p,q) = 0\}$, an element $(q,p) \in \mathbb{R}^8 \backslash \Sigma$ is said to be of order 0. According to Bonnard B. (2005), we state that every normal time-minimal trajectory is a concatenation of a finite number of arcs of order 0 such that the control u instantaneously rotates by an angle π at junction points.

Such extremal solutions can be computed using a shooting method which relies on determining a right initial condition (q_0, p_0) so that the corresponding solution of the Hamiltonian system $(\dot{q}(t), \dot{p}(t)) = \overrightarrow{H_r}(q(t), p(t))$ satisfies the required final condition. Since the value of q_0 is fixed, our goal is to determine an appropriate value of p_0. Rewriting the required boundary conditions as $R(q(0), p(0), q(t_f), p(t_f)) = 0$, the boundary values problem we have to solve is

$$\begin{cases} (\dot{q}(t), \dot{p}(t)) = \overrightarrow{H_r}((q(t), p(t))) \\ R(q(0), p(0), q(t_f), p(t_f)) = 0. \end{cases} \tag{2.5}$$

which is equivalent to finding a zero of the so-called shooting function E defined by

$$E : (p_0, t_f) \longrightarrow R(q(0), p(0), q(t_f), p(t_f)). \tag{2.6}$$

In that case, the condition $H_r = 0$ is crucial and provide one of the needed equations so that solving the shooting equation is a well-posed problem. Moreover, since $u_r(q,p)$ is smooth, so is E and we can find such zeros using a Newton type algorithm, provided we can approximate them accurately enough. The higher the maximum control bound ϵ, the shorter the corresponding transfer time and the Newton algorithm converges easily to a solution of the shooting method whichever initial guess is used. Therefore, we first compute a reference extremal whose projection on the phase space is a candidate to be a high-thrust time-minimal transfer. A discrete homotopic method on the parameter ϵ, based on following a smooth path of zeros (Bonnard B. (2010)), is then used to determine solutions of the shooting function for smaller control bound and thus candidates to be low-thrust time-minimal transfers from $\mathcal{O}_g$ to $\mathcal{TCO}$. Finally, the local optimality of such transfers is checked using the second order condition connected to the geometric notion of conjugate point (Bonnard B. (2007)). Let us define t_c^1 the first conjugate time along a transfer $q(t)$. It is known that $q(t)$ is locally optimal on $[0, t_c^1)$ in L^∞ topology; if $t > t_c^1$ then $q(.)$ is not locally optimal on $[0,t]$. The conjugate times can be easily numerically computed as they correspond to the times at which the exponential mapping $\exp_{q_0,t} : p_0 \longrightarrow q(t, q_0, p_0)$ is not an immersion at p_0 (Bonnard B. (2007)).

3 METHODOLOGY AND RESULTS

Among a database of 16923 TCO numerically simulated spatial trajectories, we selected the 383 that come within 0.1 lunar units of the well-known collinear equilibrium point L_1 of the Earth-Moon system (Szebehely V. (1967)), a Lunar unit being the distance between the Earth and the Moon. The value 0.1 is arbitrary but small enough to represent a neighborhood of L_1 in which such trajectories can be approximated by a solution of the spatial restricted 3-body problem. Among them, we examined the 100 with the smallest absolute perpendicular coordinate to the plane defined by the rotation of the Moon around the Earth, at the time they are nearest L_1. This choice was made, as a first approach, to guarantee that the dynamic of the considered trajectories with respect to this coordinate could be neglected so that they could be approximated by their two-dimensional projection on the plane of rotation of the Moon for a significant interval of time. The projections on this plane have been calculated at every time by taking into account the position of the Moon in its orbit and the inclination of this orbit. The resulting bi-dimensional trajectories have then been expressed as trajectories in the planar restricted 3-body problem by using the usual change of variable from the inertial to the rotating frame.

We set the point q_0=(0.0947, 0, 0, 2.8792), expressed in the distance and time units of the restricted 3-body problem, as the initial position and velocity of the spacecraft on the geostationary orbit. For each of the 100 TCO trajectories that have been selected, we fix the final condition as the position and velocity as the one of the TCO when it is nearest to L_1. Our computations were based on using the planar time-minimal transfer from the geostationary orbit to L_1 (Picot G. (2012)), associated with a maximum thrust of 1N for a spacecraft of 350kg, to initialize the shooting method. This provided us with a collection of 2-dimensional extremal transfers to the TCO. Each one of these was then used as the starting point of a discrete continuation method, whose homotopic parameter was the engine propulsion, so as to compute iteratively extremal transfers to TCO with lower thrust. At each step of the continuation algorithm, the first conjugate time along every generated extremal was computed to ensure, according to the second order condition, that it was locally time-optimal. These computations were carried out using the software HAMPATH (Caillau J-B. (2012)) that computes solutions of indirect methods in optimal control and checks the second order optimality condition when smooth optimal control problems are considered. As a result, the continuation algorithm provided 16 time-optimal transfers to distinct TCO associated with a propulsion bound of 0.2N, that we arbitrarily defined to be the lowest acceptable thrust to match the TCOs position and velocity. The minimum time needed to reach a TCO has been found to be 9.9921 days with a maximum thrust of 1N and 46.7203 days with a maximum thrust of 0.2N. Let us point out that 14 of the computed optimal transfer times associated with a maximum thrust of 0.2N turned out to be shorter than the interval of time over which the motion to the TCO had been originally simulated. This remark is crucial from the practical point of view since it suggests that the detection time of some TCO, before they reach a small neighborhood of the point L_1, would be large enough to envision low-thrust time-optimal rendezvous missions with such objects.

Figure 5.1 displays several examples of locally time-minimal transfers from the geostationary orbit to TCO, associated with thrust of 1N and 0.2N, represented in the rotating frame of the restricted 3-body problem. Comparisons between the transfer time t_f and the first conjugate time t_c^1 along extremals are provided in Figure 5.2.

4 CONCLUSION

This preliminary study provides with the first numerical approximations of time-optimal transfers to TCO. Providing a precise-enough initial guess is always the biggest issue when using indirect methods to solve optimal control problems. In that sense, the transfers we computed here are extremely valuable since they can be used as initializations for future work. We can use these initializations to study transfers to the entire database of TCO, and to widen our target set to including whole segments of the TCOS trajectories, rather than only the points closest to L_1. Furthermore, we have already begun to develop more realistic studies of optimal transfers to TCO, such as the three-dimensional problem. We are also working to consider the more-general N-body problem to take into account the influence of others planets of the solar system on the motion of the spacecraft.

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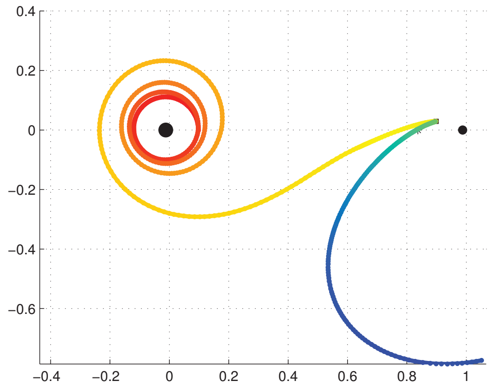
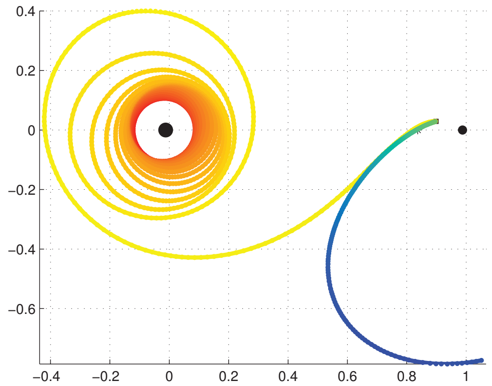
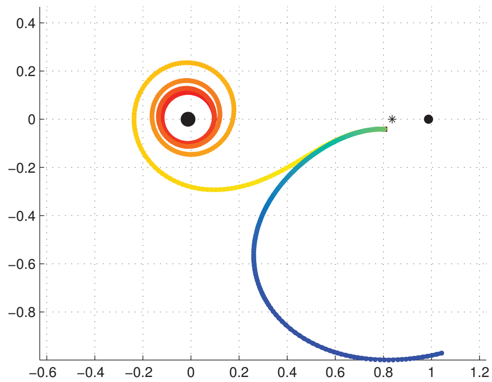


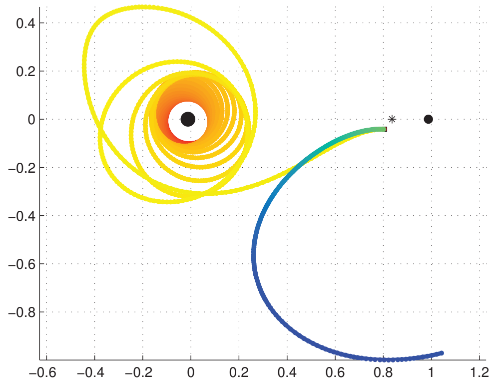
(a) TCO 00057, 1N, t_f=11.76 days



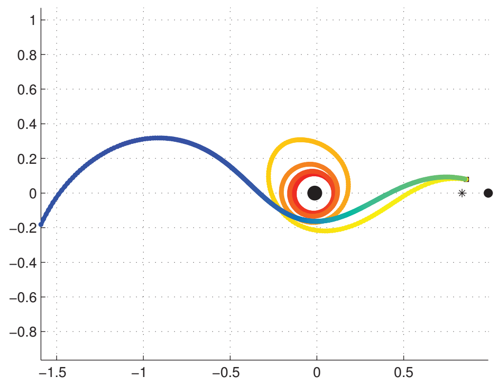
(b) TCO 00057, 0.2N, t_f=46.7203 days



(c) TCO 03813 , 1N, t_f=9.982 days



(d) TCO 03813, 0.2N, t_f=55.3592 days



(e) TCO 16803, 1N, t_f=10.588 days

Figure 3.1 Locally time-minimal transfers to 3 distinct TCO, associated with different thrusts, in the rotating frame. In each figure, the Earth (left) and the Moon (right) are shown as black circles and the point L_1 by a star. The blue/green trajectories represent neighborhoods of the TCO trajectories, displayed in the restricted 3-body problem. The red/yellow trajectories represent the optimal rendezvous from the geostationary orbit to the TCO.

TCO	t_f for thrust=1N (d)	t_c^1 for thrust=1N (d)	t_f for thrust=0.2N (d)	t_c^1 for thrust=0.2N (d)
13933	10.606	13.924	58.768	64.426
11249	9.901	13.747	57.636	66.105
10585	9.997	30.474	55.011	119.644
16803	10.588	14.815	65.751	90.762
5481	10.900	25.722	58.781	100.056
8962	11.717	∞	59.384	76.056
12028	13.237	22.468	60.851	147.604
3813	9.982	13.895	55.359	∞
14487	10.012	22.308	57.866	∞
57	11.015	12.372	46.720	49.266
7548	12.027	17.460	58.685	∞
3867	10.122	16.427	54.450	61.157
4980	11.305	∞	57.314	61.572
10979	9.999	∞	58.316	∞
651	19.179	∞	72.196	∞

Figure 3.2 Validation of the second order condition for several values of the thrust, by comparing the transfer time t_f and the first conjugate time t_c^1 along extremals. The symbol ∞ is used when no conjugate time appears on the interval $[0, 15t_f]$.

Proceedings of the 5th International Conference on Optimization and Control with Applications (OCA2012), Beijing, China, December 4-8, 2012

13 THE EXTRAGRADIENT METHOD FOR SOLVING VARIATIONAL INEQUALITIES

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Abstract: The aim of this article is to investigate the new direction of the subgradient extragradient method for solving variational inequalities. In our algorithm, the new direction is related with the fixed point of a nonexpansive mapping and the current point. Simultaneously, the weak convergence theorem of the algorithm is established.

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14 PERFORMANCE OF A DIFFERENTIAL EVOLUTION ALGORITHM: BEST PARAMETERS SEARCH THROUGH PARALLEL COMPUTING AND DESIGN OF EXPERIMENTS

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Abstract: Differential Evolution is a population-based stochastic evolutionary optimization algorithm with a high convergence speed. The choice of parameters values for the algorithm influences its performance, i. e., the number of iterations necessary to reach a satisfactory result. A modified ED/rand/1/Bin strategy, which includes a high initial population that rapidly diminishes to a pre-defined target value by using elimination criteria, was implemented in the LabVIEWTM 'G' language. This algorithm was run on a parallel computing basis, using a network of desktop computers in which the master coordinator splits the computational task and distributes it to the enabled stations. The system was set to repeatedly solve the Ackley benchmark function problem in the —R20, using combinations of parameters defined through the application of a Design of Experiments methodology. The algorithm performance under the different conditions is analyzed, and a best set of parameters could be drawn.

Key words: Differential evolution; Optimization; Design of experiments; Parallel computing.

Proceedings of the 5th International Conference on Optimization and Control with Applications (OCA2012), Beijing, China, December 4-8, 2012

15 STATIC FORCE CAPACITY MODELING AND OPTIMIZATION OF HUMANOID ROBOTS

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Abstract: Robots are created to track a desired path or follow instructions. When their structures are developed to be similar to the human body and to replicate human activities, to understand the force capacities of humanoid robots are essential to perform optimal movements. Once contact forces are applied by a humanoid robot the static equilibrium is affected and, depending on the magnitude of this force, the robot can lose its stability and fall down. To correctly understand this dynamic it is necessary to create a static model, to optimize the force capacity and to develop a software program capable of simulating tasks and calculating the mechanic efforts on the robot's joints. Based on previous studies where humanoid robots models are discussed and information about geometry and structure were provided, this work presents a generic model of a robot. The model proposed in this work was created by defining equations that operates the static balance. Additionally, the differential evolution method was adopted to optimize the force capacity of the robot. Considering the mathematical complexity involving the equations, this paper also describes the development of a simulation tool to determine the acting forces and moments in generic situations. Evaluating the simulation results, it is possible to analyze the most relevant factors on robots performance when employed to accomplish tasks that need interaction efforts with the environment. This study could be used as a reference to new project developments where the efforts distributions on the joints should be taken into account to build new robots models. Moreover, the generic equations presented in this work can be applied as well as on civil construction until to help senior citizens or with some kind of physical incapacity.

Key words: Human robot; Static balance; Optimization.

1 INTRODUCTION

The development of robots that could replicate the human movements is constantly in evidence into scientific and industrial researches and the possibilities to reduce human efforts, especially those associated to dangerous and to high strength activities are evident. Moreover, humanoid robots can be used to help seniors citizens or with some kind of physical incapacity.

Due to the wide applicability range of humanoid robots, distinct approaches to solve the movement and force capacities problems appear in the literature and many research groups are currently been working on this subject (Wu *et al.*, 2011; Kim, 2011; Alouloua and Boubaker, 2012; Tax *et al.*, 2012).

The approaches to develop humanoids are strongly different in their mechanical structure, their sensing and actuation capabilities and the way they process data to maintain the movement stability and to optimize force. Many techniques from conventional mathematical methods, such as quadratic programming (Zhang *et al.*, 2004) and nonlinear programming, have been proposed to deal with force optimization problem in robotics. Conventional optimization techniques take in advantages in computing speed and convergence with the objective function of continuous, differentiable and single peak value. However, most methods often give a local rather than a global optimum solution.

Increasing interests in biology motivated the use of adaptive systems to solve real world optimization problems. These techniques are been adopted to overcome the inherent limitations of conventional optimization approaches, metaheuristics like evolutionary algorithms (EAs) (Kicinger *et al.*, 2005; Fleming and Purshouse, 2002; Gen and Yun, 2006; Tiwari *et al.*, 2002) have been developed to solve many types of optimization problems in robotics, see details in (Wang et al., 2008; Rout and Mittal, 2008; Alici *et al.*, 2006; Bergamaschi *et al.*, 2008; Kalra *et al.*, 2006).

This work proposes the solution of the static force capability optimization problem of humanoid robots by using the Differential Evolution (DE) technique. The solution takes into account the robot joints actuation limits and all possible configurations. Next section presents the humanoid robot model and the main characteristics of the Differential Evolution approach are addressed in the sequence. Section 4 reports the aspects of the software that was developed to integrate the robot's model and the optimization algorithm. The next two sections illustrate tests and results using the software in order to provide the best position for the robot's parts in order to optimize the force considering a specific case study. Finally, the conclusions are addressed at the last section of the paper.

2 HUMANOID ROBOT MODEL

This section describes the structural characteristics of the humanoid robot presented in this paper. It also shows how the geometry of a limb is set and how the static balance can be calculated.

2.1 Geometry

A humanoid robot is a mechanical structure that contains upper and lower limbs. The limbs are composed by revolute joints and links. The joints are supposed to provide torque while links need to support all the forces internally transferred up. The resulting wrenches in the joints are caused by contact, reaction and gravity forces.

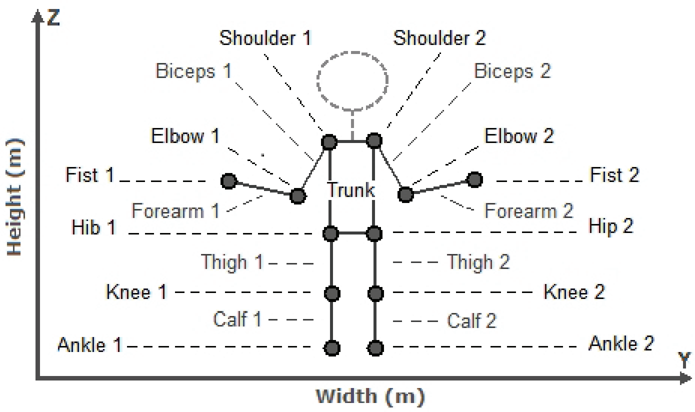

Figure 2.1 Humanoid robot model.

The model adopted in this study simulates a human body, which contains two upper limbs and two lower ones. Moreover, it contains a trunk that connects both shoulders and hips as it can be verified in Fig. 2.1. All approaches in this study are based on the two-dimensional sagittal plane as presented in Fig. 2.2. Considering a fixed trunk and free contact point or in the same way fixed contact point and free trunk position, it can be noticed that in both cases there are infinite settings configurations (position and orientation) for the robot.

In order to define a robot configuration, it is necessary to specify the contact points. After that, the middle joints need to be set. It can be made by using the trigonometric relations as showed in Fig. 2.3, where d_1, d_2 e d_3 are the lengths of the links, D is the distance between the contact point and the limb's

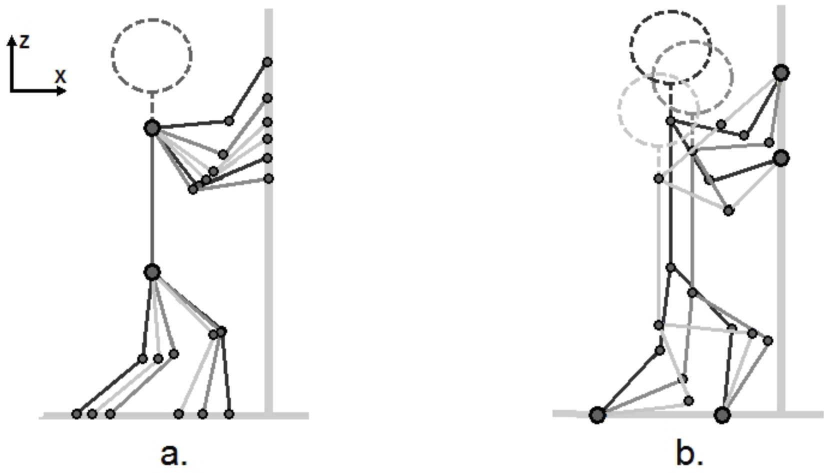

Figure 2.2 a) Infinite configurations of the robot considering a fixed trunk angle; b) Infinite configurations of the robot considering fixed contact points.

source and θ_1, θ_2 and θ_3 are the angles. It can also be verified that there are two possible settings for the limbs, it depends if the configuration is for the upper or for the lower one. If the considered limb is the upper, then the middle joint is set backward and for the latter case it is set forward.

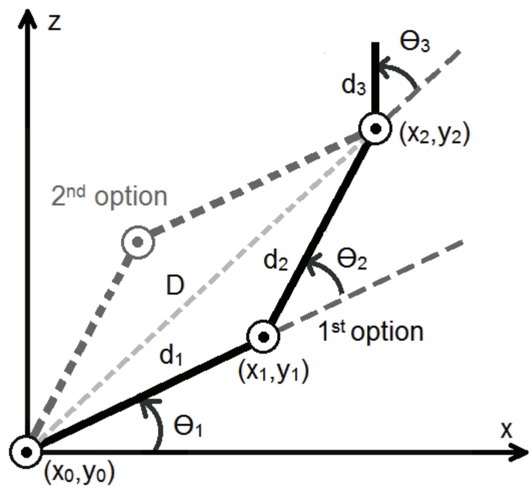

Figure 2.3 Geometric limb model.

2.2 Static Balance

After the geometry definition, it is necessary to evaluate the static balance. In this way, first is calculated the mass center of the robot. As each link is considered as a rectangle, the mass center has matched with the geometric center. Thus, there is a weight to be considered in the middle of each link that affects the balance. As a result, there is an equivalent mass center around the robot (not necessarily inside of it) which generates a resulting force that represents the entire robot weight.

In the sequence, the next step is to find the Zero Moment Point (ZMP), which is defined as a reference point for the balance of the bipedal. The ZMP is defined by the projection of the center of gravity on the ground. The sum of the moments generated by all the external forces acting on the robot in relation to the ZMP is equal to zero (Torres, 2006). The Zero Moment Point can be expressed by:

$$X_{ZMP} = \frac{MgD + \sum_{i=1}^{n} F_i d_i}{Mg + \sum_{i=1}^{n} F_i}, \tag{2.1}$$

where M is the total mass of the robot, g is the gravitational acceleration, D is the mass center of the robot, i is the index that represents each of the n forces in the contacts, F is the contact forces and d is the distance between the force and the reference point.

By using the humanoid geometry, mass center and the Zero Moment Point, the static balance can be established through the free-body diagram of all joints or also by the Davies' Method (Tsai, 1999).

2.2.1 Davies' Method. First of all, the Davies' Method consists on the use of graphs for representing systems. The balance equations are gotten from the Kirchhoff-Davies cutset law, which establishes that the algebraic sum of the efforts in each cut is zero (Erthal, 2010). Further, each cut provides a subset which preserves the balance.

As a result of the method, the balance equations are written in the matrical format where the vectors of the efforts have the same reference point. In addition, the already known efforts are called primary variables, while the secondary ones are calculated forward for keeping the balance (Weihmann, *et al.*, 2012).

The advantages of the Davies' Method when compared to the Free-Body Diagram technique are: the clear formalism to solve the static analysis, the easiness of including new additional external forces and also the matrical representation. Thus, these characteristics are used to justify the employ of the Davies' Method along this work. More information about the Davies' Method can be found in (Cazangi, 2008).

3 DIFFERENTIAL EVOLUTION

This section provides a brief overview about the Differential Evolution (DE) optimization method, which is a population-based stochastic method created by Ken Price and Rainer Storn (Storn and Price, 1997; Price *et al.*, 2005). The DE is a widespread method with an approach that can be easily learned by any engineer or student looking for a simple optimization method. It was chosen due to the fact that the modeling procedures of a humanoid robot generates a group of equations that results in a non-linear function based on equations to define the geometry, the static balance and the force capability optimization. Furthermore, the cost function evaluated inside the DE contains local minimums that difficult the optimization. For solving this problem the crossover rate is set and the DE method can be used to find the global minimum.

The method requires the setup of some parameters and the definition of an objective function that describes the problem to be optimized (Storn and Price, 2005). The first step to apply the method is to ascertain the decision variables that should be modified during the solution of the optimization problem. The main idea is to find the best values to minimize the objective function. Further, it is required to specify the boundary conditions, that are important to avoid unexpected results.

In the next step, it is necessary to define the key parameters accountable to handle the execution of the method. The first one is the population size (N), which means the number of individuals, each of them containing a set of decision variables. The following parameter are: the mutation factor (MF), the crossover rate (CR) and the stopping criterion (t_{max}).

Finally, it is needed to generate the cost (or objective) function which must be minimized. It contains a sequence of treatments for the population being tested. These treatments are both the calculation of the result for population and the preset penalties for those results. These penalties are applied when the population yields unfavorable results.

The execution sequence of the method proceeds by following steps:

1. N groups of randomly-generated solutions are created between its boundary constraints. It composes a population and each group is called individual;

2. The cost function value is calculated for each individual;

3. A mutation is executed and a new population is generated, where each new member is descendent from a member of the last population;

4. The cost function value is calculated to the mutant population;

5. Both values calculated are compared and only the best individuals are kept;

6. Update the generation's counter and;

7. Loop to step 2 while the stopping criterion is not met or until the maximum number of iterations be reached.

4 SOFTWARE DEVELOPMENT

Based on the robot model described in previous sections an object-oriented software was created to manage and process all information about the humanoid robot structure, its static balance and its static force capacity. By using object-oriented concepts it is possible to develop larger codes and reduce the complexity in order to model real situations of interactions between a humanoid robot and the environment. Considering this application, both environment and robot became a class (objects) in the software and its instances were used to compose and simulate a real task.

The best advantage of object-oriented programming is the possibility to separate the objects by their features. It makes the software flexible and fitter to get updated (McLaughlin *et al.*, 2006). Furthermore, it provides a clean script that avoids time-loss during the development process.

The interface was designed in MATLAB in order to simplify the usage. Forms that allow changes in the robot features by the users were also been provided. Further, it is possible to lay the humanoid under external forces with different values and establish the robot's static balance. Moreover, the user can order the optimization of the humanoid's static force. The results are shown in graphics and text fields to facilitate the analysis.

5 TESTS AND SIMULATIONS

With the objective of testing the software and evaluating the optimization technique on this problem, a case study was proposed. It consists in the search for the largest force on the horizontal axis to push an object. Its configurations are shown in Tab. 5.1, where the mass and the length of the links and the maximum torque of the joints are specified.

To optimize the static force, the key parameters and the decision variables was chosen strategically as it is shown in Subsection 5.1. The results are carefully described in Subsection 5.2.

Table 5.1 Setup of the humanoid.

Links	Mass (kg)	Length (m)	Joints	Maximum Torque ($N.m$)
Forearms	1.0	1.0	Fists	absent
Biceps	1.0	1.0	Elbows	10.0
Trunk	2.0	2.0	Shoulders	20.0
Thighs	2.0	1.2	Hips	40.0
Calfs	1.5	1.2	Knees	45.0
			Ankles	absent

5.1 Simulation Parameters

The setup of the optimization method follows the sequence described in Section 3. Thus, the decision variables defined are the contact points between the robot and the environment and the force on the horizontal axis. Assuming that both shoulders have the same position in the plane, the geometric limits of the contacts are equal. The same situation occurs with the hips and consequently with the ankles.

As an initial approach, the friction forces between fists and the object was neglected. Hence, the only vertical forces are the weight of the robot and the reaction forces on the ground. Considering that the friction force on the ground is the support of the robot horizontally, it is possible to evaluate the force limit: static friction coefficient multiplied by normal force. As the coefficient adopted is $\mu = 0.5$ and the normal force calculated is $N = 127.4N$, the maximum sum of the forces sustained by feet is $F_{Xmax} = 63.7N$.

After defining the forces described above, the control parameters could be set (Storn and Price, 1997). The population size adopted was ten times the number of decision variables (Liu and Lampinen, 2005), resulting in $N = 60$. The mutation factor adopted was $MF = 0.8$ and the crossover rate was $CR = 0.5$ (Weihmann, *et al.*, 2012). The stopping criterion t_{max} was chosen as 500 and can be changed if convergence is not achieved.

The next step is the cost function definition, which is composed by the optimization function F_{wrench} and its constraints $F_{penalties}$. As the objective is to optimize the maximum horizontal forces F_{X_1} and F_{X_2} and considering that the DE method searches the global minimum, the main equation for this problem can be defined as presented on Eq. 5.1.

$$F_{wrench} = \frac{1}{|F_{X_1}|} + \frac{1}{|F_{X_2}|} \tag{5.1}$$

where F_{X_1} and F_{X_2} are the horizontal forces in the contact.

The conditional penalty installment is composed by the sum of forces in the opposite way to the intended (Eq. 5.2), the joints overloaded (Eq. 5.3), the contact points that are out of the range limits of the limbs (Eq. 5.4), the middle joints through the object or the ground (Eq. 5.5), ZMP out of polygon of stability (Eq. 5.6) and the insufficient friction force on the ground (Eq. 5.7).

$$F_{forces} = K_w(|F_{x_1}| + |F_{x_2}|) \tag{5.2}$$

$$F_{torques} = K_t(\sum_{i=1}^{n} |T_i|) \tag{5.3}$$

$$F_{contacts} = K_c(\sum_{i=1}^{n} |CP_i|) \tag{5.4}$$

$$F_{joints} = K_j(\sum_{i=1}^{n} |MJ_i|) \tag{5.5}$$

$$F_{zmp} = K_z|ZMP - x_1| \tag{5.6}$$

$$F_{friction} = K_f \mu \tag{5.7}$$

where K_w, K_t, K_c, K_j, K_z and K_f weight the contributions, T are the torques in the joints, CP are the contact points, MJ are the middle joints, ZMP is the Zero Moment Point, x_1 the position of the back foot on the ground and μ is the static friction coefficient. Assuming Eq. 5.8, the cost function can be described by Eq. 5.9.

$$F_{penalties} = F_{forces} + F_{torques} + F_{contacts} + F_{joints} + F_{zmp} + F_{friction} \tag{5.8}$$

$$F_{cost} = F_{wrench} + F_{penalties} \tag{5.9}$$

In this work, K_w=K_t=K_c=K_j=K_z=K_f=100, that is just a high valued constant used to emphasize the penalty because all the situations quoted are worthless. The strategy employed was $rand/1/bin$ and seven different angles between trunk and horizontal axis were adopted in order to understand how this parameter affects the efforts and the force capability. The method was executed on a $3.40GHz$ $i7$ processor with $8GB$ of random access memory.

5.2 Results

In this section the results of the DE method applied in the static force capability of the humanoid robot are reported and discussed. The adopted case study was described above on Section 5.1.

According to the results reported in Tab. 5.2, it can be seen the mean minimum value found in each situation, the total horizontal force achieved and also the total time consumed by each of the 700 optimization runs (totaling $350,000$ cost function evaluations).

The best result was reached with the angle 1.047198 radians and the resultant torque in each joint associated to this case is shown in Tab. 5.3. It can also be seen in Tab. 5.3 the usage percentage of each joint. Notice that some of it are almost fully loaded while others are nearly unladen. It is not a profitable circumstances in a robot project design view.

The final configuration (position) of the robot is shown in Fig. 5.1, where it can be seen that the Zero Moment Point appears in its limit, which is the same location where the lower limb 1 touches the ground. It means that the robot uses all force capacity offered by the current configuration. Considering that the robot mass does not change, there are two ways to upgrade the horizontal resultant force: by improving the friction on the ground or by changing the geometry, probably by tilting the trunk.

Table 5.2 Optimization results using Differential Evolution (DE).

Angle (rad)	Minimum ($)	Total force (N)	Time elapsed (m)
1.047198	0.0734	48.3378	17.7179
1.134464	0.0768	46.3040	17.7157
1.221730	0.0827	44.2641	17.7094
1.308997	0.0921	39.8545	17.7278
1.396263	0.1057	36.7777	17.6923
1.483530	0.1107	34.2297	17.6992
1.570796	0.1250	30.2353	17.7287

Table 5.3 Resultant torques of the best angle.

Joints	Available torque ($N.m$)	Required torque (N)	Availed Torque(%)
Elbow 1	10	8.0957	80.9567
Elbow 2	10	9.9122	99.1228
Shoulder 1	20	-19.9712	99.8560
Shoulder 2	20	-19.8315	99.1577
Hip 1	40	8.1443	20.3607
Hip 2	40	-2.4975	6.2438
Knee 1	45	-4.1487	9.2195
Knee 2	45	32.5462	72.3248

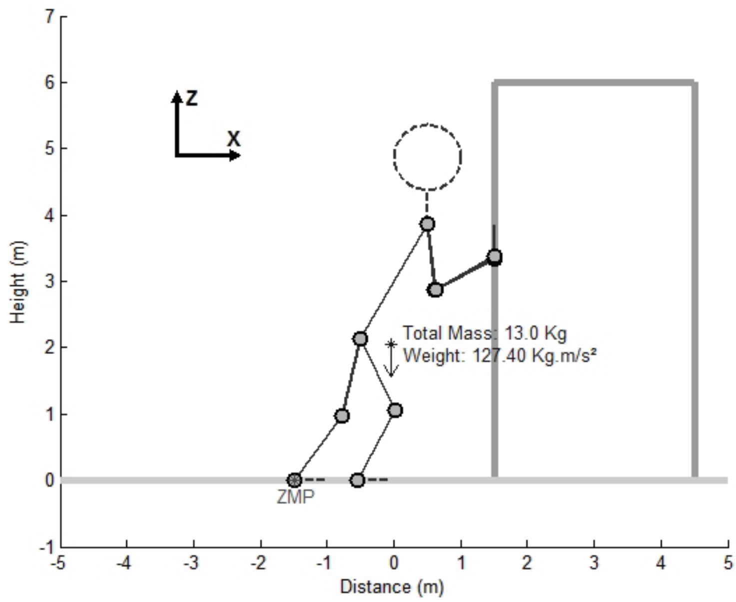

Figure 5.1 Robot's optimal position.

6 CONCLUSION

This work presented an initial approach for humanoid robots modeling with the purpose of describing its interactions to the environment by using the static balance equations. Considering external forces, the optimization of the static force capacity by using the Differential Evolution method were performed in order to determinate the optimal robot position to maximize the efforts performed by its joints and links within the structure limitation.

Although the DE is a highly assured and widespread method, it is evident the importance of comparing the DE results in terms of computational effort and optimization efficiency to another optimization method, mainly because it's just the beginning of a wide range of research on humanoid robots, which can also have a non-static approach. Moreover, the DE optimization method showed high percentage of efficiency in this application considering that nearly values were found in several simulations.

In the sequence of this work, both comparisons to another optimization method and reduction of the algorithm processing time will be the main objectives of the authors' research. Parallel programming will be used to almost reach the entire processor capacity and to reduce the algorithm processing time. In a project design view, the joint torques optimization to reduce production costs can be identified as a potential application possibility of the software developed in this research.

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16 USING PENALTY IN MATHEMATICAL DECOMPOSITION FOR PRODUCTION–PLANNING TO ACCOMMODATE CLEARING FUNCTION CONSTRAINTS OF CAPACITY

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Abstract: Linear Programming models (LP) is a widely used framework to address the problem of production-planning, which has been studied for several decades. However, standard LP models presents a number of drawbacks in that their recommendations are inconsistent with the queueing behavior observed in most production facilities. One such drawback is that the dual variables associated with capacity constraints will only take nonzero values when the resource is fully utilized, contradicting both theoretical results from queueing theory and practical experience from the shop floor. Another serious drawback is related to lead time. Most LP models assume that a resource can maintain a constant lead time regardless of its workload, again contradicting basic queuing insights. Practical experience and queuing theory show that the performance of production system is affected by the loading of the system related to its capacity, in particular the lead time, i.e., the mean time between the release of work for production and its completion, increase nonlinearly with increasing resources utilization. Thus, deterministic models for production planning suffer from a problem that in order to match demand and supply they need to consider lead times to plan releases, but in doing so, they determine levels of resources utilization, which, in turn, determines the realized lead time of production. A number of iterative approaches has been proposed to deal with this circularity. These involve assuming fixed lead times to obtain a release plan, and then simulating the release plan to obtain realized lead times estimates. The estimates are then used to generate a new release plan, until the procedure converges. Nevertheless, at the best of our knowledge the convergence of these approach is not yet very well established. In this paper we address the pointed circularity through the use of clearing function, first introduced by Graves, Karmarkar, and Srinivasan et alli, and more recently studied by Asmundsson, Kefeli et alli, Irdem at alli, and Uzsoy and Sampaio, just to name a few. The clearing function is an increasing bounded concave function that expresses the expected throughput of a capacitated resource over a period of time as a function of the average Work–In–Process (WIP) level in the system over that period. The classical scheme used to decompose this problem, however, does not works properly in presence of nominal capacity constraints, and since CF function is a kind of capacity constraint, the classical scheme does not work properly in presence of clearing function, as we can see in Sampaio, thus requiring a new decomposition scheme, which is actually the problem we address here. The scheme we present, which works properly in presence of clearing function, combines decomposition with penalty function for the subproblems, which assure a quite balanced production related to capacity at each period in the planning horizon. The whole idea of the approach is to penalize unbalanced production: the more a product of high profitability is produced the more a product less profitable becomes attractive to be produced.

Key words: Clearing Function; Production–Planning; Convex Programming; Mathematical Decomposition.

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17 EXPERIMENTAL OPTIMIZATION OF THE MACHINING P ARAMETERS

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Abstract: The surface roughness is one most important requirement in the machining process. The determination of machining parameters is an important stage in the manufacturing process and tool life. In this paper we propose to optimize machining parameters of iron in milling process in an industrial plant agricultural machine using response surface methodology (RSM). We calculated the effects of machining parameters: speed, feed and depth of cut on the response variables about surface roughness and cycle time of the milling operation. Applying this RSM allowed to plan and execute test efficiently, modeling the response variables according to adjustment of process parameters and thus determine the optimal setting of parameters that optimizes simultaneously the surface roughness and cycle time, providing to evaluate the material loss and cost of operation.

Key words: Optimization; Response surface methodology; Machining; Milling process.

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18 MODELING AND OPTIMIZATION OF EXPERIMENTAL DATA IN MANUFACTURE PROCESSES

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Abstract: The modeling and optimization of processes is a quite powerful strategy, since it allows investigating variables that influence quality characteristics of the product or manufacture processes. The literature seems to be more focused on the data analysis and regression model for the problem the researcher is confronted with optimization. This paper proposes a approach for modeling and optimization of experimental data in manufacture processes, reinforcing idea that planning and conducting data modeling are so important as formal design and analysis. The manufacture case studies illustrate the strong relationship of the results with incorporating presented approach into practice. Moreover, in this case study consolidating a fundamental advantage of regression models and optimization provide more knowledge about products, processes and technologies, even in unsuccessful case studies.

Key words: Optimization; Regression models; Mining process.

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19 USING EXPERT SYSTEMS AS TOOL FOR MAINTENANCE PLANNING AND OPTIMIZATION IN MANAGEMENT

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Abstract: In manufacturing process there is the seasonality and it is an important point to be considered in any decision or planning. Professionals of maintenance face tasks that range since the specification about the more adequate maintenance modality for the system (a line production or part of it, a function of machine), the determination and choose of critical item, the necessity of redundancy, fault cause finding until the optimal moment of the system stop for intervention. In order to assure the accomplishment of such tasks assertively, professionals search for tools that help them in decision-making processes. Some tools comprehend adoption of matrix or complex mathematical models. Efficiency of these mathematical models depends on considering all the process variables, such as: predictable or not, quantitative or qualitative, as well as, objective or subjective. When dealing with diagnosis about the operation conditions of equipment, failure identification and anticipation one, it is not easy to apply an algorithmic solution, because the universe of discourse is so wide. The decision making must be based on consistent diagnosis, which must foresee the actions or repair. These diagnostics can be based on information, heuristic knowledge, linguistic variables, and intuitive/deductive feelings. As far as engineering is concerned, instrumentation is used to measure, register, and control of physical variables (e.g.: temperature, strength, displacement, motion, time, speed, acceleration, pressure, etc.) that have influence on any process or system. In this context, system is an object to be care, an industrial process, part of this process, and machine or even a small part of it. In current industrial society, control theory and application are of the most important technologies, since primary processes levels moved by steam engine until current state when there is great interaction among information systems and production processes. During all the Industrial Revolution, promoted by control theory, the mathematical modeling of processes had been based on linear aspects. However, this transformation success depends on the methodology of mathematical modeling used, and in the case of control theory, it came to a stage where precision became a hard or even impossible task. Human being usually strive for modeling mathematically the nature processes and, often, is impossible for a human operator controls several systems without understanding about mathematics, or all involved physics details. This operator is, however, able to handle input variables that influence the process outputs. This fundamental realization leads to a new focus in (complex) industrial process theory where the concept of "artificial intelligence" (AI), through of characteristics emulation of human behavior, arose as an alternative of control and modeling. An important fact is that mathematical modeling of industrial processes has been substituted for a modeling that makes possible the handling of control variables in a way to reach the aimed outputs. This article proposes the use of Fuzzy Logic as an underlying support tool for planning and decision-making for convenient moment for maintenance in production process equipments (optimization). The work focuses that planning and intervention decisions involve technical aspects (quantitative variables), management aspects, and cultural values (qualitative variables). The Fuzzy rules are used as a basis for diagnosis and decision making to determine opportune moments to apply maintenance, in order to avoid profit loss. In this way, it is shown that the non-availability of processes is minimized and maintenance can be set for a suitable moments in time.

Key words: Expert systems; Maintenance strategy; Optimization; Fuzzy logic.

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20 HYBRID OPTIMIZATION OF OPERATION PROCESS FOR MINERAL PROCESSES

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Abstract: To pursuit the plant-wide optimization of mineral process, a hybrid intelligent optimization approach is proposed. The objective of optimization is that the production indices defined as the performance related to the final product quality and production rate fall into their target ranges; whilst the decision variables are operational indices of each unit, which is related to units' intermediate product quality, efficiency and consumption. In this context, the domain knowledge of process engineers are mimicked and combined with the framework in terms of feedback, prediction and feed-forward schemes so as to realize the required optimization. The effectiveness of the proposed approach has been demonstrated by the practical application results.

Key words: Mineral process; Operational indices; Multiobjective optimization; Dynamic tuning; Performance prediction.

1 INTRODUCTION

Mineral processing is a production process consisting of multiple units. These units have different purposes and perform their own manufactory tasks to make their objectives fall into the target ranges, where these objectives are defined as the operational indices and characterize the intermediate product quality, efficiency, consumption of each unit process. On the other hand, these units also cooperate together to fulfill the mission of the multiple units composed process and to ensure the global production indices, which characterize the final product quality, yield, consumption and costs, fall into their target ranges.

In recent years, the optimal operation and control with objectives of the operational indices of units has attracted much attentionNevertheless, there is no unified method for optimal operation and control that is applicable for all industrial processes, because it is closely connected with industry technique knowledge.

In petroleum and chemical industries, a two layered structure consisting of real-time optimization (RTO) and single input single output (SISO) control has been widely used to perform the optimal operation of unit-process. RTO is a model-based method, where the precise static process models usually are adopted, and its performance objective is the operating profit of a unit, which is a function of controlled variables, whilst the operational indices are taken as the constraints. The decision variables of RTO are the set-points of SISO control system and these set-points are followed by the outputs of controlled variables to ensure the production process running on an economically optimal point, so as to achieve the profits as high as possible . Since RTO uses static models of the concerned process, the optimization routine could not start until the process reaches a new steady state when there are changes in its operating conditions or when a system disturbance occurs. Therefore, the whole optimization

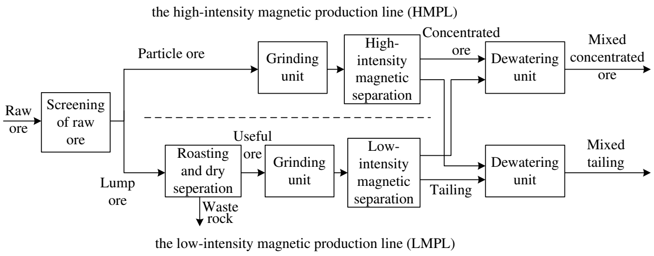

Figure 2.1 Scheme of mineral processing production process

process is subjected to some time delays. Indeed, the integration of RTO and model predictive control can be used to solve the problems using long execution period and would therefore lead to inconsistency with respect to relatively fast response of control layer during the integration between the RTO and the control systems .

The aforementioned method needs to establish the static or dynamic models of a process and its constraints. And its optimization is performed in an open loop manner. Furthermore, in the complicated industrial processes, there always exist some unmodelled dynamics and uncertainty disturbances. Therefore, all of these make it very difficult to apply in the complicated industrial processes. In this context, existing process operation optimal control research is very much problem based and is focused on specific plant. For instance, proposed a hybrid supervision system which combines a nominal control with an intelligent control, leading to a much improved final product quality of a laminar cooling process. A hybrid intelligent control approach for process optimal operation is proposed, which adjusts the set-points of the control loops according to the operational indices and the operation condition in real-time and the fault working situation diagnosis and tolerant control are considered as wellThe successful application in the shaft furnace roasting process of mineral processing shows the effectiveness of the proposed approach.

The decision optimization of the operational indices for the industrial processed composed of multiple units involves multi-objectives, such as product quality, yield, consumption of energy and raw materials. At the same time, the performance, operational indices, are closely related to the domain knowledge. And the dynamic model between the operational indices and the global production indices and the process constraints cannot achieve easily. Also there are many uncertainties in the production conditions. Therefore, it is very difficult to realize the optimal decision making of the operation indices using existed optimization methods. That leads to the result that the decision making of the operational indices can only be performed manually according to the operator's experience, which, however, cannot ensure optimization of the global production indices.

Motivated by this problems, a novel multi-objective hybrid intelligent optimization of operational indices for industrial mineral processing is proposed in this paper, which combines the multi-objective optimization, performance indices prediction and dynamic turning. The proposed approach has been applied to a hematite iron ore mineral processing successfully and the effectiveness is proved by the application results.

2 PROBLEM DESCRIPTION

The procedures of a mineral processing production process of hematite iron ore (as shown in Fig. 2.1) can be divided into two sub-production line, namely the weak magnetic production line (WMPL) and the strong magnetic production line (SMPL), according to the magnetic density of the magnetic separation process. The weak magnetic sub-production line includes the shaft furnace roasting process, grinding process, and weak magnetic separation process whilst the strong magnetic sub-production line consists of the grinding process, strong magnetic separation process. In addition, there are the concentrated ores condensing and tailing condensing also for sharing by the two sub-processes.

As shown in Fig. 2.2, the decision making of operational indices for the mineral processing of hematite iron ore, which composed of multiple units that perform certain tasks, involves scheduling, technical department, operational optimization and control system, and process control system. Scheduling determines the global production indices of whole production line,$Q_k(k = 1, 2)$, $Q_k \in [Q_{k,min}, Q_{k,max}]$, where Q_1 is the product quality, the daily mixed concentrate grade, and Q_2 is the daily yield of concentrated ore. $Q_{k,min}$ and $Q_{k,max}$ are the lower and upper limits of Q_k. According to the obtained global production indices, $Q_k(k = 1, 2)$, the technical department generates the operational indices of

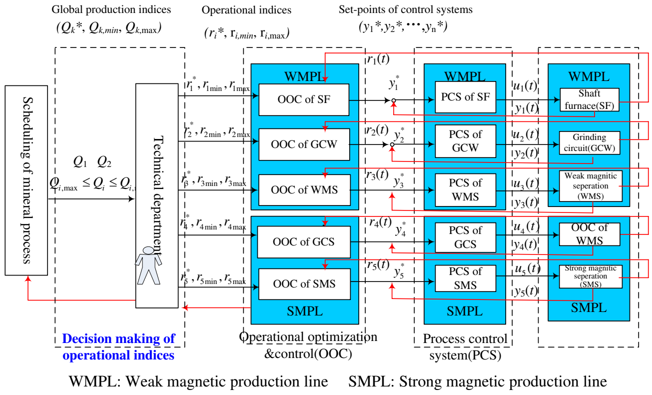

Figure 2.2 Decision making process of operational indices for the mineral processing

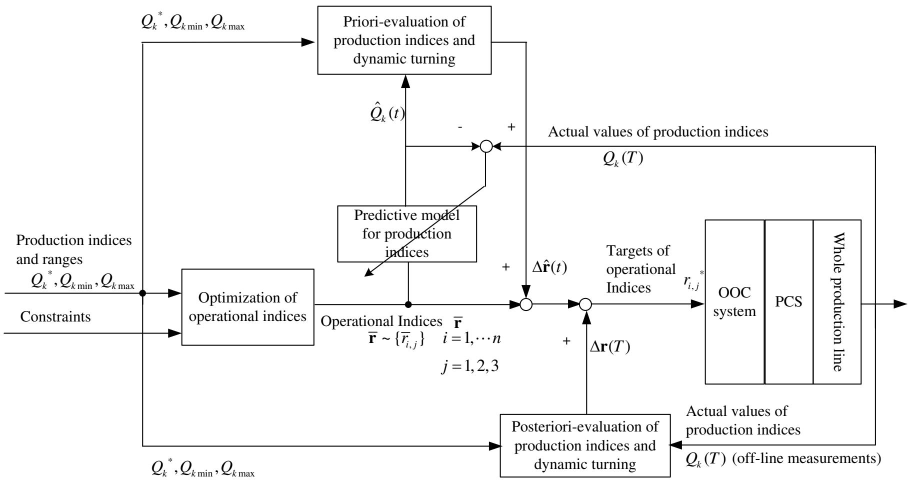

Figure 3.1 Decision optimization strategy of operational indices

each unit process r_{ij}, $r_{ij} \in [r_{ij,min}, r_{ij,max}]$, where represents the number of units and represents the product quality, efficiency and consumption index, respectively. Taking the operational indices as the target of operational optimization and control system of ith unit, it calculates the set-points of control loops for process control system, and control system ensures the output of the controlled variables to follow up prescribed set-points, so as to make the actual global production indices into its target range.

In this case study, the global production indices provided by the scheduling system of mineral process are daily mixed concentrate grade and daily yield of concentrated ore . The operational indices decided by the technical department are the magnetic tube recovery rate (MTRR) , the grinding particle size of weak magnetic production line (PSWMPL) , the concentrate grade of WMPL (CGWMPL) , the tailing grade of WMPL (TGWMPL) , the grinding particle size of strong magnetic production line (PSSMPL) , the concentrate grade of SMPL (CGSMPL) , the tailing grade of SMPL (TGSMPL) . Then these operational indices are sent to the operational optimization and control system and transferred into the set-points of control systems. The set-points of control systems are shown in Table 1.

3 OPTIMIZATION OF OPERATIONAL INDICES

To solve the problem, a novel strategy of operational indices optimization is proposed as shown in Fig. 3.1, where the evolutionary algorithm is combined with the case-based reasoning (CBR) to generate the near-optimal solution. Then the performance prediction, evaluation and dynamic tuning are adopted to solve this optimization problem. It is composed of four modules (see Fig. 3.1), including a module of optimization operational indices , a predictive model for production indices, a priori-evaluation and a posteriori-evaluation with dynamic tuning module. The purpose of this structure is to cope with the uncertainty caused by v_k.

Table 2.1 The operational indices and set-points of control system of mineral processing

Unit	Performance
Shaft furnace of WMPL	The operational indices: r_1: Magnetic tube recovery rate (MTRR) The set-points of control system: y_{11}^*: Temperature ($°C$) y_{12}^*: Flow rate of reducing gas (m^3/s) y_{13}^*: Discharge time (s)
Grinding of WMPL	The operational indices: r_2: Grinding particle size (PSWMPL) The set-points of control system: y_{21}^*: Feed rate of raw ore (T/h) y_{22}^*: Flow rate of feed water (m^3/s) y_{23}^*: Density (%)
Weak magnetic separation	The operational indices: r_{31}: Concentrate grade (CGWMPL) r_{32}: Tailing grade (TGWMPL) The set-points of control system: y_{31}^*: Density of feed ore pulp (%) y_{32}^*: Electricity currency (A) y_{33}^*: Flow rate of water (m^3/s)
Grinding of SMPL	The operational indices: r_4: Grinding particle size (PSSMPL) The set-points of control system: y_{41}^*: Feed rate of raw ore (T/h) y_{42}^*: Flow rate of feed water (m^3/s) y_{43}^*: Density (%)
Strong magnetic separation	The operational indices: r_{51}: Concentrate grade (CGSMPL) r_{52}: Tailing grade (TGSMPL) The set-points of control system: y_5^*: On/off of the machines

A detailed description for each module is discussed in the following.

1) Optimization of operational indices: Determines the near-optimal operational indices according to the targets of production indices Q_k^* and the constraints (**??**)-(**??**).

2) Predictive model for production indices: Predicts the production indices $\hat{Q}_k$ according to the near-optimal operational indices $\overline{r}_{ij}$ from the optimization part and the boundary conditions $\boldsymbol{B}$.

3) Priori-evaluation and dynamic tuning: Produces the regulation value for the operational indices according to the targets Q_k^* and predictions $\hat{Q}_k(t)$ of the production indices.

4) Posteriori-evaluation and dynamic tuning: This unit calculates the correcting value $\Delta\mathbf{r}(T)$ for the operational indices using the error between the targets Q_k^* and the actual values $Q_k(T)$ of the production indices. Here, T is the sampling (or assay?) period of the production indices.

Finally, the operational indices of each unit can be generated as shown in Fig. 3.1.

4 APPLICATION RESULTS AND ANALYSIS

The decision making system of operational indices for the mineral processing of hematite iron ore is developed. This system consists of the optimization of operational indices and five operational optimization & control systems of unit-processes which are the shaft furnace, the grinding unit of SMPL and WMPL, the strong magnetic separation unit and the weak magnetic separation unit. The operational indices of each unit and the set-points of control systems are shown in Table 2.1.

The industrial application results over one week's operation are given when the 8 series of the production are all in the normal condition. The sampling and statistical periods of all the indices are of two hours.

Over one week's operation, it is can be seen that when the production conditions vary the proposed approach can provide the optimal operational indices and are taken as the targets of the lower level systems. The performance of the proposed approach is superior to that of the manual decision making. The operational indices r_1, r_2, r_{31}, r_4 and r_{51} are enhanced by 2, 1.98, 1.26, 1.49 and 0.57, and r_{32} and r_{52} are cut down by 0.69 and 0.31, respectively. This improvement(or reduction) leads to the finally improvements of the daily mixed concentrate grade and daily yield of concentrated ore of the whole production line. The statistical analysis results of one month show the averages of the daily mixed concentrate grade and daily yield are improved by 0.57% and 132.37t/d, respectively, as compared to those of manual operation.

5 CONCLUSION

Manual decision making of the operational indices cannot ensure global optimization of the industrial process. To solve this problem, a hybrid intelligent operational indices optimization approach is proposed. If the operating points vary or uncertain disturbances occur, the proposed approach will automatically adjust the operational indices of each unit-process. The modified operational indices are then taken as the targets and are tracked by the lower level systems to realize production's global optimization of mineral process. The real application results shows the effectiveness of the proposed approach and the high potential of being further applied in the operational indices optimization of other complex industrial processes under dynamic environment.

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21 TYPICAL OPTIMIZATION PROBLEMS AND ITS ENGINEERING APPLICATIONS IN CHEMICAL PROCESS

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Abstract: Due to the shortage of energy, increasingly tight regulation on environment control and globalization of competition on the product price and quality, the development of chemical industries are facing great pressure. Optimization technique is the critical engineering method to solve such problems, which is able to effectively decrease production cost and satisfy various constrains simultaneously by improving the plant design and operation related parameters, and therefore promote the efficiency and profit. However, with the increase of the optimization scope, a great deal of new problems and difficulties concerning the optimization objective and problem have been brought up, such as, that the frequent variation of the feedstock and the raw material composition and property have the significant influence on the production efficiency. Another difficulty could be that in the optimization process not only the requirements on the maximization of the product quality and efficiency should be satisfied, but the short or long term performance indexes concerning the equipment safety and operation efficiency should also be taken into consideration. Since most of these types of problem are NP-hard, it is always difficult to solve them especially in the case of large-scale, complex constrains and uncertainty. Therefore, searching of the optimal operation condition for the large scale industrial production process puts forward the new challenge for the real time optimization theory and method. Regarding to the typical problems in chemical production process together with practical industrial application cases, this paper gives the application state of the typical cases including the multi-objective based optimization for the p-xylene oxidization process, MILP (Mixed Integer Linear Programming) problem based optimization for energy usage of the steam network, and grade change optimization for TE process on the basis of intelligent approach for dynamic optimization. In the last, the key problems and difficulties existing in the current engineering optimization are analyzed, while the research direction in future are proposed, that is, the research on system modeling with merge of process mechanism, knowledge and operation information, research on integration of industrial process control and optimization, and research on multi-objective optimization method dealing with general behavior of the industrial process system.

Key words: Engineering Optimization; Real Time Optimization; Optimization Algorithm; Chemical Process.

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22 PARAMETER IDENTIFICATION FOR ZINC SOLUTION PURIFICATION: AN OPTIMAL CONTROL APPROACH

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Abstract: In this talk, we will introduce an interacting continuously stirred tank (ICSTR) model with multiple time delays for zinc solution purification process. In the ICSTR model, the time delays and the reaction kinetic parameters are unknown. To identify these unknown parameters, a dynamic optimization problem, whose cost function measures the discrepancy between predicted and observed system output, is formulated. Then a computational approach based on optimal control techniques is proposed to determine the decision variables, namely, the unknown parameters. Finally, a numerical simulation is carried out based on experimental data collected from a zinc production factory in China. The results obtained indicate that the proposed parameter identification approach is highly effective.

Key words: Zinc solution purification; Interacting continuously stirred tank reactors; Time delay systems; Reaction kinetic parameter; Parameter identification; Optimal control.

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23 A PREDICTIVE-MODEL-BASED EXPERT CONTROL SYSTEM FOR THE CONTINUOUS CARBONIZATION PROCESS OF SODIUM ALUMINATE SOLUTION

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Abstract: Continuous carbonization of sodium aluminate solution is a key process in the sintering alumina production, whose product $Al(OH)_3$ determines the output and the quality of alumina directly. Precipitation ratios of all troughs should be stabilized in expected ranges and maximal ratio of the last trough is expected to obtain $Al(OH)_3$ particles with certain strength, size and purity. In this paper, based on the analysis of the process mechanism, an expert control system based on neural network predictive model is designed to stabilize and optimize the process. The expert controller is composed of main expert rules and compensation expert rules that are based on the output of the predictive model, which are coordinated to get the regulation values of the operation variables. An objective is established to make sure that the regulation value is optimal so that expected maximal ratio of the last trough is tracked. And the predictive model is based on variables clustering and PCA. Finally, the results of actual runs using the system are presented. They show that great benefits can be obtained both in $Al(OH)_3$ output and quality.

Key words: Sintering alumina production; Continuous carbonization process; Precipitation ratio; Expert control; Predictive control.

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24 INTELLIGENT OPTIMIZATION FOR THE RAW SLURRY PREPARING PROCESS OF ALUMINA SINTERING PRODUCTION

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Abstract: As a key process in alumina sintering production, the raw slurry preparing is very important to guarantee quality and improve benefits, which includes a blending sub-process and three re-mixing sub-processes. In the blending sub-process, the first-time raw slurry is made by blending and wet grinding all kinds of raw materials including bauxites, limestone, anthracite, alkali, etc. Because the qualities of the first-time raw slurry always cannot meet the requirements of the following sintering process, the first-time raw slurry in different tanks are re-mixed in the re-mixing sub-process. Due to instability of mine sources, data incompleteness and large time-delay of measurement, there are many uncertainties in the raw slurry preparing process. Considering the uncertainties and long process characteristics, an intelligent optimization system is developed to make the raw slurry preparing process run in the optimal state.

The intelligent optimization system consists of three parts: a raw material proportioning optimization subsystem, a re-mixing operation optimization subsystem, and an intelligent coordinator.

The raw material proportioning optimization aims to improve the first-time raw slurry quality as close to its quality indices as possible by optimizing the raw slurry proportioning. In which, the quality indices of the first-time raw slurry are the intermediate target which should be determined by the intelligent coordinator. For this, in the subsystem, an integrated prediction model is first proposed to predict the quality of the first-time raw slurry based on the compositions of raw materials and their proportioning. The prediction model is mainly composed of the mechanical model based on the mass balance principle and the intelligent residual compensation model based on back propagation neural networks. If there exist errors between the quality prediction and the quality indices, a multi-objective hierarchical expert reasoning strategy based on the integrated prediction model and the expert knowledge is employed to adjust the set point of raw material proportioning for the distributed control system, where the knowledge base with precedence level is constructed to improve the reasoning efficiency, and the bias between the model prediction and the intermediate target is used to evaluate the reasoning result.

The purpose of the re-mixing operation optimization is to provide an optimal combination of selected tanks for the re-mixing of raw slurry so as to make the quality of re-mixing raw slurry meet the requirements of sintering process and to stabilize the re-mixing sub-process. Here, the optimal re-mixing model with uncertainty is built and genetic algorithm is adopted to solve the nonlinear combination optimization problem.

The coordinator is the key part which is used to cooperate the two optimization sub systems and take charge of real-time regulation of the introduced intermediate target.

The proposed intelligent optimization system was applied to the raw slurry preparing process of an alumina smeltery in China. By using the optimization system, the process performances are greatly improved in two aspects: the quality of raw slurry and the simplification of two re-mixing operation. The simplification increases the throughout of raw slurry, raises the eligibility rate and reduces energy consumption. Furthermore, the improvement of raw slurry quality makes downstream sintering process more stable.

Key words: Intelligent optimization; Raw slurry preparing process; Integrated prediction model; Expert reasoning.

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25 UNIFIED DUALITY THEORY FOR CONSTRAINED EXTREMUM PROBLEMS

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Abstract: By virtue of the image space analysis approach, a unified duality scheme for a constrained extremum problem based on the class of regular weak separation functions in the image space is proposed. Some equivalent characterizations to the zero duality property are obtained under an appropriate assumption. Moreover, some necessary and sufficient conditions for the zero duality property are also established in the form of the perturbation function. Simultaneously, the Lagrange-type duality, Wolfe duality and Mond-Weir duality are discussed as special duality schemes in a unified interpretation. As applications, some special cases of the class of regular weak separation functions are discussed.

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26 A MULTIOBJECTIVE MULTICLASS SUPPORT VECTOR MACHINE BASED ON ONE-AGAINST-ONE METHOD

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Abstract: Recently, some kinds of extensions of the binary support vector machine (SVM) to multiclass classification have been proposed. In this paper, we focus on an all-together method called the multiobjective multiclass support vector machine (MMSVM), which is aimed at maximizing the geometric margins for the generalization ability, while it requires a large amount of computational resources because it is formulated as a large-scale multiobjective optimization problem. In this paper, we propose a new all together method which constructs a classifier by making use of binary SVMs obtained by the one-against-one method which requires less computational resources. Moreover, in order to solve the multiobjective optimization problem, we derive a single objective optimization problem by using a reference point method. The proposed model can be expected to have the high generalization ability and reduce computational resources.

Key words: Support vector machine(SVM); Multiclass classification; One-against-one method; All-together method; Reference point.

1 INTRODUCTION

The support vector machine (SVM) is one of popular methods for the machine learning because it has high generalization ability to solve binary classification problems. However, its extension for multiclass classification is still an ongoing research issue. These extended methods can be roughly grouped into two categories: the first method constructs a discriminant function by combining multiple SVMs for binary classification problems which are derived from the original multiclass classification problem, such as *one-against-one* and *one-against-all* methods. Many kinds of combination techniques such as the majority voting, the directed binary tree structure and the error correcting output code, have been proposed. The second method called *all-together* directly finds a discriminant function by solving one optimization problem, where all patterns are classified into the corresponding classes (Ratliff and Rosenthal (1983)). The one-against-one and one-against-all methods require less computational resources than the all-together method for training, because the former two methods train multiple binary SVMs while the latter needs solving a large-scale optimization problem. Moreover, through some numerical experiments it is reported that the latter method is not exceptionally superior to the former ones in the sense of classification performance. Therefore, the former method have been mainly investigated so far.

On the other hand, in (P. Tseng and S. Yun. (2009)) a new all-together method is proposed, which is called multiclass multiobjective SVM (MMSVM). The proposed model is formulated as an optimization problem with new objective functions which maximize the geometric margins which are the minimal distances of training patterns to the corresponding discriminant hyperplanes. Then, in order to maximize multiple geometric margins simultaneously, it is formulated as a multiobjective optimization (MO) problem (Z.Q.Luo and P. Tseng. (1992), P. Tseng and S. Yun. (2009)). Moreover, a single-objective optimization problem is derived by using scalarization approaches to solve the MO, and it is reported that it has high generalization ability. However, since the model still requires a large amount of computational resources, it is difficult to apply it to a large-scale classification problem.

Therefore, in this paper, we limit the feasible region of the optimization problem in MMSVM by exploiting binary SVMs obtained by the one-against-one method. The proposed model finds weights of combination of binary SVMs that maximizes the geometric margins, which can reduce the computational resources. At the same time, the proposed model is a multiobjecive optimization problem similarly to the MMSVM, and thus, we derive a single-objective optimization model based on reference point method to obtain a Pareto optimal solution of the proposed model approximately.

2 MULTICLASS CLASSIFICATIION

In this paper, we consider the following multiclass classification problem. For given data: $D = \{x^i, y_i\}, i \in I := \{1, ..., l\}$, where $x^i \in R^n$ denotes an input pattern and $y^i \in P := \{1, ..., m\}$ denotes the corresponding class, we construct a classifier $f(x)$ which divides all patterns into the corresponding classes such that $y_i = f(x^i)$. Let us consider linearly separable data D. Although in this paper, for the sake of simplicity of argument, we mainly consider linear models, which can be easily extended into nonlinear models by mapping input patterns into an appropriate high-dimensional feature space.

Some kinds of extensions of the binary SVM to multiclass classification have been proposed. In this paper, we focus on the one-against-one and all-together methods. In the one-against-one method, a P-class problem is transformed into $P(P-1)$ binary classification problems. Namely, we determine the decision variables by training binary SVMs for all the combinations of class pairs. In determining decision variables for a pair, we use the training data for the corresponding two classes. The decision function for classes p and q is given by

$$f_{pq}(x) = w^{pq\top} x + b_{pq}, \tag{2.1}$$

where $w^{pq}, b_{pq},\ p, q \in P$ are decision variables. The decision function, which is used to classify patterns, is constructed by combining obtained binary SVMs. As a popular combination method, the decision directed acyclic graph (DDAG) SVM (P.Tseng. (2001)) is often used, which classifies pattern by using binary SVMs in the order of the priority given by an appropriate graph. This method can obtain high generalization by making the structure of a graph. However, the generalization depends heavily on the selection of the graph, and its appropriate selection is often difficult.

In the all-together method, the aim is finding the following function:

$$f(x) = \operatorname*{argmax}_{p} \{w^{p\top} x + b_p\}, \tag{2.2}$$

where $(w^p, b_p) \in R^{n+1}, p \in P$ are decision variables. The linear function $w^{p\top} x + b_p$ indicates the degree of confidence when a point x is classified into class p. Then,

$$(w^p - w^q)^\top x + b_p - b_q = 0,\ p \neq q,\ p, q \in P, \tag{2.3}$$

is the discriminant hyperplane which distinguishes between classes p and q. Decision variables are directly found by solving a single optimization problem, as follows:

$$\text{(O)} \quad \begin{array}{ll} \displaystyle\min_{w,b} & \displaystyle\frac{1}{2}\sum_{p=1}\sum_{q>p} \|w^p - w^q\|^2 \\ \text{s.t} & (w^p - w^q)^\top x^i + (b^p - b^q) \geq 1,\ \ i \in I_p,\ \ q > p,\ \ p, q \in P, \end{array} \tag{2.4}$$

where I_p denotes an index set defined by $I_p := \{i \in I \mid y_i = p\}$. The model maximizes $1/ \parallel w^p - w^q \parallel$ for all pairs $(p, q), q \neq p, p, q \in P$ for the generalization ability. Then, $1/ \parallel w^p - w^q \parallel$ denotes the functional margin, which is the distance between the discriminant hyperplane (2.3) and two normalized support hyperplanes parallel to (2.3),

$$(w^p - w^q)^\top x + b_p - b_q = 1 \text{ and } (w^p - w^q)^\top x + b_p - b_q = -1.$$

In binary classification the functional margin can exactly denote the distance between the discriminant hyperplane and the nearest corresponding pattern, while in multiclass classification the margin cannot necessarily do. The distance is defined an exact margin $d(w, b)$ called a geometric margin which is the distance of the nearest pattern in the corresponding classes to the discriminant hyperplane as follows:

$$d_{pq}(w,b) = \min_{i \in I_p \cup I_q} \frac{\mid (w^p - w^q)^\top x^i + (b_p - b_q) \mid}{\parallel w^p - w^q \parallel},\ q > p,\ p, q \in P. \tag{2.5}$$

It cannot guarantee that margins obtained by minimizing $\parallel w^p - w^q \parallel$, $q \neq p \in P$ in model (O) are equal to the geometric margins. Here, note that maximizing the geometric margins guarantees the generalization ability. Therefore, in (P. Tseng and S. Yun. (2009)) a piecewise linear hardmargin multiobjective multiclass SVM (MMSVM) was proposed. The problem maximizes all geometric margins simultaneously by adding a vector $\sigma \in R^{m(m-1)/2}$:

$$\text{(M)} \quad \begin{array}{ll} \max\limits_{w,b,\sigma} & \left(\dfrac{\sigma_{12}}{\parallel w^1 - w^2 \parallel}, \ldots, \dfrac{\sigma_{(m-1)m}}{\parallel w^{m-1} - w^m \parallel} \right) \\ \text{s.t.} & (w^p - w^q)^\top x^i + (b^p - b^q) \geq \sigma_{pq}, \quad i \in I_p, \quad q > p, \quad p, q \in P, \\ & (w^q - w^p)^\top x^i + (b^q - b^p) \geq \sigma_{pq}, \quad i \in I_q, \quad q > p, \quad p, q \in P, \\ & \sigma_{pq} \geq 1, \quad q > p, \quad p, q \in P. \end{array}$$

Then, in order to solve the multiobjective optimization problem (M), two kinds of single objective optimization problems were proposed by scalarization approaches to multiobjective optimization, ε-constraint approach and Benson's method (Z.Q.Luo and P. Tseng. (1992), P. Tseng and S. Yun. (2009)). These problems can be regarded as single-objective second-order cone programming (SOCP) problems, which are easily solvable due to convexity. MMSVM was shown to have high generalization abilities in numerical experiments.

Since MMSVM requires a large amount of computational resources, it is difficult to apply it to a large-scale classification problems. However, since MMSVM can obtain high generalization, it will be an effective model if computational resources are reducible. Therefore, in this paper we use weights obtained by the one-against-one method to reduce computational resources in solving (M).

3 MULTIOBJECTIVE MULTICALASS MODEL BASED ON ONE-AGAINST-ONE METHOD

In this section, we propose a new MMSVM model which requires less computational resources by using binary SVMs of the one-against-one method.

Now, suppose that $\bar{w}^{pq},\ p, q \in P$ are weights obtained by training a binary SVM which distinguishes classes p and q in the one-against-one method. We propose a new model where w^p in MMSVM are restricted as a linear sum of $\bar{w}^{pq}$ to reduce computational resources. Then, by introducing additional variables $\alpha^{pq},\ p, q \in P$, and adding constraints

$$w^p = \sum_{q=1}^{m} \alpha^{pq} \bar{w}^{pq} \tag{3.1}$$

to the MMSVM model (M), discriminant function (2.2) changes to the following unified discriminant function:

$$f(x) = \underset{p}{\operatorname{argmax}} \left\{ \sum_{q=1}^{m} \alpha^{pq} \bar{w}^{pq\top} x + b_p \right\}, \tag{3.2}$$

and thus, the optimization problem (M) changes to the following problem:

$$\text{(M} - \text{OAO)} \quad \begin{array}{ll} \max\limits_{\alpha,b,\sigma} & \left(\dfrac{\sigma_{12}}{\left\| \sum\limits_{r}^{m} \alpha^{1r} \bar{w}^{1r} - \sum\limits_{r}^{m} \alpha^{2r} \bar{w}^{2r} \right\|}, \ldots, \dfrac{\sigma_{(m-1)m}}{\left\| \sum\limits_{r}^{m} \alpha^{(m-1)r} \bar{w}^{(m-1)r} - \sum\limits_{r}^{m} \alpha^{mr} \bar{w}^{mr} \right\|} \right) \\ \text{s.t.} & \left(\sum\limits_{r=1}^{m} \alpha^{pr} \bar{w}^{pr} - \sum\limits_{r=1}^{m} \alpha^{qr} \bar{w}^{qr} \right)^\top x^i + (b^p - b^q) \geq \sigma_{pq},\ i \in I_p,\ q > p,\ p, q \in P, \\ & \left(\sum\limits_{r=1}^{m} \alpha^{qr} \bar{w}^{qr} - \sum\limits_{r=1}^{m} \alpha^{pr} \bar{w}^{pr} \right)^\top x^i + (b^q - b^p) \geq \sigma_{pq},\ i \in I_q,\ q > p,\ p, q \in P, \\ & \sigma_{pq} \geq 1,\ q > p,\ p, q \in P. \end{array}$$

This problem means finding $(\alpha^{pq}, b_p),\ p, q \in P$ to maximizes geometric margins in the discriminant function (3.2).

Let us compare the sizes of problems of (M-OAO) with (M). The number of variables of them are m^2 and $m(m+2n+1)/2$, respectively. Since $m << n$ in general, the proposed model can be expected to require a less amount of computational resources than (M).

Next, in order to solve the multiobjective optimization problem (M-OAO), we propose single objective optimization problem by a similar scalarization approach, which can be used for (M). In this case, we use a reference point method which finds the Pareto optimal solution which is the nearest point from the reference point. Then, we can derive the following single objective optimization problem:

$$
(\mathrm{RP-OAO}) \quad
\begin{array}{ll}
\min\limits_{\alpha,b,\sigma,l} & l_M \\
\text{s.t.} & l_M \geq l_{pq},\ \ q > p,\ \ p, q \in P, \\
& \bar{d}_{pq} - \dfrac{\sigma_{pq}}{\left\| \sum\limits_{r=1}^{m} \alpha^{pr} \bar{w}^{pr} - \sum\limits_{r=1}^{m} \alpha^{qr} \bar{w}^{qr} \right\|} = l_{pq},\ \ q > p,\ \ p, q \in P, \\
& l_{pq} \geq 0,\ \ q > p,\ \ p, q \in P, \\
& \left(\sum\limits_{r=1}^{m} \alpha^{pr} \bar{w}^{pr} - \sum\limits_{r=1}^{m} \alpha^{qr} \bar{w}^{qr} \right)^{\top} x^i + (b^p - b^q) \geq \sigma_{pq},\ \ i \in I_p,\ \ q > p,\ \ p, q \in P, \\
& \left(\sum\limits_{r=1}^{m} \alpha^{qr} \bar{w}^{qr} - \sum\limits_{r=1}^{m} \alpha^{pr} \bar{w}^{pr} \right)^{\top} x^i + (b^q - b^p) \geq \sigma_{pq},\ \ i \in I_q,\ \ q > p,\ \ p, q \in P, \\
& \sigma_{pq} \geq 1,\ \ q > p,\ \ p, q \in P,
\end{array}
$$

where $\bar{d}_{pq},\ p, q \in P$ denotes the reference point. In this paper, we use the margins obtained by binary SVMs trained for binary problem between class pairs as the reference point because the margins express the upper bounds of margins which can be obtained in MMSVM. Since the (RP-OAO) is difficult to solve directly because of fractional constrains, we transformed it into a SOCP problem whose optimal solution is an approximate Pareto optimal one for (M-OAO). Moreover, we can exactly obtain the Pareto optimal solution by using the ε-constraint approach or Benson's method furthermore. The computational complexity for solving the SOCP problem is less than it for the SOCP problem (M) because of the number of variables.

On the other hand, we need to decide constants $\bar{d}$ and ε in the SOCP problems of (RP-OAO) and (M), respectively. In (P. Tseng and S. Yun. (2009)), to solve a SOCP problem of (M) on the basis of ε-constraint approach, the optimal solution of (O) is used in order to determine ε. Thus, all training data are necessary to solve (O). While, the one-against-one method used by the proposed method needs only two classes training data at a time. Therefore, we conclude that the proposed model reduces the computational complexity with comparison of (M).

4 CONCLUSION

In this paper, we have focused on the one-against-one and the all-together method of the support vector machine (SVM) for multiclass classification, and proposed a multiobjective model (M-OAO) which construct a multiclass classifier as a weighted combination of binary SVMs obtained by the one-against-one method. Since the proposed models are formulated as a multiobjective optimization problem, we have proposed a single-objective problem (RP-OAO) based on a reference point method. The proposed model is required less computational resources.

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27 AN IMPROVED TWO-STEP METHOD FOR SOLVING GENERALIZED NASH EQUILIBRIUM PROBLEMS

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Abstract: The generalized Nash equilibrium problem (GNEP) is a non-cooperative game in which the strategy set of each player, as well as his payoff function, depend on the rival players strategies. As a generalization of the standard Nash equilibrium problem (NEP), the GNEP has recently drawn much attention due to its capability of modeling a number of interesting conflict situations in, for example, an electricity market and an international pollution control. In this paper, we propose an improved two-step (a prediction step and a correction step) method for solving the quasi-variational inequality (QVI) formulation of the GNEP. Per iteration, we first do a projection onto the feasible set defined by the current iterate (prediction) to get a trial point; then, we perform another projection step (correction) to obtain the new iterate. Under certain assumptions, we prove the global convergence of the new algorithm. We also present some numerical results to illustrate the ability of our method, which indicate that our method outperforms the most recent projection-like methods of Zhang et al. (2010).

Key words: Convex programming; Generalized Nash equilibrium; Quasi-variational inequality problems; Two-step methods; Projection methods.

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28 EXISTENCE AND BOUNDEDNESS OF SOLUTIONS TO THE HEMIVARIATIONAL INEQUALITIES OF HARTMAN-STAMPACCHIA TYPE

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Abstract: We consider the existence and boundedness of solution for variational-hemivariational inequalities of Hartman-Stampacchia type, when the mapping has a certain kind of monotonicity and when the mapping is only upper semicontinuous, respectively. On the one hand, assuming that the mapping is set-valued, lower hemicontinuous and stably quasimonotone with respect to a certain set, when the constraint set is bounded, we prove the existence of solution; when the constraint set is not bounded, we derive a suffcient condition for the existence of solutions and a suffcient condition for the existence and boundedness of solution. On the other hand, assuming that the mapping is set-valued and upper semicontinuous and the space is Euclidean space, when the constraint set is bounded, we prove the existence of solutions; when the constraint set is unbounded, we give a suffcient condition for the existence of solutions and a suffcient condition for the existence and boundedness of solutions.

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29 A MODIFIED DOUBLE PROJECTIVE ALGORITHM FOR SOLVING QUASIMONOTONE VARIATIONAL INEQUALITIES

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Abstract: The modified double projective method for solving quasimonotone variational inequalities is studied. Motivated by the solutions of variational inequality can be divided into nontrivial solutions and trivial solutions in the discussion of the existence of quasimonotone variational inequalities. Our method is proven to be globally convergent under the assumption of nontrivial solutions is nonempty. A necessary and sufficient conditions of trivial solutions is presented under the assumption of the interior of the closed and convex subset is nonempty. To our best of knowledge, this is the first discussion of projection method for solving quasimonotone variational inequalities.

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30 ON THE SUBSPACE MINIMIZATION CONJUGATE GRADIENT METHOD

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Abstract: The linear conjugate gradient method is an *optimal* method for solving symmetrical and positive definite linear equations. The proposition of limited-memory BFGS method and Barzilai-Borwein gradient method, however, heavily restricted the use of conjugate gradient method in large-scale optimization. This is, to the great extent, due to the requirement of a relatively exact line search at each iteration and the loss of conjugacy property of the search directions in various occasions. In this talk, I shall pay much attention on the subspace minimization conjugate gradient method by Yuan and Stoer (1995). Some nice theoretical properties of the method will be explored and some promising numerical results will be provided. Consequently, we can see that the subspace minimization conjugate gradient method can become a strong candidate for large-scale optimization.

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31 MATHEMATICAL MODELS FOR SOME PROBLEMS IN ELECTRICITY MARKET IN CHINA AND CORRESPONDING DISCUSSIONS

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Abstract: This paper talk about the power pricing in electricity market in general. There are some key points for the process of the power pricing. Then the models for it is set up. Secondly transmission congestion management is very important to the security of the power network. It gives out some kinds of optimal models for the problem and has some discussions.

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32 AN EXACT PENALTY METHOD FOR GENERALIZED NASH EQUILIBRIUM PROBLEMS

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Abstract: In this paper, we present an exact penalty function method to reduce the GNEP into a Nash equilibrium problem. Then, it is solved by using the projection method. We also report some numerical results so as to illustrate the efficiency of the method proposed.

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33 DESIGN OF ALLPASS VARIABLE FRACTIONAL DELAY FILTER WITH SIGNED POWERS OF TWO COEFFICIENTS

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Abstract: This paper investigates the optimal design of allpass variable fractional delay (VFD) filters with coefficients expressed as sums of signed powers-of-two terms, where the weighted integral squared error is the cost function to be minimized. A new optimization procedure is proposed to generate a reduced discrete search region. Then, a new exact penalty function method is developed to solve the optimal design problem for allpass VFD filter with signed powers-of-two coefficients. Design examples show that the proposed method is highly effective. Compared with conventional methods, our method can achieve a higher accuracy with less computation.

Key words: Allpass variable fractional delay filter; Signed powers-of-two; Exact penalty function method; Integer programming.

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34 OPTIMAL MACHINE MAINTENANCE SCHEDULING WITH RANDOM BREAKDOWN TIMES

Yufei Sun*[a], Grace Aw[a], Kok Lay Teo, Ryan Loxton
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Curtin University, Perth, Australia

Abstract: We investigate the optimal time to replace and overhaul a machine while minimizing cost and ensuring the minimum output is met. We model machine breakdowns using an appropriate probability distribution, and include a probabilistic constraint to limit the probability of breakdowns to be below a specified level. We show that the problem of determining an optimal maintenance schedule can be formulated as a stochastic optimal control problem governed by an Ito differential equation. The problem is then transformed into a deterministic optimal control problem with continuous inequality constraints. This equivalent problem can be solved using the time-scaling transformation and the constraint transcription method.

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35 CORPORATE BOND PORTFOLIO OPTIMIZATION WITH DEFAULT RISKS

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Abstract: We investigate the optimal bond portfolio a pension fund should hold in order to meet all outgo and to maximize the cash of the fund at the end of the period considered. We consider two situations: the first is where bond defaults are assumed not to occur, and the second is where we allow bond defaults. This leads to two stochastic discrete-time optimal control problems. We show that these problems can be transformed into standard deterministic problems and then solved using conventional methods. We then compare the results obtained by solving both problems.

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36 ON INVESTIGATION OF PRICE CHANGES IN PRICING AND PRODUCTION PLANNING

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Abstract: Joint pricing and production decisions are crucial to the competitiveness of a manufacturing company. A common assumption in coordination of pricing and production planning decisions is that the price-adjustments are costless. However, those costs are not negligible, as they may take up a significant part of the firms reported profit. In this work, we consider multi-product multi-period production planning systems with costly price-adjustments. A capacitated setting is investigated and a demand-based model in which the demand is a function of the price is introduced. Effective computational models will be developed for both deterministic and stochastic price dependent demand. Both fixed and variable price-adjustment costs will be considered. For the uncertain case, we focus on an additive demand model for which the underlying random variable is normally distributed. By using a chance constrained programming approach, we show that the model is still solvable. The aim of the work is to utilize the existing commercial packages for modelling and optimization to compare the effectiveness of various models for large-scale realistic problems.

Key words: Price-Adjustment Cost; Pricing; Production Planning; Uncertain Demand.

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37 A POWER PENALTY METHOD FOR A FINITE-DIMENSIONAL OBSTACLE PROBLEM ARISING IN FINANCIAL ENGINEERING

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Abstract: In this talk we present a power penalty method for solving a nonlinear optimization problem in R^n arising from the discretization of a financial option pricing problem under transaction costs. This problem is first formulated as a mixed complementarity problem of which the nonlinear mapping involved is not strongly monotone. We then approximate the mixed complementarity problem by a penalty equation containing a power penalty term with a penalty constant $\lambda > 1$ and a power constant $\alpha \leq 1$. We show that the solution to the penalty equation converges to that of the original problem at the exponential rate of order $\lambda^{-1/\alpha}$ as λ goes to infinity. We will present some numerical results to demonstrate the usefulness and convergence rates of the proposed method.

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38 AN INTELLIGENT REACTIVE POWER OPTIMIZATION METHOD FOR DFIG WIND FARM

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Abstract: We study a reactive power optimization model and algorithm in power system network regarding of reactive power control capability of DFIG wind farm, so as to provide sufficient reactive power compensation for the power system when the node voltage sags. First, the effect of wind speed fluctuation on DFIG wind farm reactive power capacity is analyzed. The reactive power capability limits of the DFIG wind farm are used as the constraints and the DFIG wind farm is regarded as a continuous reactive power source to participate in the reactive power optimization. Then, the summation of active power loss and node voltage deviation is chosen as the objective function. This reactive power optimization problem can be transformed into a nonlinear mixed integer optimization problem. Finally, as an example we consider an IEEE 33-node power system. We solve this numerical example using a genetic optimization algorithm to validate the effectiveness of the proposed method.

Key words: Reactive power; Wind farm; DFIG; Genetic optimization.

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39 A CLASS OF MAX-MIN OPTIMAL CONTROL PROBLEMS WITH APPLICATIONS TO CHROMATOGRAPHY

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Abstract: In this paper, we consider a class of non-standard optimal control problems in which the objective function is in max-min form and the state variables evolve over different time horizons. Such problems arise in the control of gradient elution chromatography—an industrial process used to separate and purify multi-component chemical mixtures. We develop a computational method for solving this class of optimal control problems based on the control parameterization technique, a time-scaling transformation, and a new exact penalty method.

Key words: Optimal control; Control parameterization; Time-scaling transformation; Exact penalty function.

1 PROBLEM STATEMENT

Consider a master system consisting of m coupled subsystems. The dynamics of the ith subsystem are described by the following set of ordinary differential equations:

$$\dot{\boldsymbol{x}}^i(t) = \boldsymbol{f}^i(\boldsymbol{x}^1(t), \ldots, \boldsymbol{x}^m(t), \boldsymbol{u}(t))\chi_{[0,\tau_i]}(t), \quad t \geq 0, \tag{1.1}$$

$$\boldsymbol{x}^i(0) = \boldsymbol{\zeta}^i, \tag{1.2}$$

where $\boldsymbol{x}^i(t) \in \mathbb{R}^n$ is the *state* of the ith subsystem, τ_i is the *terminal time* of the ith subsystem (a free decision variable), $\boldsymbol{\zeta}^i \in \mathbb{R}^n$ is the initial state of the ith subsystem (a given vector), $\boldsymbol{u}(t) \in \mathbb{R}^r$ is the *control input*, and the indicator function $\chi_{[0,\tau_i]} : \mathbb{R} \to \mathbb{R}$ is defined by

$$\chi_{[0,\tau_i]}(t) = \begin{cases} 1, & \text{if } t \in [0, \tau_i], \\ 0, & \text{if } t \notin [0, \tau_i]. \end{cases}$$

We assume that $\boldsymbol{f}^i : \mathbb{R}^{mn} \times \mathbb{R}^r \to \mathbb{R}^n$ in (1.1) is a given continuously differentiable function.

The control function in (1.1) is subject to the following bound constraints:

$$a_j \leq u_j(t) \leq b_j, \quad t \geq 0, \quad j = 1, \ldots, r, \tag{1.3}$$

where $u_j(t)$ is the jth element of $\boldsymbol{u}(t)$ and a_j and b_j are given constants such that $a_j < b_j$. Any measurable function $\boldsymbol{u} : [0, \infty) \to \mathbb{R}^r$ satisfying (1.6) is called an *admissible control*. Let $\mathcal{U}$ denote the class of all such admissible controls.

We collect the subsystem terminal times into a vector $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_m] \in \mathbb{R}^m$. Let $\mathcal{T}$ denote the set of all such vectors with components satisfying $\tau_i \geq 0$, $i = 1, \ldots, m$. Furthermore, let T denote the terminal time of the overall system. Then clearly,

$$T = \max\{\tau_1, \ldots, \tau_m\}.$$

The subsystems described by (1.1)-(1.2) are subject to the following *terminal state constraints*:

$$\Phi_i(\boldsymbol{x}^i(\tau_i)) = 0, \quad i = 1, \ldots, m, \tag{1.4}$$

where each $\Phi_i : \mathbb{R}^n \to \mathbb{R}$ is a given continuously differentiable function.

Our optimal control problem is stated below.

Problem 1 Choose $\boldsymbol{\tau} \in \mathcal{T}$ and $\boldsymbol{u} \in \mathcal{U}$ to maximize the objective functional

$$J(\boldsymbol{\tau}, \boldsymbol{u}) = \min_{i \neq j} \Psi(\tau_i, \tau_j, T, \boldsymbol{x}^i(\tau_i), \boldsymbol{x}^j(\tau_j)), \tag{1.5}$$

subject to the dynamic system (1.1)-(1.2) and the constraints (1.4), where $\Psi : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is a given continuously differentiable function.

Problem 1 presents two major challenges for existing optimal control methods: (i) the objective functional is non-smooth; and (ii) the state variables are defined over different time horizons.

Farhadinia et al. (see Farhadinia B. (2009)) developed a computational method for solving Problem 1 with $\Psi = -T$. In this case, Problem 1 reduces to a time-optimal control problem in which the aim is to minimize the terminal time of the overall system. Our goal in this paper is to develop a new method that is applicable to more general problems.

2 APPLICATIONS TO CHROMATOGRAPHY

Optimal control problems in the form of Problem 1 arise in chromatography—a separation and purification process that plays an important role in many industrial settings. A typical chromatography system consists of a column containing an absorbent (called the stationary phase) and a liquid that flows through the column (called the mobile phase). The mixture to be separated is injected into the mobile phase and flows through the column. Because the different components in the mixture are attracted to the stationary phase in different degrees, they travel through the column at different speeds, and thus they exit the column at different times (called retention times). Therefore, the mixture is gradually separated while moving through the column.

Jennings et al. (Jennings L. S. (1995)) and Chai et al. (Chai Q. (2012)) have considered a special case of Problem 1 in which the aim is to maximize separation efficiency in a chromatography system. In this problem, the subsystems correspond to the different components in the mixture, and the terminal times are the retention times. The terminal state constraints (1.4) arise because of a requirement that the concentration of each component reach a given value at the corresponding retention time. Meanwhile, the objective functional, which is obtained by setting $\Psi = (\tau_j - \tau_i)^2/T$ in equation (1.5), measures the minimum duration between successive retention times—a quantity that should be maximized.

3 CONTROL PARAMETERIZATION

In this section, we apply the control parameterization method (see Teo K. L. (1991)) to approximate Problem 1 by a finite-dimensional optimization problem.

Let $p \geq 1$ be a given integer. We approximate the control $\boldsymbol{u}$ in (1.1)-(1.2) by a piecewise-constant function that switches value at each terminal time and at $p-1$ locations between each pair of consecutive terminal times. The approximate control is defined as follows:

$$\boldsymbol{u}^p(t) = \sum_{k=1}^{mp} \boldsymbol{\sigma}^k \chi_{[t_{k-1}, t_k)}(t), \tag{3.1}$$

where t_k is the kth control switching time, $\boldsymbol{\sigma}^k \in \mathbb{R}^r$ is the control value on subinterval $[t_{k-1}, t_k)$, and the characteristic function $\chi_{[t_{k-1}, t_k)} : \mathbb{R} \to \mathbb{R}$ is as defined in Section 1. The control switching times satisfy

$$0 = t_0 \leq t_1 \leq t_2 \leq \cdots \leq t_{mp-1} \leq t_{mp} = T. \tag{3.2}$$

Furthermore, every pth control switching time coincides with one of the subsystem terminal times.

In view of (1.6), we have the following constraints on the control values:

$$a_j \le \sigma_j^k \le b_j, \quad j = 1, \dots, r, \quad k = 1, \dots, mp, \tag{3.3}$$

where σ_j^k is the jth component of $\boldsymbol{\sigma}^k$.

Let

$$v_{ij} = \begin{cases} 1, & \text{if subsystem } i \text{ has the } j\text{th earliest terminal time,} \\ 0, & \text{otherwise.} \end{cases}$$

Hence, if $v_{ij} = 1$, then subsystem i terminates at time $t = t_{jp}$. Clearly,

$$\sum_{j=1}^{m} v_{ij} = 1, \quad i = 1, \dots, m, \tag{3.4}$$

and

$$\sum_{i=1}^{m} v_{ij} = 1, \quad j = 1, \dots, m. \tag{3.5}$$

Substituting the approximate control defined by (3.1) into the dynamic system (1.1)-(1.2) yields

$$\dot{\boldsymbol{x}}^i(t) = \sum_{j=1}^{m} v_{ij} \boldsymbol{f}^i(\boldsymbol{x}^1(t), \dots, \boldsymbol{x}^m(t), \boldsymbol{\sigma}^k) \chi_{[0,t_{jp}]}(t), \quad t \in [t_{k-1}, t_k), \quad k = 1, \dots, mp, \tag{3.6}$$

$$\boldsymbol{x}^i(0) = \boldsymbol{\zeta}^i. \tag{3.7}$$

Furthermore, the terminal constraints (1.4) become

$$\sum_{j=1}^{m} v_{ij} \Phi(\boldsymbol{x}^i(t_{jp})) = 0, \quad i = 1, \dots, m. \tag{3.8}$$

The binary constraints $v_{ij} \in \{0, 1\}$ are difficult to enforce explicitly. Hence, we replace $v_{ij} \in \{0, 1\}$ with the following set of *non-discrete* constraints:

$$\sum_{j=1}^{m} v_{ij}(j^2 - j + \tfrac{1}{3}) - \left\{ \sum_{j=1}^{m} v_{ij}(j - \tfrac{1}{2}) \right\}^2 = \frac{1}{12}, \quad i = 1, \dots, m, \tag{3.9}$$

and

$$0 \le v_{ij} \le 1, \quad i = 1, \dots, m, \quad j = 1, \dots, m. \tag{3.10}$$

The following theorem, proved in Chai et al. (Chai Q. (2012)), shows that (3.9) and (3.10) imply $v_{ij} \in \{0, 1\}$.

Theorem 3.1 *Suppose that v_{ij}, $i = 1, \dots, m$, $j = 1, \dots, m$ satisfy* (3.4) *and* (3.10). *Then for each $i = 1, \dots, m$, equation* (3.9) *holds if and only if there exists a $q \in \{1, \dots, m\}$ such that $v_{iq} = 1$ and $v_{ij} = 0$ for all $j \neq q$.*

The subsystem terminal times occur at $t = t_{jp}$, $j = 1, \dots, m$. Let q_j denote the unique index satisfying $v_{q_j j} = 1$. Then

$$\boldsymbol{x}^{q_j}(t_{jp}) = v_{1j}\boldsymbol{x}^1(t_{jp}) + \dots + v_{mj}\boldsymbol{x}^m(t_{jp}). \tag{3.11}$$

We can now state the following finite-dimensional approximation of Problem 1.

Problem 2 Choose t_k and $\boldsymbol{\sigma}^k$, $k = 1, \dots, mp$ and v_{ij}, $i = 1, \dots, m$, $j = 1, \dots, m$ to maximize the objective function

$$J^p = \min_{i \neq j} \Psi(t_{ip}, t_{jp}, t_{mp}, \boldsymbol{x}^{q_i}(t_{ip}), \boldsymbol{x}^{q_j}(t_{jp}))$$

subject to the dynamic system (1.7)-(3.7) and the constraints (3.2)-(3.5) and (3.8)-(3.10).

4 TIME-SCALING TRANSFORMATION

Standard optimization algorithms will struggle with Problem 2 because the switching times in (1.7)-(3.7) are variable (see Loxton R. (2008)). Thus, in this section, we will apply a novel time-scaling transformation to map the switching times to fixed points in a new time horizon. To do this, we introduce a new time variable $s \in [0, mp]$ and relate s to t through the following differential equation:

$$\frac{dt(s)}{ds} = \omega(s), \quad t(0) = 0, \tag{4.1}$$

where $\omega : [0, mp] \to [0, \infty)$ is a piecewise-constant function with fixed switching times at $s = 1, \ldots, mp-1$. We express ω mathematically as follows:

$$\omega(s) = \sum_{k=1}^{mp} \theta_k \chi_{[k-1,k)}(s),$$

where $\theta_k = t_k - t_{k-1}$ is the duration between consecutive switching times in the original time horizon. Clearly,

$$\theta_k \geq 0, \quad k = 1, \ldots, mp. \tag{4.2}$$

For $s \in [k-1, k]$, integrating (4.1) gives

$$t(s) = \int_0^s \omega(\eta) d\eta = \sum_{l=1}^{k-1} \theta_l + \theta_k (s - k + 1).$$

Thus, for each $k = 1, \ldots, mp$,

$$t(k) = \sum_{l=1}^{k} \theta_l = \sum_{l=1}^{k} (t_l - t_{l-1}) = t_k.$$

This shows that the time-scaling transformation defined by (4.1) maps $t = t_k$ to the fixed integer $s = k$. In particular, the terminal time $t = T$ is mapped to $s = mp$:

$$t(mp) = \sum_{k=1}^{mp} \theta_k = t_{mp} = T = \max\{\tau_1, \ldots, \tau_m\}.$$

After applying the time-scaling transformation, the approximate control (3.1) becomes

$$\tilde{\boldsymbol{u}}^p(s) = \boldsymbol{u}^p(t(s)) = \sum_{k=1}^{mp} \boldsymbol{\sigma}^k \chi_{[k-1,k)}(s).$$

Furthermore, the dynamic system (1.7)-(3.7) becomes

$$\dot{\tilde{\boldsymbol{x}}}^i(s) = \sum_{j=1}^{m} v_{ij} \theta_k \boldsymbol{f}^i(\tilde{\boldsymbol{x}}^1(s), \ldots, \tilde{\boldsymbol{x}}^m(s), \boldsymbol{\sigma}^k) \chi_{[0,jp]}(s), \quad s \in [k-1, k), \quad k = 1, \ldots, mp, \tag{4.3}$$

$$\tilde{\boldsymbol{x}}^i(0) = \boldsymbol{\zeta}^i, \tag{4.4}$$

where $\tilde{\boldsymbol{x}}^i(s) = \boldsymbol{x}^i(t(s))$. Constraints (3.8) become

$$\sum_{j=1}^{m} v_{ij} \Phi(\tilde{\boldsymbol{x}}^i(jp)) = 0, \quad i = 1, \ldots, m. \tag{4.5}$$

Also, (3.11) becomes

$$\tilde{\boldsymbol{x}}^{q_j}(jp) = v_{1j} \tilde{\boldsymbol{x}}^1(jp) + \cdots + v_{mj} \tilde{\boldsymbol{x}}^m(jp).$$

We now state the following transformed optimal control problem, which is equivalent to Problem 2.

Problem 3 Choose θ_k and $\boldsymbol{\sigma}^k$, $k = 1, \ldots, mp$ and v_{ij}, $i = 1, \ldots, m$, $j = 1, \ldots, m$ to maximize the objective function

$$\tilde{J}^p = \min_{i \neq j} \tilde{\Psi}(\boldsymbol{\theta}, \tilde{\boldsymbol{x}}^{q_i}(ip), \tilde{\boldsymbol{x}}^{q_j}(jp))$$

subject to the dynamic system (4.3)-(4.4) and the constraints (3.3)-(3.5), (3.9), (3.10), (4.2), and (4.5), where

$$\tilde{\Psi}(\boldsymbol{\theta}, \tilde{\boldsymbol{x}}^{q_i}(ip), \tilde{\boldsymbol{x}}^{q_j}(jp)) = \Psi(\theta_1 + \cdots + \theta_{ip}, \theta_1 + \cdots + \theta_{jp}, \theta_1 + \cdots + \theta_{mp}, \tilde{\boldsymbol{x}}^{q_i}(ip), \tilde{\boldsymbol{x}}^{q_j}(jp)).$$

5 TRANSFORMATION INTO SMOOTH FORM

In this section, we transform Problem 3 into a smooth optimization problem. Let ξ be a new decision variable, where

$$\xi = \min_{i \neq j} \tilde{\Psi}(\boldsymbol{\theta}, \tilde{\boldsymbol{x}}^{q_i}(ip), \tilde{\boldsymbol{x}}^{q_j}(jp)).$$

Then we have the following set of inequality constraints:

$$\tilde{\Psi}(\boldsymbol{\theta}, \tilde{\boldsymbol{x}}^{q_i}(ip), \tilde{\boldsymbol{x}}^{q_j}(jp)) \geq \xi, \quad i \neq j. \tag{5.1}$$

It is clear that Problem 3 is equivalent to the following smooth optimization problem.

Problem 4 Choose ξ, θ_k, $\boldsymbol{\sigma}^k$, and v_{ij} to maximize the objective function $\bar{J}^p(\xi) = \xi$ subject to the dynamic system (4.3)-(4.4) and the constraints (3.3)-(3.5), (3.9), (3.10), (4.2), (4.5), and (5.1).

Standard optimization algorithms will typically struggle with Problem 4 because constraints (3.4), (3.5), (3.9), and (3.10) restrict v_{ij} to be binary decision variables. In the next section, we will describe an exact penalty method for solving Problem 4.

6 AN EXACT PENALTY METHOD

Define

$$\boldsymbol{\gamma} = \left[\xi, \theta_1, \ldots, \theta_{mp}, (\boldsymbol{\sigma}^1)^\top, \ldots, (\boldsymbol{\sigma}^{mp})^\top, (\boldsymbol{v}^1)^\top, \ldots, (\boldsymbol{v}^m)^\top\right]^\top, \tag{6.1}$$

where

$$\boldsymbol{v}^i = [v_{i1}, \ldots, v_{im}]^\top.$$

Furthermore, define a *constraint violation function* as follows:

$$\begin{aligned}\Delta(\boldsymbol{\gamma}) = &\sum_{i=1}^{m} \left\{ \sum_{j=1}^{m} v_{ij} - 1 \right\}^2 + \sum_{j=1}^{m} \left\{ \sum_{i=1}^{m} v_{ij} - 1 \right\}^2 + \sum_{i=1}^{m} \left\{ \sum_{j=1}^{m} v_{ij} \Phi(\tilde{\boldsymbol{x}}^i(jp)) \right\}^2 \\ &+ \sum_{i=1}^{m} \left\{ \sum_{j=1}^{m} v_{ij}(j^2 - j + \tfrac{1}{3}) - \left[\sum_{j=1}^{m} v_{ij}(j - \tfrac{1}{2}) \right]^2 - \tfrac{1}{12} \right\}^2 + \sum_{\substack{i,j=1,\ldots,m \\ i \neq j}} \max\{\xi - \tilde{\Psi}(\boldsymbol{\theta}, \tilde{\boldsymbol{x}}^i(ip), \tilde{\boldsymbol{x}}^j(jp)), 0\}^2,\end{aligned}$$

where $\boldsymbol{\gamma}$ is defined by (6.1). Clearly, $\Delta(\boldsymbol{\gamma}) = 0$ if and only if the current values of the decision variables are feasible for Problem 4.

Define a penalty function as follows:

$$G^p_\mu(\epsilon, \boldsymbol{\gamma}) = -\xi + \epsilon^{-\alpha}\Delta(\boldsymbol{\gamma}) + \mu\epsilon^\beta, \tag{6.2}$$

where ϵ is a new decision variable, $\mu > 0$ is the penalty parameter, and α and β are fixed constants satisfying $1 \leq \beta \leq \alpha$. The new decision variable ϵ is subject to the following bound constraints:

$$0 \leq \epsilon \leq \bar{\epsilon}, \tag{6.3}$$

where $\bar{\epsilon} > 0$ is a given constant.

In the penalty function (6.2), the last term $\mu\epsilon^\beta$ is designed to penalize large values of ϵ, while the middle term $\epsilon^{-\alpha}\Delta(\boldsymbol{\gamma})$ is designed to penalize constraint violations. When μ is large, minimizing (6.2) forces ϵ to be small, which in turn causes $\epsilon^{-\alpha}$ to become large, and thus constraint violations are penalized very severely. Hence, minimizing the penalty function for large values of μ will likely lead to feasible points. On this basis, we can approximate Problem 4 by the following *penalty problem.*

Problem 5 Choose ϵ and $\boldsymbol{\gamma}$ to minimize the penalty function $G^p_\gamma(\epsilon, \boldsymbol{\gamma})$ subject to the dynamic system (4.3)-(4.4) and the bound constraints (3.3), (3.10), (4.2), and (6.3).

Problem 5 can be viewed as an optimal parameter selection problem with multiple characteristic times. Such problems can be solved effectively using the computational method described in Loxton R. (2008), which uses gradient-based optimization techniques. It can be shown that under mild assumptions, when the penalty parameter μ is sufficiently large, any local solution of Problem 5 generates a corresponding local solution for Problem 4 (see Lin Q. (2012)). Thus, the penalty function (6.2) is *exact* in the sense that feasibility is attained for finite values of the penalty parameter.

Solving Problem 2 amounts to solving Problem 5 for an increasing sequence of penalty parameters, where the solution at each iteration is used as the initial guess for the next iteration. The solution of Problem 2 can be used to generate a suboptimal control for Problem 1 through equation (3.1). See Chai Q. (2012) and Lin Q. (2012) for more details.

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40 STABILITY ANALYSIS AND OPTIMAL CONTROL OF SINGULAR STOCHASTIC SYSTEMS

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Abstract: In this paper, the control problem of the class of singular stochastic systems is studied. Firstly, under some appropriate assumptions, results on mean-square admissibility are developed and the corresponding LMI sufficient condition is given. Secondly, the finite time horizon linear quadratic regulation (LQR) problem of the singular stochastic system is investigated, in which the coefficients are allowed to be random in control input and quadratic criterion. Some results involving new generalized stochastic Riccati equation are discussed as well. Finally, the proposed singular stochastic LQR control model provides to be an appropriate and effective framework to study the portfolio selection problem in light of the recent development on general stochastic LQR problems.

Key words: Singular stochastic systems; Mean-square stability; Linear quadratic control; Generalized stochastic Riccati equation.

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41 THE LS-SVM APPROXIMATE SOLUTION TO AFFINE NONLINEAR SYSTEMS WITH PARTIALLY UNKNOWN FUNCTIONS

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Abstract: By using the Least Squares Support Vector Machines (LS-SVMs), we develop a numerical approach to give an approximate solution for affine nonlinear system with unknown function part but solutions of a discrete point sequence of the system are known. From the point of view of control theory, it makes sense to consider the approximate solution for such nonlinear systems. This approach can obtain continuous and differential approximate solutions of the nonlinear differential equations and avoids knowing the structure of unknown nonlinear part and identifying its coefficients, and approximate solutions for the unknown and known parts can also be given. Technically, we first transform the unknown and known parts of the affine nonlinear system into feature spaces with nonlinear feature maps. Then we formulate the original problem as an approximation problem via kernel trick with LS-SVMs. Furthermore,the original approximate solution can be expressed as some linear forms whose coefficient matrices are coupling square matrices, and then solve them as a linear regression problem. Finally, some examples are presented to illustrate the validity of the proposed method.

Key words: Least Squares Support Vector Machines (LS-SVMs); Affine nonlinear systems; Coupling square matrices; Approximate solutions.

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42 H$_\infty$ CONTROL OF DISCRETE-TIME SINGULARLY PERTURBED SYSTEMS VIA STATIC OUTPUT FEEDBACK

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Abstract: This paper concentrates on H$_\infty$ control problems of discrete-time singularly perturbed systems via static output feedback. Two methods of designing H$_\infty$ controllers, which ensure the closed-loop system is asymptotically stable and meets a prescribed H$_\infty$ norm bound, are presented in terms of LMIs. Based on some matrix transformations, the proposed approaches are derived by optimizing γ and ϵ respectively. Furthermore, the stability upper bound of ϵ is also obtained. The validity of the proposed results is demonstrated by a numerical example.

Key words: discrete-time singularly perturbed system; static output feedback; asymptotical stability; H$_\infty$ performance; linear matrix inequality (LMI).

1 INTRODUCTION

Singularly perturbed systems, described by parameter related state-space models in control theory, widely exist in industrial processes. The existence of the small parameter causes high dimensionality and ill-conditioning problems. Thus, stability bounds of the singular perturbation parameter have been extensively studied. A traditional method of decomposing the original system into fast and slow subsystems was presented in (Sen. (1993)). In (Liu W. Q.(1996)), two algorithms to compute and improve the stability bound were developed. Moreover, (Liu W. Q.(1997)) established a method to testify the stability of singularly perturbed systems without fast-slow decomposition, which revealed the close relationship between stability and the system matrix. More details are discussed in (Geromel. J. C . (1998)), (Shi. (1999)).

Recent years, state feedback control of singularly perturbed systems have attracted much attention, see (Dong J. X. (2007)), (Xu S. Y. (2009)), (Ivan M. (2012)), (Vrabel R. (2012)) and (Chen J. (2011)). Though state feedback can achieve desired properties, it requires the availability of all state variables, which can not be satisfied in most of the practical systems. While dynamic output feedback usually increases the dimension of the original system. Therefore, static output feedback plays an important role in control theory with its simpleness and low cost. The primal point involved in static output feedback is the decoupling problem. In (Han Q. L. (2008)), special inequality and new variables were used to deal with the nonlinear inequality. An effective technique of introducing a stabilizing state feedback controller and some matrix transformations were proposed in (Boukas. E. K. (2005)), which is adopted in this paper due to its easy implementation.

Motivated by the above studies, we design an H_∞ controller via static output feedback to stabilize a discrete-time singularly perturbed system and guarantee that the resulting closed-loop system satisfies a prescribed H_∞ norm bound. The rest of this paper is organized as follows. Section 2 states the system description and some useful lemmas. In section 3, two LMI-based methods are proposed to design a static output feedback controller for the system presented in section 2. A numerical example is given to demonstrate the effectiveness of the proposed results in section 4. Finally, conclusions are given in section 5.

Throughout this paper, the following notations will be adopted. A^{T} denotes the transpose of matrix A. Blocks induced by symmetry is denoted by *. Sym{A} denotes $A + A^{\mathrm{T}}$.

2 PROBLEM DESCRIPTION

Consider a class of linear fast sampling discrete-time singularly perturbed systems of the following form:

$$\begin{cases} x_{k+1} = A_\epsilon x_k + B_{1\epsilon} w_k + B_{2\epsilon} u_k \\ z_k = C_1 x_k + D_{11} w_k + D_{12} u_k \\ y_k = C_2 x_k \end{cases} \tag{2.1}$$

where $x_k = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix}$, $A_\epsilon = \begin{bmatrix} I_{n1} + \epsilon A_{11} & \epsilon A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $B_{1\epsilon} = \begin{bmatrix} \epsilon B_{11} \\ B_{12} \end{bmatrix}$, $B_{2\epsilon} = \begin{bmatrix} \epsilon B_{21} \\ B_{22} \end{bmatrix}$, $C_1 = \begin{bmatrix} C_{11}^{\mathrm{T}} \\ C_{12}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, $C_2 = \begin{bmatrix} C_{21}^{\mathrm{T}} \\ C_{22}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, $x_k \in R^n$ is the state vector, in which $x_{1k} \in R^{n_1}$, $x_{2k} \in R^{n_2}$ *and* $n_1 + n_2 = n$; $u_k \in R^{m_1}$ is the control input; $w_k \in R^{m_2}$ is the disturbance input which belongs to $L_2[0,\infty)$; $y_k \in R^{q_l}$ is the measurement output; $z_k \in R^{q_2}$ is the controlled output. The scalar $\epsilon > 0$ denotes the singular perturbation parameter.

In the rest of this paper, we will assume system (2.3) is completely controllable and observable. We will also assume not all of its state variables are available. Introduce the following static output feedback control law:

$$u_k = F y_k, \tag{2.2}$$

then the resulting closed-loop system can be obtained as follows

$$\begin{cases} x_{k+1} = \tilde{A}_\epsilon x_k + B_{1\epsilon} w_k \\ z_k = \tilde{C}_1 x_k + D_{11} w_k \\ y_k = C_2 x_k \end{cases} \tag{2.3}$$

where $\tilde{A}_\epsilon = \begin{bmatrix} I_{n1} + \epsilon \tilde{A}_{11} & \epsilon \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}$, with $\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} + \begin{bmatrix} B_{21} \\ B_{22} \end{bmatrix} F \begin{bmatrix} C_{21} & C_{22} \end{bmatrix}$,
$\tilde{C}_1 = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} \end{bmatrix}$, with $\begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} + D_{12} F \begin{bmatrix} C_{21} & C_{22} \end{bmatrix}$.

The H_∞ problem studies in this paper can be described as: Given a scalar $\gamma > 0$ and a discrete-time singularly perturbed system (2.3), design a static output feedback controller in form of (2.4) such that the closed-loop system (3.4) is asymptotically stable and its transfer function from ω to z:

$$G(z) = \tilde{C}_1 (zI - \tilde{A}_\epsilon)^{-1} B_{1\epsilon} + D_{11}, \tag{2.4}$$

satisfies $\|G\|_\infty < \gamma$.

The following lemmas will be used in establishing our main results:

Lemma 2.1 *(Boukas. E. K. (2005)) The following two statements are equivalent:*

- *Let A, B and C be given such that the following LMI is feasible in X and Y.*

$$\begin{bmatrix} A & B \\ B^{\mathrm{T}} & 0 \end{bmatrix} + Sym\left\{ \begin{bmatrix} X \\ Y \end{bmatrix} \begin{bmatrix} C^{\mathrm{T}} & -I \end{bmatrix} \right\} < 0$$

- *Let A, B and C be given such that the following LMI holds.*

$$A + BC^{\mathrm{T}} + CB^{\mathrm{T}} < 0$$

Lemma 2.2 *(Boukas. E. K. (2005)) The following two statements are equivalent:*

- *Let A, B and C be given such that the following LMI is feasible in G.*

$$\begin{bmatrix} A & B+CG^{\mathrm{T}} \\ B^{\mathrm{T}}+GC^{\mathrm{T}} & -G-G^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} A & B \\ B^{\mathrm{T}} & 0 \end{bmatrix} + Sym\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} G[\ C^{\mathrm{T}} \quad -I\ ] \right\} < 0$$

- *Let A, B and C be given such that the following LMIs hold.*

$$A < 0, \quad A + BC^{\mathrm{T}} + CB^{\mathrm{T}} < 0$$

3 MAIN RESULTS

In this section, two LMI-based methods of designing static output feedback controllers are proposed to ensure asymptotical stability of system (3.4). The first result is given in form of an $\epsilon-$independent LMI, while the second is presented by two LMIs which are related to ϵ. Furthermore, the stability upper bound of ϵ is obtained.

Before giving the results, let F_0 be a stabilizing state feedback controller gain of system (2.3). In other words, the matrix F_0 is chosen to make $A_{\epsilon 0} = A_\epsilon + B_{2\epsilon}F_0$ stable. Then we introduce the following transformations which will be used in the rest of this paper:

$$A_0 = A + B_{2\epsilon}F_0, \quad C_{20} = C_2 + D_{12}F_0, \quad FC_2 = S + F_0,$$

where $A_0 = \begin{bmatrix} A_{110} & A_{120} \\ A_{210} & A_{220} \end{bmatrix}$, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $C_{20} = \begin{bmatrix} C_{210}^{\mathrm{T}} \\ C_{220}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, $S = \begin{bmatrix} S_1^{\mathrm{T}} \\ S_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, $F_0 = \begin{bmatrix} F_{01}^{\mathrm{T}} \\ F_{02}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$.

Theorem 3.1 *For a discrete-time singularly perturbed system (2.3), given a stabilizing state feedback controller with gain F_0, if there exists matrices $P_{11} > 0$, $P_{22} > 0$, $G > 0$, and matrices P_{12}, L, X_{ij}, Y_{kj}, (i=1,...,5, k,j=1,2) with appropriate dimensions, such that the following LMI holds:*

$$\begin{bmatrix} \Xi - XF_0^{\mathrm{T}}T - (XF_0^{\mathrm{T}}T)^{\mathrm{T}} & \Phi - X - T^{\mathrm{T}}F_0Y^{\mathrm{T}} & XC_2^{\mathrm{T}} + T^{\mathrm{T}}L \\ * & -Y-Y^{\mathrm{T}} & YC_2^{\mathrm{T}} \\ * & * & -G-G^{\mathrm{T}} \end{bmatrix} < 0, \tag{3.1}$$

where $X = [X_{ij}]$, $Y = [Y_{kj}]$, $\quad i = 1, \ldots, 5, k, j = 1, 2,$
$\Delta_{110} = P_{11}A_{110}^{\mathrm{T}} + P_{12}A_{120}^{\mathrm{T}} + (P_{11}A_{110}^{T} + P_{12}A_{120}^{\mathrm{T}})^{\mathrm{T}}$, $\Delta_{120} = P_{11}A_{210}^{\mathrm{T}} + P_{12}A_{220}^{\mathrm{T}} - P_{12}$,

$$\Phi = \begin{bmatrix} 0 & P_{22} \\ P_{11} & P_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ T = \begin{bmatrix} 0 \\ B_{21} \\ B_{22} \\ D_{12} \\ 0 \end{bmatrix}^{\mathrm{T}}, \ \Xi = \begin{bmatrix} -P_{22} & P_{22}A_{120}^{\mathrm{T}} & P_{22}A_{220}^{\mathrm{T}} & P_{22}C_{120}^{\mathrm{T}} & 0 \\ * & \Delta_{110} & \Delta_{120} & P_{11}C_{110}^{\mathrm{T}} + P_{12}C_{120}^{\mathrm{T}} & B_{11} \\ * & * & -P_{22} & 0 & B_{12} \\ * & * & * & -\gamma I & D_{11} \\ * & * & * & * & -\gamma I \end{bmatrix},$$

then there exists a scalar $\epsilon^* > 0$ such that for every $\varepsilon \in (0, \epsilon^*]$, system (3.4) is asymptotically stable and its transfer function satisfies $\|G\|_\infty < \gamma$ with the static output feedback controller gain $F = LG^{-\mathrm{T}}$.

Proof: For all $\varepsilon \in (0, \epsilon^*]$, let $P(\epsilon) = \begin{bmatrix} \epsilon P_{11} & \epsilon P_{12} \\ * & P_{22} \end{bmatrix} > 0$ satisfy discrete bounded real lemma in (Gahinet P. (1994)). By some matrix transformations and simplification, we have

$$\Pi = \begin{bmatrix} -P_{22} & P_{22}\widetilde{A}_{12}^{\mathrm{T}} & P_{22}\widetilde{A}_{22}^{\mathrm{T}} & P_{22}\widetilde{C}_{12}^{\mathrm{T}} & 0 \\ * & \Delta_{11} + \Delta_{11}^{\mathrm{T}} & \Delta_{12} - P_{12} & \Gamma_1 & B_{11} \\ * & * & -P_{22} & 0 & B_{12} \\ * & * & * & -\gamma I & D_{11} \\ * & * & * & * & -\gamma I \end{bmatrix} < 0. \tag{3.2}$$

To turn (3.2) into an LMI, we will use the same idea proposed in (Mehdi, D. (2003)). By substituting the transformations defined before, we get another form of (3.2):

$$\Xi + \Phi(T^{\mathrm{T}}S)^{\mathrm{T}} + (T^{\mathrm{T}}S)\Phi^{\mathrm{T}} < 0, \tag{3.3}$$

where Ξ, Φ, S and T are given as before.
Applying lemma 2.1 and lemma 2.1 to (3.3) respectively, we get

$$\begin{bmatrix} \Xi - XF_0^{\mathrm{T}}T - (XF_0^{\mathrm{T}}T)^{\mathrm{T}} & \Phi - X - T^{\mathrm{T}}F_0Y^{\mathrm{T}} & XC_2^{\mathrm{T}} \\ * & -Y-Y^{\mathrm{T}} & YC_2^{\mathrm{T}} \\ * & * & 0 \end{bmatrix} + Sym\left\{ \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} G\left[\ F^{\mathrm{T}}T \quad 0 \quad -I\ \right] \right\} < 0, \tag{3.4}$$

By letting $L = FG^{\mathrm{T}}$ we can finally conclude (3.1). This completes the proof. $\square$

Both the static output feedback gain matrix F and the minimum H_∞ norm γ can be obtained by solving the following optimization problem:

OP1.

$$\min_{P_{mj}, X_{ij}, Y_{kj}, L, G,} \gamma$$

where $i = 1, \ldots, 5,\ m \le j,\ m, k, j = 1, 2$, subject to $\gamma > 0, P_{11} > 0, P_{22} > 0, G > 0$ and the LMI in (3.1).

Note that the result in **Theorem 3.1** is finally obtained by solving a γ related optimal problem. While in the following part, a different designing approach is given by solving an optimal problem in which ϵ is involved.

Theorem 3.2 *Given a scalar $\gamma > 0$ and a stabilizing state feedback controller with gain F_0 , if there exists matrices $P_{11} > 0,\ P_{22} > 0,\ P_{12},\ L,\ G > 0,\ Q_{mm} > 0,\ Q_{mn},\ n < m,\ m, n = 1, 2, 3, 4,$ and matrices $X_i, i = 1, \ldots, 4,\ Y,\ \widetilde{X}_l, \widetilde{X}_2,\ \widetilde{Y}$, with appropriate dimensions, such that the following set of LMIs hold:*

$$\begin{bmatrix} \widetilde{Q}_1 & \widetilde{Q}_2 & -\widetilde{X}_1 & \widetilde{X}_1 B_{21} + C_2^{\mathrm{T}} L^{\mathrm{T}} - F_0^{\mathrm{T}} G^{\mathrm{T}} \\ \widetilde{Q}_3 & \widetilde{Q}_4 & p_{12}^{\mathrm{T}} - \widetilde{X}_2 & \widetilde{X}_2 B_{21} \\ -\widetilde{X}_1^{\mathrm{T}} & p_{12} - \widetilde{X}_2^{\mathrm{T}} & -\widetilde{Y} - \widetilde{Y}^{\mathrm{T}} & \widetilde{Y} B_{21} \\ B_{21}^{\mathrm{T}} \widetilde{X}_1^{\mathrm{T}} + LC_2 - GF_0 & B_{21}^{\mathrm{T}} \widetilde{X}_2^{\mathrm{T}} & B_{21}^{\mathrm{T}} \widetilde{Y}^{\mathrm{T}} & -G - G^{\mathrm{T}} \end{bmatrix} < 0, \tag{3.5}$$

$$\begin{bmatrix} \Pi_{11} & * & * & * & * & * \\ 0 & -\gamma I & * & * & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * & * & * \\ \Pi_{41} & \Pi_{42} & \Pi_{43} & \Pi_{44} & * & * \\ -X_1^{\mathrm{T}} & -X_2^{\mathrm{T}} & \Phi - X_3^{\mathrm{T}} & -X_4^{\mathrm{T}} & -Y - Y^{\mathrm{T}} & * \\ \Psi^{\mathrm{T}} X_1^{\mathrm{T}} - LC_2 + GF_0 & \Psi^{\mathrm{T}} X_2^{\mathrm{T}} & \Psi^{\mathrm{T}} X_3^{\mathrm{T}} & \Psi^{\mathrm{T}} X_4^{\mathrm{T}} & \Psi^{\mathrm{T}} Y^{\mathrm{T}} & -G - G^{\mathrm{T}} \end{bmatrix} < 0, \tag{3.6}$$

where $Q = Q^{\mathrm{T}} = [Q_{in}]$, $\begin{bmatrix} \Pi_{41} & \Pi_{42} & \Pi_{43} & \Pi_{44} \end{bmatrix} = \begin{bmatrix} Q & 0_{2*2} & -\epsilon^* Q \end{bmatrix}$, $\begin{bmatrix} \widetilde{Q}_1 & \widetilde{Q}_2 \end{bmatrix} = -[Q_{kn}], k = 1, 2, 3,$
$\widetilde{Q}_3 = \begin{bmatrix} -Q_{41} + p_{12}^{\mathrm{T}} A_{110} & -Q_{44} \end{bmatrix}$, $\widetilde{Q}_4 = \begin{bmatrix} -Q_{43} + p_{12}^{\mathrm{T}} B_{11} & -Q_{44} \end{bmatrix}$, $\Psi = \begin{bmatrix} B_{22}^{\mathrm{T}} & B_{21}^{\mathrm{T}} & D_{12}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$,
$\Pi_{11} = \begin{bmatrix} -\epsilon^* P_{11} & * \\ -P_{12}^{\mathrm{T}} & -P_{22} \end{bmatrix}$, $\Pi_{31} = \begin{bmatrix} P_{12}^{\mathrm{T}} + P_{22} A_{210} & P_{22} A_{220} \\ \epsilon^* P_{11} + P_{11} A_{110} + P_{12}^{\mathrm{T}} A_{210} & P_{11} A_{120} + P_{12} A_{220} \\ C_{110} & C_{120} \end{bmatrix}$,
$\Pi_{32} = \begin{bmatrix} P_{22} B_{12} \\ P_{11} B_{11} + P_{12} B_{12} \\ D_{11} \end{bmatrix}$, $\Pi_{33} = \begin{bmatrix} -P_{22} & * & * \\ -P_{12} & -\epsilon^* P_{11} & * \\ 0 & 0 & -\gamma I \end{bmatrix}$, $\Phi = \begin{bmatrix} P_{22} & 0 & 0 \\ P_{12} & P_{11} & 0 \\ 0 & 0 & I \end{bmatrix}$,
then for any singular perturbation parameter $\epsilon \in (0, \frac{1}{\epsilon^*}]$, by introducing a static output feedback controller in form of (2.4), the closed-loop fast sampling system (3.4) is asymptotically stable and its transfer function satisfies $\| G \|_\infty < \gamma$ with the controller gain $F = G^{-1} L$.

Proof: For any $\epsilon \in (0, \frac{1}{\epsilon^*}]$, let $P(\epsilon) = \begin{bmatrix} \frac{1}{\epsilon} P_{11} & * \\ P_{12}^{\mathrm{T}} & P_{22} \end{bmatrix} > 0$ satisfy discrete bounded real lemma in (Gahinet P. (1994)), after some matrix transformations we get

$$\begin{bmatrix} \Theta_{11}(\frac{1}{\epsilon}) & * \\ \Theta_{21}(\frac{1}{\epsilon}) & \Theta_{22}(\frac{1}{\epsilon}) \end{bmatrix} + \begin{bmatrix} \epsilon \Omega & 0 \\ 0 & 0 \end{bmatrix} < 0, \tag{3.7}$$

where

$$\Omega = \begin{bmatrix} 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \\ P_{12}^{\mathrm{T}} \widetilde{A}_{11} & P_{12}^{\mathrm{T}} \widetilde{A}_{12} & P_{12}^{\mathrm{T}} B_{11} & 0 \end{bmatrix}, \Theta_{11}(\tfrac{1}{\epsilon}) = \begin{bmatrix} -\frac{1}{\epsilon} P_{11} & * & * & * \\ -P_{12}^{\mathrm{T}} & -P_{22} & * & * \\ 0 & 0 & -\gamma I & * \\ P_{12}^{\mathrm{T}} + P_{22} \widetilde{A}_{21} & P_{22} \widetilde{A}_{22} & P_{22} B_{12} & -P_{22} \end{bmatrix},$$

$$\Theta_{21}(\tfrac{1}{\epsilon}) = \begin{bmatrix} \frac{1}{\epsilon} P_{11} + \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & -P_{12} \\ \widetilde{C}_{11} & \widetilde{C}_{12} & D_{11} & 0 \end{bmatrix}, \Theta_{22}(\tfrac{1}{\epsilon}) = \begin{bmatrix} -\frac{1}{\epsilon} p_{11} & * \\ 0 & -\gamma I \end{bmatrix},$$

$\Upsilon_{11} = P_{11} \widetilde{A}_{11} + P_{12} \widetilde{A}_{21}$, $\Upsilon_{12} = P_{11} \widetilde{A}_{12} + P_{12} \widetilde{A}_{22}$, $\Upsilon_{13} = P_{11} B_{11} + P_{12} B_{12}$.

If there exists a positive definite matrix $Q = [Q_{ij}], (i, j = 1, \ldots, 4)$ satisfies the following two matrix inequalities

$$\Omega - Q < 0, \tag{3.8}$$

$$\begin{bmatrix} \Theta_{11}(\epsilon^*) & * & * \\ \Theta_{21}(\epsilon^*) & \Theta_{22}(\epsilon^*) & * \\ Q & 0 & -\epsilon^* Q \end{bmatrix} < 0, \tag{3.9}$$

Applying the Schur complement to (3.9) and considering $\epsilon \in (0, \frac{1}{\epsilon^*}]$, we get

$$\begin{bmatrix} \Theta_{11}(\frac{1}{\epsilon}) & * \\ \Theta_{21}(\frac{1}{\epsilon}) & \Theta_{22}(\frac{1}{\epsilon}) \end{bmatrix} + \begin{bmatrix} \epsilon Q & 0 \\ 0 & 0 \end{bmatrix} < 0, \tag{3.10}$$

Taking (3.8) into consideration, we have (3.7).

To turn (3.8) and (3.10) into LMIs, we still adopt the same technique used in (Mehdi, D. (2003)). Finally we can conclude (3.5) and (3.6) by considering the expression of S. This completes the proof. □

We can obtain the static output feedback controller gain matrix F and the stability upper bound of the singular perturbation parameter which we record as $\frac{1}{\epsilon^*}$ by solving the following optimal problem: **OP2**.

$$\min_{P_{ij},\ Q_{mn},\ X_k,\ Y,\ \widetilde{X}_l,\ \widetilde{Y}, L,\ and\ G} \epsilon^*$$

where $i \leq j,\ i,j,l = 1,2,\ m,n,k = 1,\ldots,4$, subject to $\epsilon^* > 0, P_{ii} > 0, Q_{nn} > 0, G > 0$, and LMIs in (3.5), (3.6).

Remark: The result in **Theorem 3.1** finally turns into **OP1**, which is solved by optimizing γ. While results in **Theorem 3.2** are obtained by solving **OP2**, which optimizes the stability upper bound $\frac{1}{\epsilon^*}$.

4 A NUMERICAL EXAMPLE

In this section, a numerical example is presented to illustrate the effectiveness of the proposed results.

Example: Consider a discrete-time singularly perturbed system described by (2.3) with

$$A_\epsilon = \begin{bmatrix} 1 + 0.2129\epsilon & 1.8140\epsilon \\ -0.1814 & 0.8179 \end{bmatrix}, B_{1\epsilon} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, B_{2\epsilon} = \begin{bmatrix} 0.1874\epsilon \\ 0.1812 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$D_{11} = \begin{bmatrix} 0.01 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, D_{12} = \begin{bmatrix} 0.31 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, C_2 = \begin{bmatrix} 0.4394 & 0.1372 \end{bmatrix}.$$

By solving the optimal problem **OP1**, we can get the static output feedback controller gain F_γ and the minimum H_∞ norm of the closed-loop system γ:

$$F_\gamma = -0.1078, \quad \gamma = 0.8557.$$

According to **OP2**, we can solve the corresponding LMIs with $\gamma = 1$. The obtained static output feedback controller gain F_ϵ and the stability upper bound of ϵ are:

$$F_\epsilon = -23.3354, \quad \frac{1}{\epsilon^*} = 0.2775.$$

Remark: Though the performance of the closed-loop system is not as good as the state feedback case, static output feedback controller plays a more important role in implemental sense with proper performance.

5 CONCLUSION

In this paper, H_∞ control problems for fast sampling singularly perturbed systems via static output feedback have been discussed. Rather than adopting the traditional design method of decomposing the original system into fast and slow subsystems, two LMI-based sufficient conditions have been given to guarantee the existence of static output feedback controllers and the asymptotical stability of the closed-loop system with a transfer function whose H_∞ norm is less than γ. The obtained LMI results can be solved easily with matlab. The proposed methods simplify the controller design procedure.

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43 ADMISSIBILITY ANALYSES FOR DYNAMIC INPUT-OUTPUT ECONOMIC MODELS WITH MULTIPLE DELAYS

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Abstract: Two admissibility conditions of two dynamic input-out economic models with multiple delays and with probably singular capital coefficient matrices are addressed respectively. By simple transformation, the economic models are written as two discrete-time singular systems with commensurate delays, then a delay-independent admissibility condition and a delay-dependent admissibility condition are presented and expressed in terms of linear matrix inequalities (LMIs) by Lyapunov approach. Two numerical examples for the dynamic input-out economic models are illustrated, which show the effectiveness of the proposed methods.

Key words: Dynamic input-output economic model; Multiple delays; Admissibility; Discrete-time singular system with commensurate delays.

1 INTRODUCTION

A dynamic Leontief model of a multi-sector economy (Leontief W. (1936)) has the form:

$$x(k) = Ax(k) + B[x(k+1) - x(k)] + y(k) \tag{1.1}$$

where k is a time index, $x(k) = [x_1(k), \dots, x_n(k)]^{\mathrm{T}}$ and $y(k) = [y_1(k), \dots, y_n(k)]^{\mathrm{T}}$ denote the total outputs and final net demands of n sectors respectively for the kth year. $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are respectively the technical coefficient matrix and the capital coefficient matrix, where $a_{ij} \geq 0, b_{ij} \geq 0$. System (1.1) is assumed to operate for the period $k = 0, 1, \dots, K-1$. (1.1) can be rewritten as:

$$Bx(k+1) = (I - A + B)x(k) - y(k)$$

I is an identity matrix. Commonly, some rows of the matrix B are only zero elements since not every sector produces capital goods (agriculture being a typical example in many models). Therefore, matrix B may be singular, $rank(B) \leq n$. Studies have been done for system (1.1) when B is singular in the literature. Stability analyses of dynamic input-output economic model were performed in (P.Tseng. (2001)) and (Z.Q.Luo and P. Tseng. (1992)). The problem of positive solutions of discrete dynamic Leontief input-output model has been studied in (P. Tseng and S. Yun. (2009)). However, these papers do not consider the case of multiple delays. This paper discusses the admissibility conditions of the Leontief model (1.1) with multiple delays and with possibly singular capital matrices.

2 ADMISSIBILITY CONDITION OF DYNAMIC INPUT-OUTPUT ECONOMIC MODEL (I) WITH MULTIPLE DELAYS

2.1 Dynamic Input-Output Economic Model (I) *with Multiple Delays*

Consider a dynamic input-output economic model with multiple delays described by:

$$x(k) = Ax(k) + \sum_{\tau=1}^{T} B_\tau v_\tau [x(k+\tau) - x(k+\tau-1)] + y(k) \tag{2.1}$$

where $B_\tau \in \mathbb{R}^{n\times n}$ is the capital coefficient matrix for the kth year investment and effectiveness after τ year. Matrix B_τ may be singular, $rank(B_\tau) \le n$. $v_\tau \in \mathbb{R}^{n\times n}$ is the decision coefficient matrix with a diagonal structure. The diagonal element v_τ^i represents the ratio of the $(k-\tau)$th year investment and increased production after τ year to the total increased production in the kth year $(\tau = 1, 2, \ldots, T)$ for the ith sector. Obviously, for a certain k year, there is

$$\begin{aligned} &0 \le v_\tau^i \le 1 \quad (i = 1, 2, \ldots, n,\ \tau = 1, 2, \ldots, T) \\ &\sum_{\tau=1}^{T} v_\tau^i = 1 \quad (i = 1, 2, \ldots, n) \end{aligned}$$

T is the maximum delay of the investment effectiveness, and n is the number of the sectors. In the market economy, the final net demands $y(k)$ are related to the wage level and the price level, and the total outputs $x(k)$ are decided by profits which are relevant to price and wage rate. Thus, it is reasonable to assume

$$y(k) = Wx(k) \tag{2.2}$$

where $W = [w_{ij}]_{n\times n}$ is a square matrix, with $w_{ij} \ge 0$. Replacing $y(k)$ in (2.1) with (2.2) and by simple identical transformation, we get

$$H_T x(k+T) = H_{T-1}x(k+T-1) + \sum_{\tau=1}^{T-2} H_\tau x(k+\tau) + Gx(k), \quad T \ge 3 \tag{2.3}$$

where

$$\begin{aligned} &H_T = B_T v_T, \quad H_{T-1} = B_T v_T - B_{T-1}v_{T-1}, \quad H_\tau = B_{\tau+1}v_{\tau+1} - B_\tau v_\tau, \\ &G = B_1 v_1 - A + I - W \end{aligned}$$

I is an identity matrix. Letting $k + T = k_0 + 1$, (2.3) is rewritten as:

$$H_T x(k_0+1) = H_{T-1}x(k_0) + \sum_{\tau=1}^{T-2} H_\tau x[k_0 - (T-1-\tau)] + Gx[k_0 - (T-1)], \quad T \ge 3 \tag{2.4}$$

It is noted that matrix H_T may be singular, so the economy model (2.4) is a discrete-time singular system with commensurate delays. We will derive an admissibility condition of model (2.4), which is very important to ensure the normal operation of the economic model.

2.2 Problem Statement and Preliminaries

Consider the discrete-time singular system with commensurate delays described by

$$\begin{aligned} Ex(k+1) &= Ax(k) + \sum_{i=1}^{m} A_i x(k-d_i) \\ x(k) &= \phi(k),\ k = -d_m, -d_m+1, \ldots, 0 \end{aligned} \tag{2.5}$$

where $x(k) \in \mathbb{R}^n$ is the state vector. $E \in \mathbb{R}^{n\times n}$ may be singular, $rank(E) = r \le n$. A, A_i are constant matrices with appropriate dimensions. The scalar $d_i = i$, $i = 1, 2, \ldots, m$ is the commensurate delay of the system. $\phi(k)$ is a compatible initial condition.

2.3 Delay-Independent Admissibility Condition for Singular System with Commensurate Delays

Theorem 2.1 *Discrete-time singular system (2.5) with commensurate delays is admissible if there exist matrices $P > 0$, $Q_i > 0$, $i = 1, 2, \ldots, m$, $P, Q_i \in \mathbb{R}^{n\times n}$ and a symmetric matrix $\Phi \in \mathbb{R}^{(n-r)\times(n-r)}$ satisfying*

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \cdots & \Theta_{1,m} & \Theta_{1,m+1} \\ * & \Theta_{22} & \cdots & \Theta_{2,m} & \Theta_{2,m+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & \Theta_{m,m} & \Theta_{m,m+1} \\ * & * & \cdots & * & \Theta_{m+1,m+1} \end{bmatrix} < 0 \tag{2.6}$$

where

$$\begin{aligned} &\Theta_{11} = A^{\mathrm{T}}XA - E^{\mathrm{T}}PE + Q_1 \\ &\Theta_{1,j} = A^{\mathrm{T}}XA_{j-1}, \quad j = 2, \ldots, m+1 \\ &\Theta_{i,i} = A_{i-1}^{\mathrm{T}}XA_{i-1} + Q_i - Q_{i-1}, \quad i = 2, \ldots, m \\ &\Theta_{i,j} = A_{i-1}^{\mathrm{T}}XA_{j-1}, \quad i = 2, \ldots, m, \; j = i+1, \ldots, m+1 \\ &\Theta_{m+1,m+1} = A_m^{\mathrm{T}}XA_m - Q_m \\ &X = P - S^{\mathrm{T}}\Phi S \end{aligned}$$

Matrix $S \in \mathbb{R}^{(n-r)\times n}$ is of full row rank and satisfying $SE = 0$.

Remark 2.1 *Theorem 2.1 presents a delay-independent admissibility condition for system (2.5). This criterion is also applicable to economic model (2.1). Compared with system (2.5), the element values of the coefficient matrices in economic model (2.1) are nonnegative due to economic significance.*

2.4 Numerical Example

Example 2.1 *Consider a dynamic input-output economic model with multiple delays described by:*

$$x(k) = Ax(k) + \sum_{\tau=1}^{3} B_\tau v_\tau[x(k+\tau) - x(k+\tau-1)] + y(k)$$

where

$$A = \begin{bmatrix} 1.7 & 0.12 \\ 0.1 & 1.2 \end{bmatrix}, B_1 = \begin{bmatrix} 5 & 0.1 \\ 0.5 & 4 \end{bmatrix}, B_2 = \begin{bmatrix} 2.5 & 0.5 \\ 1 & 2 \end{bmatrix}, B_3 = \begin{bmatrix} 5 & 2.5 \\ 0 & 0 \end{bmatrix},$$
$$v_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, v_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, v_3 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, y(k) = Wx(k), W = \begin{bmatrix} 0.4 & 0.5 \\ 0.8 & 1 \end{bmatrix}$$

By simple computation, the model is rewritten as a discrete-time singular system with commensurate delays described by:

$$Ex(k+1) = A_0x(k) + A_1x(k-1) + A_2x(k-2)$$

where

$$E = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, A_0 = \begin{bmatrix} -0.2 & 0.2 \\ -0.4 & -0.8 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0.18 \\ 0.3 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1 & -0.6 \\ -0.8 & -0.4 \end{bmatrix}$$

Assuming $S = [0, 1]$, by Theorem 2.1, LMI (2.6) is feasible. It means that the discrete-time singular system with commensurate delays is admissible. Thus, the economic model with multiple delays and with singular capital coefficient matrix is admissible.

3 ADMISSIBILITY CONDITION OF DYNAMIC INPUT-OUTPUT ECONOMIC MODEL (II) WITH MULTIPLE DELAYS

3.1 Dynamic Input-Output Economic Model (II) with Multiple Delays

Consider a dynamic input-output economic model with multiple delays described by:

$$x(k) = Ax(k) + \sum_{\tau=1}^{T} B_\tau v_\tau[x(k+\tau d) - x(k+(\tau-1)d)] + B_0v_0[x(k+Td+1) - x(k+Td)] + y(k) \tag{3.1}$$

where d is a positive integer, presenting the delay. B_τ, $B_0 \in \mathbb{R}^{n\times n}$ are the capital coefficient matrices for the kth year investment and effectiveness after τd $(\tau = 1, 2, \ldots, T)$, $Td+1$ year, respectively. Matrix B_τ, B_0 may be singular, $rank(B_\tau) \le n$, $rank(B_0) \le n$. v_τ, $v_0 \in \mathbb{R}^{n\times n}$ are the decision coefficient matrices with diagonal structures. The diagonal element v_τ^i, v_0^i represent the ratio of the $(k-\tau d)$th, $(k-Td-1)$th year investment and increased production after τd, $Td+1$ year to the total increased production in the kth year $(\tau = 1, 2, \ldots, T)$ for the ith sector. Obviously, for a certain k year, there is

$$0 \le v_\tau^i \le 1,\ 0 \le v_0^i \le 1 \quad (i = 1, 2, \ldots, n,\ \tau = 1, 2, \ldots, T)$$
$$\sum_{\tau=1}^{T} v_\tau^i + v_0^i = 1 \quad (i = 1, 2, \ldots, n)$$

T is the maximum multiple of delay of the investment effectiveness, and n is the number of the sectors. It is assumed $y(k) = Wx(k)$ similarly as (2.2). Substituting $y(k) = Wx(k)$ into (3.1) and by simple identical transformation, we get

$$\mathscr{B}_0 x(k+Td+1) = \mathscr{B}_T x(k+Td) + \sum_{\tau=1}^{T-1} \mathscr{B}_\tau x(k+\tau d) + Gx(k) \tag{3.2}$$

where

$$\mathscr{B}_0 = B_0 v_0, \quad \mathscr{B}_T = B_0 v_0 - B_T v_T, \quad \mathscr{B}_\tau = B_{\tau+1} v_{\tau+1} - B_\tau v_\tau, \quad G = B_1 v_1 - A + I - W$$

I is an identity matrix. Letting $k + Td = k_0$, (3.2) is rewritten as:

$$\mathscr{B}_0 x(k_0+1) = \mathscr{B}_T x(k_0) + \sum_{\tau=1}^{T-1} \mathscr{B}_\tau x(k_0 - (T-\tau)d) + Gx(k_0 - Td) \tag{3.3}$$

It is noted that matrix $\mathscr{B}_0$ may be singular, so the economy model (3.3) is a discrete-time singular system with commensurate delays. Specially, if $d = 1$, model (II) is turned into model (I).

3.2 Problem Statement and Preliminaries

Consider the discrete-time singular system with commensurate delays described by

$$Ex(k+1) = Ax(k) + \sum_{i=1}^{m} A_i x(k-d_i) \tag{3.4}$$
$$x(k) = \phi(k),\ k = -d_m, -d_m+1, \ldots, 0$$

where $x(k) \in \mathbb{R}^n$ is the state vector. $E \in \mathbb{R}^{n\times n}$ may be singular, $rank(E) = r \le n$. A, A_i are constant matrices with appropriate dimensions. The scalar $d_i = id$ $(i = 1, 2, \ldots, m)$ is the commensurate delay where d is a positive integer presenting the delay. $\phi(k)$ is a compatible initial condition.

3.3 Delay-Dependent Admissibility Condition for Singular System with Commensurate Delays

Theorem 3.1 *Given positive integer d, discrete-time singular system (3.4) with commensurate delays is admissible if there exist matrices $P > 0$, $Q_i > 0$, $R_i > 0$, $i = 1, 2, \ldots, m$, $P, Q_i, R_i \in \mathbb{R}^{n\times n}$ and a symmetric matrix $\Phi \in \mathbb{R}^{(n-r)\times(n-r)}$ such that*

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \cdots & \Theta_{1,m} & \Theta_{1,m+1} \\ * & \Theta_{22} & \cdots & \Theta_{2,m} & \Theta_{2,m+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & \Theta_{m,m} & \Theta_{m,m+1} \\ * & * & \cdots & * & \Theta_{m+1,m+1} \end{bmatrix} < 0 \tag{3.5}$$

where

$$\Theta_{11} = A^{\mathrm{T}}XA - E^{\mathrm{T}}PE + Q_1 - E^{\mathrm{T}}R_1E + d^2(A-E)^{\mathrm{T}}\sum_{f=1}^{m} R_f(A-E)$$

$$\Theta_{12} = A^{\mathrm{T}}XA_1 + d^2(A-E)^{\mathrm{T}}\sum_{f=1}^{m} R_fA_1 + E^{\mathrm{T}}R_1E$$

$$\Theta_{1,i} = A^{\mathrm{T}}XA_{i-1} + d^2(A-E)^{\mathrm{T}}\sum_{f=1}^{m} R_fA_{i-1}, \qquad i = 3,\ldots,m+1$$

$$\Theta_{i,i} = A_{i-1}^{\mathrm{T}}XA_{i-1} + Q_i - Q_{i-1} + d^2A_{i-1}^{\mathrm{T}}\sum_{f=1}^{m} R_fA_{i-1} - E^{\mathrm{T}}R_{i-1}E - E^{\mathrm{T}}R_iE$$

$$\Theta_{i,i+1} = A_{i-1}^{\mathrm{T}}XA_i + d^2A_{i-1}^{\mathrm{T}}\sum_{f=1}^{m} R_fA_i + E^{\mathrm{T}}R_iE$$

$$\Theta_{i,j} = A_{i-1}^{\mathrm{T}}XA_{j-1} + d^2A_{i-1}^{\mathrm{T}}\sum_{f=1}^{m} R_fA_{j-1}, \qquad i = 2,\ldots,m,\ j = i+2,\ldots,m+1$$

$$\Theta_{m+1,m+1} = A_m^{\mathrm{T}}XA_m - Q_m + d^2A_m^{\mathrm{T}}\sum_{f=1}^{m} R_fA_m - E^{\mathrm{T}}R_mE$$

$$X = P - S^{\mathrm{T}}\Phi S$$

Matrix $S \in \mathbb{R}^{(n-r)\times n}$ is of full row rank and satisfying $SE = 0$.

Remark 3.1 *Theorem 3.1 shows a delay-dependent admissibility condition for system (3.4). This condition is applicable to economic model (3.1). Specially, it is noted that with d = 1, system (3.4) becomes system (2.5). Correspondingly, Theorem 2.1 is derived from Theorem 3.1.*

3.4 Numerical Example

Example 3.1 *Consider a dynamic input-output economic model with multiple delays described by:*

$$x(k) = Ax(k) + \sum_{\tau=1}^{2} B_\tau v_\tau[x(k+\tau d) - x(k+(\tau-1)d)] + B_0v_0[x(k+2d+1) - x(k+2d)] + y(k)$$

where

$$A = \begin{bmatrix} 4 & 0 \\ 0.2 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 8.25 & 0 \\ 1.25 & 0.25 \end{bmatrix}, B_2 = \begin{bmatrix} 6 & 0 \\ 0 & 0.6 \end{bmatrix}, B_0 = \begin{bmatrix} 20 & 0 \\ 0 & 0 \end{bmatrix},$$
$$v_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, v_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, v_0 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$
$$y(k) = Wx(k), W = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}, d = 2$$

By simple computation, the model is rewritten as a discrete-time singular system with commensurate delays described by:

$$Ex(k+1) = A_0x(k) + A_1x(k-2) + A_2x(k-4)$$

where

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, A_0 = \begin{bmatrix} -1 & 0 \\ 0 & -0.3 \end{bmatrix}, A_1 = \begin{bmatrix} -0.3 & 0 \\ -0.5 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} -0.2 & 0 \\ -0.2 & 0 \end{bmatrix}$$

Assuming $S = [0,\ 1]$, by Theorem 3.1, LMI (3) is feasible. Thus, the economic model with multiple delays and with singular capital coefficient matrix is admissible.

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44 THE ACQUISITION OF MINIMAL DECISION RULES BASED ON DISCERNIBILITY FUNCTION FOR ORDERED INFORMATION SYSTEMS

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Abstract: In an ordered decision information system, objects are preference-ordered by the condition criteria, and also they are assigned to decision classes with preference-ordered. In practice, it is always assumed that the "dominance principle" should be observed in an ordered decision information system, i.e., an object not worse than the other on all the considered criteria should be assigned to a better decision class than others. However, because of limited discriminatory power of the criteria and hesitation of the decision maker, the "dominance principle" is often violated for some objects (S. Greco, B. Matarazzo, R. Slowinski, 2002), and in this case, one calls the ordered decision information system inconsistent. Since the Pawlak rough set model, which concerns the indiscernibility between objects, can not cope with the inconsistency in ordered decision information systems, the dominance-based rough set approach (DRSA) was proposed to deal with inconsistency and draw "at least" and "at most" decision rules (Greco, Matarazzo, and Slowinski, 1998, 2001, 2002).

Nowadays, the most representative approaches and software system JAMM (http://idss.cs.put.poznan.pl, 2006) for computing minimal decision rules in ordered decision information systems employ the ideas of system LERS (Learning from Examples based on Rough Sets) (J. W. Grzymala-Busse, 1992). Motivated by the ideas in LEM2 algorithm (J. W. Grzymala-Busse, 1992) and the MODLEM algorithm (J. Stefanowski, 1998) developed for Pawlak rough set model, a heuristic algorithm, called DOMLEM(Greco, Matarazzo, and Slowinski, 2001, 2002), was proposed to induce minimal decision rules from the ordered decision information systems. The DOMLEM algorithm is of polynomial time complexity, and it can obtain a minimal set of the minimal certain (possible) decision rules, which can cover all the objects in the lower (upper) approximation of the upward (downward) unions of decision classes. However, it can not compute all the minimal decision rules. Two algorithms, which are used respectively to induce all rules and subset of all rules, were proposed (S. Greco, R. Slowinski, J. Stefanowski1 and M. Zurawski, 2004; J. Stefanowski, 2001; M. Zurawski, 2001), and their extended versions are included in the software systems JAMM. The former algorithm can induce all minimal decision rules for a given data table, but its computational complex is exponential with respect to the number of attributes. The last algorithm can satisfy user's requirements, e.g., sufficiently high strength and confidence, or with a limited number of elementary conditions, but its computational cost is higher than that of the DOMLEM.

As we know, in Pawlak rough set model, discernibility function approach was successfully used in drawing minimal decision rules. However, up to now, we have not found any computational approach for minimal decision rules by using discernibility function and Boolean reasoning technique for ordered decision information systems. In this paper, we will employ discernibility function to compute minimal "at least" decision rules and minimal "at most" decision rules.

Let us give a logical interpretation of the minimal decision rules. In a symbolic decision information system $S = (U, C \cup \{d\}, V, f)$, the rule $\wedge_{b\in B}(b, = v_b) \to (d, = k)$ is called minimal one if it is true in S and $\wedge_{q\in Q}(q, = v_q) \to (d, = k)$ is false in S for any sub-formula $\wedge_{q\in Q}(q, = v_q)$ of $\wedge_{b\in B}(b, = v_b)$ with

$Q \subset B$ (J. Bazan, M. Szczuka, M. Wojna and M. Wojnarski, 2004; Z. Pawlak, 2007). For two decision rules $\wedge_{p \in P}(p, = v_p) \to (d, = s)$ and $\wedge_{q \in Q}(q, = v_q) \to (d, = t)$, if the objects covered by the former rule are also covered by the latter rule, then the former rule can be implied by the latter one. And in this case, $s = t$, $P \supseteq Q$, and $v_q = u_q$ for any $q \in Q$. So, one can see that, two rules possess implication relation if and only if they have the same decision part and the condition part of one rule is a sub-formula of the other one. Therefore, the minimal decision rule is one which can not be implied by any other rules. By constructing a decision-relative discernibility function of object x, and computing all prime implicants of the discernibility function, one can obtain the reducts of x and get the optimal decision rules corresponding to x (Z. Pawlak, 2007), where the reduct of x is the minimal set of condition attributes satisfying $[x]_B = [x]_C$, and the optimal decision rule is actually a minimal decision rule.

In the dominance-based rough set approach, the "at least" and "at most" decision rules, which are presented respectively as the form of $\wedge_{b \in B}(b, \geq v_b) \to (d, \geq k)$ and $\wedge_{b \in B}(b, \leq v_b) \to (d, \leq k)$, are implications. The minimal "at least" ("at most") decision rule was defined as such an implication that there is no other implication with a left-hand side (LHS) of at least the same weakness (in other words, a rule using a subset of elementary conditions and/or weaker elementary conditions) and a right-hand side (RHS) of at least the same strength (in other words, a rule assigning objects to the same union or sub-union of classes (S. Greco, B. Matarazzo and R. Slowinski, 2002). We give a logical interpretation of the minimal "at least" decision rule as follows.

For two "at least" decision rules $\wedge_{q \in Q}(q, \geq v_q) \to (d, \geq t)$ and $\wedge_{p \in P}(p, \geq v_p) \to (d, \geq s)$, we call the former one implies the latter one, if the following three conditions are satisfied:

C1: $Q \subseteq P$;

C2: $\forall q \in Q, v_p \leq u_p$;

C3: $t \geq s$.

Then, the minimal rule can be conceived as a rule that can not be implied by any other rules.

The implication relationship between the "at least" decision rules is obviously more complicated than that between the decision rules in Pawlak rough set model. We can define and compute the optimal decision rules in ordered decision information systems, as did in the Pawlak rough set model established for symbolic decision information systems. Unfortunately, different from the case in the Pawlak rough set model, in DRSA, the optimal decision rules are not necessarily minimal decision rules. So, to compute the minimal decision rules for ordered decision information systems, we will propose a method to compute the minimal decision rules by deleting the redundant decision rules.

For an ordered decision information system $S = (U, C \cup \{d\}, V, f)$, in order to optimize the "at least" decision rule $\wedge_{c \in C}(c, \geq c(x_i)) \to (d, \geq s)$ generated by $x_i \in \underline{R_C^{\succ}}(Cl_s^{\geq})$, one should simplify as much as possible its conditional part by deleting some conjunctive terms while keeping its decision part unchanged, i.e., delete as many conjunctive terms in $\wedge_{c \in C}(c, \geq c(x_i))$ as possible with the constraint that the set B, which consists of condition attributes in the remainder $\wedge_{b \in B}(b, \geq b(x_i))$, satisfies $\min[x_i]_B^{\succ} \geq s$. We call such B as a reduct of x_i with respect to $(d, \geq s)$. $\wedge_{c \in B}(c, \geq c(x_i)) \to (d, \geq s)$ is called an optimal decision rule of $\wedge_{c \in C}(c, \geq c(x_i)) \to (d, \geq s)$, and it satisfies the condition C1 of the minimal decision rule. We can demonstrate that, the reduct of x_i with respect to $(d, \geq s)$ can be computed by constructing the discernibility function $\Delta_s^{\succ}([x_i]_C^{\succ}) = \wedge_{d(y) < s}(\vee \alpha^{\succ}(y, x_i))$.

To discard those optimal decision rules which can be implied by others, we propose a new concept called rule-vector. For a rule $r([x_i]_B^{\succ}, s)$: $\wedge_{b \in B}(b, \geq b(x_i)) \to (d, \geq s)$, we define a corresponding $n+1$ dimensional vector, which is denoted as: $V([x_i]_B^{\succ}, s) = (u_1, u_2, \cdots, u_n, s)$, where $u_l = c_l(x_i)$ if $c_l \in B$, and u_l is denoted as "*" if $c_l \notin B$, for $1 \leq l \leq n$. We call $V([x_i]_B^{\succ}, s)$ a rule-vector corresponding to the rule $r([x_i]_B^{\succ}, s)$. By computing minimal rule-vectors of the set of all rule-vector corresponding to the optimal decision rules, we can obtain the minimal decision rules which satisfy both condition C1, C2 and C3. This can be done by comparing each $V([x_i]_B^{\succ}, s)$ with every other $V([x_j]_P^{\succ}, s)$ and deleting the larger one if the two rule-vectors can be compared.

In a similar way, we can compute all the minimal "at most" decision rules.

Experimental results show that our approach costs much less time than the famous JAMM algorithm does in computing minimal "at least" and "at most" decision rules, especially for the ordered decision information systems containing more condition attributes.

Key words: Rough set; Ordered information system; Dominance relation; Decision rule; Discernibility function.

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45 TO LEARN THE UNCORRELATED AND DISCRIMINANT COLOUR SPACE FOR FACIAL EXPRESSION RECOGNITION

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Abstract: Recent research has shown improved performance by embedding the colour information into facial expression recognition. However, the RGB colour space may not always be the most desirable space for representing the colour information. This paper addresses the problem of how to learn an optimum colour space for facial expression recognition based on the given training sample set. There are two typical learning colour spaces which have been used for face recognition. The uncorrelated colour space (UCS) decorrelates the three component images of RGB colour space using principal component analysis, and the discriminant colour space (DCS) creates three new component images by applying discriminant analysis. We will investigate these two colour spaces for facial expression recognition. First, colour face images are transformed into these colour spaces and represented by concatenating their component vectors. Secondly, facial expression recognition is achieved by utilizing Fisher Linear Discriminant (FLD) and support vector machines. We test these colour spaces with two classifiers on Curtin-Kinect dataset in three ways: person-independent, person-dependent and crossing image datasets. The results reveal that the uncorrelated colour space is more effective than RGB space in colour information representation for facial expression recognition, but the discriminant colour space is not as expected.

Key words: Color facial expression recognition; FLD; DCS.

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46 CONTENT BASED IMAGE RETRIEVAL USING LOCAL DIRECTIONAL PATTERN AND COLOR CUMULATIVE HISTOGRAM

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Abstract: Content based image retrieval (CBIR) has many important practical applications in database management and computer vision, especially due to ever-increasing of the easily captured digital color images. In fact, texture, color and shape are three important features in a CBIR system. So the main focus of CBIR research is to develop feature extraction method in terms of expressing effective texture, color and shape features in a similar way of human visual perception. In this paper, a new feature extraction method is developed by using Local Directional Pattern (LDP) and color cumulative histogram which not only can capture color, texture and shape properties, but also utilize different color space effectively due to special properties in these two color spaces. First the RGB image is converted into HSV model, and LDP descriptor is used to describe visual texture and geometrical features using the V (value) image. Then color texture is extracted from color cumulative histogram in RGB color space with color quantization. At last these features are combined as final image features for image retrieval with different distance measures. The WANG image database is used to validate the proposed method effectively. The performance has been evaluated in comparison with some existing methods, and the results demonstrate that the proposed approach is more effective for image retrieval and can be used directly on natural images without any segmentation and preprocessing.

Key words: Color Image retrieval; LDP; HSV; RGB.

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47 MODIFIED PEDESTRIAN MODEL WITH ITS APPLICATION TO EXIT DESIGN

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Abstract: Pedestrian dynamics is an important issue for the design of evacuation strategy in emergency situation. One of the central issues in emergency management is how to establish an appropriate model for pedestrians. Therefore, it is very necessary to investigate various pedestrian models and come up with a new one with high mimicry of real pedestrian dynamics. Typical social force model is widely studied in recent years, concerning more about psychological and physical factors. This paper proposed a modified social force model, considering the socio-psychological force has a relationship with pedestrians distance, vision field and degree of excitement. The proposed model can significantly improve the computational efficiency in the simulation process. Numerical simulations are also carried out to analyze the effect of the different exit width over different pedestrian densities using the modified social force model, which indicates a moderate exit width should be chosen for different situations to realize the expected effect.

Key words: Social force model; Pedestrian dynamics; Pedestrian evacuation.

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48 DISSIPATIVE CONTROL FOR A CLASS OF LINEAR OUTPUT TRACKING SYSTEMS

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Abstract: In this paper, a class of tracking systems is transformed into the augmented form, and then, the problem of dissipative control and passive control for the augmented system is investigated. An improved storage function is constructed and the subsequent analysis provides some new sufficient conditions in the form of LMIs for nominal and time-delay representations. Dissipative state feedback controllers are designed. A numerical simulation example is given to illustrate the effectiveness of the theoretical result.

Key words: Tracking systems; Dissipative control; State feedback controller; LMIs.

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49 UNIFIED OPTIMIZATION OF H_∞ INDICES AND STABILITY BOUNDS FOR SINGULARLY PERTURBED SYSTEMS

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Abstract: In this paper, unified optimization problem for the stability upper bound ϵ^* and the H_∞ index γ based on state feedback is considered for singularly perturbed systems. First, a sufficient condition for the existence state feedback controller is presented in terms of linear matrix inequalities such that the resulting closed-loop system is asymptotically stable if $0 < \epsilon < \epsilon^*$ and guarantees H_∞ performance index. Furthermore, a new algorithm to optimize these two indices simultaneously is proposed based on Nash game theory which transfers multi-objective problem into a single objective problem and develops a new way to determine the objective weights. Then an optimal state feedback controller can be obtained. Finally, some numerical examples are provided to demonstrate the effectiveness and correctness of the proposed results.

Key words: Singularly perturbed systems; Stability; H_∞ performance; State feedback control; Linear matrix inequality (LMI); Nash game approach.

1 INTRODUCTION

Singularly perturbed systems often occur naturally in many branches of applied mathematics, see (Kumar M. (2011)). Some popular approaches are adopted to deal with these systems, like the reduced technique (Cao L. (2005)), separate designs for slow and fast subsystems and descriptor systems approach(Zhong N. (2007)). Optimization problems for singular perturbed system have become popular research topics in recent years. Here, we only discuss the optimal control for stability upper bound ϵ^* and H_∞ index. (Liu W. (1996)) gives a characterization for upper bound ϵ^* of the parasitic parameter ϵ. (Tan W. (1998)) derived a set of ϵ-independent sufficient and necessary conditions for the H_∞ control problem in a different way (via dynamic output feedback). All these results are "one-player" optimal problem that has only one performance considered. (Xu S. (2009)) gave some sufficient conditions for optimizing H_∞ norm bound γ and stability upper bound ϵ, which searched optimal γ for the given ϵ and vice versa. The proper algorithm research is an important task in future. (Zhang G. (2012)) considered $\mathrm{H}_2/\mathrm{H}_\infty$ optimal problem for descriptor system based on Nash game approach, which provided some effective ideas for our subject.

In this paper, unified optimization problem for the upper bound ϵ^* and the H_∞ index γ based on state feedback is studied. A sufficient condition for the existence state feedback controller is presented in terms of LMIs such that the resulting closed-loop system is asymptotically stable if $0 < \epsilon < \epsilon^*$ and also guarantees H_∞ performance index. The main contribution of this paper is to derive an algorithm to optimize these two indices based on Nash game theory. We translate multi-objective problem into single objective problem and develop a new way to determine the objective weights. Finally, some numerical examples are provided to demonstrate the effectiveness and feasibility of the proposed results.

2 PROBLEM FORMULATION

Consider a linear time-invariant singularly perturbed system described by

$$\left\{\begin{array}{l} E_\epsilon \dot{x}(t) = Ax(t) + B_1u(t) + B_2w(t) \\ z(t) = C_1x(t) + D_1u(t) \\ y(t) = C_2x(t) + D_2w(t) \end{array}\right. \tag{2.1}$$

where $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $E_\epsilon = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \epsilon I_{n_2} \end{bmatrix}$ and the scalar $\epsilon > 0$ is a small singular perturbed parameter. $x(t) \in \mathbf{R}^n$ is the state vector with $x_1(t) \in \mathbf{R}^{n_1}$ (slow state) and $x_2(t) \in \mathbf{R}^{n_2}$, $n_1 + n_2 = n$ (fast state). $u(t) \in \mathbf{R}^m$ is the input vector, $w(t) \in \mathbf{R}^l$ is the disturbance input which belongs to $L_2[0, \infty)$. And $z(t), y(t) \in \mathbf{R}^p$ are the controlled output and the measurement output respectively.

For the system (2.3) consider the state feedback control law

$$u(t) = Kx(t) \tag{2.2}$$

where $K \in \mathbf{R}^{m\times n}$ is the control gains to be solved.

Substituting (2.4) into (2.3), one can obtain the resulting closed-loop system given by

$$\left\{\begin{array}{l} E(\epsilon)\dot{x}(t) = (A + B_1K)x(t) + B_2w(t) \\ z(t) = (C_1 + D_1K)x(t) \\ y(t) = C_2x(t) + D_2w(t) \end{array}\right. \tag{2.3}$$

The purpose of this paper is to design state feedback controller (2.4) for the system (2.3), such that for any $\epsilon \in (0, \epsilon^*]$ the closed-loop system (2.3) is asymptotically stable and satisfies the H_∞ performance γ.

3 MAIN RESULTS

3.1 State Feedback Controller Design for Singularly Perturbed Systems

Theorem 3.1 *The closed-loop system (2.3), which is constituted by system (2.3) and state feedback controller (2.4), will be asymptotically stable and guarantees H_∞ performance index $\gamma > 0$ for any $\epsilon \in (0, \epsilon^*]$, if there exist matrices $P_i (i = 1, 2, 3, 4, 5)$ and $P_i = P_i^{\mathrm{T}} (i = 1, 2, 3, 4)$ and K such that the following matrix inequalities hold*

$$P_1 > 0, \begin{bmatrix} P_1 + \epsilon P_3 & \epsilon P_5^{\mathrm{T}} \\ P_5 & P_2 + \epsilon P_4 \end{bmatrix} > 0, \begin{bmatrix} P_1 + \epsilon P_3 & \epsilon P_5^{\mathrm{T}} \\ \epsilon P_5 & \epsilon P_2 + \epsilon^2 P_4 \end{bmatrix} > 0 \tag{3.1}$$

$$\begin{bmatrix} P_0^{\mathrm{T}}\hat{A} + \hat{A}^{\mathrm{T}}P_0 & P_0^{\mathrm{T}}B_2 & (C_1 + D_1K)^{\mathrm{T}} \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{3.2}$$

$$\begin{bmatrix} P_0^{\mathrm{T}}\hat{A} + \hat{A}^{\mathrm{T}}P_0 & P_0^{\mathrm{T}}B_2 & (C_1 + D_1K)^{\mathrm{T}} \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} + \epsilon \begin{bmatrix} \hat{P}_0^{\mathrm{T}}\hat{A} + \hat{A}^{\mathrm{T}}\hat{P}_0 & \hat{P}_0^{\mathrm{T}}B_2 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0, \tag{3.3}$$

where $*$ is denoted the symmetric terms in a symmetric matrix, $P_0 = \begin{bmatrix} P_1 & 0 \\ P_5 & P_2 \end{bmatrix}$, $\hat{P}_0 = \begin{bmatrix} P_3 & P_5^{\mathrm{T}} \\ 0 & P_4 \end{bmatrix}$, $\hat{A} = A + B_1K$.

Proof: First of all, we define $P(\epsilon) = P_0 + \epsilon\hat{P}_0$, then

$$E^{\mathrm{T}}(\epsilon)P(\epsilon) = \begin{bmatrix} P_1 + \epsilon P_3 & \epsilon P_5^{\mathrm{T}} \\ \epsilon P_5 & \epsilon P_2 + \epsilon^2 P_4 \end{bmatrix} \tag{3.4}$$

Because of (3.4), it is obviously that $E^{\mathrm{T}}(\epsilon)P(\epsilon) = P^{\mathrm{T}}(\epsilon)E(\epsilon) > 0$. In this case, we choose a Lyapunov Functional

$$V(t,\epsilon) = x^{\mathrm{T}}(t)E^{\mathrm{T}}(\epsilon)P(\epsilon)x(t) > 0 \tag{3.5}$$

Then, the time-derivative of $V(t,\epsilon)$ along the solution of (2.3) gives

$$\dot{V}(t,\epsilon) = x^{\mathrm{T}}(t)(P^{\mathrm{T}}(\epsilon)(A+B_1K) + (A+B_1K)^{\mathrm{T}}P(\epsilon))x(t) + x^{\mathrm{T}}(t)P^{\mathrm{T}}(\epsilon)B_2w(t). \tag{3.6}$$

When $w(t) = 0$, substituting P_0 and $\hat{P}_0$ into (3.6), from matrix inequalities (3.5) and (3.3), we can obtain

$$\dot{V}(t,\epsilon) < 0.$$

Therefore the resulting closed-loop system (5) is asymptotically stable for any $\epsilon \in (0,\epsilon^*]$.

Secondly, interpret $\|z(t)\|_2 < \gamma\|w(t)\|_2$ into integral form. And it is equivalent to

$$J_{zw} = \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(t,\epsilon)]\mathrm{d}t + V(t,\epsilon)|_{t=0} - V(t,\epsilon)|_{t=\infty}. \tag{3.7}$$

Since $V(t,\epsilon)|_{t=0} = 0$ under zero initial condition and $V(t,\epsilon)|_{t=\infty} \geq 0$, we can derive

$$J_{zw} \leq \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(t,\epsilon)]\mathrm{d}t = \int_0^\infty \xi^{\mathrm{T}}(t)\Pi\xi(t)\mathrm{d}t. \tag{3.8}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \Pi = \begin{bmatrix} P(\epsilon)^{\mathrm{T}}\hat{A} + \hat{A}^{\mathrm{T}}P(\epsilon) + (C_1+D_1K)^{\mathrm{T}}(C_1+D_1K) & P^{\mathrm{T}}(\epsilon)B_2 \\ * & -\gamma^2 I \end{bmatrix}$$

Take P_0, $\hat{P}_0$ into (3.8). Since the matrix inequalities (3.5) and (3.3) hold, then $\Pi < 0$ hold. □

Theorem 3.2 *The closed-loop system (2.3) will be asymptotically stable and guarantees H_∞ performance index $\gamma > 0$ for any $\epsilon \in (0,\epsilon^*]$, if there exist matrices $P_i (i = 1,2,3,4,5)$ with $P_i = P_i^{\mathrm{T}} (i = 1,2,3,4)$ and V such that (3.4) and the following LMIs hold*

$$\begin{cases} \varphi_1 < 0, \\ \varphi_1 + \epsilon\varphi_2 < 0, \end{cases} \tag{3.9}$$

where P_0 and $\hat{P}_0$ are defined as in the Theorem 3.1.

$$\varphi_1 = \begin{bmatrix} P_0^{\mathrm{T}}A^{\mathrm{T}} + AP_0 + V^{\mathrm{T}}B_1^{\mathrm{T}} + B_1V & B_2 & P_0^{\mathrm{T}}C_1 + V^{\mathrm{T}}D_1^{\mathrm{T}} \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix}, \varphi_2 = \begin{bmatrix} \hat{P}_0^{\mathrm{T}}A^{\mathrm{T}} + A\hat{P}_0 & 0 & \hat{P}_0^{\mathrm{T}}C_1^{\mathrm{T}} \\ * & 0 & 0 \\ * & 0 & 0 \end{bmatrix}.$$

In this case, the controller gain is given by $K(\epsilon) = V(P_0 + \epsilon\hat{P}_0)^{-1}$.

In order to obtain the optimal $\{\epsilon,\gamma\}$, we should solve the minimization problem described by

$$\begin{array}{c} \min\gamma,\ \max\epsilon \\ \text{s.t.}(3.4),(3.9) \end{array} \tag{3.10}$$

4 OPTIMIZATION ALGORITHM

The optimal control problem can be converted into finding the Nash equilibria point u^*. u^* can be regarded as trade-off or proper controller such that ϵ and γ achieve "win-win" situation.

- Algorithm 1 is given to obtain a set of $\{\epsilon_i,\gamma_i\}$ by solving two convex optimization problems.

  Algorithm 1

  Step 1: Set $i = 1$ and $\gamma_i = \gamma_{max}$, $\epsilon_i = \epsilon^{min}$ ($\gamma^{max} > 0$ and $\epsilon^{min} > 0$ are real constant).

  Step 2: Let $i = i+1$, $\epsilon_i = \epsilon_{i-1}$, solving the optimization problem (OP1):

$$\begin{array}{c} \min\gamma_i, \\ \text{s.t.}(3.4),(3.9) \end{array} \tag{4.1}$$

Step 3: Let $i = i + 1$, $\gamma_i = \gamma_{i-1}$, solving the optimization problem (OP2):

$$\begin{array}{l} \min -\epsilon_i, \\ \text{s.t.}(3.4), (3.9) \end{array} \tag{4.2}$$

Step 4: If $\epsilon_i > \epsilon_{i-1}$, $\gamma_i < \gamma_{i-1}$ and $i < N$ (N is the upper bound for the iteration number), then go back to Step 2. Otherwise, stop.

- Then go to normalized processing.

Algorithm 2

There are n strategies set $K = \{K_1, K_2, \cdots, K_n\}$. Let the objects set $\{\epsilon, \gamma\}$ or $A = \{a_j\}, j = 1, 2$, that is $a_1 = \epsilon, a_2 = \gamma$. Then the object matrix is $B = [b_{ij}]_{n\times 2}$, where b_{ij} means that the object a_j of the strategy K_i. Define the membership matrix $R = \{r_{ij}\}, i = 1, 2, \cdots, n, j = 1, 2$, and $r_{ij} = \frac{b_{ij} - b_{i\mathbf{min}}}{b_{i\mathbf{max}} - b_{i\mathbf{min}}}$. Determine the desired method $E = \{e_j\}, j = 1, 2$, and $e_1 = \mathbf{max}\{r_{i\mathbf{1}}\}, e_2 = \mathbf{min}\{r_{i\mathbf{2}}\}, 1 \le i \le n$.

We select $f_i(\mu) = \sum_{j=1}^{2} \mu_j (e_j - r_{ij})$, where μ_1, μ_2 is the weight vectors for the objects. For the given weight vectors μ_j, the strategy K_i is optimal when the $f_i(\mu)$ achieves minimum.

$$\begin{array}{l} \min\{\sum_{i=1}^{n} f_i(\mu)\}, \\ \text{s.t.} \left\{ \begin{array}{l} \sum_{j=1}^{2} \mu_j^2 = 1, \\ \mu_j \ge 0. \end{array} \right. \end{array} \tag{4.3}$$

Using the nonlinear programming principle, introduce the Lagrange function. Then the expression for μ_j is obtained. Compute $f_i(\mu)$, then the minimum $f_i^*(\mu)$ can be found and the corresponding strategy K_i^* is the optimal controller gain. Now, ϵ_i^* and γ_i^* is optimal value which we expected.

5 NUMERICAL EXAMPLE

Example 1: [Zhong N. (2007)] Consider a singularly perturbed system (2.3) with the the following given parameter

$$E_\epsilon = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{bmatrix}, A = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0.1 & 0 \\ 1 & 0 & -0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}, D_1 = D_2 = 0.$$

From Tab 2.1, one can see that the H_∞ performance index γ with those obtained in this paper is less conservative.

Table 5.1 Comparison between Zhong N. (2007) and Theorem 3.2

ϵ	γ	
	Zhong N. (2007)	Theorem 3.2
0.001	0.1692	0.1150
0.01	0.2384	0.0987
0.05	4.7725	0.0451
0.1	None	0.0285

Example 2: Consider magnetic tape control system [Nguyen T. (2012)] with the system matrices given by

$$E_\epsilon = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \epsilon & 0 \\ 0 & 0 & 0 & \epsilon \end{bmatrix}, A = \begin{bmatrix} 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.345 & 0 \\ 0 & -0.524 & -0.465 & 0.262 \\ 0 & 0 & 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.11 & 0.12 & 0.13 & 0.1 \end{bmatrix}, C_2 = \begin{bmatrix} 1.8 & 0 & 3.7 & 0 \end{bmatrix}, D_1 = 0.1, D_2 = 0.1.$$

- In [Nguyen T. (2012)], one sets $\epsilon = 0.1$. Using the result Theorem 3.2, we get the H_∞ performance index

$$\gamma = 0.2006, K = \begin{bmatrix} -1.1 & -1.2 & -1.3 & -1 \end{bmatrix}.$$

- Then unified optimize γ and ϵ based on Algorithm 1, one can obtain

$$\gamma_{min} = 0.0020, \epsilon_{max} = 0.7846, K = \begin{bmatrix} -1.4816 & -1.4602 & -1.4203 & -1.7433 \end{bmatrix}.$$

Set the initial value of the state vectors and the disturbance input

$$x_0 = \begin{bmatrix} -2 & 3 & -4 & -1 \end{bmatrix}, w(t) = 0.1 \sin t.$$

Now the trajectories of the output vectors $y(t)$ are shown in Figure.5.1.

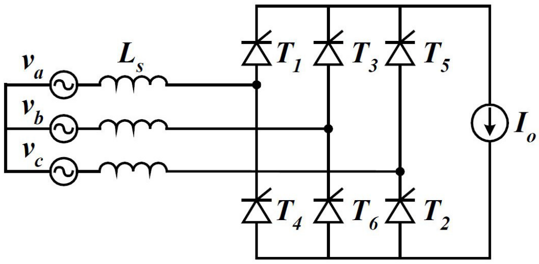

Figure 5.1 Comparison with $y(t)$ under the case of different ϵ.

Example 3: Consider a singularly perturbed system (2.3) with the the following given parameter

$$E_\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & \epsilon \end{bmatrix}, A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, B_1 = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, D_1 = 0.1, D_2 = 0.$$

In light of Algorithm 1 and Algorithm 2, one can get

$$\gamma_{min} = 0.3055, \epsilon_{max} = 1.5, K = \begin{bmatrix} 94.7127 & -787.8431 \end{bmatrix}.$$

Now the trajectories of the state vectors $x(t)$ are shown in Figure.5.2.

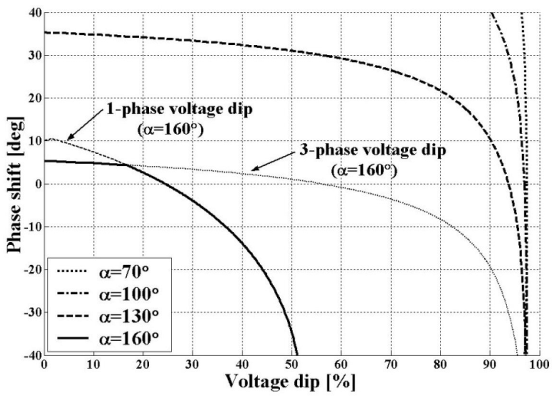

Figure 5.2 state curves with $\epsilon = 1.5$.

6 CONCLUSION

In this paper, we have considered the unified optimization problem for the upper bound stability ϵ^* and the H_∞ index γ based on state feedback. A sufficient condition for the existence state feedback controller has been presented in terms of LMIs such that the resulting closed-loop system is asymptotically stable if $0 < \epsilon < \epsilon^*$ and guarantees H_∞ performance index. Then an algorithm to optimize these two indices based on Nash game theory has been derived. It can translate multi-objective into single objective and develops a new way to determine the objective weights. Using this algorithm, the corresponding optimal state feedback controller with the upper bound of singular perturbation parameter with meeting a prescribed H_∞ performance bound requirement can be obtained. Finally, several numeral examples have demonstrated the feasibility, the effectiveness and the less conservatism of the new results.

Acknowledgment

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50 MODEL PREDICTIVE TRAJECTORY TRACKING OF A REENTRY HYPERSONIC VEHICLE BASED ON CONVEX OPTIMIZATION

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1 ABSTRACT

The trajectory-tracking problem of a reentry hypersonic vehicle is studied in this paper. Due to the complicated aerodynamics characteristic and tight integration configuration, the model of the vehicle is highly nonlinear and multi-constrained, which poses significant challenge to the control problem. This paper develops a trajectory tracking method based on the model predictive control (MPC). In order to improve the speed of MPC, the optimization problem at each time step is transformed into a convex form and, accordingly, convex optimization is used to obtain the control action. The resulting model predictive control method is able to take into consideration the constraints on heat, dynamic pressure, and control efforts, which sets it aside from most classical methods. The newly developed convex optimization solver `CVXGEN` is utilized successfully in the evaluation of the method, and derives the solutions in milliseconds. This makes real-time applications of the MPC feasible. Simulation results show that the proposed trajectory tracking strategy possesses good performance in the presence of aerodynamic uncertainties. All the constraints are satisfied even when the given nominal trajectory has large perturbations.

Hypersonic vehicles have large lift-to-drag ratio and good maneuverability, which makes related research topics more and more popular recently. This paper studies the control problem of a reentry hypersonic vehicle which reentries the atmosphere at the altitude of 80 km and follows the nominal trajectory until it hits the target with a certain velocity. However, the trajectory-tracking problem of reentry hypersonic vehicles is always tricky. Firstly, the control model of the system is highly nonlinear due to the unique configuration of hypersonic vehicles. Secondly, there are all kinds of constraints to the states and control signals due to considerations of aerodynamic stability, thermal condition and mechanical requirements, which are nonlinear too. Thirdly, the uncertainties of the model parameters could be large because, by now, we only have limited knowledge about the aerodynamic characteristics under hypersonic conditions.

The dynamics of the hypersonic vehicle considered is represented as:

$$\dot{h} = v \sin\gamma, \tag{1.1}$$

$$\dot{v} = -\frac{\rho_0}{2m} e^{-\beta h} S v^2 (c_{D0} + C_{D1}\alpha^2 + C_{D2} e^{C_{D3} v}) - \frac{\mu}{(R_e + h)^2} \sin\gamma, \tag{1.2}$$

$$\dot{\gamma} = -\frac{\rho_0}{2m} e^{-\beta h} S v (c_{L0} + C_{L1}\alpha + C_{L2} e^{C_{L3} v}) + \frac{v\cos\gamma}{R_e + h} - \frac{\mu}{(R_e + h)^2 v} \cos\gamma. \tag{1.3}$$

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The trajectory of the vehicle has to satisfy the following constraints:

$$K_Q(\rho_0 e^{-\beta h})^{\frac{1}{2}} v^3 \le Q_{d\max}, \tag{1.4}$$

$$\frac{1}{2}\rho_0 e^{-\beta h} v^2 \le P_{d\max}, \tag{1.5}$$

$$\frac{\sqrt{L^2 + D^2}}{m g_0} \le n_{L\max}, \tag{1.6}$$

$$\alpha_{\min} \le \alpha \le \alpha_{\max}, \tag{1.7}$$

$$\dot{\alpha}_{\min} \le \dot{\alpha} \le \dot{\alpha}_{\max}. \tag{1.8}$$

According to the idea of model predictive control, the control action at each time step is obtained by solving an online optimization problem. Solving a nonconvex optimization problem using general purpose methods can be slow, and this has limited MPC to applications with sampling time measured in seconds or minutes. In order to speed up the computation, the optimization problem in this paper is transformed into a convex form as follows:

$$\begin{aligned} &\min_{\substack{x_1,\dots,x_{T+1}\\ u_0,\dots,u_T}} \; J \\ &\text{s.t.} \quad \begin{cases} x(i+1) = A(i)x(i) + B(i)u(i) + w(i), \\ u(i) \le u_{\max}, & \qquad i = 0, ..., T, \\ C_a x(i) + C_b u(i) + C_c \le 0, \end{cases} \end{aligned} \tag{1.9}$$

where the performance index J is defined by

$$\begin{aligned} J = &\sum_{i=1}^{T} \{(x(i) - x_r(i))^T Q(x(i) - x_r(i)) + u(i)^T R u(i) + (u(i) - u(i-1))^T R_d (u(i) - u(i-1))\} \\ &+ (x(T+1) - x_r(T+1))^T Q_f (x(T+1) - x_r(T+1)). \end{aligned}$$

When we choose $T = 20$, the above optimization problem has 1803 non-zero KKT matrix entries. It takes less than 2 milliseconds to solve this optimization problem by the solver `CVXGEN`, which is fast enough to satisfy the requirement of real-time computation.

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51 EXACT SPARSE LS-SVM

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Abstract: For the support vector machine(SVM) learning, the least squares SVM(LS-SVM) model derived by duality is a widely used model since it has an explicit solution. But its limitation is that the solution lacks sparseness. In this paper, we derive another equivalent LS-SVM model by the representer theorem, and prove that the new model can be solved exactly at some sparse solutions, but not the approximate sparse solution as some researchers(Suykens et al., 2000; Suykens et al., 2002b; Kruif and Vries, 2003; Zeng and Chen, 2005; Jiao et al., 2007; Kuh and Wilde, 2007) did. For the linear leaning problem, our theoretical analysis and experimental results support the new model gives the *sparsest* solutions in all SVM models.

Key words: Support vector machine(SVM); representer theorem; RKHS; sparseness

1 INTRODUCTION

In a learning problem, an input set $\mathbb{T} = \{(x_1, y_1), \cdots, (x_m, y_m)\}$ for samples $x_i \in \mathbb{R}^n$ along with corresponding targets y_i is given, and a deterministic function that best represents the relation between input vectors and class labels is learned form the input samples.

Based on the Vanpnik and Chervonkis' structural risk minimization principle (Vapnik, 1999; Vapnik, 2000), support vector machine(SVM) is a computationally powerful and successful machine learning method. It is widely used in classification and regression problems, such as character identification, disease diagnoses, face recognition, the time serial prediction. For the classification problem, the SVM model solves the following optimization problem

$$\min_{w\in\mathcal{H}, b\in\mathbb{R}, \xi\in\mathbb{R}^m} \quad \frac{\lambda}{2}\|w\|_{\mathcal{H}}^2 + \sum_{i=1}^{m} L(\xi_i), \tag{1.1a}$$

$$s.t. \quad y_i(\langle w, k(\cdot, x_i)\rangle_{\mathcal{H}} + b) + \xi_i = 1, i = 1, 2, \cdots, m, \tag{1.1b}$$

to find the optimal classification function

$$f(x) = \langle w, k(\cdot, x)\rangle_{\mathcal{H}} + b, \tag{1.2}$$

where $\lambda > 0$ is the regularization parameter, the loss function $L : \mathbb{R} \to \mathbb{R}_+ \cup \{+\infty\}$ has some typical forms for different learning problems, $k : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is a kernel function with a good generalized capacity and $\mathcal{H}$ is a reproduced kernel Hilbert spaces(RKHS) corresponding to the kernel function k with the reproducing property that admits $g(x) = \langle g(\cdot), k(\cdot, x)\rangle_{\mathcal{H}}$ and especially $\langle k(\cdot, x), k(\cdot, z)\rangle_{\mathcal{H}} = k(x, z)$ for all $g \in \mathcal{H}$ and $x, z \in \mathbb{R}^m$. Commonly, model (1.1) can be convert to a finite dimensional problem by representer theorem (Schölkopf et al., 2001; Keerthi et al., 2006; Chapelle, 2007) or duality (Joachims, 1998; Platt, 1999; Suykens and Vandewalle, 1999; Vapnik, 2000; Cristianini and Shawe-Taylor, 2000; Fine and Scheinberg, 2001; Ferris and Munson, 2004; Zhou et al., 2007), which admits the solution $w \in \mathcal{H}$ to model (1.1) can be represented as

$$w = \sum_{j=1}^{m} \alpha_j k(\cdot, x_j). \tag{1.3}$$

Based on model (1.1), one simple and efficient method is the least squares support vector machine(LS-SVM) (Suykens and Vandewalle, 1999; Suykens et al., 2002b) with the least squared loss $L(u) = \frac{1}{2}u^2$ in the model (1.1). Experimental comparisons (Van Gestel et al., 2004) show that LS-SVM has good performance on many applications, but one obvious limitation is that the solution to LS-SVM lacks the sparseness, and hence its its test speed is significantly slower than others SVM models.

Suykens et al.(Suykens et al., 2000) proposed a pruning approach to improve the sparseness of LS-SVM by iteratively removing some samples(such as 5%) with smallest the support value spectrum, i.e., the absolute value of the current solution of LS-SVM, and some other studies on pruning method (Kruif and Vries, 2003; Zeng and Chen, 2005; Kuh and Wilde, 2007 etc.) are reported. All these pruning algorithms require solving a system of linear equations (slowly decreasing in size) many times, which incurs a large computational cost, and only give an approximate solution at last.

Jiao et al.(Jiao et al., 2007) present another fast sparse approximation scheme for LS-SVM to deal with the sparseness, in which the approximated decision function is built iteratively by adding one basis function from a kernel-based dictionary at one time until the ε criterion satisfied, and a probabilistic speedup scheme is used to further improve the speed of their algorithms.

All the works for LS-SVM mentioned above are based on solving a system of linear equations $B\alpha = b$ with a *nonsingular* dense matrix B and a dense vector b. The original LS-SVM (Suykens and Vandewalle, 1999) solves it exactly and must meet an unique dense solution, the others (Suykens et al., 2000; Kruif and Vries, 2003; Zeng and Chen, 2005; Kuh and Wilde, 2007; Jiao et al., 2007) are to iteratively reach an approximate sparse solution. Of course, they will meet a non-sparse solution if the approximation errors is small.

In this paper, we focus on an equivalent LS-SVM model induced by the representer theorem, which is also based on solving a system of linear equation $B\alpha = b$, but the coefficients matrix B is always *singular*. Hence the new model may have multi-solutions, which include some sparse solutions. For linear learning problem with $n \ll m$, we can prove the model has a sparse solution at most with n non-zero components, which is sparser than the solutions obtained by other SVM learners–So the model can be called *sparest* LS-SVM at this situation.

For nonlinear kernel learning problem, if its kernel matrix has low rank or is approximated with a low-rank matrix, we also prove that the model can give a sparse solution with r non-zero components where r is the rank of kernel matrix or the rank of approximated kernel matrix.

The rest of the paper is organized as follows. We review the LS-SVM model induced by duality in Section 2.1 and derive the new LS-SVM model by representer theorem in Section 2.1. In Section 2.3, we prove the equivalence of two models and we prove that the new model may meet the sparse solution in Section 3. Section 4 gives some elementary experiments and Section **??** concludes the paper.

2 TWO RELATED LS-SVM MODELS

There are two kinds of LS-SVM models. One is induced by the duality and the other is induced by the representer theorem.

2.1 LS-SVM models induced by duality

Many researcher focus on the LS-SVM models are induced by duality, such as Suykens and Vandewalle, 1999; Kruif and Vries, 2003; Zeng and Chen, 2005; Kuh and Wilde, 2007; Jiao et al., 2007 and the references therein. They study the Wolf duality of (1.1) with the least squared loss $L(u) = \frac{1}{2}u^2$, which is equivalent to

$$\min_{\beta} \tfrac{1}{2}\beta^\top K\beta + \tfrac{\lambda}{2}\beta^\top\beta - y^\top\beta, \quad s.t. \;\; \boldsymbol{e}^\top\beta = 0, \tag{2.1}$$

where the kernel matrix K satisfies $K_{i,j} = k(x_i, x_j)$ and $\boldsymbol{e}$ is a vector whose components are all 1. It is solved by the following system of linear equations:

$$\begin{bmatrix} 0 & \boldsymbol{e}^\top \\ \boldsymbol{e} & \lambda\boldsymbol{I} + K \end{bmatrix} \begin{bmatrix} b \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} \tag{2.2}$$

where $\boldsymbol{I}$ is identity matrix. The duality relationship maintains $w = \sum_{j=1}^m \beta_j k(\cdot, x_j)$. The corresponding output of a new sample x is predicted by $f(x) = \sum_j \beta_j k(x_j, x) + b$.

For the nonlinear problem in a large feature kernel space reproduced by a kernel function such as Gaussian kernel, the offset b in (1.2) can be omitted without loss the generalization performance (Steinwart, 2003; Steinwart and Christmann, 2008), then one simpler LS-SVM model is proposed to solve the following system of linear equations

$$(\lambda \boldsymbol{I} + K)\,\beta = y. \tag{2.3}$$

The output of a new input x is predicted as the sign of $f(x) = \sum_j \beta_j k(x_j, x)$.

2.2 LS-SVM models induced by representer theorem

By representer theorem(Schölkopf et al., 2001; Keerthi et al., 2006; Chapelle, 2007), the solution w of (1.1) admits (1.3). Plugging (1.3) in (1.1) and eliminating ξ_i with the equality constraint, we have

$$\min_{\alpha} \tfrac{\lambda}{2}\alpha^\top K\alpha + \tfrac{1}{2}\alpha^\top KK^\top\alpha + \tfrac{m}{2}b^2 + b\boldsymbol{e}^\top K\alpha - y^\top K\alpha - b\boldsymbol{e}^\top y + \tfrac{m}{2}. \tag{2.4}$$

Its solution is reached by solving the following system of linear equations:

$$\begin{bmatrix} m & \boldsymbol{e}^\top K^\top \\ K\boldsymbol{e} & \lambda K + KK^\top \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} \boldsymbol{e}^\top y \\ Ky \end{bmatrix} \tag{2.5}$$

The variation form without offset b in (1.2) is to solve the following system of linear equations:

$$\left[\lambda K + KK^\top\right]\alpha = Ky. \tag{2.6}$$

2.3 The properties of the related models

Comparing the two models, we have the following conclusions:

Proposition 2.1 *Since the kernel matrix is always a dense matrix, so:*

i) *No matter the kernel matrix is full rank or not, the solution of the LS-SVM models* (2.2) *and* (2.3) *induced by the duality are unique and dense(non-sparse);*

ii) *If the kernel matrix is not full rank, the solution of the LS-SVM models* (2.5) *and* (2.6) *induced by the representer theorem may have multi-solutions;*

iii) *If the rank of the kernel matrix is $r(\leq m)$, the LS-SVM models* (2.5) *and* (2.6) *induced the representer theorem have a sparse solution with r non-zero components.*

All of those can be proven by Gaussian elimination procession.

Theorem 2.2 *Those two types models are equivalent to each other correspondingly. Precisely,*

i) *the unique solution of* (2.2) *is always the solution of* (2.5)*, and the unique solution of* (2.3) *is always the solution to* (2.6)*;*

ii) *the classification function corresponding to the unique solution of* (2.2) *meets the same classification function corresponding to the all solutions of* (2.5)*, and the classification function corresponding to the unique solution of* (2.3) *meets the same classification function corresponding to the all solutions of* (2.6).

Proof: i) If let $(\bar\beta, \bar b)$ solves (2.2), we have $(\lambda\boldsymbol{I} + K)\bar\beta + \bar b\boldsymbol{e} = y$ and $\boldsymbol{e}^\top\bar\beta = 0$. Then

$$m\bar b + \boldsymbol{e}^\top K^\top\bar\beta = \boldsymbol{e}^\top\boldsymbol{e}\bar b + \lambda\boldsymbol{e}^\top\bar\beta + \boldsymbol{e}^\top K\bar\beta = \boldsymbol{e}^\top((\lambda\boldsymbol{I} + K)\bar\beta + \boldsymbol{e}\bar b) = \boldsymbol{e}^\top y,$$

and

$$K\boldsymbol{e}\bar b + (\lambda K + KK^\top)\bar\beta = K\left((\lambda\boldsymbol{I} + K)\bar\beta + \bar b\boldsymbol{e}\right) = Ky.$$

Namely,

$$\begin{bmatrix} m & \boldsymbol{e}^\top K^\top \\ K\boldsymbol{e} & \lambda K + KK^\top \end{bmatrix} \begin{bmatrix} \bar b \\ \bar\beta \end{bmatrix} = \begin{bmatrix} \boldsymbol{e}^\top y \\ Ky \end{bmatrix}$$

Hence $(\bar\beta, \bar b)$ solves (2.5) and the proof of the solution of (2.3) $\bar\beta$ solving (2.6) is similar.

ii) It is true since all solutions to the models induced by representer theorem are equivalences. ∎

3 THE SPARSENESS OF THE MODELS INDUCED BY REPRESENTER THEOREM

In this section, we study how to obtain the sparsest solution of the new model induced by representer theorem.

Lemma 3.1 *Given inputs set $S = \{x_i \in \mathbb{R}^n | i = 1, 2, ..., m\}$ and its semi-positive definite kernel matrix $K \in \mathbb{R}^{m\times m}$, if $rank(K) = r$, then there exists a basic set $B \in S$ with r inputs(and the non-basic subset N including the rest $m - r$ inputs) such that the kernel matrix K can be rearranged as:*

$$K = \begin{bmatrix} K_{BB} & K_{BN} \\ K_{NB} & K_{NN} \end{bmatrix} = \begin{bmatrix} K_{BB} & K_{BB}T^\top \\ TK_{BB} & TK_{BB}T^\top \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_r \\ T \end{bmatrix} K_{BB} \left[\boldsymbol{I}_r \quad T^\top\right] \tag{3.1}$$

where $K_{BB} \in \mathbb{R}^{r\times r}$ is the full rank kernel matrix of B and $T \in \mathbb{R}^{(m-r)\times r}$ satisfies $K_{NB} = TK_{BB}$.

Proof: Since $rank(K) = r$, there have r linear independent rows of K. Put the inputs corresponding to those r rows into basic set B. Then kernel matrix can be rearranged as $K = \begin{bmatrix} K_{BB} & K_{BN} \\ K_{NB} & K_{NN} \end{bmatrix}$ and the first r rows of K are linear independent. Hence the rest $r - m$ rows can be resented by the first r rows. Namely, there exists matrix $T \in \mathbb{R}^{(m-r)\times r}$ such that $[K_{NB} \;\; K_{NN}] = T\,[K_{BB} \;\; K_{BN}]$. Then $K_{NB} = TK_{BB}$, $K_{BN} = K_{NB}^\top = K_{BB}^\top T^\top = K_{BB}T^\top$ and $K_{NN} = TK_{BN} = TK_{BB}T^\top$. Then (3.1) is proven and hence K_{BB} is a full rank matrix. This completes the proof. ∎

Theorem 3.2 *Let the kernel matrix K satisfy $\mathbf{rank}(K) = r$. Then,*

i) *LS-SVM model (2.6) has a solution α at most with r non-zero components corresponding to basic set B in Lemma 3.1 and $\alpha_N = 0$.*

ii) *LS-SVM model (2.5) has a solution (α, b) such that α at most with r non-zero components corresponding to basic set B in Lemma 3.1 and $\alpha_N = 0$.*

Proof: i) Plugging (3.1) in (2.6), we can show that (2.6) is equivalent to

$$\begin{cases} Q[\boldsymbol{I}_r \;\; T^\top]\alpha &= \;\; K_{BB}[\boldsymbol{I}_r \;\; T^\top]y, \\ TQ[\boldsymbol{I}_r \;\; T^\top]\alpha &= \;\; TK_{BB}[\boldsymbol{I}_r \;\; T^\top]y, \end{cases} \tag{3.2}$$

where

$$Q := \lambda K_{BB} + K_{BB}(\boldsymbol{I}_r + T^\top T)K_{BB} = \lambda K_{BB} + K_{BB}K_{BB} + K_{BN}K_{BN}^\top.$$

Obviously the second equality in (3.2) is redundant for solving these linear equations and the first equality can be solvable at

$$\begin{cases} \alpha_B &= \;\; Q^{-1}[K_{BB} \;\; K_{BN}]y, \\ \alpha_N &= \;\; 0. \end{cases} \tag{3.3}$$

ii) LS-SVM model in (2.5) can be simplified by eliminating b. With $b = \frac{1}{m}(\boldsymbol{e}^\top y - \boldsymbol{e}^\top K\alpha)$, we have

$$\left[\lambda K + KK^\top - \tfrac{1}{m}K\boldsymbol{e}\boldsymbol{e}^\top K\right]\alpha = Ky - \tfrac{1}{m}K\boldsymbol{e}\boldsymbol{e}^\top y. \tag{3.4}$$

Then we can similarly show that (3.4) is equivalent to the following system of linear equations

$$\begin{cases} \bar{Q}[\boldsymbol{I}_r \;\; T^\top]\alpha &= \;\; K_{BB}[\boldsymbol{I}_r \;\; T^\top]\left(y - \frac{\boldsymbol{e}^\top y}{m}\boldsymbol{e}\right), \\ T\bar{Q}[\boldsymbol{I}_r \;\; T^\top]\alpha &= \;\; TK_{BB}[\boldsymbol{I}_r \;\; T^\top]\left(y - \frac{\boldsymbol{e}^\top y}{m}\boldsymbol{e}\right), \end{cases} \tag{3.5}$$

where

$$\bar{Q} := \lambda K_{BB} + K_{BB}[\boldsymbol{I}_r \;\; T^\top](\boldsymbol{I}_m - \frac{1}{m}\boldsymbol{e}\boldsymbol{e}^\top)[\boldsymbol{I}_r \;\; T^\top]^\top K_{BB} = Q - \frac{1}{m}pp^\top,$$

with $p = [K_{BB} \;\; K_{BN}]\boldsymbol{e}$. Again noticed than the second equality in (3.5) is redundant and hence the model (2.5) can be solved at

$$\begin{cases} \alpha_B &= \;\; \bar{Q}^{-1}[K_{BB} \;\; K_{BN}]\left(y - \frac{\boldsymbol{e}^\top y}{m}\boldsymbol{e}\right), \\ \alpha_N &= \;\; 0, \\ \;\; b &= \;\; \frac{1}{m}(\boldsymbol{e}^\top y - p^\top \alpha_B). \end{cases} \tag{3.6}$$

This completes the proof. ∎

Furthermore, for linear problem, we have

Theorem 3.3 *For linear classification problems with offset b on input set $T = \{(x_i, y_i) | x_i \in \mathbb{R}^n, y_i \in \{-1, 1\}, i = 1, 2, ..., m\}$ ($n \ll m$), let $X = [x_1 \;\; x_2 \;\; \cdots \;\; x_m]^\top$ and X_B comprised by any r (usually the* *same as n)linear independent inputs as its rows. Then the output classification function of LS-SVM model* (2.5) *is*

$$f(x) = (x - \bar{x})^\top w + \frac{1}{m}\boldsymbol{e}^\top y,$$

where $\bar{x} = \frac{1}{m}X^\top \boldsymbol{e}$ *and* $w = X_B^\top \bar{Q}^{-1} X_B(X^\top y - \boldsymbol{e}^\top y\bar{x})$ *satisfies* $\bar{Q} = X_B(\lambda \boldsymbol{I}_n + X^\top X - m\bar{x}\bar{x}^\top)X_B^\top$.

The total computational complexity for linear classification problems with LS-SVM model (2.5) is less than $O(mn^2)$, and memory cost is $O(mn)$. This is similar as the model (2.2) induce by duality, where SMW identity (Golub and Loan, 1996) $(\boldsymbol{I}_m + \lambda XX^\top)^{-1} = \boldsymbol{I}_m - X\left(\lambda \boldsymbol{I}_n + X^\top X\right)^{-1} X^\top$ is used to reduce the computational complexity. However, LS-SVM model (2.5) has a sparse solution at most with n non-zeros components–This is the **least** number of inputs to present a typical hyperplane in $\mathbb{R}^n$. Comparing with other SVM models(such as C-SVM(Vapnik, 2000), ν-SVM(Schölkopf et al., 2000)etc.), we should call this model ***the sparest LS-SVM***.

4 EXPERIMENTAL RESULTS FOR LINEAR CLASSIFICATION PROBLEM

In this section, we perform some elementary experiments to illustrate our LS-SVM models induced by representer theorem can achieve sparser **exact** solution while others methods(Suykens et al., 2000; Zeng and Chen, 2005; Kuh and Wilde, 2007; Jiao et al., 2007 are to achieve the approximated sparse solution of LS-SVM).

Firstly we give a toy example with 8 separable input samples in $\mathbb{R}^2$ assigned two class as Figure 4.1 shows. We train this set by the new sparsest LS-SVM model (2.5) and the traditional non-sparse LS-SVM (2.2). For the sparsest model, the first sample at upper-left corner is assigned to the basic set B , and the second sample in B is chosen corresponding to Figure 4.1(a)-(f) respectively. The sample at lower-right corner can not be the second basic sample because it is linear independent with the first one. The result of traditional non-sparse LS-SVM is plotted in Figure 4.1(g), where all samples are the "support vectors" and the size of bullet(• or •) is in proportion to its weight. It needs to mention that, for LS-SVM, those samples with non-zero weight have already lost the meaning as the support vectors. However, for consistent with normal SVM, we still call them "support vectors".

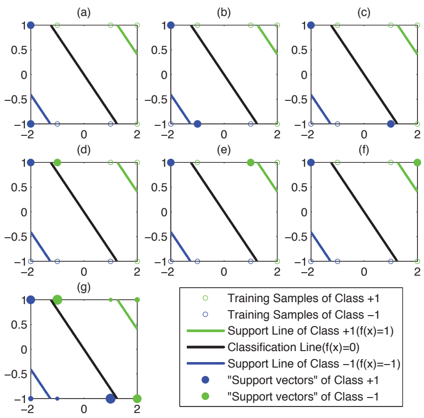

Figure 4.1 Sparse LS-SVMs (a)-(f) with different basic sets B comparing to non-sparse LS-SVM(g).

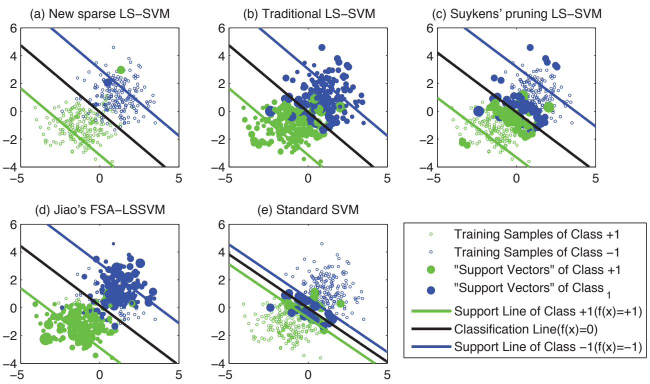

Figure 4.2 Linear classification experiments to show that the new proposed sparse LS-SVM is the ***sparsest*** SVM learner.

In Figure 4.1, it clearly shows that all the classification functions are the same and any 2 linear independent inputs, no matter within same class or not, can represent the solution of our sparse LS-SVM (2.5), and the non-sparse LS-SVM use all input samples to represent its solution. This is consistent with our theoretical results(Theroem 2.2, Theorem 3.2 and Theorem 3.3).

Another experiment is on an inseparable linear classification problem. 400 training inputs are randomly sampled as Figure 4.2 shows. Here we compare our new LS-SVM model with traditional non-sparse LS-SVM, Suykens et al.'s sparse approximation LS-SVM(Suykens et al., 2000) by pruning method and Jiao et al.'s fast sparse approximation LS-SVM(FAS-LSSVM)(Jiao et al., 2007) as well as the standard SVM.

In Figure 4.2, it observes that our new models has only two support vectors and the no-sparse LS-SVM model has all 400 training samples as support vectors and they meet the same classification hyperplane. For approximation method with the similar accuracy, Suykens et. al.'s pruning LS-SVM has 130 support vectors while Jiao et. al.'s FSA-LS-SVM has 347 support vectors. The standard SVM achieves 67 support vectors. It is clear that our new model is the sparsest SVM learner. Some other experiments reveal the similar conclusion.

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52 SEMI-DEFINITE PROGRAMMING BASED APPROACHES FOR REAL- TIME TRACTOR LOCALIZATION IN PORT CONTAINER TERMINALS

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Abstract: In order to effectively manage and deploy internal tractors in a port container marine terminal, real-time information concerning the location of the tractors is required so that timely scheduling and planning of tractors control and dispatching can be derived. This paper propose a wireless sensor network-based Truck Flow Management System (TFMS) to help tracking the real-time location of internal tractors in terminal so as to streamline the management of the terminal operation. Focusing on the real-time localization, the semi-definite programming (SDP) based approaches are employed by introducing the terminal context information, including prior known road constraints and available time-serial data recorded in the network, into the traditional SDP formulation. Experimental results are presented to show that the proposed formulation and treatments to the problem can greatly decrease the estimated errors compared to the traditional formulation.

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53 STATIONARY ANALYSIS OF A FLUID MODEL DRIVEN BY AN M/M/C MULTIPLE VACATIONS QUEUE

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Abstract: A fluid queue is an input-output system where a continuous fluid enters and leaves a storage device, called a buffer, according to a randomly varying rate influenced by an underling stochastic environment or background. This paper considers a fluid flow model driven by a multi-server M/M/c queue with classical vacation. We obtain the sets of differential equations satisfied by the stationary joint distribution of the buffer content, by which we gain the simple structure of the Laplace Transform (LT) for the stationary distribution of the buffer content. Furthermore, we give the probability of an empty buffer and the mean of the buffer content based on the relationship between the LT and the Laplace-Stieltjes Transform (LST) of the stationary distribution.

Key words: Fluid model; Buffer content; Multiple vacation; LT; LST.

1 INTRODUCTION

Owing to the application of fluid queueing models in the field of wireless communications, transport, storage and computer systems, or others fields, the model has recently attracted interest from probability researchers as a research subject.

As is well known, the distribution function of any buffer content satisfies a set of differential equations. Spectral analysis method has been the most traditional and commonly used method to find the solution to these equations. Kulkarni (1997) presented the spectral method to deal with a fluid model driven by a Markov process with finite state. Moreover, Doorn (1997) used orthogonal polynomials to express the stationary distribution of the buffer content, which is driven by an infinite-state birth-death process. Ramaswami (1999) advanced a matrix analytic method, Neuts (1981) extended a geometric solution method into a multi-dimensional matrix geometric solution method. Lenin (2000) studied the fluid model driven by an M/M/1/N queue, while Parthasarathy (2002) and many other researchers have learned from indicators of fluid models driven by an M/M/1 queue with different methods. However, fluid models driven by queues with different vacation policies, such as the fluid model driven by an M/M/1/N queue with multiple exponential vacations (Mao (2010)) and the fluid model driven by an M/G/1 queue with multiple exponential vacations (Mao (2011)), have just began to be studied.

In this paper, we have mainly studied some indices relating to fluids models driven by an M/M/c queue with multiple vacations. Firstly, we discuss the drive system and the stationary distribution of the drive system is obtained. Then, we introduce the Laplace Transform (LT) and Laplace-Stieltjes Transform (LST) of the distribution functions. The LST of the stationary distribution of the buffer content is given on the basis of the relationship between the LT and the LST. Furthermore, we obtain the brief expressions of the mean of the buffer content, as well as the probability of the buffer being empty.

2 AN M/M/C QUEUE WITH MULTIPLE VACATIONS

In this system, the inter-arrival times and service times follow an exponential distribution with parameters λ and μ, respectively. When there is no customer in the system after a service completion, the server will take a vacation of a random length which follows an exponential distribution with parameters θ. If there are customers in the system when a vacation comes to an end, the servers enter a busy period. Otherwise, the servers take another vacation. This model is identified as the M/M/c queue with multiple vacations, abbreviated as the M/M/c/MV queue.

We assume that inter-arrival times, service times and vocation times are all independent, and the service discipline is First-Come First-Served (FCFS).

Let $L(t)$ be the number of customers in the system at time t, and $J(t) = 0$ or 1, decided according to whether the system stays in a vacation period or a busy period at time t. Then, the stochastic process $\{(L(t), J(t)), t \geq 0\}$ is a Quasi-Birth-and-Death(QBD) process with the state space $\Omega = \{(0,0)\} \bigcup \{(k,j), k \geq 1, j = 0, 1\}$.

Arranging the state space in lexicographic order, the infinitesimal generator for the process $\{(L(t), J(t)), t \geq 0\}$ can be expressed as a block tridiagonal matrix form, that is

$$Q = \begin{pmatrix} A_0 & C_0 & & & & & \\ B_1 & A_1 & C & & & & \\ & B_2 & A_2 & C & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & B_{c-1} & A_{c-1} & C & \\ & & & & B & A & C \\ & & & & & \ddots & \ddots & \ddots \end{pmatrix}$$

where

$$A_0 = -\lambda, \quad C_0 = (\lambda, 0), \quad B_1 = (0, \mu)^T,$$
$$A_k = \begin{pmatrix} -(\lambda+\theta) & \theta \\ 0 & -(\lambda+k\mu) \end{pmatrix}, \quad k = 1, 2, ..., c-1, \quad B_k = \begin{pmatrix} 0 & 0 \\ 0 & k\mu \end{pmatrix}, \quad k = 2, 3, ..., c-1,$$
$$B = \begin{pmatrix} 0 & 0 \\ 0 & c\mu \end{pmatrix}, \quad A = \begin{pmatrix} -(\lambda+\theta) & \theta \\ 0 & -(\lambda+c\mu) \end{pmatrix}, \quad C = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}.$$

It is easy to conclude that the M/M/c/MV system is stable if and only if the system workload $\rho_c = \lambda/(c\mu) < 1$. Let $\pi_{kj} = \lim\limits_{t\to+\infty} P\{L(t) = k, J(t) = j\}$, $(k,j) \in \Omega$, then $\{\pi_{kj}, (k,j) \in \Omega\}$ is the stationary distribution of process $\{(L(t), J(t)), t \geq 0\}$.

In order to get the expression of the stationary distribution of process $\{(L(t), J(t)), t \geq 0\}$, it is necessary to obtain the minimal non-negative solution of the matrix equation $R^2B + RA + C = 0$. This solution R is called the rate matrix, which plays an important role in the analysis of QBD process.

Lemma 2.1 *If $\rho_c < 1$, the quadratic matrix equation $R^2B + RA + C = 0$ has the minimal non-negative solution*

$$R = \begin{pmatrix} \frac{\lambda}{\lambda+\theta} & \rho_c \\ 0 & \rho_c \end{pmatrix}. \tag{2.1}$$

It is well known that the stationary distribution of $\{(L(t), J(t)), t \geq 0\}$ exists if and only if $SP(R) < 1$ and the homogeneous linear equation $XB[R] = 0$ has a positive solution, where $SP(R)$ is the spectral radius of R and

$$B[R] = \begin{pmatrix} A_0 & C_0 & & & & \\ B_1 & A_1 & C & & & \\ & B_2 & A_2 & C & & \\ & & \ddots & \ddots & \ddots & \\ & & & B_{c-1} & A_{c-1} & C \\ & & & & B & RB+A \end{pmatrix}.$$

Theorem 2.1 *If $\rho_c < 1$, the stationary distribution of $\{(L(t), J(t)), t \geq 0\}$ are as follows:*

$$\pi_{k0} = \begin{cases} (\frac{\lambda}{\lambda+\theta})^k \pi_{00}, & 1 \leq k \leq c-1 \\ (\frac{\lambda}{\lambda+\theta})^{k-c} \pi_{c0}, & k \geq c, \end{cases}$$

$$\pi_{k1} = \begin{cases} \frac{\lambda}{\mu}\pi_{00}, \quad k=1 \\ \frac{1}{k!}\pi_{00}\left\{(\frac{\lambda}{\mu})^k + \frac{\lambda+\theta}{\mu}(\frac{\lambda}{\lambda+\theta})^c \sum\limits_{j=1}^{k-1}(\frac{\lambda}{\mu})^j(k-j)! + \frac{\theta}{\mu}\sum\limits_{v=k+1-j}^{c-1}(\frac{\lambda}{\lambda+\theta})^v \sum\limits_{j=1}^{k-1}(\frac{\lambda}{\mu})^j(k-j)!\right\}, \quad 2 \le k \le c-1 \\ \pi_{c0}\sum\limits_{v=1}^{k-c}(\frac{\lambda}{\lambda+\theta})^{k-c-v}{\rho_c}^v + \pi_{c1}{\rho_c}^{k-c}, \quad k \ge c \end{cases}$$

where $\pi_{c1} = \frac{\lambda}{c\mu(c-1)!}\pi_{00}\left[(\frac{\lambda}{\mu})^{c-1} + \frac{\lambda+\theta}{\mu}(\frac{\lambda}{\lambda+\theta})^c \sum\limits_{j=1}^{c-2}(\frac{\lambda}{\mu})^{j-1}(c-1-j)! + \frac{\theta}{\mu}\sum\limits_{v=c-j}^{c-1}(\frac{\lambda}{\lambda+\theta})^v \sum\limits_{j=1}^{c-2}(\frac{\lambda}{\mu})^{j-1}(c-1-j)!\right] + \frac{\lambda^c}{c\mu(\lambda+\theta)^{c-1}}\pi_{00}$, $\pi_{c0} = (\frac{\lambda}{\lambda+\theta})^c\pi_{00}$, π_{00} *can be determined by the normalization condition as follows:*

$$\sum_{k=0}^{\infty}\pi_{k0} + \sum_{k=1}^{\infty}\pi_{k1} = 1.$$

3 DESCRIPTION AND ANALYSIS FOR THE FLUID MODEL

Let $X(t)$ be the content of the buffer at time t, which is a non-negative random variable. Assume that the net input rate of fluid (the input rate minus the output rate) to the buffer is the function of the process $\{(X(t), L(t), J(t)), t \ge 0\}$, we have

$$\sigma[X(t), L(t), J(t)] = \frac{dX(t)}{dt} = \begin{cases} 0, & (L(t), J(t)) = (0,0), X(t) = 0 \\ \sigma, & (L(t), J(t)) = (0,0), X(t) > 0 \\ \sigma_0, & (L(t), J(t)) = (k,0), k \ge 1 \\ \sigma_1, & (L(t), J(t)) = (k,1), k \ge 1. \end{cases}$$

Now, the fluid model driven by the M/M/c queue with multiple vacations is a three-dimensional Markov process with state space $\Omega' = \Omega \times [0, +\infty)$. Let $d = \sigma\pi_{00} + \sigma_0\sum_{k=1}^{+\infty}\pi_{k0} + \sigma_1\sum_{k=1}^{+\infty}\pi_{k1}$, then d is called the average drift of the fluid model. It is not difficult to prove that the fluid model is stable if and only if $d < 0$ and $\rho_c < 1$ when the buffer capacity is infinite (see Kulkarni (1997)).

Let $F_{k0}(t,x) = P\{L(t) = k, J(t) = 0, X(t) \le x\}$, $k \ge 0$ and $F_{k1}(t,x) = P\{L(t) = k, J(t) = 1, X(t) \le x\}$, $k \ge 1$, which are called the instantaneous joint probability distribution function of the three-dimensional Markov process. When the process achieves balance, $\{(X(t), L(t), J(t)), t \ge 0\}$ converges to the random vector (X, L, J). Here X is the stationary distribution of the buffer content. The joint distribution of (X, L, J) is denoted by $F_{k0}(x) = \lim\limits_{t\to+\infty} F_{k0}(t,x)$, $F_{k1}(x) = \lim\limits_{t\to+\infty} F_{k1}(t,x)$. Then, the steady-state inventory has a distribution function $F(x) = P\{X \le x\} = F_{00}(x) + \sum_{k=1}^{+\infty}F_{k0}(x) + \sum_{k=1}^{+\infty}F_{k1}(x), x \ge 0$. $F(x) = (F_{00}(x), F_1(x), F_2(x), ...)$, where $F_k(x) = (F_{k0}(x), F_{k1}(x)), k \ge 1$.

Using standard methods (see Mitra (1998), Parthasarathy (2002)), we can prove that $F(x)$ satisfies the matrix differential equation as follows:

$$\frac{d}{dx}F(x)\Lambda = F(x)Q \tag{3.1}$$

and the boundary condition

$$F(0) = (a, 0, 0, 0, ...) \tag{3.2}$$

where $\Lambda = diag(\sigma, \Sigma, \Sigma, ...)$, $\Sigma = diag(\sigma_0, \sigma_1)$. The probability $a = F_{00}(0) = P\{X = 0, L = 0, J = 0\}$ is called the stationary probability of the empty buffer content, which will be determined in the follow analysis.

In order to solve the differential Eq. (3.1), we have to get help from the LT of the joint distribution, denoted by $\hat{F}_{kj}(s) = \int_0^{+\infty} e^{-sx}F_{kj}(x)dx, s > 0, (k,j) \in \Omega$. Then $\hat{F}(s) = (\hat{F}_{00}(s), \hat{F}_1(s), ...)$ and $\hat{F}_k(s) = (\hat{F}_{k0}(s), \hat{F}_{k1}(s)), k \ge 1$.

Taking the LT on both sides of Eq. (3.1) and combining the boundary conditions, we can found that $\hat{F}(s)$ satisfies the equation as in the following structure:

$$\hat{F}(s)(Q - s\Lambda) = -F(0)\Lambda = (-a\sigma, 0, 0, 0, ...). \tag{3.3}$$

Next we introduce a crucial quadratic equation, whose roots play an important role in the following analysis.

Lemma 3.1 *For any* $s \ge 0$, $c\mu z^2 - (\lambda + c\mu + s\sigma_1)z + \lambda = 0$ *has two real roots* $\gamma_0(s)$ *and* $\gamma_1(s)$, *where*

$$\gamma_0(s)(\gamma_1(s)) = \frac{(\lambda+c\mu+s\sigma_1)-(+)\sqrt{(\lambda+c\mu+s\sigma_1)^2-4c\lambda\mu}}{2c\mu}.$$

It is easy to verify $0<\gamma_0(s)<1$, $\gamma_1(s)>1$, $\gamma_0(0)=\frac{\lambda}{c\mu}=\rho_c$, $\gamma_1(0)=1$, $c\mu\gamma\prime_0(0)=\frac{\lambda\sigma_1}{\lambda-c\mu}$ *and* $c\mu(\gamma_0(s)+\gamma_1(s))=\lambda+c\mu+s\sigma_1$, $c\mu\gamma_0(s)\gamma_1(s)=\lambda$.

Lemma 3.2 *If* $\rho_c<1$, *the quadratic matrix equation* $R(s)^2B+R(s)(A-s\Sigma)+C=0$ *has the minimal non-negative solution as follows:*

$$R(s)=\begin{pmatrix} \frac{\lambda}{\lambda+\theta+s\sigma_0} & \frac{\gamma_0(s)\theta}{\lambda+\theta+s\sigma_0-c\mu\gamma_0(s)} \\ 0 & \gamma_0(s) \end{pmatrix}.$$

Define a series of functions as

$$\begin{cases} \phi_0(s)=1 \\ \phi_k(s)=\frac{\lambda-c\mu\gamma_0(s)}{\lambda}+\frac{(c-k)\mu}{\lambda}\phi_{k-1}(s)+\frac{s\sigma_1}{\lambda}\sum\limits_{v=0}^{k-1}\phi_v(s), \quad 1\le k\le c-1 \end{cases} \tag{3.4}$$

and

$$\begin{cases} \varphi_k(s)=\frac{\lambda+(k+1)\mu+s\sigma_1}{\lambda}\varphi_{k+1}(s)-\frac{(k+2)\mu}{\lambda}\varphi_{k+2}(s)+\frac{\theta}{\lambda}(\frac{\lambda}{\lambda+\theta+s\sigma_0})^{k+1}\hat{F}_{00}(s), \quad 1\le k\le c-3 \\ \varphi_{c-2}(s)=\frac{(\lambda+\theta+s\sigma_0)\theta K_0(s)}{\lambda(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))} \\ \varphi_{c-1}(s)=0. \end{cases} \tag{3.5}$$

Using the above functions, matrix equation (3.3) can be rewritten as

$$\begin{cases} -(\lambda+s\sigma)\hat{F}_{00}(s)+\mu\hat{F}_{11}(s)=-a\sigma \\ \theta\hat{F}_{10}(s)-(\lambda+\mu+s\sigma_1)\hat{F}_{11}(s)+2\mu\hat{F}_{21}(s)=0 \\ \lambda\hat{F}_{k-1,0}(s)-(\lambda+\theta+s\sigma_0)\hat{F}_{k0}(s)=0, \quad 1\le k\le c-1 \\ \lambda\hat{F}_{k-1,1}(s)+\theta\hat{F}_{k0}(s)-(\lambda+k\mu+s\sigma_1)\hat{F}_{k1}(s)+(k+1)\mu\hat{F}_{k+1,1}(s)=0, \quad 2\le k\le c-1 \\ \hat{F}_{k-1}(s)C+\hat{F}_k(s)(A-s\Sigma)+\hat{F}_{k+1}(s)B=0, \quad k\ge c. \end{cases} \tag{3.6}$$

Then we get

Theorem 3.1 *If* $d<0$, $\rho_c<1$, $\hat{F}_{k0}(s)$, $\hat{F}_{k1}(s)$ *and* $\{\hat{F}_k(s),k\ge 1\}$ *can be expressed as*

$$\begin{cases} \hat{F}_{k0}(s)=(\frac{\lambda}{\lambda+\theta+s\sigma_0})^k\hat{F}_{00}(s), \quad 0\le k\le c-1 \\ \hat{F}_{k1}(s)=K_1(s)\phi_{c-1-k}(s)-\varphi_k(s), \quad 1\le k\le c-1 \\ \hat{F}_k(s)=K(s)R^{k-c+1}(s), \quad k\ge c \end{cases} \tag{3.7}$$

where $K(s)=(K_0(s),K_1(s))=(\hat{F}_{c-1,0}(s),\hat{F}_{c-1,1}(s)),\hat{F}_{00}(s)=\frac{a\sigma}{\lambda+s\sigma}+\frac{\mu}{\lambda+s\sigma}\phi_{c-2}(s)K_1(s)-\frac{\mu}{\lambda+s\sigma}\varphi_1(s),K_0(s)=(\frac{\lambda}{\lambda+\theta+s\sigma_0(s)})^{c-1}\hat{F}_{00}(s),K_1(s)=\frac{[\mu\lambda\theta-(\lambda+\mu+s\sigma_o)(\lambda+\mu+s\sigma_1)(\lambda+s\sigma)]\varphi_1(s)+2\mu(\lambda+s\sigma)(\lambda+\theta+s\sigma_o)\varphi_2(s)-a\sigma\lambda\theta}{[\mu\lambda\theta-(\lambda+\mu+s\sigma_o)(\lambda+\mu+s\sigma_1)(\lambda+s\sigma)]\phi_{c-2}(s)+2\mu(\lambda+s\sigma)(\lambda+\theta+s\sigma_o)\phi_{c-3}(s)}$.

Theorem 3.2 *If* $d<0$, $\rho_c<1$, *the stable buffer content* X *has the LST and the mean as*

$$\begin{aligned} F^*(s) \quad = \quad & \frac{[\lambda(\sigma_1-\sigma_0)+(\sigma_1-\sigma)(\theta+s\sigma_0)]s\hat{F}_{00}(s)}{\sigma_1(\theta+s\sigma_0)}+\frac{c\mu\gamma_0(s)-\lambda}{\sigma_1}K_1(s)+\frac{a\sigma}{\sigma_1} \\ & -\frac{(\theta+s\sigma_1)\lambda^c\hat{F}_{00}(s)}{\sigma_1(\theta+s\sigma_0)(\lambda+\theta+s\sigma_0)^{c-1}}+\frac{c\mu\theta\gamma_0(s)K_0(s)}{\sigma_1(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))} \\ & +\frac{s\lambda K_0(s)}{\theta+s\sigma_0}+\frac{s\gamma_0(s)\theta(\lambda-\theta-s\sigma_0)K_0(s)}{(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))(\theta+s\sigma_0)(\gamma_0(s)-1)}+\frac{s\gamma_0(s)K_1(s)}{1-\gamma_0(s)}, \end{aligned} \tag{3.8}$$

$$\begin{aligned} E(X) \quad = \quad & \frac{\lambda^c[(\sigma_1-\sigma_0)\hat{F}_{00}(0)+\theta\hat{F}_{00}'(0)]}{\sigma_1\theta(\lambda+\theta)^{c-1}}-\frac{(c-1)\sigma_0\lambda^c\hat{F}_{00}(0)}{\sigma_1(\lambda+\theta)^c}-\frac{\lambda^2(3\sigma_1-\sigma_0)K_0(0)}{\sigma_1\theta(\lambda-c\mu)} \\ & -\frac{[\lambda(\sigma_1-\sigma_0)+\theta(\sigma_1-\sigma)]\hat{F}_{00}(0)}{\sigma_1\theta}-\frac{\lambda K'_0(0)}{\sigma_1}+\frac{\lambda c\mu(\sigma_1-\sigma_0)K_0(0)}{\sigma_1\theta(\lambda-c\mu)}. \end{aligned} \tag{3.9}$$

Proof. Taking sum on both sides of the forth line of Eq. (3.6) from 2 to *c*-1, we have

$$\theta\sum_{k=2}^{c-1}\hat{F}_{k0}(s)-s\sigma_1\sum_{k=2}^{c-1}\hat{F}_{k1}(s)+\lambda\hat{F}_{11}(s)-\lambda\hat{F}_{c-1,1}(s)-2\mu\hat{F}_{21}(s)+c\mu\hat{F}_{c1}(s)=0. \tag{3.10}$$

Summing up Eq. (3.10) and the second line of Eq. (3.6), we obtain

$$s\sigma_1\sum_{k=1}^{c-1}\hat{F}_{k1}(s)=\theta\sum_{k=1}^{c-1}\hat{F}_{k0}(s)-\mu\hat{F}_{11}(s)-\lambda\hat{F}_{c-1,1}(s)+c\mu\hat{F}_{c1}(s). \tag{3.11}$$

Using the first line of Eq. (3.7), Eq. (3.11), the first line of Eq. (3.6) and the expressions of $\hat{F}_{c1}(s), \hat{F}_{c-1,1}(s)$, then

$$\begin{aligned}\sum_{k=1}^{c-1}\hat{F}_{k1}(s) &= \tfrac{\hat{F}_{00}(s)}{s\sigma_1}[\tfrac{\lambda\theta(1-(\frac{\lambda}{\lambda+\theta+s\sigma_0})^{c-1})}{\theta+s\sigma_0} - (\lambda+s\sigma)] + \tfrac{c\mu\theta\gamma_0(s)K_0(s)}{s\sigma_1(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))} \\ &+\tfrac{a\sigma}{s\sigma_1} + \tfrac{c\mu\gamma_0(s)-\lambda}{s\sigma_1}K_1(s).\end{aligned}$$

Hence, the LT of the stationary distribution $F(x)$ of the buffer content can be given by

$$\begin{aligned}\hat{F}(s) = \int_0^{+\infty} e^{-sx}F(x)dx &= \sum_{k=0}^{c-1}\hat{F}_{k0}(s) + \sum_{k=1}^{c-1}\hat{F}_{k1}(s) + \sum_{k=c}^{+\infty}\hat{F}_k(s)e \\ &= \tfrac{\hat{F}_{00}(s)}{\theta+s\sigma_0}[(\lambda+\theta+s\sigma_0) - \tfrac{\lambda^c}{(\lambda+\theta+s\sigma_0)^{c-1}}] + \tfrac{c\mu\theta\gamma_0(s)K_0(s)}{s\sigma_1(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))} \\ &+\tfrac{\hat{F}_{00}(s)}{s\sigma_1}[\tfrac{\lambda\theta(1-(\frac{\lambda}{\lambda+\theta+s\sigma_0})^{c-1})}{\theta+s\sigma_0} - (\lambda+s\sigma)] + \tfrac{a\sigma}{s\sigma_1} + \tfrac{c\mu\gamma_0(s)-\lambda}{s\sigma_1}K_1(s) \\ &+K(s)R(s)(I-R(s))^{-1}e\end{aligned}$$

where $e = (1,1)^T$.

For the spectral radius $SP[R(s)] = \max(\gamma_0(s), \frac{\lambda}{\lambda+\theta+s\sigma_0}) < 1$, so $I - R(s)$ is invertible, and

$$(I-R(s))^{-1} = \begin{pmatrix} \frac{\lambda+\theta+s\sigma_0}{\theta+s\sigma_0} & \frac{\gamma_0(s)\theta(\lambda+\theta+s\sigma_0)}{(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))(\theta+s\sigma_0)(\gamma_0(s)-1)} \\ 0 & \frac{1}{1-\gamma_0(s)}. \end{pmatrix}.$$

After calculation, we have

$$\begin{aligned}\hat{F}(s) \quad = \quad & \tfrac{[\lambda(\sigma_1-\sigma_0)+(\sigma_1-\sigma)(\theta+s\sigma_0)]\hat{F}_{00}(s)}{\sigma_1(\theta+s\sigma_0)} - \tfrac{(\theta+s\sigma_1)\lambda^c\hat{F}_{00}(s)}{s\sigma_1(\theta+s\sigma_0)(\lambda+\theta+s\sigma_0)^{c-1}} \\ &+\tfrac{c\mu\theta\gamma_0(s)K_0(s)}{s\sigma_1(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))} + \tfrac{a\sigma}{s\sigma_1} + \tfrac{c\mu\gamma_0(s)-\lambda}{s\sigma_1}K_1(s) + \tfrac{\lambda K_0(s)}{\theta+s\sigma_0} \\ &+\tfrac{\gamma_0(s)\theta(\lambda-\theta-s\sigma_0)K_0(s)}{(\lambda+\theta+s\sigma_0-c\mu\gamma_0(s))(\theta+s\sigma_0)(\gamma_0(s)-1)} + \tfrac{\gamma_0(s)K_1(s)}{1-\gamma_0(s)}.\end{aligned}$$

Next, we define the LST of the stationary joint distribution for the fluid model and the stationary distribution of the buffer content as

$$F^*_{kj}(s) = \int_0^{+\infty} e^{-sx}dF_{kj}(x), F^*(s) = \int_0^{+\infty} e^{-sx}dF(x), \quad (k,j) \in \Omega.$$

It is easy to prove $F^*_0(s) = -a + s\hat{F}_0(s)$, $F^*_k(s) = s\hat{F}_k(s)$, $k \geq 1$. Taking the expression of $\hat{F}(s)$ into $F^*_k(s) = s\hat{F}_k(s)$ and after calculation and arrangement, we can obtain the LST of the stationary distribution of the buffer content as in Eq. (3.8).

With the normalization condition of the LST $\lim_{s\to 0} F^*(s) = 1$, we can acquire the expression of a as

$$a = \frac{1}{\sigma}[\frac{\lambda^c\hat{F}_{00}(0)}{(\lambda+\theta)^{c-1}} + \sigma_1 - \lambda K_0(0)].$$

Now, taking the derivatives on both sides of Eq. (3.8) with respect to s, then letting $s \to 0$, we get the mean of the buffer content as in Eq. (3.9), and theorem 3.2 is proved.

4 CONCLUSION

In this paper we discussed the fluid model driven by the M/M/c multiple vocations queue, where the input rate and output rate are determined by the drive system. That is the queue length of the M/M/c multiple vacations queue. Using a QBD process and a matrix-geometric solution method, the steady state distribution of the queue length was derived. Furthermore, we obtained the brief expressions for the LST of the stationary distribution and the mean of the buffer content.

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54 PERFORMANCE ANALYSIS OF A DISCRETE-TIME GIX/GEO/1/N QUEUE WITH NEGATIVE CUSTOMERS AND SINGLE WORKING VACATION

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Abstract: Using the supplementary variable and embedded Markov chain method, we consider a discrete time batch arrival finite capacity queue with negative customers and single working vacation, where the RCH killing policy and partial batch rejection policy are adopted. First, we obtain steady-state system length distributions at pre-arrival, arbitrary and outside observers observation epochs. Based on the various system length distributions, the blocking probability of the first, an arbitrary and the last customer in a batch, the analysis of actual waiting time distributions measured in slots of the first, an arbitrary and the last positive customer in an accepted batch have been investigated. Finally, we consider the influence of system parameters on several performance measures to demonstrate the correctness of the theoretical analysis.

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55 OPTIMAL BALKING STRATEGIES IN AN M/G/1 QUEUING SYSTEM WITH A REMOVABLE SERVER UNDER N-POLICY

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Abstract: In this paper, we consider the balking behavior of customers in an M/G/1 queueing system with a removable server under N-policy, where the server may be turned off when no customers are present, and be turned on when the queue length reaches size N. Arriving customers decide whether to join the system or balk, based on a linear reward-cost structure that incorporates their desire for service, as well as their unwillingness for waiting. We study the balking behavior of customers in an unobservable case and derive Nash equilibrium strategies and socially optimal strategies. We also compare the resulting equilibrium and socially optimal strategies.

Key words: Queuing system; Optimization; Balking behavior; Nash equilibrium strategy; Socially optimal strategy.

1 INTRODUCTION

In past decades, there is an emerging tendency in the literature to study queueing systems from an economic viewpoint, where a reward-cost structure is imposed on the system that reflects the customers' desire for service and their unwillingness for waiting. Customers are allowed to take their own decisions and therefore the system can be modeled as a game among the customers. The basic problem is to find equilibrium strategies and (or) socially optimal strategies.

Burnetas and Economou [1] study an Markovian single-server queueing system with setup times. They derive equilibrium strategies for the customers under the various levels of information and analyze the stationary behavior of system under these strategies. Economou and Kanta [2] consider the Markovian single-server queue that alternates between on and off periods. They derive equilibrium threshold balking strategies in two cases, according to the information for the server's state. Guo and Hassin [3] study a vacation queue with N-policy and exhaustive service. They present the equilibrium and socially optimal strategies for unobservable and observable queues. This work is extended by Guo and Hassin [4] to heterogenous customers. They study both unobservable and observable queues and consider two situations regarding customers' delay sensitivity. However, in all aforementioned papers, the queueing models are studied under Markovian assumptions. Recently, Economou et al. [5] analyze the optimal balking strategies in single-sever queues with general service and vacation times.

In this paper, we consider M/G/1 queueing model with a removable server under N-policy, where the server may be turned off when no customers are present, and be turned on when the queue length reaches size N. We study the balking behavior of customers in an unobservable case and derive Nash equilibrium strategies and socially optimal strategies.

2 MODEL DESCRIPTION

Consider the N-policy M/G/1 queueing system with a single removable server. It is assumed that customers arrive according to a Poisson process with parameter λ. Service times are assumed to be independent and generally distributed according to a common probability distribution function $H(t)(t \geq 0)$ of finite first and second moments, $E(H) < \infty$ and $E(H^2) < \infty$. The server employs an N-policy, i.e., the server may be turned off when no customers are present, and be turned on when the queue length reaches size N.

Arriving customers are assumed to be identical. Our interest is in the customers' strategic response as they can decide whether to join or balk upon arrival. Assume that a customers' utility consists of a reward for receiving service minus a waiting cost. Specifically, every customer receives a reward of R units for completing service. There is a waiting cost of C units per time unit that the customer remains in the system. Customers are risk neutral and maximize their expected net benefit.

We consider an unobservable queue, i.e., an arriving customer either joins the system or balks, but it is not possible for him to observe the system state before making this decision. Also, the decisions are irrevocable and, consequently, retrials of balking customers and reneging of entering customers are not allowed.

3 EQUILIBRIUM BALKING STRATEGIES

Under the unobservable assumption, an arriving customer has two pure balking strategies: to balk or to join. With a mixed strategy, an arriving customer joins the system with a certain probability $q \in (0, 1)$.

If all arriving customers follow a strategy q, then the effective arrival rate is λq. Then, using the results of Tian and Zhang (2006), the mean sojourn time $W(q)$ of a customer who joins the system is given by

$$W(q) = \frac{N-1}{2\lambda q} + E(H) + \frac{\lambda q E(H^2)}{2[1 - \lambda q E(H)]}. \tag{3.1}$$

for $\lambda q E(H) < 1$. So, the function $W(q)$ is strictly convex in q, with minimum value

$$W(\tilde{q}) = \frac{N+1}{2} E(H) + \sqrt{(N-1)E(H^2)} \tag{3.2}$$

where

$$\tilde{q} = \left\{ \lambda E(H) + \lambda \sqrt{\frac{E(H^2)}{N-1}} \right\}^{-1}. \tag{3.3}$$

Consider a tagged customer. If this tagged customer decides to balk, then his benefit equals 0. On the contrary, if he joins the system, his expected utility is given by

$$U(q) = R - CW(q) = R - C \left\{ \frac{N-1}{2\lambda q} + E(H) + \frac{\lambda q E(H^2)}{2[1 - \lambda q E(H)]} \right\}, \tag{3.4}$$

for $q < 1/[\lambda E(H)]$.

From Eq. (3.4), it is easy to see that $U(q) = 0$ may have two different roots if $\Delta > 0$, where

$$\Delta = \left[\frac{R}{C} - \frac{N+1}{2} E(H) \right]^2 - (N-1)E(H^2). \tag{3.5}$$

The two roots are given by

$$q_1 = \frac{(N-1)E(H) + y - 2\sqrt{\Delta}}{2\lambda[E(H^2) + yE(H)]} \tag{3.6}$$

and

$$q_2 = \frac{(N-1)E(H) + y + 2\sqrt{\Delta}}{2\lambda[E(H^2) + yE(H)]} \tag{3.7}$$

where $y = 2[R/C - E(H)]$.

Theorem 3.1 *(i) If $\Delta < 0$, there exists unique equilibrium strategy is $q_e = 0$.*

(ii) If $\Delta = 0$, there are two cases: (a) If $\tilde{q} > 1$, there exists unique equilibrium strategy $q_e = 0$; (b) If $\tilde{q} \le 1$, there exists unique equilibrium strategy is $q_e = \tilde{q}$.

(iii) If $\Delta > 0$, there are three cases: (a) If $q_1 > 1$, there exists unique equilibrium strategy $q_e = 0$; (b) If $q_1 \le 1 < q_2$, there exist two positive equilibrium strategies $q_e^1 = q_1$ and $q_e^2 = 1$; (c) If $q_2 \le 1$, there exist two positive equilibrium strategies $q_e^1 = q_1$ and $q_e^2 = q_2$.

4 SOCIALLY OPTIMAL BALKING STRATEGIES

In this section, we analyze the problem of social profit optimization, where the decision problem for the social planner is to obtain a socially optimal strategy q^*, which maximizes customers' overall expected net benefit pre unit time, defined by $S(q) = \lambda q[R - CW(q)]$. For simplicity, we assume that $\lambda q E(H) < 1$.

By Eq. (3.1), the objective function $S(q)$ is given by

$$S(q) = \lambda q R - C\left\{\frac{N-1}{2} + \lambda q E(H) + \frac{(\lambda q)^2 E(H^2)}{2[1-\lambda q E(H)]}\right\}. \tag{4.1}$$

Then, this social objective function is concave and the unique optimal solution that maximize the objective function $S(q)$ is

$$\overline{q} = \frac{1}{\lambda E(H)}\left\{1 - \sqrt{\frac{E(H^2)}{yE(H) + E(H^2)}}\right\}. \tag{4.2}$$

Therefore, the social net benefit at $\overline{q}$ is given by

$$S(\overline{q}) = \lambda \overline{q} R - C\frac{N-1}{2} - C\left\{\lambda \overline{q} E(H) + \frac{(\lambda \overline{q})^2 E(H^2)}{2[1-\lambda \overline{q} E(H)]}\right\}. \tag{4.3}$$

It is interesting that $\overline{q}$ does not depend on N, but $SW(\overline{q})$ does. The following theorem gives an upper and a lower bounds on $\overline{q}$.

Theorem 4.1 *If $\Delta > 0$, then $\tilde{q} < \overline{q} < q_2$.*

The following theorem shows that there may exist unique optimal strategy or two optimal strategies depending on the different conditions in parameters.

Theorem 4.2 *(i) If $\Delta < 0$, there exists unique optimal strategy $q^* = 0$.*

(ii) If $\Delta = 0$, there exist two optimal strategies $q^* = 0$ and $q^* = \tilde{q}$.

(iii) If $\Delta > 0$, there exists unique optimal stragtey $q^* = min\{\overline{q}, 1\}$.

If $\Delta > 0$, by Theorem 4.1, we have $q_1 < \tilde{q} < \overline{q} < q_2$. Then, comparing Theorem 3.1 and Theorem 4.2, It is easy to observe that: (i) $q^* > q_e$ if $q_1 > 1$, i.e, the optimal strategy is greater than the equilibrium strategy q_e, and (ii) $q_e^1 < q^* < q_e^2$ if $q_1 \le 1$, i.e., the optimal strategy is greater than the the equilibrium strategy q_e^1 and smaller than the equilibrium strategy q_e^2.

5 CONCLUSION

The balking behavior of customers has been considered in an N-policy M/G/1 queueing system with a removable server. Upon arrival, the customers can not observe the system state and decide whether to join or balk based on a linear reward-cost structure that incorporates their desire for service, as well as their unwillingness for waiting. We have derived the equilibrium strategies and compared them to socially optimal strategies. We have found that the equilibrium strategy may be smaller or greater than the social optimal strategy for some cases.

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56 A MODEL OF NONSMOOTH STOCHASTIC SYSTEMS FOR SPONTANEOUS SIGNALING NETWORKS

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Abstract: In order to understand the phenomenon of delay-time balance at the users' side which is emerged in the spontaneous signaling networks, we propose a model to formalize the generation mechanism of delay-time balance by a stochastic linear system in a nonsmooth space. The coefficient matrix of the stochastic linear system is designated by the reverse matrix of a load-balanced Birkhoff-von Neumann (LB-BvN) switch network. The model can explain the generation mechanism of delay-time balance occurred in a spontaneous signaling network of networked control systems (NCS).

Key words: Networked control system (NCS); time-delay balance; Stochastic linear system; Filter.

1 INTRODUCTION

In networked control systems (NCS), the communication channels are embedded into the control systems. When such distributed control systems are scaled up, the nonlinear dynamics of NCS in which the stochastic characteristics of packet flow have great influence on its robustness, makes it difficult to be controlled. In the case of the Internet, there is no central controller. But the dominant principle of the Internet operations, which is the so-called end-to-end principle, allows the users to access the Internet with a high degree of transparency provided by the network protocols. With the transparency, we can analyze the dynamics mechanism of a communication network. In a networked control system, various regulations for packet flow have been used in the network protocol. When the load (service) for multiple users is heavy, e.g., in the case of the "big data" service provided by cloud computing, the balance of multiple users is necessary with respect to quality of service (QoS). The requirement on the balance in network architecture is reflected in the fairness of multiple routes, which is a required measure for Internet architecture. The direct observation of such fairness at the users' side is the delay time, which is suppressed to be proportional to their packet flow under certain threshold. For instance, the fluctuation of the delay time of multiple receivers will be constrained below certain threshold. With the flow control (a generalized form of the packet control), we can use the model of stochastic linear systems to explain the delay-time balance phenomenon of traffic flow among receivers by their delay time.

2 MODELING A SPONTANEOUS SIGNALING NETWORK BY A NONSMOOTH STOCHASTIC SYSTEM

2.1 Network abstraction

The framework of the network architecture of NCS is an abstract type of network architecture called network abstraction extracted by comparing the window size control of TCP dynamics with the signal control of networked control systems. Owing to the hierarchical structure of network protocol, we use the flow dynamics of network abstraction for the theoretical analysis of the network architecture generating the balance phenomenon in a spontaneous way. By observing the autonomic traffic flow of the Internet, we can quantitatively analyze the intrinsic characteristics of the Internet. Because of the existence of autonomy in the autonomic network architecture (Bouabene et al. (2010)), we can design a network protocol for NCS with flexibility and test new ideas independent on physical configuration of the communication systems. With the self-organization operations of NCS that regulates the traffic flow, the network abstraction for a spontaneous signaling network can be a reference for the design of the network architecture in a real world scenario, especially the sensor network whose switch structure is physically applicable to optical devices.

2.2 Delay in network abstraction

The flow of network abstraction can be measured by the observed queue, which is used for network control and supported by analysis tools in information theory. Among flow measures, the delay time is the major factor for performance evaluation. The investigation on the realistic delay in Choi et al.'s research (Choi et al. (2004)) emphasizes the significance of the delay time on the performance evaluation of resilient network control even though the delay time is often hidden by the measurable signals from terminals. Thus, the dynamical mechanism of adaptive switching in terms of an information flow network under uncertainty of environment is crucial in improving the adaptation performance of the relay network, esp., the disturbance tolerant network, based on the simulated flow dynamics of network abstraction, whose theoretical formulation is the store-and-forward principle in information theory. In switch networks, the scheduler is designed in advance. But in adaptive routing (switching), the network configuration is *ad hoc*. The empirical study can provide a test-bed to explore the new design principle for adaptive routing (switching). Based on the empirical study of network abstraction of NCS, the delay time at users is used as the measure for performance evaluation of the balance in a spontaneous signaling network.

2.3 Computational complexity of the adaptive routing processes reaching balance

To search the expected balance time is a NP problem because there is not any central control in spontaneous signaling networks. Assuming that n senders and m relay nodes are fully connected by the link with the unitary capacity, in which it equals to the relay capacity, the flow per relay node will be n. Then, the time to be used to find the configuration of the network architecture that causes the flow balance (i.e., the network structure allows all the flow will pass through the relay nodes) can be transformed as a m-CNF problem. According to Schöning's result (Schöning (1999)), the *sup* of the fastest time is

$$[2(1-1/m)]^n. \tag{2.1}$$

The quantity given above is exponential. But it can be reduced when we introduce the dynamical constraint into the problem. In order to achieve the maximum balance efficiency of transmission, it is necessary to avoid possible conflict on the relay nodes.

2.4 Delay-time balance

According to the Zipf's law with one order, the delay-time balance is formulated as follows:

$$\text{Delay Time} = \sum_i [a(i)/C(i)] + b[E(w1) - E(w2)] \tag{2.2}$$

where i is the route index; $a(i)$ is the constant; $C(i)$ is relay capacity; $w1$ and $w2$ are the random variables that refer to the chance of the flows falling into the underlying route and other routes, respectively.

The first item is given according to the formula in Choi et al.'s conclusion (Choi et al. (2004) obtained from the delay measurement in the Internet. $w1$ and $w2$ are caused by the conflicts of the flows, where b is an constant. $w1$ shows the increase of the delay by coming-in flows and $w2$ shows the decrease of the delay by leaving-away flows. Because these two items can be expressed by a Bernoulli distribution, they will approach to the equal value when the time is sufficient long. The average values of $w1$ and $w2$ are adopted by using the expectation E.

2.5 *Filtering mechanism of networked control systems*

From the viewpoint of the network abstraction, the state of network flow is a generalized term of the packet number sent by the sender in TCP, e.g., cwnd (congestion window size in TCP control). The feedback (acknowledgment signal of the receiver for the transmission) of the related communication channels is a random variable. In order to reflect two factors (the packet number sent by the sender and the packet number received by the receiver) in the network dynamics of NCS, we use Kalman filter to map the flow signaling mechanism constrained by the network abstraction into a filtering mechanism from the viewpoint of complex systems.

An abstract description of a model of NCS is defined as follows:

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t) + N(t) \\ Y(t) = CX(t) + N'(t) \end{cases} \tag{2.3}$$

where $X(t)$ is the state vector of the NCS system (the flow of the communication channel), $U(t)$ is the input vector of the NCS system, $N(t)$ is noise, and $Y(t)$ is the output of the NCS system, respectively. $N(t)$ and $N'(t)$ are noise in the communication channel. A, B and C are constant matrices.

In a spontaneous signaling network, input U is 0. We have that

$$\begin{cases} \dot{X}(t) = AX(t) + N(t) \\ Y(t) = CX(t) + N'(t) \end{cases} \tag{2.4}$$

where $X(t)$ refers to the flow of the communication channel; $Y(t)$ refers to the detectable signal for performance evaluation. When we measure the delay time at the user side, $X(t)$ is the flow controlled at the sender; $Y(t)$ is the flow measured at the receiver. A and C describe the characteristics of flow control and channel, respectively.

The delay-time balance can be observed when the system is in the steady states. Two classes of the steady states are taken into consideration in our study. One is the class that refers to the connected areas of the stable steady states described by continuous functions. The stability of the underlying dynamics defined by these steady states implies the network can still stay in a steady state under the changes of the corresponding parameters of the network dynamics. The other is the class that refers to the disconnected areas of unstable steady states whose transitions are defined in a nonsmooth space. The channel parameters such as bandwidth, loss (channel loss), and delay are the crucial factors for the network analysis. In order to systematically understand the complexity of the network flow control processes, it is necessary to use a generic filtering method to formalize the effect of the channel parameters on the performance of the flow control with respect to the complex behavior emerged from the autonomous network architecture.

The above-mentioned state representation can be rewritten as follows:

$$X(k+1) = AX(k) + N_x(t) \tag{2.5}$$

where $A(k)$ is a matrix, N_x is noise. $X(k)$ is the state which is the same as the one defined for the NCS, i.e., the flow controlled at the sender; the input is absent for a spontaneous network, k is the current moment. This equation is consistent with the flow control in network protocol.

The value of observation for $X(k)$ is given as follows:

$$Z(k) = HX(k) + N_z(k) \tag{2.6}$$

where H is the matrix, N_z is noise, $Z(k)$ is the observed value of the state $X(k)$, i.e., the value of the flow of the communication channel.

By using the Kalman filter, the estimated value of the state $X(k)$ can be obtained and is denoted as X_F.

Corresponding to the linear representation of the system, a stochastic process that describes the connection matrix of the system is used for the theoretical explanation of the generation mechanism of the balance phenomenon in a spontaneous signaling network.

Under the condition of balanced flow, the filtering mechanism given above becomes a reverse Birkhoff-von Neumann (BvN) switch (Chao and Liu (2007)) in which the connection is configured by a random process. The BvN network architecture configured by constant matrix exists. In the case of stochastic linear systems, the configuration of the connection matrix becomes a parameter estimation process of system identification. The load balance can be inferred from the BL-BvN switch structure, where the values of the connection matrix needs to be identified by using the stochastic signal processing technology.

We write the time-variant representation of NCS as follows:

$$\begin{cases} X(k+1) = A(k)X(k) + N(k) \\ Y(k) = C(k)X(k) + N'(k) \end{cases} \tag{2.7}$$

where $A(k)$ and $C(k)$ are the matrices whose parameters are random variables. These matrices reflect the stochastic characteristics of communication channels.

When the spontaneous signaling network reaches the balance of delay time, the matrix $C(k)$ equals to the reverse matrix of the connection matrix that is the multiplication of $M_1(k)$ and $M_2(k)$ of a LB-BvN switch network given in the following equation:

$$\begin{cases} S_1(k) = M_1(k)V(k) \\ S_2(k) = M_2(k)S_1(k) \end{cases} \tag{2.8}$$

where the $V(k)$, $S_1(k)$, and $S_2(k)$ are the state vectors of the input of load-balance module of a LB-BvN switch network, the queue of the intermediate node of a LB-BvN switch network, and the output of BvN switch in a LB-BvN switch network.

Let $G(k)$ be the reverse matrix of the connection matrix $M_1(k)M_2(k)$, we can write that

$$\begin{cases} X_F(k+1) = A(k)X_F(k) + N(k) \\ Y(k) = G(k)X_F(k) + N'(k). \end{cases} \tag{2.9}$$

After the value of $A(k)$ is estimated by using the LMI method in control theory under the condition of the robustness of $X_F(k)$ (defined in stochastic signal processing), we can design a NCS controller with the balance of delay time corresponding to $Y(k)$.

3 CONCLUSION

We select the balance phenomenon measured by delay time as an object to study the dynamics of the communication processes in NCS and use a Kalman filter to formulate the basic signal transmission of packet flow defined by network abstraction. The generation mechanism of the balance of delay time in a spontaneous signaling network can be explained by the filter model through the stochastic modeling of the flow dynamics, from which we expect to find a numerical calculation method for performance analysis of communication networks (Liu (2010), Liu, Yue and Umehara (2011a), Liu, Yue and Umehara (2011a), Yue and Matsumoto (2002), Walsh et al. (2011)) through the relation between delay time and channel capacity.

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57 A GATED-POLLING BASED SPECTRUM ALLOCATION MECHANISM FOR COGNITIVE USERS IN CRNS AND ITS PERFORMANCE ANALYSIS

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Abstract: In Cognitive Radio Networks (CRNs), the Primary Users (PUs) enjoy the preemptive priority on using the spectrum, and the cognitive users are able to opportunistically access the spectrum, so the packet transmission of the cognitive users may be interrupted. Considering the fairness of spectrum usage, a gated polling strategy is introduced, and a novel centralized spectrum allocation mechanism is proposed in this paper. Accordingly, a gated vacation queueing system with non-zero switchover procedure and interrupted service is built. By applying the method of a regeneration cycle, the performance of the proposed spectrum allocation mechanism is evaluated analytically, the formulas for the performance measures in terms of the average response time of SU packets, the throughput for SUs and the spectrum utility are derived. Numerical results with analysis and simulation are provided in order to investigate the dependence of the system performance on different parameters in CRNs.

Key words: CRNs; spectrum allocation; gated polling strategy; preemptive priority; non-zero switchover procedure; interrupted service.

1 INTRODUCTION

With the development of wireless technology and the proliferation of wireless applications, there has been a dramatic increase in the demand for radio spectrum. As most spectrum has been assigned to the Primary Users (PUs), for exclusive use, the spectrum has become a scare resource. However, most of the already allocated radio bands are either not used, or are sporadically used. Existing spectrum measurement reports indicate that up to 85% of the spectrum remains unoccupied at any given time and location (Gao et al. (2011)). Therefore, developing an efficient spectrum allocation mechanism with high utilization becomes a matter of great importance. Recently, many experts have probed related research on spectrum allocation strategies and performance analysis in the context of Cognitive Radio Networks (CRNs).

Do et al. (2012) considered a sensitive delay network, and proposed an algorithm to distribute the packets of the Secondary Users (SUs) to the only group of channels which satisfy the delay constraint. By applying an M/G/1 queueing model, the performance of SU was analyzed. Wu et al. (2012) offered a channel hopping defense strategy through the interaction between SUs and stackers, and then proposed two learning schemes. The Nash equilibrium for the Colonel Blotto game to minimize the worst-case damage was derived accordingly. In order to guarantee the usage fairness of spectrum, by pre-selecting a set of SUs based on their interference to PUs, Li et al. (2012) offered a modified hybrid opportunistic scheduling method, and optimized the throughput of SUs.

In this paper, for the purpose of guaranteeing every SU access to the unoccupied spectrum fairly, we propose a novel spectrum allocation mechanism based on a gated polling strategy. By establishing a gated vacation queueing model with non-zero switchover procedure and interrupted service, we evaluate the system performance for SUs in the proposed spectrum allocation mechanism, and optimize the arrival rate of PU packets in CRNs.

2 A GATED-POLLING BASED SPECTRUM ALLOCATION MECHANISM AND SYSTEM MODEL

In CRNs with centralized spectrum allocation mechanisms, PUs enjoy preemptive priority at all times, and a Central Scheduler (CS) allocates the spectrum band that is temporarily unused by PUs to SUs. Due to the existence of the CS, the centralized spectrum allocation mechanism can satisfy diverse needs of different users and restrain users from mutual interference as much as possible. In order to guarantee fairness in the use of the spectrum bands and to capture the digital nature of modern communication, by introducing a gated polling strategy, a novel centralized spectrum allocation mechanism with a slotted structure is proposed.

In this system model, we consider that there is one spectrum band, one PU, and several SUs in the CRNs. The time axis is divided into a sequence of fixed length intervals, called slots. Moreover, we present the detailed system model for the performance analysis as follows:

(1) At the beginning instant of each slot, the SUs sense the spectrum bands for PU activity (idle or busy) and send the sensing results to the CS. Each SU is polled in a fixed cyclic order with slotted structure. If there are no packets ready for transmission in the tagged SU at the polling beginning instant, the CS will experience a non-zero switchover procedure, and try to allocate the spectrum band to the next SU. If the SU being polled has packets to be transmitted, these packets will be transmitted following a gated polling strategy.

(2) For each SU, after all the packets present at the polling beginning instant finish their transmission, the CS will allocate the spectrum band to the next SU with a non-zero switchover procedure, while all the SU packets that arrived during the ongoing transmission procedure will be waiting in the buffer of the SU. If there is no transmission interruption due to the arrival of PUs, the tagged SU will occupy one spectrum band throughout its transmission period. Otherwise, the tagged SU will drop the packet being transmitted at the interruption instant, and apply for another spectrum band from the CS, then all the remaining SU packets will be transmitted following a gated polling strategy. In this way, fairness for user Quality of Service (QoS) can be guaranteed.

Based on the mentioned description about the system model presented in this paper above, we can make the following assumptions. The SU packet to be transmitted is regarded as a customer, the spectrum band is regarded as a server, the transmission of an SU packet is regarded as a service, and the transmission interruption is regarded as a service interruption. Moreover, the time period elapsed from the instant that the tagged SU occupies the spectrum band to the instant that the tagged SU releases the spectrum band is regarded as a service period, denoted as S_p. The time period elapsed from the instant that the tagged SU releases the spectrum band, to the instant that the tagged SU occupies the spectrum band again, is regarded as a vacation period, denoted as V. The time interval that begins at the end of a vacation and terminates at the end of the next vacation is defined as a service cycle, denoted as R. A service cycle consists of a service period and a vacation period.

Therefore, the proposed spectrum allocation mechanism can be modelled as a gated vacation queue with non-zero switchover procedure and interrupted service.

3 PERFORMANCE ANALYSIS

Considering the slotted structure of the gated-polling based spectrum allocation mechanism, we assume that the SU packets arrive at the end of a slot, the initiation and termination of the transmission for an SU packet occurs at the beginning of a slot. We assume that SU packet arrivals follow a Bernoulli process with arrival rate p, and the transmission time S of an SU packet follows a geometric distribution with transmission rate μ. Furthermore, we also assume that the packets are transmitted according to a First-In First-Out (FIFO) strategy in an SU.

In CRNs, PU packets are allowed to arrive at any slot, and has preemptive priority on using the spectrum. Let α be the arrival rate of PU packets. This means the transmission of SU packets may be interrupted by PU packets, and the interruption probability in a slot can be considered as the arrival rate α of PU packets.

When a PU accesses to a spectrum band during the transmission procedure of an SU packet, the transmission of this SU packet will be interrupted, and the interrupted SU packet will be dropped from the system immediately. Therefore, the actual transmission time T of an SU packet is shorter than the transmission time S. The probability distribution and the Probability Generating Function (PGF) $T(z)$ of T can be represented as follows:

$$P\{T=k\}=\bar{\mu}^{k-1}\mu\bar{\alpha}^{k-1}+\bar{\mu}^{k}\bar{\alpha}^{k-1}\alpha,\ k\geq 1,\qquad T(z)=\sum_{k=1}^{\infty}z^{k}P\{T=k\}=\frac{(\mu+\bar{\mu}\alpha)z}{(1-\bar{\mu}\bar{\alpha}z)} \tag{3.1}$$

where $\bar{\mu}=1-\mu$, $\bar{\alpha}=1-\alpha$.

Differentiating Eq. (3.1) with respect to z at $z=1$, we can give the average value $E[T]$ of T as follows:

$$E[T]=T'(z)|_{z=1}=\frac{1}{1-\bar{\mu}\bar{\alpha}}. \tag{3.2}$$

We can also obtain the secondary factorial moment $E[T(T-1)]$ of T from Eq. (3.1).

Let P_I be the probability that the transmission of an SU packet is interrupted by the arrival of PU, and let P_{NI} be the probability that the transmission of an SU packet is transmitted successfully without interruption. P_I and P_{NI} can be obtained as follows:

$$P_I=\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}P\{S=k+m\}\bar{\alpha}^{k-1}\alpha=\frac{\bar{\mu}\alpha}{1-\bar{\mu}\bar{\alpha}},\qquad P_{NI}=\sum_{k=1}^{\infty}P\{S=k\}\bar{\alpha}^{k-1}=\frac{\mu}{1-\bar{\mu}\bar{\alpha}}.$$

Let Q_b be the number of SU packets which exist in the system at the end instant of a vacation V. Namely, Q_b is the number of the SU packets that have arrived during the previous service cycle. Let T_{Sp} be the time length of a service period S_p, and $T_{Sp}(z)$ be the PGF of T_{Sp}. Let T_V be the time length of vacation period V, and $T_V(z)$ be the PGF of T_V. Then the PGF $Q_b(z)$ of Q_b can be given as follows:

$$Q_b(z)=T_{Sp}(\lambda(z))T_V(\lambda(z)) \tag{3.3}$$

where $\lambda(z)=1-p(1-z)$ is the PGF for the number of SU packets arrived within a single slot.

Suppose there are N SUs in the CRN discussed in this paper. Meanwhile, we define the traffic load ρ for SUs as $\rho=Np/\mu$. Considering the identical stochastic and symmetry characteristics of all the SUs, $T_{Sp}(z)$ and $T_V(z)$ can be given as follows:

$$T_{Sp}(z)=Q_b(T(z)),\qquad T_V(z)=z^{Nw}T_{Sp}^{N-1}(z) \tag{3.4}$$

where w is the non-zero switchover time as a system parameter.

Substituting Eq. (3.4) into Eq. (3.3), $Q_b(z)$ follows that

$$Q_b(z)=Q_b^N(T(\lambda(z)))\lambda^{Nw}(z). \tag{3.5}$$

Differentiating Eq. (3.5) with respect to z at $z=1$ and combining with Eq. (3.2), we can give the average value $E[Q_b]$ of Q_b as follows:

$$E[Q_b]=Q_b'(z)|_{z=1}=\frac{Npw}{1-NpE[T]}. \tag{3.6}$$

In this gated vacation queueing model, the number Φ of SU packets transmitted during a service cycle is identical to the number Q_b of SU packets which exist in the system at the end of a vacation.

Differentiating Eq. (3.4) with respect to z at $z=1$, combining with Eqs. (3.2) and (3.6), the average value $E[T_{Sp}]$ of T_{Sp} and the average value $E[T_V]$ of T_V can be given respectively as follows:

$$E[T_{Sp}]=T_{Sp}'(z)\big|_{z=1}=\frac{NwpE[T]}{1-NpE[T]},\qquad E[T_V]=T_V'(z)|_{z=1}=\frac{Nw(1-pE[T])}{1-NpE[T]}.$$

Therefore, the average value $E[T_R]$ of T_R is $E[T_R]=E[T_{Sp}]+E[T_V]=(Nw)/(1-NpE[T])$.

Letting L_n be the number of SU packets in the system immediately after the transmission termination of the nth SU packet, and letting A_i $(i=1,2,...,n)$ be the number of SU packets arrived during the actual transmission time of the ith SU packet, L_n can be obtained as follows:

$$L_n=Q_b-n+A_1+A_2+\cdots+A_n,\qquad n=1,2,...,Q_b. \tag{3.7}$$

Let L be the number of SU packets in steady state. By using the method of a regeneration cycle, the PGF $L(z)$ of L is given as follows:

$$L(z) = \frac{1}{E[\Phi]} E\left[\sum_{n=1}^{\Phi} z^{L_n}\right] = \frac{(1-pE[T])(1-z)T(\lambda(z))}{T(\lambda(z))-z} \times \frac{1-T_V(\lambda(z))}{pE[T_V](1-z)} \times Q_b(T(\lambda(z))). \quad (3.8)$$

Differentiating Eq. (3.8) with respect to z at $z=1$, we can give the average value $E[L]$ of L as follows:

$$E[L] = L'(z)|_{z=1} = pE[T] + \frac{p^2 E[T(T-1)]}{2(1-pE[T])} + \frac{pE[T_V(T_V-1)]}{2E[T_V]} + \frac{p^2 E[T]E[T_V]}{1-pE[T]}. \quad (3.9)$$

4 PERFORMANCE MEASURES

We define the average response time σ of SU packets as the time period in slots that has elapsed from the arrival instant of an SU packet to the transmission termination instant for that SU packet. Combining with Eq. (3.9), and applying Little's Law (Jin et al. (2011) and Xu et al. (2008)), σ can be given as follows:

$$\sigma = \frac{E[L]}{p} = \frac{pE[T(T-1)]}{2(1-pE[T])} + \frac{E[T_V(T_V-1)]}{2E[T_V]} + \frac{NwpE[T]}{1-NpE[T]} + E[T]. \quad (4.1)$$

The throughput η for SUs is defined as the average number of SU packets transmitted successfully per slot, excluding the SU packets interrupted by PUs. The throughput η for SUs is given as follows:

$$\eta = NpP_{NI} = \frac{Np\mu}{1-\bar{\mu}\bar{\alpha}}. \quad (4.2)$$

We define the spectrum utility γ as the fraction of the time spent on transmitting SU packets normally without being interrupted to the time length T_R of a service cycle R. γ can be given as follows:

$$\gamma = \frac{E[\Phi]P_{NI}E[T]}{E[T_R]} = \frac{Np\mu}{(1-\bar{\mu}\bar{\alpha})^2}. \quad (4.3)$$

5 NUMERICAL RESULTS AND DISCUSSIONS

We set the system parameters as follows: the number N of SUs in the system is $N = 8$, the transmission rate μ of an SU packet is $\mu = 5/6$, the time length w of the switchover procedure between SUs is $w = 4$ in slots.

Figure 5.1 depicts the average response time σ of SU packets as a function of the offered traffic load ρ for SUs with different arrival rates α of PU packets.

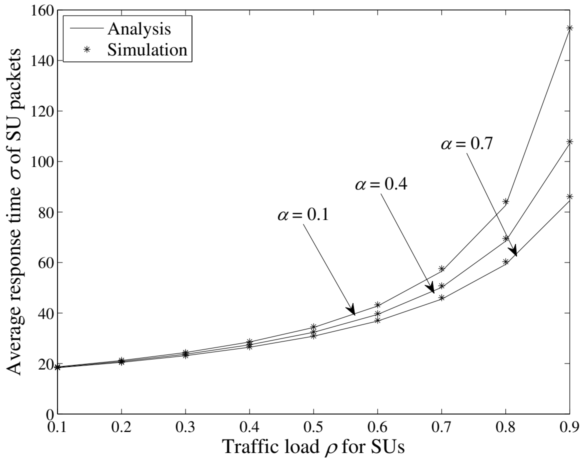

Figure 5.1 Average response time σ of SU packets vs. traffic load ρ for SUs.

It is interesting to observe that for the same traffic load ρ for SUs, the higher the arrival rate α of PU packets is, the less the average response time σ of SU packets will be. On the other hand, for the same arrival rates α of PU packets, the larger the traffic load ρ for SUs is, the greater the average response time σ of SU packets will be.

In Fig. 5.2, we show the throughput η for SUs versus the traffic load ρ for SUs with different arrival rates α of PU packets.

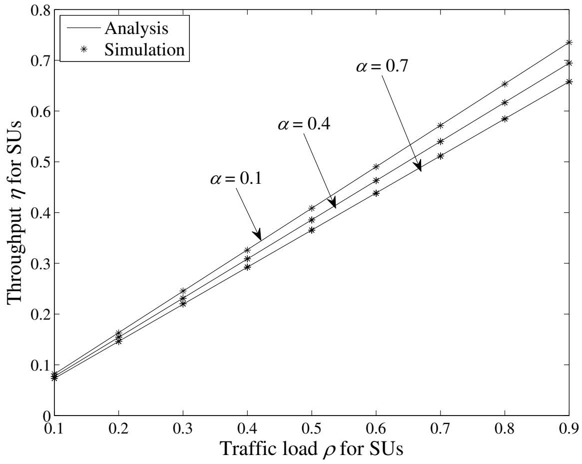

Figure 5.2 Throughput η for SUs vs. traffic load ρ for SUs.

From Fig. 5.2, we can find that for a certain traffic load ρ for SUs, the throughput η for SUs will decrease along with an increase in the arrival rate α of PU packets. On the other hand, for the same arrival rates α of PU packets, the throughput η for SUs will increase as the traffic load ρ for SUs increases.

In Fig. 5.3, we plot the function of the spectrum utility γ versus the traffic load ρ for SUs with respect to different arrival rates α of PU packets.

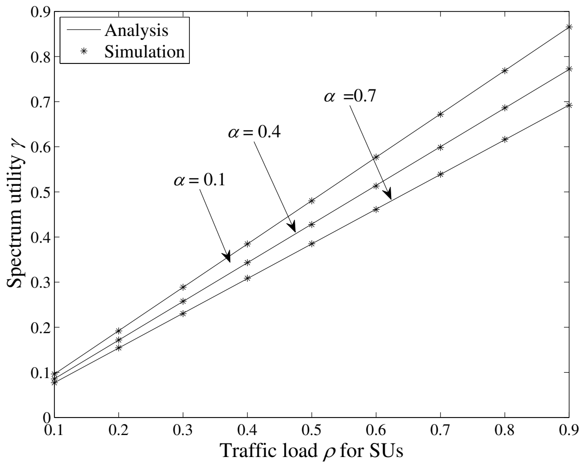

Figure 5.3 Spectrum utility γ vs. traffic load ρ for SUs.

It is demonstrated that when the traffic load ρ for SUs takes the same value, the spectrum utility γ will decrease as the arrival rate α of PU packets increases. On the other hand, for the same arrival rates α of PU packets, the spectrum utility γ will increase as the traffic load ρ for SUs increases.

6 CONCLUSION

How to improve the spectrum utilization in cognitive radio networks is currently one of the most important issues in wireless communication systems. In this paper, to improve the spectrum utilization as well as to guarantee the usage fairness of spectrum bands, we proposed a gated-polling based spectrum allocation mechanism in Cognitive Radio Networks (CRNs), and built a gated vacation queueing model accordingly. By using the method of regeneration cycle, we gave the formulas of the average response time of SU packets, the throughput for SUs and the spectrum utility. Moreover, we provided numerical results and demonstrated how the traffic load for SUs impacts the system performance for different arrival rates of PU packets. The numerical results show that there is a tradeoff among different performance measures when setting the arrival rate of PU packets.

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58 A NOVEL DYNAMIC CHANNEL BONDING STRATEGY IN COGNITIVE RADIO NETWORKS AND ITS PERFORMANCE EVALUATION

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Abstract: In conventional cognitive radio networks with primary users and secondary users, all the channels will be active even when there are no packets to be transmitted, obviously, this will lead to a waste of network resources. In order to conserve the network resources and to guarantee the Quality of Service (QoS) for secondary users, we propose a dynamic channel bonding strategy, in which channels are bonded dynamically based on the traffic in cognitive radio networks. We consider the digital nature of modern communication and the preemptive priority of the primary users in cognitive radio networks. Based on the working principle of a dynamic channel bonding strategy, we build a discrete-time preemptive priority queueing model by regarding the time period when part channels are bonded as a working vacation period. To get the steady state distribution of the queueing model, we construct a three-dimensional Markov chain, and give the state transition probability matrix of the Markov chain. Correspondingly, we derive the performance measures in terms of the blocking ratio, the throughput, the average latency of the secondary users, and the channel closed ratio. Moreover, numerical results are provided to shown the influence of the proportion of the closed channels during the Part Bonding Period on the system performance.

Key words: Cognitive radio networks; channel bonding; discrete-time priority queue; working vacation.

1 INTRODUCTION

Nowadays, the increasing demand for radio spectrum stimulated the study for the efficient use of the spectrum recourses. However, a great number of research studies have indicated that the utilization of the spectrum is very low in practical networks (Marinho (2012)). For example, most of the spectrum utilization was no more than 6% (Zhao (2007)). As a promising technology for improving the spectrum utilization, cognitive radio networks have emerged (Jha (2011)).

There are two types of users in cognitive radio networks, namely, primary users (PUs) and secondary users (SUs) (Wang (2011)). The network spectrum is licensed to the PUs while the SUs access to the spectrum opportunistically when the spectrum is not occupied by any PUs.

In research of cognitive radio networks, a channel bonding strategy is one spectrum enhancement technology with which the available channels are aggregated into one channel (Ren (2012)). There have been several researches focused on the study of cognitive radio networks with channel bonding strategy.

Lee et al. (Lee (2010)) considered a kind of channel bonding scheme in which an SU can utilize the bandwidth consisting of multiple available channels. The loss probability and throughput were obtained with a continuous-time Markov chain. Jiao et al. (Jiao (2010)) assumed the channel occupancy time of an

SU is inversely proportional to the number of bonded channels, and investigated the loss probability and throughput of SUs. Su et al. (Su (2008)) developed a Markov chain model and an M/G/1 queueing model to analyze the cognitive radio networks with two channel sensing policies, and obtained the aggregate throughput.

As shown above, most of the researches about cognitive radio networks with channel bonding strategy was performed under the condition that all the channels are active even when there is no packet to be transmitted. Obviously, this will lead to the waste of network resources. For this, by introducing a Part Bonding Period in this paper, we propose a dynamic channel bonding strategy, in which only part channels are bonded to be active for a time period when there is no packet to be transmitted. In this paper, taking into account the working principle of the dynamic channel bonding strategy and the digital nature of modern networks, we build a discrete-time queueing model with preemptive priority and working vacation. Accordingly, we derive the performance measures of the system such as the blocking ratio, the throughput, the average latency of SUs and the channel closed ratio are obtained. To the best of our knowledge, this is the first paper related to the channel bonding strategy with dynamic channel closed scheme in cognitive radio networks.

2 A DYNAMIC CHANNEL BONDING STRATEGY IN COGNITIVE RADIO NETWORKS

We consider a cognitive radio network with a licensed spectrum that is equally divided into N channels. Normally, all the N channels will be aggregated into one bonding channel. When there is no packet to be transmitted, a part of the channels will be closed, and the unclosed channels will be aggregated into one bonding channel for a random period. We call the time period that the part of the channels are closed as a “Part Bonding Period”. During the Part Bonding Period, the packets will be transmitted with a lower transmission rate. On the other hand, the time period when all channels are bonded so as to become active is called a “Full Bonding Period”. During the Full Bonding Period, the packets can be transmitted with a higher transmission rate.

If there has been no any packet arrival during a Part Bonding Period, another Part Bonding Period will be continued. If a PU packet arrives at the system during a Part Bonding Period, due to the priority of the PUs, the Part Bonding Period will terminate immediately, and a Full Bonding Period will begin. Then this PU packet will be transmitted with higher transmission rate. During a Part Bonding Period, SU packets will be transmitted with a lower transmission rate. If the transmission of an SU packet is not finished before the end instant of a Part Bonding Period, a Full Bonding Period will begin after this Part Bonding Period is over, and the SU packet will be transmitted with a higher transmission rate sequentially. If there is no SU packet to be transmitted when a Part Bonding Period is over, another Part Bonding Period will begin.

3 SYSTEM MODEL AND PERFORMANCE ANALYSIS

We assume the arriving intervals and transmission times of the packets are independent and identically distributed (i.i.d) random variables. The arriving intervals of PU packets and SU packets are supposed to follow geometrical distributions with parameters λ_1 ($\overline{\lambda}_1 = 1 - \lambda_1$) and λ_2 ($\overline{\lambda}_2 = 1 - \lambda_2$), respectively. The transmission time of a PU packet is assumed to follow a geometrical distribution with parameter μ_1 ($\overline{\mu}_1 = 1 - \mu_1$). The transmission times of an SU packet during a Part Bonding Period and a Full Bonding Period are supposed to follow geometrical distributions with rates μ_{2v} ($\overline{\mu}_{2v} = 1 - \mu_{2v}$) and μ_{2b} ($\overline{\mu}_{2b} = 1 - \mu_{2b}$), respectively. Additionally, the time length T_V of a Part Bonding Period is assumed to follow a geometrical distribution with parameter θ. θ is called “Part Bonding Rate” in this paper. During a Part Bonding Period, the proportion of the closed channels is defined as α. The buffer capacity of the SUs is defined to be finite with size H ($H > 0$), and the PUs are supposed to have no buffer. Moreover, the transmissions of the SU packets are supposed to follow a First-Come First-Served (FCFS) strategy.

We suppose the time axis is divided into slots with equal length. The slot boundaries are marked by $t = 1, 2, \ldots$. The packets are supposed to arrive immediately after the beginning instant of a slot, and depart just prior to the end of a slot. We consider the instant $t = n$ ($n = 1, 2, \ldots$) and suppose the arrivals of packets can only occur in (n, n^+), and the departures of packets can only occur in (n^-, n). Let L_n and $L_n^{(1)}$ be the number of all packets and the number of PU packets in the system at the instant $t = n^+$, respectively. Let K_n indicate the system stage. K_n can be described as follows:

$$K_n = \begin{cases} 0, & \text{the system is in the Part Bonding Period at the time } t = n^+ \\ 1, & \text{the system is in the Full Bonding Period at the time } t = n^+. \end{cases}$$

Therefore, $\{L_n, L_n^{(1)}, K_n\}$ constitutes a three-dimensional Markov chain. The state space of this three-dimensional Markov chain can be given as follows:

$$\boldsymbol{\Omega} = \{(i,j,k) : 0 \le i \le H+1,\ j=0,\ k=0\} \cup \{(i,j,k) : 1 \le i \le H+1,\ j=0,1,\ k=1\}.$$

Using the lexicographical sequence for the states, the state transition probability matrix of the three-dimensional Markov chain can be written as follows:

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{A}_0 & \boldsymbol{B}_0 & \boldsymbol{C}_0 & & & & \\ \boldsymbol{D}_0 & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & & & \\ & \boldsymbol{D} & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ & & & \boldsymbol{D} & \boldsymbol{A} & \boldsymbol{B}+\boldsymbol{C} & \\ & & & & \boldsymbol{D} & \boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C} & \end{pmatrix}_{(H+1)\times(H+1)}$$

where $\boldsymbol{A}_0 = \overline{\lambda}_1\overline{\lambda}_2,\ \boldsymbol{B}_0 = (\overline{\lambda}_1\lambda_2\overline{\theta}, \overline{\lambda}_1\lambda_2\theta, \lambda_1\overline{\lambda}_2),\ \boldsymbol{C}_0 = (0,0,\lambda_1\lambda_2),\ \boldsymbol{D}_0 = (\overline{\lambda}_1\overline{\lambda}_2\mu_{2v}, \overline{\lambda}_1\overline{\lambda}_2\mu_{2b}, \overline{\lambda}_1\overline{\lambda}_2\mu_1)^T$,

$$\boldsymbol{A} = \begin{pmatrix} \overline{\lambda}_1(\overline{\lambda}_2\overline{\mu}_{2v} + \lambda_2\mu_{2v})\overline{\theta} & \overline{\lambda}_1(\overline{\lambda}_2\overline{\mu}_{2v} + \lambda_2\mu_{2v})\theta & \lambda_1\overline{\lambda}_2\mu_{2v} \\ 0 & \overline{\lambda}_1(\overline{\lambda}_2\overline{\mu}_{2b} + \lambda_2\mu_{2b}) & \lambda_1\overline{\lambda}_2\mu_{2b} \\ 0 & \overline{\lambda}_1\lambda_2\mu_1 & \overline{\lambda}_2(\lambda_1\mu_1 + \overline{\mu}_1) \end{pmatrix},$$

$$\boldsymbol{B} = \begin{pmatrix} \overline{\lambda}_1\lambda_2\overline{\mu}_{2v}\overline{\theta} & \overline{\lambda}_1\lambda_2\overline{\mu}_{2v}\theta & \lambda_1(\overline{\lambda}_2\overline{\mu}_{2v} + \lambda_2\mu_{2v}) \\ 0 & \overline{\lambda}_1\lambda_2\overline{\mu}_{2b} & \lambda_1(\overline{\lambda}_2\overline{\mu}_{2b} + \lambda_2\mu_{2b}) \\ 0 & 0 & \lambda_2(\lambda_1\mu_1 + \overline{\mu}_1) \end{pmatrix},$$

$$\boldsymbol{C} = \begin{pmatrix} 0 & 0 & \lambda_1\lambda_2\overline{\mu}_{2v} \\ 0 & 0 & \lambda_1\lambda_2\overline{\mu}_{2b} \\ 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{D} = \begin{pmatrix} \overline{\lambda}_1\overline{\lambda}_2\mu_{2v}\overline{\theta} & \overline{\lambda}_1\overline{\lambda}_2\mu_{2v}\theta & 0 \\ 0 & \overline{\lambda}_1\overline{\lambda}_2\mu_{2b} & 0 \\ 0 & \overline{\lambda}_1\overline{\lambda}_2\mu_1 & 0 \end{pmatrix}.$$

The structure of the transition probability matrix $\boldsymbol{P}$ indicates that the three-dimensional Markov chain $\{(L_n, L_n^{(1)}, K_n)\}$ is non-periodic, irreducible and positive recurrent. The steady-state distribution $\pi_{i,j,k}$ of the three-dimensional Markov chain is defined as follows:

$$\pi_{i,j,k} = \lim_{n\to\infty} P\{L_n = i, L_n^{(1)} = j, K_n = k\}. \tag{3.1}$$

Let $\boldsymbol{\Pi}_i$ be the steady-state probability vector for the system being at level i. $\boldsymbol{\Pi}_i$ can be given as follows:

$$\boldsymbol{\Pi}_i = \begin{cases} \pi_{0,0,0}, & i=0 \\ (\pi_{i,0,0},\ \pi_{i,0,1},\ \pi_{i,1,1}), & 1 \le i \le H+1. \end{cases} \tag{3.2}$$

$\boldsymbol{\Pi}_i$ can be calculated by solving the following equilibrium equations with the normalization condition:

$$\begin{cases} (\boldsymbol{\Pi}_0, \boldsymbol{\Pi}_1, \ldots, \boldsymbol{\Pi}_H, \boldsymbol{\Pi}_{H+1})\boldsymbol{P} = (\boldsymbol{\Pi}_0, \boldsymbol{\Pi}_1, \ldots, \boldsymbol{\Pi}_H, \boldsymbol{\Pi}_{H+1}) \\ (\boldsymbol{\Pi}_0, \boldsymbol{\Pi}_1, \ldots, \boldsymbol{\Pi}_H, \boldsymbol{\Pi}_{H+1})\boldsymbol{e} = 1 \end{cases} \tag{3.3}$$

where $\boldsymbol{e}$ is a column vector with $H+1$ elements, all of which equal 1.

By substituting Eq. (3.2) to Eq. (2.2) and using a Gaussian elimination method, we can obtain the steady-state distribution $\pi_{i,j,k}$ of Eq. (3.1).

Let ${L_n}^{(2)}$ be the number of SU packets in the system at the instant $t = n^+$ and let $L^{(2)} = \lim_{n\to\infty} L_n^{(2)}$ be the steady-state distribution of $L_n^{(2)}$. The average number $E[L^{(2)}]$ of SU packets in steady state can be given by

$$E[L^{(2)}] = \sum_{j=0}^{H+1} jP\{L^{(2)} = j\} = \sum_{j=1}^{H+1} j(\pi_{j,0,0} + \pi_{j,0,1}) + \sum_{j=0}^{H} j\pi_{j+1,1,1}. \tag{3.4}$$

4 PERFORMANCE MEASURES

The blocking ratio P_B of SUs is defined as the probability that a new arrival SU packet is blocked by the system. P_B can be given as follows:

$$P_B = \lambda_2((\overline{\mu}_{2v}+\mu_{2v}\lambda_1)\pi_{H+1,0,0}+(\overline{\mu}_{2b}+\mu_{2b}\lambda_1)\pi_{H+1,0,1}+(\overline{\mu}_1+\mu_1\lambda_1)\pi_{H+1,1,1}+\overline{\mu}_{2v}\lambda_1\pi_{H,0,0}+\overline{\mu}_{2b}\lambda_1\pi_{H,0,1}). \tag{4.1}$$

The throughput S of SUs is defined as the number of SU packets transmitted successfully per slot by the licensed spectrum. S can be given as follows:

$$S = \lambda_2 - P_B - \lambda_1(\overline{\mu}_{2v}\pi_{H+1,0,0} + \overline{\mu}_{2b}\pi_{H+1,0,1}). \tag{4.2}$$

The latency of an SU packet is the sojourn time of that SU packet. By using Little's formula, the average latency $E[T]$ of SUs can be given as follows:

$$E[T] = \frac{E[L^{(2)}]}{S}. \tag{4.3}$$

The channel closed ratio β of the system is defined as the probability that one channel is closed during a Part Bonding Period. β can be given as follows:

$$\beta = \alpha \sum_{i=0}^{H+1} \pi_{i,0,0}. \tag{4.4}$$

5 NUMERICAL RESULTS

In the numerical results, the parameters of the system are set as follows: The Part Bonding Rate is assumed to be $\theta = 0.05, 0.10, 0.15$. The data set for the proportion of the closed channels is supposed to be $\alpha = \{0,\ 0.05,\ 0.1,\ \ldots,\ 0.95,\ 1\}$, and the buffer capacity of SUs is set to be $H = 5$.

In Fig. 5.1, we show how the throughput S of SUs changes as a function of the proportion α of the closed channels for different Part Bonding Rates θ.

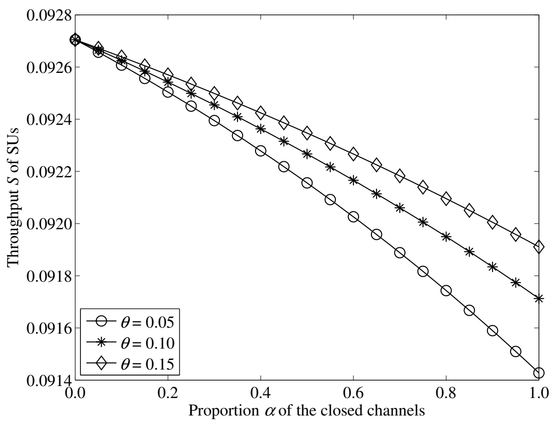

Figure 5.1 Throughput S of SUs vs. proportion α of the closed channels.

From Fig. 5.1, we observe that, for the same Part Bonding Rate θ, the throughput S of SUs will decrease as the proportion α of the closed channels increases. On the other hand, for the same proportion α of the closed channels, the larger the Part Bonding Rate θ is, the greater the throughput S of SUs will be.

We examine the effect for the proportion α of the closed channels on the average latency $E[T]$ of SUs for different Part Bonding Rates θ in Fig. 5.2.

In Fig. 5.2, we can conclude that for the same Part Bonding Rate θ, the average latency $E[T]$ of SUs will increase as the proportion α of the closed channels increases. On the other hand, for the same proportion α of the closed channels, the larger the Part Bonding Rate θ is, the shorter the average latency $E[T]$ of SUs will be.

In Fig. 5.3, we show how the channel closed ratio β changes versus the proportion α of the closed channels for different Part Bonding Rates θ.

As illustrated in Fig. 5.3, for the same Part Bonding Rate θ, the channel closed ratio β will increase as the proportion α of closed channel increases. On the other hand, for the same proportion α of the closed channels, the higher the Part Bonding Rate θ is, the smaller the channel closed ratio β will be.

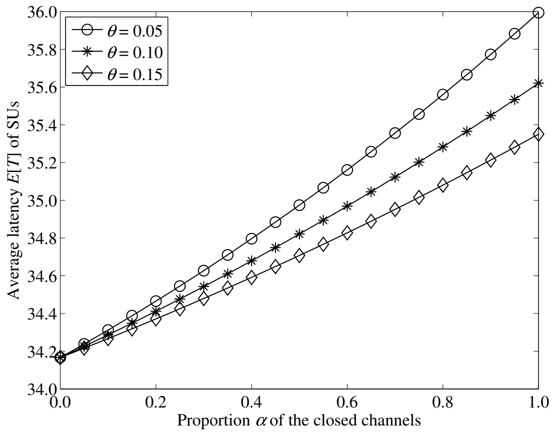

Figure 5.2 Average latency $E[T]$ of SUs vs. proportion α of the closed channels.

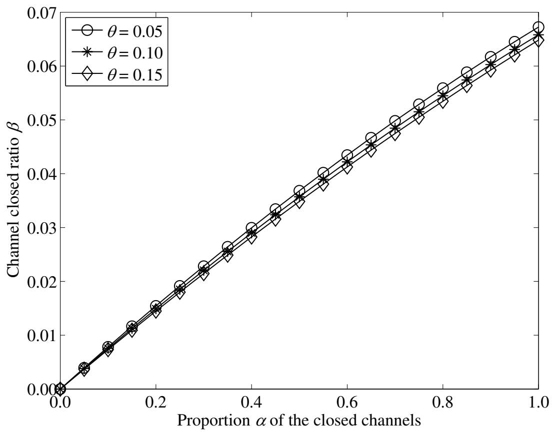

Figure 5.3 Channel closed ratio β vs. proportion α of the closed channels.

6 CONCLUSIONS

In this paper, when considering the conservation of network resources and the guarantee of QoS for the secondary users (SUs), we proposed a novel dynamic channel bonding strategy in cognitive radio networks. Based on the working principle of the dynamic channel bonding strategy and the priority of the primary users (PUs) in cognitive radio networks, a discrete-time preemptive priority queueing model with a working vacation was built. The steady-state distribution of the system model was analyzed with a three-dimensional Markov chain. The formulas for the blocking ratio, the throughput, the average latency of the secondary users, and the channel closed ratio were derived to evaluate the system performance. The numerical results show that the dynamic channel bonding strategy proposed in this paper can effectively improve the system performance.

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59 NEW PRODUCT DEVELOPMENT IN A SUPPLY CHAIN

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Abstract: New product development is an important role in enterprises. Today, firms competition becomes chains competition. Then, how to cooperate in a supply chain for the new product development? In this paper, we study three main R & D cooperation modes for new product development: that is dominated by the upstream, by the downstream, and collaborated by both members, in a supply chain with one upstream (supplier) and one downstream (manufacturer). We present a mathematical model and derive equilibrium for each of them. Numerical analysis further compares the three modes. We find that the new product will have the highest quality level and then the highest customers total welfare in the mode that is dominated by the downstream. While the mode dominated by the upstream has the smallest ones. Moreover, the firm will obtain the highest profit among the three modes if it is the dominator of the R & D activities.

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60 OPTIMAL SELECTION OF ENVIRONMENTALLY FRIENDLY PRODUCTS IN A GREEN SUPPLY CHAIN WITH RISK AVERSION

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Abstract: In this study, we investigate the selection of environmentally friendly products (EFPs) with the consideration of the tradeoff between risk and return of players in two cases of supply chain structures: vertical integration and a decentralized setting. The objectives are to maximize the utilities of players, subject to particular constraints. A numerical example of a green supply chain on household electrical appliance in China is presented to illustrate related issues. The results suggest that both supply chain structure and risk attitude have significant impacts on environmental performance.

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61 COIL RETRIEVAL PLANNING WITH SUBSTITUTION

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Abstract: In process industry, warehouse management decouples the rhythm among each production process. In a planning horizon, a set of coils is required to be retrieved from the warehouse due to downstream production process. Coils are usually stored in two levels in the warehouse. Retrieval of the lower level coil will lead to shuffling. As a result of the similarity of coils, substitution decisions between required coils and candidate coils will be considered. We formulate the problem as a mixed integer linear program and solve it based on some analysis results.

Key words: Warehouse; Coil retrieval; Substitution; Shuffle; Properties.

1 INTRODUCTION

Warehouse management in steel industry decouples the rhythm among each production process. Fig. 1.1 shows a main production process of steel production. Products in each warehouse are produced from the upstream process and will be raw materials of the downstream process. One of the operations in the warehouse that highly affect the efficiency of production and transportation is the retrieval request performed by crane. If required coils cannot be retrieved in time, downstream production will be intermitted which will bring a large cost. On the other hand, the warehouse may be lack of enough space for incoming products which will obstruct the upstream production progress.

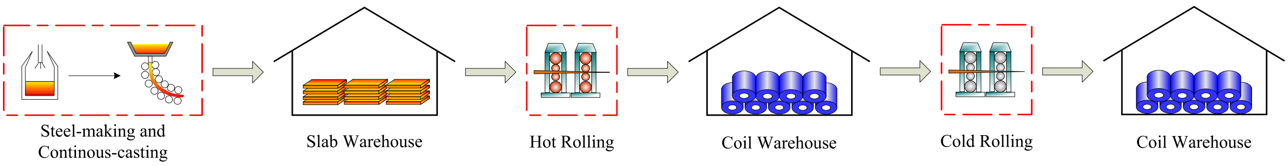

Fig. 1.1 Main process of steel production

In the warehouse, coils are arranged in rows. According to technological requirements, coils are stored in at most two levels for an upper level coil is on top of two lower level coils, and a lower level coil may be blocked by two upper level coils which seem like triangles. A retrieval request can be performed directly if the target coil is stored at the upper level or stored at the lower level and not being blocked by coils at the upper level. If the coil is stored at the lower level and there are one or two coils at the upper level blocking it, the retrieval request can be performed after the blocking coils shuffled to other empty positions.

The collision between large-scale production of steel enterprises and multi-variety with small batch of customer demand results in the surplus products. They also storage in the warehouse, but there is no requirement for them. Some of them are same or similar with a required coil in steel grade and other technical specifications. Downstream production will be satisfied with such a substitution, while sometimes the manufacturers may pay a cost for the substitution which will decease the shuffling times for retrieve the substitute coil. For a given set of required coils and candidate set of each required coil, we make decisions on which required coil will be substituted and by which candidate coil. The objective of our problem is minimize the total shuffling times and substitution cost. Good solutions of this problem will accelerate the retrieval operations and reduce abrasions of the coils so that provide better logistics support for the steel production process and better qualities for steel products.

There is little attention on scheduling for steel coils. K. Koh. (2007) investigate a single crane scheduling problem in a distribution centre of steel coils to store incoming coils and retrieve coils required by customers. The problem is viewed as a job shop scheduling problem and formulated as a nonlinear integer programming model which is hard to solve. A local search based heuristic is proposed and tested through computation. A. Gholami. (2010) study the plate and coil shuffling problems in the logistics system of steel production. They formulate the two problems respectively and construct several valid inequalities for them. They also derive some properties of optimal solutions. Algorithmic studies are performed on both special cases and general problem. Numerical experiments show the effectiveness of the proposed algorithms.

Among warehousing operation management problems, the one most relevant to our problem is the order picking problem. There are a great deal of research on order picking problem, e.g. Ratliff and Rosenthal (1983), P.Tseng. (2001), Z.Q.Luo and P. Tseng. (1992), P. Tseng and S. Yun. (2009). However, the general order picking problems do not involve substitution decision and shuffling operations which will be main considerations in our problem.

Research involving shuffling operations usually appears in container terminals. Many researchers focus on the handling of containers. Interested readers are provided to see an overview by Steenken *et al.* (2004). The difference with our problem is that substitution is not permitted for containers. So research in container terminals doesnt consider the substitution decision.

The remaining part of the paper is organized as follows. A detailed problem description and a mathematical formulation of the problem are provided in Section 2. In Section 3 we give some properties of the problem and reduce the dimension. Experimental results based on practical data are presented in Section 4. We conclude this paper in Section 5.

2 FORMULATION OF THE PROBLEM

In this section, we provide a detailed description of the problem and formulate it as an integer linear program which will be solved conveniently by optimization software.

In the warehouse, steel coils are stored in R rows for each can stack at most two levels. We use l to represent the level number such that $l = 1$ denotes the lower level and $l = 2$ denotes the upper level. There are P_l storage positions at level l in each row. Since any coil stored at the upper level must be supported by two coils at the lower level, P_1 must be larger than P_2 by 1. Each position in the warehouse can be identified uniquely by its row-level-position coordinates (r, l, p). Fig.2.1 shows a top view of a warehouse where a dotted inclined coil indicates an empty position. The number in the coil center is the number for that position at that level in that row.

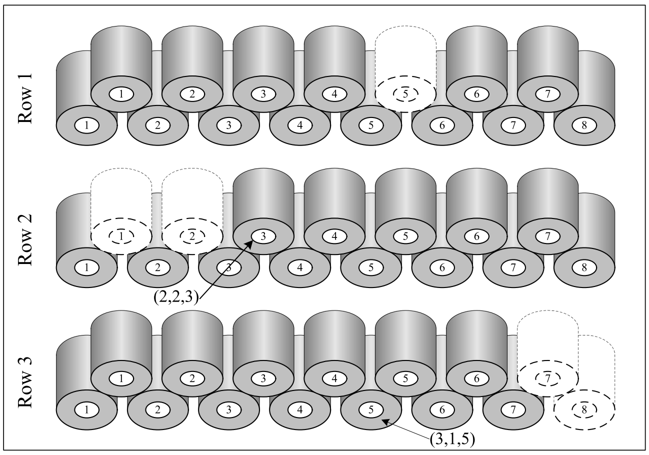

Fig. 2.1 Top view of a coil warehouse

In a planning horizon, a set of coils is required to be retrieved from the warehouse due to downstream production process or delivery demand. Let Ω denote the set of all the coils in the warehouse. The set of coils required to be retrieved is marked as Ω_r and set of other coils in the warehouse is marked as Ω_o, i.e. $\Omega = \Omega_r + \Omega_o$. For one required coil, there may be one or more candidate coils in the warehouse. For each pair of a required coil and a candidate coil, there is a matching degree which is characterized with substitution cost. The higher matching degree indicates the lower substitution cost. We define F_{ik} as the substitution cost if a required coil i is substituted by a non-required coil k. If coil i is forbidden to be substituted by coil k, $F_{ik} = \infty$. Let C denote the cost for shuffling one blocking coil. We use a binary parameter X_{irlp} to denote the initial position of coil i, such that $X_{irlp} = 1$ represent the initial position of coil i is (r, l, p), otherwise $X_{irlp} = 0$.

As mentioned above, the aim of our problem is to decide substitution of required coils. Next, we define the decision variables of the problem as follows:

$$E_{ik} = \begin{cases} 1, & \text{if coil } i \text{ is substituted by coil } k \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } i \in \Omega_r \text{ and } k \in \Omega_o$$

As a result of the substitution decision, some coils will be retrieved as the required coils, regardless of their original status. To distinguish with the former one, we call the required coils determined by substitution target coils. Some auxiliary variables determined by substitution decision are defined as follows:

$$T_i^t = \begin{cases} 1, & \text{if coil } i \text{ is a target coil} \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } i \in \Omega$$

$$T_i^b = \begin{cases} 1, & \text{if coil } i \text{ blocks a target coil} \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } i \in \Omega$$

$$T_{ik}^b = \begin{cases} 1, & \text{if coil } i \text{ blocks target coil } k \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } i \in \Omega \text{ and } k \in \Omega$$

$$T_i^n = \begin{cases} 1, & \text{if coil } i \text{ is not a target coil nor a blocking coil} \\ 0, & \text{otherwise} \end{cases}, \qquad \text{for } i \in \Omega$$

Based on the parameters and variables defined above, the problem can be formulated as the following model.

$$min \quad \sum_{i \in \Omega} C T_i^b + \sum_{i \in \Omega_r} \sum_{k \in \Omega_o} F_{ik} E_{ik} \tag{2.1}$$

s.t.

$$\sum_{k \in \Omega_o} E_{ik} \leq 1, \quad i \in \Omega_r \tag{2.2}$$

$$\sum_{i \in \Omega_r} E_{ik} \leq 1, \quad k \in \Omega_o \tag{2.3}$$

$$T_i^t = 1 - \sum_{k \in \Omega_o} E_{ik}, \quad i \in \Omega_r \tag{2.4}$$

$$T_k^t = \sum_{i \in \Omega_r} E_{ik}, \quad k \in \Omega_o \tag{2.5}$$

$$T_{ik}^b = T_k^t (1 - T_i^t) \sum_{r=1}^{R} \sum_{p=1}^{P_2} X_{ir2p} (X_{ir1p} + X_{ir1(p+1)}), \quad i \in \Omega, k \in \Omega \tag{2.6}$$

$$T_i^b = \sum_{k \in \Omega} T_{ik}^b, \quad i \in \Omega \tag{2.7}$$

$$T_i^t + T_i^b + T_i^n = 1, \quad i \in \Omega \tag{2.8}$$

The objective function (2.1) in the model is to minimize the sum of shuffling cost and substitution cost. Constraints (2.2) guarantee that each required coil can be substituted by at most one candidate coil. Constraints (2.3) guarantee that each candidate coil can substitute at most one required coil. Constraints (2.4) and (2.5) express the set of target coils gained from substitution. Constraints (2.6) express the set of coils blocking target coil k gained from substitution. Constraints (2.7) express the set of all the blocking coils gained from substitution. Constraints (2.8) show that there is only one status for each coil in the warehouse.

Since constraints (2.6) are non-liner, transformations are made as follows:

$$T_{ik}^{b} \geq 1 - T_{i}^{t} - M(1 - A_{ik}T_{k}^{t}), \quad i \in \Omega, k \in \Omega \tag{2.9}$$

$$T_{ik}^{b} \leq 1 - T_{i}^{t} + M(1 - A_{ik}T_{k}^{t}), \quad i \in \Omega, k \in \Omega \tag{2.10}$$

$$T_{ik}^{b} \geq -MA_{ik}T_{k}^{t}, \quad i \in \Omega, k \in \Omega \tag{2.11}$$

$$T_{ik}^{b} \leq MA_{ik}T_{k}^{t}, \quad i \in \Omega, k \in \Omega \tag{2.12}$$

where $A_{ik} = \sum_{r=1}^{R}\sum_{p=1}^{P_2} X_{ir2p}(X_{ir1p} + X_{ir1(p+1)})$ for writing convenience, and M is a large number.

3 PROPERTY OF THE PROBLEM

Benefited from the special stacking structure of the coils, we can gain some properties of the problem. Generally, the required coils can be categorized into two classes.

(1) The required coils not blocked by non-required coils. We denote this class as Ω_1 which implies three cases: a) the required coil is located on the upper level; b) the required coil is located on the lower level and there is no coil on top of it; c) the required coil is located on the lower level and there are one or two required coils on top of it.

For this class of required coils, no shuffling is needed with retrieving them from the warehouse. Therefore, there is no need to substitute them with any candidate coil. Based on this consideration, we can add the following equations to the model.

$$\sum_{k \in \Omega_o} E_{ik} = 0, \quad i \in \Omega_1 \tag{3.1}$$

(2) The required coils located on the lower level and blocked by one or two non-required coils. We denote this class as Ω_2. Similarly with the notations in Section 2, we define the set of coils blocking required coils as Ω_b. Define the set of coils blocking required coil i as Ω_{bi}. Define the set of non-blocking coils as Ω_n.

In order to retrieve a coil in Ω_2, the coils blocking it need to be shuffled first to other positions. To avoid shuffling, substitution can be made between required coils in Ω_2 and other coils. When the substitution is happened, the initial required coil will turn to be a non-blocking coil. We can observe the effects of substitution in the following situations.

(1) If a required coil i in Ω_2 is substituted by a blocking coil j, shuffling times will be reduced.

a. if coil j belongs to Ω_{bi}, shuffling times will be reduced by 1 at least;

b. if coil j doesn't belong to Ω_{bi}, tt must be in Ω_{bk} for some other required coil k. If there is a coil blocking both coil i and another required coil i', shuffling times will be reduced by 1 at least; Otherwise, shuffling times will be reduced by 2 at least.

(2) If a required coil i in Ω_2 is substituted by a non-blocking coil j, we can see the effect in the following cases.

a. If coil j has same number of blocking coils with coil i, there is no need to substitute;

b. If coil j has less number of blocking coils than coil i and each blocking coil in Ω_{bi} also blocks another required coil, there is no need to substitute; Otherwise, shuffling times will be reduced by 1 at least.

From the above argument we can fix some decision variables based on the situation "no need to substitute" as follows.

$$E_{ij} = 0, \quad i \in \Omega_2, j \in \Omega_n, |\Omega_{bi}| = |\Omega_{bj}| \tag{3.2}$$

$$E_{ij} = 0, \quad i \in \Omega_2, j \in \Omega_n, |\Omega_{bi}| > |\Omega_{bj}|, k \in \Omega_{bi} \cap \Omega_{bi'} \tag{3.3}$$

The situations which can reduce shuffling times may be contributed to construct effective heuristics to solve the problem.

4 EXPERIMENTAL RESULTS

In this section, computational experiments are conducted with practical data in order to check the performance of the proposed model and problem properties. 100 instances within 10 problem settings are collected from an advanced iron and steel enterprise in China. The model proposed in Section 2 and that with equations (3.1) to (3.3) are solved by ILOG CPLEX 12.4 respectively. Comparison between the objective from models and the original shuffling cost is presented in Table 4.1. Computing time comparison between the two models is presented in Table 4.2.

Table 4.1 Comparison between the objective from models and the original shuffling cost

Problem	Objective from models	Original shuffling cost
$\lvert\Omega_r\rvert = 200, \lvert\Omega_o\rvert = 400$	47	330
$\lvert\Omega_r\rvert = 150, \lvert\Omega_o\rvert = 250$	29	195
$\lvert\Omega_r\rvert = 70, \lvert\Omega_o\rvert = 150$	15	114
$\lvert\Omega_r\rvert = 50, \lvert\Omega_o\rvert = 90$	10	60
$\lvert\Omega_r\rvert = 30, \lvert\Omega_o\rvert = 60$	9	54

Table 4.2 Computing time comparison between the two models

Problem	General model(s)	Add equation (3.1)-(3.3)(s)
$\lvert\Omega_r\rvert = 200, \lvert\Omega_o\rvert = 400$	463	417
$\lvert\Omega_r\rvert = 150, \lvert\Omega_o\rvert = 250$	154	126
$\lvert\Omega_r\rvert = 70, \lvert\Omega_o\rvert = 150$	30	14
$\lvert\Omega_r\rvert = 50, \lvert\Omega_o\rvert = 90$	8	5
$\lvert\Omega_r\rvert = 30, \lvert\Omega_o\rvert = 60$	4	3

We present 5 different problem settings in which 10 instances are tested. Results in the tables are average of the 10 instances for each problem setting. Results in Table 4.1 show the formulation of our problem can reduce the shuffling cost for coil retrieval effectively. Results in Table 4.2 show the equations constructed based on properties can reduce the dimension of the problem efficiently.

5 CONCLUSION

We considered a retrieval scheduling problem with substitution in coil warehouse. We modeled this problem with an integer linear program to minimize the shuffling cost and substitution cost. We got some properties of the problem and then reduce the dimension of the problem based on them. Experimental results based on practical data show that the model is reasonable and the properties are very effective for solving the problem.

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62 A STUDY ON THE OPTIMAL CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENT IN A REGRESSION MODEL WITH TWO-FOLD ERROR STRUCTURE

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Abstract: In applications using a linear regression model with a balanced two-fold nested error structure, interest focuses on inferences concerning the regression coefficient. This article derives exact and approximate confidence intervals on the regression coefficient in the simple regression model with a balanced two-fold nested error structure. Several methods are considered for constructing the confidence intervals. The methods are compared using computer simulation and recommendations are provided for selecting an appropriate method.

Key words: Inference; Mixed model; Regression coefficient.

1 INTRODUCTION

This article considers the simple linear regression model with a balanced two-fold nested error structure. This model is appropriate to use when there is subsampling within secondary sampling units within primary sampling units. The model therefore includes one error term associated with the first-stage sampling unit, a second error term associated with the second-stage sampling unit, and a third error term associated with the last-stage sampling unit. These three error terms are assumed independent and normally distributed with zero means and constant variances. This model extends the simple linear regression model with a balanced on-fold nested error structure that Park and Burdick (1994) studied.

2 A REGRESSION MODEL WITH A BALANCED TWO-FOLD NESTED ERROR STRUCTURE

The regression model with a balanced two-fold nested error structure is written as

$$\begin{gathered} Y_{ijk} = \mu + \beta X_{ijk} + P_i + O_{ij} + E_{ijk} \\ i = 1, \ldots, a; j = 1, \ldots, b; k = 1, \ldots, r \end{gathered} \tag{2.1}$$

where Y_{ijk} is the kth random observation within the jth secondary level within the ith primary level, μ and β are unknown constants, X_{ijk} is a fixed predictor variable, P_i, O_{ij}, and E_{ijk} are respectively error terms associated with the first-stage, second-stage, and last-stage sampling unit, and P_i, O_{ij}, and E_{ijk} are jointly independent normal random variables with zero means and variances σ_P^2, σ_O^2, and σ_E^2, respectively.

Model (2.1) is written in matrix notation as

$$\mathbf{y} = \mathbf{X}\underline{\alpha} + \mathbf{B}_1\mathbf{p} + \mathbf{B}_2\mathbf{o} + \mathbf{B}_3\mathbf{e} \tag{2.2}$$

Table 2.1 A Partition for Source of Variability of Model (2.1)

SV	DF	SS
Among Primaries	n_1+1	S_{yy1}
Among Primaries Regression	1	$\hat{\beta}_1^2 S_{xx1}$
Among Primaries Residual	n_1	R_1
Among Secondaries	n_2+1	S_{yy2}
Among Secondaries Regression	1	$\hat{\beta}_2^2 S_{xx2}$
Among Secondaries Residual	n_2	R_2
Within Secondaries	n_3+1	S_{yy3}
Within Secondaries Regression	1	$\hat{\beta}_3^2 S_{xx3}$
Within Secondaries Residual	n_3	R_3
Adjusted Total	$n_{123}+3$	S_{yy123}

where $\mathbf{y}$ is an $abr \times 1$ random vector of observations, $\mathbf{X}$ is an $abr \times 2$ matrix with a column of 1's in the first column and a column of known X_{ijk}'s in the second column, $\underline{\alpha}$ is a 2×1 vector with elements μ and β, $\mathbf{B}_1 = \overset{a}{\underset{i=1}{\oplus}} \mathbf{1}_{br}$, $\mathbf{B}_2 = \overset{a}{\underset{i=1}{\oplus}}\overset{b}{\underset{j=1}{\oplus}} \mathbf{1}_r$, and $\mathbf{B}_3 = \overset{a}{\underset{i=1}{\oplus}}\overset{b}{\underset{j=1}{\oplus}}\overset{r}{\underset{k=1}{\oplus}} 1 = \mathbf{I}_{abr}$ are design matrices, $\oplus$ is the direct sum operator, $\mathbf{1}_{br}$ and $\mathbf{1}_r$ are respectively $br \times 1$ and $r \times 1$ column vectors of 1's, $\mathbf{I}_{abr}$ is an $abr \times abr$ identity matrix, $\mathbf{p}$ is an $a \times 1$ vector of random P_i effects, $\mathbf{o}$ is an $ab \times 1$ vector of random O_{ij} effects, and $\mathbf{e}$ is an $abr \times 1$ vector of random error terms, E_{ijk}. Under the distributional assumptions of (2.1), $\mathbf{y}$ has a multivariate normal distribution with mean $\mathbf{X}\underline{\alpha}$ and covariance matrix $\sigma_P^2\mathbf{B}_1\mathbf{B}_1' + \sigma_O^2\mathbf{B}_2\mathbf{B}_2' + \sigma_E^2\mathbf{I}_{abr}$.

A partition for source of variability of model (2.1) that is useful for subsampling is shown in Table 2.1. The notation for the sums of squares and the estimators of β in Table 2.1 is defined as follows: $n_1 = a-2$, $n_2 = a(b-1)-1$, $n_3 = ab(r-1)-1$, $n_{123} = n_1+n_2+n_3$, $\bar{Y}_{ij.} = \Sigma_k Y_{ijk}/r$, $\bar{Y}_{i..} = \Sigma_j\Sigma_k Y_{ijk}/br$, $\bar{Y}_{...} = \Sigma_i\Sigma_j\Sigma_k Y_{ijk}/abr$, $\bar{X}_{ij.} = \Sigma_k X_{ijk}/r$, $\bar{X}_{i..} = \Sigma_j\Sigma_k X_{ijk}/br$, $\bar{X}_{...} = \Sigma_i\Sigma_j\Sigma_k X_{ijk}/abr$, $S_{yy1} = br\Sigma_i(\bar{Y}_{i..}-\bar{Y}_{...})^2$, $S_{yy2} = r\Sigma_i\Sigma_j(\bar{Y}_{ij.}-\bar{Y}_{i..})^2$, $S_{yy3} = \Sigma_i\Sigma_j\Sigma_k(Y_{ijk}-\bar{Y}_{ij.})^2$, $S_{xx1} = br\Sigma_i(\bar{X}_{i..}-\bar{X}_{...})^2$, $S_{xx2} = r\Sigma_i\Sigma_j(\bar{X}_{ij.}-\bar{X}_{i..})^2$, $S_{xx3} = \Sigma_i\Sigma_j\Sigma_k\,(X_{ijk}-\bar{X}_{ij.})^2$, $S_{xy1} = br\Sigma_i(\bar{X}_{i..}-\bar{X}_{...})(\bar{Y}_{i..}-\bar{Y}_{...})$, $S_{xy2} = r\Sigma_i\Sigma_j(\bar{X}_{ij.}-\bar{X}_{i..})(\bar{Y}_{ij.}-\bar{Y}_{i..})$, $S_{xy3} = \Sigma_i\Sigma_j\Sigma_k(X_{ijk}-\bar{X}_{ij.})(Y_{ijk}-\bar{Y}_{ij.})$, $S_{yy12} = S_{yy1}+S_{yy2}$, $S_{yy123} = S_{yy12}+S_{yy3}$, $S_{xx12} = S_{xx1}+S_{xx2}$, $S_{xx123} = S_{xx12}+S_{xx3}$, $S_{xy12} = S_{xy1}+S_{xy2}$, $S_{xy123} = S_{xy12}+S_{xy3}$, $\hat{\beta}_1 = S_{xy1}/S_{xx1}$, $\hat{\beta}_2 = S_{xy2}/S_{xx2}$, $\hat{\beta}_3 = S_{xy3}/S_{xx3}$, $R_1 = S_{yy1}-\hat{\beta}_1^2 S_{xx1}$, $R_2 = S_{yy2}-\hat{\beta}_2^2 S_{xx2}$, and $R_3 = S_{yy3}-\hat{\beta}_3^2 S_{xx3}$.

The estimators of β and the sums of squares in Table 2.1 are now described in the context of a standard linear regression model. The estimator $\hat{\beta}_1 = S_{xy1}/S_{xx1}$ is obtained from the least squares regression of $\bar{Y}_{i..}$ on $\bar{X}_{i..}$. The sum of squares R_1 is written in a quadratic form as $R_1 = S_{yy1}-\hat{\beta}_1^2 S_{xx1}$. The estimator $\hat{\beta}_2 = S_{xy2}/S_{xx2}$ is obtained from the least squares regression of $\bar{Y}_{ij.}$ on $\bar{X}_{ij.}$ and grouping variables that represent i primary levels. The sum of squares R_2 is written as $R_2 = S_{yy2}-\hat{\beta}_2^2 S_{xx2}$. The estimator $\hat{\beta}_3 = S_{xy3}/S_{xx3}$ is obtained from the least squares regression of Y_{ijk} on X_{ijk} and grouping variables that represent i primary and j secondary levels. The sum of squares R_3 is written as $R_3 = S_{yy3}-\hat{\beta}_3^2 S_{xx3}$. The estimator $\hat{\beta}_S = S_{xy12}/S_{xx12}$ is obtained from the least squares regression of $\bar{Y}_{ij.}$ on $\bar{X}_{ij.}$. The sum of squares R_S is written as $R_S = \hat{\beta}_1^2 S_{xx1}+\hat{\beta}_2^2 S_{xx2}-\hat{\beta}_S^2 S_{xx12}$ where $R_{12} = S_{yy12}-\hat{\beta}_S^2 S_{xx12}$. The estimator $\hat{\beta}_T = S_{xy123}/S_{xx123}$ is obtained from the least squares regression of Y_{ijk} on X_{ijk}. The sum of squares R_T is written as $R_T = \hat{\beta}_S^2 S_{xx12}+\hat{\beta}_3^2 S_{xx3}-\hat{\beta}_T^2 S_{xx123}$ where $R_{123} = S_{yy123}-\hat{\beta}_T^2 S_{xx123}$.

3 DISTRIBUTIONAL RESULTS AND CONFIDENCE INTERVALS FOR β

In order to construct confidence intervals on the regression coefficient, the ordinary least square(OLS) estimators of β are examined.

Theorem 3.1 *Under the assumptions in (2.1), five OLS estimators have following properties:* $\hat{\beta}_1 \sim N(\beta, (br\sigma_P^2+r\sigma_O^2+\sigma_E^2)/S_{xx1})$, $\hat{\beta}_2 \sim N(\beta, (r\sigma_O^2+\sigma_E^2)/S_{xx2})$, $\hat{\beta}_3 \sim N(\beta, \sigma_E^2/S_{xx3})$, $\hat{\beta}_S \sim N(\beta, (k_{12}br\sigma_P^2+$

$r\sigma_O^2 + \sigma_E^2)/S_{xx12})$, $\hat{\beta}_T \sim N(\beta, (k_{13}br\sigma_P^2 + k_{23}r\sigma_O^2 + \sigma_E^2)/S_{xx123})$, *where* $\hat{\beta}_S = k_{12}\hat{\beta}_1 + (1-k_{12})\hat{\beta}_2$, $\hat{\beta}_T = k_{13}\hat{\beta}_1 + (k_{23}-k_{13})\hat{\beta}_2 + (1-k_{23})\hat{\beta}_3$, $k_{12} = S_{xx1}/S_{xx12}$, $k_{13} = S_{xx1}/S_{xx123}$, *and* $k_{23} = S_{xx12}/S_{xx123}$.

Park (2012) showed that the sums of squares R_1, R_2, and R_3 are independently chi-squared distributed using Searle (1987). It can be shown that linear functions of the sums of squares are also chi-squared random variables and Theorem 3.2 hold.

Theorem 3.2 *Under the assumptions in (2.1), linear functions of the sums of squares have following properties:* $R_{12}/(k_{12}br\sigma_P^2 + r\sigma_O^2 + \sigma_E^2) \sim \chi^2_{n_{12}}$, *and* $R_{123}/(k_{13}br\sigma_P^2 + k_{23}r\sigma_O^2 + \sigma_E^2) \sim \chi^2_{n_{123}}$, *where* $R_{12} = \kappa_1 R_1 + \kappa_2 R_2$, $R_{123} = \tau R_1 + \tau_2 R_2 + \tau_3 R_3$, $\kappa_1 = (k_{12}br\sigma_P^2 + r\sigma_O^2 + \sigma_E^2)/(br\sigma_P^2 + r\sigma_O^2 + \sigma_E^2)$, $\kappa_2 = (k_{12}br\sigma_P^2 + r\sigma_O^2 + \sigma_E^2)/(r\sigma_O^2 + \sigma_E^2)$, $\tau_1 = (k_{13}br\sigma_P^2 + k_{23}r\sigma_O^2 + \sigma_E^2)/(br\sigma_P^2 + r\sigma_O^2 + \sigma_E^2)$, $\tau_2 = (k_{13}br\sigma_P^2 + k_{23}r\sigma_O^2 + \sigma_E^2)/(r\sigma_O^2 + \sigma_E^2)$, $\tau_3 = (k_{13}br\sigma_P^2 + k_{23}r\sigma_O^2 + \sigma_E^2)/\sigma_E^2$, $n_{12} = n_1 + n_2$, *and* $n_{123} = n_1 + n_2 + n_3$.

Based on standard results of linear model theory, it can be shown that the OLS estimators and the sums of squares presented in Section 2 are independent.

Theorem 3.3 *Under the assumptions in (2.1), OLS estimators and sums of squares are independent as follows:* $\hat{\beta}_1$ *and* R_1 *are independent,* $\hat{\beta}_2$ *and* R_2 *are independent,* $\hat{\beta}_3$ *and* R_3 *are independent,* $\hat{\beta}_S$ *and* R_{12} *are independent, and* $\hat{\beta}_T$ *and* R_{123} *are independent.*

The confidence intervals for β are constructed using basic mathematical statistics theory. Since $\hat{\beta}_1$ and R_1 are independent, it follows that an exact $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_1 \;\pm\; t_{(\alpha/2\,:\,n_1)}\sqrt{\frac{S_1^2}{S_{xx1}}} \tag{3.1}$$

where $S_1^2 = R_1/n_1$, and $t_{(\delta:\nu)}$ is the t-value for ν degrees fo freedom with δ area to the right. This interval is referred to as EX1. Using independence of $\hat{\beta}_2$ and R_2, it follows that an exact $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_2 \;\pm\; t_{(\alpha/2\,:\,n_2)}\sqrt{\frac{S_2^2}{S_{xx2}}} \tag{3.2}$$

where $S_2^2 = R_2/n_2$. This interval is referred to as EX2. Using independence of $\hat{\beta}_3$ and R_3, it follows that an exact $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_3 \;\pm\; t_{(\alpha/2\,:\,n_3)}\sqrt{\frac{S_3^2}{S_{xx3}}} \tag{3.3}$$

where $S_3^2 = R_3/n_3$. This interval is referred to as EX3. Using independence of $\hat{\beta}_S$ and R_{12}, it follows that an exact $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_S \;\pm\; t_{(\alpha/2\,:\,n_{12})}\sqrt{\frac{S_{12}^2}{S_{xx12}}} \tag{3.4}$$

where $S_{12}^2 = R_{12}/n_{12}$. This interval is referred to as EXS. Using independence of $\hat{\beta}_T$ and R_{123}, it follows that an exact $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_T \;\pm\; t_{(\alpha/2\,:\,n_{123})}\sqrt{\frac{S_{123}^2}{S_{xx123}}} \tag{3.5}$$

where $S_{123}^2 = R_{123}/n_{123}$. This interval is referred to as EXT. By substituting $(S_1^2 - S_2^2)/br$, $(S_2^2 - S_3^2)/r$, and S_3^2 for σ_P^2, σ_O^2, and σ_E^2 and using independence of $\hat{\beta}_S$ and R_{12}, it follows that an approximate $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_S \;\pm\; Z_{\alpha/2}\sqrt{\frac{k_{12}S_1^2 + (1-k_{12})S_2^2}{S_{xx12}}} \tag{3.6}$$

where $Z_{\delta/2}$ is the z-value with δ area to the right. This interval is referred to as LSS. By substituting $(S_1^2 - S_2^2)/br$, $(S_2^2 - S_3^2)/r$, and S_3^2 for σ_P^2, σ_O^2, and σ_E^2 and using independence of $\hat{\beta}_T$ and R_{123}, it follows that an approximate $100(1-\alpha)\%$ confidence interval on β is

$$\hat{\beta}_T \;\pm\; Z_{\alpha/2}\sqrt{\frac{k_{13}S_1^2 + (k_{23} - k_{13})S_2^2 + (1 - k_{23})S_3^2}{S_{xx12}}} \tag{3.7}$$

This interval is referred to as LST.

4 SIMULATION STUDY

The performance of confidence intervals proposed in Section 3 is examined using a simulation study. Sixty four designs are formed by taking all combinations of $a = 3, 5, 10, 15$, $b = 2, 5, 10, 15$, and $r = 2, 5, 10, 15$. The values of σ_P^2 are selected from the set of values (0.01, 0.2, 0.4, 0.6, 0.8, 0.98) and the values of σ_O^2 and σ_E^2 are determined to set $\sigma_P^2 + \sigma_O^2 + \sigma_E^2 = 1$. Recall that the mean squares in Section 3 are chi-squared random variables. In particular, $S_1^2 \sim [(br\sigma_P^2 + r\sigma_O^2 + \sigma_E^2)/n_1]\chi^2_{n_1}$, $S_2^2 \sim [(r\sigma_O^2 + \sigma_E^2)/n_2]\chi^2_{n_2}$, $S_3^2 \sim [\sigma_E^2/n_3]\chi^2_{n_3}$, $S_S^2 \sim [br(1-k_{12})\sigma_P^2 + r\sigma_O^2 + \sigma_E^2]\chi^2_1$, and $S_T^2 \sim [br(k_{12} - k_{13})\sigma_P^2 + r(1-k_{23})\sigma_O^2 + \sigma_E^2]\chi^2_1$. These mean squares are generated by the RANGAM function of the SAS by substituting the specific values in Table 4.1 for each design. Simulated values for S_1^2, S_2^2, S_3^2, S_S^2, and S_T^2 are substituted into the appropriate formula and the confidence intervals are computed. The values of S_{xx1} are selected from the set of values (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8) and the values of S_{xx2} and S_{xx3} are determined to set $S_{xx1} + S_{xx2} + S_{xx3} = 1$. The values of the sums of squares are used to calculate the constants k_{12}, k_{13}, and k_{23}. For each design 2000 iterations are simulated and two-sided confidence intervals on variance components are computed for each proposed method. Confidence coefficients are determined by counting the number of the intervals that contain variance components. The average lengths of the two-sided confidence intervals are computed.

Tables 4.1 reports the results of the simulation for stated 90% confidence intervals on β for $a = 3, b = 2$ and $a = 3, b = 15$. The EX1, EX2, EX3, EXS, EXT, LSS, and LST methods refer to the intervals in (3.1), (3.2), (3.3), (3.4), (3.5), (3.6), and (3.7), respectively. Using the normal approximation to the binomial, if the true confidence coefficient is 0.90, there is less than a 2.5% chance that a simulated confidence coefficient based on 2000 replications will be less than 0.88685. The comparison criteria are: i) the ability to maintain the stated confidence coefficient and ii) the average length of two-sided confidence intervals. Although shorter average lengths are preferable, it is necessary that an interval first maintain the stated confidence level.

The EX1, EX2, EX3, EXS, and EXT methods generally maintain the stated confidence level in Table 4.1. The LSS and LST methods are too liberal for $a = 3, b = 2$ since the simulated confidence coefficients of two methods fall below the 0.88685. However, the simulated confidence coefficients of LST method become close to 0.9 when $a = 3, b = 2$, and $r = 5$. This is because LST method combines the information of the sums of squares R_1, R_2, and R_3 that are based on $n_1 = a-2$, $n_2 = a(b-1)-1$, and $n_3 = ab(r-1)-1$ degrees of freedom. However, LSS and LST methods start to maintain the stated confidence level when b becomes 15.

Table 4.1 Ranges of Simulated Confidence Coefficients for 90 % Two-sided Intervals on β

a	b	r		EX1	EX2	EX3	EXS	EXT	LSS	LST
3	2	2	max	0.9175	0.9190	0.9180	0.9175	0.9210	**0.8380**	**0.8825**
			min	0.8825	0.8810	0.8780	0.8830	0.8790	**0.7770**	**0.8395**
3	2	5	max	0.9210	0.9205	0.9160	0.9180	0.9150	**0.8350**	0.9040
			min	0.8770	0.8810	0.8750	0.8830	0.8770	**0.7735**	0.8685
3	15	2	max	0.9180	0.9185	0.9220	0.9195	0.9195	0.9160	0.9165
			min	0.8800	0.8830	0.8835	0.8795	0.8810	0.8715	0.8775
3	15	5	max	0.9210	0.9180	0.9190	0.9160	0.9150	0.9100	0.9135
			min	0.8825	0.8740	0.8780	0.8825	0.8810	0.8750	0.8810

5 NUMERICAL EXAMPLE

Belsley et al. (1980) analyzed the relationship of house prices on quality of the environment using 506 observations on census tracts belonging to 92 towns in the Boston Standard Metropolitan Statistical Area(SMSA) in 1970. The response variable used in the study is logarithm of the median value of owner-occupied homes(Y) and one of the predictor variables is the per capita crime rate(X).

A subset of these data was selected to conform to the design with $a = 3$, $b = 2$, and $r = 5$ of our simulation. We selected six towns(Salem, Woburn, Natick, Winchester, Belmont, and Arlington) and 5 observations from each town. We assume that Salem and Woburn, Natick and Winchester, and Belmont and Arlington belong to the same district, respectively, and that observations are nested within towns(secondary units) that are nested within districts(primary units) in order to correspond with a balanced two-fold nested error structure. The data are shown in Table 5.1.

Table 5.1 Selected Data Set from Boston SMSA in 1970

Salem		Woburn		Natick	
Y	X	Y	X	Y	X
10.0389	0.08829	10.3735	0.07875	10.0732	0.08244
10.2073	0.14455	10.3023	0.12579	10.0562	0.09252
9.71112	0.21124	10.4602	0.08370	9.99880	0.11329
9.84692	0.17004	10.5187	0.09068	9.90848	0.10612
9.61581	0.22489	10.3255	0.06911	10.0078	0.10290
Winchester		**Belmont**		**Arlington**	
Y	X	Y	X	Y	X
10.2541	0.12204	10.5241	0.05780	10.0732	0.13914
9.97115	0.11504	10.5914	0.06588	10.0690	0.09178
10.5636	0.12083	10.4968	0.06888	10.0257	0.08447
10.6874	0.08187	10.5427	0.09103	10.2888	0.06664
10.4103	0.06860	10.3890	0.10008	10.0519	0.07022

Using the data in Table 5.1 we computed $S_1^2 = 0.0163$, $S_2^2 = 0.3068$, $S_3^2 = 0.0199$, $S_S^2 = 0.0820$, $S_T^2 = 0.0796$, $k_{12} = 0.3970$, $k_{13} = 0.2174$, and $k_{23} = 0.5477$. The resulting 90% two-sided confidence intervals on β are shown in Table 5.2. LSS method is not recommended because it did not maintain stated confidence level when $a = 3$, $b = 2$, and $r = 5$ in the simulation study.

EX3, EXT, and LST methods are recommended to construct 90% two sided confidence interval for β in this example since EX3 and EXT methods use $n_3 = 23$ or $n_{123} = 26$ degrees of freedom, respectively, and LST can also be applied for this example. The the confidence intervals for three methods contain negative values for lower and upper limits. Therefore the null hypothesis $H_0 : \beta = 0$ is rejected using $\alpha = 10\%$

6 CONCLUSIONS

This paper presents an approach for constructing confidence intervals for regression coefficient in a simple linear regression model with a balanced two-fold nested error structure. When $a = 3$ and $b = 2$, LSS method is not recommended to compute confidence intervals on regression coefficient. When $a = 3$, $b = 2$, and $r = 2$, LST method is not recommended. Except these cases, LSS and LST methods can be applied to construct confidence interval regression coefficient. Other methods can be used across all combinations of a, b, and r.

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Table 5.2 Confidence Intervals on β for Example Data

Method	Estimates	Lower bound	Upper bound	Interval length
EX1	$\hat{\beta}_1 = -3.484$	-11.40	4.43	15.83
EX2	$\hat{\beta}_2 = -7.103$	-19.98	5.78	25.76
EX3	$\hat{\beta}_3 = -3.072$	-4.72	-1.43	3.29
EXS	$\hat{\beta}_S = -5.666$	-12.03	0.70	12.73
EXT	$\hat{\beta}_T = -4.493$	-7.13	-1.86	5.27
LSS	$\hat{\beta}_S = -5.666$	-10.12	-1.22	8.90
LST	$\hat{\beta}_T = -4.493$	-7.03	-1.95	5.08

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63 OPTIMIZATION STUDY ON THE RESPONSE TIME OF DVR FOR 3-PHASE PHASE-CONTROLLED RECTIFIER

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Abstract: This paper presents relation between the response time of DVR (Dynamic Voltage Restorer) and the possible compensation range of voltage dip by the DVR system which protects the 3-phase phase-controlled rectifier from voltage dip. The permissible range of voltage dip is presented in the 3-phase phase-controlled rectifier. Using the proposed method, the DVR's response time can be determined from the parameters of 3-phase phase-controlled rectifier and the possible compensation range of voltage dip. It showed good stability of overall system.

Key words: DVR; Response speed; Voltage dip; 3-phase phase-controlled rectifier.

1 INTRODUCTION

The voltage dip primarily causes malfunction of control unit, and it also causes commutation failure on switching element kept at large plants that use inverter, resulting in overall system failure (Arora A. (1998)). The problems caused by voltage dip are especially severe on locations that consist of SCR converter and inverter (Chellali B. (2008)). Recently, DVR (Dynamic Voltage Restorer) has been utilized as a solution for voltage dip that causes severe damages to rolling process at iron mill, and semi-conductor factories.

Choi (Choi S. S. (2000)) suggested a method to minimize compensation energy by setting the effective power supply provided by DVR at 0, and Zhan (Zhan C. (2001)) designed phase-lock loop to detect the optimal phase angle from unbalanced power source. Although many researches have conducted on DVR, no researches have done to find out the range of voltage dip that can be compensated by the DVR at specific system.

In this paper, we presents optimization study on the response time of DVR (Dynamic Voltage Restorer) and the possible compensation range of voltage dip by the DVR system. It also researches the magnitude and phase range of voltage dip that can be compensated according to the response time of DVR.

2 CIRCUIT AND MODELING

Phase-controlled rectifier is a power converter that uses SCR to convert AC power into variable DC power, and the switching-ON time of SCR can be changed to adjust the level of DC output. Fig. 1 shows the

Table 2.1 The parameters of phase-controlled rectifier in A steelworks B facility

AC input power		460[V]	Frequency	60[Hz]
Rectifier rating	Voltage	300[V]	Overload ratio	150 %
	Current	360[A]	Power Source Inductance	60[μH]

main component of 3-phase phase-controlled rectifier. Since phase-controlled rectifier can operate in both rectification mode and inverter mode, it is possible to restore the regeneration power that is produced during deceleration of electric motor connected to the loading side. In Fig. 1, L_s represents inductance of power line due to the effect of transformer. Since this inductance limits sudden changes in the current passing through line voltage, current overlapping phenomenon occurs.

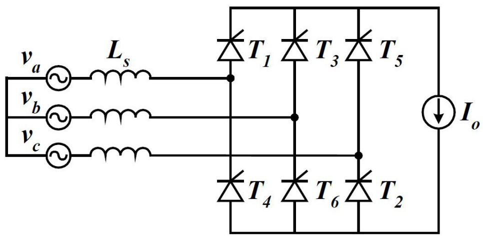

Figure 2.1 3-phase phase-controlled rectifier

Table 1 shows the parameters of phase-controlled rectifier in facility B of A steelworks, and Fig. 2 shows the relationship between voltage dip and commutation failure following the conditions of Table 1.

Fig. 2 shows possible occurrence of commutation failure depending on phase shifts, and the intensity of input voltage when firing angle is 70°, 100°, 130°, and 160°. Horizontal axis is the voltage dip rate where the amount of voltage reduction by voltage dip is compared to the normal voltage, and then it is converted into percentile value, while vertical axis is phase shift of input voltage. The graph in Fig. 2 shows each area where commutation failure is observed as a result of 3 phase parallel voltage dip and single phase voltage dip. Normal state of input voltage shows 0% of voltage dip and 0 phase shift, and the right side of the graph is the region which causes commutation failure. From Fig. 2, we observe the range of voltage dip for various firing angles of phase-controlled rectifier that does not cause commutation failure, and we also observe that as firing angle and phase shift of input voltage increase, even though minor changes in input power can cause commutation failure.

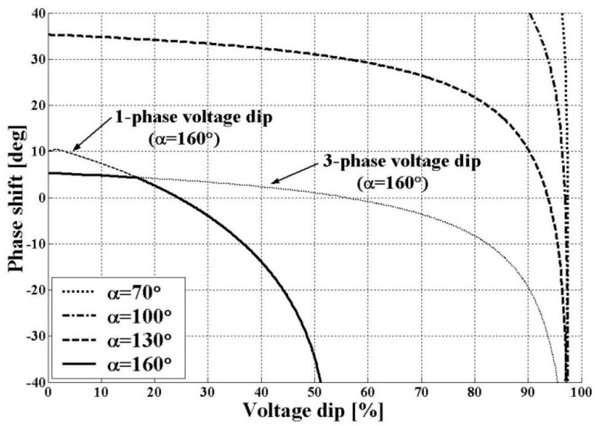

Figure 2.2 The permissible range of voltage dip according to the firing angle

3 RESULTS

Fig. 3 shows occurrence of voltage dip on line voltage of 3-phases. The setting was done for cases where normal voltage on phase A drops by 80% from 460V to 92V. The phase changes were not considered. Fig. 4 shows commutation failure due to voltage dip. The firing angle is 140°. In Fig. 4(a) and 4(b), phase current and output voltage of rectifier are depicted respectively. The output voltage decreases to 0[V] because of commutation failure in SCR T_1 and T_2. As shown in Fig. 4, the proposed rectifier showed good stability as compared to conventional rectifier.

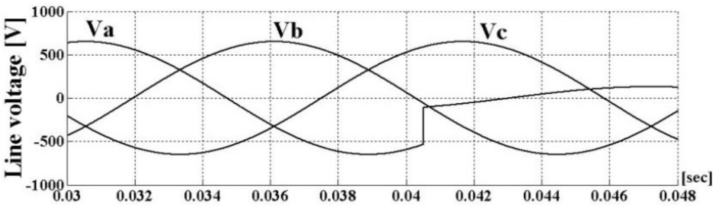

Figure 3.1 The 3-phase line voltage occurred 80% voltage dip in the A phase

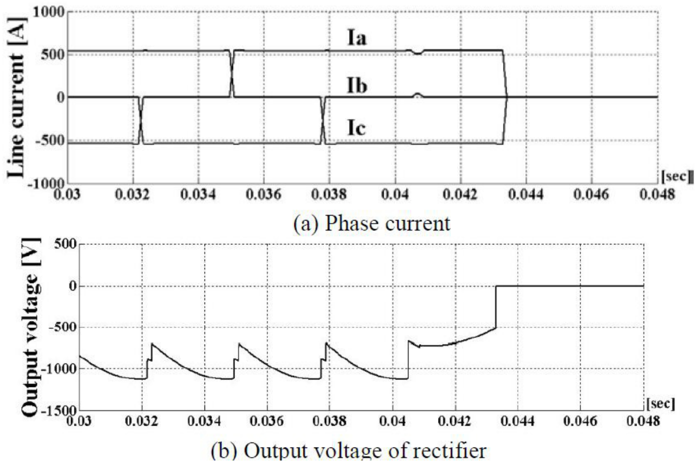

Figure 3.2 Waveform of phase-controlled rectifier when firing angle is $155°$

Fig. 5 shows waveform of the phase-controlled rectifier when DVR response time is set at $366\mu s$. Fig. 5(a) represents waveform of the line voltage that is compensated by DVR, which shows the reduction of line voltage at phase A by the voltage dip, and its restoration state by the DVR after $366\mu s$. Fig. 5(b) shows phase current of the rectifier. It was set to cause voltage dip after trigger signal is sent to SCR T_3, thus commutation does not conduct between phase A and phase B when the voltage dip occurs, but commutation successfully performs after line voltage is compensated for DVR response time. Output waveform of phase-controlled rectifier is presented on Fig. 5(c), which shows that DVR effectively restores voltage dip for normal operation of rectifier.

4 CONCLUSION

This paper proposed a solution to set the optimum response time of DVR, which is an important factor to consider when designing DVR suitable to promote 3-phase phase-controlled rectifier from voltage dip. The proposed method suggested to calculate optimum DVR response time required for certain intensity of voltage dips that may protect the characteristics of 3-phase phase-controlled rectifier. It showed good stability and reliability.

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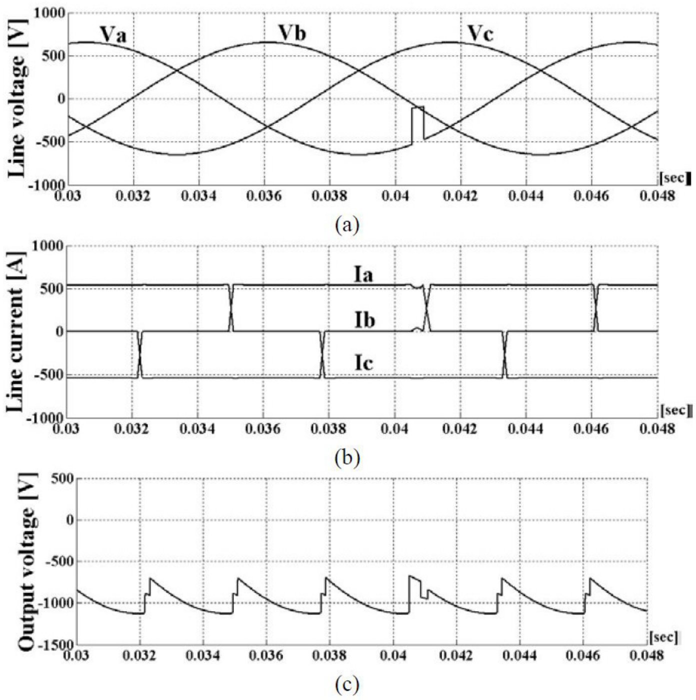

Figure 3.3 Waveform of phase-controlled rectifier when the response time of DVR is $366\mu s$, (a) compensated line voltage, (b) phase current, (c) output voltage

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64 USING DATA ENVELOPMENT ANALYSIS IN EVALUATION OF SEWERAGE SYSTEMS

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Abstract: Generally, a standard life cycle of sewer drainpipe is 50 years, and most of sewerage systems in Japan have been deteriorating. Moreover, more than 5000 traffic accidents occur due to deteriorated sewerage systems. In this paper, we suggest a method to evaluate sewerage systems using data envelopment analysis (DEA). DEA is a method to measure relative efficiencies of decision making units performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. Distinctive features of DEA are that it can measure a relative distance of each alternative to so-called DEA-efficient frontier and give information on reference points which an alternative is dominated by. To begin with, we survey DEA and discuss on several DEA models. Next, we apply DEA to evaluation of sewerage systems for preventive maintenance and investigate the effectiveness of using DEA.

Key words: Data envelopment analysis; Deteriorated degree; Preventive maintenance.

1 INTRODUCTION

Data envelopment analysis (DEA) was suggested by Charnes, Cooper and Rhodes (1978, 1979) as a method for measuring relative efficiencies of decision making units (DMUs) performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. CCR model is the first model in DEA, and main characteristics of DEA are that

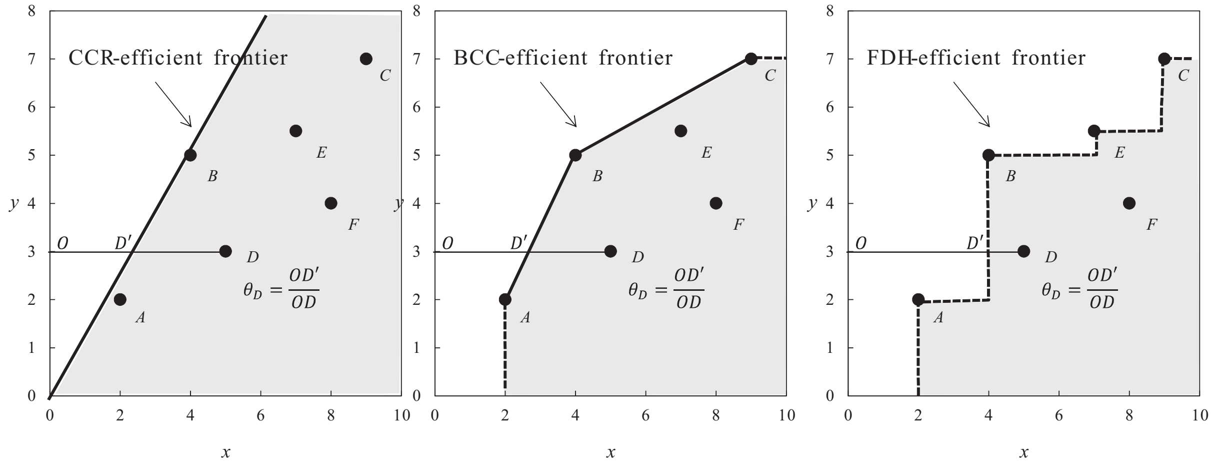
- it can be applied to analyze efficiency for multiple outputs and multiple inputs without preassigned weights
- it can be used for measuring a relative efficiency based on the observed data without knowing information on the production function with respect to inputs and outputs
- decision makers' preferences (value judgments) can be incorporated in DEA model

Utilizing the above characteristics of DEA, we elicit relevant attributes (factors) to collapse of roads from basic data and inspection for the sewerage systems in which road subsidence has occurred due to damaged sewer pipes. Calculating latent risk degree of each sewerage system, we decide a priority for maintaining preventively, properly and efficiently.

2 DATA ENVELOPMENT ANALYSIS

To begin with, we give an overview of DEA models, and summarize the following notations used commonly:

- $q,\ m,\ p$: the number of DMUs, input and output, respectively
- o : an index of DMU to be evaluated
- x_{ij} : i-th input of DMU$_j$, $j = 1, \ldots, q$; $i = 1, \ldots, m$



(a) CCR model (b) BCC model (c) FDH model

Figure 2.1 DEA-efficient frontiers (solid line : efficient frontier and shadowed area : production possibility set)

- y_{kj} : k-th output of DMU_j, $j = 1, \ldots, q$; $k = 1, \ldots, p$
- ε : sufficiently small positive number (e.g., 10^{-7}), $\mathbf{1} = (1, \ldots, 1)^T$

We assume that $x_{ij} > 0$ for each $i = 1, \ldots, m$ and $y_{kj} > 0$ for each $k = 1, \ldots, p$, and there are no duplicated units in the observed data. The $m \times q$ input matrix for the q DMUs is denoted by $\boldsymbol{X}$, and the $p \times q$ output matrix for the q DMUs is denoted by $\boldsymbol{Y}$.

For better understanding, we explain DEA models along a simple example of one input and one output as shown in Table 2.1.

Table 2.1 Case of a Single Input and a Single Output

DMU	A	B	C	D	E	F
x (input)	2	4	9	5	7	8
y (output)	2	5	7	3	5.5	4

Figure 2.1 shows the production possibility set generated by the given data in Table 2.1 of CCR model (Charnes *et al.*, 1978), BCC model (Baker *et al.*, 1984) and FDH model (Tulkens, 1993), which are representative DEA models. Pareto frontier[1] on the production possibility set is called DEA-efficient frontier, and DEA measures a relative distance to DEA-efficient frontier from each point. For example, the efficient value θ_D of DMU_D is given by the ratio of OD' to OD. As seen from Figure 2.1, clearly a DMU on DEA-efficient frontier has its efficient value $\theta = 1$, and if a DMU is far from DEA-efficient frontier, DEA-efficient value is closely to 0. Table 2.2 shows DEA-efficient values for the data in Table 2.1 by DEA models.

Table 2.2 DEA-Efficiencies for the Data in Table 2.1

DMU	A	B	C	D	E	F
CCR	0.8	1.0	0.62	0.48	0.63	0.4
BCC	1.0	1.0	1.0	0.53	0.75	0.42
FDH	1.0	1.0	1.0	0.8	1.0	0.5

Extending to the case of multiple inputs and multiple outputs, an efficiency can be evaluated by maximizing the following ratio for non-negative input weights ν_i, $i = 1, \ldots, m$ and output weights μ_j, $j = 1, \ldots, p$:

$$\frac{\sum_{j=1}^{p} \mu_j y_{jo}}{\sum_{i=1}^{m} \nu_i x_{io}} \quad \longrightarrow \quad \max \tag{2.1}$$

Imposing the normality condition that the ratio (2.1) is not larger than one for all DMUs, CCR model determines optimal weights $\boldsymbol{\mu} = (\mu_i, \dots, \mu_p)^T$, $\boldsymbol{\nu} = (\nu_i, \dots, \nu_m)^T$ and an efficient value θ by solving the following linear programming problem (CCR)

$$\begin{aligned}
\underset{\boldsymbol{\mu},\,\boldsymbol{\nu}}{\text{maximize}} \quad & \sum_{j=1}^{p} \mu_j y_{jo} \\
\text{subject to} \quad & \sum_{i=1}^{m} \nu_i x_{io} = 1, \\
& \sum_{j=1}^{p} \mu_j y_{jk} - \sum_{i=1}^{m} \nu_i x_{ik} \leqq 0, \;\; k = 1, \dots, q, \\
& \mu_j \geqq \varepsilon, \;\; j = 1, \dots, p; \;\; \nu_i \geqq \varepsilon, \;\; i = 1, \dots, m,
\end{aligned} \tag{CCR}$$

and its dual problem (CCR_D)

$$\begin{aligned}
\underset{\theta,\,\boldsymbol{\lambda},\,\boldsymbol{s}_x,\,\boldsymbol{s}_y}{\text{minimize}} \quad & \theta - \varepsilon \left(\mathbf{1}^T \boldsymbol{s}_x + \mathbf{1}^T \boldsymbol{s}_y \right) \\
\text{subject to} \quad & \boldsymbol{X\lambda} - \theta \boldsymbol{x}_o + \boldsymbol{s}_x = \mathbf{0}, \\
& \boldsymbol{Y\lambda} - \boldsymbol{y}_o - \boldsymbol{s}_y = \mathbf{0}, \\
& \boldsymbol{\lambda} \geqq \mathbf{0}, \;\; \boldsymbol{s}_x \geqq \mathbf{0}, \;\; \boldsymbol{s}_y \geqq \mathbf{0}, \\
& \theta \in \mathbb{R}, \;\; \boldsymbol{\lambda} \in \mathbb{R}^q, \;\; \boldsymbol{s}_x \in \mathbb{R}^m, \;\; \boldsymbol{s}_y \in \mathbb{R}^p.
\end{aligned} \tag{CCR$_D$}$$

Elementary and detailed description of DEA can be referred to the book (Cooper *et al.*, 2007). Normally, we give the definition of DEA-efficiency as follows:

Definition 2.1 (DEA-efficiency) *For the optimal solution θ^*, $\boldsymbol{s}_x^*$ and $\boldsymbol{s}_y^*$ in CCR_D problem, a DMU is said to be* DEA-efficient *if and only if $\theta^* = 1$, $\boldsymbol{s}_x^* = \mathbf{0}$ and $\boldsymbol{s}_y^* = \mathbf{0}$.*

3 EVALUATION OF SEWERAGE SYSTEMS USING DEA

From the results of Table 2.2, DEA-efficiency means relative inefficiency for each DMU, while we cannot know how efficient a DMU is. To this end, CCR model can be reformulated as follows:

$$\begin{aligned}
\underset{\boldsymbol{\mu},\,\boldsymbol{\nu}}{\text{maximize}} \quad & \theta := \sum_{j=1}^{p} \mu_j y_{jo} \\
\text{subject to} \quad & \sum_{i=1}^{m} \nu_i x_{io} = 1, \\
& \sum_{j=1}^{p} \mu_j y_{jk} - \sum_{i=1}^{m} \nu_i x_{ik} \leqq 0, \;\; k = 1, \dots, q; k \neq o \\
& \mu_j \geqq \varepsilon, \;\; j = 1, \dots, p; \;\; \nu_i \geqq \varepsilon, \;\; i = 1, \dots, m.
\end{aligned} \tag{E-CCR}$$

The above E-CCR model evaluates a relative distance to the CCR-frontier which is generated by the data set $\{(\boldsymbol{x}_1, \boldsymbol{y}_1), \dots, (\boldsymbol{x}_q, \boldsymbol{y}_q)\} \setminus (\boldsymbol{x}_o, \boldsymbol{y}_o)$.

DEA-inefficient DMUs do not contribute to the generation of CCR-efficient frontier. Conversely, CCR-efficient frontier is determined by only CCR-efficient DMUs, and thus may be altered by excluding $(\boldsymbol{x}_o, \boldsymbol{y}_o)$ of DMU_o from the data set. Figure 3.1 shows CCR-efficient frontier except for the DMU_B from the data of Table 2.1. And, its efficient value for CCR-efficient DMU_B changes into

$$\theta_\text{B}^* = 1.25.$$

The above efficient value with larger than 1 means the degree how far $(\boldsymbol{x}_o, \boldsymbol{y}_o)$ is outward from CCR-efficient frontier generated without the DMU to be evaluated, and it can be regarded that DMUs with larger values of θ are more efficient among CCR-efficient DMUs in CCR model. Therefore, CCR-efficient value by E-CCR model can constitute a measure of degree how CCR-efficient DMUs are with respect to the current efficient frontier. In this paper, an optimal value θ by DEA (CCR and E-CCR) models is called a *ratio value*. From the analysis of using DEA, we obtain also optimal weights $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ corresponding to outputs and inputs, respectively.

In this research, we utilize the information of a ratio value θ, and optimal weights $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ in evaluating sewerage systems.

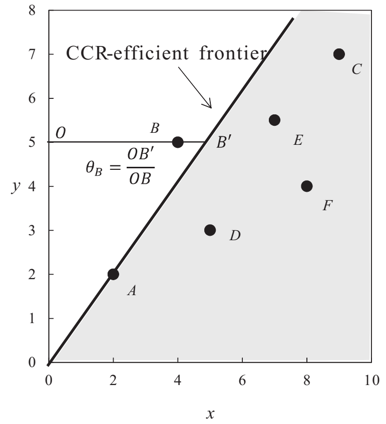


Figure 3.1 Extended CCR-efficient frontier for Table 2.1 (points : DMUs, solid line : CCR-efficient frontier generated by the data except for B)

3.1 Use of Optimal Weights $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$

Here, we describe our idea using DEA with the case for two outputs (attribute$_1$ and attribute$_2$), as shown in Table 3.1. In this example, furthermore, we assume that points with larger values are more dangerous. Figure 3.2 shows the frontier generated by CCR model for the given data set, and the frontier is regarded as the boundary of safety area.

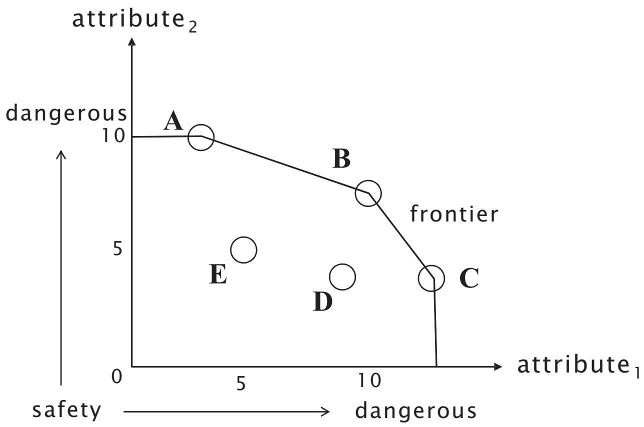


Figure 3.2 Frontier for case with two outputs

A, B and C with a ratio value 1 are on the frontier (solid line in Figure 3.2), and are relatively more dangerous than D and E. As seen from Table 3.1 of DEA results, we can know which attribute is effective from the optimal weights μ_1 and μ_2. For example, A has the first weight 0.0 and the second weight 1.0, which means attribute$_2$ is more effective than attribute$_1$ for A that has a relatively large value for attribute$_2$. Conversely, attribute$_1$ is more effective than attribute$_2$ for C, and both attributes are effective for B.

Table 3.1 Ratio Values and Weights for the Data in Figure 3.2 by CCR

ID	attribute$_1$	attribute$_2$	θ	μ_1	μ_2
A	3	10	1	0.00	1.00
B	10	8	1	0.50	0.50
C	12	3	1	1.00	0.00
D	5	5	0.56	0.22	0.78
E	9	4	0.80	0.71	0.29

Therefore, we can think that the attributes with a large weight are more effective than small ones, and it is interpreted as that a large weight has a strong influence on evaluation. So, we utilize weight information on eliciting relevant attributes to collapse of roads from basic and inspection data.

3.2 Use of a Ratio Value θ

In this subsection, we give an explanation for the use of a ratio value θ. We consider two test data F and G, as shown in Figure 3.3. As can be seen from the figure, a ratio value for F is larger than 1, and F is further than G from the frontier. This means that F is more dangerous than G from the viewpoint of collapse of roads.

Therefore, we can think that the points with a large ratio value are more dangerous than small ones. Also, we utilize a ratio value on deciding a priority for preventive maintenance of sewerage systems.

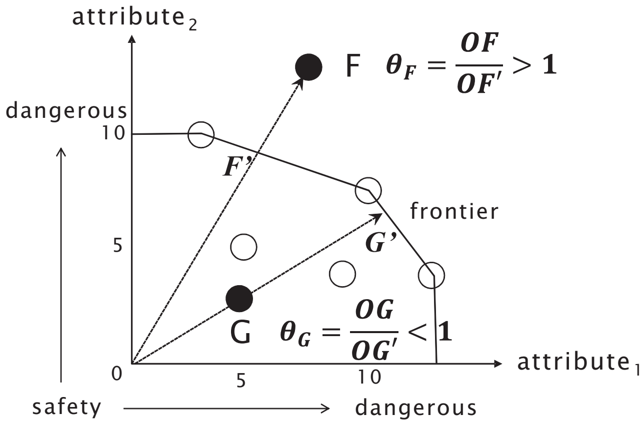


Figure 3.3 Evaluation for Test Data F and G by E-CCR

4 CONCLUSION

Significant benefits of DEA are that it can measure a relative distance of each alternative to the so-called DEA-efficient frontier and give information on the optimal weight corresponding to each attribute. From these facts, one may see how efficient or inefficient each alternative is, and also which attribute is important or relevant to the evaluation. This is important in supporting decision making with multiple criteria. Therefore, DEA models can be regarded to be helpful in problems with multiple criteria, and also applicable to a wide range of real problems.

It can be expected that DEA for evaluation of sewerage systems is useful and especially, in this research, we applied DEA to the evaluation of sewerage systems based on inspection data. Due to the page limitation, the details for experimental results are ommited, and will be described at the presentation. Through the data for Chuo-ward in Osaka City, DEA can find important and relevant factors to collapse of roads due to deteriorated sewerage systems, and calculating latent risk degree of each sewerage system, we can decide a priority for maintaining preventively, properly and effectively deteriorated sewerage systems. As the future works, we are going to investigate the effectiveness of using DEA for the other area of Osaka City.

Acknowledgment

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Notes

1. In multi-objecctive optimization, a point $\hat{\boldsymbol{x}} \in X$ is is said to be a Pareto optimal solution if there exists no $\boldsymbol{x} \in X$ such that $\boldsymbol{f}(\boldsymbol{x}) \leq \boldsymbol{f}(\hat{\boldsymbol{x}})$, where $\boldsymbol{f}$ is a vector of objective functions and X is a set of feasible solutions. And the set of Pareto optimal solutions in objective function space is called as Pareto frontier (Pareto, 1906).

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65 VECTOR OPTIMIZATION PROBLEM AND VECTOR VARIATIONAL-LIKE INEQUALITIES

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Abstract: Some properties of α-weakly preinvex and pseudoinvex functions via Clarke-Rockafellar and limiting subdifferentials are obtained. Furthermore, the equivalence between vector variational-like inequalities and vector optimization problems are studied under pseudoinvexity condition. We also consider two generalized Minty vector variational-like inequalities and investigate the relations between their solutions and vector optimization problems for non-differentiable α-invex functions.

Key words: Nonsmooth functions; Limiting subdifferential; Pseudoinvex functions; Vector variational-like inequalities; Vector optimization problems.

1 INTRODUCTION

The study of vector variational inequalities has become an important research direction of vector optimization problems. In particular, various relationship between vector variational inequalities and vector optimization problems have been established ((Giannessi (1980)), (Ward (2002) , (Yang-Teo (2004)), (Giannessi (1997)), (Jabarootian (2007)),(Ansari (2010)). In (Giannessi (1980)) introduced and studied the concept of vector variational inequality in finite dimensional spaces. (Giannessi (1997)) gave a direct application of Minty vector variational inequalities to establish the necessary and sufficient conditions for a point to be a solution of vector optimization problems for differentiable and convex functions are that the point to be a solution of Minty vector variational inequalities.The vector variational-like inequalities (VVLI), a generalization of (VVI) was studied in ((Lin (1976)), (Chiang (2005)), (Yang (2006)), (Fang (2003)), (Jabarootian (2008)), (Chinaie (2008)), (Rezaie (2009)), (Al-Homidan (2010))). (Yang (2006)) gave some relationships between Minty vector variational-like inequalities and vector optimization problems for differentiable pseudoinvex vector-valued functions. After these works, (Fang (2003)) obtained similar results for pseudoconvex functions with lower Dini directional derivative and Rezaie and Zafarani (Rezaie (2009)) obtained some of the above results for non-differentiable pseudoinvex functions. Very recently, (Al-Homidan (2010)) obtained these results for invex functions with Clarke's generalized directional derivative and then, (Ansari (2010)) showed that for pseudoconvex functions with upper Dini directional derivative, similar results hold.

In recent years, characterizations and applications for generalized preinvexity and generalized invexity were studied by many authors, see ((Yang-Teo (2003), (Garzon (2003)), (Yang-Teo (2005)), (Jabarootian (2006))) and references therein.((Yang-Teo (2003)) and (Yang-Teo (2005)) studied the relations between generalized invexity of a differentiable function and generalized monotonicity of its gradient mapping. For nondifferentiable locally Lipschitz functions similar results are obtained in terms of monotonicity of their Clarke subdifferentials in (Jabarootian (2006)). The concept of generalized differentials plays a fundamental role in modern variational analysis (Mordukhovich (2006)). In (Mordukhovich (1976)) presented and introduced a new subdifferential that it is defined by the limit of the other subdifferentials

which is called the limiting subdifferential.
Here, the equivalence between vector variational-like inequalities and vector optimization problems for non-differentiable functions under pseudoinvexity condition, are established. Furthermore, we consider two generalized Minty vector variational-like inequalities and compare their solutions with solutions of vector optimization problems. We also give some properties of the solution sets of non-differentiable vector optimization

2 PRELIMINARIES

This section deals with some known definitions and results which will be used in the sequel. Let X be a Banach space endowed with a norm $||.||$ and X^* its dual space with a norm $||.||_*$. We denote $\langle .,.\rangle, [x,y]$ and $]x,y[$ the dual pair between X and X^*, the line segment for $x,y \in X$, and the interior of $[x,y]$, respectively. Let K be a nonempty subset of X and $f : K \subset X \to \mathbb{R}$ be a non-differentiable real valued function. Now, we recall some concepts of subdifferentials that we need in the next sections.

Definition 2.1 *Let X be a normed vector space, Ω be a nonempty subset of X, $x \in \Omega$ and $\varepsilon \geq 0$. The set of ε-normals to Ω at x is*

$$\widehat{N}_\varepsilon(x;\Omega) := \{x^* \in X^* | \limsup_{u \xrightarrow{\Omega} x} \frac{\langle x^*, u-x\rangle}{||u-x||} \leq \varepsilon\}.$$

Let $\overline{x} \in \Omega$, the limiting normal cone to Ω at $\overline{x}$ is

$$N(\bar{x};\Omega) := \limsup_{x\to\bar{x},\varepsilon\downarrow 0} \widehat{N}_\varepsilon(x;\Omega).$$

Let $f : X \to \bar{\mathbb{R}}$ be finite at $\bar{x} \in X$; the limiting subdifferential of f at $\bar{x}$ is defined as follows

$$\partial_M f(\bar{x}) := \{x^* \in X^* | (x^*, -1) \in N((\bar{x}, f(\bar{x})); \mathrm{epi} f)\}.$$

Remark 2.1 *One can see easily that the set-valued mapping $x \mapsto \partial_M f(x)$ has closed graph for locally Lipschitz functions.*

Definition 2.2 *Let $\eta : X \times X \to X$. A subset K of X is said to be invex with respect to η if for any $x, y \in K$ and $\lambda \in [0,1]$, $y + \lambda\eta(x,y) \in K$.*

Definition 2.3 *Let K be an invex set with respect to η and $f : K \to \mathbb{R}$.*
(i) *f is said to be α-preinvex with respect to η on K, if there exists a constant α such that for any $x, y \in K$ and $\lambda \in [0,1]$, one has*

$$f(y + \lambda\eta(x,y)) \leq \lambda f(x) + (1-\lambda)f(y) - \alpha\lambda(1-\lambda)||\eta(x,y)||^2;$$

(ii) *f is said to be α-invex with respect to η on K, if there exists a constant α such that for any $x, y \in K$ and $\zeta \in \partial_M f(y)$, one has*

$$\langle \zeta, \eta(x,y)\rangle + \alpha||\eta(x,y)||^2 \leq f(x) - f(y);$$

(iii) *f is said to be α-weakly invex with respect to η on K, if there exists a constant α such that for any $x, y \in K$ there exist $\zeta \in \partial_M f(y)$ such that*

$$\langle \zeta, \eta(x,y)\rangle + \alpha||\eta(x,y)||^2 \leq f(x) - f(y).$$

Remark 2.2 *If $\alpha > 0$, then case*(i) *reduces to strong preinvexity and case*(ii) *to strong invexity. If $\alpha = 0$, then case(i) reduces to preinvexity, case(ii) to invexity and case(iii) to weak invexity.*

Definition 2.4 *Let $K \subset X$, $T : K \to 2^{X^*}$ be a set-valued mapping. T is said to be invariant α-monotone on K with respect to η if there exists a constant α such that for any $x, y \in K$ and any $u \in T(x), v \in T(y)$, one has*

$$\langle v, \eta(x,y)\rangle + \langle u, \eta(y,x)\rangle \leq -\alpha(||\eta(x,y)||^2 + ||\eta(y,x)||^2).$$

The following conditions are useful in the sequel.

Condition A. Let K be an invex set with respect to η and $f : K \to \mathbb{R}$. Then,

$$f(y + \eta(x,y)) \leq f(x) \qquad for\ any \quad x, y \in K.$$

Condition C . Let η: $X \times X \to X$. Then for any $x, y \in X, \lambda \in [0,1]$

$$\eta(y, y + \lambda\eta(x,y)) = -\lambda\eta(x,y), \qquad \eta(x, y + \lambda\eta(x,y)) = (1-\lambda)\eta(x,y).$$

For undefined terminologies, we refer to (Mordukhovich (2006)).

3 MAIN RESULTS

In this section, we consider Minty vector variational-like inequality (MVLI) and its weak version (MWVLI) and compare their solutions with vector optimization problems's solutions (VOP) for pseudoinvex functions. Furthermore, we also consider two generalizations of (MVLI) and (MWVLI) and we obtain relationship between of their solutions and (VOP) solutions for the α-weaky invex fuctions.
Suppose $f_i : K \to \mathbb{R}$, the components of $f : K \to \mathbb{R}^n$ are non-differentiable functions, $C := \mathbb{R}^n_+ \backslash \{0\}$ and int $C :=$ int $\mathbb{R}^n_+$. In the sequel we have the following ordering relation:

$$x \geq_C y \Leftrightarrow x - y \in C; \qquad x \not\geq_C y \Leftrightarrow x - y \notin C, \ \ \forall x, y \in K.$$

The definition of generalized subdifferential can be extended to vector-valued functions. Generalized limiting subdifferential of f at $x \in X$ is the set

$$\partial_M f(x) = \partial_M f_1(x) \times \partial_M f_2(x) \times \cdots \times \partial_M f_n(x).$$

Definition 3.1 *Let K be an invex set with respect to η and $f : K \to \mathbb{R}^n$.*
(1) Minty vector variational-like inequality (MVLI) *consists of finding a vector $x \in K$ such that*

$$\langle \partial_L f(y), \eta(x, y)\rangle \nsubseteq C \setminus \{0\}, \qquad \forall y \in K;$$

(2) Minty weak vector variational-like inequality (MWVLI) *consists of finding a vector $x \in K$ such that*

$$\langle \partial_L f(y), \eta(x, y)\rangle \nsubseteq \text{int } C, \qquad \forall y \in K.$$

(3) Stampacchia weak vector variational-like inequality (SWVLI) *consists of finding a vector $x \in K$ such that*

$$\langle \partial_L f(x), \eta(y, x)\rangle \nsubseteq -\text{int } C, \qquad \forall y \in K.$$

We consider now the following vector-minimization problem (VOP):

$$\min_{y \in K} {}_C \, f(y).$$

Solving a (VOP) means finding all the (weakly) efficient solutions, which are defined as follows.

Definition 3.2 *(1) $x \in K$ is said to be an efficient solution (Pareto solution) of* (VOP) *iff*

$$f(x) - f(y) \notin C \setminus \{0\}, \quad \forall y \in K;$$

(2) $x \in K$ is said to be a weak efficient solution (weak Pareto solution) of (VOP) *iff*

$$f(x) - f(y) \notin \text{int } C, \quad \forall y \in K.$$

We denote by $\bar{S}$ the set of all weak efficient solutions of (VOP).

Theorem 3.1 *Let X be an Asplund space and $f : K \to \mathbb{R}^n$ be locally Lipschitz on K and $C = \mathbb{R}^n_+$. Assume that $f_i : K \to \mathbb{R}$ for $1 \leq i \leq n$ are pseudoinvex with respect to function η, η is continuous with respect to the second argument and satisfies Condition* C. *Then $x_0 \in K$ is a solution of* (MVLI) *if and only if it is a efficient solution of* (VOP).

By a similar way, we can establish the weak version of Theorem 3.1

Theorem 3.2 *Let X be an Asplund space and $f : K \to \mathbb{R}^n$ be locally Lipschitz on K and $C = \mathbb{R}^n_+$. Assume that $f_i : K \to \mathbb{R}$ for $1 \leq i \leq n$, are pseudoinvex with respect to function η, η is continuous with respect to the second argument and satisfies Condition* C. *Then $x_0 \in K$ is a solution of* (WMVLI) *if and only if it is a weak efficient solution of* (VOP).

Remark 3.1 *Theorems 3.1, 3.2 also hold for the Clarke subdifferential in any Banach space.*

We introduce now two types of generalized Minty vector variational-like inequalities. Let K be an invex set with respect to η and $f : K \to \mathbb{R}^n$.
Problem (I): Generalized Minty vector variational-like inequality consists of finding a vector $x \in K$ and a real constant β such that

$$\langle \partial_M f(y), \eta(x, y)\rangle + \beta ||\eta(x, y)||^2 e \nsubseteq C;$$

Problem (II): Generalized weak Minty vector variational-like inequality consists of finding a vector $x \in K$ and a real constant β such that

$$\langle \partial_M f(y), \eta(x,y) \rangle + \beta ||\eta(x,y)||^2 e \nsubseteq \text{int } C;$$

where, $e = (\underbrace{1, 1, \cdots, 1}_{n})$.

Remark 3.2 (1) *In Problem* (I) *(resp.* (II)*) if $\beta = 0$, then it reduces to Minty (resp. weak) vector variational-like inequality* (MVLI) (2) *Notice that, if x_0 is a solution of Problem* (I)*, then it is also a solution of Problem* (II)*; furthermore, if x_0 is either a solution of Problem* (I) *or* (II) *with constant β, then x_0 is also their solution for all parameters $\beta' \leq \beta$. Hence, we have*

There are many examples of vector optimization problems that their solutions are not a solution of Minty vector variational-like inequality. Hence, by a minor modification in Minty vector variational-like inequalities, we get Problems (I) and (II). Notice that the role of term $\beta||\eta(x,y)||^2$ in Problems (I) and (II) is similar to a kind of perturbation in Minty vector variational-like inequalities. Because β can be chosen in $\mathbb{R}$, the solution set of Problems (I) and (II) are larger than the solution set of Minty vector variational-like inequalities.

We present now a necessary and sufficient condition for being an efficient solution of (VOP) a solution of Problem (I).

Theorem 3.3 *Let X be an Asplund space, $K \subset X$ be invex with respect to η and $f = (f_1, \ldots, f_n) : K \to \mathbb{R}^n$ be l.s.c. on K. If each f_i, $1 \leq i \leq n$ is α-weakly invex and $x_0 \in K$ is an efficient solution of (VOP), then it is also a solution of Problem* (I)*. Conversely, suppose that η satisfies Condition* C*, f is locally Lipschitz on K, each f_i satisfies Condition* A *and is α-invex with constant $\alpha_i > 0$ and x_0 is a solution of Problem* (I)*, then x_0 is a solution of (VOP).*

By a similar way of Theorem 3.3, we can deduce the relation between solutions of Problem (II) and weak efficient solutions of (VOP).

Theorem 3.4 *Let X be an Asplund space, $K \subset X$ be invex with respect to η and $f = (f_1, \ldots, f_n) : K \to \mathbb{R}^n$ be l.s.c. on K. If each f_i, $1 \leq i \leq n$ is α-weakly invex and $x_0 \in K$ is a weak efficient solution of (VOP), then it is also a solution of Problem* (II)*. Conversely, suppose that η satisfies Condition* C*, f is locally Lipschitz on K, each f_i satisfies Condition* A *and is α-invex with constant $\alpha_i > 0$ and x_0 is a solution of Problem* (II)*, then x_0 is a weak efficient solution of (VOP).*

Remark 3.3 *Theorems 3.1 and 3.4 also hold for the Clarke's subdifferential in any Banach space.*

Now, we obtain an existence result for the solution of Problem (II) and therefore a weak efficient solution of (VOP).

Theorem 3.5 *Let X be an Asplund space and $f = (f_1, \ldots, f_n) : X \to \mathbb{R}^n$ be locally Lipschitz such that each f_i, $1 \leq i \leq n$ be α-invex with constant α_i. Assume that the following conditions are satisfied:*
(1) *η is affine and continuous in the first argument and skew,*
(2) *there are a nonempty compact set $M \subset X$ and a nonempty compact convex set $B \subset X$ such that for each $x \in X \setminus M$, there exists $y \in B$ such that*

$$\langle \partial_M f(y), \eta(x,y) \rangle + \beta ||\eta(x,y)||^2 \subseteq \text{int } C,$$

where $\beta = 2\min\{\alpha_1, \ldots, \alpha_n\}$. Then Problem (II) *has a solution and the set of solutions is compact.*

Here, we obtain a relation between solutions of (VOP) and solutions of (SWVLI) for an arbitrary locally Lipschitz function without any convexity.

Theorem 3.6 *Let X be an Asplund space, $K \subset X$ be invex with respect to η and $f = (f_1, \ldots, f_n) : K \to \mathbb{R}^n$ be locally Lipschitz on K. Suppose that η satisfies Condition* C *and continuous in the second argument. Then any point in $\bar{S}$ is a solution of (SWVLI).*

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66 MODELING AND ANALYSIS OF THE CYBER INFRASTRUCTURE FOR LOGISTICS DISTRIBUTION ROUTE OPTIMIZATION

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Abstract: Using differential-algebraic equations, finite automata, stochastic process, queuing theory, this paper proposes a steady-state and a dynamic model for quantitatively analyzing the information transmission performance in different communication networks. Based on the proposed model, this paper also proposes a mathematical model for real-time vehicle route optimization considering the information system. The information system design for logistics vehicle dispatching is also given based on numerical simulations on different communication networks.

Key words: Logistics distribution; Information system; Vehicle route; Real-time optimization.

1 INTRODUCTION

Vehicle route problem has been attracting continuous research attention in the operations research field. In (A. Serdar Tasan, (2010)) established a mixed integer programming model for vehicle route optimization and applied genetic algorithm (GA) to solve the model. In (Cao Erbao, (2009)), proposed a chance constrained programming model under the fuzzy demand conditions for the vehicle routing, and a stochastic simulation-based hybrid differential evolution (DE) was used to effectively solve the problem. In (Jean-Yves Potvin, (2006)), proposed the dynamic vehicle route problem considering real-time customer demand and dynamic travel time, and compared the effects of different kinds of planning strategies. However, they did not consider the impact and limitation of the information system and only assumed that the required information is accurate and timely. With the continuous development of the logistics system, bringing a variety of challenges to the information transmission, such as information loss, and delay, etc. So, the research on the modeling of the information system and its impact on the logistics optimization can be of great importance.

2 MODELING OF THE LOGISTICS INFORMATION SYSTEM

Logistics information system consists of the real-time collection, transmission, and processing of the customer demand, vehicle status, and traffic status information. It can provide the basic data for the vehicle route optimization. The communication network is the core of the whole information system, and its model is presented below.

2.1 STEADY-STATE MODEL OF THE COMMUNICATION NETWORK

The steady-state model of the communication network can be formulated as a network flow model as follows:

1. Node information balance equation

   For any node $v \in V$, its information inflow and outflow should be equal, i.e.

$$\sum_{(i,v)\in E} S_{i,v} + \sum_{k=1}^{N_v} S_v\left(k\right) = \sum_{(v,j)\in E} S_{v,j} + \sum_{k=1}^{M_v} O_v\left(k\right) \tag{1}$$

   where $(i, v) \in E$ and $(v, j) \in E$ indicate that nodes i and j are connected with node v directly. $S_{i,v}$ and $S_{v,j}$ represent respectively the data rate of the information flow from node i to node v, and from node v to node j. N_v denotes the number of information sources located at node v. $S_v(k)$ is the data rate of the information flow injected by source k at node v. M_v represents the information flow ended at node v. $O_v(k)$ is the data rate of the information flow k which is ended at node v.

2. Node information flow limit constraint

   For any node $v \in V$, its information inflow should not exceed its information exchange ability as described by

$$0 \le \sum_{(i,v)\in E} S_{i,v} + \sum_{k=1}^{N_v} S_v\left(k\right) \le C_v \tag{2}$$

3. Link information flow limit constraint

   For any link$(i, j) \in E$, its information flow should not exceed its bandwidth

$$0 \le S_{i,j} \le B_{i,j} \tag{3}$$

   where $B_{i,j}$ indicate the bandwidth of link (i, j).

The feasible working state can be obtained by solving the model denoted by Eqns. (1)-(3) directly.

2.2 DYNAMIC MODEL OF THE COMMUNICATION NETWORK

The dynamic model of the communication network can be formulated based on the network layer and the transport layer in the open system interconnection (OSI) model.

In the network layer and transport layer, we will mainly model routers, communication links, and the congestion control protocol acting as a key control mechanism for handling communication network congestion. Routers and communication links usually have memory buffers, the memory buffer can usually be modeled as a queue. The size of the memory buffer is called the maximum queue size. Take the TCP/IP network as an example, the congestion control protocol can mitigate the congestion in two ways. First, the information volume injected into the network can be decreased, and this can be implemented by reducing the congestion window size of some nodes. Second, some data packets with a lower priority can be deleted proactively from the memory buffer to avoid the memory buffer overflow, and further the data loss. Because of the congestion control protocol, the congestion window size and queue size will vary dynamically when network congestion occurs; thus they can be selected as the state variables of the dynamic model. The outputs of the cyber system can usually be set as the communication delays and data loss rates of information flows. Denote $X\left(t\right)$ as the state variable vector, $Y\left(t\right)$ as system output vector and $u\left(t\right)$ as the control signal vector, then the dynamic model of the communication network can be expressed as

$$\dot{X}\left(t\right) = f\left(X, u\right) \tag{4}$$

$$\dot{Y}\left(t\right) = g\left(X, u\right) \tag{5}$$

For different working states of the communication network, the dynamic model will be different. Still take the TCP/IP network as an example, a queue can transit between three different states: empty, non-empty, and full. Also, corresponding to different congestion conditions, the TCP protocol has three different working states: slow start, congestion avoidance, fast recovery. For all these working states, the differential equations should be different. Therefore, in order to handle the state transition of the communication network, we can introduce the finite automaton, and combine it with differential equations to form the mathematical model of the communication network.

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67 WEIGHTED VEHICLE ROUTING PROBLEMS: A NEW MODELING APPROACH AND BEAM SEARCH COMBINED WITH MAX-MIN ANT SYSTEMS

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Abstract: The vehicle routing problem (VRP) is a well-known operations research model for a class of transportation and logistics management. A typical vehicle routing problem (VRP) aims to find the optimal tours for several homogeneous vehicles from a depot to a lot of customers with known demands and return to the depot. All vehicles must visit all customers exactly once and cant exceed the capacity constraints. The classical VRP and its variants assume that the weight loaded in the vehicle keep unchanged through the visiting route, which results minimum cost measure equivalent to minimum distance one. That is to say, the costs associated with the amount of the loads in the vehicle are neglected in the objective function of the most VRP models; as a result, the optimal routes may not be the minimum cost one.

In real-world cargo transportation practice, the charge is not only on the traveling distance, but also on the loading weight. For example, a toll-by-weight measure for China Expressway is proposed, by which the cargo-truck are charged based on its loading quantity (sum of truck weight and cargo quantity) and traveling distance, rather than on distance only. As a result, a truck is charged different when it is empty, normally loaded or overloaded. In practical delivery of perishable/fruit food, a dedicated container/vehicle is equipped to monitor its status towards minimum delivery loss rather than traveling distance when optimizing routes. The delivery loss brought by the risk of perished, during the routing, is measured not only on traversing distance, but also on the quantity of the perished or hazardous products. As an instance, for some perishable food, even 70% of the price is used for loss compensation during transport. When delivering perishable food, one need to pay not only the distance cost, but also the extra loading cost caused by the cooling devices to keep food fresh during transport. To this end, people tend to delivery customers with large quantities and close location first to balance the cost of distance and loading, towards minimum loss caused or the total costs. as possible. Typically, for another example, large trucks such as gasoline tankers and containers, their fuel consumption are related to the weight of the cargo. Oil consumption is not the same for a truck with full loaded and unloaded. Trucks with full loads produce carbon emissions significantly more than empty trucks do. On lower carbon emission obligation policy, it is better to serve customers with large demand firstly to decrease the load on the truck and bring the economic benefits and environmental protection when the two customers have the same distance.

To modeling the above problems precisely occurred in general logistics management, the weight loaded in the vehicle should be considered as a variable part of the objective when optimizing the vehicle routine, rather than as a constant from a customer to another in a routine. For this purpose, this paper considers a new type of VRP with consideration of cargo weight, which we call weighted VRP, short for WVRP. The weight here is a terminology in general means, and may be extended to represent the number or values of objects (cargo/goods/passenger) to be delivered, importance/priority of customers (or points); and, correspondingly, the terminology cost to measure the objective here has as well broad meaning. Apart from the costs itself, it may represent the minimum emission/petrol consumption when transporting general goods, minimum risk of loss brought by transporting perishable foods, livestock or dangerous goods, maximum utility, satisfaction degree, values or benefits of delivering customers, etc. In this aspect, the WVRP represent a new modeling approach to solving practical vehicle routing problem (VRP).

There are large volumes of models and algorithm for CVRP and its variants, such as capacitated vehicle routing problem (CVRP) requires the load on one vehicle cannot exceed the capacity, vehicle routing problem with time windows (VRPTW) adds the service time constraints of every customer on CVRP; multi-depot vehicle routing problem (MDVRP) has multiple depots instead of a single depot; periodic vehicle routing problem (PVRP) considers the service time as a period instead of a day; VRP with pickup and delivery(VRPPD) has both delivery and pickup; split delivery vehicle routing problem (SDVRP) allows each customer can be served more than once and so on.

This paper aims to consider weighted VRP (short for WVRP) by generalizing the VRP with loading costs in two aspects. One is that the weight may represent not only quantity in absolute way, but also priority of the customers in relative way. The other is the formulation of objective function and its interpretation. To demonstrate the effectiveness of the WVRP, computational experiments were carried out on benchmark problems of capacitated VRP with seven types of distribution. For this purpose and effectively solving WVRP, a beam search combined with ant colony optimization algorithm (short for BEAM-MMAS) is developed as a means to show how effectiveness of WVRP more than VRP and for which types of VRP instances WVRP has more cost-saving than classical VRP. The effectiveness of the BEAM-MMAS algorithm is tested on large sizes of benchmarking instances in comparison with general ACO and MMAS. Full factorial experiments are conducted to suggest a set of better parameters for implementing the algorithm.

Key words: Optimization; Regression models; Mining process.

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68 ELECTRIC VEHICLE ROUTE OPTIMIZATION CONSIDERING THE TIME-OF-USE ELECTRICITY PRICE

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Abstract: Based on the traveling salesman problem (TSP), this paper proposes an optimal EV route model considering the rapid-charging and the depot charging in the context of the time-of-use (TOU) electricity price. The proposed model considers the impact of the vehicle load on the electricity consumption per mile, and aims to minimize the total distribution costs of the EV route while satisfying the EV battery capacity limits. The immune algorithm (IA) is employed to solve the proposed model.

Key words: Electric vehicle (EV); Vehicle route optimization; Time-of-use electricity price; Immune algorithm (IA).

1 INTRODUCTION

The route optimization of the EVs is a relatively new research topic. In (Sevgi Erdogan. (2012)), a route optimization model of the alternative energy-fueled vehicles is reported. In (Michael Schneider. (2012)), the EV charging station is introduced and some constraints such as battery capacity is considered in addition to the conventional VRP constraints. However, this model does not consider the EV charging costs. In (Owen Worley. (2012), Ryan G. Conrad. (2011)), the EV charging cost is reflected, but it is based on the assumption that the charging price is constant, which is however not realistic. It can be seen that the different charging modes are not considered and the cost function of the problem is relatively rude. In particular, the time-of-use (TOU) electricity price in the power system is not considered. In practical logistics distribution system, although the reported model can provide a reasonable vehicle rout, it is difficult to minimize the distribution costs.

2 PROPOSED MODEL

2.1 PROBLEM DESCRIPTIONS

The routing optimization of EV can be described as: an EV with the battery capacity of Q starts from the depot v_0. then deliver goods to n customers and finally return back to v_0. The customer vertex is denoted as $I = \{v_1, v_2, \cdots, v_n\}$, customer demand is denoted as $\mu_i\,(i \in I)$, the loading at vertex j is l_j, and the initial loading is the sum of all the individual customer demand.

Assume there are k rapid-charging stations in one area, $\{v_{n+1}, v_{n+2}, \cdots, v_{n+k}\}$, each station can accommodate s fast charging service. When the EV returns back to the depot, the regular charging is then applied. It is also assumed that after each charging, either fast or regular, the battery is fully charged. The TOU electricity price is considered for the charging stations and the depot. $M(t)$ denotes the electricity price of the rapid-charging at time t, $m(t)$ denotes the electricity price of the regular charging in the depot at time t, and $M(t) > m(t)$.

In the proposed model, the charging stations can be visited many times or never be visited, while the customer vertex must and only be visited once. For a revisited charging station, the battery level, loading level, and the visiting time can be different, in such case, dummy vertices are used to distinguish a revisited station. If a charging station v_{n+i} is revisited s times, the dummy vertices are denoted as $F^{(i)} = \left\{v_{n+i}^{(0)}, v_{n+i}^{(1)}, \cdots, v_{n+i}^{(s)}\right\}$, $i = 1, 2, \cdots, k$. The full set of the dummy vertices of all the charging stations is $F = F^{(1)} \cup F^{(2)} \cup \cdots \cup F^{(k)}$. The collection of the system vertices is $V = I \cup F \cup \{v_0\}$, $|V| = n + k \times s + 1$.

The state-of-health (SOH) is an important parameter of the EV battery. Let C denotes the battery replacement cost and N denotes the maximum cycle life of a battery, the SOH cost of each rapid-charging is C/N.

2.2 OBJECTIVE FUNCTION

The objective function in the proposed model is as follows:

$$min \quad P_{fast} \cdot \sum_{i \in V, j \in F, i \neq j} \left(\int_{t_j}^{t_j + T_j} M(t)\, dt \right) \cdot x_{ij} + P_{slow} \cdot \int_{t_{v_0}}^{t_{v_0} + T_{v_0}} m(t)\, dt + G_{total} \cdot \frac{C}{N} \tag{1}$$

where the first and second terms respectively correspond to the charging cost of rapid-charging and regular charging, and the third term represents the SOH cost by the rapid-charging. Note that the rapid-charging power P_{fast} and regular charging power P_{slow} are constants; T_j denotes the duration time of rapid-charging at dummy vertices j; T_{v_0} represents the duration time of regular charging at the depot; G_{total} denotes the total number of rapid-charging, i.e., the total number of the visit to the dummy vertices; t_j denotes the visiting time to vertices j; q_j denotes the battery level when the EV arrives at vertices j.

2.3 CONSTRAINTS

The objective function is subject to the following constraints:

$$\sum_{j \in V, i \neq j} x_{ij} = 1 \quad \forall i \in I \cup \{v_0\} \tag{2}$$

$$\sum_{j \in V, i \neq j} x_{ij} \leq 1 \quad \forall i \in F \tag{3}$$

$$\sum_{i, j \in F^{(h)}, i \neq j} x_{ij} = 0 \quad h = 1, 2, \cdots, k \tag{4}$$

$$\sum_{i \in V, i \neq j} x_{ji} - \sum_{i \in V, i \neq j} x_{ij} = 0 \quad \forall j \in V \tag{5}$$

$$\sum_{i \in Z, j \in \overline{Z}} x_{ij} \geq 1 \tag{6}$$

$$0 \leq q_j \quad \forall j \in V \tag{7}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V, j \in V, i \neq j \tag{8}$$

where Eq. (2) restricts that each customer vertex has exactly one successor; Eq. (3) means that each dummy vertex can only be visited once at most; Eq. (4) denotes that the dummy vertices of the same charging station are not connected; Eq. (5) means the EV arrives at and leaves from each vertex is the same one; Eq. (6) is the subtour breaking constraint, which ensures that the EV route is a circuit; Eq. (7) means the EV battery level is positive when the EV arrives at a vertex. Eq. (8) denotes the binary decision variable, 0 or 1.

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69 BOUNDEDLY RATIONAL ROUTE CHOICE UNDER TRAVEL TIME UNCERTAINTY

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Abstract: In reality, route choice is one of the daily decisions made by travelers in an uncertain environment. This study formulates the route choice as a process of decision among alternative routes, on which the travel time is uncertain (or ambiguous). We also analyze the boundedly rational route choice problem based on the non-expect utility theory. Numerical example is presented to illustrate the key concepts and results of the model.

Key words: Route choice; Bounded rationality; Travel time uncertainty; Decision theory; Non-expected utility.

1 INTRODUCTION

The route choice problem refers to the selection of a route for a particular origin-destination pair given a set of alternative routes by the motorists. The question of interest is how these motorists will be distributed among the possible routes. For the past several decades, travelers decision making has been studied within the framework of expected utility theory (Von Neumann and Morgenstern, 1944). In reality, travel time is uncertain and it is not always reasonable to assume that travel times are deterministic and exactly known by all travelers because travel times are unknown to users at the time when they make their routing decisions. In this paper, we analyze the boundedly rational route choice problem under travel time uncertainty based on the non-expect utility theory.

2 DECISION THEORY UNDER UNCERTAINTY

Uncertainty usually takes two forms: risk or ambiguity (Knight, 1921). An uncertain environment is referred as risky if the set of outcomes and probability distribution of the outcomes is known, while it is called ambiguous if uncertainties cannot be reduced to simple probabilities or when the true probability distribution of outcomes is unknown. Hsu et al. (2005) extend the study of the neural basis of decision under risk to encompass ambiguity using functional brain imaging. Bernoulli(1738) present the expected value (e.g. travel time, money) theory to account for Petersburg paradox. von Neumann and Morgenstern(1944) provide a sound theoretical foundation for using expected utility (EU) as a guide to decision making under risk.

3 ROUTE CHOICE MODEL BASED ON α-MAXMIN EXPECTED UTILITY(α-MEU)

In De Palma and Picard (2005) empirical study, they conclude that the exponential function, which appeals to travelers with Constant Absolute Risk Aversion (CARA), aptly characterizes travelers preference for travel time under uncertainty. Followed by Ahn et al. (2011), we assume the utility function $u(x)$ with constant absolute risk aversion (CARA):$u(x) = -e^{-\rho x}$. The ambiguity-aware CARA utility is given by:

$$U = \alpha - MEU = \alpha e^{-\rho t_0} + (1-\alpha)e^{-\rho(t_0+t)}$$

4 NUMERICAL EXAMPLE

This section presents some numerical examples to demonstrate the above route choice model.

5 CONCLUDING REMARKS

This paper analyzed the boundedly rational route choice problem under travel time uncertainty based on the non-expect utility theory. Numerical example is given to illustrate the key concepts and results of the model. In the future, we can study the departure time choice or mode choice or the integral model under uncertainty. Dynamical process of route choice problem is also our one of future research topics.

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70 RESEARCH ON OPEN VEHICLE ROUTING PROBLEM WITH DEMAND UNCERTAINTY BASED ON ROBUST OPTIMIZATION

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Abstract: We firstly describe the customer demand as specific bounded uncertainty sets with expected demand value and nominal value, and propose the model of robust optimization counterpart. Then we present an improved differential evolution algorithm to solve the robust model, and analyze the performance by considering the extra costs and unmet demand. Finally, the computational experiments indicate that the trade-offs between the extra cost related to robust solution and unmet demand related to deterministic best solution when the customer demand is uncertain.

Key words: Logistics distribution; Open vehicle routing problem; Fuzzy credibility; Monte Carlo simulation; Genetic algorithm.

1 INTRODUCTION

The OVRP differs from the well-known vehicle routing problem (VRP) in that the vehicles do not necessarily return to their original locations after delivering goods to customer (Sariklis and Powell,2000). The major difference in theory between the OVRP and the VRP is that the routes in the OVRP consist of Hamiltonian paths while the routes in the VRP are Hamiltonian cycles (Fu et al., 2005). Brandão (2004) and Fu et al. (2005) implemented a tabu search (TS) heuristic to solve the OVRP. Li et al. (2007) proposed a record-to-record travel heuristic and a deterministic variant of simulated annealing to solve the OVRP. However, traditional studies of the OVRP assumed that the demands of all customers visited on its route by any vehicle were deterministic. To the best of our knowledge, no research has considered uncertain customer demand in an OVRP framework.

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71 RESEARCH ON FRESH AGRICULTURE PRODUCT BASED ON OVERCONFIDENCE UNDER OPTIONS AND SPOT MARKETS DOMINATED BY THE RETAILER

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1 INTRODUCTION

The fresh agricultural products are the necessities in our daily life, which is essential for our healthy life. With the development of our economy, more fresh food is in largely demand. Besides, we require more in the freshness. Furthermore, developing fresh agricultural products is also an important method to improve the income of the peasants. As one kind of special products, fresh agriculture products are featured with deteriorated and perished. Some may be wasted when circulating. In order to speed up the circulation of them and reduce wastage, some scholars research them in the point of supply chain. Nowadays the wholesalers and retailers are more and more powerful than before, the dominance of the market has changed to some extent. However, some scholars model designs lack rationality. Furthermore, former studies are based on the assumption of rational people, ignoring their behavioral characteristics and other irrational factors, such as overconfidence and so on. In actual transactions, the participants tend to be over-confident and more optimistic of their own abilities, knowledge, and predictions about the future performance. Based on the above considerations, this paper attempts to establish a more reasonable model to analyze the optimal ordering policy of fresh agricultural product in the view of wholesalers.

2 HYPOTHESES

1. The initial inventory is zero, and the fresh agricultural products are not returnable, which is determined by their characteristics, and the remaining residual value is so little that it can be considered to be zero.

2. When the retailer is in the dominant position, shortage cost is transferred to the supplier by virtue of its strong negotiation skills.

3 FRESH AGRICULTURE SUPPLY CHAIN MODEL ANALYSIS

3.1 The suppliers outcome policies

As the follower, the supplier determines its own production according to the retailer's order quantity. Due to the uncertainty of market demand, when the options market and the spot market exist at the same time, the supplier first meets the retailer's stock Order.

Proposition 1: when the supplier supports the shortage cost, its optimal outcome of fresh agricultural product is

$$Q_s^* = \frac{(\theta\mu + \theta\sigma R)}{1-\beta}, \quad \Phi(R) = \frac{(g + c_e - c_0)(1-\beta) - c}{(c_e + g)(1-\beta)}$$

3.2 The retailers optimal centralized decision with the supply chain dominated by itself

Centralized decision is designed to study the performance of the supply chain as a whole. At this time, the retailer decides variable c0 , variable ce , variable Qs . To maximize the overall profit of the supply chain, the retailer tries to make optimal order quantity and the outcome of the supplier.

Proposition 2: with the centralized decision, the optimal outcome of the supplier which could maximize the profit of whole supply chain is written as

$$Q^{**} = \frac{1}{1-\beta}(k\theta\mu + \theta\sigma A), \quad \Phi(A) = 1 - \frac{c}{(p+g)(1-\beta)}$$

Proposition 3: when the retailer decides the option premium and option executive price, in order to maximize its profit, there will be

$$Q_1^* = \frac{1}{1-\beta}(k\theta\mu + \theta\sigma B), \quad \Phi(B) = \frac{c_1 + c_0 - w_0}{c_e}$$

4 CONTROL AND ADJUSTMENT OF RETAILERS OVERCONFIDENCE LEVEL

When the retailer is overconfident about the market demand of fresh agriculture product, the interests of both sides will change with the level of retailer's overconfidence. At this time, the supply chain as a whole does not reach the best situation. What's more, the fluctuation in the level of overconfidence is not conducive to the cooperation between the two sides. It is necessary to find a suitable way to eliminate the impact of retailer's overconfidence. Croson [12] designed a buy-back contract and wholesale-price contract to eliminate the retailer's overconfidence in general supply chain. Due to the higher requirements of fresh agriculture product, the perishable feature determines the products cannot be returned. As a flexible tool, options can be used to achieve the coordination of the supply chain. In this article, we attempt to use the option contract to control the retailer's overconfidence, so as to coordinate the fresh agriculture supply chain at last.

Proposition 4: when c_0 and c_e satisfy

$$c_0 = \frac{c - [1 - \Phi(A - \frac{\mu(k-1)}{\sigma})](g + c_e)(1-\beta)}{1-\beta}$$

the outcome is optimal.

5 CONCLUSION

Generally speaking, this paper changes the mode of the traditional supply chain that the seller develops options and the buyer purchases options, and introduces this new mode into the research about the ordering strategy of fresh agricultural product. Furthermore, based on the influence of the retailers overconfidence on the prediction of market demand, we analyzed the case of the wholesaler setting options, how the overconfident coefficient affects the optimal quantities and the profits as well as how the optimal ordering quantity varies. Finally, we could draw the conclusion that the optimal order quantities are related to the level of overconfidence. When the prospect of market is good, overconfidence of the retailer has a positive effect, and vise versa.

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72 NETWORK INFORMATION SERVICES PRICING: CONSIDERING CONSUMERS' OVERCONFIDENCE

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1 INTRODUCTION

With the gradual application of Internet of Things technology in the service industry, the pricing of information services will be inevitably involved when it provides information. Therefore, an appropriate pricing scheme service providers, intermediaries and consumers are willing to accept through consumer overconfidence has gained a significant amount of scholars attention. For the characteristics of information service of Internet of Things is different from traditional service, this makes marginal cost pricing in traditional service is not suitable for the pricing of information service in the Internet of Things, the existing study mainly concentrated on why and how to pricing level, the studies that consider how consumer overconfidence has an effect on the pricing of information service are rare, while consumer overconfidence behavior is a kind of common phenomenon. Based on this, this article put overconfidence theory of behavioral economics into information service pricing model. We consider the operators have the ubiquity of network coverage, the strong network infrastructure as well as reliable public service provider status, the ability to develop on the whole Internet of Things is operators, so we focus on the information service pricing strategy between operators and consumers. First of all, using the method of mathematical modeling, we construct the pricing model of information service of Internet of Things based on the consumer overconfidence, under the condition of the monopoly and the duopoly, it then discuss consumer overconfidence to the influence of operator returns and pricing strategy between connect-time-based pricing and search-based pricing and between the combination of connect-time-based and search-based pricing and subscription pricing. The results show that whether operators adopt connect-time-based pricing depends on the cost under the monopoly condition, when the cost is less than a certain value, the search-based strategy dominates the connect-time-based pricing strategy, if the cost is greater than a certain value, the connect-time strategy dominates. If the distribution of consumer overconfidence is the same across both the high- and the low-demand consumer, then the operators does not make any additional profits by offering a subscription plan, and we deduce that the degree of overconfidence of high demand of consumers is lower than the low demand of consumers. the distribution of consumer overconfidence for the low-demand consumers stochastically dominates (in the first-order sense) the distribution of consumer overconfidence for the high-demand consumers, and the valuation of a unit of information service lie in a certain range, the ratio between high demand and low demand is large enough, the operator is strictly better off by offering a subscription plan, it deduced that If all the low demand of consumer buy the information service, then all the high demand of consumers will buy. Then it consider how the operator make a price when the operator faces the two different degree of overconfidence of consumers, using only a pricing strategy, operator should use a connect-time-based pricing strategy for the high degree overconfidence for consumers, and adopt the a search-based strategy for the low degree overconfidence for consumers. Using two different strategies, the operator should consider the difference of consumer overconfidence and adopt different pricing strategy, and it also can provide operators with the following reference: with the number of using service of IoT increase, the consumer know more about the service of IoT and the relevant services of time will be less and less, then the degree of overconfidence will tends to be more and more rational, the operator should adjust the corresponding pricing strategy and makes its profit increase, In more advanced search facility or faster access speed cases, the time consumer spend on the search correlative services will be reduced, the degree of overconfidence will tend to rational, the operator needs to change the pricing strategy to adapt to consumer overconfidence degree change. It mainly discuss the operator pricing strategy under the monopoly condition, but in reality, there are many operators coexist, the competition between operators will make the price changes, thus consumer strategy will change, and prompted operators and other participant's income change. So we need to discuss the competition pricing for the several operators, therefore this part mainly take about how the duopoly competition among operators affect their pricing strategy. We assume the following decision structure. In Stage 1, operators choose a pricing scheme. We will restrict the types of pricing schemes to pure connect-time-based pricing or a pure search-based pricing. In Stage 2, after observing each other's pricing strategy, each server decides on the specific prices. This decision structure is reasonable since operators find it much easier to change specific prices than changing the pricing strategy. In Stage 3, consumers make their purchase decisions. Under the condition of Duopoly, Assume that operators choose pricing policy and then choose specific prices, and consumers are differentiated only in terms of the degree of overconfidence. When operators make a choice between the connect-time-based pricing and search-based pricing, there exists an equilibrium in which both operators offer different pricing policies. Further, there exist parameters such that both operators make positive profits by choosing different pricing strategies, and consequently in equilibrium, both operators choose different pricing policies. It indicates that once we consider the degree of consumer overconfidence, the oligarchs will choose asymmetric pricing strategy and will get different benefits. This is because the same type of operators can differentiate themselves by using different pricing schemes. This argument will also hold if we allow operators to offer subscriptions. As long as the information services are undifferentiated, operators will select different pricing mechanism so that the pricing mechanism can be used as a way to differentiate their services. At the same time, operators promote information service of IoT, they should integrate the advantage resources, develop different service, and pay attention to different development strategy. Except for the differentiation service in the same field, operators also adopt differentiated industry positioning in different fields, and select key industries and applications, make efforts to expand the market, exercise differentiated operation. So it can form rival differentiation services, and ultimately improve the quality of service. When operators choose between the combination of connect-time-based and search-based pricing and subscription pricing, there must be exist pure strategy equilibrium that two operators adopt the combination pricing strategy, here there are also asymmetric game equilibrium that one operators use the subscription strategy, the another operator adopt the combination pricing strategy, the equilibrium condition is that the consumer overconfidence need to meet certain conditions. Use Matlab7.0 program, under the condition of the monopoly and the duopoly, this article analysis that the consumer overconfidence has an effect on the operators information service pricing strategy and profit and implement the inspection of pricing model, through the application of intelligent traffic information service. Finally this article summarizes the conclusion also points out further research direction.

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73 EQUILIBRIUM PROPERTIES OF COMMUTING ON A MASS TRANSIT SYSTEM WITH HETEROGENEOUS COMMUTERS

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Abstract: This Paper will analyze the equilibrium properties of the morning peak-period commuting pattern on a multiple origins and single destination transit system with in-vehicle crowding effect, the value of travel time and schedule delay cost. In this paper the total generalized travel cost equals to sum of fare, crowding effect , value of travel time and schedule delay cost. Commuters are assumed to choose their optimal time-of-use decision from home locations to a single destination by trading off the travel time and the crowding cost against the schedule delay cost. An equivalent mathematical programming model proposed is formulated from the commuters point of view, based on minimizing their own travel costs. The model is proposed to characterize the equilibrium state, in which no commuter can reduce his/her total commuting cost by unilaterally changing his/her departure time or train service. Assuming that all commuters are homogeneous with the same crowding effect function, travel time value function and schedule delay cost function. In this assumption, the insight of model solution includes the following: (1) the farther a station is from the workplace, the longer is the peak-period departure duration from that station; (2) the peak-period exists for each station during which the departure rate of commuters is identical and maximal. When commuters are assumed heterogeneous, which means different commuters have different crowding effect function, travel time value function and schedule delay cost function, we can receive some different conclusions against homogeneous Assumption. For example, the peak-period exists for each station during which the departure rate of commuters is bell distribution, but not identical and maximal. This paper analyzes the equilibrium properties of three factors that influence commuters selection. At last this paper applies statistics of Beijing metro to verify effectiveness of the model. Thereby it offers useful information for optimal transit service planning and operations.

Key words: Public transport; Congestion cost function; Wardrop UE equilibrium; Equilibrium properties.

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74 A LEARNABLE GENETIC ALGORITHM TO THE RESOURCE OPTIMIZATION PROBLEM

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Abstract: A learnable genetic algorithm is proposed to the resource optimization problem. The learnable genetic algorithm extracts some knowledge from obtained solutions, and applies the knowledge to guide the subsequent optimization process. Experimental results suggest that this approach is efficient to the resource optimization problem.

Key words: Genetic algorithm; Software engineering; Allocation of resources.

1 INTRODUCTION

Recently, more and more scholars have studied applications of the interaction between evolution and learning (Xing et al., 2008a, 2008b and 2010a). Normally, these approaches keep useful features of previous individuals to improve the performance of current individuals (Xing et al., 2006a, 2007, 2009). In fact, such approaches outperform traditional evolutionary algorithms on several benchmarks (e.g., flexible job shop scheduling problem, traveling salesman problem, capacitated arc routing problem) (Xing et al., 2006b, 2010b; Ho et al., 2007; Louis & McDonnell, 2004). In the similar fashion, an Learnable Genetic Algorithm (LGA) is proposed in this work.

2 PROBLEM FORMULATION

Activity is the fundamental element in the practical engineering, suppose there are totally N activates and M persons. Suppose that T_{di}, T_{di}^{c} and T_{di}^{n} denote the period of activities, the time remainder of activities and the actual completion time of activities Respectively. $Role_{i,j}$ means the i^{th} activity can be done by the j^{th} person. $RoleNum_i$ is the completion person number of activities. T_D and T_N are the planned completion time and the actual completion time of projects. C_D and C_N denote the planned cost and the actual cost of projects. $f_T(EM)$ and $f_C(EM)$ denote the completion date function and the cost function of projects.

The cost of software process contains: management fee and development fee. The former is daily cost C_1 which maintaining software development and cost of administrators C_2. The latter points at cost of device resources C_3 and development cost C_4 of different developers with various abilities and labor-hours.

$$\begin{cases} CN = C_1 + C_2 + C_3 + C_4 \\ C_4 = \sum\limits_{i=1}^{M} c_i T_i \end{cases} \tag{2.1}$$

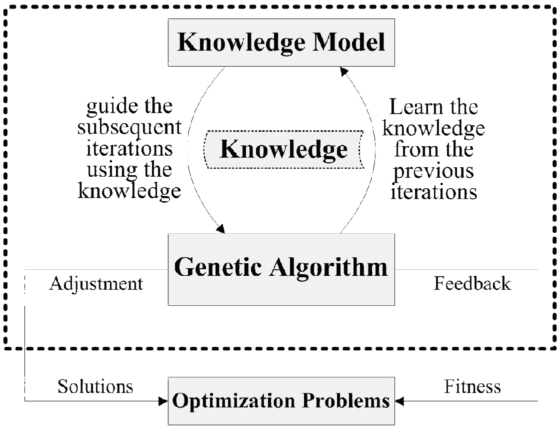

Figure 3.1 The basic framework of LGA.

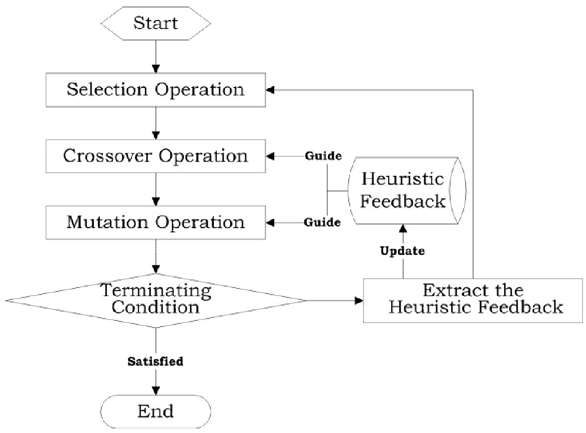

Figure 3.2 The computational flow of LGA.

Here, c_i and T_i denote the development cost and labor-hour of developers respectively. Cost and construction period is the main attributes of software process. Assume the sum weight of cost and period as optimized objective function, the definition of which is as shown in the following equation. In which, C means cost, T signifies the weight value of construction period x_1, $x_2$determined by demand of decision makers, EM is the executive matrix.

$$\max \ Fitness\,(C,T) = x_1 \ * C + x_2 * T$$
$$s.t. \begin{cases} C = f_C\,(EM) \\ T = f_T\,(EM) \\ 0 \le x_1 \le 1 \\ 0 \le x_2 \le 1 \\ x_1 + x_2 = 1 \end{cases} \tag{2.2}$$

3 LEARNABLE GENETIC ALGORITHM

The Learnable genetic algorithm is characterized by the extraction and application of knowledge in the whole evolution process. In this paper, the near-optimal solutions obtained throughout the search are analyzed to extract the knowledge, and then the obtained knowledge is used to guide the subsequent search. The basic framework of LGA is displayed as Fig. 1. The computational flow of LGA is shown in Fig. 2.

3.1 Definition of knowledge

The first kind of knowledge is called the activity assignment position which is applied to establish a beneficial order for the given activity. A matrix HF_1 with size $N \times N$ is defined for the activity assignment position, $HF_1(i,j)$ denotes the total number of times of assigning the activity i to the j^{th} position among the near-optimal solutions obtained throughout the search. The second kind of knowledge is called the activity assignment person which is applied to establish the beneficial person for one given activity. A matrix HF_2 with size $N \times M$ is defined for the activity assignment person, $HF_2(i,j)$ denotes the total number of times of assigning the activity i to the j^{th} person among the near-optimal solutions obtained throughout the search.

3.2 Application of Knowledge

In LGA, the activity assignment position is applied to guide the crossover operation. The activity assignment position is employed to determine one beneficial position for the given activity. To the activity assignment position matrix displayed in Table 1, if we want to determine the beneficial position for activity 3, then we can obtain the following probabilities, and the beneficial position to activity 3 is decided by a random way with the following probability distribution.

Table 1 An Example of Activity Assignment Position

3	5	4	2	1	0
1	5	6	2	1	0
0	1	2	8	2	2
0	2	2	6	5	1
0	0	4	5	6	0
0	0	2	3	4	6

$$\begin{array}{lll}
\text{Position} \quad 1: & \dfrac{0}{1+2+2+8+2} & = 0 \\
\text{Position} \quad 2: & \dfrac{1}{1+2+2+8+2} & = 0.07 \\
\text{Position} \quad 3: & \dfrac{2}{1+2+2+8+2} & = 0.13 \\
\text{Position} \quad 4: & \dfrac{8}{1+2+2+8+2} & = 0.53 \\
\text{Position} \quad 5: & \dfrac{2}{1+2+2+8+2} & = 0.13 \\
\text{Position} \quad 6: & \dfrac{2}{1+2+2+8+2} & = 0.13
\end{array}$$

In LGA, the activity assignment person is applied to guide the mutation operation. The activity assignment person is employed to determine one beneficial person for the given activity. To the activity assignment person matrix displayed in Table 2, if we want to determine the beneficial person for activity 6, then we can obtain the following probabilities, and the beneficial person to activity 6 is decided by a random way with the following probability distribution.

Table 2 An Example of Activity Assignment Person

4	2	5	2	4	6
5	7	5	11	6	6
6	6	5	2	5	3

$$\begin{array}{lll}
\text{Person} \quad 1: & \dfrac{6}{6+6+3} & = 0.40 \\
\text{Person} \quad 2: & \dfrac{6}{6+6+3} & = 0.40 \\
\text{Person} \quad 3: & \dfrac{3}{6+6+3} & = 0.20
\end{array}$$

3.3 Knowledge Updating

After each generation, if the global optimal solution (the best solution from the start) was obtained at this iterative, then the knowledge level will be updated by the following rule, which is based on the optimal solution to accomplish knowledge updating. If the activity i to the j^{th} position among the best solution, then

$$HF_1(i,j) = HF_1(i,j) + 1 \tag{3.1}$$

If the activity i to the j^{th} person among the best solution, then

$$HF_2(i,j) = HF_2(i,j) + 1 \tag{3.2}$$

4 EXPERIMENTAL RESULTS

The LGA was implemented using Visual C++ language, and executed on a personal computer with the 2 GHz processor and 2 GB memory. In this paper, the final experimental results were averaged over 30 trials, and 10 testing instances were randomly produced to validate the performance of our approach. The optimal objectives obtained by the IGA are summarized in Table 3. From the experimental results of Table 3, we can see that, there exists the small gap among different trials. Experimental results suggest that it is efficient to the given problem.

Table 3. The Final Experimental Results

SN	*N*	*M*	*Cost*		*Time*	
			Avg.	*Std.*	*Avg.*	*Std.*
1	20	5	55.6	0.28	22.9	0.15
2	20	5	86.9	0.15	25.6	0.19
3	20	5	75.3	0.31	26.7	0.21
4	50	10	128.6	1.26	42.5	0.59
5	50	10	135.6	2.14	43.8	0.61
6	50	10	129.4	2.09	46.9	0.63
7	200	30	658.1	12.58	59.8	1.25
8	200	30	682.7	15.49	61.2	1.38
9	200	30	648.2	10.87	62.7	1.09
10	200	30	684.1	11.68	59.4	1.53

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75 DESIGN AND OPTIMIZATION OF THE THIRD PARTY LOGISTICS COLLABORATIVE SERVICE DYNAMIC EVALUATION SYSTEM

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Abstract: In this paper, we try to design a system include a serial of indexes to evaluate the service of third party logistics collaborative corporations in long-term. And find a method to optimize it.

Key words: Logistics service; Collaborative; Evaluate system; Rough set

1 INTRODUCTION

The third party logistics is an important form of professional logistics, there are also many theory about evaluating its service quality. The NDSERV quality table (by Spiros Gounaris) is among the top of them, which could evaluate the service quality of the corporations that in B2B environment effectively. Ackerman and some other people have also designed some systems about it, but they all focus on evaluating an independent firm. Considering the cooperative environment, the majority of research is about the supply chain. Gunnar Stefansson derives and verify a collaborative framework that specifies the role of different parties in contemporary logistics setups. There are also some researches about evaluate the collaborative ability of firms in the environment of collaborative business, but they didnfocus on logistics service. Collaborative logistics is an effective way to improve the logistics efficiency and reduce the cost, it is the future of modern logistics. But studies are less involved in the third party logistics collaborative service evaluation system temporarily.

2 DYNAMIC SYSTEM

The cooperation of logistics corporations involves not only the cooperation of internal departments of an individual enterprise but also the the cooperation among several enterprise. There are many collaborative models among enterprises. Enterprises with complementary resources or techniques can promote competitiveness through cooperation. Those with similar core competitiveness can expand scale or raise funds. To view the collaborative firms as a whole, the development of each partner may have significant influence on it and their positions are also variable. That means the collaborative state among these enterprises is always in a dynamic progress. In order to achieve good overall benefit, the quality of collaborative service is a very important index. As the collaborative state isnstatic, we need a dynamic index system which could adjust the core indexes base on the changes of the collaborative state. So we can always catch the most important elements that the collaborative enterprises should care if they want to maintain or improve the collaborative state. Based on the above considerations, we design the third party logistics collaborative service dynamic evaluation system.

3 EVALUATION SYSTEM

Principles: In selecting the indexes, we mainly take basic service quality customer perception collaborative level and sustainability of collaborative state into consideration. In Basic service quality we select the indexes that could reflect the service quality of logistics enterprises directly, it is the basic level of evaluation and reflect the utility that cooperative state brings to enterprises objectively. Customer perception evaluates it from the aspect of consumers feeling, Modern logistics is a buyer's market, Logistics service is a typical experience goods, so customers feeling could reflect the service quality of enterprises sufficiently. Then determine the performance of the enterprises and the competitiveness of collaborative enterprises. Collaborative level reflects the communication, coordination and sharing level among collaborative enterprises. High standard of collaboration will bring the enterprises good benefit as well as high level of collaborative service. Sustainability of collaborative state means the stability of the collaborative state, it has a strong influence on the quality of service. For a system that evaluate the service of collaborative enterprises in long-term, if it canevaluate the sustainability of its target, it would be meaningless. When we choose the specific indexes, we grasp the following principles emphatically. Those are,

- Purpose: our purpose is to design a system that could evaluate the logistics service effectively.
- Scientificity: there must be theories to support the indexes, and the indexes should reflect the quality of service from certain aspect.
- Systematicness: the system must be perfected, could reflect the service quality comprehensively.
- prospect: the system should be able to evaluate its target in long-term and reflect the dynamic process.

4 SYSTEM OPTIMIZATION

We use the method of rough set to optimize the indexes. Rough set is proposed by a Polish scholar z. Paw Lak in 1982. It could deal with vague and uncertain problems, with the advantage that it doesn't need a priori knowledge. And have been applied in many fields. Consider the service of collaborative logistics enterprises in a certain stage. If we processing the indexes of evaluation system by the rough set, we can eliminate redundant indexes and then sort the rest of indexes by their importance by calculating their weight. After that we can find the important indexes for the collaborative enterprises in this stage. So we can improve the quality of collaborative service pertinently and guarantee the sustainable development of the collaborative state.

5 SUMMARY AND CONCLUSION

Based on the existing researches, we design the third party logistics cooperative service dynamic evaluation system, and optimize it by the method of rough set. Then we make a numerical analysis with 5 collaborative logistics enterprises. Evaluating the quality of their service and give some advices.

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76 AN INEXACT COORDINATE DESCENT METHOD FOR THE WEIGHTED L_1-REGULARIZED CONVEX OPTIMIZATION PROBLEM

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Abstract: For solving the weighted l_1-regularized convex optimization problem with a box constraint, we propose an inexact coordinate descent (ICD) method. The proposed algorithm solves a subproblem inexactly at each iteration. We give criteria of the inexactness under which the sequence generated by the proposed method converges to an optimal solution and its convergence rate is at least R-linear without assuming the uniqueness of the solution.

Key words: l_1-regularized convex optimization; Inexact coordinate descent method; Linear convergence; Error bound.

1 INTRODUCTION

We consider the following weighted l_1-regularized convex optimization problem:

$$\begin{aligned} &\text{minimize} \;\; F(x) := g(Ax) + \langle b, x\rangle + \sum_{i=1}^{n} \tau_i |x_i| \\ &\text{subject to} \;\; l \le x \le u, \end{aligned} \tag{1.1}$$

where $g : \mathcal{R}^m \to (-\infty, \infty]$ is a strictly convex function on dom$g$, $A \in \mathcal{R}^{m\times n}$ and $b \in \mathcal{R}^n$. Moreover, τ, l and u are n-dimensional vectors such that $l_i \in [-\infty, \infty)$, $u_i \in (-\infty, \infty], \tau_i \in [0, \infty)$ and $l_i < u_i$ for each $i = 1, \cdots, n$. The nonnegative scalar constant τ_i is called weight and the term $\sum_{i=1}^{n}\tau_i|x_i|$ is called the l_1-regularization function.

This kind of optimization problems arise usually as an approximation of intractable problems in real life such as the compressed sensing (W. Yin. (2008)), the feature selection in the data classification (K. Koh. (2007)), the data mining (M. Y. Park.(2007)), geophysics (A. Gholami. (2010)) and so on. It is nondifferentiable and large scale. Moreover, the optimal solutions are possibly not unique.

To solve this kind of problem, recently, some methods have been presented, such as the block-coordinate gradient (BCG) method (P. Tseng and S. Yun. (2009)), the block coordinate descent (BCD) method (P.Tseng. (2001)), the interior-point (IP) method (K. Koh. (2007)). However, the convergence rate of the BCG method with the almost cycle rule has not been shown. In fact, the BCD method is difficult to implement when the problem is not a lasso. In this paper, we propose an inexact coordinate descent (ICD) method which is an extension upon the results of (Z.Q.Luo and P. Tseng. (1992)). Roughly, we extend in the following three respects:

- The smooth convex problem is extended to that with the l_1-regularized function.

- We extend the exact CD method to the inexact solution.
- The convergence and convergence rate is extended to the almost cycle rule.

2 PRELIMINARIES

Throughout the paper, we make the following basic assumptions for the problem (1.1).

Assumption 2.1 *For the problem (1.1),*

(a) A_j *is a nonzero vector for all* $j \in \{1, 2, \cdots, n\}$.

(b) $l_i < 0 < u_i$ *for all* $i \in \{1, 2, \cdots, n\}$.

(c) The set of the optimal solutions, denoted by X^**, is nonempty.*

(d) The effective domain of g*, denoted by* $\mathrm{dom}g$*, is nonempty.*

(e) $\mathrm{dom}g$ *is open.* g *is twice continuously differentiable on* $\mathrm{dom}g$.

(f) $\nabla^2 g(Ax^*)$ *is positive definite for every optimal solution* $x^* \in X^*$.

In Part (a), if A_j is zero, then x_j of the optimal solution can be easily determined. Thus we can remove x_j from the problem (1.1). Part (b) is just for simplification. If both l_i and u_i are positive for some $i \in \{1, 2, \cdots, n\}$, then we may replace x_i, l_i and u_i by $\bar{x}_i + \frac{l_i+u_i}{2}$, $\frac{l_i-u_i}{2}$ and $\frac{u_i-l_i}{2}$. Part (c) and (d) are standard. If g is strongly convex and twice differentiable on $\mathrm{dom}g$, then Part (e) and (f) are satisfied automatically. For example, a quadratic function, an exponential function, and even some complicate functions in the l_1-regularized logistic regression problem satisfy (e) and (f).

Note that under this assumption, X^* may be not bounded.

We define the following two mappings:

$$(T_\tau(x))_i := (|x_i| - \tau_i)_+ \mathrm{sgn}(x_i), \tag{2.1}$$

and

$$P_{\tau,l,u}(x) := [T_\tau(x - \nabla f(x))]^+_{[l,u]}, \tag{2.2}$$

where $(a)_+ := \max(0, a)$, $\mathrm{sgn}(a)$ is a sign function, $[x]^+_{[l,u]}$ denote the orthogonal projection of vector x onto the box $[l, u]$. Then optimal solution can be described as a fixed point of the mapping $P_{\tau,l,u}$.

Theorem 2.2 *For the problem (1.1),* x *belongs to the optimal solution set* X^* *if and only if* $x = P_{\tau,l,u}(x)$*, i.e.,* $X^* = \{x : x \in \mathrm{dom}g, x = P_{\tau,l,u}(x)\}$.

For convenience, we let $f(x) := g(Ax) + \langle b, x\rangle$. The following definition will be used in establishing the ICD method.

Definition 2.1 *We say that the* ε*-optimality conditions for the problem (1.1) hold at* x *if one of the following statements holds for each* i.

(i) $\nabla_i f(x) - \tau_i \geq -\varepsilon$ *and* $|x_i - l_i| \leq \varepsilon$.

(ii) $|\nabla_i f(x) - \tau_i| \leq \varepsilon$ *and* $l_i - \varepsilon \leq x_i \leq \varepsilon$.

(iii) $|\nabla_i f(x)| \leq \tau_i + \varepsilon$ *and* $|x_i| \leq \varepsilon$.

(iv) $|\nabla_i f(x) + \tau_i| \leq \varepsilon$ *and* $-\varepsilon \leq x_i \leq u_i + \varepsilon$.

(v) $\nabla_i f(x) + \tau_i \leq \varepsilon$ *and* $|x_i - u_i| \leq \varepsilon$.

Theorem 2.3 *The* ε*-optimality conditions hold at* x *if and only if* $|x_i - (P_{\tau,l,u}(x))_i| \leq \varepsilon$ *holds for each* i.

3 INEXACT COORDINATE DESCENT (ICD) METHOD

A general framework of the ICD method can be described as follows:

Inexact coordinate descent (ICD) method
Step 0: Initial setting. Choose an initial point $x^0 \in [l, u]$ and let $r := 0$.
Step 1: Check the termination condition.
Step 2: Choose an index $i \in \{1, \cdots, n\}$, get an inexact solution x_i^{r+1} of the following subproblem:

$$\text{minimize}_{x_i \in \{l_i \le x_i \le u_i\}} F(x_1^r, x_2^r, \cdots, x_{i-1}^r, x_i, x_{i+1}^r, \cdots, x_n^r). \tag{3.1}$$

Step 3: Set $x_j^{r+1} := x_j^r$ for all $j \in \{1, \cdots, n\}$ such that $j \neq i$ and let $r := r + 1$. Go to Step 1.

For the global convergence of the ICD method, it is important to define the inexactness of the subproblem (3.1) and to chose the index i in Step 2.

We call a solution of the subproblem (3.1) is an inexact solution if it satisfies the following assumptions.

Assumption 3.1 *(i)* $x_i^{r+1} \in [l_i, u_i]$.
(ii) $F(x_1^r, x_2^r, \cdots, x_{i-1}^r, x_i^{r+1}, x_{i+1}^r, \cdots, x_n^r) \le \min\limits_{x_i \in \{l_i, 0, u_i, x_i^r\}} F(x_1^r, x_2^r, \cdots, x_{i-1}^r, x_i, x_{i+1}^r, \cdots, x_n^r)$;
(iii) $|x_i^{r+1} - (P_{\tau,l,u}(x^{r+1}))_i| \le \varepsilon^{r+1}$ *where* $\varepsilon^{r+1} \le \min\{\delta_r, \alpha|x_i^{r+1} - x_i^r|, \varepsilon^r\}$, $0 < \alpha < \dfrac{\sigma \min\limits_j \|A_j\|^2}{L \max\limits_j \|A_j\|^2 + 1}$, $\sigma > 0$, *L is the Lipschitz constant of ∇g. δ_r monotonically decreases as r increases and* $\lim\limits_{r\to\infty} \delta_r = 0$.

Part (ii) enforces not only that $\{F(x^r)\}$ is decreasing but also is less than $F(x_1^r, x_2^r, \cdots, x_{i-1}^r, x_i, x_{i+1}^r, \cdots, x_n^r)$ at a point where F is nonsmooth. It plays a key role for the convergence of $\{x^r\}$. In Part (iii) implies that x_i^{r+1} is an ε^{r+1}-approximate solution.

For the choice of the index i, we adopt the following almost cycle rule.

Almost cyclic rule
There exists an integer $B \ge n$, such that every coordinate is iterated upon at least once every B successive iterations.

4 GLOBAL AND LINEAR CONVERGENCE

Theorem 4.1 *Suppose that $\{x^r\}$ is generated by the ICD method with the almost cycle rule. Let F^* denote the optimal value of the problem (1.1). Then $\{F(x^r)\}$ converges to F^* at least B-step Q-linearly. Moreover, there exists an optimal solution x^* of the problem (1.1) such that $\{x^r\}$ converges to x^* at least R-linearly.*

Note that this theorem shows that the global and linear convergence of the sequence $\{x^r\}$ holds even if A is not full column rank.

5 CONCLUSION

In this paper, we have presented a general framework of the ICD method for solving l_1-regularized convex optimization (1.1). We also have established the R-linear convergence rate of this method under the almost cycle rule. On each iteration step, we only need to find an approximate solution of the subproblem, that raises the possibility in theory to solve general l_1-regularized convex problem with the idea of the CD method.

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77 SENSITIVITY ANALYSIS OF GAP FUNCTIONS FOR VECTOR VARIATIONAL INEQUALITY VIA CODERIVATIVES

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Abstract: The aim of this article is to investigate codifferential properties of a class of set-valued maps and gap function involving vector variational inequality. Relationships between their coderivatives are discussed. Formulae for computing coderivatives of the gap function are established. Optimality conditions of solutions for vector variational inequalities are obtained. The finite-dimensional cases are also discussed.

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78 MINIMAX INEQUALITIES FOR SET-VALUED MAPPINGS

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Abstract: In this paper, by using the finite intersection property, we first obtain two types of minimax inequalities for set-valued mappings are obtained, which improve and generalize the corresponding results in the literatures. Then, by using the Ky Fan lemma and the Kakutani-Fan-Glicksberg fixed point theorem, we also investigate some Ky Fan minimax inequalities for set-valued mappings.

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79 A VISION-BASED OPTICAL CHARACTER RECOGNITION SYSTEM FOR REAL-TIME IDENTIFICATION OF TRACTORS IN A PORT CONTAINER TERMINAL

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Abstract: Automation has been seen as a promising solution to increase the productivity of modern sea port container terminals. The potential of increase in throughput, work efficiency and reduction of labor cost have lured stick holders to strive for the introduction of automation in the overall terminal operation. A specific container handling process that is readily amenable to automation is the deployment and control of gantry cranes in the container yard of a container terminal where typical operations of truck identification, loading and unloading containers, and job management are primarily performed manually in a typical terminal. To facilitate the overall automation of the gantry crane operation, we devised an approach for the real-time identification of tractors through the recognition of the corresponding number plates that are located on top of the tractor cabin. With this crucial piece of information, remote or automated yard operations can then be performed. A machine vision-based system is introduced whereby these number plates are read and identified in real-time while the tractors are operating in the terminal. In this paper, we present the design and implementation of the system and highlight the major difficulties encountered including the recognition of character information printed on the number plates due to poor image integrity. Working solutions are proposed to address these problems which are incorporated in the overall identification system.

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80 AN ALGORITHM FOR THE LARGEST EIGENVALUE OF NONHOMOGENEOUS NONNEGATIVE POLYNOMIALS

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Abstract: Eigenvalue has applications in many fields from locating oil reserve in the ground, car design (in order to damp out the noise so that the ride is quiet) to checking for cracks or deformities in solid by constructors. In this paper, we propose an iterative method for calculating the largest eigenvalue of nonhomogeneous nonnegative polynomials. Perron-Frobenius Theorem plays an important part in this method.

Key words: Eigenvalue; Nonhomogeneous; Polynomial; Iterative method.

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81 ON THE OPTIMAL DESIGN OF BEAMFORMING SYSTEM

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Abstract: In this paper, the design of broadband beamforming system is studied. The performances are to select the coefficients of the FIR filters such that the errors between the actual responses and the desired responses are minimized. When there are a large number of microphones deployed, the performance limit is studied and sought as a lower bound for all possible designs. An efficient algorithm is proposed to take advantages of the derived lower bound to design for the filter coefficients for the beamformers. In addition, using the performance limit of very long filters, the locality of the microphones is also optimized. We show that much better beamformers can be designed and we illustrate the proposed method by several designs.

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82 INDOOR BEAMFORMER DESIGN BASED ON ROOM SIMULATION WITH POST-FILTER FOR FURTHER ENHANCEMENT

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Abstract: Beamforming techniques have demonstrated their ability to greatly enhance the speech from background inter- ference, while in the reverberant environments, the signal of in- terest is both corrupted by interference noise and distorted by room acoustics. In this paper, we will study to design an indoor beamformer for noise reduction and reverbera- tion suppression based on room simulation, and the post-filter technique is introduced to further enhance the output of the beamformer, which can improve the performance of the speech in some heavier reverberant cases. The numerical experiments will be provided to illustrate our proposed method.

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83 JOB SHOP SCHEDULING WITH ARTIFICIAL IMMUNE SYSTEMS

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Abstract: The job shop scheduling is complex due to the dynamic environment. When the information of the jobs and machines are pre-defined and no unexpected events occur, the job shop is static. However, the real scheduling environment is always dynamic due to the constantly changing information and different uncertainties. This study discusses this complex job shop scheduling environment, and applies the AIS theory and switching strategy that changes the sequencing approach to the dispatching approach by taking into account the system status to solve this problem. AIS is a biological inspired computational paradigm that simulates the mechanisms of the biological immune system. Therefore, AIS presents appealing features of immune system that make AIS unique from other evolutionary intelligent algorithm, such as self-learning, long-lasting memory, cross reactive response, discrimination of self from non-self, fault tolerance, and strong adaptability to the environment. These features of AIS are successfully used in this study to solve the job shop scheduling problem. When the job shop environment is static, sequencing approach based on the clonal selection theory and immune network theory of AIS is applied. This approach achieves great performance, especially for small size problems in terms of computation time. The feature of long-lasting memory is demonstrated to be able to accelerate the convergence rate of the algorithm and reduce the computation time. When some unexpected events occasionally arrive at the job shop and disrupt the static environment, an extended deterministic dendritic cell algorithm (DCA) based on the DCA theory of AIS is proposed to arrange the rescheduling process to balance the efficiency and stability of the system. When the disturbances continuously occur, such as the continuous jobs arrival, the sequencing approach is changed to the dispatching approach that involves the priority dispatching rules (PDRs). The immune network theory of AIS is applied to propose an idiotypic network model of PDRs to arrange the application of various dispatching rules. The experiments show that the proposed network model presents strong adaptability to the dynamic job shop scheduling environment.

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84 OPTIMAL INVENTORY STRATEGIES UNDER VALUE-AT-RISK CONSTRAINT

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Abstract: In this work, we consider the raw material inventories of a manufacture as a part of investment. Then the inventory control problem can be treated as an optimal portfolio problem. A portfolio consisting of the risky raw material inventory and the risk free bank account is studied and the VaR of the portfolio is analyzed and imposed as a risk control constraint. The objective function is to maximize the utility of total portfolio value. In this model, the ordering cost is assumed to be fixed and the selling cost is proportional to the value. The optimality conditions and transaction regions are derived by using stochastic optimal control theory and the method of Lagrange multiplier. Under this formulation, the optimal inventory level is reviewed and adjusted continuously. A numerical method is proposed and the results illustrate how the material price, inventory level and VaR are interrelated.

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85 STABILITY FOR DIFFERENTIAL MIXED VARIATIONAL INEQUALITIES

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Abstract: In this paper, an existence of Caratheodory weak solution for a differential mixed variational inequality is presented under some suitable conditions. Furthermore, an upper semicontinuity of Caratheodory weak solution sets for the differential mixed variational inequality is established when both the mapping and the constraint set are perturbed by two different parameters. Finally, the continuity of Caratheodory weak solution sets of the differential mixed variational inequality is also established when both the mapping and the constraint set are perturbed by two different parameters.

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86 DERIVATIVE VARIATION APPROACH FOR SOME CLASS OF OPTIMAL CONTROL

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1 INTRODUCTION

Consider the problem:

$$\begin{cases} J(u) = \varphi(x(t_1)) + \int_{t_0}^{t_1} F(x,u,t)dt \to \min \\ \dot{x} = f(x,u,t) \\ x(t_0) = x^0 \end{cases} \tag{1.1}$$

where $t \in T = [t_0, t_1]$, $\varphi : R^n \to R$ is a differentiable function.The functional $F: R^n \times R^r \times R \to R$ is continuous in all arguments and continuously differentiable in x and u. Also, assume that

$$u(t) \in C_1^r([t_0,\, t_1]),\ u(t) \neq Const,\ \|\dot{u}(t)\| \leq M < +\infty,\ t \in T$$

Introduce the set as:

$$S = \{\delta \in C_1([t_0,\, t_1]) \,|\, \delta(t_0) = \delta(t_1) = 0\}$$

Consider derivative variations of $u(t) \in C_1^r([t_0,\, t_1])$ as

$$\tilde{u} = u(t) + \varepsilon\delta(t)\dot{u}(t),$$

for all $\delta \in S$, $\varepsilon \in R$ and $t \in [t_0,\, t_1]$.

After introducing a notion of weak optimal process in problem (1.1), we prove the following:

Theorem 1.1 *Assume that an admissible process (x^*, u^*) is weak optimal to problem (1.1). Then the following conditions*

$$\langle \frac{\partial H(\psi^*, x^*, u^*, t)}{\partial u}, \dot{u}^*(t) \rangle = 0,$$

are satisfied for all $t \in [t_0, \ , t_1]$, where $H(\psi, x, u, t) =< \psi(t), f(x,u,t) > -F(x,u,t)$ and the function $\psi^* = \psi^*(t)$ is the solution of the conjugate system

$$\dot{\psi} = -\frac{\partial H(\psi, x^*, u^*, t)}{\partial x},\ \psi(t_1) = -\frac{\partial \varphi(x^*(t_1))}{\partial x}.$$

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87 A COMPUTATIONAL METHOD FOR THE QUASICONVEX MAXIMIZATION PROBLEM

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Abstract: We consider the problem of maximizing a quasiconvex function over a convex set. Based on the global optimality conditions [1], we develop an algorithm for solving the problem which generates a relaxation sequence. Some numerical results are presented

Key words: Global optimality conditions; Quasiconvex function; Algorithm.

1 INTRODUCTION

Problem Formulation and Optimality Conditions

Consider a problem:

$$\max_{x \in \mathbb{D}} f(x), \tag{1}$$

$$\mathbb{D} = \{x \in \mathbb{R}^n \mid Ax \leq b\}, \tag{2}$$

where $f : \mathbb{D} \longrightarrow \mathbb{R}$ is a differetiable quasiconvex function, A is a matrix of (m x n) , $b \in \mathbb{R}^n$. Problem (1)-(2) is nonconvex and belonds to a class of global optimization problems.
Introduce the level set of the function f as follows

$$L_c(f) = \{y \in \mathbb{R}^n \mid f(y) = c\}, c \in \mathbb{R}.$$

Global optimality conditions for problem (1)- (2) were given in [1] by the following proposition.

Theorem 1 ([1]) If $z \in \mathbb{D}$ is a global solution to problem (1)-(2) then

$$\langle f'(y), x - y \rangle \leq 0, \forall y \in E_{f(z)}(f). \tag{3}$$

If in addition,

$$f'(y) \neq 0, \forall y \in E_{f(z)}(f)$$

then condition (3) is sufficient for z to be global.

2 APPROXIMATION SET

We introduce an approximation set of the level set of the function f at a point $z \in \mathbb{D}$ for a given number $m \in \mathbb{N}$.

$$A_z^m = \{y^1, y^2, ..., y^m \mid f(y^i) = f(z), i = 1, 2, ..., m\}.$$

Define θ_m as follows
$$\theta_m = \max_{1\le i\le m} \langle f'(y^i), u^i - y^i\rangle,$$

where $u^i = argmax_{x\in\mathbb{D}}\langle f'(y^i), x\rangle.$

Lemma 1 If $\theta_m > 0$ then there exit a $j \in \{1, 2, ..., m\}$ such that $f'(u^j) > f(z)$.

3 ALGORITHM

Based on global optimality conditions and lemma 1, we develop the following algorithm which provides an approximate global solution .

Algorithm QMAX

step 1. $x^k \in \mathbb{D}, k = 0$ is an initial feasible point.
step 2. Find a local maximizer z^k by the conditional gradient method starting with x^k.
step 3. Construct an approximation set $A^k_{z^k}$ at the point z^k :

$$A^m_{z^k} = \{y^1, y^2, ..., y^m \mid f'(y^i) = f(z^k), i = 1, 2, .., m\}.$$

step 4. Solve linear programming problems

$$u^i = argmax_{x\in\mathbb{D}}\langle f'(y^i), x\rangle, i = 1, 2, ..., m$$

step 5. Compute θ_m as
$$\theta_m = \max_{1\le i\le m} \langle f'(y^i), u^i\rangle = \langle f'(y^j), u^j\rangle$$

step 6. If $\theta_m > 0$ then set $k := k + 1$ and $x^k = u^j$. Goto step 2
step 7. Terminate. z^k is an approximate global solution

4 NUMERICAL EXPERIMENT

Algorithm QMAX was examined on some test problems of the following type

$$\max_{x\in\mathbb{D}} \left\{ f(x) = \frac{\langle Ax, x\rangle + \langle B, x\rangle}{\langle Cx, x\rangle + \langle D, x\rangle + M} \right\},$$

where A_{nxn} and B_{nxn} are positive and and negative defined martrices, respectively.

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88 BARRIER-PROJECTION QP SOLVER WITH MATRIX LIBRARY FOR AN ARRAY PROCESSOR

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Abstract: Machine learning with SVM involves solving a large dense Hassian matrix in the QP problem. The barrier-projection with quadratic transformation and relaxation is an effective solver. The array processor that is optimized for matrix computations can provide the computationally intensive machine learning results in real time. Currently a gap exists between the barrier-projection QP algorithm and the low-level computing machine code. This paper presents a matrix library methodology that expresses general matrix operations into array processor operations for implementing the BP-QP algorithm.

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89 COORDINATING SUPPLY CHAIN WITH A NEW CREDIT CONTROL

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Abstract: The paper studies the supply chain coordination with trade credit under symmetric and asymmetric information. Both the centralized and decentralized scenarios are studied, and the analytical results are provided. In the decentralized setting, we propose a new credit contract to coordinate the supply chain under symmetric and asymmetric information. We show that the win-win outcome is achieved by redistributing the profit in an appropriate way. Numerical examples are given to illustrate our results.

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90 GAP FUNCTIONS AND ERROR BOUNDS FOR WEAK VECTOR VARIATIONAL INEQUALITIES

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Abstract: In this paper, by using the image space analysis, a gap function for weak vector variational inequalities is obtained. Its lower semicontinuity is also discussed. Then, these results are applied to obtain the error bounds for weak vector variational inequalities. These bounds provide effective estimated distances between a feasible point and the solution set of the weak vector variational inequalities.

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91 ROBUST SELF-SCHEDULING OPTIMIZATION UNDER MOMENT UNCERTAINTY OF ELECTRICITY PRICES

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Abstract: This paper studies a worst-case profit self-schedules of price-taker generators in pool-based electricity markets. A distributionally robust self-scheduling optimization model describes uncertainty of prices in both distribution form and moments (mean and covariance matrix), where the knowledge of the prices is solely derived from historical data. It is proved that the proposed robust self-scheduling model can be solved to any precision in polynomial time. These arguments are confirmed in a practical example on the IEEE 30 bus test system. Numerical results validate the efficiency of our proposed model and method.

Key words: Robust optimization; Generation self-scheduling; Moment uncertainty; Data-driven.

1 INTRODUCTION

Generation self-scheduling in a pool-based electricity market has been recently studied in the power systems literature (Yamin H.Y. (2004); Jabr R.A. (2010); Jabr R.A. (2005)). The self-schedules are required in developing successful bidding strategies and constructing hourly bidding curves for consideration by the independent system operator . In order to obtain successful generation bids, the generation companies have to self-schedule their unit by maximizing the expected profit based on the forecasted location marginal prices and accounting for the network security constraints.

The issue of interest for this work is that, since the electricity prices are of stochastic nature, the generation company cannot be certain about the revenue. Measuring the underlying risk due to this uncertainty is crucial not only for assessing profitability but also for generation self-scheduling. Stochastic programming can effectively describe self-scheduling problems in uncertain environments. Unfortunately, although the self-scheduling problem is a convex optimization problem, to solve it one must often resort to Monte Carlo approximations, which can be computationally challenging. A more challenging difficulty that arises in practice is the need to commit to a distribution given only limited information about the stochastic parameters (Delage E. (2010)).

In an effort to address these issue, robust formulations for self-scheduling problems were proposed, see (Jabr R.A. (2010); Jabr R.A. (2005)). Jabr R.A. (2010) considers a generation self-scheduling model based on a worst-case conditional robust profit with partial information on the probability distribution of prices. It is assumed that the nominal distribution and a set of possible distribution were given. Uncertainty of prices is represented by box and ellipsoidal uncertainty sets. However, in practice, true probability distribution of prices can not be known exactly. Their solution can be misleading when there is ambiguity in the choice of a distribution for the random prices.

Recently, Delage and Ye (Delage E. (2010)) proposed a distributionally robust optimization model that describes uncertainty in both the distribution form and moments (mean and covariance matrix). By deriving a new form of confidence region for the mean and the covariance matrix of a random vector, it was showed how the proposed distribution set can be well justified when addressing data-driven problems (i.e., problems where the knowledge of the random parameters is solely derived from historical data).

Motivated by the work of Delage and Ye (Delage E. (2010)), in this paper, we propose a distributionally robust self-scheduling optimization model under moment uncertainty of the prices, where the knowledge of the prices is solely derived from historical data. Then we prove that the proposed robust self-scheduling model can be solved to any precision in polynomial time. These arguments are confirmed in a practical example on the IEEE 30 bus test system.

2 ROBUST SELF-SCHEDULING PROBLEM WITH MOMENT UNCERTAINTY

It is often the case in practice that one has limited information about the locational marginal prices ξ driving the uncertain parameters that are involved in the decision making process. In such situations, it might instead be safer to rely on estimates of the mean μ_0 and covariance matrix Σ_0 of the random vector-e.g., using empirical estimates. However, we believe that in such problems, it is also rarely the case that one is entirely confident in these estimates. For this reason, following the work in Delage E. (2010), we propose representing this uncertainty using two constraints parameterized by $\gamma_1 \geq 0$ and $\gamma_2 \geq 1$:

$$(\mathbb{E}[\xi] - \mu_0)^T \Sigma_0^{-1} (\mathbb{E}[\xi] - \mu_0) \leq \gamma_1, \tag{2.1}$$

$$\mathbb{E}[(\xi - \mu_0)(\xi - \mu_0)^T] \preceq \gamma_2 \Sigma_0, \tag{2.2}$$

where constraint (2.1) assumes that the mean of price ξ lies in an ellipsoid of size γ_1 centered at the estimate μ_0, constraint (2.2) forces the centered second-moment matrix of ξ to lie in a positive semidefinite cone defined with a matrix inequality. In other words, it describes how likely ξ is to be close to μ_0 in terms of the correlations expressed in Σ_0. Finally, the parameters γ_1 and γ_2 provide natural means of quantifying ones's confidence in μ_0 and Σ_0, respectively.

Denote the distributional set as

$$\mathscr{D}_1(\mathscr{F}, \mu_0, \Sigma_0, \gamma_1, \gamma_2) = \left\{ F \in \mathscr{U} \;\middle|\; \begin{array}{l} \mathbb{P}(\xi \in \mathscr{F}) = 1, \\ (\mathbb{E}[\xi] - \mu_0)^T \Sigma_0^{-1} (\mathbb{E}[\xi] - \mu_0) \leq \gamma_1, \\ \mathbb{E}[(\xi - \mu_0)(\xi - \mu_0)^T] \preceq \gamma_2 \Sigma_0 \end{array} \right\}, \tag{2.3}$$

where $\mathscr{U}$ is the set of all probability measures on the measurable space$(\mathbb{R}^m, \mathscr{B})$, with $\mathscr{B}$ the Borel σ-algebra on $\mathbb{R}^m$, and $\mathscr{F} \in \mathbb{R}^m$ is any closed convex set known to contain the support of F. The set $\mathscr{D}_1(\mathscr{F}, \mu_0, \Sigma_0, \gamma_1, \gamma_2)$ which will also be referred to in shorthand notation as $\mathscr{D}_1$, can be seen as a generalization of many previously proposed sets.

In what follows, we will study a worst-case expected results over the choice of a distribution in the distributional set $\mathscr{D}_1$. This leads to solving the distributionally robust self-scheduling optimization with moment uncertainty of prices (DRSSO):

$$\max_{x \in X} \quad \min_{F \in \mathscr{D}_1} \quad \mathbb{E}_F[\xi^T x - \sum_{i=1}^{m} C_i(x_i)]. \tag{2.4}$$

which is equivalent to

$$-\min_{x \in X} \quad \max_{F \in \mathscr{D}_1} \quad \mathbb{E}_F[-\xi^T x + \sum_{i=1}^{m} C_i(x_i)]. \tag{2.5}$$

First, we consider the question of solving the inner maximization problem of a DRSSO that uses the set $\mathscr{D}_1$.

Definition 2.1 *Given any fixed $x \in X$, let $\Phi(x; \gamma_1, \gamma_2)$ be the optimal value of the moment problem:*

$$\max_{F \in \mathscr{D}_1} \quad \mathbb{E}_F[-\xi^T x + \sum_{i=1}^{m} C_i(x_i)] \tag{2.6}$$

where $\mathbb{E}_F[\cdot]$ is the expectation taken with respect to the random vector ξ given that it follows the probability distribution $F \in \mathscr{D}_1$.

Applying duality theory and robust optimization method (Bertsimas D. (2011); Ben-tal A. (2001)), we can circumvent the difficulty of finding the optimal value of the problem (2.6).

Lemma 2.1 *For a fixed $x \in \mathbb{R}^n$, suppose that $\gamma_1 \geq 0, \gamma_2 \geq 1, \Sigma_0 \succ 0$. Then $\Phi(x;\gamma_1,\gamma_2)$ must be equal to the optimal value of the problem (2.7):*

$$\begin{aligned}
\min_{Q.q.r.t} \quad & r+t \\
s.t. \quad & t \geq (\gamma_2\Sigma_0 + \mu_0\mu_0^T)\bullet Q + \mu_0^T q + \sqrt{\gamma_1} \parallel \Sigma_0^{1/2}(q+2Q\mu_0) \parallel \\
& r \geq -\xi^T x + \sum_{i=1}^{m} C_i(x_i) - \xi^T Q \xi - \xi^T q, \quad \forall \xi \in \mathscr{F} \\
& Q \succeq 0
\end{aligned} \tag{2.7}$$

where $A \bullet B$ refers to the Frobenius inner product between matrices, $Q \in R^{m\times m}$ is a symmetric matrix, the vector $q \in R^m$ and $r, t \in R$. In addition, if $\Phi(x;\gamma_1,\gamma_2)$ is finite, then the set of optimal solutions to problem (2.7) must be nonempty.

Since $f(x,\xi) = -\xi^T x + \sum_{i=1}^{n} C_i(x_i)$ is convex in x and concave in ξ, X is convex and compact, satisfies the Assumption 1 and Assumption 2 in Delage E. (2010), a straightforward application of (Delage E. (2010), Proposition2) shows that the DRSSO model presented in problem (2.4) can be solved in polynomial time. To show that the DRSSO problem (2.4) is a tractable problem, one needs to take a closer look at the dual formulation presented in lemma 3.1.

Theorem 2.1 *The DRSSO problem (2.4) is equivalent to the quadratic cone program:*

$$\begin{aligned}
-\min_{x.Q.q.r.t.\eta} \quad & r+t \\
s.t. \quad & t \geq (\gamma_2\Sigma_0 + \mu_0\mu_0^T)\bullet Q + \mu_0^T q + \sqrt{\gamma_1} \parallel \Sigma_0^{1/2}(q+2Q\mu_0) \parallel \\
& \begin{bmatrix} Q & q/2 + x/2 \\ q^T/2 + x^T/2 & r-\eta \end{bmatrix} \succeq 0 \\
& Q \succeq 0 \\
& \eta \geq \sum_{i=1}^{m} b_i x_i + y \qquad for \quad i = 1,\ldots,m \\
& y \geq \sum_{i=1}^{m} \omega_i^2 \\
& \omega_i = \sqrt{c_i} x_i \\
& x \in X
\end{aligned} \tag{2.8}$$

In addition, the DRSSO problem (2.4) can be solved to any precision ϵ in time polynomial in $log(1/\epsilon)$ and the size of x and ξ.

We can use the optimization software of SeDuMi 1.21 (Sturm J.F. (1999)) to solve this quadratic cone program.

3 NUMERICAL RESULT

We present our simulation results on the IEEE 30 bus test system, and get the results by using the SeDuMi conic optimization software running on an $Intel^{\circledR} Core(TM)$ i3-2350M (2.30GHz) PC with 2 GB RAM. Alsac O. (1974) gives the network and load data for this system. The generator data and a historical data set of 100 prices vector ξ is shown in (Jabr R.A. (2010)). There are 6 power generations with coal as their fuel in this system. And we assume that the generating units are belong to the same generation company.

In implementing our method, the distributional set is formulated as $\mathscr{D}_1(\mathbb{R}^6, \mu_0, \Sigma_0, \gamma_1, \gamma_2)$, where μ_0 and Σ_0 are the empirical estimates of the mean $\mu_0 = m^{-1}\sum_{i=1}^{m}\xi_i$ and covariance matrix $\Sigma_0 = m^{-1}\sum_{i=1}^{m}(\xi_i - \mu_0)(\xi_i - \mu_0)^T$ of ξ.

We fix the γ_1 to 0.5, and let γ_2 range from 1 to 11. The simulation results show that the objective profit function is almost invariable when γ_2 is taken from 1 to 11. However, if we fix the $\gamma_2 = 1.2$, and let γ_1 vary from 0 to 2, it can be shown that the object decreases when γ_1 increases, and the object is almost invariable after $\gamma_1 > 1.2$.

The following Table 1 illustrates the generation self-scheduling result for γ_1 from 0 to 1 and $\gamma_2 = 1.2$.

γ_1	Bus no.1	Bus no.2	Bus no.5	Bus no.8	Bus no.11	Bus no.13	profit
0	135.00	35.37	16.45	35.00	30.00	38.67	229.52
0.1	112.27	34.07	16.27	35.00	30.00	37.34	199.89
0.2	103.36	33.44	16.20	35.00	30.00	36.77	188.78
0.3	96.78	32.92	16.13	35.00	30.00	36.33	180.68
0.4	91.42	32.45	16.08	35.00	30.00	35.94	174.14
0.5	86.86	32.02	16.02	35.00	30.00	35.60	168.58
0.6	82.87	31.62	15.98	35.00	30.00	35.29	163.71
0.7	79.32	31.24	15.94	35.00	30.00	35.00	159.35
0.8	76.13	30.88	15.90	35.00	30.00	34.73	155.41
0.9	73.23	30.54	15.86	35.00	30.00	34.48	151.79
1	70.55	30.20	15.82	35.00	30.00	34.23	148.44

Table 1: Generation self-scheduling result for γ_1 from 0 to 2 ($\gamma_2 = 1.2$)

4 CONCLUSION

This paper studies a worst-case profit self-schedules of price-taker generators in pool-based electricity markets. A distributionally robust self-scheduling optimization model describes uncertainty of prices in both distribution form and moments (mean and covariance matrix), where the knowledge of the prices is solely derived from historical data. It is proved that the proposed robust self-scheduling model can be solved to any precision in polynomial time. These arguments are confirmed in a practical example on the IEEE 30 bus test system, where our framework leads to good-performing policies on the "true" distribution underlying locational marginal prices. Numerical results show that parameter γ_1 of mean is sensitive to the solution and parameter γ_2 of covariance is not sensitive to the solution.

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92 TOLL RATES FOR HIGHWAY CONSTRUCTION: THE CONCESSIONARY PROJECTS

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Abstract: This study assesses the profitability of a private entity going into partnership with a public entity for the construction and management of highways. The study derives the conditions under which such an arrangement can be viable incorporating the damage to the highway caused by a vehicular class. A mathematical model that can be used by any businessman or an organization (or a concessionaire) to determine the risk exposure to optimal profit, the optimal number of different vehicle sizes, and the optimal toll rates for a given concessionary period are developed. Using data from the Ghana Highway Authority, we demonstrate that public private partnership financing can work for a number of highways through the right mix of two variables, the concession period and the road toll rate. Our findings can assist investors interested in partnering with government in highway financing on the type of highway to choose and its accompanying cost. Furthermore, this study will provide the government with better insight when partnering with the private sector in highway financing.

Key words: Asset replacement cost; Concessionary period; Damage weight; Mathematical model; Optimization; Reconstruction; Toll rate.

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93 SOLVING NONLINEAR SECOND-ORDER CONE PROGRAMS VIA EXACT PENALTY FUNCTIONS

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Abstract: We propose a continuously differentiable exact penalty function for non- linear second-order cone programs (SOCPs). To construct the penalty func- tion, we incorporate a Lagrange multipliers estimate in the augmented La- grangian for SOCPs. Under the nondegeneracy assumption and the strong second-order sufficient condition, we show that a generalized Newton method has global and superlinear (or quadratic) convergence. We conclude with some numerical experiments.

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94 CANONICAL DUAL THEORY FOR SOLVING NONCONVEX OPTIMIZATION PROBLEMS WITH LOG-SUM-EXP FUNCTION

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Abstract: n this talk, the speaker will present a detailed study on a general nonconvex global optimization problemwith a log-sum-exp function, which arises in many research regions, such as minimax problems, combinatorial network optimisation and geometric programming. Based on the canonical duality theory, the perfect dual problem without duality gap is constructed. The triality theory shows that calculating the global solution for the primal problem is equivalent to solving a convex optimisation problem in the dual space. Some examples are given to illustrate the efficiency of our approach.

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95 ON THE SOLUTION OF EIGENVALUE COMPLEMENTARITY PROBLEMS

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Abstract: In this talk, we discuss the solution of linear and quadratic eigenvalue complementarity problems (LEiCP and QEiCP). It is shown that the LEiCP always has a solution while the existence of a solution for the QEiCP is only guaranteed under some sufficient conditions. The symmetric and asymmetric cases are considered, which differ on the matrices of these problems to be all symmetric or not. For the sym- metric case, it is shown that both problems can be solved by finding a stationary point of an appropriate merit function. A projected-gradient algorithm is introduced for processing the symmetric LEiCP and QEiCP by exploiting these formulations. Interior-point, path-following methods and semi-smooth algorithms can be recommended for the asymmetric LEiCP and QEiCP. Both the problems can also be reduced to Finite-Dimensional Variational Inequal- ity Problems and solved by using these formulations. However, these approaches are not always able to find a solution for the LEiCP and QEiCP. An enumerative algorithm is introduced for solving the LEiCP and QEiCP by finding a global minimum of special nonlinear programs (NLP). It is shown that the algorithm always converges to a solution of the corresponding eigenvalue problem when it exists. The algorithm requires procedures for the computation of an interval containing all the eigen- values and a local solver for finding stationary points of the NLP. These techniques are introduced together with necessary and sufficient condi- tions for stationary points of each one of these NLPs to be solutions of the corresponding EiCPs. Some computational experience is reported to highlight the efficiency and efficacy of the projected-gradient and enumerative algorithms for solv- ing the symmetric and asymmetric LEiCP and QEiCP respectively.

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96 CONTROL AUGMENTATION DESIGN OF UAV BASED ON DEVIATION MODIFICATION OF AERODYNAMIC FOCUS

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1 ABSTRACT

Due to the difference between the scaled model and real aircraft, and the difference between wind tunnel blowing and real wind, the blowing data is different from the real data. The design data can be modified for the aircraft with traditional configuration by engineering experience, since there is rich experience in engineering. However, for these novel flight vehicles [Y. J. Shi (2011)], there is insufficiency of engineering experience to follow. An efficient method is developed for such problem to modify the error between the blowing data and real data by virtue of identification the real flight data and compensating the blowing data. In recent years there has been increasing research interest in the identification problem of aircrafts, for example, see Y. L. Nong (2011); J. Suk (2003); Z. K. Shi (2012); S. A. Salman (2006); M. K. Samal (2008); H. J. Lin (2011). However, much of the published works focus on the identification of the system models. Few work has carried out for identification of aircraft focus according to flight qualities, although the the focus is an important variable for the performance analysis of the aircraft since the aerodynamic moment of focus does not change with the variation of the angle of attack. In this paper, we will study how to identify the focus of the aircraft according to the real flight date, by which the real flight pneumonia can be reproduced and design basis of control law can be given for the further improvement of flight qualities.

Considering the longitudinal flight, and when the stability and maneuverability is measured, the following formulation is always used:

$$C_m = C_{m0} + C_m^{C_L} C_L + C_m^{\delta_e}\delta_e \tag{1.1}$$

where, C_m is the pitching moment coefficient; C_{m0} is the Zero lift pitching force moment coefficient; C_L is lift coefficient; δ_e is the deflection angle of the elevator; $C_m^{C_L}$ the partial derivative of the pitching moment with respect with lift; $C_m^{\delta_e}$ is the partial derivative of the pitching moment with respect with elevator deflection angle. By analysis, we know that when we reproduce the real flight qualities, identification can be carried out for parameter $C_m^{C_L}$, by which the original blowing data is compensated. According to theoretical mechanics, the aerodynamic moment M around aircraft centroid is:

$$M = F_z(x_{c,g} - x_{pc}).$$

If aerodynamic coefficient method is applied, we have

$$C_m = C_z \left(\frac{x_{c,g} - x_{pc}}{c} \right) = C_z \left(\overline{x}_{c,g} - \overline{x}_{pc} \right)$$

where C_m is pitching moment coefficient; C_z is normal moment coefficient; c is average aerodynamic chord. Usually, both normal force coefficient C_z and location of pressure center x_{pc} will change with the variation of angle of attack α, which is inconvenient for study of the variation character of longitudinal moment. Thus, the concept of focus is introduced. Focus is represented by $\circ$, the characteristic of which is: when the angle of attack α changes during a certain range, aerodynamic moment of focus keeps as a constant. Thus, focus can be viewed as the action point of lift increment produced by the change of the angle of attack.

By analysis of a flight curve, we know that the effect of aerodynamic focus brought by undercarriage cabin and undercarriage door is insufficient of consideration, resulting the large difference between the blowing focus data and real focus data. In the following, we will show how to identify the static stability degree deviation factor K. In this paper, the attitude angle is considered as an investigated vector. Due to the uncertainty of the model, we first approximate expression $\theta = f(k)$ by using cubic spline interpolation. During the identification course, norm of longitudinal altitude angle deflection is taken as the optimization index, i.e.,

$$J = \|\theta - \theta^*\| = \left(\sum_{i=1}^{n} (\theta_i - \theta_i^*)^2 \right)^{\frac{1}{2}}$$

where, θ is the function value of the altitude angle about deviation factor K, and θ_i is corresponding to the i component; θ^* is the investigated vector taken from the real flight data recorded by FTI, θ_i^* is corresponding to the i component. For clarification, the length of investigated vector θ_i^* cannot be too long, otherwise the optimization course will be long. Then the SSDD factor K is obtained as 5.5SA. After identification, pitching angle rate feedback is applied to achieve augmentation, since the aircraft flies with low speed. The transfer function between pitching angle rate and elevator deflection angle is [Y. J. Shi (2011)]

$$\frac{q(s)}{\delta_e(s)} = \frac{K_\alpha (T_\theta S + 1)}{T_{nsp}^2 S^2 + 2\zeta_{sp} T_{nsp} S + 1}$$

where K_α is static gain; T_θ is time constant; T_{nsp} is pitching short period motion cycle of aircraft, satisfying $T_{nsp} = 1/\omega_{nsp}$, ω_{nsp} is undamped natural frequency of pitching short period motion of the aircraft; ζ_{sp} is the damping ratio of pitching short period motion of the aircraft. Introduce pitching angle rate feedback and suppose that the feedback gain is K_q, then we have

$$\frac{q(s)}{\delta_e(s)} = \frac{K_\alpha (T_\theta S + 1)}{T_{nsp}^2 S^2 + (2\zeta_{sp} T_{nsp} + K_q K_\alpha T_\theta) S + (1 + K_q K_\alpha)}.$$

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97 ON ZERMELO'S NAVIGATION PROBLEMS WITH MOVING OBSTACLES

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Abstract: A Zermelo's navigation problem with moving obstacles is studied is this paper. We formulate this problem as an optimal control problem subject to equality terminal state constraints and continuous inequality constraints. By applying the control parametrization technique together with a time scaling transformation, the formulated problem is approximated by a sequence of optimal parameter selection problems with equality terminal state constraints and continuous inequality constraints. Then, a new exact penalty function method is used to append the continuous inequality constraints to the cost function, yielding a sequence of unconstrained optimal parameter selection problems. These problems can then be solved as nonlinear optimization problems by using the gradient based methods, such as Sequential Quadratic Programming (SQP) method. For this, the gradient formulas for the objective function and the terminal state equality constraints are derived.

Key words: Zermelo; time optimal control; control parametrization; time scaling transform; exact penalty function

1 STATEMENT OF THE PROBLEM

Given three agents in a 2D flow field, two of the slower agents follow navigated trajectories and third one is faster and can be autonomously controllable. We denote the trajectories of slower agents as $\vec{z}_i(t) = [x_i(t), y_i(t)]^{\mathrm{T}}$, $i = 1, 2$, and $t \geq 0$. We assume that the velocity components at any point (x, y) in the 2D flow field can be denoted by $G(x, y, t)$ and $H(x, y, t)$, respectively. The flow dynamics $[G(x, y, t), H(x, y, t)]^{\mathrm{T}}$ can be modeled by the famous Navier-Stokes equation.

The motion of the controlled agent can be modeled by

$$\begin{cases} \dfrac{dx(t)}{dt} = V\cos\left[u(t)\right] + G\left[x(t), y(t), t\right], \\ \dfrac{dy(t)}{dt} = V\sin\left[u(t)\right] + H\left[x(t), y(t), t\right], \end{cases} \tag{1.1}$$

where V is assumed to be a unit propulsion velocity ($V = 1$) of the controlled agent and $u(t)$ the steering angle which is considered the control variable, subject to a magnitude constraint

$$|u(t)| \leq U, \quad \forall t \geq 0. \tag{1.2}$$

By defining $\vec{z}(t) = [x(t), y(t)]^{\mathrm{T}}$ and introducing the following vector

$$\vec{f} := \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \cos[u(t)] + G[x(t), y(t), u] \\ \sin[u(t)] + H[x(t), y(t), t] \end{pmatrix}, \tag{1.3}$$

we can use the following abstract dynamical system to represent the fast agent motion

$$\begin{cases} \dfrac{d\vec{z}(t)}{dt} = \vec{f}(\vec{z}(t), u(t), t), \quad t \geq 0, \\ \vec{z}(0) = \vec{z}_0, \end{cases} \tag{1.4}$$

where $\vec{z}_0$ is the initial position of the fast agent.

We now use Fig. 1.1 to state the so-called Zermelo's navigation problem of the fast agent. A_1 and A_2 represent the slower agents and A_3 represents the fast agent in Fig. 1.1. The harbor is the target area that all agents are supposed to arrive at within finite time horizon. The Zermelo's problem of the fast agent A_3 is to find an optimal trajectory (e.g., shortest route, fastest route, least fuel consumption, or etc.) to arrive at the harbor region without colliding with the other slower agents.

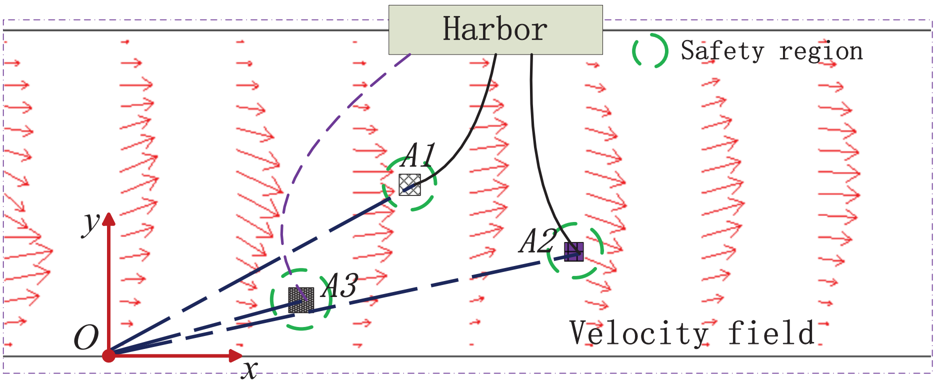

Figure 1.1 Schematic of planar agent navigation problem.

In this work, we only consider the minimum time problem, which can be stated mathematically as

$$\begin{cases} \min\limits_{|u|\leq U} T \\ \dfrac{d\vec{z}(t)}{dt} = \vec{f}(\vec{z}(t), u(t), t), \quad 0 \leq t \leq T, \, \vec{z}(0) = \vec{z}_0, \\ \sqrt{[x(t) - x_i(t)]^2 + [y(t) - y_i(t)]^2} \geq \max\{R, R_i\}, \quad 0 \leq t \leq T, \, i = 1, 2, \\ \vec{z}(T) \in \mathcal{N} = \{x_- \leq x \leq x_+, y = 2h\}, \end{cases} \tag{1.5}$$

where T represents the time instant that the fast agent arrive at the harbor region. Noting that the terminal time T depends implicitly on the control function which is defined as the first time when the agent enters the target set $\mathcal{N}$. R_i is the safety radius of each slow agent and R is the safety radius of the fast agent. The safety region of each agent is marked by a green dash circle in Fig. 1.1.

2 COMPUTATIONAL METHOD

2.1 Parameterization and time scaling

The time horizon $[0, T]$ is partitioned by a monotonically increasing sequence $\{\tau_0, \tau_1, \ldots, \tau_p\}$, where $\tau_0 = 0$ and $\tau_p = T$. The control function is then approximated by a piecewise constant function

$$u_p(t) = \sum_{k=1}^{p} \sigma_k \chi_{[\tau_{k-1}, \tau_k)}(t), \tag{2.1}$$

where $\tau_{k-1} \leq \tau_k$, $k = 1, 2, \ldots, p$ and the characteristic function $\chi_I(t)$ is defined as

$$\chi_I(t) = \begin{cases} 1, & \text{if } t \in I, \\ 0, & \text{otherwise}. \end{cases} \tag{2.2}$$

The switching times $\tau_i,\ 1 \le i \le p-1$, are also regarded as decision variables. We shall employ the time scaling transform introduced in Teo K.L. (1997) to map these switching times into fixed time points $\frac{k}{p}$, $k = 1, \dots, p-1$, on a new time horizon $[0,1]$. This is easily achieved by the following differential equation

$$\frac{\mathrm{d}t(s)}{\mathrm{d}s} = \upsilon^p(s),\ \ s \in [0,1], \tag{2.3a}$$

with initial condition

$$t(0) = 0, \tag{2.3b}$$

where

$$\upsilon^p(s) = \sum_{k=1}^{p} \theta_k \chi_{\left[\frac{k-1}{p},\frac{k}{p}\right)}(s). \tag{2.4}$$

Here, $\theta_k \ge 0,\ k = 1, \dots, p$.

Let $\theta = [\theta_1, \dots, \theta_p]^\top$ and let Θ be the set containing all such θ.

Taking integration of (2.3a) with initial condition (2.3b), it is easy to see that, for $s \in \left[\frac{k-1}{p}, \frac{k}{p}\right),\ k = 1, \dots, p$,

$$t(s) = \sum_{i=1}^{k-1} \frac{\theta_i}{p} + \frac{\theta_k}{p}(ps - k + 1), \tag{2.5}$$

where $k = 1, \dots, p$. Clearly, for $k = 1, \dots, p-1$,

$$\tau_k = \sum_{i=1}^{k} \frac{\theta_i}{p}, \tag{2.6}$$

and

$$t(1) = \sum_{i=1}^{p} \frac{\theta_i}{p} = T. \tag{2.7}$$

Now we can rewrite the dynamical system using the new time scale via the chain's law

$$\frac{d\vec{z}(t)}{dt}\frac{dt(s)}{ds} = \frac{dt(s)}{ds}\vec{f}(\vec{z}(t(s)), u(t(s)), t(s)) = v(s)\vec{f}(\vec{z}(t(s)), u(t(s)), t(s)). \tag{2.8}$$

Similarly, we apply the time scaling transform to the continuous state inequality constraints

$$\|\vec{z}(t(s))) - \vec{z}_i(t(s))\|^2 \ge \max\{R^2, R_i^2\}, \quad i = 1, 2, \tag{2.9}$$

$$\vec{z}(t(1)) \in \mathcal{N} = \{x_- \le x(t(1)) \le x_+, y(t(1)) = 2h\}, \tag{2.10}$$

We let $\vec{\mathcal{X}}(s) = \vec{z}(t(s))$, then we obtain the optimization control problem as the following form:

$$\min_{\{\sigma_k,\theta_k\}_{k=1}^{p}} \sum_{k=1}^{p} \frac{\theta_k}{p} \tag{2.11}$$

subject to:

$$\begin{cases} \dfrac{d\vec{\mathcal{X}}(s)}{ds} = \theta_k \vec{f}(\vec{\mathcal{X}}(s), \sigma_k, t(s)), \quad s \in \left[\dfrac{k-1}{p}, \dfrac{k}{p}\right), k = 1, 2, \dots, p, \\ \dfrac{dt(s)}{ds} = \theta_k, \quad s \in \left[\dfrac{k-1}{p}, \dfrac{k}{p}\right), k = 1, 2, \dots, p, \\ \left\|\vec{\mathcal{X}}(s) - \vec{\mathcal{X}}_i(s)\right\|^2 \ge \max\{R^2, R_i^2\}, \quad i = 1, 2; \quad s \in \left[\dfrac{k-1}{p}, \dfrac{k}{p}\right), k = 1, 2, \dots, p, \\ \vec{\mathcal{X}}(1) \in \mathcal{N} = \left\{x_- \le \vec{\mathcal{X}}_1(1) \le x_+, \vec{\mathcal{X}}_2(1) = 2h\right\}, \\ t(1) = T \end{cases} \tag{2.12}$$

where

$$\vec{\mathcal{X}}(s) = \left[\vec{\mathcal{X}}_1(s), \vec{\mathcal{X}}_2(s)\right]^{\mathrm{T}} = [x(t(s)), y(t(s))]^{\mathrm{T}}$$

$$\vec{\mathcal{X}}(0) = \left[\vec{\mathcal{X}}_1(0), \vec{\mathcal{X}}_2(0)\right]^{\mathrm{T}} = [x(0), y(0)]^{\mathrm{T}}$$

$$t(0) = 0$$

This Problem is referred to as Problem (P(p)).

2.2 A new exact penalty function method

Problem (P(p)) is an optimization problem subject to both the terminal equality constraints and the continuous inequality constraints. To solve this problem, a new exact penalty function method introduced in Lin Q. (2012) is used.

By introducing this exact penalty function, the new cost function can be written as follows:

$$J_\delta(\sigma,\theta,\epsilon) = \begin{cases} \sum_{k=1}^{p} \frac{\theta_k}{p}, & \text{if } \epsilon = 0,\ G(\vec{\mathcal{X}}(s)) = 0 \\ \sum_{k=1}^{p} \frac{\theta_k}{p} + \epsilon^{-\alpha}(G(\vec{\mathcal{X}}(s))) + \delta\epsilon^{\beta}, & \text{if } \epsilon > 0, \\ +\infty, & \text{if } \epsilon = 0,\ G(\vec{\mathcal{X}}(s)) \neq 0. \end{cases} \tag{2.13}$$

where

$$\begin{aligned} G(\vec{\mathcal{X}}(s)) &= \sum_{i=1}^{2}\sum_{k=1}^{p}\int_{\frac{k-1}{p}}^{\frac{k}{p}} \theta_k \max\left\{\max\{R^2, R_i^2\} - \left\|\vec{\mathcal{X}}(s) - \vec{\mathcal{X}}_i(s)\right\|^2, 0\right\}^2 ds \\ &+ \max\left\{x_- - \vec{\mathcal{X}}_1(1), 0\right\}^2 + \max\left\{\vec{\mathcal{X}}_1(1) - x_+, 0\right\}^2 \\ &+ \left\{\sum_{i=1}^{p}\frac{\theta_i}{p} - T\right\}^2 \end{aligned}$$

This yields a sequence of unconstrained optimal parameter selection problems as follows, which are referred to as Problem $P_\delta(p)$: Given a δ and the following system

$$\begin{cases} \frac{d\vec{\mathcal{X}}(s)}{ds} = \theta_k \vec{f}(\vec{\mathcal{X}}(s), \sigma_k, t(s)), \quad s \in \left[\frac{k-1}{p}, \frac{k}{p}\right),\ k = 1, 2, \ldots, p, \\ \frac{dt(s)}{ds} = \theta_k, \quad s \in \left[\frac{k-1}{p}, \frac{k}{p}\right),\ k = 1, 2, \ldots, p, \\ \vec{\mathcal{X}}(0) = [x(0), y(0)]^{\mathrm{T}} \\ t(0) = 0 \end{cases} \tag{2.14}$$

find a (σ,θ,ϵ) such that $J_\delta(\sigma,\theta,\epsilon)$ is minimized. Problem $P_\delta(p)$ is an unconstrained optimal control problem in canonical form. To solve it as a nonlinear optimization problem by using the optimal control software MISER 3.3 (Jennings L.S. (2004)), we need the gradient formula of the objective function. The gradient formula is given below. It's proof is similar to that given for Theorem 5.2.1 reported in Teo K.L. (1991).

Theorem 2.1

For each $\delta > 0$, the gradients of the cost function $J_\delta(\sigma,\theta,\epsilon)$ with respect to σ and θ are:

$$\frac{\partial J_\delta(\sigma,\theta,\epsilon)}{\partial\sigma} = \frac{\partial\Phi_0\left(\theta,\epsilon,\vec{\mathcal{X}}(s)\right)}{\partial\sigma} + \int_0^1 \frac{\partial H_0\left(s,\vec{\mathcal{X}}(s),\sigma,\theta,\lambda(s)\right)}{\partial\sigma}\mathrm{d}s, \tag{2.15}$$

$$\frac{\partial J_\epsilon(\sigma,\theta,\epsilon)}{\partial\theta} = \frac{\partial\Phi_0\left(\theta,\epsilon,\vec{\mathcal{X}}(s)\right)}{\partial\theta} + \int_0^1 \frac{\partial H_0\left(s,\vec{\mathcal{X}}(s),\sigma,\theta,\lambda(s)\right)}{\partial\theta}\mathrm{d}s, \tag{2.16}$$

where $H_0\left(s,\vec{\mathcal{X}}(s),\sigma,\theta,\lambda(s)\right)$ is the Hamiltonian function for the cost function given by

$$H_0\left(s,\vec{\mathcal{X}}(s),\sigma,\theta,\lambda(s)\right) = \sum_{i=1}^{2}\sum_{k=1}^{p}\mathcal{L}_{0,ik}\left(\vec{\mathcal{X}}(s),\theta\right) + \lambda_0(s)\upsilon^p(s)\vec{f}\left(\vec{\mathcal{X}}(s),\sigma,t(s)\right), \tag{2.17}$$

$\Phi_0\left(\theta,\epsilon,\vec{\mathcal{X}}(s)\right)$ and $\mathcal{L}_{0,ik}\left(\vec{\mathcal{X}}(s),\theta\right)$ are defined as

$$\begin{aligned} \Phi_0\left(\theta,\epsilon,\vec{\mathcal{X}}(s)\right) &= \sum_{k=1}^{p}\frac{\theta_k}{p} + \delta\epsilon^{\beta} + \epsilon^{-\alpha}\left\{\max\left\{x_- - \vec{\mathcal{X}}_1(1), 0\right\}^2\right. \\ &+ \left.\max\left\{\vec{\mathcal{X}}_1(1) - x_+, 0\right\}^2 + \left\{\sum_{i=1}^{p}\frac{\theta_i}{p} - T\right\}^2\right\} \end{aligned}$$

$$\mathcal{L}_{0,ik}\left(\vec{\mathcal{X}}(s),\theta\right)=\theta_k \max\left\{\max\{R^2,R_i^2\}-\left\|\vec{\mathcal{X}}(s)-\vec{\mathcal{X}}_i(s)\right\|^2,0\right\}^2$$

and $\lambda_0(s)$ is the solution of the following co-state differential equation

$$\dot{(\lambda_0(s))}^{\mathrm{T}}=-\frac{\partial H_0\left(s,\vec{\mathcal{X}}(s),\sigma,\theta,\lambda(s)\right)}{\partial\vec{\mathcal{X}}(s)} \tag{2.18a}$$

with the boundary condition

$$(\lambda_0(1))^{\mathrm{T}}=\frac{\partial\Phi_0\left(\theta,\epsilon,\vec{\mathcal{X}}(s)\right)}{\partial\vec{\mathcal{X}}(s)}. \tag{2.18b}$$

2.3 Algorithm

With the results in the previous sections, we provide the following algorithm for solving Problem (P).
Algorithm 2.1
Step 1 Set $\delta^{(1)}=10$, $\epsilon^{(1)}=0.1$, $\epsilon^*=10^{-9}$, $\beta>2$, choose an initial point $(\sigma^0,\theta^0,\epsilon^0)$, the iteration index $k=0$. The values of γ and α are chosen depending on the specific structure of Problem (P) concerned.
Step 2 Solve Problem (P_{δ_k}), and let $\left(\sigma^{(k),*},\theta^{(k),*},\epsilon^{(k),*}\right)$ be the minimizer obtained.
Step 3 If $\epsilon^{(k),*}>\epsilon^*$, $\delta^{(k)}<10^8$,
set $\delta^{(k+1)}=10\times\sigma^{(k)}$, $k:=k+1$. Go to **Step 2** with $\left(\sigma^{(k),*},\theta^{(k),*},\epsilon^{(k),*}\right)$ as the new initial point in the new optimization process
Else set $\epsilon^{(k),*}:=\epsilon^*$, then go to **Step 4**
Step 4 Check the feasibility of $\left(\sigma^{(k),*},\theta^{(k),*}\right)$ (i.e., check whether or not
$\max_{1\le i\le N}\max_{s\in[0,1]}\bar{g}_i\left(s,\tilde{y}\left(s|\sigma^{(k),*},\theta^{(k),*}\right),\sigma^{(k),*}\right)\le 0$.
If $\left(\sigma^{(k),*},\theta^{(k),*}\right)$ is feasible, then it is a local minimizer of Problem (P(p)). Exit.
Else go to **Step 5**
Step 5: Adjust the parameters α,β and γ such that the conditions of Lemma **??** are satisfied. Set $\delta^{(k+1)}=10\delta^{(k)}$, $\epsilon^{(k+1)}=0.1\epsilon^{(k)}$, $k:=k+1$. Go to **Step 2**.

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98 PDE APPROACH FOR SENSOR DEPLOYMENTS FOR LEAKAGE DETECTION

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Abstract: We consider a multi-agent control problem using PDE techniques for a novel sensing problem arising in the leakage detection and localization of offshore pipelines. A continuous protocol is proposed using parabolic PDEs and then a boundary control law is design using the maximum principle.

Key words: Pipeline leakage; Leakage detection; Floating sensors; Parabolic PDEs; Optimal control.

1 INTRODUCTION

Pipeline transport is the transportation of mass from one place to another through a pipeline network. Generally, any chemically stable substance can be sent through a pipeline. Pipelines are probably the most economical way to transport large quantities of oil, refined oil products or natural gas over land. In the offshore oil industry, pipeline is also an important choice to transport oil from the platform to the tanker ships or directly to the land refinery factories. Till the end of 2008, there exist more than 6000 kilometers of pipelines in China and more than 2000 kilometers of them are offshore pipelines.

Since oil and gas pipelines are an important asset of the economic development of almost any country, the enhancement of asset safety and security is highly required by government regulations and policies. For example, it is a mandatory rule for pipeline operators in the State of Washington to be able to detect and locate leaks of 8 percent of maximum flow within no more than 15 mins. Any failure in identifying pipeline leakage and delivering appropriate repairs in time may lead to serious environmental pollution and economical loss.

In this work, we propose using floating sensors to detect leakage inside the pipeline. In order to make cooperative sensing, it is necessary to consider the coordination control of large number of autonomous sensors. A continuous method using PDE control is applied to design the protocols for autonomous sensors.

2 FORMULATION OF THE PROBLEM

$$\begin{cases} \dfrac{\partial x(\theta,t)}{\partial t} = \dfrac{\partial}{\partial \theta}\left[\alpha \dfrac{\partial x(\theta,t)}{\partial \theta}\right] + \beta \dfrac{\partial x(\theta,t)}{\partial \theta} + \gamma x(\theta,t), \quad \theta \in (0,2\pi), \\ \left.\dfrac{\partial x(\xi,t)}{\partial t}\right|_{\theta=0} = u_l(t), \quad \left.\dfrac{\partial x(\xi,t)}{\partial t}\right|_{\theta=2\pi} = u_r(t), \\ x(\theta,0) = x_0(\theta), \end{cases} \tag{2.1}$$

If we integrate the boundary condition of (2.1) over $[0,t)$, then the boundary condition becomes

$$x(0,t) = x(0,0) + \int_0^t u_l(\tau)d\tau = v_l(t), \tag{2.2}$$

$$x(2\pi,t) = x(2\pi,0) + \int_0^t u_r(\tau)d\tau = v_r(t), \tag{2.3}$$

where $v_l(t)$ and $v_r(t)$ are auxilliary control functions. We consider a standard Dirichlet boundary control problem

$$\begin{cases} \dfrac{\partial x(\theta,t)}{\partial t} = \alpha \dfrac{\partial^2 x(\theta,t)}{\partial \theta^2} + \beta \dfrac{\partial x(\theta,t)}{\partial \theta} + \gamma x(\theta,t), \quad \theta \in (0,2\pi), \\ x(0,t) = v_l(t), \quad x(2\pi,t) = v_r(t), \\ x(\theta,0) = x_0(\theta). \end{cases} \tag{2.4}$$

where $v_l(t)$ and $v_r(t)$ represent the boundary control variables. We consider the following cost functional

$$\begin{aligned} J_c = \frac{1}{2}\int_0^T & \left[\langle x(\theta,t), Q\,[\![x(\theta,t)]\!]\rangle + R_l v_l^2(t) + R_r v_r^2(t)\right] dt \\ & + \frac{1}{2}\langle x(\theta,T), S\,[\![x(\theta,T)]\!]\rangle, \end{aligned} \tag{2.5}$$

where the inner product and linear operator are defined as

$$\langle x_1(\theta), x_2(\theta)\rangle_{L_2} := \int_0^{2\pi} x_1(\theta)x_2(\theta)d\theta, \tag{2.6}$$

$$\mathcal{A}\,[\![x(\theta,t)]\!] := \int_0^{2\pi} \mathcal{A}(\theta,\eta)x(\eta,t)d\eta. \tag{2.7}$$

3 OPTIMALITY CONDITIONS

Let $x^*(\theta,t)$,$v_{l,r}^*(t)$ and $\lambda(\theta,t)$ denote the optimal state, control and co-state that minimize the quadratic cost functional, then we assume the following perturbed representation with respect to the optimal trajectories,

$$x(\theta,t) = x^*(\theta,t) + \epsilon\delta x(\theta,t), \tag{3.1}$$

$$v_l(t) = v_l^*(t) + \epsilon\delta v_l(t), \tag{3.2}$$

$$v_r(t) = v_r^*(t) + \epsilon\delta v_r(t), \tag{3.3}$$

where $\delta x(\theta,0) = 0$ and δ represents the perturbation operator and ϵ is an arbitrary constant. Then, the perturbed cost functional is

$$\begin{aligned} & J_c\left[x^*(\theta,t) + \epsilon\delta x(\theta,t), v_l^*(t) + \epsilon\delta v_l(t), v_r^*(t) + \epsilon\delta v_r(t)\right] \\ =& \frac{1}{2}\int_0^T \langle x^*(\theta,t) + \epsilon\delta x(\theta,t), Q\,[\![x^*(\theta,t) + \epsilon\delta x(\theta,t)]\!]\rangle\, dt \\ & + \frac{1}{2}\int_0^T \left\{R_l\left[v_l^*(t) + \epsilon\delta v_l(t)\right]^2 + R_r\left[v_r^*(t) + \epsilon\delta v_r(t)\right]^2\right\} dt \\ & + \frac{1}{2}\langle x^*(T) + \epsilon\delta x(T), S\,[\![x^*(T) + \epsilon\delta x(T)]\!]\rangle\,. \end{aligned} \tag{3.4}$$

We introduce a function of ϵ based on the perturbed cost functional by incorporating the PDE system using the Lagrangian multiplier $\lambda(\theta, t)$,

$$
\begin{aligned}
g(\epsilon) := {} & J_c\left[x^*(\theta,t) + \epsilon\delta x(\theta,t), v_l^*(t) + \epsilon\delta v_l(t), v_r^*(t) + \epsilon\delta v_r(t)\right] \\
& + \int_0^T \left\langle \lambda(\theta,t), \alpha\left[\frac{\partial^2 x^*(\theta,t)}{\partial\theta^2} + \epsilon\delta\frac{\partial^2 x(\theta,t)}{\partial\theta^2}\right]\right\rangle dt \\
& + \int_0^T \left\langle \lambda(\theta,t), \beta\left[\frac{\partial x^*(\theta,t)}{\partial\theta} + \epsilon\delta\frac{\partial x(\theta,t)}{\partial\theta}\right] + \gamma\left[x^*(\theta,t) + \epsilon\delta x(\theta,t)\right]\right\rangle dt \\
& - \int_0^T \left\langle \lambda(\theta,t), \frac{\partial}{\partial t}\left[x^*(\theta,t) + \epsilon\delta x(\theta,t)\right]\right\rangle .
\end{aligned}
\tag{3.5}
$$

Then, the necessary condition for optimality is

$$
\left.\frac{dg(\epsilon)}{d\epsilon}\right|_{\epsilon=0} = 0, \tag{3.6}
$$

i.e.,

$$
\begin{aligned}
& \frac{dg}{d\epsilon}(\epsilon) \\
& = \frac{1}{2}\int_0^T \left\langle \delta x(\theta,t), Q\left[\!\left[x^*(\theta,t) + \epsilon\delta x(\theta,t)\right]\!\right]\right\rangle dt \\
& + \frac{1}{2}\int_0^T \left\langle x^*(\theta,t) + \epsilon\delta x(\theta,t), Q\left[\!\left[\delta x(\theta,t)\right]\!\right]\right\rangle dt \\
& + \int_0^T R_l\left[v_l^*(t) + \epsilon\delta v_l(t)\right]\delta v_l(t) dt \\
& + \int_0^T R_r\left[v_r^*(t) + \epsilon\delta v_r(t)\right]\delta v_r(t) dt \\
& + \frac{1}{2}\left\langle \delta x(\theta,T), S\left[\!\left[x^*(\theta,T) + \epsilon\delta x(\theta,T)\right]\!\right]\right\rangle \\
& + \frac{1}{2}\left\langle x^*(\theta,T) + \epsilon\delta x(\theta,T), S\left[\!\left[\delta x(\theta,T)\right]\!\right]\right\rangle \\
& + \int_0^T \left\langle \lambda(\theta,t), \alpha\frac{\partial^2\delta x(\theta,t)}{\partial\theta^2} + \beta\frac{\partial\delta x(\theta,t)}{\partial\theta} + \gamma\delta x(\theta,t) - \frac{\partial\delta x(\theta,t)}{\partial t}\right\rangle dt.
\end{aligned}
\tag{3.7}
$$

Using integration by parts, we can obtain the following simplification

$$
\begin{aligned}
& \left\langle \lambda(\theta,t), \alpha\frac{\partial^2\delta x(\theta,t)}{\partial\theta^2}\right\rangle \\
& = \int_0^{2\pi} \lambda(\theta,t)\alpha\frac{\partial^2\delta x(\theta,t)}{\partial\theta^2} d\theta \\
& = \int_0^{2\pi} \alpha\lambda(\theta,t) d\left[\frac{\partial\delta x(\theta,t)}{\partial\theta}\right] \\
& = \alpha\lambda(\theta,t)\left.\frac{\partial\delta x(\theta,t)}{\partial\theta}\right|_0^{2\pi} - \int_0^{2\pi}\alpha\frac{\partial\lambda(\theta,t)}{\partial\theta}\frac{\partial\delta x(\theta,t)}{\partial\theta} d\theta \\
& = \alpha\lambda(\theta,t)\left.\frac{\partial\delta x(\theta,t)}{\partial\theta}\right|_{\theta=0}^{\theta=2\pi} - \alpha\left.\frac{\partial\lambda(\theta,t)}{\partial\theta}\delta x(\theta,t)\right|_{\theta=0}^{\theta=2\pi} + \int_0^{2\pi}\alpha\frac{\partial^2\lambda(\theta,t)}{\partial\theta^2}\delta x(\theta,t) d\theta \\
& = \alpha\lambda(2\pi,t)\left.\frac{\partial\delta x(\theta,t)}{\partial\theta}\right|_{\theta=2\pi} - \alpha\lambda(0,t)\left.\frac{\partial\delta x(\theta,t)}{\partial\theta}\right|_{\theta=0} \\
& \quad + \alpha\left.\frac{\partial\lambda(\theta,t)}{\partial\theta}\right|_{\theta=2\pi}\delta x(2\pi,t) - \alpha\left.\frac{\partial\lambda(\theta,t)}{\partial\theta}\right|_{\theta=0}\delta x(0,t) + \alpha\left\langle\frac{\partial^2\lambda(\theta,t)}{\partial\theta^2}, \delta x(\theta,t)\right\rangle .
\end{aligned}
\tag{3.8}
$$

We choose the multiplier to satisfy $\lambda(0,t) = \lambda(2\pi,t) = 0$ and note that

$$
\delta x(0,t) = \delta v_l(t), \quad \delta x(2\pi,t) = \delta v_r(t), \tag{3.9}
$$

then we have

$$
\begin{aligned}
&\left\langle \lambda(\theta,t), \alpha \frac{\partial^2 \delta x(\theta,t)}{\partial \theta^2} \right\rangle \\
&= \alpha \left. \frac{\partial \lambda(\theta,t)}{\partial \theta} \right|_{\theta=2\pi} \delta v_r(t) - \alpha \left. \frac{\partial \lambda(\theta,t)}{\partial \theta} \right|_{\theta=0} \delta v_l(t) + \alpha \left\langle \frac{\partial^2 \lambda(\theta,t)}{\partial \theta^2}, \delta x(\theta,t) \right\rangle.
\end{aligned} \tag{3.10}
$$

Similarly, we can have

$$
\begin{aligned}
&\left\langle \lambda(\theta,t), \beta \frac{\partial \delta x(\theta,t)}{\partial \theta} \right\rangle \\
=&\beta \lambda(2\pi,t) \delta v_r(t) - \beta \lambda(0,t) \delta v_l(t) - \beta \left\langle \frac{\partial \lambda(\theta,t)}{\partial \theta}, \delta x(\theta,t) \right\rangle,
\end{aligned} \tag{3.11}
$$

and

$$
\begin{aligned}
&\int_0^T \left\langle \lambda(\theta,t), \frac{\partial \delta x(\theta,t)}{\partial t} \right\rangle dt \\
&= \langle \lambda(\theta,T), \delta x(\theta,T) \rangle - \langle \lambda(\theta,0), \delta x(\theta,0) \rangle - \int_0^T \left\langle \frac{\partial \lambda(\theta,t)}{\partial t}, \delta x(\theta,t) \right\rangle dt \\
&= \langle \lambda(\theta,T), \delta x(\theta,T) \rangle - \int_0^T \left\langle \frac{\partial \lambda(\theta,t)}{\partial t}, \delta x(\theta,t) \right\rangle dt,
\end{aligned} \tag{3.12}
$$

where we have noted that $\delta x(\theta,0) = 0$. Now we are ready to compute

$$
\left. \frac{dg(\epsilon)}{d\epsilon} \right|_{\epsilon=0} = 0. \tag{3.13}
$$

Thus, we obtain the co-state equation

$$
\begin{cases}
-\dfrac{\partial \lambda(\theta,t)}{\partial t} = \alpha \dfrac{\partial^2 \lambda(\theta,t)}{\partial \theta^2} - \beta \dfrac{\partial \lambda(\theta,t)}{\partial \theta} + \gamma \lambda(\theta,t) + Q \left[\!\left[x^*(\theta,t) \right]\!\right], \\
\lambda(0,t) = \lambda(2\pi,t) = 0, \\
\lambda(\theta,T) = S \left[\!\left[x^*(\theta,T) \right]\!\right] = \displaystyle\int_0^{2\pi} S(\theta,\eta) x(\eta,t) d\eta, \\
v_l^*(t) = -R_l^{-1} \alpha \left. \dfrac{\partial \lambda(\theta,t)}{\partial \theta} \right|_{\theta=0}, \\
v_r^*(t) = R_r^{-1} \alpha \left. \dfrac{\partial \lambda(\theta,t)}{\partial \theta} \right|_{\theta=2\pi}.
\end{cases} \tag{3.14}
$$

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99 SOLVING ECONOMIC LOT SCHEDULING PROBLEM WITH DETERIORATING ITEMS USING AN EFFICIENT SEARCH ALGORITHM

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Abstract: The Economic Lot Scheduling Problem (ELSP) is concerned with the lot sizing and scheduling decision of n items. The focus of this study is to employ the extended basic period (EBP) approach for solving the ELSP with deteriorating items. In order to propose our solution approach, we first conduct full analysis on the mathematical model. By utilizing our theoretical results, we propose an efficient search algorithm to obtain (feasible) candidate solutions. Finally, among those candidate solutions, we pick the 'best' solution with the minimum average total costs. Based on our numerical experiments, we show that the proposed search algorithm is a more efficient and reliable solution approach than the existing approach in the literature.

Key words: Lot size; Scheduling; Deterioration; Extended basic period; Search algorithm.

1 INTRODUCTION

The Economic Lot Scheduling Problem (ELSP) is concerned with the lot sizing, production cycle adjustments and scheduling decision of n items to derive feasible solutions that satisfy the customer demand and minimize the average total cost.

The conventional ELSP models did not consider deterioration of inventory. However, it is common that inventory of finished product deteriorates before meeting customers' demand. For instances, packaged food-products such as vegetable, fruits, seafood, and chemical products such as gasoline, might become rotten, evaporating, deteriorating and spoiling. Since the deteriorating inventory will cause extra cost, the mathematical model could mislead the decision-maker if it does not take into account deteriorating inventory. Therefore, we are motivated to study the ESLP with deteriorating items and to propose a solution approach for solving the problem.

One may refer to Elmaghraby(1978) and Lopez & Kingsman(1991) for the reviews of the studies on the ELSP. Heuristics (e.g, Park & Yun(1984), Boctor(1987), Geng & Vickson(1988)) are more popular solution approaches for solving the ELSP in the literature.

On the deterioration of inventory, we consider the ELSP refer to the time value category of Raafat(1991) with the characteristic of utilization decline and the lifetime category of Nahmias(1978) with random decreasing lifetime in this study. We further assume that the decreasing of lifetime follows the exponential distribution.

Most of the inventory control models with deterioration considered single product in their decision-making scenario. Recently, some researchers studied lot-sizing problem with deteriorating items. For instance, Yang & Wee(2001) discuss a two-stage supply chain system with single supplier and multiple retailers. Huang & Yao(2005) further modified their model, presented theoretical analysis and proposed an effective searching algorithm for solving the problem.

Furthermore, only the study of Yao & Huang(2005) assume that the multiple products are deteriorating and use the derived base period method to find the optimal solution of ELSP with deteriorating items. They search for the possible optimal solutions of ELSP with deteriorating items by using genetic algorithm (GA). The Feasibility Testing Procedure for the ELSP (Proc. FT) is further applied to judge whether the solutions derived from genetic algorithm are feasible or not. However, the method in Yao & Huang's study requires longer time to derive the optimal solution. In this study, we refer to some studies with similar issues such as Huang & Yao(2005), Huang & Yao(2006) and apply the ELSP with deteriorating items and the feasibility testing procedure for the ELSP to solve the problem effectively.

We introduce the mathematical model for the ELSP with deteriorating items in Section 2. Section 3 presents our theoretical analysis for the optimal cost function curve. In Section 4, we propose a searching algorithm for obtaining candidate solutions. Our numerical experiments in Section 5 compare the effectiveness of the proposed search algorithm and GA in Yao & Huang(2005). Finally, Section 6 gives concluding remarks.

2 THE MATHEMATICAL MODEL

In this section, we introduce the model of the economic lot scheduling problem (ELSP) with deteriorating items. Our assumptions for the concerned problem are presented as follows. We assume that all items are produced by a facility. In each given time, the demand rate, production rate, setup time, setup costs and inventory holding costs are known and will not change through the passage of time. The production facility can only produce one product at a single given time. The product demand is continuous. The setup cost and setup time are independent of the production sequence. The production cost of the unit product is not related to the lot size.

In the ELSP with deteriorating items, we consider the following cost terms. They are, namely, (1) setup cost; (2) inventory holding costs; (3) the deteriorating costs of items. We define the notation that is used for the formulation of the model as follows.

n: the number of products

d_i: demand rate for product i in each unit period.

p_i: production rate for product i in each unit period ($p_i > d_i$).

a_i: setup cost for product i in each lot.

h_i: inventory holding cost for product i in each unit period.

θ_i: the deteriorating coefficient of product i. (It is assumed as a constant.)

s_i: setup time for product i in each lot.

B: basic period.

k_i: the time multiplier of product i.

ρ_i: $\rho_i = d_i/p_i \left(\sum_{i=1}^{n} \rho_i < 1\right)$

Our mathematical model for the ELSP with deteriorating items is as follows.

$$\min_{k_i \in \mathbb{N}^+, \forall i, B>0} TC(k_1, ..., k_n, B) = \sum_{i=1}^{n} \left\{ \frac{a_i}{k_i B} + \frac{1}{2} H_i k_i B \right\} \tag{2.1}$$

subject to

$$\sum_{i=1}^{n} \left[\left(s_i + \rho_i \left(1 + \frac{\theta_i k_i B}{2}\right) k_i B \right) \right] w_{i\varphi(i,t)} \leq B, \text{ for } t = 1, ..., K \tag{2.2}$$

$$\sum_{t=1}^{k_i} w_{it} = 1, \text{for} i = 1, ..., n \tag{2.3}$$

$$\left\{ \begin{array}{ll} w_{it} = 1, & \text{if product } i \text{ is produced in the } t^{th} \text{ basic period,} \\ w_{it} = 0, & \text{otherwise,} \end{array} \right\} \tag{2.4}$$

$$K = \text{lcm}\{k_1, ..., k_n\} \tag{2.5}$$

$$\varphi(i,t) = \left\{ \begin{array}{ll} t \bmod k_i, & \text{if } t \neq \gamma k_i, \gamma \in \mathbb{N} \\ k_i, & \text{if } t = \gamma k_i, \gamma \in \mathbb{N} \end{array} \right\} \tag{2.6}$$

where $H_i = d_i\left(\theta_i \xi_i + h_i\right)$.

Equation (2.2) denotes the capacity constrains that the total capacity must less than the length of basic period in each basic period t. Equation (2.3) indicates that product i would be produced once in each k_i basic period. When product i is produced in the tth basic period $w_{it} = 1$ as indicated in equation (2.4). In equation (2.5), the total number of basic period in the whole planning horizon is denoted by K which is the least common multiple of all $k_i' s$. Equation (2.6) presents that the cyclic scheduling of the production lots after the first production lot is determined.

3 THEORETICAL ANALYSIS

In this section, we first conduct theoretical analysis on the unconstrained optimal total cost function. Our theoretical results shall establish important foundation for the design of the proposed search algorithm.

We define the cost function of individual product i, $\Gamma_i(k_i, B)$, as:

$$\Gamma_i(k_i, B) = \frac{a_i}{k_i B} + \frac{1}{2} H_i k_i B. \tag{3.1}$$

For each B, we have the optimal cost function $\Gamma_i\left(B\right) = \min_{k_i \in \mathbb{N}} \left\{ \frac{a_i}{k_i B} + \frac{1}{2} H_i k_i B \right\}$ by taking the optimal k_i. We assert that $\Gamma_i\left(B\right)$ has an interesting property in Lemma 1.

Lemma 3.1 *$\Gamma_i(B)$ function is piece-wise convex with respect to B.*

We define $\Gamma\left(B\right)$ as the unconstrained optimal total cost function of $TC\left(k_1, ..., k_n, B\right)$, namely,

$$\Gamma\left(B\right) = \inf \sum_{i=1}^{n} \left\{\Gamma_i\left(B\right)\right\} \tag{3.2}$$

Then, we confirm an important property in Theorem 1.

The function $\Gamma(B)$ is piece-wise convex with respect to B.

Theorem 3.1 *The function $\Gamma(B)$ is piece-wise convex with respect to B.*

Next, we define a "junction point" as a particular point where two consecutive convex curves intersect. For individual product i, the junction point of function $\Gamma\left(B\right)$ can be derived by

$$\delta_i(k_i) = \sqrt{\frac{2a_i}{H_i(k_i+1)k_i}} \tag{3.3}$$

Note that $\delta_i(k_i)$ represents the k_i^{th} junction point of $\Gamma_i\left(B\right)$. Lemma 2 indicates the relationship between the optimal multipliers and a junction point w.

Suppose that k_L^* and k_R^*, respectively, are the set of optimal multipliers for the left-side and right-side convex curves with regard to a junction point w in the plot of the $\Gamma_i\left(B\right)$ function. Then, $k_L^* = k_R^* + 1$.

Lemma 3.2 *Suppose that k_L^* and k_R^*, respectively, are the set of optimal multipliers for the left-side and right-side convex curves with regard to a junction point w in the plot of the $\Gamma_i\left(B\right)$ function. Then, $k_L^* = k_R^* + 1$.*

Since $\Gamma\left(B\right)$ is a sum of $\Gamma_i\left(B\right)$ function, the following corollary is also a by-product of Lemma 2.

Corollary 3.1 *For any given B, the optimal multiplier of function is given by*

$$k_i(B) = \left\{ \begin{array}{cc} 1, & B > \delta_i(1) \\ m, & B \in [\delta_i(m), \delta_i(m-1)) \end{array} \right\} \tag{3.4}$$

Next, we discuss the structure of optimal total cost curve. First, to simply the symbols, we define the multiplier vector $\mathbf{k} \equiv (k_1, ..., k_n)$.

The following theorem results from Theorem 1 and Lemma 2.

Theorem 3.2 *Suppose $\mathbf{k}^{(L)}$ and $\mathbf{k}^{(R)}$ are the set of optimal multiplier vectors for the left-side and right-side convex curves, thus, $\mathbf{k}^{(L)}$ can be obtained from $\mathbf{k}^{(R)}$ by $k_i^{(L)} = k_i^{(R)} + 1$ for some product i.*

4 THE PROPOSED ALGORITHM

We consider four key issues in the design of the proposed search algorithm, namely, (1) finding the local minimum for a set of multipliers, (2) setting the range of the search, (3) proceeding with the search, and (4) generating a feasible production schedule, in this section.

We define $\breve{B}(\mathbf{k})$ as the local minimum with respect to a set of multipliers $\mathbf{k}$ in the function $TC\,(k_1, ..., k_n, B)$. If $(\mathbf{k}, \breve{B}(\mathbf{k}))$ is able to generate a feasible production schedule, it serves as a candidate solution. By taking the first-order derivative of eq. (1), we have the closed form of $\breve{B}(\mathbf{k})$ by

$$\breve{B}(\mathbf{k}) = \sqrt{\left(\sum_{i=1}^{n} \frac{a_i}{k_i B}\right) / \left(\sum_{i=1}^{n} \frac{1}{2} H_i k_i\right)} \tag{4.1}$$

Then, we would settle the upper bound and lower bound of the search range. We would like to apply the Rotational Cycle (RC) approach for obtaining the upper bound. In the RC approach, it assumes that the cycle times of all products are equal. Thus, the objective function of the RC approach is as follows.

$$TC^{RC} = \sum_{i=1}^{n} \left\{ \frac{a_i}{T_{RC}} + \frac{1}{2} H_i T_{RC} \right\} \tag{4.2}$$

One may refer to Yao & Huang(2005) for the approach of solving the optimal cycle time (T^*_{RC}) of the RC approach. On the other hand, we consider the value of $\max_i \{(1+\rho_i)\, s_i\}$ as the lower bound since there exists no feasible solution as $B < \max_i \{(1+\rho_i)\, s_i\}$.

Next, the searching algorithm proceeds as follows. First, we obtain T^*_{RC} and $\max_i \{(1+\rho_i)\, s_i\}$ as an upper bound as a lower bound of the search range. We find the first junction point w and the closest (largest) point w_{p+1}. Then, we obtain the set of optimal multiplier $\mathbf{k}(w_p)$ and its (unconstrained) minimum $\breve{B}(\mathbf{k}(w_p))$. When $\breve{B}(\mathbf{k}(w_p) > \max_i \{(1+\rho_i)\, s_i\}$ and $w_{p+1} < \breve{B}(\mathbf{k}(w_p) < w_p$, we test the feasibility of $(\breve{B}(\mathbf{k}(w_p)), \mathbf{k}(w_p))$. When $(\breve{B}(\mathbf{k}(w_p)), \mathbf{k}(w_p))$ is feasible, it serves as a candidate solution. When it is not feasible, test whether $\breve{B}(\mathbf{k}(w_p))$ and the prior $\mathbf{k}(w_{p-1})$ are feasible. When $(\breve{B}(\mathbf{k}(w_p)), \mathbf{k}(w_{p-1}))$ is still not feasible, we need to adjust one multiplier by changing k_{i+1} to k_i to seek for the lowest cost increment while meeting all the constraints. We shall repeat the search process is until it reaches the lower bound, and we pick the candidate solution with the lowest objective function value as the optimal solution.

Note that it is an important issue of trying to generate a feasible production schedule for a candidate solution. Here, the proposed search algorithm simply applied the procedure in Yao & Huang(2005), namely, *Proc. FT*, that determines the basic period where of the first production lot starts for each product. The major concern is that the total capacity must less than the length of basic period in each basic period t, as indicated in eq. (2). Actually, *Proc. FT*, keeps "shuffling" the starting time of the first production lot of all the products until meeting all the capacity constraints in the planning horizon.

5 NUMERICAL EXPERIMENTS

Here, we use the six benchmark problems in Fujita(1978) for numerical experiments. We would compare the computation results of the proposed search algorithm with the genetic algorithm (GA) in Yao & Huang(2005).

To verify the solution quality, we define an Error Measure Index (EMI) as $EMI = \left(TC\,(\mathbf{k}^*, B^*) - TC^{IS}\right)/TC^{IS}$ where TC^{IS} is the Independent Solution (IS) from the sum of the optimal EPQ with deteriorating item of all the products, which is a well-known lower bound of the ELSP with deteriorating items.

Since there exists some random mechanism in Prof. FT and GA, we solve each benchmark problem using both solution approaches for 30 times and collect its solutions and run time data. Table 1 summarizes our numerical experiments on the six benchmark problems.

We may refer to the values of Best EMI and Average (Avg) EMI for the solution quality and the average run time (AvgRT) for computational efficiency in Table 1. For most of the (Best or Average) cases, the proposed search algorithm is better than the GA in its solution quality. But, the solution quality of the proposed search algorithm is insignificantly inferior (by less than 0.2%) to the GA for the benchmark problems 2 (in Best EMI), 4 (in Best EMI) and 5 (in Best and Avg EMI). The proposed search algorithm extensively outperforms the GA in the run-time aspect for all the cases.

We conclude that the proposed search algorithm is a more efficient and reliable solution approach for solving the ELSP with deteriorating items following our numerical experiments on the six benchmark problems

	The proposed search algorithm			GA in Yao & Huang(2005)		
Benchmark problem	Best EMI(%)	Avg EMI(%)	Avg RT(s)	Best EMI(%)	Avg EMI(%)	Avg RT(s)
1	0.33	0.38	1.06	1.83	1.94	26.66
2	0.81	0.81	1.21	0.70	0.94	105.89
3	1.00	1.00	2.51	1.93	2.96	27.45
4	0.48	0.48	13.02	0.46	0.68	79.17
5	0.43	0.43	2.58	0.08	0.36	26.92
6	0.06	0.07	1.32	0.26	0.56	50.58

Table 5.1 A summary of our numerical experiments

6 CONCLUSION

In this study, we first conduct theoretical analysis on the ELSP with deteriorating items using the extended basic period approach. Our theoretical results provide us more insights into the structure of the optimal objective function curve. Utilizing our theoretical results as foundation, we propose a new solution approach, which is an efficient search algorithm. We compare the proposed search algorithm with the GA in Yao & Huang(2005). Our numerical experiments show that the proposed search algorithm is a more efficient and reliable solution approach than GA for solving the ELSP with deteriorating items.

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100 THE OPTIMAL REPLENISHMENT AND PAYMENT POLICY FOR EPQ MODELS UNDER CONDITIONS OF PERMISSIBLE DELAY IN PAYMENTS AND CASH DISCOUNT

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Abstract: In order to stimulate the ordered quantity, the suppliers often provide the strategy: the permissible delay in payments, to their retailer. That is, before of the permissible delay period, the retailer can accumulate revenue and deposit it into an account for more profit. However, for suppliers, it is better to obtain the payments from their retailers as soon as possible. So the suppliers also provide another strategy: the cash discount. When a retailer choose this strategy and settle the payments before the end of a given period, the retailer can get cash discount. But the retailers can choose exact one from these two policies. In this paper, we propose a new EPQ model which simultaneously considers both conditions of cash discount and permissible delay in payment. In this new model, the retailers are allowed to divide the payments into two parts: one part is settled in the end of period of cash discount, the other part is settled in the end of permissible delay period. This policy is called two-stage payments and first discussed in Li(2012). Besides providing this new model, we also discuss some properties of its objective functions.

Key words: EPQ; Cash discount; Permissible delay in payments.

1 INTRODUCTION

In practical, supplier often offer two kinds of trade credit for the goods to his retailer. If the retailer settle the payment within a permissible delay period, then no penalty will be charged by his supplier. On the other side, in order to encourage the retailer to settle the payment as soon as possible, supplier often offer cash discount if the retailer can settle the payment within the period of cash discount(which is smaller than permissible delay period). The retailer can accumulate the money obtained from selling goods and depot it in a bank for earning the interest before he settle the payment. But if the retailer settles the payment after the permissible delay period, then he will be charged some interest. Hence the cost function of retailer not only considers the setup cost and holding cost but also cash discount, charged interest or earned interest. There are more details in Chung & Huang(2003), Huang & Lin(2005), Huang & Chung(2003), Huang & Hsu(2007), Huang & Lai(2007).

Although supplier provides two kinds of trade credit, retailer can only choose one, that is, if retailer wants to choose the cash discount, he must settle the full payment before the period of cash discount. Otherwise, he only can settle the full payment between the period of cash discount and permissible delay period. In this paper, we divide the payment into two parts. Retailer can settle partial payment within the period of cash discount and unpaid payment between the period of cash discount and permissible delay period. This is a new strategy. Based on different strategies, some cost functions are given and compared in this paper. We also provide a numerical example which show that dividing the payment into two parts can reduced more cost.

2 BEST PAYMENT POLICY AND OPTIMAL REPLENISHMENT CYCLE OF EPQ MODELS UNDER CONDITIONS OF PERMISSIBLE DELAY IN PAYMENTS AND CASH DISCOUNT

3 NUMERICAL EXAMPLE

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101 SINGLE-MACHINE SCHEDULING WITH SUPPORTIVE TASKS

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Abstract: This paper investigates a single-machine scheduling problem with a set of supportive tasks and a set of jobs. A job cannot start its processing until all of its supportive tasks are finished. The objective functions studied involve job completion times, but not task completion times. For the two objective functions of the total weighted completion time and the number of late jobs, we classify the complexity of several special cases. Our study adds new complexity results to the two standard objective functions under precedence constraints.

Key words: Scheduling; Supportive precedence; Total weighted completion time; Number of late jobs.

1 INTRODUCTION

This paper studies a new scheduling model: single-machine scheduling with supportive tasks. Two disjoint sets of activities $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ are to be processed on a single machine. The elements of set A are called *supportive tasks*, and the elements of set B are called *regular jobs*. The processing times of task a_i and job b_j are denoted by α_i and β_j, respectively. A supportive relation $\mathcal{R} : A \to B$ is a mapping from set A to set B such that for $a_i \in A$ and $b_j \in B$, if (a_i, b_j) belongs to $\mathcal{R}$ then job b_j cannot start unless task a_i is completed. Each job b_j is associated with a weight w_j and a due date d_j specifying the time at which it is expected to be completed. Denote the completion times of job $b_j \in B$ in a particular schedule by C_j. If job b_j is late, that is, $C_j - d_j > 0$, then we set binary variable $U_j = 1$; 0, otherwise. While all supportive tasks are required to be processed, they are not included in the objective functions because their roles are simply preparatory operations for the jobs they support. This paper discusses two min-sum objective functions: the total weighted completion time ($\sum_j w_j C_j$) and the number of late jobs ($\sum_j U_j$). Our studied model can be reduce to regular one by setting the weight of all supportive tasks to 0 for the problem of minimizing total weighted completion time, and setting the due dates of all supportive tasks to infinity for the problem of minimizing the number of late jobs. The problem is denoted by the three-field notation $1|s-prec|\gamma$, where $s-prec$ dictates the supportive precedence and γ is $\sum w_j C_j$ or $\sum U_j$.

The proposed model is motivated by real-world applications where preparatory operations are required before jobs can be processed. In multi-media scheduling, a playback is comprised of several media objects, including for example, audio, video, and text objects. Once a media object is prepared, either downloaded from a digital archive or created at the local site, it can be embedded in several playbacks. In this context, media objects and playbacks are referred to as supportive tasks and regular jobs, respectively, in the defined scheduling setting. Another scenario describing the setting is tool or material preparation in a manufacturing environment. A job may require a set of tools installed before processing. On the other hand, a tool, once installed, may be required by several jobs. In summary, the scheduling setting is applicable to scheduling contexts where preparatory operations occupy a limited resource (the machine) but are not taken into account in the objective functions.

2 PRELIMINARY PROPERTIES

This section presents several structural properties of the scheduling setting studied. An objective function is called regular if it is non-decreasing in the job completion times. The objectives considered in this paper are all regular. Although there are only n regular jobs in the problem setting, we have to sequence $m+n$ operations abiding by the supportive precedence. Since no idle time is assumed, a schedule is uniquely implied by a given sequence. Hereafter, we use the terms sequence and schedule interchangeably if no confusion results. The following property reduces the number of candidate sequences considered for producing an optimal schedule. For each task $a_i \in A$, denote $\mu_i = \{b_j \in B | (i,j) \in \mathcal{R}\}$ as the set of jobs supported by a_i. Similarly, for each job $b_j \in B$, denote $\nu_j = \{a_i \in A | (i,j) \in \mathcal{R}\}$ the set of tasks that support b_j.

Lemma 2.1 *For any regular objective, there exists an optimal schedule such that if there are supportive tasks scheduled between two consecutive jobs b_{j_1} and b_{j_2}, then these supportive tasks all belong to ν_{j_2}.*

Proof: If this is not the case for some optimal sequence, then we consider the supportive tasks scheduled between b_{j_1} and b_{j_2} but do not belong to ν_{j_2}. Since all jobs supported by these tasks are scheduled after job b_{j_2} we move these tasks to the right at the positions immediately following b_{j_2}. Such a schedule is still feasible. Moreover, the move will not increase the completion time of any job. The lemma thus follows.

Hereafter we consider only sequences of the form specified in Lemma 2.1. Next, we investigate a special case where the job sequence is known or fixed a priori. Let $S = (S_1, S_2, \ldots, S_n)$ be a given job sequence. The following algorithm produces an optimal sequence $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{m+n})$ for $1|s{-}prec|\gamma$ subject to job sequence S.

Algorithm Fixed_Job_Sequence(S)
Step 1: Let $t = 1$.
Step 2: For $j = 1$ to n do
    Arrange the tasks of ν_{S_j} in arbitrary order as $\sigma_t, \ldots, \sigma_{t+|\nu_{S_j}|-1}$.
    Let $\sigma_{t+|\nu_{S_j}|} := b_{S_j}; t := t + |\nu_{S_j}| + 1$.
    For each $\ell \neq S_j$ let $\nu_\ell := \nu_\ell - \nu_\ell \cap \nu_{S_j}$.
Step 3: Return sequence σ.

Theorem 2.1 *Given a job sequence S, the $1|s{-}prec|\gamma$ problem can be solved in $O(|\mathcal{R}|)$ time, where $\gamma = \sum_j w_j C_j$ or $\sum_j U_j$.*

Proof: The correctness of Algorithm Fixed_Job_Sequence(S) directly follows from Lemma 2.1. As for the running time, we note that each task is scheduled once. When task a_i is scheduled, it is removed from each ν_j for $j \in \mu_i$. Deletion of an element from a set can be carried out in constant time. Therefore, after scheduling task a_i, $O(\mu_i) = O(n)$ time is required to adjust the sets ν_j for $j \in \mu_i$. The overall running time of Algorithm Fixed_Job_Sequence(S) is thus $O(mn)$. In fact, the upper bound of the running time is $|\mathcal{R}| \leq mn$ since each ordered pair is removed only once.

3 TOTAL WEIGHTED COMPLETION TIME ($\sum_J W_J C_J$)

This section discusses the minimization of the total weighted completion time. Existing relevant results and our results are summarized in Table 3.1. Single-machine scheduling to minimize the total weighted completion time ($1||\sum w_j C_j$) can be tackled using the weighted shortest processing time (WSPT) rule (Smith. W.E. (1956)). When precedence constraints are introduced, the problem becomes hard to solve. Lawler proved the strong NP-hardness of cases with $p_j = 1$ and $w_j \in \{k, k+1, k+2\}$ for any integer k, see (Lawler, E.L. (1978)). The proof was adapted for the case with $w_j = 1$ and $p_j \in \{1, 2, 3\}$. Lawler (Lawler, E.L. (1978)) and Lenstra and Rinnooy Kan (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1978)) also showed the case with $w_j = 1$ and $p_j \in \{0, 1\}$ to be strongly NP-hard. The problem is polynomially solvable if the precedence graph is a forest, see (Horn, W.A. (1972)), or a generalized series-parallel graph, see (Adolphson, D.L. (1977)) and (Lawler, E.L. (1978)).

Table 3.1 Complexity Results of $\sum w_j C_j$.

Conditions	Complexity
series-parallel graph	$O(n \log n)$ (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1978))
$prec, p_j = 1, w_j \in \{k+1, k+2, k+3\}$	snp-h (Lawler, E.L. (1978))
$prec, p_j \in \{1, 2, 3\}, w_j = 1$	snp-h (Lawler, E.L. (1978))
$prec, p_j \in \{0, 1\}, w_j = 1$	snp-h (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1978))
$s{-}prec, \alpha_i = 1, \beta_j = 0, w_j = 1$	snp-h (Hassin, R. and Levin, A. (2005))
$s{-}prec$, task sequence is fixed	$O((m+n)\log(m+n))$ (Theorem 3.1)

snp-h: Strongly NP-hard; np-h: NP-hard

The supportive model can be considered a scheduling problem with precedence constraints where all supportive tasks have zero weight.In the proofs presented by Lawler (Lawler, E.L. (1978)) and Lenstra and Rinnooy Kan (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1978)), the constructed precedence graphs have long chains of unit execution time (or unit-weight) jobs. The precedence graph defined in our problem is however in the form of bipartite precedence, where the edges are all oriented from the nodes of the one part into the nodes of the other. Therefore, not all known complexity results can be applied to our studied problem.

The minimum latency set cover problem investigated by Hassin and Levin, see (Hassin, R. and Levin, A. (2005)), is relevant to our problem with $\alpha_i = 1$ for all $a_i \in A$ and $\beta_j = 0$ for all $b_j \in B$. Therefore, the $1|s{-}prec, \alpha_i = 1, \beta_j = 0|\sum C_j$ problem is strongly NP-hard, as Hassin and Levin (Hassin, R. and Levin, A. (2005)) obtained a reduction from $1|prec, p_j = 1|\sum w_j C_j$ to show the strong NP-hardnes.

We address a special case in which the sequence of all supportive tasks is given and fixed, in contrast to the results of Algorithm Fixed_Job_Sequence(S). Let $s = (s_1, s_2, \ldots, s_m)$ be a task sequence. For any job b_j, let $a_{s_{i_1}}, a_{s_{i_2}}, \ldots, a_{s_{i_{|\nu_j|}}}$ be its supportive tasks as they appear in sequence s. We can eliminate the ordered pairs $(a_{s_{i_1}}, b_j), \ldots, (a_{s_{i_{|\nu_j|-1}}}, b_j)$ from the instance without altering the precedence constraints.

We thus come up with a new instance, in which $|\nu_j| = 1$ for all b_j. Next, we consider the jobs supported by each task in the new instance. For the $1|s{-}prec|\sum w_j C_j$ problem, if a task sequence is given, then the problem is reduced to $1||\sum w_j C_j$ with out-trees, which can be solved in $O((m+n)\log(m+n))$ time.

Theorem 3.1 *The $1|s-prec|\sum w_j C_j$ problem with a fixed task sequence can be solved in $O((m+n)\log(m+n))$ time.*

4 NUMBER OF LATE JOBS ($\sum_J U_J$)

This section discusses the objective function of minimizing the number of late jobs. See Table 2 for a summary of the results.

Karp (Karp, R.M. (1972)) showed that the single-machine problem of minimizing the weighted number of late jobs ($1||\sum w_j U_j$) is NP-hard and proposed a pseudo-polynomial dynamic programming algorithm. The case where all jobs are equally weighted can be solved in polynomial time by Moore-Hodgson algorithm, see (Moore, J.M (1968)). The problem with precedence constraints is strongly NP-hard, even if all jobs are equally weighted and have a unit processing time (Garey, M.R. and Johnson, D.S. (1979)). Lenstra and Rinnooy Kan further proved the strong NP-hardness of the much restricted case $1|chains, p_j = 1|\sum U_j$, see (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1980)). Note that chains are not more restricted than bipartite precedence. In the proof of Lenstra and Rinnooy Kan (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1980)), long chains were created. The problem studied in this paper has bipartite precedences. We start with the earliest due date (EDD) arrangement of the early jobs, see (Jackson, J.R. (1955)). A job is called 'early' if it is completed before its due date.

Lemma 4.1 *There is an optimal schedule to $1|s{-}prec|\sum U_j$ in which the early jobs are sequenced by the EDD rule.*

Proof: The proof is similar to the argument deployed in the proof of Theorem 2.1.

By a reduction similar to that from Maximum Clique to the $1|p_j = 1, prec|\sum U_j$ problem, see (Garey, M.R. and Johnson, D.S. (1979), pp. 73-74), we can prove the strong NP-hardness of the case with

Table 4.1 Complexity Results of $\sum U_j$.

Conditions	Complexity
$chains, p_j = 1$	snp-h (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1980))
$s{-}prec, \|\nu_j\| = 2, \alpha_i = \beta_j = 1$	snp-h (Observation 1)
$s{-}prec, \|\nu_j\| = 2, \alpha_i = 1, \beta_j = 0$	snp-h (Observation 1)
$s{-}prec, \|\mu_i\| = 3, \|\nu_j\| = 1, \alpha_i = 1, \beta_j = 1$	snp-h (Theorem 4.1)
$s{-}prec, \|\mu_i\| = 2, \|\nu_j\| = 1$	np-h (Theorem 4.2)

snp-h: Strongly NP-hard; np-h: NP-hard

$\alpha_i = \beta_j = 1$ for all a_i and all b_j. The case with $\alpha_i = 1$ for all a_i and $\beta_j = 0$ for all b_j can be proved to be strongly NP-hard as well. Indeed, the latter case corresponds to the well-known Dense-K-Subgraph problem, see (Feige, U., Peleg, D. and Kortsarz, G. (2001)), when each job requires exactly two supportive tasks.

The Observation 1 *Problems* $1|s{-}prec, \alpha_i = \beta_j = 1|\sum U_j$ *and* $1|s{-}prec, \alpha_i = 1, \beta_j = 0|\sum U_j$ *are strongly NP-hard.*

In the following, we consider the case with out-tree precedences. We show that the $1|s{-}prec|\sum U_j$ problem remains strongly NP-hard even when the precedence relation consists of out-trees with $|\mu_i| = 3$ for all tasks a_i and $|\nu_j| = 1$ for all jobs b_j. Under the specified conditions, each job is preceded by exactly one task and the out-trees are disjoint. Our proof is based on a reduction from the "Exact Cover by 3-Sets" problem (Garey, M.R. and Johnson, D.S. (1979)). It will in the mean time strengthen the results of Lenstra and Rinnooy Kan (Lenstra, J.K. and Rinnooy Kan, A.H.G. (1980)).

Exact Cover by 3-Sets: Given a set $T = \{1, \ldots, 3t\}$ and a family $\mathbf{T} = \{T_1, T_2, \ldots, T_r\}$ of three-element subsets of T. Does there exist a subfamily $\mathbf{T}' \subset \mathbf{T}$ such that $|\mathbf{T}'| = t$ and each element of T is involved in exactly one of the subset of $\mathbf{T}'$?

Theorem 4.1 *Problem* $1|s{-}prec|\sum U_j$ *is NP-hard in the strong sense even if* $|\mu_i| = 3, |\nu_j| = 1$, *and* $\alpha_i = \beta_j = 1$ *for all tasks* $a_i \in A$ *and all jobs* $b_j \in B$.

Proof: Given instance I of *Exact Cover by 3-Sets*, we construct instance I' of $1|s{-}prec|\sum U_j$ with r supportive tasks and $N = \sum_{i=1}^{r}\sum_{j\in T_i} t(3t-j+1)$ jobs as follows. For each subset $T_i = \{j_1, j_2, j_3\} \in \mathbf{T}$ with $1 \le j_1 < j_2 < j_3 \le 3t$, define a task a_i that supports (1) $t(3t-j_1+1)$ jobs, denoted by subset $B_{i,j_1} = \{b_{i,j_1,1}, b_{i,j_1,2}, \ldots, b_{i,j_1,t(3t-j_1+1)}\}$, (2) $t(3t-j_2+1)$ jobs, denoted by subset $B_{i,j_2} = \{b_{i,j_2,1}, b_{i,j_2,2}, \ldots, b_{i,j_2,t(3t-j_2+1)}\}$ and (3) $t(3t-j_3+1)$ jobs, denoted by subset $B_{i,j_3} = \{b_{i,j_3,1}, b_{i,j_3,2}, \ldots, b_{i,j_3,t(3t-j_3+1)}\}$. All tasks and all jobs require a unit time for processing. Define due dates $d_j = t + \sum_{l=1}^{j} t(3t-l+1)$ for all $1 \le j \le 3t$. For any i and any $j \in T_i$, the jobs of $B_{i,j} = \{b_{i,j,1}, b_{i,j,2}, \ldots, b_{i,j,t(3t-j+1)}\}$ are associated with due date d_j. To facilitate the following discussion, define an auxiliary due date $d_0 = t$. For different subsets T_i and $T_{i'}$, $B_{i,j} = B_{i',j}$. In the following discussion, if no confusion would arise, we call subset $B_{i,j}$ a B_j-type job set.

With the two instances I and I', we claim that the answer to instance I of Exact Cover by 3-Sets is affirmative if and only if there is a feasible schedule of instance I' with *exactly* $N - \sum_{j=1}^{3t} t(3t-j+1)$ late jobs.

Assume sets $T_1, T_2, \ldots, T_t$ constitute a partition of Exact Cover by 3-Sets. We obtain a feasible schedule with exactly $N - \sum_{j=1}^{3t} t(3t-j+1)$ late jobs as follows. Tasks $a_1, \ldots, a_t$ are arranged in the interval $[0, t)$. After the execution of the t supportive tasks, the available jobs defined by element $j \in T$ are scheduled in the interval $[d_{j-1}, d_j)$. The remaining $r - t$ tasks start at time d_{3t} and the remaining jobs follow. It is easy to see that the derived schedule is feasible and the jobs that start before d_{3t} are early. $(N - \sum_{j=1}^{3t} t(3t-j+1) = N - (d_{3t} - t))$

Assume that there exists a feasible schedule with *exactly* $N - \sum_{j=1}^{3t} t(3t-j+1)$ late jobs. Two basic properties can be easily established: (1) All early jobs precede all late jobs. (2) The early jobs are sequenced in the EDD order.

If, for $0 \le k < t$, $t - k$ tasks are scheduled within $[0, d_{3t}]$, then $\sum_{j=1}^{3t} t(3t-j+1)$ early jobs and k late jobs are scheduled within $[0, d_{3t})$. We analyze the case with $k = 0$. Other cases with $k > 0$ can be similarly discussed. Now, we have exactly t tasks scheduled before d_{3t}. So, every time slot in $[0, d_{3t})$ is occupied by either a task or an early job.

The t tasks allow the processing of exactly $3t$ subsets of jobs, among which some may be defined by the same $j \in T$. Let $x_j (\geq 0)$ denote the number of B_j-type subsets scheduled within $[0, d_{3t})$. We have $\sum_{j=1}^{3t} x_j = 3t$. If $x_1 = x_2 = \cdots = x_{3t} = 1$, then a partition is found.

Assume $x_1 = x_2 = \cdots = x_{3t} = 1$ is not satisfied in the schedule. We want to show that under this assumption, some slots before d_{3t} will not be occupied, implying that the number of early jobs is less than $d_{3t} - t$. Let $\ell, 1 \leq \ell \leq 3t$ be the largest integer such that $x_\ell = 0$. It is clear that $\sum_{j=\ell+1}^{3t} x_j \geq 3t - \ell$, which further implies $\sum_{j=1}^{\ell-1} x_j = \sum_{j=1}^{\ell} x_j \leq \ell$.

Note that $|B_1| > |B_2| > \cdots > |B_{3t}|$ by the definition of instance I'. Assume $\sum_{j=1}^{\ell-1} x_j = \ell - 1 - k$ for $k > 1$. Within interval $[0, d_\ell)$, the number of slots not occupied by the t tasks and the jobs of $B_1 \cap B_2 \cap \cdots \cap B_{\ell-1}$ is no less than $k \times |B_\ell|$. The fact that $\sum_{j=\ell+1}^{3t} x_j = 3t - \ell + k$ implies that no more than $k \times |B_{\ell+1}|$ jobs of $B_{\ell+1} \cap B_{\ell+2} \cap \cdots \cap B_{3t}$ can be allocated to fill up the empty slots before d_ℓ. Since $|B_\ell| > |B_{\ell+1}|$, some slots must be empty. Therefore, $x_1 = x_2 = \cdots = x_{3t} = 1$ must hold and a partition is found.

The following theorem gives the complexity result of another case where each task supports two jobs.

Theorem 4.2 *The* $1|s{-}prec|\sum U_j$ *remains NP-hard even when* $|\mu_i| = 2$ *for all tasks* a_i *and* $|\nu_j| = 1$ *for all jobs* b_j.

Proof: We give the proof by a polynomial-time reduction from the NP-hard Equal-Size-Partition problem.

Equal-Size-Partition: Given a set of $2t$ integers $X = \{1, 2, \ldots, 2t\}$ and, for each i, a size of x_i such that $\sum_{i \in X} x_i = 2M$, does there exist a partition of X into X_1 and X_2 such that $\sum_{i \in X_1} x_i = \sum_{i \in X_2} x_i$ and $|X_1| = |X_2| = t$?

Given an instance I of Equal-Size-Partition, we create an instance I' of problem $1||\sum U_j$ that consists of $2t$ tasks and $4t$ jobs. For each element $i \in X$, define two jobs $b_{i,1}$ and $b_{i,2}$ and their common supportive task a_i. The associated parameters are given as processing time $\alpha_i = \theta^3$; processing time $\beta_{i,1} = \theta + x_i$ and due date $d_{i,1} = t\theta^3 + t\theta + M$; processing time $\beta_{i,2} = \theta^2 - 2x_i$ and due date $d_{i,2} = t\theta^3 + t\theta^2 + t\theta - M$, where θ is an appropriate large positive number.

Denote $B_1 = \{b_{i,1} | 1 \leq i \leq 2t\}$ and $B_2 = \{b_{i,2} | 1 \leq i \leq 2t\}$. To prove the theorem, it suffices to show that if the answer to instance I is affirmative, then there is a feasible solution to instance I' with at most $2t$ late jobs, and vice versa.

Let X_1 and X_2 be a partition of instance I. We schedule the tasks and jobs of instance I' as follows. The sequence starts with t tasks a_i defined by the elements of X_1. The t jobs $b_{i,1}$ released by the t scheduled tasks follow. We then schedule the corresponding t jobs $b_{i,2}$. The remaining tasks and jobs are scheduled late. The t jobs $b_{i,1}$ are completed at time $t\theta^3 + t\theta + \sum_{i \in X_1} x_i = t\theta^3 + t\theta + M = d_{i,1}$. The completion time of the t jobs $b_{i,2}$ is $(t\theta^3 + t\theta + M) + t\theta^2 - 2\sum_{i \in X_1}^{t} x_i = t\theta^3 + t\theta^2 + t\theta - M = d_{i,2}$. Therefore, we obtain a feasible schedule with exactly $2t$ early jobs.

We now consider a schedule of instance I' with no more than $2t$ late jobs. Because of the large number θ^3 in the processing times of tasks $\{a_i\}$ and the due date $d_{i,2}$, no more than t tasks a_i can be completed before $d_{i,2}$. This implies that at most t jobs of B_1 and at most t jobs of B_2 are eligible to be processed as early. Combining the observations with the fact that at least $2t$ jobs are scheduled early in schedule $\sigma(I')$, we conclude that exactly t jobs of B_1 and t jobs of B_2 are early.

By Lemma 4.1, we assume that the jobs of B_1 precede the jobs of B_2. In other words, the schedule starts with t tasks a_i, followed by a block of t jobs of B_1 and a block of t jobs of B_2. We examine the completion times of the $2t$ jobs. The t jobs of B_1 are completed at time $t\theta^3 + t\theta + \sum_{i \in X_1} x_i$, where $X_1 \subset X$ contains the elements defining the t tasks. To ensure the early status of the first t jobs, $t\theta^3 + t\theta + \sum_{i \in X_1} x_i$ must be smaller than or equal to the due date $d_{i,1}$. Thus, we have $\sum_{i \in X_1} x_i \leq M$. Similarly, the completion time of the t early $\{b_{i,2}\}$ jobs is $(t\theta^3 + t\theta + \sum_{i \in X_1} x_i) + t\theta^2 - 2\sum_{i \in X_1} x_i = t\theta^3 + t\theta^2 + t\theta - \sum_{i \in X_1} x_i$, which must be smaller than or equal to the due date $d_{i,2}$. Inequality $\sum_{i \in X_1} x_i \geq M$ follows. The two inequalities lead to $\sum_{i \in X_1} x_i = M$. A desired partition is found.

5 CONCLUSION

This paper proposes a new scheduling problem by introducing supportive tasks, whose completion times are not considered in the objective functions. Two standard min-sum objectives, namely, the total weighted completion time and the number of late jobs, are investigated. We analyzed the complexity status of these objectives under various restrictions. The study extends the relevant results in the literature.

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102 NEW MODEL AND EFFICIENT SOLUTION ALGORITHM FOR TWO-PRODUCT NEWSVENDOR PROBLEM BASED ON THEORY OF COPULAS

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Abstract: Newsvendor Problem has been attracting extensive research interest in the field of inventory man- agement, and found a wide application in the other fields such as production planning and finance management. As an important extension of the newsvendor problem, the model of multi-product newsvendor problem plays an important role on reducing the investment risk of enterprise by providing a suitable product portfolio.

In the existent results on the problem of multi-product newsvendor problem, it is often that the market demands of products are assumed to be mutually independent random variables with identical distributions. With this assumption, it is easy to get the joint distribution function for the random demands in the model of multi-product newsvendor. However, in the many real-world problems, this assumption may not hold since there exists a kind of correlation amongst the productss demands.

In this paper, the theory of Copulas is employed to describe the correlation among the random market demands, and a new optimization model is constructed to formulate the classical multi-product newsvendor problem to find an optimal ordering decision. Because the objective function in this model is in the form of integral, we will first study the differential properties of the objective functions, especially the evaluation of the objective function, the calculation of the derivative. On the basis of this theoretical analysis, an efficient numerical algorithm is developed to solve the original model. Finally, the constructed model and the proposed solution method are applied to solve a multi-product newsvendor problem from the real-world management engineering, the obtained results shows that both the model and the solution method are promising.

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103 A DIAGONAL QUASI NEWTON SPECTRAL CONJUGATE GRADIENT ALGORITHM FOR NONCONVEX UNCONSTRAINED OPTIMIZATION PROBLEMS

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Abstract: In this paper, a diagonal quasi Newton spectral conjugate gradient algorithm is developed for solving nonconvex unconstrained optimization problems. Different from the existent similar methods, the spectral parameter in the new method is a specific diagonal matrix chosen such that it owns the features of quasi-Newton method. The obtained search direction is then a sum of a spectral direction and a conjugate gradient direction, which is proved to be descent. With a modified Armijo-type line search, the global convergence of the algorithm is established. Numerical experiments are employed to demonstrate the efficiency of the algorithm for solving large-scale benchmark test problems.

Key words: Unconstrained optimization; Spectral conjugate gradient method; Quasi Newton method; Global convergence; Inexact line search; Descent algorithm.

1 INTRODUCTION

Consider the following unconstrained optimization problem:

$$\min f(x), \quad x \in \mathbb{R}^n, \tag{1.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable such that its gradient is available. Let $g : \mathbb{R}^n \to \mathbb{R}^n$ denote the gradient function of f, and let g_k denote the value of g at x_k.

Recently, as an improvement of conjugate gradient method, spectral conjugate gradient methods have been attracting extensive research interest, where the features of the well known spectral methods such as the Newton method are incorporated into the determination of search direction with the following form:

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -\theta_k g_k + \beta_k d_{k-1}, & \text{if } k > 0. \end{cases} \tag{1.2}$$

In this paper, we study how to replace θ_k by a suitable diagonal matrix $B_k = \mathrm{diag}(b_k^1, ..., b_k^n)$ such that d_k owns the features of the quasi-Newton direction.

2 NEW DIAGONAL QUASI-NEWTON SPECTRAL CONJUGATE GRADIENT ALGORITHM

Similar to the idea in (Shi Z. J. (2006)) to design a modified quasi-Newton method, we obtain such a matrix B_k by solving a subproblem as follows:

$$\min_{L_k \le b_k^i \le U_k} \frac{1}{2}\|B_k y_{k-1} - s_{k-1}\|^2,$$

where $\|\cdot\|$ is Euclidean norm, L_k and U_k ($L_k < U_k$) are given the upper and the lower bounds of b_k^i for $i = 1, 2, \cdots, n$. To ensure the descent property of $d_k = -B_k g_k$ for the objective function f at x_k, it is required that $0 < L_k \le b_k^i \le U_k$. Thus, a suitable B_k is a solution of the following problem:

$$(SP) \quad \min \|B_k y_{k-1} - s_{k-1}\|^2 = \min_{L_k \le b_k^i \le U_k} \sum_{i=1}^{n} (b_k^i y_{k-1}^i - s_{k-1}^i)^2.$$

We present a method to determine a search direction as follows:

$$d_k = -B_k g_k + \beta_k d_{k-1}, \tag{2.1}$$

where

$$B_k = \begin{cases} \operatorname{diag}(b_k^1, ..., b_k^n), & k > 0, \\ I, & k = 0 \end{cases} \tag{2.2}$$

is a diagonal matrix, whose components are computed by

$$b_k^i = \begin{cases} \dfrac{s_{k-1}^i}{y_{k-1}^i}, & \text{if } y_{k-1}^i \ne 0 \text{ and } L_k \le \dfrac{s_{k-1}^i}{y_{k-1}^i} \le U_k \\ L_k, & \text{if } y_{k-1}^i \ne 0 \text{ and } \dfrac{s_{k-1}^i}{y_{k-1}^i} < L_k \\ U_k, & \text{if } y_{k-1}^i \ne 0 \text{ and } \dfrac{s_{k-1}^i}{y_{k-1}^i} > U_k \\ \dfrac{L_k + U_k}{2}, & \text{if } y_{k-1}^i = 0 \end{cases} \quad i = 1, ..., n. \tag{2.3}$$

$$L_k = c_1 \|g_k\|, \tag{2.4}$$

$$U_k = L_k + c_2, \tag{2.5}$$

and the conjugate parameter

$$\beta_k = \begin{cases} \beta_k^{PRP}, & \text{if } \beta_k^{PRP} g_k^T d_{k-1} < 0, \\ 0, & \text{if } k = 0 \text{ or } \beta_k^{PRP} g_k^T d_{k-1} \ge 0. \end{cases} \tag{2.6}$$

To ensure the global convergence of the presented algorithm, the following line search rule is employed:

$$f(x_k + \alpha_k d_k) \le f(x_k) + \delta_1 \alpha_k g_k^T d_k - \delta_2 \alpha_k^2 \|d_k\|^2. \tag{2.7}$$

With above preparation, a new diagonal quasi-Newton spectral PRP conjugate gradient algorithm (DQNSCG) is presented as follows.

Algorithm 2.1 (DQNSCG):

Step 0. Given constants $0 < \delta_1 < 1$, $\delta_2 > 0$, $c_1 > 0$, $c_2 > 0$ and $\epsilon > 0$. Choose an initial point $x_0 \in \mathbb{R}^n$. Set $k := 0$.

Step 1. If $\|g_k\|_\infty \le \epsilon$, then the algorithm stops. Otherwise, compute d_k by (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6). Go to Step 2.

Step 2. Determine a step length $\alpha_k = \max\{\alpha_l | \alpha_l = \rho^l, l = 0, 1, \cdots,\}$ such that α_k satisfies the following inequality (2.7).

Step 3. Set $x_{k+1} := x_k + \alpha_k d_k$, and $k := k + 1$. Return to Step 1.

In Algorithm 2.1, $\|\cdot\|_\infty$ denotes the infinity norm of a vector, defined by $\|x\|_\infty = \max_{1 \le k \le n} |x_k|$.

3 GLOBAL CONVERGENCE

In this section, we are going to study the global convergence of Algorithm 2.1.

We first state the following mild assumptions.

Assumption 1 The level set $\Omega = \{x \in \mathbb{R}^n \mid f(x) \leq f(x_0)\}$ is bounded.

Assumption 2 In some neighborhood N of Ω, f is continuously differentiable and its gradient is Lipschitz continuous, namely, there exists a constant $L > 0$ such that $\|g(x) - g(y)\| \leq L\|x - y\|, \quad \forall x, y \in N$.

Since $f(x_k)$ is decreasing, it is clear that the sequence $\{x_k\}$ generated by Algorithm 2.1 is contained in a bounded region from Assumption 1. So, there exists a convergent subsequence of $\{x_k\}$. Without loss of generality, it is supposed that $\{x_k\}$ is convergent. On the other hand, from Assumptions 1 and 2, it is easy to see that there is a constant $\gamma_1 > 0$ such that $\|g(x)\| \leq \gamma_1$, $\forall x \in \Omega$. Hence, the sequence $\{ g_k \}$ is bounded.

Proposition 3.1 *Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function. Suppose that d is a descent direction of f at x. Then, there exists a nonnegative integer number j_0 such that*

$$f(x + \alpha d) \leq f(x) + \delta_1 \alpha g^T d - \delta_2 \alpha^2 \|d\|^2, \tag{3.1}$$

where $\alpha = \rho^{j_0}$, g is the gradient vector of f at x, δ_1, $\rho \in (0, 1)$, and $\delta_2 > 0$ are given constant scalars.

Proof Actually, we only need to prove that a step length α is obtained in finitely many steps. If it is not true, then for all sufficient large positive integer m, we have

$$f(x + \rho^m d) - f(x) > \delta_1 \rho^m g^T d - \delta_2 \rho^{2m} \|d\|^2. \tag{3.2}$$

Thus, by the mean value theorem, there is a $\theta \in (0, 1)$ such that

$$\rho^m g(x + \theta\rho^m d)^T d > \delta_1 \rho^m g^T d - \delta_2 \rho^{2m} \|d\|^2. \tag{3.3}$$

It reads $(g(x + \theta\rho^m d) - g)^T d > (\delta_1 - 1) g^T d - \delta_2 \rho^m \|d\|^2$.

When $m \to \infty$, it is obtained that $(\delta_1 - 1) g^T d < 0$. From $\delta_1 \in (0, 1)$, it follows that $g^T d > 0$. This contradicts the condition that d is a descent direction.□

Remark 3.1 *From Proposition 3.1, if d_k is a descent direction, Step 2 of Algorithm 2.1 is well-defined. In addition, by the modified Armjio-type line search (2.7), it is clear that more descent magnitude is required for the obtained step length at each iteration than that by the standard Armjio line search .*

Next, we will prove that d_k generated by (2.1) is actually descent.

Lemma 3.1 *Let d_k be given by (2.1). Then, the following result $g_k^T d_k < 0$ holds for any $k \geq 0$.*

Proof Actually, if $k = 0$, then $g_k^T d_k = -g_0^T d_0 = -\|g_0\|^2 < 0$.

If $k > 0$ and $\beta_k^{PRP} g_k^T d_{k-1} < 0$, then $g_k^T d_k = g_k^T(-B_k g_k + \beta_k^{PRP} d_{k-1}) = -g_k^T B_k g_k + \beta_k^{PRP} g_k^T d_{k-1} < 0$.

Otherwise, $g_k^T d_k = g_k^T(-B_k g_k) = -g_k^T B_k g_k < 0$ since B_k is positive definite and $g_k \neq 0$. □

Lemma 3.2 *Let $\{ \alpha_k \}$ and $\{ d_k \}$ be the sequences of the step length and the search direction generated by Algorithm 2.1, respectively. Then, $\lim\limits_{k\to\infty} \alpha_k^2 \|d_k\|^2 = 0$.*

Proof Firstly, from Assumption 1, there exists $M > 0$ such that $|f(x)| < M$ on the level set Ω.

By (2.7), we know $-\delta_1 \alpha_k g_k^T d_k + \delta_2 \alpha_k^2 \|d_k\|^2 \leq f(x_k) - f(x_{k+1})$. It is obtained that

$\delta_1 \sum\limits_{k=1}^{n} \alpha_k |g_k^T d_k| + \delta_2 \sum\limits_{k=1}^{n} \alpha_k^2 \|d_k\|^2 \leq \sum\limits_{k=1}^{n} (f(x_k) - f(x_{k+1})) = f(x_1) - f(x_{n+1}) \leq 2M.$

It follows that the series $\sum\limits_{k=1}^{\infty} \alpha_k^2 \|d_k\|^2$ is convergent. It yields $\lim\limits_{k\to\infty} \alpha_k^2 \|d_k\|^2 = 0$. The desired result is proved.□

Lemma 3.3 *Let d_k be defined by (2.1). If $\|g_k\| > \epsilon$ for every k, then $\|d_k\|$ is bounded.*

Proof From (2.1), we have

$$\begin{aligned}\|d_k\| &\le \|B_k g_k\| + |\beta_k^{PRP}|\|d_{k-1}\| \\ &\le U_k\|g_k\| + \frac{|g_k^T(g_k - g_{k-1})|}{\|g_{k-1}\|^2}\|d_{k-1}\| \\ &< U_k\|g_k\| + \frac{\|g_k\|L\alpha_{k-1}\|d_{k-1}\|}{\|g_{k-1}\|^2}\|d_{k-1}\| \\ &< (c_1\gamma_1 + c_2)\gamma_1 + \frac{\gamma_1 L\alpha_{k-1}\|d_{k-1}\|}{\epsilon^2}\|d_{k-1}\|.\end{aligned}$$

For $0 < r < 1$, from Lemma 3.2, it follows that $\lim\limits_{k\to\infty} \alpha_{k-1}\|d_{k-1}\| = 0$,
hence there exists $K > 0$, as $k \ge K$, such that $\alpha_{k-1}\|d_{k-1}\| < \dfrac{r\epsilon^2}{\gamma_1 L}$.
Consequently,

$$\begin{aligned}\|d_k\| &< M_1 + r\|d_{k-1}\| \\ &< M_1 + r(M_1 + r\|d_{k-2}\| \\ &= M_1(1+r) + r^2\|d_{k-2}\| \\ &< ... \\ &< M_1(1 + r + ... + r^{k-K-1}) + r^{k-K}\|d_K\| \\ &< \frac{M_1}{1-r} + \|d_K\|,\end{aligned}$$

where $M_1 = (c_1\gamma_1 + c_2)\gamma_1$. Let $M_2 = \max\{\|d_0\|, \|d_1\|, ..., \|d_K\|, \dfrac{M_1}{1-r} + \|d_K\|\}$, then we have $\|d_k\| < M_2$. □

Lemma 3.4 *With Assumption 2, there exists a constant $m > 0$ such that the following inequality*

$$\alpha_k \ge m\frac{|g_k^T d_k|}{\|d_k\|^2} \tag{3.4}$$

holds for all k sufficiently large.

Proof From the line search rule (2.7), we know that $0 < \alpha_k \le 1$.
From the line search rule, it follows that $\rho^{-1}\alpha_k$ does not satisfy the inequality (2.7). So, we have

$$f(x_k + \rho^{-1}\alpha_k d_k) - f(x_k) > \delta_1\alpha_k\rho^{-1}g_k^T d_k - \delta_2\rho^{-2}\alpha_k^2\|d_k\|^2. \tag{3.5}$$

On the other hand,

$$\begin{aligned}f(x_k + \rho^{-1}\alpha_k d_k) - f(x_k) &= \rho^{-1}\alpha_k g(x_k + t_k\rho^{-1}\alpha_k d_k)^T d_k \\ &= \rho^{-1}\alpha_k g_k^T d_k + \rho^{-1}\alpha_k(g(x_k + t_k\rho^{-1}\alpha_k d_k) - g_k)^T d_k \\ &\le \rho^{-1}\alpha_k g_k^T d_k + L\rho^{-2}\alpha_k^2\|d_k\|^2,\end{aligned} \tag{3.6}$$

where $t_k \in (0,1)$ satisfies $x_k + t_k\rho^{-1}\alpha_k d_k \in N$.
Combined with (3.5) and (3.6), it is obtained that $\delta_1\alpha_k\rho^{-1}g_k^T d_k - \delta_2\rho^{-2}\alpha_k^2\|d_k\|^2 < \rho^{-1}\alpha_k g_k^T d_k + L\rho^{-2}\alpha_k^2\|d_k\|^2$.
It reads $(1-\delta_1)\alpha_k\rho^{-1}g_k^T d_k + (L+\delta_2)\rho^{-2}\alpha_k^2\|d_k\|^2 > 0$,
i.e. $(L+\delta_2)\rho^{-1}\alpha_k\|d_k\|^2 > (\delta_1 - 1)g_k^T d_k$.
Therefore, $\alpha_k > \dfrac{(\delta_1-1)\rho g_k^T d_k}{(L+\delta_2)\|d_k\|^2}$. From Lemma 3.1, it follows that $\alpha_k > \dfrac{(1-\delta_1)\rho|g_k^T d_k|}{(L+\delta_2)\|d_k\|^2}$.
Let $m = \dfrac{\rho(1-\delta_1)}{L+\delta_2}$. Then the desired inequality (3.4) holds. □
Next, we are going to prove the Zoutendijk condition, originally given in (Zoutendijk G.(1970)), also holds for Algorithm 2.1.

Lemma 3.5 *Under Assumptions 1 and 2, it holds that* $\sum\limits_{k=0}^{\infty} \dfrac{|g_k^T d_k|^2}{\|d_k\|^2} < \infty$.

Proof From the line search rule (2.7) and assumption 1, there exists a constant M such that

$$\sum_{k=0}^{n-1}(-\delta_1\alpha_k g_k^T d_k + \delta_2\alpha_k^2\|d_k\|^2) \le \sum_{k=0}^{n-1}(f(x_k) - f(x_{k+1})) = f(x_0) - f(x_n) < 2M.$$

Then, from Lemmas 3.1 and 3.4, we have

$$\begin{aligned} 2M \geq & \sum_{k=0}^{n-1}(-\delta_1\alpha_k g_k^T d_k + \delta_2\alpha_k^2\|d_k\|^2) \\ = & \sum_{k=0}^{n-1}(\delta_1\alpha_k|g_k^T d_k| + \delta_2\alpha_k^2\|d_k\|^2) \\ \geq & \sum_{k=0}^{n-1}(\delta_1 m\frac{|g_k^T d_k|}{\|d_k\|^2}|g_k^T d_k| + \delta_2\cdot m^2\cdot\frac{|g_k^T d_k|^2}{\|d_k\|^4}\cdot\|d_k\|^2) \\ = & (\delta_1+\delta_2 m)m\sum_{k=0}^{n-1}\frac{|g_k^T d_k|^2}{\|d_k\|^2}. \end{aligned}$$

Thus, the desired conclusion is proved.□

With the above preparation, we are in a position to state the main result in this paper.

Theorem 3.1 *Let* $\{g_k\}$ *be the sequence of the gradient values at the iterate points generated by Algorithm 2.1. Under Assumptions 1 and 2, the following result* $\liminf_{k\to\infty}\|g_k\| = 0$ *holds.*

Proof For arbitrary $\epsilon > 0$, suppose that $\|g_k\| > \epsilon$ for each k. From Lemma 3.3, it is obtained that

$$\begin{aligned} g_k^T d_k &= g_k^T(-B_k g_k + \beta_k d_{k-1}) \\ &= -g_k^T B_k g_k + \beta_k g_k^T d_{k-1} \\ &\leq -\|g_k\|\|B_k\|\|g_k\| + \frac{\|g_k\|\|g_k - g_{k-1}\|\|g_k\|}{\|g_{k-1}\|^2}\|d_{k-1}\| \\ &< -U_k\|g_k\|^2 + \frac{\|g_k\|^2 L\alpha_{k-1}\|d_{k-1}\|}{\epsilon^2}\|d_{k-1}\| \\ &< -U_k\|g_k\|^2 + \frac{\|g_k\|^2 L\alpha_{k-1}\|d_{k-1}\|}{\epsilon^2}M_2 \\ &< -(c_1\gamma_1 + c_2)\|g_k\|^2 + \frac{L\alpha_{k-1}\|d_{k-1}\|}{\epsilon^2}M_2\|g_k\|^2 \\ &= -M_3\|g_k\|^2 + \frac{L\alpha_{k-1}\|d_{k-1}\|}{\epsilon^2}M_2\|g_k\|^2, \end{aligned}$$

where $M_3 = c_1\gamma_1 + c_2 > 0$.

By Lemma 3.2, $\lim_{k\to\infty}\alpha_{k-1}\|d_{k-1}\| = 0$,for k large enough, we have $\frac{L\alpha_{k-1}\|d_{k-1}\|}{\epsilon^2}M_2 < \frac{M_3}{2}$

It follows that $g_k^T d_k < -c\|g_k\|^2$ where $c = \frac{M_3}{2} > 0$. i.e. $|g_k^T d_k| > c\|g_k\|^2$.

By Lemma 3.5 and 3.3, we have $\frac{c^2\|g_k\|^4}{M_2^2} < \frac{|g_k^T d_k|^2}{\|d_k\|^2} \to 0$,

Hence $\lim_{k\to\infty}\|g_k\| = 0$, which means there exists $k_1 > 0$, as $k > k_1$, $\|g_k\| < \epsilon$. This contradicts to the assumption $\|g_k\| > \epsilon, \forall\epsilon > 0$. It says that $\liminf_{k\to\infty}\|g_k\| = 0$. □

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104 MODIFIED NONMONOTONE BFGS ALGORITHM FOR SOLVING SMOOTH NONLINEAR EQUATIONS

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Abstract: In this paper, a modified nonmonotone BFGS algorithm is developed for solving a smooth system of nonlinear equations. Different from the existent techniques of nonmonotone line search, the value of an algorithmic parameter controlling the magnitude of nonmonotonicity is updated at each iteration by the known information of the system of nonlinear equations such that the numerical performance of the developed algorithm is improved. Under some suitable assumptions, the global convergence of the algorithm is established for solving a generic nonlinear system of equations. Implementing the algorithm to solve some benchmark test problems, the obtained numerical results demonstrate that it is more effective than some similar algorithms available in the literature.

Key words: smooth nonlinear equations; nonmonotone technique; modified BFGS algorithm; global convergence

1 INTRODUCTION

Consider a system of smooth nonlinear equations:

$$F(x) = 0, \tag{1.1}$$

where $F : \mathbf{R}^n \to \mathbf{R}^n$ is assumed to be a continuously differentiable function.

Theoretically and algorithmically, it is shown that the quasi-Newton method is a fundamental and efficient method to compute a solution of Equation (1.1). For recent advances in this research field, one can see, for example, (Gu G.Z. (2003), Li D.H. (2001), Li D.H. (1999), Li D.H. (2007), Liu J.G. (2004), Zhou W.J. (2009)) and the references therein.

In (Gu G.Z. (2003)), a modified BFGS formula is proposed for solving a system of symmetric nonlinear equations. Different from the standard BFGS method, B_k constructed by this method can inherit the positive definiteness of B_{k-1} for all $k \geq 1$. In (Zhang H.C. (2004)), a nonmonotone line search technique is firstly proposed for solving unconstrained optimization problems, where η_k(See (2.7)) takes a fixed value at each iteration.

In this paper, taking the fundamental role of η_k in controlling the magnitude of nonmonotonicity into account, we are going to update the value of η_k at each iteration by the known information of the system function such that the efficiency of algorithm is improved. In this paper, we shall extend a modified BFGS formula (Gu G.Z. (2003)) to solve a generic smooth system of nonlinear equations by incorporating this new nonmonotone line search method.

The rest of this paper is organized as follows. In the next section, we will present a new modified BFGS method for solving a system of nonlinear equations. By incorporating an improved line search technique, a new BFGS algorithm will be developed. In Section 3, the global convergence of the proposed method is proved. The results of numerical experiment will be reported in Section 4. Final remarks will be given in the last section.

2 MODIFIED NONMONOTONE BFGS ALGORITHM

In this section, a modified nonmonotone BFGS algorithm for solving nonlinear system of equations is developed in detail. We first address how to determine the search direction at each step of the algorithm.

Recall that a standard BFGS formula reads

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k}.$$

It is shown that B_{k+1} is positive definite with a positive definite matrix B_k in the case that $y_k^T s_k > 0$. If we use BFGS method to solve an unconstrained optimization problem, the condition $y_k^T s_k > 0$ is guaranteed by employing the Wolfe line search rule. However, for a nonlinear system equations, the Wolfe line search is no longer popular since it needs to compute the gradient of F, i.e., its Jacobian matrix. In (Gu G.Z. (2003)), a modified BFGS formula is proposed where y_k is replaced by $\bar{y}_k$, which is defined by

$$\bar{y}_k = y_k + \left(\max\left\{0, -\frac{y_k^T s_k}{\|s_k\|^2}\right\} + \phi(\|F(x_k)\|) \right) s_k, \tag{2.1}$$

where $s_k = x_{k+1} - x_k$, $y_k = F(x_{k+1}) - F(x_k)$, and $\phi : \mathbf{R} \to \mathbf{R}$ is a function given by

$$\phi(t) = \begin{cases} 10^{-5} t^2 & \text{if } t \leq 1, \\ 10^{-5} t^{0.1} & \text{otherwise.} \end{cases} \tag{2.2}$$

It is not difficult to show that $\bar{y}_k$ satisfies

$$\bar{y}_k^T s_k \geq \phi(\|F(x_k)\|)\|s_k\|^2 > 0.$$

Consequently, if B_{k+1} is taken as:

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{\bar{y}_k \bar{y}_k^T}{\bar{y}_k^T s_k}, \tag{2.3}$$

then it is clear that the obtained B_{k+1} inherits the positive definiteness of B_k. Thus, a descent search direction d at x_k is obtained by solving a linear system equations

$$B_k d = -F(x_k). \tag{2.4}$$

Define a merit function $f : R^n \to R$ given by

$$f(x) = \frac{1}{2}\|F(x)\|^2 \tag{2.5}$$

then the linear equations (1.1) is equivalent to the following global optimization problem (2.5).

Next, we come to state a new nonmonotone line search strategy. Different from the method proposed in (Zhang H.C. (2004)), we shall update the parameter η_k, which plays fundamental role in controlling the magnitude of nonmonotonicity, by employing the known information of nonlinear system.

Let $C_0 = f(x_0)$, $Q_0 = 1$, $\alpha_0 = 1$, $0 < \rho < 1$, $\sigma \in (0, 1)$. Our nonmonotone line search is to find a stepsize α_k satisfying

$$f(x_k + \alpha_k d_k) \leq C_k + \sigma \alpha_k d_k^T B_k F(x_k), \tag{2.6}$$

where

$$\begin{aligned} \alpha_k &= \alpha_0 \rho^{h_k}, \\ C_k &= \frac{\eta_{k-1} Q_{k-1} C_{k-1} + f(x_k)}{Q_k}, \\ Q_k &= \eta_{k-1} Q_{k-1} + 1, \end{aligned} \tag{2.7}$$

h_k is the smallest integer such that $\alpha = \alpha_k$ satisfies (2.6), and

$$\eta_k = \frac{\|F(x_{k+1})\|}{\|F(x_{k+1}) - F(x_k)\| + \|F(x_k)\|}. \tag{2.8}$$

Remark 2.1 *In (Zhang H.C. (2004)), η_k takes a fixed value at each iteration. However, in this paper, with the adjustable parameter η_k in (2.8), it is possible to improve the efficiency of Algorithm 2.1.*
It is easy to see that η_k in (2.8) satisfies $0 \leq \eta_k \leq 1$.

Remark 2.2 *From (2.7), it follows that C_{k+1} is a convex combination of the known values of C_k and $f(x_{k+1})$. If $\eta_k = 0$ for some k, then x_{k+1} is a solution of Problem (1.1). If $\eta_k = 1$ for some k, then*

$$C_k = A_k = \frac{1}{k+1}\sum_{i=0}^{k} f(x_i)$$

is the average function value. Thus, the line search is similar to that in (Zhang H.C. (2004)).

Now, we are in a position to present an overall framework of our algorithm.

Algorithm 2.1 (New Modified Nonmonotone BFGS Algorithm): **Step 0.** Choose an initial point $x_0 \in \mathbf{R}^n$. Take the values of the parameters $\varepsilon > 0$, $0 < \sigma < 1$, $0 < \rho < 1$. Set $k := 0$.

Step 1. If $\|F(x_k)\| \leq \varepsilon$, the algorithm stops. Otherwise, go to Step 2.

Step 2. Find d_k which solves the following linear equations $B_k d = -F(x_k)$ in the unknown vector $d \in R^n$.

Step 3. Determine a step length α_k by (2.6)-(2.8).

Step 4. Set $x_{k+1} := x_k + \alpha_k d_k$. Update B_k as B_{k+1} by (2.3).

Step 5. Set $k := k+1$. Return to Step 1.

3 GLOBAL CONVERGENCE

In this section, we analyze the global convergence of Algorithm 2.1. We first make the following assumptions, which are also used to establish the global convergence of BFGS algorithm in many relevant literatures.

Assumption 3.1 *The level set $\Omega = \{x \in \mathbf{R}^n | f(x) \leq f(x_0)\}$ is bounded*

Assumption 3.2 *$F : R^n \to R^n$ is continuously differentiable on an open convex set Ω_1 containing Ω, and there exists $x_* \in \mathbf{R}^n$ such that $F(x_*) = 0$.*

Assumption 3.3 *There exists a positive constant scalar msuch that $\lambda_k^{min} > m$, where λ_k^{min} is the minimal eigenvalue of B_k.*

Assumption 3.4 *B_k is a good approximation to $F'(x_k)$,i.e. $\|[F'(x_k) - B_k]d_k\| \leq \epsilon \|F(x_k)\|$, where ϵ is a positive constant scalar and $\epsilon \in (0, 1)$.*

Lemma 3.1 *Let $\{x_k\}$, $\{B_k\}$, $\{d_k\}$ be sequences generated by Algorithm 2.1. Then*

$$d_k^T B_k F(x_k) = -\|F(x_k)\|^2 < 0. \tag{3.1}$$

Lemma 3.2 *Let $\{x_k\}$, $\{f(x_k)\}$, $\{C_k\}$, $\{A_k\}$ be sequences generated by Algorithm 2.1. Then,*

$$f(x_k) \leq C_k \leq A_k. \tag{3.2}$$

Lemma 3.3 *Let $\{x_k\}$ be a sequence generated by Algorithm 2.1. Then*

$$x_k \in \Omega,\ \forall k \in \{1, 2, \cdots\}.$$

Lemma 3.4 *Let $\{x_k\}$ be a sequence generated by Algorithm 2.1 and $F(x_k) \neq 0$. If Assumption 4 holds, then,*

$$F(x_k)^T F'(x_k) d_k \leq -(1-\epsilon)\|F(x_k)\|^2. \tag{3.3}$$

Lemma 3.5 *Let Assumptions 1,2 and 4 hold. Then Algorithm 2.1 produces an iterate $x_{k+1} = x_k + \alpha_k d_k$ in a finite number of backtracking steps for a suitable σ.*

Theorem 3.5 *Let $\{\alpha_k\}$ be the steplength sequence generated by Algorithm 2.1. With Assumptions 3.1, 3.2 and 3.3, the following inequality*

$$\alpha_k \geq \min\left\{\alpha_0\rho, \frac{2(1-\epsilon-\sigma)\rho m^2}{M^2}\right\} \tag{3.4}$$

holds for k sufficiently large.

Corollary 3.1 *Let $\{x_k\}$, $\{C_k\}$ and $\{F(x_k)\}$ be sequences generated by Algorithm 2.1. Under Assumptions 3.1, 3.2 and 3.3, the inequality*

$$f(x_{k+1}) \leq C_k - c\|F(x_k)\|^2 \tag{3.5}$$

holds, where

$$c = \min\left\{\sigma\nu\rho, \frac{2(1-\epsilon-\sigma)\sigma\rho m^2}{M^2}\right\}$$

is a constant scalar.

Theorem 3.6 *Let $\{x_k\}$, $\{F(x_k)\}$ be sequences generated by Algorithm 2.1. Under Assumptions 3.1, 3.2 and 3.3, the result*

$$\lim_{k\to\infty} \inf \|F(x_k)\| = 0 \tag{3.6}$$

is true.

4 NUMERICAL EXPERIMENTS

In this section, we test Algorithm 2.1 by solving five benchmark problems, four of them are nonsymmetric equations and one is symmetrical.

Take $\varepsilon = 10^{-5}$, $\sigma = 0.25$ and $\rho = 0.01$. For given initial values, we obtain the solutions of these problems. For the following problems 6-4, which are nonsymmetric system of nonlinear equations, the numerical performance of Algorithm is reported in Table 1. For a symmetric problem 5 that has been solved in Gu G.Z. (2003), the numerical performance of Algorithm 2.1 is reported in Table 2.

Problem 1 *Generalized function of Rosenbrock (Luksan L. (1994)).*

Problem 2 *Five-diagonal system Luksan L. (1994).*

Problem 3 *Trigonometric-Exponential System Luksan L. (1994).*

Problem 4 *Tridiagonal System Luksan L. (1994).*

Problem 5 *The discretized two-point boundary value problem (Gu G.Z. (2003),Li D.H. (1999)).*

In the above table, the following notations are used.
Pro: the problem;
Dim: the dimension of the problem;
x_i^0: the initial point, $i = 1, 2, \cdots, n$;
x_i^*: the optimal solutions, $i = 1, 2, \cdots, n$;
k/N: the number of the iterations/the number of function evaluations;
$Algo$: the algorithms
$Fnorm$: the final value of$\|F(x_k)\|$
NEW: the new algorithm;
$-$: unsuccessfully obtaining the optimal solution;
$\eta_k = 0.2$, $\eta_k = 0.25$, ..., $\eta_k = 0.85$: the numerical performance of the nonmonotone line search strategy for fixed value of η_k;
$DBFGS$ and $CBFGS$: the algorithm of Gu G.Z. (2003).

The results in Table 1 indicate that the computational efficiency of the developed algorithm is better than that of the existing method.

5 FINAL REMARKS

We have presented a new nonmonotone modified BFGS algorithm for solving smooth nonlinear equations by incorporating a new nonmonotone line search technique. We have established the theory of global convergence for the proposed method. Numerical experiments demonstrated that the developed algorithm is more effective than the similar algorithms available in the literature.

Table 4.1 Comparison with different strategies of η_k

Pro	*Dim*	$\eta_k = 0.2$	$\eta_k = 0.25$	$\eta_k = 0.5$	$\eta_k = 0.75$	$\eta_k = 0.85$	*NEW*	x_i^0	x_i^*
Problem 1	10	20/33	20/33	20/33	21/32	22/34	20/32	1.2	1
	50	76/145	76/145	76/145	76/143	76/142	76/142		
	100	139/273	139/272	139/271	139/270	139/270	139/270		
	150	207/408	207/408	207/405	210/402	204/393	204/398		
	200	268/530	268/530	268/528	268/526	268/526	268/528		
Problem 2	10	86/154	85/149	–	–	–	69/116	-2	1
	50	191/358	190/354	180/329	–	–	157/275		
	100	312/621	312/618	301/582	290/559	275/520	262/488		
	150	371/734	371/734	370/739	337/654	339/655	323/617		
	200	439/889	439/887	439/876	410/815	401/789	394/767		
Problem 3	10	36/56	36/56	35/54	36/53	36/53	36/53	0	1
	50	104/194	104/194	104/194	104/194	104/194	103/190		
	100	187/362	187/362	187/362	187/362	187/362	189/359		
	150	270/531	270/531	270/531	270/531	270/531	270/520		
	200	353/697	353/697	353/697	353/697	352/691	354/684		
Problem 4	10	–	–	548/1048	85/103	85/103	389/722	8	1
	50	648/1277	653/1285	588/1108	564/1025	225/300	506/900		
	100	775/1548	736/1456	728/1397	704/1311	334/479	561/990		
	150	863/1719	856/1703	1165/2367	959/1834	949/1811	698/1267		
	200	958/2140	–	1398/ 2930	920/1764	839/1604	622/1106		

Table 4.2 Comparison between DBFGS and CBFGS

Dim	*Algo*	k	N	*Fnorm*	*Dim*	k	N	*Fnorm*	x_i^0
10	DBFGS	128	144	9.83e-05	50	2131	2177	9.96e-05	10
	CBFGS	127	159	9.69e-05		2416	2450	9.98e-05	
	NEW	11	14	1.87e-05		104	186	6.23e-05	
	DBFGS	-	-	-		2121	2164	9.99e-05	-10
	CBFGS	138	170	9.42-05		1984	2062	9.96e-05	
	NEW	11	14	2.04e-05		98	173	7.99e-05	
	DBFGS	155	179	9.54e-05		2742	2798	9.99e-05	100
	CBFGS	145	219	9.64e-05		2534	2622	9.98e-05	
	NEW	12	15	7.63e-05		112	200	3.88e-05	
	DBFGS	157	173	9.53e-05		2735	2791	9.96e-05	-100
	CBFGS	164	222	9.31e-05		2677	2772	1.00e-04	
	NEW	12	15	7.17e-05		120	217	7.54e-05	
	DBFGS	179	200	9.55e-05		3410	3461	9.97e-05	1000
	CBFGS	171	252	9.90e-05		3021	3067	9.97e-05	
	NEW	13	16	4.63e-06		120	215	5.28e-05	
	DBFGS	184	212	9.88e-05		3337	3420	9.98e-05	-1000
	CBFGS	165	202	9.36e-05		3382	3419	1.00e-05	
	NEW	13	16	1.39e-05		121	217	4.36e-05	

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105 A NUMERICAL APPROACH TO OPTIMAL CONTROL PROBLEMS WITH SMOOTH CONTROL

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Abstract: In this research, we consider a class of optimal control problems involving inequality continuous-state constraints in which the control is smooth. The requirement for this type of control is more stringent than that for the control considered in standard optimal control problems in which the controls are usually taken as bounded measurable function. In this research, we give control parametrization by using quadratic B-spline functions. We shall then use it to devise a computational algorithm for solving this equivalent dynamic optmization problem. Furthermore, convergence analysis is will be given to support this numerical approach. For illustration, two nontrivial optimal control problems, involving transferring cargo via a container crane will be solved using the proposed approach.

Key words: Optimal control; Smooth control, Spline function, Optimal parameter selection problem, Safety requirement.

1 PROBLEM STATEMENT

Consider a system described by the following state differential equations defined on the fixed time interval (0, T]:

$$\dot{x}(t) = f(t, x(t), u(t)) \tag{1.1a}$$

where

$$x = [x_1, \dots, x_n]^T \in \mathbb{R}^n, \quad u = [u_1, \dots, u_r]^T \in \mathbb{R}^r$$

are, respectively, the state and control vectors; and $f = [f_1, ..., f_n]^T \in \mathbb{R}^n$ is a given real-valued function.

The initial condition for the difierential equation (1.1) is:

$$x(0) = x^0 \tag{1.1b}$$

where $x^0 = [x_1^0, \dots, x_n^0]$ is a given vector. Define

$$U = \{v = [v_1, \dots, v_r]^T \in \mathbb{R}^r : \ \alpha_i \le v_i \le \beta_i, \ i = 1, \dots, r\} \tag{1.2}$$

where α_i, $i = l, \dots, r$, and β_i, $i = l, \dots, r$, are given real numbers. Note that U is a compact and convex subset of $\mathbb{R}^r$.

Definition 1.1 *A function $u : [0,T] \to \mathbb{R}^r$ is said to be smooth if it is continuously differentiable on $[0,T]$.*

Let u be a smooth function defined on $[0,\ T]$ with values in U, and let $\dot{u}$ denote the derivative of u. If

$$c_i \le \dot{u}_i(t) \le d_i, \quad \forall t \in [0,T], \quad i = 1,\dots,r, \tag{1.3}$$

where $c_i,\ i = l,\dots,r$, and $d_i,\ i = 1,\dots,r$, are given real numbers, then the u is called an admissible control. Let $\mathscr{U}$ be the class of all such admissible controls. Furthermore, let $\mathring{\mathscr{U}}$ be a subset of the set $\mathscr{U}$ defined by

$$\mathring{\mathscr{U}} = \{u \in \mathscr{U} :\ \alpha_i < u_i(t) < \beta_i,\ \forall t \in [0,T],\ i = 1\dots,r\} \tag{1.4}$$

Note that the derivative $\dot{u}$ of u is, in fact, only defined almost everywhere in $[0,T]$. However, we may assign appropriate values for the function $\dot{u}$ at those points in $[0,T]$ at which the functional is not defined so that the extended function satisfies the condition (1.3). Throughout remainder of this research, the function $\dot{u}$ is to be understood that it is in its extended form. Note also that the class $\mathscr{U}$ of admissible controls considered in this section is more restrictive than that considered in [K.L. Teo and L.S. Jennings], where bounded measurable functions are taken as admissible controls. For each $u \in \mathscr{U}$, let $x(\cdot|u)$ be the corresponding solution of the system (1.1). The inequality terminal state constraints and inequality continuous state constraints are specified as follows:

$$\Phi_i(x(T|u)) \ge 0,\ i = 1,\dots,N_T \tag{1.5}$$

where $\Phi_i,\ i = 1,\dots,N_T$, are given real-valued functions defined on $\mathbb{R}^n$, and

$$h_i(t, x(t|u), u(t)) \ge 0, \quad \forall t \in [0,T], \quad i = 1,\dots,N_S \tag{1.6}$$

where $h_i,\ i = 1,...,N_S$, are given real valued functions defined on $[0,T] \times \mathbb{R}^n \times \mathbb{R}^r$. Note that the admissible control is required to be smooth. Thus, it is allowed to appear in the inequality continuous state constraints (1.6). This is a slight generalization of that considered in [K.L. Teo and L.S. Jennings]. Define

$$\Theta = \{u \in \mathscr{U} :\ \Phi_i(x(T|u)) \ge 0,\ i = 1,\dots,N_T\} \tag{1.7}$$

and

$$\mathcal{F} = \{u \in \Theta :\ h_i(t, x(t|u), u(t)) \ge 0,\ \forall \in [0,T],\ i = 1,\dots,N_S\} \tag{1.8}$$

Elements from $\mathcal{F}$ are called feasible controls, and $\mathcal{F}$ is called the class of feasible controls. We may now state the optimal control problem as follows:

Problem (P). Given the system (1.1), find control $u \in \mathcal{F}$ such that the cost functional

$$g_0(u) = \Phi_0(x(T|u)) + \int_0^T \mathcal{L}_0(t, x(t|u), u(t))dt \tag{1.9}$$

is minimized over $\mathcal{F}$, where Φ_0 and $\mathcal{L}_0$ are given real valued functions, and T is the terminal time of the problem. The following conditions are assumed throughout:

(A1) $f : [0,\ T] \times \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$ is piecewise continuous on $[0,\ T]$ for each $(x,u) \in \mathbb{R}^\times \mathbb{R}^r$, and continuously differentiable with respect to each of the components of x and u for each $t \in [0,\ T]$; and furthermore, for any given compact subset $C \subset \mathbb{R}^r$, there exists a constant $K > 0$ such that

$$|f(t,x,u)| \le K(1 + |x|)$$

fopr all $(t,x,u) \in [0,\ T] \times C \times \mathbb{R}^n$, where $|\cdot|$ denotes the usual Euclidean norm;

(A2) For each $i = 1,\dots,N_T$, $\Phi_i : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable;

(A3) For each $i = 1,\dots,N$, $h_i : [0,T] \times \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}$ is continuously differentiable;

(A4) $\Phi_0 : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable;

(A5) $\mathcal{L}_0 : [0,T] \times \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}$ is piecewise continuous on $[0,T]$ for each $(x,u) \in \mathbb{R}^n \times \mathbb{R}^r$, and continuously differentiable with respect to each of the components of x and u for each $t \in [0,\ T]$. Define

$$\Theta = \{u \in \mathscr{U} :\ \Phi_i(x(T|u)) \ge 0,\ i = 1,\dots,N_T\} \tag{1.10}$$

and

$$\mathscr{F} = \{u \in \Theta : h_i(t, x(t|u), u) \geq 0,\ \forall t \in [0, T],\ i = 1, \ldots, N\}. \tag{1.11}$$

Let $\mathring{\Theta}$ and $\mathring{\mathscr{F}}$ be, respectively, subsets of the set Θ and $\mathscr{F}$ defined by

$$\mathring{\Theta} = \{u \in \mathring{\mathscr{U}} :\ \Phi_i(x(T|u)) > 0,\ i = 1, \ldots, N\} \tag{1.12}$$

To continue, we assume that the following condition is satisfied.

(A6) For any $u \in \mathscr{F}$, there exists a $\bar{u} \in \mathring{\mathscr{F}}$ such that

$$\alpha\bar{u} + (1-\alpha)u \in \mathring{\mathscr{F}} \quad \forall \alpha \in (0,\ 1].$$

2 SPLINE FUNCTIONS

Definition 2.1 *Let m be integer satisfying $m \geq 0$ and let $S = \{t_k\}_{k\in\mathbb{Z}}$ be sequence of nondecreasing real numbers t_k of length at least $m+2$. The k-th basis B-spline of degree m with knots S is defined recursively by*

$$\Omega_{k,m}(t) = \frac{t - t_k}{t_{k+m} - t_k}\Omega_{k,m-1}(t) + \frac{t_{k+m+1} - t}{t_{k+m+1} - t_{k+1}}\Omega_{k+1,m-1}(t)$$

for all $m \geq 1$ and

$$\Omega_{k,0}(t) = \begin{cases} 1, & \text{if } t_k \leq t < t_{k+1} \\ 0, & \text{otherwise} \end{cases}$$

for $m = 0$. Here, for convenience it is assumed that $0/0 = 0$.

Definition 2.2 *Let $\{t_k\}_{k=1}^{N+m+1}$ be a nondecreasing sequence of real numbers and let w be a linear combination of B-splines $\Omega_{1,m}, \ldots, \Omega_{N,m}$, i.e.*

$$w(\sigma, t) = \sum_{k=1}^{N} \sigma^k \Omega_{k,m}(t), \tag{2.1}$$

where $\sigma \in \mathbb{R}^N$ and $\sigma = (\sigma^1, \ldots, \sigma^N)$. Then it is called as a spline function, or spline of degree m with knots $\{t_k\}_{k=1}^{N+m+1}$ and $\{\sigma^k\}_{k=1}^{N}$ are called the B-spline coefficients of it.

Bounds on spline function and its derivative

Consider spline function w with knots $\{t_k\}_{k=1}^{N+m+1}$ defined by the following

$$w(\sigma, t) = \sum_{k=1}^{N} \sigma^k \Omega_{k,2}(t). \tag{2.2}$$

Lemma 2.1 *T.Lyche et al Spline function w given by (2.2) is bounded by its smallest and largest B-spline coefficients, i.e. it holds*

$$\min_{1\leq k\leq N} \sigma^k \leq w(\sigma, t) \leq \max_{1\leq k\leq N} \sigma^k \quad \forall t \in [0, T]. \tag{2.3}$$

Corollary 1 *Let α and β be real numbers satisfying $\alpha \leq \beta$. For spline function w given by (2.2) if inequalities*

$$\alpha \leq \sigma^k \leq \beta \tag{2.4}$$

are true for all $k = 1, \ldots, N$, then it holds

$$\alpha \leq w(\sigma, t) \leq \beta \quad \forall t \in [0, T]. \tag{2.5}$$

Lemma 2.2 *Let c and d be real numbers satisfying $c \leq d$. For spline function w given by (2.2) if inequalities*

$$\frac{c}{m}(t_{k+m} - t_k) \leq \sigma^k - \sigma^{k-1} \leq \frac{d}{m}(t_{k+m} - t_k) \tag{2.6}$$

are true for all $k = 2, \ldots, N$, then it holds

$$c \leq w'(\sigma, t) \leq d \quad \forall t \in [0, T]. \tag{2.7}$$

3 CONTROL PARAMETRIZATION

In this section, our aim is to convert the problem (P) into a form solvable by the optimal control software MISER. We look for smooth optimal control. Let S^p denote set of $n_p + 5$ knot points defining partition on interval $[0, T]$ and satisfying the following conditions

$$t^p_{-1} < t^p_0 < \ldots < t^p_{n_p+3} \qquad (3.1)$$
$$t_1 = 0,\ t_{n_p} = T$$

for all $p \in \mathbb{N}$. Notice that, a quadratic spline function is continuously differentiable, when knots points are strictly increasing and we assume it in (3.1). Then each control is approximated by quadratic spline function with knots S^p as it follows:

$$u(t) = w(\sigma^p, t) = \sum_{k=-1}^{n_p} \sigma^{p,k}\Omega_{k,2}(t) \quad \forall t \in [0, T], \qquad (3.2)$$

where $\sigma^{p,k} \in \mathbb{R}^r$ are parameters to be determined optimally and their expansion is given by

$$(\sigma^{p,k})^\top = [\sigma^{p,k}_1, \ldots, \sigma^{p,k}_r]$$

for all $k = 1, \ldots, n_p$. Moreover, σ^p denotes a matrix composed of these parameters, i.e.

$$\sigma^p = [(\sigma^{p,1})^\top, \ldots, (\sigma^{p,n_p})^\top]^\top.$$

In view of the definition of $\mathscr{U}$, the following constraints must be satisfied

$$\alpha_i \le u_i(t) \le \beta_i, \quad i = 1, \ldots, r, \qquad (3.3)$$

and

$$c_i \le \dot{u}_i(t) \le d_i, \quad \forall t \in [0,\ T], \quad i = 1, \ldots, r. \qquad (3.4)$$

Control approximation (3.2) gives relaxation to these conditions. From Corrollary 1 it implies that inequality (3.3) is true, if it holds

$$\alpha_i \le \sigma^{p,k}_i \le \beta_i$$

for all $k = -2, \ldots, n_p$ and $i = 1, \ldots r$. Due to Lemma 2.2, the inequality (3.4) is relaxed in terms of knot points S^p as it follows

$$\frac{c_i}{3}\left(t^p_{k+3} - t^p_k\right) \le \sigma^{p,k}_i - \sigma^{p,k-1}_i \le \frac{d_i}{3}\left(t^p_{k+3} - t^p_k\right) \qquad (3.5)$$

for all $i = 1, \ldots, r$ and $k = 2, \ldots, n_p$. Parametrization given by (3.2) implies that

$$\dot{x}(t) = f(t, x(t), w(\sigma^p, t)) \qquad (3.6a)$$

where

$$x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n, \quad w(\sigma^p, t) = \begin{pmatrix} w_1(\sigma^p_1, t) \\ \vdots \\ w_r(\sigma^p_r, t) \end{pmatrix}, \qquad (3.6b)$$

where σ^p_i denotes i-th row of the matrix σ^p for all $i = 1, \ldots, r$.

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106 A PROXIMITY ALGORITHMS BASED ON LAGRANGIAN FUNCTION FOR IMAGE MODELS

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Abstract: Motivated by proximity method and Lagrangian function, we investigate a novel framework for study of the total-variation model for image denoising and obtain a scheme with close form for each iteration. Under some stronger assumptions, we can prove the global convergence of the proposed methods. We also give the connection of the algorithms with other proximity methods. Our numerical experience indicates that the proposed methods perform favourably.

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107 ZERO FORCING BEAMFORMING IN RURAL BROADBAND WIRELESS NETWORK WITH SIDELOBE CONSTRAINTS

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Abstract: We investigate an optimal zero-forcing beamforming design problem in multiple antenna channels with restrictions on the total power or per-antenna element power constraints. Additional sidelobe constraints aimed at controlling the sensitivity to the interference from unknown but non-user directions will be introduced. These constraints also reduce the level of radiation from the base station array in non-user directions. The design problem is formulated as an optimization problem, which will be solved by using a recent developed exact penalty function method. Numerical results are given to illustrate the efficiency of the optimization method and the effect of the additional constraints on the user information rate that can be achieved. The numerical results are for two circular array geometries.

Key words: Zero-forcing beamformer; Total power constraints; Per-element power constraints; Exact penalty function method.

1 INTRODUCTION

The challenges of providing broadband services in rural areas can be tackled through the utilization of a range of communication techniques, which include advanced adaptive multicarrier modulation and coding; QOS based on cross-layer scheduling; and multi-user multiple-input multiple-output (MU-MIMO) systems to increase spectral efficiency. There has been much research on MU-MIMO in multipath environments that naturally provide the channel diversity exploited by MU-MIMO. In rural areas, in order to maximize range and coverage, the base-station/user channels will be dominated by line-of-sight (LOS) components resulting from direct and ground reflections. Thus, we adopt a deterministic LOS channel model in this research.

Zero-forcing beamforming (ZFBF) is a reduced complexity linear pre-coding strategy to serve multiple users. Each user stream is coded independently and multiplied by a beamforming weight vector for transmission through multiple BS antennas. ZFBF has been shown to achieve a large fraction of Dirty Paper Coding (DPC) capacity when the base station has multiple antennas and each user has a single antenna. DPC achieves a sum rate capacity that is linear in the number of transmit antennas.

In this paper, we investigate the optimal zero-forcing beamforming design problem with restrictions on the total power or per-antenna element power constraints in addition to the sidelobe constraints. A recent developed exact penalty function method is employed for solving the resulting optimization problems. The effect of additional sidelobe constraints on the power and the user rate will also be investigated.

2 OPTIMIZATION PROBLEM AND SOLUTION METHOD

Consider a system equipped with M antenna array elements transmitting information to N mobile users. For $1 \leq n \leq N$, the received signal y_n for the n^{th} mobile user is $y_n = \mathbf{h}_n^H \mathbf{x} + w_n$ where $\mathbf{h}_n$ is the n^{th} mobile's complex channel vector, $\mathbf{x}$ denotes the antenna outputs and w_n is the complex Gaussian noise with mean 0 and variance σ^2. By using the linear precoding method, the transmitted vectors $\mathbf{x}$ are coded by precoding vectors. For $1 \leq n \leq N$, denote by $\mathbf{w}_n = [w_{n1}, \ldots, w_{nM}]^T$ the weight vector that maps the n^{th} mobile data symbol b_n to the M antenna outputs. Assume that the channel vector $\mathbf{h}_n$ can be estimated. The problem of optimizing the weight vectors $\{\mathbf{w}_1, \ldots, \mathbf{w}_N\}$ to maximize the minimum mobile achievable rate subject to per-element power and zero-forcing constraints can be formulated as

$$\begin{cases} \max\limits_{\mathbf{w}_1,\cdots,\mathbf{w}_N} & r \\ \text{s. t.} & \log_2\left(1 + \frac{|\mathbf{h}_n^H \mathbf{w}_n|^2}{\sigma^2}\right) \geq r, \quad \forall\, 1 \leq n \leq N \\ & \sum\limits_{n=1}^{N} |\mathbf{e}_m^T \mathbf{w}_n|^2 \leq P_{\max}, \qquad \forall\, 1 \leq m \leq M \\ & \mathbf{h}_j^H \mathbf{w}_n = 0, \qquad\qquad \forall\, j \neq n,\ 1 \leq j, n \leq N \end{cases} \tag{2.1}$$

where $P_{\max}$ is a maximum per-element power and $\mathbf{e}_m$ the m^{th} standard unit-basis vector in $\mathcal{R}^N$, which has 1 for the m^{th} component and 0 for other components. The problem of optimizing the weight vectors to maximize the minimum mobile achievable rate subject to the total power and zero-forcing constraints can be formulated as

$$\begin{cases} \max\limits_{\mathbf{w}_1,\cdots,\mathbf{w}_N} & r \\ \text{s. t.} & \log_2\left(1 + \frac{|\mathbf{h}_n^H \mathbf{w}_n|^2}{\sigma^2}\right) \geq r, \quad \forall\, 1 \leq n \leq N \\ & \sum\limits_{m=1}^{M} \sum\limits_{n=1}^{N} |\mathbf{e}_m^T \mathbf{w}_k|^2 \leq P, \\ & \mathbf{h}_j^H \mathbf{w}_n = 0, \qquad\qquad \forall\, j \neq n,\ 1 \leq j, n \leq N \end{cases} \tag{2.2}$$

where P is the maximum allowable total power.

We now consider the introduction of additional constraints aimed at controlling the sensitivity to interference from unknown but non-user directions. These constraints will also reduce the level of radiation from the base station array in no-nuser directions. We consider sidelobe constraints in K directions of the form

$$|\mathbf{w}_n^H \mathbf{S}_{kn}| \leq \gamma_{kn},\ 1 \leq k \leq K,\ 1 \leq n \leq N$$

where $\mathbf{w}_n^H \mathbf{S}_{kn}$ is the array response for a specific k^{th} direction and γ_{mk} is a small sidelobe level. The optimization problem (2.1) with sidelobe constraints can be expressed as

$$\begin{cases} \max\limits_{\mathbf{w}_1,\cdots,\mathbf{w}_N} & r \\ \text{s. t.} & \log_2\left(1 + \frac{|\mathbf{h}_n^H \mathbf{w}_n|^2}{\sigma^2}\right) \geq r, \quad \forall\, 1 \leq n \leq M \\ & \sum\limits_{n=1}^{N} |\mathbf{e}_m^T \mathbf{w}_n|^2 \leq P_{\max}, \qquad \forall\, 1 \leq m \leq M \\ & \mathbf{h}_j^H \mathbf{w}_n = 0, \qquad\qquad \forall\, j \neq n,\ 1 \leq j, n \leq M \\ & |\mathbf{w}_n^H \mathbf{S}_{kn}| \leq \gamma_{kn}, \qquad\quad 1 \leq k \leq K,\ 1 \leq n \leq N. \end{cases} \tag{2.3}$$

The optimization problem (2.2) with additional sidelobe constraints can be formulated similarly. The problems (2.1)–(2.3) are not convex optimization problems. However, by using the fact that the optimum beamforming vectors are invariant to phase-shifts, i.e. if $\mathbf{w}_n^*$ is an optimum solution, then the vector $e^{j\theta}\mathbf{w}_n^*$ is also an optimum solution, it is possible to find an equivalent optimal solution so that $\mathbf{h}_n^H \mathbf{w}_n$ is a real number by rotating the optimum solution. Consequently, the above optimization problems can be transformed into convex optimization problems by restricting $\mathbf{h}_n^H \mathbf{w}_n$ to real numbers. A recently developed exact penalty function method is then employed to solve the resulting convex problems. Simulation results show the effect of the sidelobe constraints on the per-element power constraints and the total power constraints; and the trade-off between mobile user rate and the sidelobe levels.

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108 STOCHASTIC OPTIMAL CONTROL FOR NONLINEAR MARKOV JUMP SYSTEMS

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Abstract: As a special kind of hybrid system, Markov jump systems (MJSs) are appropriate and reasonable to describe systems subject to abrupt and random variation in structures or parameters. In recent years, MJSs have been studied extensively due to their comprehensive application in many areas, such as in manufacturing systems, economic systems, electrical systems and communication systems, and the existing results cover a large variety of problems such as stochastic stabilization, robust control and filtering. In spite of these developments, there is little work done on nonlinear Markov jump systems (NMJS). Actually, random changes or sudden variations in structures or parameters are commonly encountered in many practical dynamic nonlinear systems, so the investigation of control problems on nonlinear Markov jump systems is a topic worthy of investigation. This paper concerns the problem of stochastic optimal control for a class of nonlinear systems subject to Markov jump parameters. Gradient linearization procedure is employed and the nonlinear system is described by several linear Markov jump systems. Next, a mode-dependent Lyapunov function is constructed for these linear systems, and a sufficient condition is derived to make them stochastically stable. Then, a continuous gain-scheduled approach is applied to design a continuous nonlinear optimal controller on the entire extended nonlinear jump system. A simulation example is given to illustrate the effectiveness of the developed techniques.

Key words: Markov jump system; Stochastically stable; Optimal control; Gain scheduling.

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109 A MODIFIED DISCRETE FILLED FUNCTION ALGORITHM FOR SOLVING NONLINEAR DISCRETE OPTIMIZATION PROBLEMS

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Abstract: The discrete filled function method is a global optimization tool for searching for the best solution amongst multiple local optima. This method has proven useful for solving large-scale discrete optimization problems. In this paper, we consider a standard discrete filled function algorithm in the literature and then propose a modification to increase its efficiency.

Key words: Discrete filled function; Global optimization; Discrete optimization.

1 BACKGROUND

The discrete filled function method is one of the more recently developed global optimization tools for discrete optimization problems. Once a local minimum has been determined by an ordinary descent method, the discrete filled function approach involves the introduction of an auxiliary function, called a filled function, to avoid entrapment in the basin associated with this minimum. The local minimizer of the original function becomes a local maximizer of the filled function. By minimizing the filled function, the search moves away from the current local minimizer in the hope of escaping the basin associated with this minimizer and finding an improved solution.

The first filled function was introduced by Ge in the late 1980s Ge R. (1990) in the context of solving continuous global optimization problems. Zhu Zhu W. (1998) is believed to be the first researcher to introduce a discrete equivalent of the continuous filled function method in the late 1990s. This discrete filled function method overcomes the difficulties encountered in using a continuous approximation of the discrete optimization problem. However, the filled function proposed by Zhu contains an exponential term, which consequently makes it difficult to determine a point in a lower basin Ng C.K. (2007). Since the introduction of the original discrete filled function by Zhu, several new types of discrete filled functions with improved theoretical properties have been proposed, such as in Ng C.K. (2007); Yang Y. (2008); Shang Y. (2008), to enhance computational efficiency. A comprehensive survey of several discrete filled functions in the literature has been given in Woon S.F. (2010). The study showed that the discrete filled function developed in Ng C.K. (2007) seems to be the most reliable one since it guarantees that a local minimizer of the filled function is also a local minimizer of the original function, whereas other filled functions do not share this property. The goal of this paper is to propose an improved filled function algorithm based on the work in Ng C.K. (2007).

2 CONCEPTS & APPROACH

Consider the following nonlinear discrete optimization problem:

$$\min f(\mathbf{x}), \quad \text{s.t. } \mathbf{x} \in X, \tag{2.1}$$

where $X = \{\mathbf{x} \in \mathbb{Z}^n : \mathbf{x}_{i,\min} \leq \mathbf{x}_i \leq \mathbf{x}_{i,\max},\ i = 1, \ldots, n, \}$, $\mathbb{Z}^n$ is the set of integer points in $\mathbb{R}^n$, and $\mathbf{x}_{i,\min},\ \mathbf{x}_{i,\max},\ i = 1, \ldots, n$, are given bounds. Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be any two distinct points in the box constrained set X. Since X is bounded, there exists a constant $\mathcal{K}$ such that

$$1 \leq \max_{\substack{\mathbf{x}_1, \mathbf{x}_2 \in X \\ \mathbf{x}_1 \neq \mathbf{x}_2}} \| \mathbf{x}_1 - \mathbf{x}_2 \| \leq \mathcal{K} < \infty, \tag{2.2}$$

where $\| \cdot \|$ is the Euclidean norm. We make the following assumption.

Assumption 2.1 *There exists a constant $\mathcal{L}$, $0 < \mathcal{L} < \infty$, such that*

$$|f(\mathbf{x}_1) - f(\mathbf{x}_2)| \leq \mathcal{L} \| \mathbf{x}_1 - \mathbf{x}_2 \|, \quad (\mathbf{x}_1, \mathbf{x}_2) \in X \times X.$$

We now recall some familiar definitions and concepts used in the discrete optimization area.

Definition 2.1 *A sequence $\{\mathbf{x}^{(i)}\}_{i=0}^{k+1}$ in X is a discrete path between two distinct points $\mathbf{x}^*$ and $\mathbf{x}^{**}$ in X if $\mathbf{x}^{(0)} = \mathbf{x}^*$, $\mathbf{x}^{(k+1)} = \mathbf{x}^{**}$, $\mathbf{x}^{(i)} \in X$ for all i, $\mathbf{x}^{(i)} \neq \mathbf{x}^{(j)}$ for $i \neq j$, and $\| \mathbf{x}^{(i+1)} - \mathbf{x}^{(i)} \| = 1$ for all i. Let A be a subset of X. If, for all $\mathbf{x}^*, \mathbf{x}^{**} \in A$, $\mathbf{x}^*$ and $\mathbf{x}^{**}$ are connected by a discrete path, then A is called a pathwise connected set.*

Definition 2.2 *For any $\mathbf{x} \in X$, the neighbourhood of $\mathbf{x}$ is defined by*

$$N(\mathbf{x}) = \{\mathbf{w} \in X : \ \mathbf{w} = \mathbf{x} \pm \boldsymbol{e}_i, \ \ i = 1, \ldots, n\},$$

where $\boldsymbol{e}_i$ denotes the i-th standard unit basis vector of $\mathbb{R}^n$ with the i-th component equal to one and all other components equal to zero.

Definition 2.3 *The set of feasible directions at $\mathbf{x} \in X$ is defined by*

$$\mathcal{D}(\mathbf{x}) = \{\mathbf{d} \in \mathbb{R}^n : \mathbf{x} + \mathbf{d} \in N(\mathbf{x})\} \subset E = \{\pm \boldsymbol{e}_1, \ldots, \pm \boldsymbol{e}_n\}.$$

Definition 2.4 *$\mathbf{d} \in \mathcal{D}(\mathbf{x})$ is a descent direction of f at $\mathbf{x}$ if $f(\mathbf{x} + \mathbf{d}) < f(\mathbf{x})$.*

Definition 2.5 *$\mathbf{d}^* \in \mathcal{D}(\mathbf{x})$ is a steepest descent direction of f at $\mathbf{x}$ if it is a descent direction and $f(\mathbf{x} + \mathbf{d}^*) \leq f(\mathbf{x} + \mathbf{d})$ for any $\mathbf{d} \in \mathcal{D}(\mathbf{x})$.*

Definition 2.6 *$\mathbf{x}^* \in X$ is a local minimizer of X if $f(\mathbf{x}^*) \leq f(\mathbf{x})$ for all $\mathbf{x} \in N(\mathbf{x}^*)$. If $f(\mathbf{x}^*) < f(\mathbf{x})$ for all $\mathbf{x} \in N(\mathbf{x}^*) \setminus \mathbf{x}^*$, then $\mathbf{x}^*$ is a strict local minimizer of f.*

Definition 2.7 *$\mathbf{x}^*$ is a global minimizer of f if $f(\mathbf{x}^*) \leq f(\mathbf{x})$ for all $\mathbf{x} \in X$. If $f(\mathbf{x}^*) < f(\mathbf{x})$ for all $\mathbf{x} \in X \setminus \mathbf{x}^*$, then $\mathbf{x}^*$ is a strict global minimizer of f.*

Definition 2.8 *$\mathbf{x}$ is a vertex of X if for each $\mathbf{d} \in \mathcal{D}(\mathbf{x})$, $\mathbf{x} + \mathbf{d} \in X$ and $\mathbf{x} - \mathbf{d} \notin X$. Let $\tilde{X}$ denote the set of vertices of X.*

Definition 2.9 *$B^* \subset X$ is a discrete basin of f corresponding to the local minimizer $\mathbf{x}^*$ if it satisfies the following conditions:*

- *B^* is pathwise connected.*
- *B^* contains $\mathbf{x}^*$.*
- *For each $\mathbf{x} \in B^*$, any connected path starting at $\mathbf{x}$ and consisting of descent steps converges to $\mathbf{x}^*$.*

Definition 2.10 *Let $\mathbf{x}^*$ and $\mathbf{x}^{**}$ be two distinct local minimizers of f. If $f(\mathbf{x}^{**}) < f(\mathbf{x}^*)$, then the discrete basin B^{**} of f associated with $\mathbf{x}^{**}$ is said to be lower than the discrete basin B^* of f associated with $\mathbf{x}^*$.*

Definition 2.11 *Let $\mathbf{x}^*$ be a local minimizer of $-f$. The discrete basin of $-f$ at $\mathbf{x}^*$ is called a discrete hill of f at $\mathbf{x}^*$.*

Definition 2.12 *For a given local minimizer $\mathbf{x}^*$, define the discrete sets $S_L(\mathbf{x}^*) = \{\mathbf{x} \in X : f(\mathbf{x}) < f(\mathbf{x}^*)\}$ and $S_U(\mathbf{x}^*) = \{\mathbf{x} \in X : f(\mathbf{x}) \geq f(\mathbf{x}^*)\}$. Note that $S_L(\mathbf{x}^*)$ contains the points lower than $\mathbf{x}^*$, while $S_U(\mathbf{x}^*)$ contains the points higher than $\mathbf{x}^*$.*

Let $\mathbf{x}^*$ be a local minimizer of f. In Ng C.K. (2007), the discrete filled function $G_{\mu,\rho,\mathbf{x}^*}$ at $\mathbf{x}^*$ is defined as follows:

$$G_{\mu,\rho,\mathbf{x}^*}(\mathbf{x}) = A_\mu(f(\mathbf{x}) - f(\mathbf{x}^*)) - \rho \parallel \mathbf{x} - \mathbf{x}^* \parallel, \tag{2.3}$$

$$A_\mu(y) = \mu y\left[(1-c)\left(\frac{1-c\mu}{\mu - c\mu}\right)^{-y/\omega} + c\right],$$

where $\omega > 0$ is a sufficiently small number, $c \in (0,1)$ is a constant, $\rho > 0$, and $0 < \mu < 1$. It can be shown that the function $G_{\mu,\rho,\mathbf{x}^*}(\mathbf{x})$ is a discrete filled function when certain conditions on the parameters μ and ρ are satisfied, as detailed by the following properties proved in Ng C.K. (2007):

- $\mathbf{x}^*$ is a strict local maximizer of $G_{\mu,\rho,\mathbf{x}^*}$ if $\rho > 0$ and $0 < \mu < \min\{1, \rho/\mathcal{L}\}$.
- If $\mathbf{x}^*$ is a global minimizer of f, then $G_{\mu,\rho,\mathbf{x}^*}(\mathbf{x}) < 0$ for all $\mathbf{x} \in X \setminus \mathbf{x}^*$.
- Let $\bar{\mathbf{d}} \in \mathcal{D}(\bar{\mathbf{x}})$ be a feasible direction at $\bar{\mathbf{x}} \in S_U(\mathbf{x}^*)$ such that $\parallel \bar{\mathbf{x}} + \bar{\mathbf{d}} - \mathbf{x}^* \parallel > \parallel \bar{\mathbf{x}} - \mathbf{x}^* \parallel$. If $\rho > 0$ and $0 < \mu < \min\{1, \frac{\rho}{2\mathcal{K}^2\mathcal{L}}\}$, then
$$G_{\mu,\rho,\mathbf{x}^*}(\bar{\mathbf{x}} + \bar{\mathbf{d}}) < G_{\mu,\rho,\mathbf{x}^*}(\bar{\mathbf{x}}) < 0 = G_{\mu,\rho,\mathbf{x}^*}(\mathbf{x}^*).$$
- Let $\mathbf{x}^{**}$ be a strict local minimizer of f with $f(\mathbf{x}^{**}) < f(\mathbf{x}^*)$. If $\rho > 0$ is sufficiently small and $0 < \mu < 1$, then $\mathbf{x}^{**}$ is a strict local minimizer of $G_{\mu,\rho,\mathbf{x}^*}$.
- Let $\acute{\mathbf{x}}$ be a local minimizer of $G_{\mu,\rho,\mathbf{x}^*}$ and suppose that there exists a feasible direction $\bar{\mathbf{d}} \in \mathcal{D}(\acute{\mathbf{x}})$ such that $\parallel \acute{\mathbf{x}} + \bar{\mathbf{d}} - \mathbf{x}^* \parallel > \parallel \acute{\mathbf{x}} - \mathbf{x}^* \parallel$. If $\rho > 0$ is sufficiently small and $0 < \mu < \min\{1, \frac{\rho}{2\mathcal{K}^2\mathcal{L}}\}$, then $\acute{\mathbf{x}}$ is a local minimizer of f.
- Assume that every local minimizer of f is strict. Suppose that $\rho > 0$ is sufficiently small and $0 < \mu < \min\{1, \frac{\rho}{2\mathcal{K}^2\mathcal{L}}\}$. Then, $\mathbf{x}^{**} \in X \setminus \tilde{X}$ is a local minimizer of f with $f(\mathbf{x}^{**}) < f(\mathbf{x}^*)$ if and only if $\mathbf{x}^{**}$ is a local minimizer of $G_{\mu,\rho,\mathbf{x}^*}$.

3 THE STANDARD ALGORITHM

The discrete filled function approach can be described as follows. First, an initial point is chosen and a local search is applied to find an initial discrete local minimizer. Then, the filled function is constructed at this local minimizer. By minimizing the filled function, either an improved discrete local minimizer is found or the boundary of the feasible region is reached. The discrete local minimizer of the filled function usually becomes a new starting point for minimizing the original objective with the hope of finding an improved point compared to the first local minimizer. A new filled function is constructed at this improved point. The process is repeated until no improved local minimizer of the original filled function can be found. The final discrete local minimizer is then taken as an approximation of the global minimizer. If a local minimizer of the filled function cannot be found after repeated searches terminate on the boundary of the box constrained feasible region, then the parameters defining the filled function are adjusted and the search is repeated. This adjustment of the parameters continues until the parameters reach their predetermined bounds; the best solution obtained so far is then taken as the global minimizer. The parameter μ is reduced if $\acute{\mathbf{x}}$ is neither a vertex nor an improved point and we return to Step 4(b). When all searches terminate at vertices ($\ell > q$), ρ is adjusted. The algorithm for minimizing $G_{\mu,\rho,\mathbf{x}^*}$ exits prematurely when an improved point $\mathbf{x}_k$ with $f(\mathbf{x}_k) < f(\mathbf{x}^*)$ is found in Step 4 of Algorithm **??**. The algorithm sets $\mathbf{x}_0 := \mathbf{x}_k$ and returns to Step 2 to minimize the original function f. Note that a direction yielding the greatest improvement of $f + G_{\mu,\rho,\mathbf{x}^*}$ is chosen when minimizing $G_{\mu,\rho,\mathbf{x}^*}$, assuming that a direction for improving f and $G_{\mu,\rho,\mathbf{x}^*}$ simultaneously exists. If such a direction does not exist, the algorithm chooses the steepest descent direction for $G_{\mu,\rho,\mathbf{x}^*}$ alone.

Table 4.1 Comparison of Algorithms - Colville's Function.

Types	$E_{f,avg}$	$E_{G,avg}$	$R_{E,avg}$
Standard Algorithm	1679.5	5247.2	0.008635805
Modified Algorithm	1143.2	2954.7	0.005878038

4 A MODIFIED APPROACH

We replace the neighbourhood $N(\mathbf{x}^*)$ in Step 3 of the standard Algorithm with a set of randomly chosen points from X. Then, an additional step is added to test whether any one of these random points happens to be an improved point. The motivation for this modification is to search for improved points more efficiently by choosing points which give a broader coverage of X, similar to the approaches proposed in Shang Y. (2005); Shang Y. (2008); Yang Y. (2007). In the standard approach, the initial points are chosen as the neighbouring points of the current local solution.

We tested both the original algorithm and our modified version on Colville's function. This function has 1.94481×10^5 feasible points and a global minimum $\mathbf{x}^*_{\text{global}} = [1, 1, 1, 1]^\top$ with $f(\mathbf{x}^*_{\text{global}}) = 0$. We initialized both parameters μ and ρ as 0.1 and set $\rho_L = 0.001$. The parameter μ is reduced if $\acute{\mathbf{x}}$ is neither a vertex nor an improved point by setting $\mu := \mu/10$.

Computational results are shown in Table 4.1, where E_f is the total number of original function evaluations, E_G represents the total number of discrete filled function evaluations, and R_E denotes the ratio of the average number of original function evaluations to the total number of feasible points.

Problem 1: Colville's Function Schittkowski K. (1987)

$$\begin{aligned} \min\ f(\mathbf{x}) \quad = \quad & 100\big(x_2 - x_1^2\big)^2 + \big(1 - x_1\big)^2 + 90\big(x_4 - x_3^2\big)^2 + \big(1 - x_3\big)^2 \\ & + 10.1\Big[\big(x_2 - 1\big)^2 + \big(x_4 - 1\big)^2\Big] + 19.8\big(x_2 - 1\big)\big(x_4 - 1\big), \end{aligned}$$

$$\text{s.t.} \quad -10 \le x_i \le 10, \quad x_i \text{ integer}, \quad i = 1, 2, 3, 4.$$

Six starting points are considered, namely $[1, 1, 0, 0]^\top$, $[1, 1, 1, 1]^\top$, $[-10, 10, -10, 10]^\top$, $[-10, -5, 0, 5]^\top$, $[-10, 0, 0, -10]^\top$, and $[0, 0, 0, 0]^\top$. From Table 4.1, both algorithms succeeded in finding the global minimum from all starting points. Our algorithm succeeds in determining the global solution of Colville's function much more efficiently with an average $E_f = 1143.2$, compared with $E_f = 1679.5$ for the standard algorithm, which is a reduction of 31.9% in the average number of original function evaluations. However, the gain in efficiency for our Algorithm is offset somewhat by reduced reliability, since we sometimes needed to repeat the algorithm several times for each starting point before a global solution was attained.

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110 NEW RESULTS IN SINGULAR LINEAR QUADRATIC OPTIMAL CONTROL

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Abstract: This paper focuses on the singular infinite-horizon linear quadratic (LQ) optimal control problem for continuous-time systems. In particular, we are interested in the stabilising impulse-free solutions to this problem that can be expressed as a static state feedback. In particular we establish a link between the geometric properties of the so-called Hamiltonian system associated with the optimal control problem at hand and the so-called proper deflating subspaces of the Hamiltonian matrix pencil.

Key words: Singular LQ optimal control; Hamiltonian system; Hamiltonian matrix pencil; Deflating subspaces.

1 INTRODUCTION

This paper introduces new results on the singular linear quadratic optimal control problem for continuous-time systems. It is a well known fact that when the matrix penalising the control in the performance index to be minimised – traditionally denoted by R – is positive definite, the optimal control can be expressed as a static state-feedback, whose gain depends on the solution of a standard algebraic Riccati equation. This equation involves the inverse of matrix R. When this matrix is singular, the optimal control is guaranteed to exist for any initial condition only if the set of allowed inputs is extended to include distributions (Dirac delta and its derivatives in the sense of distributions). In this case, the standard Riccati equation is not defined, and the problem has been solved in the literature mainly by resorting to a geometric approach, see e.g. Willems *at al.* (1986); Hautus and Silverman (1983); Saberi and Sannuti (1987).

A different perspective was established in Prattichizzo *et al.* (2008), where the main focus of the geometric analysis was the Hamiltonian system. Indeed, the cornerstone of that paper was the interpretation of the LQ regulator as an output nulling problem referred to the Hamiltonian system. In particular, by writing the conditions for optimality in the form of the Hamiltonian system, whose output has to be maintained identically equal to zero, the singular LQ problem reduces to finding a state feedback such that the state-costate trajectory entirely lies on the largest stabilisability subspace of the Hamiltonian system. The analysis carried out in that paper was restricted to the *cheap* LQ problem, i.e., the one in which the matrices weighting the control in the objective function are zero.

In recent years, another important tool aimed at characterising the solutions of the so-called *generalised continuous algebraic Riccati equation* has been introduced in the literature: the Hamiltonian matrix pencil (sometimes also referred to as "extended Hamiltonian pencil" in analogy with the extended symplectic pencil of the discrete time counterpart), see van Dooren (1983); Weiss (1994); Ionescu and Oară (1996); Ionescu *et al.* (1996). The aim of this paper is to establish a link between the approach taken in Prattichizzo *et al.* (2008) based on the Hamiltonian system with that based on the Hamiltonian pencil.

Indeed, the latter is nothing more than the Rosenbrock matrix pencil associated with the Hamiltonian system, see Ionescu *et al.* (1996), p. 86. In this paper, we show that there exists a simple correspondence between the proper right deflating subspaces of the Hamiltonian matrix pencil with the output-nulling subspaces of the Hamiltonian system and that a dual correspondence exists between the proper left deflating subspaces of the Hamiltonian matrix pencil with the input-containing subspaces of the Hamiltonian system. This very simple observation is crucial as it can immediately lead to the derivation of simple conditions under which the singular LQ problem admits an impulse-free solution expressed in terms of a stabilising state feedback for any initial condition. To that end, the approach taken in Weiss (1994), which provides conditions under which an n-dimensional stable and proper right deflating subspace of the Hamiltonian matrix pencil exists, is exploited in this paper to show that under the same conditions the projection of the largest stabilisability subspace of the Hamiltonian system on the state space of the original system coincides with the state space itself. Therefore for any initial condition we can find an optimal stabilising state feedback optimal control by suitably manipulating the friend associated with such stabilisability subspace.

2 SINGULAR LQ PROBLEM

Consider the linear time-invariant (LTI) state differential equation with initial condition

$$\dot{x}(t) \;=\; A\,x(t) + B\,u(t), \qquad x(0) = x_0 \in \mathbb{R}^n, \tag{2.1}$$

where, for all $t \ge 0$, the vectors $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ represent the state and the control input, respectively, and A, B are real constant matrices of proper sizes, i.e., $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Let $Q \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times m}$ and $R \in \mathbb{R}^{m\times m}$ be such that

$$\Pi \stackrel{\text{def}}{=} \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} = \Pi^\top \ge 0. \tag{2.2}$$

In view of (2.2), matrix Π can be factored as

$$\Pi = \begin{bmatrix} C^\top & D^\top \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} \qquad \text{where} \;\; Q = C^\top C,\;\; S = C^\top D \;\; \text{and} \;\; R = D^\top D. \tag{2.3}$$

Using the nomenclature of Ionescu *et al.* (1996), matrix Π is referred to as *Popov matrix.* Notice that here we do not require R to be positive definite. We denote by Σ a quadruple (A,B,C,D) where C and D are such that (2.3) holds.

The singular LQ problem we consider in this paper can be stated as follows.

Problem 6 *Determine under which conditions for all $x_0 \in \mathbb{R}^n$ the input $u(t)$ that minimises the performance index*

$$J(x,u) = \int_0^\infty \begin{bmatrix} x^\top(t) & u^\top(t) \end{bmatrix} \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt$$

is impulse-free and can be expressed as a static state feedback $u(t) = F\,x(t)$, with the additional requirement that $A + B\,F$ be asymptotically stable.

In general, it is well known that Problem 6 is guaranteed to be solvable for any initial condition $x_0 \in \mathbb{R}^n$ only if the set of allowed inputs is extended to include distributions (Dirac delta and its distributional derivatives). In this case, the standard continuous algebraic Riccati equation is not defined, and the problem has been solved in the literature mainly by resorting to a geometric approach, see e.g. Willems *at al.* (1986); Hautus and Silverman (1983); Saberi and Sannuti (1987). Here we are interested in the impulse-free solutions, with the requirement of asymptotic stability of the closed loop. The approach based on the Hamiltonian system hinges on the fact that if $u(t)$ and $x(t)$ are optimal for Problem 6, then a costate function $\lambda(t) \in \mathbb{R}^n$, exists such that $x(t)$, $\lambda(t)$ and $u(t)$ satisfy for all $t \ge 0$ the equations

$$\dot{x}(t) = A\,x(t) + B\,u(t), \tag{2.4}$$

$$\dot{\lambda}(t) = -Q\,x(t) - S\,u(t) - A^\top \lambda(t), \tag{2.5}$$

$$R\,u(t) + S^\top x(t) + B^\top \lambda(t) = 0, \tag{2.6}$$

$$x(0) = x_0. \tag{2.7}$$

We now introduce some fundamental objects associated with the classical LQ optimal control problem.

3 THE HAMILTONIAN SYSTEM AND THE HAMILTONIAN MATRIX PENCIL

Recall that the Hamiltonian system associated with the Popov triple Σ is an LTI system defined by the equations

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -Q & -A^\top \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} + \begin{bmatrix} B \\ -S \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} S^\top & B^\top \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} + R\, u(t), \end{aligned} \tag{3.1}$$

where the variable $\lambda(t)$ is the *costate*. We define $\widehat{A} \stackrel{\text{def}}{=} \begin{bmatrix} A & 0 \\ -Q & -A^\top \end{bmatrix}$, $\widehat{B} \stackrel{\text{def}}{=} \begin{bmatrix} B \\ -S \end{bmatrix}$, $\widehat{C} \stackrel{\text{def}}{=} [\, S^\top \quad B^\top \,]$ and $\widehat{D} \stackrel{\text{def}}{=} R$. The Hamiltonian system (3.1) is identified with the quadruple $\widehat{\Sigma} \stackrel{\text{def}}{=} (\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D})$. The Hamiltonian system is a fundamental tool in the solution of continuous-time differential and algebraic Riccati equations, and it has strong relations with the corresponding optimal control problem. Indeed, comparing (3.1) with (2.4)-(2.6), it emerges that the optimal control and state trajectory satisfy the Hamiltonian system with an identically zero output for a suitable costate function λ. This consideration is crucial, and recasts the optimal control problem into an output-nulling problem for the Hamiltonian system (3.1). On the other hand, the trajectories of $\widehat{\Sigma}$ that yield an identically zero output are those and only those for which the state-costate vector $\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix}$ lies entirely on an output-nulling subspace[1] of $\widehat{\Sigma}$. Since in addition we have a stability requirement on the closed-loop system, $\begin{bmatrix} x_0 \\ \lambda_0 \end{bmatrix}$ must belong to the largest stabilisability subspace[2] of the Hamiltonian system, herein denoted by $\widehat{\mathcal{V}}_g^\star$, see also Prattichizzo *et al.* (2008). Thus, we have the following result.

Theorem 3.1 *Let $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ be a basis matrix of $\widehat{\mathcal{V}}_g^\star$. Given $x_0 \in \mathbb{R}^n$, a state and input functions $x(t)$ and $u(t)$ exist satisfying (2.4-2.7) if and only if $x_0 \in \operatorname{im} V_1$.*

Corollary 3.1 *A state and input functions $x(t)$ and $u(t)$ satisfy (2.4-2.6) for any $x_0 \in \mathbb{R}^n$ if and only if $\operatorname{im} V_1 = \mathbb{R}^n$.*

Another important tool that is often introduced in the theory of continuous-time Riccati equations is the so-called Hamiltonian matrix pencil, which coincides with the Rosenbrock matrix pencil associated with the Hamiltonian system (3.1). More explicitly, the Hamiltonian matrix pencil is $N - s\,M$, where the matrices N and M are defined as

$$M \triangleq \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \text{and} \qquad N \triangleq \begin{bmatrix} A & 0 & B \\ -Q & -A^\top & -S \\ S^\top & B^\top & R \end{bmatrix}.$$

It follows that the invariant zeros of the Hamiltonian system are the generalised eigenvalues of the Hamiltonian pencil. The Hamiltonian pencil has been used by several authors to characterise the stabilising solution of the generalised algebraic Riccati equation, see van Dooren (1983); Weiss (1994); Ionescu and Oară (1996).

Theorem 3.2 *Given a proper right deflating subspace of the Hamiltonian matrix pencil $N - s\,M$ spanned by the matrix $V = [\, V_1^\top \quad V_2^\top \quad V_3^\top \,]^\top$ partitioned conformably with M and N, the columns of $V = [\, V_1^\top \quad V_2^\top \,]^\top$ are a basis of an output-nulling subspace of (3.1). Conversely, given an output-nulling subspace of the Hamiltonian system (3.1) spanned by the basis matrix $V = [\, V_1^\top \quad V_2^\top \,]^\top$ partitioned conformably with (3.1), and given two matrices Ξ and Ω such that (3.5) holds, the matrix $V = [\, V_1^\top \quad V_2^\top \quad \Omega^\top \,]^\top$ spans a proper right deflating subspace of $N - s\,M$.*

Proof: Consider the basis matrix $V = [\, V_1^\top \quad V_2^\top \quad V_3^\top \,]^\top$ of a proper right deflating subspace of the Hamiltonian matrix pencil $N - s\,M$. By definition, a matrix Φ exists such that $M\,V\,\Phi = N\,V$ where MV is injective, so that

$$\begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \Phi = \begin{bmatrix} A & 0 & B \\ -Q & -A^\top & -S \\ S^\top & B^\top & R \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}. \tag{3.2}$$

We now notice that equation (3.2) can be equivalently written as

$$\left[\begin{array}{cc} A & 0 \\ -Q & -A^\top \\ \hline S^\top & B^\top \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \end{array}\right] = \left[\begin{array}{c} V_1 \\ V_2 \\ \hline 0 \end{array}\right] \Phi - \left[\begin{array}{c} B \\ -S \\ \hline R \end{array}\right] V_3, \tag{3.3}$$

which says that $\operatorname{im}\left[\begin{smallmatrix} V_1 \\ V_2 \end{smallmatrix}\right]$ is output-nulling for the Hamiltonian system by virtue of the equivalence of (3.4) and (3.5), see the Notes at the end of the paper. Conversely, if $\operatorname{im}\left[\begin{smallmatrix} V_1 \\ V_2 \end{smallmatrix}\right]$ is output-nulling for the Hamiltonian system, two matrices Ξ and Ω exist such that (3.5) holds with $\widehat{A}$, $\widehat{B}$, $\widehat{C}$ and $\widehat{D}$ in place of A, B, C, D, which means that (3.2) holds with $\Phi = \Xi$ and $V_3 = \Omega$. Hence, (3.2) holds under the same substitutions, which means that $V = \left[\begin{array}{ccc} V_1^\top & V_2^\top & \Omega^\top \end{array}\right]^\top$ is a right deflating subspace of the Hamiltonian matrix pencil $N - s\,M$. It is proper since $\left[\begin{smallmatrix} V_1 \\ V_2 \end{smallmatrix}\right]$ is of full column rank. ■

The dual of Theorem 3.2 is as follows.

Theorem 3.3 *Given a proper left deflating subspace* $\ker W$ *of the Hamiltonian matrix pencil* $N - s\,M$*, where* W *is partitioned as* $\left[\begin{array}{ccc} W_1 & W_2 & W_3 \end{array}\right]$ *conformably with* M *and* N*, then* $\ker\left[\begin{array}{cc} W_1 & W_2 \end{array}\right]$ *is an input-containing*[3] *subspace of (3.1). Conversely, given an input-containing subspace of (3.1) equal to* $\ker W$ *where* $W = \left[\begin{array}{cc} W_1^\top & W_2^\top \end{array}\right]^\top$ *is full row-rank and partitioned conformably with (3.1), and given two matrices* Γ *and* Λ *such that (3.7) holds, the null-space of the matrix* $W = \left[\begin{array}{ccc} W_1 & W_2 & \Lambda \end{array}\right]$ *is a proper left deflating subspace of the Hamiltonian matrix pencil* $N - s\,M$*.*

Proof: By definition, a left deflating subspace $\ker W$ is such that there exists Ψ for which $\Psi\,W\,M = W\,N$ with $W\,M$ full row-rank. The proof follows by dualising the proof of Theorem 3.2, by noticing the equivalence of

$$\left[\begin{array}{cc} W_1 & W_2 \end{array}\right] \left[\begin{array}{ccc} A & 0 & B \\ -Q & -A^\top & -S \end{array}\right] = \Gamma \left[\begin{array}{ccc} W_1 & W_2 & 0 \end{array}\right] + \Lambda \left[\begin{array}{ccc} S^\top & B^\top & R \end{array}\right]$$

with $\left[\begin{array}{ccc} W_1 & W_2 & W_3 \end{array}\right] N = \Psi \left[\begin{array}{ccc} W_1 & W_2 & W_3 \end{array}\right] \operatorname{diag}\{I_n, I_n, 0\}$ under the substitutions $\Gamma = \Psi$ and $\Lambda = -W_3$. ■

The following result, which relates the fundamental subspaces of the Hamiltonian system $\widehat{\Sigma}$ with those of the original system Σ, generalises Lemma 4.3 in Prattichizzo *et al.* (2008) in two directions. First, we are not assuming that D is zero (i.e., that the LQ problem is *cheap*). Second, Σ is not necessarily left invertible.

Theorem 3.4 *Let* $\mathcal{R}^\star$*,* $\mathcal{V}^\star$ *and* $\mathcal{S}^\star$ *denote the largest controllability subspace, the largest output-nulling and the smallest input containing subspaces of* Σ*, respectively. Moreover, let* $\widehat{\mathcal{R}}^\star$*,* $\widehat{\mathcal{V}}^\star$ *and* $\widehat{\mathcal{S}}^\star$ *denote the same subspaces referred to the Hamiltonian system* $\widehat{\Sigma}$*. The following identities hold:*

$$\begin{aligned} \dim \widehat{\mathcal{R}}^\star &= \dim \mathcal{R}^\star \\ \dim \widehat{\mathcal{S}}^\star &= 2\,\dim \mathcal{S}^\star - \dim \mathcal{R}^\star \\ \dim \widehat{\mathcal{V}}^\star &= 2\,n - 2\,\dim \mathcal{S}^\star + \dim \mathcal{R}^\star \end{aligned}$$

The following result can be found in Weiss (1994).

Theorem 3.5 *Let the pair* (A, B) *be stabilisable and let the Hamiltonian system (3.1) be devoid of invariant zeros on the imaginary axis. Then, the Hamiltonian pencil* $N - s\,M$ *has an* n*-dimensional stable proper deflating subspace. Let this deflating subspace be spanned by the matrix* $V = \left[\begin{array}{ccc} V_1^\top & V_2^\top & V_3^\top \end{array}\right]^\top$ *partitioned conformably with* M *and* N*. Then,* V_1 *is invertible.*

Proof: This statement follows from Weiss (1994), p. 679, by observing that the absence of generalised eigenvalues on the imaginary axis of the Hamiltonian pencil is equivalent to the absence of invariant zeros on the imaginary axis of the Hamiltonian system. This, in view of the stabilisability of the pair (A, B), in turn corresponds to the absence of unobservable eigenvalues of the pair (A, C), where C is any matrix which, together with a matrix D, factorises the Popov matrix as in (2.3). ■

Corollary 3.2 *Consider the factorisation (2.3), and let* $\operatorname{rank} G(j\omega) = \operatorname{rank} D$ *for every* $\omega \in \mathbb{R}$*. Let the pair* (A, B) *be stabilisable and let the Hamiltonian system (3.1) be devoid of invariant zeros on the imaginary axis. Then, the largest stabilisability subspace* $\widehat{\mathcal{V}}_g^\star$ *of the Hamiltonian system has dimension* n*.*

Proof: This result follows directly from Theorem 3.5 and Theorem 3.2. ∎

Theorem 3.6 *Let* $\operatorname{rank} G(j\omega) = \operatorname{rank} D$ *for every* $\omega \in \mathbb{R}$. *Let the pair* (A, B) *be stabilisable and let the Hamiltonian system (3.1) be devoid of invariant zeros on the imaginary axis. For all* $x_0 \in \mathbb{R}^n$, *a control exists such that (2.4-2.7) hold and the state converges to zero, and such control can be expressed as a static state feedback.*

Proof: Under the assumptions considered, the state-costate trajectory lies on $\widehat{\mathcal{V}}_g^\star$, whose dimension is equal to n. Since from Theorem 3.5 a basis matrix for $\widehat{\mathcal{V}}_g^\star$ is given by the columns of $\begin{bmatrix} V_1^\top & V_2^\top \end{bmatrix}^\top$ with V_1 square and invertible, it follows that the projection of $\widehat{\mathcal{V}}_g^\star$ on the state space $\mathbb{R}^n$ coincides with $\mathbb{R}^n$. For any $x_0 \in \mathbb{R}^n$, there exists $\lambda_0 \in \mathbb{R}^n$ such that $\begin{bmatrix} x_0 \\ \lambda_0 \end{bmatrix} \in \widehat{\mathcal{V}}_g^\star$. The corresponding control can be expressed as a feedback of the sole state $x(t)$. Let $\widehat{F} = \begin{bmatrix} F_x & F_\lambda \end{bmatrix}$ be a friend of $\widehat{\mathcal{V}}_g^\star$ that assigns stable closed-loop eigenvalues of $\widehat{A} + \widehat{B}\,\widehat{F}$ restricted to $\widehat{\mathcal{V}}_g^\star$. The control can therefore be expressed as a static feedback of the state-costate vector using $\widehat{F}$, i.e, $u(t) = \begin{bmatrix} F_x & F_\lambda \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix}$, where for all $t \geq 0$ we have $\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} \in \widehat{\mathcal{V}}_g^\star$, so that we can write $\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \alpha(t)$ for a suitable function $\alpha(t)$. Thus, the state $x(t)$ identically lies on the projection of $\widehat{\mathcal{V}}_g^\star$ on the state space, which is spanned by V_1. Hence

$$u(t) = \begin{bmatrix} F_x & F_\lambda \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \alpha(t) = \begin{bmatrix} F_x & F_\lambda \end{bmatrix} \begin{bmatrix} I_n \\ V_2\,V_1^{-1} \end{bmatrix} V_1\,\alpha(t) = (F_x + F_\lambda\,V_2\,V_1^{-1})\,x(t),$$

i.e., we have expressed the control as a state feedback of the sole state $x(t)$. Moreover, since $\widehat{F}$ drives the state-costate trajectory to the origin as $t \to \infty$, then $F_x + F_\lambda\,V_2\,V_1^{-1}$ drives the state to the origin as $t \to \infty$. ∎

Example 3.1 Consider the Popov triple characterised by the matrices

$$A = \begin{bmatrix} -9 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 9 \end{bmatrix}, \quad Q = \begin{bmatrix} 18 & 0 \\ 0 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad R = 0.$$

This system is stabilisable (the uncontrollable eigenvalue is equal to -9). Moreover, from the factorisation $\Pi = \begin{bmatrix} C^\top \\ D^\top \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix}$ with $C = \begin{bmatrix} 3\sqrt{2} & 0 \end{bmatrix}$ and $D = 0$, since $G(s) = C\,(s\,I - A)^{-1} B + D \equiv 0$, we find that indeed $\operatorname{rank} G(j\omega) = \operatorname{rank} D = 0$ for all $\omega \in \mathbb{R}$. A direct check shows that the zeros of the Hamiltonian system are $\{9, -9\}$. The largest stabilisability subspace of the Hamiltonian system is $\widehat{\mathcal{V}}_g^\star = \operatorname{im} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$. A friend $\widehat{F}$ of $\widehat{\mathcal{V}}_g^\star$ that assigns the eigenvalue -9 with double multiplicity to the spectrum of $\widehat{A} + \widehat{B}\,\widehat{F}$ restricted to $\widehat{\mathcal{V}}_g^\star$ is $\widehat{F} = \begin{bmatrix} 0 & -1 & 0 & 0 \end{bmatrix}$. The feedback matrix $F = F_x + F_\lambda\,V_2\,V_1^{-1} = \begin{bmatrix} 0 & -1 \end{bmatrix}$ assigns the spectrum $\sigma(A + B\,F) = \{-9\}$ with double multiplicity. However, since this system is not left invertible, as the largest controllability subspace[4] is $\mathcal{R}^\star = \operatorname{im} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, we can find the gains by changing the spectrum of $A + B\,F$ restricted to $\mathcal{R}^\star$. Let us compute a friend $\widehat{F}$ of $\widehat{\mathcal{V}}_g^\star$ which assigns the eigenvalues $\{-3, -9\}$ to the spectrum of $(\widehat{A} + \widehat{B}\,\widehat{F} \,|\, \widehat{\mathcal{V}}_g^\star)$. We get $\widehat{F} = \begin{bmatrix} 0 & -1/3 & 0 & 0 \end{bmatrix}$. Then, $F = F_x + F_\lambda\,V_2\,V_1^{-1} = \begin{bmatrix} 0 & -1/3 \end{bmatrix}$ assigns the spectrum $\sigma(A + B\,F) = \{-3, -9\}$.

Notes

1. We recall that, given a quadruple (A, B, C, D), an output-nulling subspace is a subspace $\mathcal{V}$ of the state-space satisfying the inclusion

$$\begin{bmatrix} A \\ C \end{bmatrix} \mathcal{V} \subseteq (\mathcal{V} \oplus \{0\}) + \operatorname{im} \begin{bmatrix} B \\ D \end{bmatrix}, \tag{3.4}$$

which is equivalent to the existence of a matrix F (referred to as a *friend* of $\mathcal{V}$) such that $(A + B\,F)\,\mathcal{V} \subseteq \mathcal{V} \subseteq \ker(C + D\,F)$, see Trentelman *et al.* (2001), p. 160. Notice that in view of (3.4), we have that given a subspace $\mathcal{V}$ and a basis matrix V (i.e., a matrix whose columns are linearly independent and that span $\mathcal{V}$), then $\mathcal{V}$ is output-nulling if and only if two matrices Ξ and Ω exist such that

$$\begin{bmatrix} A \\ C \end{bmatrix} V = \begin{bmatrix} V \\ 0 \end{bmatrix} \Xi + \begin{bmatrix} B \\ D \end{bmatrix} \Omega, \tag{3.5}$$

see Ntogramatzidis (2007); Ntogramatzidis (2008). Since the set of output-nulling subspaces of a given quadruple is closed under subspace addition, there exists a largest output-nulling subspace – denoted by $\mathcal{V}^\star$ – which represents the set of all initial states for which a control can be found that maintains the output at zero.

2. Given a quadruple (A,B,C,D), an output-nulling subspace $\mathcal{V}$ is referred to as a *stabilisability subspace* if a friend F exists such that the map $A+BF$ restricted to $\mathcal{V}$ is asymptotically stable. When the pair (A,B) is stabilisable, then also the map induced by $A+BF$ in the quotient space $\mathbb{R}^n/\mathcal{V}$ is stable, so that F exists such that the spectrum of $A+BF$ is stable. The set of stabilisability subspaces is closed under addition, and its largest element is denoted by $\mathcal{V}_g^\star$.

3. Given a quadruple (A,B,C,D), an input-containing subspace is a subspace $\mathcal{S}$ of the state-space satisfying the inclusion

$$\left[\begin{array}{cc} A & B \end{array}\right]\left((\mathcal{S}\oplus\mathbb{R}^m)\cap\ker\left[\begin{array}{cc} C & D \end{array}\right]\right)\subseteq\mathcal{S}, \tag{3.6}$$

Trentelman *et al.* (2001), p. 185. In view of (3.4), given a subspace $\mathcal{S}$ and a full row-rank matrix W such that $\ker W=\mathcal{S}$, then $\mathcal{S}$ is input containing if and only if two matrices Γ and Λ exist such that

$$W\left[\begin{array}{cc} A & B \end{array}\right]=\Gamma\left[\begin{array}{cc} W & 0 \end{array}\right]+\Lambda\left[\begin{array}{cc} C & D \end{array}\right], \tag{3.7}$$

see Ntogramatzidis (2007); Ntogramatzidis (2008). Since the set of input-containing subspaces is closed under subspace intersection, there exists a smallest input-containing subspace indicated by $\mathcal{S}^\star$.

4. The largest controllability subspace of a quadruple (A,B,C,D) can be computed as the intersection $\mathcal{R}^\star=\mathcal{V}^\star\cap\mathcal{S}^\star$, Trentelman *et al.* (2001), Theorem 8.22. This subspace represents the states that can be driven to the origin by maintaining the output identically equal to zero. If $\left[\begin{smallmatrix} B \\ D \end{smallmatrix}\right]$ is full column-rank, the quadruple (A,B,C,D) is left invertible if and only if $\mathcal{R}^\star=\{0\}$.

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111 OPTIMAL STATE FEEDBACK FOR CONSTRAINED NONLINEAR SYSTEMS

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Abstract: In this paper, we consider a general nonlinear control system that is subject to both terminal state and continuous inequality constraints. The continuous inequality constraints must be satisfied at every point in the time horizon—an infinite number of points. Our aim is to design an optimal feedback controller that yields efficient system performance and satisfaction of all constraints. We first formulate this problem as a semi-infinite optimization problem. We then show that, by using a novel exact penalty approach, this semi-infinite optimization problem can be converted into a sequence of nonlinear programming problems, each of which can be solved using standard numerical techniques. We conclude the paper with some convergence results.

Key words: Optimal control; State feedback; Exact penalty function; Nonlinear programming.

1 PROBLEM FORMULATION

We consider nonlinear control systems in the following general form:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{u}(t)), \quad t \in [0, T], \tag{1.1}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}^0, \tag{1.2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the *state*, $\boldsymbol{u}(t) \in \mathbb{R}^r$ is the *control*, $\boldsymbol{x}^0 \in \mathbb{R}^n$ is a given initial state, T is a given *terminal time*, and $\boldsymbol{f} : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$ is a given continuously differentiable function.

System (1.1)-(1.2) is subject to the following *terminal state constraints*:

$$\Psi_i(\boldsymbol{x}(T)) = 0, \quad i = 1, \ldots, p, \tag{1.3}$$

where $\Psi_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, p$ are given continuously differentiable functions.

In addition, system (1.1)-(1.2) is subject to a set of *continuous inequality constraints* defined as follows:

$$h_j(t, \boldsymbol{x}(t), \boldsymbol{u}(t)) \leq 0, \quad t \in [0, T], \quad j = 1, \ldots, q, \tag{1.4}$$

where $h_j : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}$, $j = 1, \ldots, q$ are given continuously differentiable functions. Note that control bounds can be easily incorporated into (1.4).

Our aim is to design an optimal *state feedback control* for system (1.1)-(1.2). To this end, we assume that the control takes the following form:

$$\boldsymbol{u}(t) = \boldsymbol{\varphi}(\boldsymbol{x}(t), \boldsymbol{\zeta}), \quad t \in [0, T], \tag{1.5}$$

where $\boldsymbol{\zeta} \in \mathbb{R}^m$ is a vector of feedback control parameters and $\boldsymbol{\varphi} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^r$ is a given continuously differentiable function. Typical choices for the feedback controller (1.5) include *linear state feedback* *control* (in which $\boldsymbol{\varphi}$ is a linear function—see Khalil H. K. (2002)) and *PID control* (in which $\boldsymbol{\varphi}$ is the sum of linear, integral, and derivative terms—see Li B. (2011)).

The feedback control parameters ζ_k, $k = 1, \ldots, m$ are subject to the following bound constraints:

$$a_k \le \zeta_k \le b_k, \quad k = 1, \ldots, m, \tag{1.6}$$

where a_k and b_k, $k = 1, \ldots, m$ are given constants. Let Γ denote the set of all $\boldsymbol{\zeta} \in \mathbb{R}^m$ satisfying (1.6).

Substituting (1.5) into (1.1) gives

$$\dot{\boldsymbol{x}}(t) = \tilde{\boldsymbol{f}}(t, \boldsymbol{x}(t), \boldsymbol{\zeta}), \quad t \in [0, T], \tag{1.7}$$

where

$$\tilde{\boldsymbol{f}}(t, \boldsymbol{x}(t), \boldsymbol{\zeta}) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{\varphi}(\boldsymbol{x}(t), \boldsymbol{\zeta})).$$

Let $\boldsymbol{x}(\cdot|\boldsymbol{\zeta})$ denote the solution of system (1.7) with the initial condition (1.2). Then the terminal constraints (1.3) become

$$\Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta})) = 0, \quad i = 1, \ldots, p. \tag{1.8}$$

Substituting the feedback control (1.5) into the continuous inequality constraints (1.4) gives

$$\tilde{h}_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta}) \le 0, \quad t \in [0, T], \quad j = 1, \ldots, q, \tag{1.9}$$

where

$$\tilde{h}_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta}) = h_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\varphi}(\boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})).$$

Let Λ denote the set of all $\boldsymbol{\zeta} \in \Gamma$ satisfying (1.8) and (1.9).

We now consider the problem of choosing the feedback control parameters ζ_k, $k = 1, \ldots, m$ to minimize the total system cost subject to the constraints (1.8) and (1.9).

Problem P Choose $\boldsymbol{\zeta} \in \Lambda$ to minimize the cost function

$$J(\boldsymbol{\zeta}) = \Phi(\boldsymbol{x}(T|\boldsymbol{\zeta}), \boldsymbol{\zeta}) + \int_0^T \mathcal{L}(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta}) dt,$$

where $\Phi : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $\mathcal{L} : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ are given continuously differentiable functions.

Note that (1.9) defines an infinite number of constraints—one for each point in $[0, T]$. Hence, Problem P can be viewed as a *semi-infinite optimization problem.* In the next section, we will use a novel exact penalty approach to approximate Problem P by a nonlinear programming problem.

2 AN EXACT PENALTY METHOD

Define a *constraint violation function* on Γ as follows:

$$\Delta(\boldsymbol{\zeta}) = \sum_{i=1}^{p} \Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))^2 + \sum_{j=1}^{q} \int_0^T \max\left\{\tilde{h}_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta}), 0\right\}^2 dt.$$

Clearly, $\Delta(\boldsymbol{\zeta}) = 0$ if and only if $\boldsymbol{\zeta} \in \Lambda$.

Let $\bar{\epsilon} > 0$ be a given constant. We consider the following *penalty function* defined on $\Gamma \times [0, \bar{\epsilon}]$:

$$G_\sigma(\boldsymbol{\zeta}, \epsilon) = \begin{cases} J(\boldsymbol{\zeta}), & \text{if } \epsilon = 0,\ \Delta(\boldsymbol{\zeta}) = 0, \\ J(\boldsymbol{\zeta}) + \epsilon^{-\alpha}\Delta(\boldsymbol{\zeta}) + \sigma\epsilon^{\beta}, & \text{if } \epsilon \in (0, \bar{\epsilon}], \\ \infty, & \text{if } \epsilon = 0,\ \Delta(\boldsymbol{\zeta}) \neq 0, \end{cases} \tag{2.1}$$

where $\epsilon \in [0, \bar{\epsilon}]$ is a new decision variable, α and β are fixed constants such that $1 \le \beta \le \alpha$, and $\sigma > 0$ is a penalty parameter.

In the penalty function (2.1), the last term $\sigma\epsilon^\beta$ is designed to penalize large values of ϵ, while the middle term $\epsilon^{-\alpha}\Delta(\boldsymbol{\zeta})$ is designed to penalize constraint violations. When σ is large, minimizing (2.1) forces ϵ to be small, which in turn causes $\epsilon^{-\alpha}$ to become large, and thus constraint violations are penalized very severely. Hence, minimizing the penalty function for large σ will likely lead to feasible points satisfying constraints (1.8) and (1.9). On this basis, we can approximate Problem P by the following *penalty problem.*

Problem Q Choose $(\boldsymbol{\zeta}, \epsilon) \in \Gamma \times (0, \bar{\epsilon}]$ to minimize the penalty function

$$G_\sigma(\boldsymbol{\zeta}, \epsilon) = J(\boldsymbol{\zeta}) + \epsilon^{-\alpha}\Delta(\boldsymbol{\zeta}) + \sigma\epsilon^{\beta}.$$

Problem Q only involves bound constraints and is therefore much easier to solve than Problem P. In the next section, we will present some convergence results that formally link Problem Q with Problem P. First, however, we discuss how to solve Problem Q.

Problem Q can be viewed as a nonlinear programming problem in which the feedback control parameters ζ_k, $k = 1, \ldots, m$ and the new decision variable ϵ need to be chosen to minimize the penalty function G_σ. Numerical algorithms for solving such problems typically use the gradient of the cost function to compute *descent directions* that lead to more profitable areas of the feasible region (Luenberger D. G. (2008)). Notice, however, that $\boldsymbol{\zeta}$ influences G_σ *implicitly* through the dynamic system (1.7), and thus computing the gradient of G_σ is not straightforward. Nevertheless, the techniques developed by Vincent and Grantham (Vincent (1981)) and Loxton et al. (Loxton R. (2008)) can be used to derive formulae for the gradient of G_σ. This is described below.

First, for each $k = 1, \ldots, m$, consider the following *variational system*:

$$\dot{\boldsymbol{\phi}}^k(t) = \frac{\partial \tilde{\boldsymbol{f}}(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(t) + \frac{\partial \tilde{\boldsymbol{f}}(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \zeta_k}, \quad t \in [0, T], \tag{2.2}$$

$$\boldsymbol{\phi}^k(0) = \boldsymbol{0}. \tag{2.3}$$

Let $\boldsymbol{\phi}^k(\cdot|\boldsymbol{\zeta})$ denote the solution of the variational system (2.2)-(2.3). We have the following result.

Theorem 2.1 *For each* $k = 1, \ldots, m$,

$$\frac{\partial \boldsymbol{x}(t|\boldsymbol{\zeta})}{\partial \zeta_k} = \boldsymbol{\phi}^k(t|\boldsymbol{\zeta}), \quad t \in [0, T].$$

Proof. First, note that

$$\frac{\partial}{\partial \zeta_k}\{\boldsymbol{x}(0|\boldsymbol{\zeta})\} = \frac{\partial}{\partial \zeta_k}\{\boldsymbol{x}^0\} = \boldsymbol{0}. \tag{2.4}$$

Thus, $\partial \boldsymbol{x}(\cdot|\boldsymbol{\zeta})/\partial \zeta_k$ satisfies the initial condition (2.3).

Now, by (1.7),

$$\boldsymbol{x}(t|\boldsymbol{\zeta}) = \boldsymbol{x}(0|\boldsymbol{\zeta}) + \int_0^t \tilde{\boldsymbol{f}}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})ds = \boldsymbol{x}^0 + \int_0^t \tilde{\boldsymbol{f}}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})ds, \quad t \in [0, T]. \tag{2.5}$$

It can be shown that $\boldsymbol{x}(t|\boldsymbol{\zeta})$ is a continuously differentiable function of ζ_k, $k = 1, \ldots, m$ (Loxton R. (2011)). Hence, by using Leibniz's rule to differentiate (2.5) with respect to ζ_k, we obtain

$$\frac{\partial \boldsymbol{x}(t|\boldsymbol{\zeta})}{\partial \zeta_k} = \int_0^t \left\{ \frac{\partial \tilde{\boldsymbol{f}}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}(s|\boldsymbol{\zeta})}{\partial \zeta_k} + \frac{\partial \tilde{\boldsymbol{f}}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \zeta_k} \right\} ds, \quad t \in [0, T], \tag{2.6}$$

where

$$\frac{\partial \tilde{\boldsymbol{f}}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \boldsymbol{x}} = \frac{\partial \boldsymbol{f}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\varphi}(\boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta}))}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{f}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\varphi}(\boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta}))}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{\varphi}(\boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \boldsymbol{x}}$$

and

$$\frac{\partial \tilde{\boldsymbol{f}}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \zeta_k} = \frac{\partial \boldsymbol{f}(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\varphi}(\boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta}))}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{\varphi}(\boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \zeta_k}.$$

Differentiating (2.6) with respect to time yields

$$\frac{d}{dt}\left\{ \frac{\partial \boldsymbol{x}(t|\boldsymbol{\zeta})}{\partial \zeta_k} \right\} = \frac{\partial \tilde{\boldsymbol{f}}(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}(t|\boldsymbol{\zeta})}{\partial \zeta_k} + \frac{\partial \tilde{\boldsymbol{f}}(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial \zeta_k}, \quad t \in [0, T]. \tag{2.7}$$

Equations (2.4) and (2.7) show that $\partial \boldsymbol{x}(\cdot|\boldsymbol{\zeta})/\partial \zeta_k$ is the solution of the variational system (2.2)-(2.3). This completes the proof. □

We are now ready to derive formulae for the gradient of G_σ.

Theorem 2.2 *The partial derivatives of G_σ are given by*

$$\frac{\partial G_\sigma(\boldsymbol{\zeta},\epsilon)}{\partial \zeta_k} = \frac{\partial J(\boldsymbol{\zeta})}{\partial \zeta_k} + \epsilon^{-\alpha}\frac{\partial \Delta(\boldsymbol{\zeta})}{\partial \zeta_k}, \quad k=1,\ldots,m, \tag{2.8}$$

and

$$\frac{\partial G_\sigma(\boldsymbol{\zeta},\epsilon)}{\partial \epsilon} = -\alpha\epsilon^{-\alpha-1}\Delta(\boldsymbol{\zeta}) + \beta\sigma\epsilon^{\beta-1}, \tag{2.9}$$

where

$$\frac{\partial J(\boldsymbol{\zeta})}{\partial \zeta_k} = \frac{\partial \Phi(\boldsymbol{x}(T|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(T|\boldsymbol{\zeta}) + \frac{\partial \Phi(\boldsymbol{x}(T|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k} + \int_0^T \left\{\frac{\partial \mathcal{L}(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(t|\boldsymbol{\zeta}) + \frac{\partial \mathcal{L}(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k}\right\}dt,$$

$$\begin{aligned}\frac{\partial \Delta(\boldsymbol{\zeta})}{\partial \zeta_k} &= 2\sum_{i=1}^{p}\Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))\frac{\partial \Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(T|\boldsymbol{\zeta}) \\ &\quad + 2\sum_{j=1}^{q}\int_0^T \max\left\{\tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta}),0\right\}\left\{\frac{\partial \tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^j(t|\boldsymbol{\zeta}) + \frac{\partial \tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k}\right\}dt.\end{aligned}$$

Proof. From Theorem 2.1, we have

$$\begin{aligned}\frac{\partial J(\boldsymbol{\zeta})}{\partial \zeta_k} &= \frac{\partial \Phi(\boldsymbol{x}(T|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}(T|\boldsymbol{\zeta})}{\partial \zeta_k} + \frac{\partial \Phi(\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k} + \int_0^T \left\{\frac{\partial \mathcal{L}(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}(t|\boldsymbol{\zeta})}{\partial \zeta_k} + \frac{\mathcal{L}(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k}\right\}dt \\ &= \frac{\partial \Phi(\boldsymbol{x}(T|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(T|\boldsymbol{\zeta}) + \frac{\partial \Phi(\boldsymbol{x}(T|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k} + \int_0^T \left\{\frac{\partial \mathcal{L}(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(t|\boldsymbol{\zeta}) + \frac{\mathcal{L}(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k}\right\}dt.\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial \Delta(\boldsymbol{\zeta})}{\partial \zeta_k} &= 2\sum_{i=1}^{p}\Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))\frac{\partial \Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(T|\boldsymbol{\zeta}) \\ &\quad + 2\sum_{j=1}^{q}\int_0^T \max\left\{\tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta}),0\right\}\left\{\frac{\partial \tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}(t|\boldsymbol{\zeta})}{\partial \zeta_k} + \frac{\partial \tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k}\right\}dt \\ &= 2\sum_{i=1}^{p}\Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))\frac{\partial \Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(T|\boldsymbol{\zeta}) \\ &\quad + 2\sum_{j=1}^{q}\int_0^T \max\left\{\tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta}),0\right\}\left\{\frac{\partial \tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \boldsymbol{x}}\boldsymbol{\phi}^k(t|\boldsymbol{\zeta}) + \frac{\partial \tilde{h}_j(t,\boldsymbol{x}(t|\boldsymbol{\zeta}),\boldsymbol{\zeta})}{\partial \zeta_k}\right\}dt.\end{aligned}$$

Equation (2.8) follows immediately from these equations. Equation (2.9) is obtained using standard differentiation rules. □

On the basis of Theorem 2.2, we can compute the gradient of G_σ using the following procedure: (i) Combine the original control system with the variational systems to form an expanded initial value problem; (ii) Solve this expanded initial value problem using a numerical integration method; (iii) Substitute the solution of the initial value problem into (2.8) and (2.9). This procedure can be integrated with a standard gradient-based optimization method—e.g. sequential quadratic programming (Nocedal J. (2006))—to solve Problem Q as a nonlinear programming problem.

3 CONVERGENCE RESULTS

In this section, we describe the mathematical theory relating Problem P with Problem Q. We begin with the following result proved by Lin et al. (Lin Q. (2012)).

Theorem 3.1 *Let $\{\sigma_l\}_{l=1}^{\infty}$ be an increasing sequence of penalty parameters such that $\sigma_l \to \infty$ as $l \to \infty$. Furthermore, let $(\boldsymbol{\zeta}^{l,*},\epsilon^{l,*})$ denote a global solution of Problem Q. Then the sequence $\{(\boldsymbol{\zeta}^{l,*},\epsilon^{l,*})\}_{l=1}^{\infty}$ has at least one limit point, and any limit point is a global solution of Problem P.*

Theorem 3.1 suggests that we can obtain a solution of Problem P by solving Problem Q sequentially for increasing values of the penalty parameter. As mentioned in the previous section, Problem Q is essentially a nonlinear programming problem that can be solved using standard numerical optimization techniques.

One disadvantage of Theorem 3.1 is that it requires the *global* solution of Problem Q. Problem Q is non-convex in general, and thus we will usually only be able to solve it locally. Nevertheless, by making some mild assumptions, one can show that a local solution of Problem Q converges to a local solution of Problem P as the penalty parameter increases.

We assume that for each feasible point $\boldsymbol{\zeta} \in \Lambda$ of Problem P, the following conditions are satisfied:

(A1) The vectors $\partial\Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))/\partial\boldsymbol{\zeta}$, $i = 1, \dots, p$ are linearly independent (when $p \neq 0$).

(A2) There exists a vector $[\eta_1, \dots, \eta_m]^\top \in \mathbb{R}^m$ and negative real numbers $\vartheta_1 < 0$ and $\vartheta_2 < 0$ such that

$$\sum_{k=1}^{m} \eta_k \frac{\partial\Psi_i(\boldsymbol{x}(T|\boldsymbol{\zeta}))}{\partial\zeta_k} = 0, \quad i = 1, \dots, p,$$

$$\sum_{k=1}^{m} \eta_k \left\{ \frac{\partial\tilde{h}_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial\boldsymbol{x}} \boldsymbol{\phi}^k(t|\boldsymbol{\zeta}) + \frac{\partial\tilde{h}_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}), \boldsymbol{\zeta})}{\partial\zeta_k} \right\} < \vartheta_1,$$

$$t \in \{\, s \in [0, T] : \tilde{h}_j(s, \boldsymbol{x}(s|\boldsymbol{\zeta}), \boldsymbol{\zeta}) \geq \vartheta_2 \,\}, \quad j = 1, \dots, q,$$

$$\eta_k \begin{cases} > 0, & \text{if } \zeta_k = a_k, \\ < 0, & \text{if } \zeta_k = b_k. \end{cases}$$

(A3) There exists a constant $L > 0$ and a neighbourhood $\mathcal{N}$ of $\boldsymbol{\zeta}$ such that for each $j = 1, \dots, q$,

$$\max\left\{\tilde{h}_j(t, \boldsymbol{x}(t|\boldsymbol{\zeta}'), \boldsymbol{\zeta}'), 0\right\}^2 \leq L \int_0^T \max\left\{\tilde{h}_j(s, \boldsymbol{x}(s|\boldsymbol{\zeta}'), \boldsymbol{\zeta}'), 0\right\}^2 ds, \quad (\boldsymbol{\zeta}', t) \in \mathcal{N} \times [0, T].$$

Under Assumptions (A1)-(A3), we have the following result proved by Lin et al. (Lin Q. (2012)).

Theorem 3.2 *Let $\{\sigma_l\}_{l=1}^{\infty}$ be an increasing sequence of penalty parameters such that $\sigma_l \to \infty$ as $l \to \infty$. Furthermore, let $(\boldsymbol{\zeta}^{l,*}, \epsilon^{l,*})$ denote a local solution of Problem Q. Suppose that $\{G_{\sigma_l}(\boldsymbol{\zeta}^{l,*}, \epsilon^{l,*})\}_{l=1}^{\infty}$ is bounded. Then there exists a positive integer l' such that for each $l \geq l'$, $\boldsymbol{\zeta}^{l,*}$ is a local solution of Problem P.*

Theorem 3.2 implies that when the penalty parameter σ is sufficiently large, the values of the feedback control parameters in a locally optimal solution for Problem Q will also be locally optimal for Problem P. On this basis, we propose the following algorithm for solving Problem P:

(1) Choose $\boldsymbol{\zeta}^0 \in \Gamma$ (initial guess), $\sigma^0 > 0$ (initial penalty parameter), $\rho > 0$ (tolerance), and $\sigma_{\max} > \sigma^0$ (upper bound for the penalty parameter).

(2) Set $\bar{\epsilon} \to \epsilon^0$ and $\sigma^0 \to \sigma$.

(3) Starting with $(\boldsymbol{\zeta}^0, \epsilon^0)$ as the initial guess, use a nonlinear programming algorithm (e.g. sequential quadratic programming) to solve Problem Q. Let $(\boldsymbol{\zeta}^*, \epsilon^*)$ denote the local minimizer obtained.

(4) If $\epsilon^* < \rho$, then stop: take $\boldsymbol{\zeta}^*$ as a local solution of Problem P. Otherwise, set $10\sigma \to \sigma$ and go to Step 5.

(5) If $\sigma \leq \sigma_{\max}$, then set $(\boldsymbol{\zeta}^*, \epsilon^*) \to (\boldsymbol{\zeta}^0, \epsilon^0)$ and go to Step 3. Otherwise stop: the algorithm cannot find a solution of Problem P.

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112 AN AUGMENTED LAGRANGIAN TRUST REGION METHOD FOR SOLVING A SPECIAL CLASS OF NONLINEAR PROGRAMMING PROBLEMS

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Abstract: We study a trust region method for a special class of nonlinear programming problems. In each iteration, the trust region subproblem is generated based on the augmented Lagrangian function. In addition, we apply the affine scaling technique to the bound constraints. We propose effective update strategies for the related Lagrange multiplier and penalty parameters, which are different from those applied in conventional penalty methods. Furthermore, we apply our new method to solve test problems from the CUTEr collection.

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113 ϵ-OPTIMALITY CONDITIONS OF VECTOR OPTIMIZATION PROBLEMS WITH SET-VALUED MAPS BASED ON THE ALGEBRAIC INTERIOR IN REAL LINEAR SPACES

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Abstract: In this paper, firstly, the necessary and sufficient optimality conditions for ϵ-global properly efficient elements of set-valued optimization problems, respectively, were established in linear spaces. Secondly, an equivalent characterization of ϵ-global proper saddle point is presented. Finally, the necessary and sufficient conditions for ϵ-global properly saddle point of a Lagrangian set-valued map were obtained. The results in this paper generalize some known results in the literature.

Key words: Set-valued maps; Generalized cone subconvexlikeness; ϵ-global properly efficient element; ϵ-global proper saddle point.

1 INTRODUCTION

In recent decades, some authors have been interested in studying generalized convexity of set-valued maps to overcome the restriction of convexity of set-valued maps. Borwein (Borwein J.M (1977)) and Giannessi (Giannessi F (1984)) introduced cone convexity of set-valued maps. Based on Borwein and Giannessi's work, some authors (Rong W.D (2000), Li Z.F (1998), Yang X.M (2000), Sach P.H (2005), Yang X.M (2001)) introduced some new notions of generalized convexity such as cone convexlikeness, cone subconvexlikeness, generalized cone subconvexlikeness, ic-cone-convexlikeness and nearly cone subconvexlikeness. Under the assumption of the above generalized convexity, some optimality conditions were established.

We know that there is some relation between efficiency of solutions and optimality conditions. Sometimes, it is difficult to find the exact solutions of vector optimization problems with set-valued maps. To overcome the difficulty, some authors introduced different types of approximate solutions. Recently, with the development of set-valued analysis, some new progress about approximate solutions of vector optimization problems with set-valued maps has been made. Rong and Wu (Rong W.D (2000)) firstly introduced ϵ-weakly efficient solution of vector optimization problems with set-valued maps. Li et al. (Li T.Y (2007)) and Tuan (Tuan L.A (2010)) defined ϵ-strictly efficient solution and ϵ-Benson properly efficient solution of vector optimization problems with set-valued maps, respectively.

Note that, in the above mentioned papers, vector optimization problems with set-valued maps are restricted in the setting of topological linear spaces or locally convex spaces. Since linear spaces are much wider than topological linear spaces or locally convex spaces, it is necessary to establish new optimality conditions in linear spaces. Under the assumption of generalized convexity of set-valued maps, Li (Li Z.M (1999)) and Hernández et al. (Hernández E (2007)) were devoted to establishing optimality conditions of vector optimization problems with set-valued maps in linear spaces.

In this paper, our purpose is to derive some ϵ-optimality conditions of vector optimization problems with set-valued maps by using the notion of ϵ-global proper efficiency introduced by Zhou and Peng (Zhou Z.A (2012)) in linear spaces. This paper is organized as follows. In Section 2, we give some preliminaries, including notations and lemmas. In Section 3, under the assumption of generalized cone subconvexlikeness, we present the necessary and sufficient optimality conditions for ϵ-global properly efficient elements of set-valued optimization problems in linear spaces. In Section 4, we obtain the necessary and sufficient conditions for ϵ-global proper saddle points of a Lagrangian set-valued map.

2 ϵ-OPTIMALITY CONDITIONS

In this section, we will present the necessary and sufficient optimality conditions for ϵ-global properly efficient elements of set-valued optimization problems.

We denote by $L(Z,Y)$ the set of all linear operators from Z to Y. A subset $L^+(Z,Y)$ of $L(Z,Y)$ is defined as $L^+(Z,Y) := \{T \in L(Z,Y) | T(D) \subseteq C\}$. The Lagrangian set-valued map of (VP) is defined by

$$L(x,T) := F(x) + T(G(x)), \forall (x,T) \in A \times L^+(Z,Y).$$

Consider the following unconstrained vector optimization problem with set-valued maps:

$$(\mathrm{UVP})_T \quad \mathrm{Min}\ L(x,T) \quad \text{subject to} \quad (x,T) \in A \times L^+(Z,Y).$$

Let $I(x) = F(x) \times G(x), \forall x \in A$. By Definition 2.5, the set-valued map $I : A \rightrightarrows Y \times Z$ is generalized $C \times D$-subconvexlike on A iff $cone(I(A)) + cor(C \times D)$ is a convex set in $Y \times Z$.

Theorem 3.1 Let $\epsilon \in C, \overline{x} \in S$ and $0 \in G(\overline{x})$. Suppose that the following conditions hold:
(i) $(\overline{x}, \overline{y})$ is an ϵ-global properly efficient element of (VP);
(ii) $\overline{I}(x)$ is generalized $C \times D$-subconvexlike on A, where $\overline{I}(x) = (F(x) - \overline{y} + \epsilon) \times G(x)$;
(iii) $vcl(cone(G(A) + D)) = Z$;
Then, there exists $\overline{T} \in L^+(Z,Y)$ and a nontrivial pointed convex cone C' with $C \setminus \{0\} \subseteq cor(C')$ such that $-\overline{T}(G(\overline{x}) \cap (-D)) \subseteq (cor(C) \cup \{0\}) \setminus (\epsilon + cor(C'))$ and $(\overline{x}, \overline{y})$ is an ϵ-global properly efficient element of $(\mathrm{UVP})_{\overline{T}}$.

We can give an example to illustrate Theorem 3.1.

Remark 3.1 Condition (iii) was introduced by Sach (Sach P.H (2005)) in locally convex spaces. Note that Condition (iii) can be replaced by the condition $0 \in cor(G(A) + D)$. We assert that the latter implies the former. Indeed, let $0 \in cor(G(A) + D)$. Clearly, $vcl(cone(G(A) + D)) \subseteq Z$. we only show that $Z \subseteq vcl(cone(G(A) + D))$. Let $z \in Z$. Since $0 \in cor(G(A) + D)$, there exists $\gamma > 0$ such that $\gamma z \in G(A) + D$. Clearly, $z \in \frac{1}{\gamma}(G(A) + D) \subseteq vcl(cone(G(A) + D))$. Hence, $vcl(cone(G(A) + D)) = Z$. However, we can give an example to show that Condition (iii) does not imply the condition $0 \in cor(G(A) + D)$.

Remark 3.2 If $\epsilon = 0$, then $-\overline{T}(G(\overline{x}) \cap (-D)) = \{0\}$. Clearly, $0 \in -\overline{T}(G(\overline{x}) \cap (-D)) \subseteq -\overline{T}(G(\overline{x}))$. Therefore, $0 \in \overline{T}(G(\overline{x}))$. Thus, from the proof of Theorem 3.1, the condition that $0 \in G(\overline{x})$ can be droppeded.

Theorem 3.2 Let $\epsilon \in C, \overline{x} \in S$ and $\overline{y} \in F(\overline{x})$. If there exists $\overline{T} \in L^+(Z,Y)$ such that $(\overline{x}, \overline{y})$ is an ϵ-global properly efficient element of $(\mathrm{UVP})_{\overline{T}}$, then $(\overline{x}, \overline{y})$ is an ϵ-global properly efficient element of (VP).

Remark 3.3 If $\epsilon = 0$ and the linear spaces Y and Z are replaced by the locally convex spaces, then Theorem 3.2 reduces to Theorem 3.2 in (Yu G.L. (2009)).

3 ϵ-GLOBAL PROPER SADDLE POINTS

In (Yu G.L. (2009)), Yu and Liu introduced the notion of the global proper saddle point of the Lagrangian set-valued map $L(x,T)$ in locally convex spaces. Next, we will introduce a new notion called ϵ-global proper saddle point of the Lagrangian set-valued map $L(x,T)$ in linear spaces.

Definition 4.1 $(\overline{x}, \overline{T}) \in A \times L^+(Z,Y)$ is called an ϵ-global proper saddle point of the Lagrangian set-valued map $L(x,T)$ iff

$$L(\overline{x}, \overline{T}) \cap \epsilon\text{-}GMin(\bigcup_{x \in A} L(x, \overline{T}), C) \cap \epsilon\text{-}GMax(\bigcup_{T \in L^+(Z,Y)} L(\overline{x}, T), C) \neq \emptyset.$$

The following proposition is an important equivalent characterization for an ϵ-global proper saddle point of the Lagrangian set-valued map $L(x,T)$.

Proposition 4.1 Let $vcl(D) = D$ and $\epsilon \in C$. Then, $(\overline{x}, \overline{T}) \in A \times L^+(Z,Y)$ is an ϵ-global proper saddle point of the Lagrangian set-valued map $L(x,T)$ iff there exist $\overline{y} \in F(\overline{x}), \overline{z} \in G(\overline{x})$ and a nontrivial pointed convex cone C' with $C \setminus \{0\} \subseteq cor(C')$ such that
(i) $\overline{y} + \overline{T}(\overline{z}) \in \epsilon\text{-}GMin(\bigcup_{x \in A} L(x, \overline{T}), C)$;

(ii) $G(\overline{x}) \subseteq -D$;
(iii) $-\overline{T}(\overline{z}) \in C \setminus (\epsilon + C' \setminus \{0\})$;
(iv) $(F(\overline{x}) - \overline{y} - \overline{T}(\overline{z}) - \epsilon) \cap (C' \setminus \{0\}) = \emptyset$.

We can give an example to illustrate the sufficiency of Proposition 4.1.

Remark 4.1 When we prove the sufficiency of Proposition 4.1, we use the condition $G(\overline{x}) \subseteq -D$. The condition $G(\overline{x}) \cap (-D) \neq \emptyset$ is weaker than the condition $G(\overline{x}) \subseteq -D$, but we can give an example to show that the condition $G(\overline{x}) \subseteq -D$ cannot be replaced by the condition $G(\overline{x}) \cap (-D) \neq \emptyset$.

Theorem 4.1 Let $vcl(D) = D, \epsilon \in C$ and $0 \in G(\overline{x})$. If $(\overline{x}, \overline{T}) \in A \times L^+(Z, Y)$ is an ϵ-globally proper saddle point of the Lagrangian set-valued map $L(x, T)$, then there exist $\overline{y} \in F(\overline{x})$ and $\overline{z} \in G(\overline{x})$ such that $(\overline{x}, \overline{y})$ is an $\overline{\epsilon}$-global properly efficient element of (VP), where $\overline{\epsilon} = \epsilon - \overline{T}(\overline{z})$.

Remark 4.2 If $\epsilon = 0$ and the linear spaces Y and Z becomes the locally convex spaces, then Theorem 4.1 reduces to Theorem 4.2 in (Yu G.L. (2009)).

Finally, under the assumption of generalized cone subconvexlikeness, a sufficient condition of the existence of ϵ-global proper saddle point of the Lagrangian set-valued map $L(x, T)$ will be established.

Theorem 4.2 Let $vcl(D) = D, \epsilon \in C$ and $0 \in G(\overline{x})$. Suppose that the following conditions hold:
(i) $(\overline{x}, \overline{y})$ is a ϵ-global properly efficient element of (VP);
(ii) $\overline{I}(x)$ is generalized $C \times D$-subconvexlike on A, where $\overline{I}(x) = (F(x) - \overline{y} + \epsilon) \times G(x)$;
(iii) $vcl(cone(G(A) + D)) = Z$;
(v) $\overline{y} \in \epsilon\text{-}GMax(\bigcup_{T \in L^+(Z,Y)} L(\overline{x}, T), C)$.

Then, there exists $\overline{T} \in L^+(Z, Y)$ such that $(\overline{x}, \overline{T})$ is an ϵ-global proper saddle point of L.

Remark 4.3 According to Theorems 3.1 and the sufficiency of Proposition 4.1, if $\epsilon = 0$ and $G(\overline{x}) \subseteq -D$, then the condition that $0 \in G(\overline{x})$ can be dropped and condition (v) can be replaced by the condition that there exist $\overline{y} \in F(\overline{x}), \overline{z} \in G(\overline{x})$ and a nontrivial pointed convex cone C' with $C \setminus \{0\} \subseteq cor(C')$ such that $(F(\overline{x}) - \overline{y}) \cap (C' \setminus \{0\}) = \emptyset$.

4 CONCLUSION

In this paper, we consider ϵ-optimality conditions of vector optimization problems with set-valued maps based on the algebraic interior in real linear spaces. Using a separation theorem characterized by the algebraic interior, we obtain the necessary and sufficient optimality conditions for ϵ-global properly efficient element of set-valued optimization problem. By a strong separation theorem characterized by the algebraic interior and the vector closure, we give an equivalent characterization of an ϵ-global proper saddle point for the Lagrangian set-valued map $L(x, T)$. We also establish the relation between an ϵ-global proper saddle point of $L(x, T)$ and an ϵ-global properly efficient element of (VP). Our results in this paper generalize some known results in the literature. Following the line in this paper, whether other kinds of proper efficiency such as Henig proper efficiency and super proper efficiency can be discussed in linear spaces is an interesting topic.

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114 THE LAGRANGIAN DUALITY FOR VECTOR OPTIMIZATION PROBLEM WITH SET-VALUED MAPPINGS

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Abstract: In this paper, by using a alternative theorem, we establish Lagrangian conditions and duality results for set-valued vector optimization problems when the objective and constant are nearly cone-subconvexlike multifunctions in the sense of E-weak minimizer.

Key words: Set-valued vector optimization problems; Lagrangian duality; Alternative theorem; E-weak minimizer; Nearly cone-subconvexlikeness.

1 INTRODUCTION

Optimality conditions and duality theorems for optimization problems of single-valued functions satisfying convexity or weaker conditions have been studied by many authors, see ([Corley 1981]), ([Borwein 1977]), ([Hayashi and Komiya 1982]), ([Craven and Jeyakumar 1987]), ([Jahn 1986]), ([Jeyakumar and Oettli 1993]), ([Illes and Kassay 1999]), ([Khanh and Nuong 1989]). In particular, in works of ([Hayashi and Komiya 1982]), ([Craven and Jeyakumar 1987]), ([Jahn 1986]), ([Jeyakumar and Oettli 1993]), Lagrangian conditions and duality theorems for convexlike functions and a class of quasiconvex functions were discussed.

In recent years, many authors have generalized the single-valued functions to set-valued mappings, for its extensive applications in many fields such as mathematical programming ([Aubin and Frankowska 1990]) economics ([Klein and Thompson 1984]) and differential inclusions ([Robinson 1979]). In particular, Lagrangian conditions and duality theorems were discussed when the objective and constraint are convex, preinvex, subconvexlike and nearly convexlike set-valued mappings in ([Corley 1987]), ([Luc 1989]), ([Bhatia and Mehra 1997]), ([Li and Chen 1997]), ([Rong and Wu 2000]) and ([Song,1996]), respectively.

Recently, Yang, Li and Wang ([Yang et al. 2001]) introduced a new class of generalized convexity for set-valued functions, called nearly cone-subconvexlike, which is a generalization of the set-valued functions mentioned above. They obtained a alternative theorem, a Lagrangian multiplier theorem and two scalarization theorems. Sach ([Sach 2003]) showed some characterizations of nearly cone-subconvexlikeness and established some saddle theorems under nearly cone-subconvexlikeness conditions for set-valued vector optimization. Some related works, we refer to ([Peng and Yang 2001]).

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In this paper, under nearly cone-subconvexlikeness, Lagrangian conditions and duality results for set-valued vector optimization problems are obtained in the sense of E-weak minimizer by using the alternative theorem of Yang, Li and Wang ([Yang et al. 2001]) .

2 PRELIMINARIES

Let $F : X \to 2^Y$ and $G : X \to 2^Z$ be two set-valued mappings with nonempty value. We consider the following vector optimization problem with set-valued mappings:

$$\text{(P)} \qquad \begin{array}{ll} \min & F(x) \\ s.t. & G(x) \cap (-D) \neq \emptyset. \end{array}$$

Let K denote the set of all feasible points for the problem (P), i.e.,

$$K = \{x \in X \mid G(x) \cap (-D) \neq \emptyset\}.$$

Let $E \subset Y$ be a nonempty subset, and let $\varepsilon \in C$.

Definition 2.1

(i) A point $x_0 \in K$ is said to be a weak efficient solution of problem (P), if there exists $y_0 \in F(x_0)$ such that $(F(K) - y_0) \cap (-intC) = \emptyset$. The pair (x_0, y_0) is said to be a weak minimizer of problem (P).

(ii) A point $x_0 \in K$ is said to be an ε-weak efficient solution of problem (P), if there exists $y_0 \in F(x_0)$ such that $(F(K) - y_0 + \varepsilon) \cap (-intC) = \emptyset$. The pair (x_0, y_0) is said to be ε-weak minimizer of problem (P).

(iii) A point $x_0 \in K$ is said to be an E-weak efficient solution of problem (P), if there exists $y_0 \in F(x_0)$ such that $(F(K) - y_0 + E) \cap (-intC) = \emptyset$. The pair (x_0, y_0) is said to be E-weak minimizer of problem (P).

It is clear that the set of weak efficient solutions is contained in the set of ε-weak efficient solutions. Some relationships between ε-weak efficient solutions and E-weak efficient solutions were investigated in ([Huang 2002]) as follows:

(i) if $E = \{\varepsilon\}$, then an E-weak efficient solution of problem (P) becomes a ε-weak efficient solution of problem (P);

(ii) if x_0 is an E-weak efficient solution of problem (P) and there exists $\varepsilon' \in E$ such that $\varepsilon - \varepsilon' \in C$, then x_0 is an ε-weak efficient solution of problem (P);

(iiii) if x_0 is an ε-weak efficient solution of problem (P) and $E - \varepsilon \subset C$, then x_0 is an E-weak efficient solution of problem (P).

Definition 2.3 Let X be a convex set. A set-valued function $F : X \to 2^Y$ is said to be C-convex on X if, for any $x_1, x_2 \in X$ and $\lambda \in [0, 1]$, one has

$$\lambda F(x_1) + (1 - \lambda)F(x_2) \subset F(\lambda x_1 + (1 - \lambda)x_2) + C.$$

Definition 2.4 ([Li and Chen 1997])

(i) A set-valued function $F : X \to 2^Y$ is said to be C-convexlike on X if, for all $x_1, x_2 \in X$ and $\lambda \in (0, 1)$,

$$\lambda F(x_1) + (1 - \lambda)F(x_2) \subset F(X) + C.$$

(ii) A set-valued function $F : X \to 2^Y$ is said to be C-subconvexlike on X if, there exists $\theta \in intC$ such that for all $x_1, x_2 \in X$, $\lambda \in (0, 1)$, and $\varepsilon > 0$,

$$\varepsilon\theta + \lambda F(x_1) + (1 - \lambda)F(x_2) \subset F(X) + C.$$

Remark 2.1 In Definition 2.3, X may be a nonconvex set.

Remark 2.2 From ([Li and Chen 1997]), we know that

(i) F is C-convexlike on X if and only if $F(X)+C$ is a convex set;

(ii) F is C-subconvexlike on X if and only if $F(X)+intC$ is a convex set.

Lemma 2.1 If $F: X \to 2^Y$ is C-convexlike on X, then F is C-subconvexlike on X.

Definition 2.4 ([Song,1996]) A set-valued function $F: X \to 2^Y$ is said to be nearly C-convexlike on X if $cl(F(X)+C)$ is a convex set.

Remark 2.3 If $F(X)+intC$ is a convex set, then $cl(F(X)+C)$ is a convex set, because $cl(F(X)+C) = cl(F(X)+intC)$.

In order to prove Theorem 2.1, we need the following lemma.

Lemma 2.2 ([Breckner and Kassay 1997]) Let C be a convex cone in Y with $intC \neq \emptyset$, and let S be a subset of Y. Then

$$int[cl(S+C)] = S + intC.$$

Theorem 2.1 If $F: X \to 2^Y$ is nearly C-convexlike on X, then $F(X)+intC$ is a convex set.
Proof. Since F is nearly C-convexlike on X, then $cl(F(X)+C)$ is a convex set. Noting that the interior of a convex set is convex, it follows that $int[cl(F(X)+C)]$ is covex. By Lemma 2.2, we have that $F(X)+intC$ is a convex set. This completes the proof. □
Corollary 2.1 If $F: X \to 2^Y$ is nearly C-convexlike on X if and only if $F(X)+intC$ is convex.

Definition 2.5 ([Yang et al. 2001]) A set-valued function $F : X \to 2^Y$ is said to be nearly C-subconvexlike on X if and only if $clcone(F(X)+C)$ is a convex set.

3 MAIN RESULTS

In this section, let $L(Z,Y)$ denote the set of all linear continuous operators $\Lambda : Z \to Y$ with $\Lambda(D) \subset C$ and $E \subset intC$ be a subset.

Theorem 3.1 Let $intC \neq \emptyset$ and $G(K) \cap (-intD) \neq \emptyset$. Assume that set-valued function $(F - y_0 + E, G)$ is nearly $(C \times D)$-subconvexlike on K. If (x_0, y_0) is E-weak minimizer of problem (P), then there exists $\Lambda \in L(Z,Y)$ such that (x_0, y_0) is E-weak minimizer of the following problem:

$$(\overline{P}) \quad \min_{x\in K} \quad (F(x) + \Lambda(G(x))$$

and

$$-\Lambda(G(x_0) \cap (-D)) \subset (intC \cup \{0\}) \setminus (E + intC).$$

Now, we consider the dual problem. Define a set-valued mapping $\Phi : L(Z,Y) \to 2^Y$ by

$$\Phi(\Lambda) = \{y \mid \exists x \in K \text{ such that } (x,y) \text{ is E-weak minimizer of problem } \overline{P}\}.$$

Consider the following maximum problem:

$$\begin{array}{lll} \text{(DP)} & \max & \Phi(\Lambda) \\ & s.t. & \Lambda \in L(Z,Y). \end{array}$$

A point $\Lambda \in L(Z,Y)$ is said to be a feasible point of problem (DP) if $\Phi(\Lambda) \neq \emptyset$. We say that (Λ_0, y_0) is E-weak maximizer of problem (DP) if Λ_0 is a feasible point of problem (DP), $y_0 \in \Phi(\Lambda_0)$, and there exists no feasible point $\Lambda \in L(Z,Y)$ such that

$$(y_0 - \Phi(\Lambda) + E) \cap (-intC) \neq \emptyset.$$

Theorem 3.2 (E-weak duality). If Λ_0 is a feasible point of problem (DP) and x_0 is a feasible point of problem (P), then

$$(F(x_0) - \Phi(\Lambda_0) + E) \cap (-intC) = \emptyset.$$

Theorem 3.3 (E-strong duality). Let $(F - y_0 + E, G)$ be nearly $(C \times D)$-subconvexlike on K. If (x_0, y_0) is E-weak minimizer of problem (P) and $G(K) \cap (-intD) \neq \emptyset$, then there exists $\Lambda_0 \in L(Z, Y)$ such that (Λ_0, y_0) is E-weak maximizer of problem (DP).

Remark 3.1

(i) If $E = \{\varepsilon\}$, F and G are subconvexlike, then Theorems 3.2 and 3.3 reduce to Theorems 5.1 and 5.2 of ([Rong and Wu 2000]) ;

(ii) If $E = \{0\}$, F and G are nearly convexlike, then Theorems 3.2 and 3.3 reduce to Theorems 4.5 of ([Song,1996]).

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115 APPROXIMATE PROPER EFFICIENT SOLUTIONS IN THE VECTOR OPTIMIZATION PROBLEMS WITH SET-VALUED MAPS

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Abstract: In this paper, we characterize approximate proper efficient solutions of vector optimization with set-valued maps. We present scalarization theorems for approximate proper efficient solutions with generalized cone subconvexlike set-valued maps and Lagrange multiplier theorems under generalized Slater constraint qualification.

Key words: Vector optimization problem with set-valued maps; Approximate proper efficient solutions; Generalized cone subconvexlike; Scalarizations; Lagrange multiplier theorems.

1 INTRODUCTION

In recent years, the concept of approximate solutions for vector optimization have been studied in several frameworks and with kinds of purposes. The first notions of approximation was introduced by Kutateladze [1] and have been used to establish vector variational principle, approximate Kuhn-Tucker type conditions and approximate duality theorems, etc.(see [2]-[4]). However, under Kutateladze's ϵ-efficiency concept, the approximate solutions so obtained are not metrically consistent. Indeed, it is possible to obtain a feasible sequence (x_n) of ϵ_n-efficient solutions such that $x_n \to x_0$, yet $f(x_0)$ is far from the image solution set as $\epsilon_n \to 0$ (see [5]). For this reason, several authors have proposed a number of other ϵ-efficiency concepts (see, for example, White [6]; Helbig [7]; Tanaka [8]). In [9,10], Gutierrez et al introduced new concepts of ϵ-efficiency, which extend and unify various existing notions of approximate solutions (Kutateladze [1],White [6], Helbig [7] and Tanaka [8]. More recently, motivated by the approximate efficient solutions in [9] and [10], Gao et al [11, 12] introduced a new class of approximate Benson-proper efficient solutions and a unified approximate solution for multiobjective optimization problems, and presented scalarizations theorems, existences results and Lagrange multiplier theorems. Based on the proper approximate solution in [11] , Gutierrez et al [13] presented scalarizations, Lagrange multiplier theorems and Lagrange saddle point theorems for approximate Benson-proper efficient solutions of vector optimization problems. In the last years, these concepts allowed one to obtain Ekeland's variational principle and well-posedness properties in vector optimization problems ([5], [15]-[18], multiplier rules for approximate solutions ([19]-[22]), and approximate saddle points theorems ([23]).

In the past years, necessary conditions for efficiency, weak efficiency, proper efficiency and other results related to optimization theory such as alternative theorems have been developed in several papers under some generalized convexity functions: cone-convexlike ([24]), generalized subconvexlike [25], generalized cone-subconvexlike [26] and near cone subconvexlike [27]. These generalized convexity functions can be applied successfully to derive necessary conditions for weakly efficient solutions and Benson properly efficient solutions in terms of Lagrange multipliers and saddle points([11]-[13], [28]-[33]). Recently, Gao et al [12] and Gutierrez et al [13] introduced a new notions of generalized convexity, which are more weaker than nearly subconvexlikeness, and presented linear scalarizatons and approximate Lagrange saddle point theorems for approximate solutions of vector optimization problems.

Motivated the works in [9], [11] and [13], we consider approximate proper efficient solutions for set-valued optimization problems. We obtain linear scalarizations and Lagrange multiplier theorems of proper approximate efficient solutions for set-valued optimization problems.

The paper is structured as follows. In Section 2, some basic notions and approximate proper efficient solution for set-valued optimization problems are introduced. In Section 3, generalized subconvexlike function for set-valued maps are defined. And we obtain several linear scalarization theorems for approximate proper efficient solution in set-valued optimization problems with generalized subconvexlike set-valued maps. In Section 4, under the slater constraint qualification([38]), Lagrange multiplier theorem for approximate proper efficient solution in set-valued optimization problems are obtained.

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116 AN OPTIMAL CONTROL PROBLEM FOR A SYSTEM OF ELLIPTIC HEMIVARIATIONAL INEQUALITIES

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Abstract: In this paper we deal with a system of two hemivariational inequalities which is a variational formulation of a boundary value problem for two coupled elliptic partial differential equations. The boundary conditions in the problem are described by the Clarke subdifferential multivalued and nonmonotone laws. First, we provide the results on existence and uniqueness of a weak solution to the system. Then we consider an optimal control problem for the system, we prove the continuous dependence of a solution on the control variable, and establish the existence of optimal solutions. Finally, we illustrate the applicability of the results in a study of a mathematical model which describes the static frictional contact problem between a piezoelectric body and a foundation.

Key words: Hemivariational inequality; Nonconvex potential; Optimal control; Piezoelectric frictional contact model.

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117 PARAMETER IDENTIFICATION OF NONLINEAR DELAYED DYNAMICAL SYSTEM IN MICROBIAL FERMENTATION BASED ON BIOLOGICAL ROBUSTNESS

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Abstract: In this paper, the nonxlinear enzyme-catalytic kinetic system of batch and continuous fermentation in the process of glycerol bio-dissimilation is investigated. On the basis of both glycerol and 1,3-PD pass the cell membrane by active and passive diffusion under substrate-sufficient conditions, we consider the delay of concentration changes on both extracellular substances and intracellular substances. We establish a nonlinear delay dynamical systems according to the batch and continuous fermentation of bio-dissimilation of glycerol to 1,3-propanediol(1,3-PD) and we propose an identification problem, in which the biological robustness is taken as a performance index, constrained with nonlinear delay dynamical system. An algorithm is constructed to solve the identification problem and the numerical result shows the value of time delays of glycerol, 3-HPA, 1,3-PD intracellular and extracellular substances. This work will be helpful for deeply understanding the metabolic mechanism of glycerol in batch and continuous fermentation.

Key words: Time delay; Biological robustness; Parameter identification; Nonlinear dynamical system

1 INTRODUCTION

Over the past several years, 1, 3-propanediol(1, 3-PD) has attracted much attention in microbial production throughout the world because of its lower cost, higher production and no pollution. Zeng A.P. (2002); Zeng A.P. (1996) carried out including the quantitative description of the cell growth kinetics of multiple inhibitions, the metabolic overflow kinetics of substrate consumption and product formation, Ye J.X. investigated the existence of equilibrium points and stability, Tian Y. and Wang L. considered the dynamical behavior for the models of the continuous cultures and the feeding strategy of glycerol Wang L.. Parameter identification and multistage modeling are widely discussed in fed-batch culture, see Wang L. and the references therein.

However, due to less information about intracellular behavior, less attempt has been made on the glycerol metabolic system. Wang J. prosposed a complex metabolic network and the corresponding nonlinear hybrid dynamical system to determine the most possible metabolic system based on the biological robustness.

Time delay is considered to play an important role in occurrence of oscillation, although the mechanism of oscillation is very complex. In this work, a finite time delay vector between the biomass formation and the operating conditions on both extracellular substances and intracellular substances is taken into account in the kinetic system. We establish a nonlinear delay dynamical systems according to the batch and continuous fermentation of bio-dissimilation of glycerol to 1,3-propanediol(1,3-PD) and we propose an identification problem, in which the biological robustness is taken as a performance index, constrained with nonlinear delay dynamical system. An algorithm is constructed to solve the identification problem and the numerical result shows the value of time delays of glycerol, 3-HPA, 1,3-PD intracellular and extracellular substances. This work will be helpful for deeply understanding the metabolic mechanism of glycerol in batch and continuous fermentation. The effect of time delay shows that the system can qualitatively describe oscillatory phenomena occurring in the experiment.

This paper is organized as follows. In Section 2, the model of the delay differential system in continuous and batch culture is proposed. Section 3 investigates the biological robustness and parameter identification model. In Section 4, we develop a computational approach to solve the parameter identification model and illustrates the numerical results. Finally, conclusions are provided in Section 5.

2 NONLINEAR DYNAMICAL SYSTEM WITH DELAY

In this paper, assumption that the transport of glycerol and 1,3-PD across cell membrane are both active transport coupled with passive diffusion, considering the process of substrate taking up and products secreting across the cell membrane, a finite time delay $\tau = (\tau_1, \tau_2, \tau_3, \tau_4)$ between the biomass formation and the operating conditions is taken into account in the kinetic system as follows:

$$
\begin{aligned}
\dot{x_1}(t) &= (\mu - D)x_1(t) && (2.1)\\
\dot{x_2}(t) &= D(C_{s_0} - x_2(t)) - q_2 x_1(t) && (2.2)\\
\dot{x_3}(t) &= q_3 x_1(t) - D x_3(t-\tau_1) && (2.3)\\
\dot{x_4}(t) &= q_4 x_1(t) - D x_4(t) && (2.4)\\
\dot{x_5}(t) &= q_5 x_1(t) - D x_5(t) && (2.5)\\
\dot{x_6}(t) &= \frac{1}{u^7}(u^8 \frac{x_2(t)}{x_2(t)+u^9} + u^{10}(x_2(t) - x_6(t-\tau_2))N_+(x_2(t) - x_6(t-\tau_2) - q_{20}) && (2.6)\\
&\quad -\mu x_6(t-\tau_2) &&\\
\dot{x_7}(t) &= \frac{u^{11} x_6(t-\tau_2)}{K_m^G + K_m^G(\frac{x_7(t-\tau_3)}{u^{12}}) + x_6(t-\tau_2)} - \frac{u^{13} x_7(t-\tau_3)}{K_m^P + x_7(t-\tau_3) + \frac{x_7^2(t-\tau_3)}{u^{14}}} && (2.7)\\
&\quad -\mu x_7(t-\tau_3) &&\\
\dot{x_8}(t) &= \frac{u_{13} x_7(t-\tau_3)}{K_m^P + x_7(t-\tau_3)(1 + \frac{x_7(t-\tau_3)}{u^{14}})} - \frac{u^{15} x_8(t-\tau_4)}{x_8(t-\tau_4) + u^{16}} - \mu x_8(t-\tau_4) && (2.8)\\
&\quad -u^{17}(x_8(t-\tau_4) - x_3(t-\tau_1))N_+(x_8(t-\tau_4) - x_3(t-\tau_1)) &&
\end{aligned}
$$

Here $x(t;\tau) = (x_1(t), x_2(t), x_3(t-\tau_1), x_4(t), x_5(t), x_6(t-\tau_2), x_7(t-\tau_3), x_8(t-\tau_4))^T$ are concentrations of biomass, glycerol, 1,3-PD, acetic acid, ethanol in reactor and intracellular concentrations of glycerol, 3-HPA, 1,3-PD, respectively. D and C_{s0} are, respectively, the dilution rate and substrate concentrate in feed. In the continuous culture, $C_{s0} = 675mmol/L, D = 0.15h^{-1}$. Based on the mechanism of fermentation, $C_{s0} = 0mmol/L, D = 0xh^{-1}$ in the batch culture. According to the factual experiments, we consider the properties of the system on a subset $W_a := \prod_{k=1}^{8}[x_k*, x_k^*]$ which is admissible set of state vector $x(t;\tau)$. x_k^*,x_k* are the lower and upper bounds of $x_k(t)$. Let $U = (u^1, \ldots u^{17})^T$, the value of parameters can be found in Wang L.. $\tau_1, \tau_2, \tau_3, \tau_4$ are the delay value of 1,3-PD and in reactor intracellular concentrations of glycerol, 3-HPA, 1,3-PD, respectively and $\tau_2 < \tau_3 < \tau_4 < \tau_1$. Let $\Gamma = \{\tau | \tau = (\tau_1, \tau_2, \tau_3, \tau_4)\}$ is the parameter need to be identified, $\Gamma_a := \prod_{i=1}^{4}[\tau_i*, \tau_i^*]$ is admissible set of τ, $\tau_i* = 0.001, \tau_*^i = 0.3$ are the lower and upper bounds of the delay value.

The specific cellular growth rate appeared in Eq.(2.1) can be expressed as follows:

$$\mu = \mu_m \frac{x_2(t)}{x_2(t)+K_s}(1 - \frac{x_2(t)}{x_2^*(t)})(1 - \frac{x_3(t-\tau_1)}{x_3^*(t-\tau_1)})(1 - \frac{x_4(t)}{x_4^*(t)})(1 - \frac{x_5(t)}{x_5^*(t)})$$

Using the Monod equation for describing active transport and Fick diffusion law for passive diffusion, we can express the specific substrate consumption rate q_2 and specific product formation rate q_3 as follows:

$$q_2 = u^1 \frac{x_2(t)}{x_2(t)+u^2} + u^3(x_2(t) - x_6(t-\tau_2))N_+(x_2(t) - x_6(t-\tau_2)) \tag{2.9}$$

$$q_3 = u^4 \frac{x_8(t-\tau_4)}{x_8(t-\tau_4)+u^5} + u^6(x_8(t-\tau_4) - x_3(t-\tau_1))N_+(x_8(t-\tau_4) - x_3(t-\tau_1)) \tag{2.10}$$

Here

$$N_+(y) = \begin{cases} 1, & y > 0 \\ 0, & y \le 0 \end{cases}$$

While the uptake of extracellular glycerol is considered as a black box model, its specific consumption rate q_{20}, the specific product formation rates of acetate q_4 and ethanol q_5 are shown as follows:

$$q_{20} = m_2 + \frac{\mu}{Y_2} + \triangle q_2 \frac{x_2(t)}{x_2(t)+K_2^*} \tag{2.11}$$

$$q_4 = m_4 + \mu Y_4 + \triangle q_4 \frac{x_2(t)}{x_2(t)+K_4^*} \tag{2.12}$$

$$q_5 = m_5 + \mu Y_5 \tag{2.13}$$

Then, we can describe the process of fermentation by the following nonlinear delay dynamical system.

$$\begin{cases} \dot{x}(t) = f^b(x;\tau) = (f_1^b(x;\tau), \cdots, f_8^b(x;\tau))^T, \quad t \in [t_0, t_b] \\ x(t_0) = x_0 \\ x_{0i}(t;\tau) = \phi_{0i}(t), t \in [-\tau, 0], i = 3,6,7,8 \end{cases} \tag{2.14}$$

$$\begin{cases} \dot{x}(t) = f^c(x;\tau) = (f_1^c(x;\tau), \cdots, f_8^c(x;\tau))^T, \quad t \in [t_b, t_f] \\ x(t_b) = x_b \\ x_{bi}(t;\tau) = \phi_{bi}(t), t \in [t_b-\tau, t_b], i = 3,6,7,8 \end{cases} \tag{2.15}$$

Where $[t_0, t_b] \subset R_+$ and $[t_b, t_f] \subset R_+$ be the period of time of batch and continuous stage, respectively. Apparently, $0 < t_b < t_f < \infty$. $x_0$is the initial state of the batch stage; x_b is the initial state of the continuous stage, which is also the state of the batch stage at the terminal time t_b.

Similarly to the result by Yan H.H., it is easy to verify the following property.

Proposition 2.1 *The vector-valued functions defined above $f^b : W_a \times \Gamma_a \to R_+^8$ and $f^c : W_a \times \Gamma_a \to R_+^8$ are continuous on $W_a \times \Gamma_a$.*

3 DESCRIPTION OF BIOLOGICAL ROBUSTNESS AND PARAMETER IDENTIFICATION

To determine the validity of the delay system, the computational values of the state vector should be consistent with experimental data. Since only extracellular data can be measured in experiments, we define the relative error between computational concentrations and experimental data of extracellular substances, on the other hand, how to evaluate the validity of the computational concentrations of intracellular substances becomes the heart of the matter. Firstly, we recall some fundamental definitions, which are similar with the work by Wang L.. On the basis of the factual continuous fermentation process, the solution of the system should reach the steady state, which is referred to as the approximately steady state defined as follows.

Definition 3.1 *$\forall \varepsilon > 0, \tau \in \Gamma_a$, if there is $t_\delta := inf\{t_s : \|f(x(t;\tau))\| < \varepsilon, \forall t \in [t_s, t_f]\}$ such that $x(t;\tau)$ is the solution of NDS, we call that the $x(t;\tau) \in W_a$ reaches approximately steady state at t_δ with the accuracy ε.*

Definition 3.2 *With uniform distribution, the set of sample points randomly generated in Γ_a, namely $\Gamma_l := \{\tau^k | \tau^k = (\tau_1^k, \tau_2^k, \tau_3^k, \tau_4^k), k = 1, 2, \ldots, q\}$, here q is a sufficiently large positive integer.*

Definition 3.3 *The set of solution according to* Γ_l *of the system* NDS *is defined by* $S(\tau^k) := \{x(t;\tau^k)|x(t;\tau^k)$*; Furthermore,* $S_0(\tau^k) := \{x(t;\tau^k)|x(t;\tau^k)$ *is the solution of the systemNDS with* $\tau^k \in \Gamma_0$ $\}$*; Let* $\Gamma_w \subseteq \Gamma_0$ *denotes the set of approximately steady state can be reached;* $S_w(\tau^k)$ *is set of steady state solution at* t_δ *with respect to* $\tau^k \in \Gamma_w$*.*

Next, we will prove the important property of the compactness about the sets Γ_0 and Γ_w. According to experiment process and transport mechanism, we also need the assumptions as follows:

(A1) During the process of continuous culture, the substrate added to the reactor only includes glycerol, which is exported by the dilution rate D.

(A2) The concentrations of reactants are uniform in reactor. Time nonuniform space distribution are ignored.

Theorem 3.1 *Under assumptions (A1) and (A2), the subsets* $S(\tau^k)$ *and* $S_w(\tau^k)$ *are compact in* $C([t_0,t_f],R^8_+)$*. If* Γ_0 *and* Γ_w *are both non-empty, the sets* Γ_0 *and* Γ_w *are compact in* R^4_+*.*

Proof: On the basis of assumptions, Γ_a is a non-empty bounded and closed subset in R^4_+. According to the Proposition2.1, the map $\tau^k \in \Gamma_a \to x(\cdot;\tau^k) \in S(\tau^k)$ is continuous, so $S(\tau^k)$ is compact in functional space $C([t_0,t_f],R^8_+)$. Let $\{\tau^k\}(\subset \Gamma_0 \subset \Gamma_a)$ be any sequence, W_a and Γ_a be the non-empty bounded subset of R^8_+ and R^4_+, respectively. It is easy to verify the sequence $\{\tau^k\}$ is also bounded, so it must has the convergent sequence, namely $\{\tau^k_j\}$ and while $j \to \infty$, $\{\tau^k_j\} \to \tau^*$. From the definition of Γ_0, $x(\cdot;\tau^k_j) \in S(\tau^k)$ and $x(\cdot;\tau^k_j) \in W_a$, so $x(\cdot;\tau^*) \in S(\tau^k)$ and $x(\cdot;\tau^*) \in W_a$, $\tau^* \in \Gamma_0$. Summing up the above, Γ_0 is compact in R^4_+, Similarly we can prove that $S_w(\tau^k)$ is compact in $C([t_0,t_f],R^8_+)$ and Γ_w is compact in R^4_+.

3.1 Quantitative definition of biological robustness

Definition 3.4 *Let the computational concentrations and experimental data of extracellular substances at steady stage be* $y = (y_1,y_2,y_3,y_4,y_5)^T$ *and* $x(t_\delta;\tau^k), k = 1,2,3,4$*, respectively, then the relative error of the extracellular substance concentrations is defined as*

$$SSD(\tau^k) := \frac{1}{5}\sum_{s=1}^{5}\frac{|x_s(t_\delta;\tau^k) - y_s|}{|y_s|} \tag{3.1}$$

Let $\Gamma_{sw} := \{\tau^k \in \Gamma_w : x(t_\delta;\tau^k) \in S_w(\tau^k)$ and the $SSD \le \alpha$ holds, $S_{sw} := \{x(\cdot;\tau^k)|x(\cdot;\tau^k)$ is the approximate solution of the system NDS with respect to $\tau^k \in \Gamma_{sw}$. We have the definition to describe the differences about the intracellular substances at steady stage.

Definition 3.5 $\forall \tau^m,\tau^n \in \Gamma_{sw},(m \ne n), x(t_{\delta m};\tau^m)$ *and* $x(t_{\delta n};\tau^n)$ *are approximate solutions of the system* NDS *at* $t_{\delta m}$ *and* $t_{\delta n}$ *corresponding to* τ^m,τ^n*, respectively. The average relative difference of intracellular states is defined by:*

$$MSD(\tau^m,\tau^n) := \frac{1}{3}\sum_{s=6}^{8}(\frac{|x_s(t_{\delta m};\tau^m) - x_s(t_{\delta n};\tau^n)|}{\|\tau^m - \tau^n\|})^2 \tag{3.2}$$

$\forall \tau^{p'} \in \Gamma_{sw}$, $\tau^{p'}$*. The average relative difference of intracellular states with* $\tau^{p'}$ *is defined by:*

$$MSD(\tau^{p'}) = \frac{1}{q}\sum_{p=1}^{q} MSD(\tau^{p'},\tau^p) \tag{3.3}$$

Definition 3.6 *For given* $\tau^a \in \Gamma_{sw}, a \in I_q$*, the maximum deviation of intracellular substances* $x(\cdot;\tau^a)$ *at steady stage* τ^a *in* $B(\tau^a;\delta)$ *is defined by:*

$$MSD_{max}(\tau^a) = \max MSD(\tau^b), \tau^b \in B(\tau^a;\delta), \quad for \quad all \quad k \in I_q \quad and \quad any \quad \tau^k \in \Gamma_{sw}, \tag{3.4}$$

The robust performance of the dynamical system is defined by:

$$J(\tau^*) := \min\{MSD_{\max}(\tau^k)|\tau^k \in \Gamma_{sw}\}, \tau^* = argmin\{J(\tau^k)|\tau^k \in \Gamma_{sw}\}. \tag{3.5}$$

Remark If there exists $\tau^1,\tau^2 \in \Gamma_{sw}$, and $\tau^1 \ne \tau^2$, such that $J(\tau^1) < J(\tau^2)$, then will say τ^1 is better than τ^2.

3.2 Parameter identification model

Since only extracellular data can be measured in experiments, a quantitative definition of biological robustness was proposed and it is as the part of performance index of an identification model. That is, the performance index is composed of SSD and MSD. So the parameter identification model of the nonlinear delay dynamical system is shown as follows:

$$\begin{array}{lll} (\mathrm{IP}): & \inf & J(\tau^k) \triangleq SSD(\tau^k) + \min\{MSD_{max}(\tau^k) \\ & \text{s.t.} & x(t;\tau^k) \in S_{sw}, t \in [t_0, t_f] \\ & & \tau^k \in \Gamma_{sw}, \\ & & \forall \varepsilon > 0, \exists t_\delta, s.t\|f(x(t;\tau^k);\tau^k)\| < \varepsilon, \forall t \in (t_\delta, t_f]. \end{array} \tag{3.6}$$

Theorem 3.2 *Parameter identification model IP is identifiable.*

Proof: On the basis of the compactness of S_{sw} and Γ_{sw}, we can obtain the desired result.

4 ALGORITHM AND NUMERICAL RESULT

The algorithm of $IP(3.6)$ is presented as follows:

Algorithm 1

step1 Given $N > 0$, let $n = 0, \Gamma_l = \emptyset$

step2 Let $n = n + 1$, if $n > N$, goto step5; otherwise, generate the stochastic samples τ^k following the uniform distribution Γ_a;

step3 For each τ^k, solve the nonlinear delay differential equation by Euler method. If the solution $x(t_\delta;\tau^k) \in W_a$, and $\|f^c(x(t;\tau^k))\| \le \varepsilon_f, \forall t \in [t_e, t_f]$ for $t_e \in [t_b, t_f]$, goto step4; else goto step2;

step4 Based on (3.1), compute $SSD(\tau^k)$, if $SSD(\tau^k) \le \delta_f$, then $\Gamma_{sw} = \Gamma_{sw} \bigcup\{\tau^k\}$, goto step2;

step5 Compute $MSD(\tau^m, \tau^n), \tau^m, \tau^n \in \Gamma_{sw}, \tau^m \ne \tau^n$ and $MSD_{max}(\tau^k), \tau^k \in \Gamma_{sw}$ on the basis of (3.2) and (3.3);

step6 $J(\tau^k) = MSD_{max}(\tau^k), \tau^k \in \Gamma_{sw}$, and $J(\tau^{*k}) = min\{J(\tau^k)|\tau^k \in \Gamma_{sw}\}, \tau^{*k} = argmin\{J(\tau^k)| \tau^k \in \Gamma_{sw}\}$

Where n, N stands for the sequence and maximum of the stochastic samples, respectively. Γ_{sw} is the set of the constraints are satisfied. According to the model and algorithm mentioned above, we have programmed the software and applied it to the optimal control problem of microbial fermentation in batch culture. The basic data are listed as follows:

boundary value of state vector:

$x_{*1} = 0.001\ mmol/L$, $x_1^* = 2039\ mmol/L$, $x_{*2} = 0.001\ mmol/L$, $x_2^* = 939.5\ mmol/L$, $x_{*3} = 0.01\ mmol/L$, $x_3^* = 10\ mmol/L$. $u_{*4} = 0.01\ mmol/L$, $u_4^* = 1026\ mmol/L$, $u_{*5} = 200$, $u_5^* = 360.9\ mmol/L$.

We adopt $\alpha = 0.2$ in the procedure. Then, by **Algorithm 1**, the optimal value of delay vector τ and the approximately steady solution $x = (x_1, x_2, \cdots, x_8)^T$ are $(0.27956, 0.0150799, 0.0185566, 0.0249532)^T$ and $(4.00206, 4.56946, 277.769, 121.471, 130.227, 4.55788, 136.994, 0.420106)^T$, respectively.

5 CONCLUSION

In this paper, the non-linear enzyme-catalytic kinetic system of batch and continuous fermentation in the process of glycerol bio-dissimilation is investigated. The delay of concentration changes on both extracellular substances and intracellular substances is considered to establish a nonlinear delay dynamical systems. According to the batch and continuous fermentation of bio-dissimilation of glycerol to 1,3-propanediol(1,3-PD), we propose an identification problem, in which the biological robustness is taken as a performance index, constrained with nonlinear delay dynamical system. An algorithm is constructed to solve the identification problem and the numerical result shows the value of time delays of glycerol, 3-HPA, 1,3-PD intracellular and extracellular substances.

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118 A CHAOTIC PARTICLE SWARM OPTIMIZATION EXPLOITING SNAP-BACK REPELLERS OF A PERTURBATION-BASED SYSTEM

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Abstract: The particle swarm optimization (PSO) is a population-based optimization technique, where a number of candidate solutions called particles simultaneously move toward the tentative solutions found by particles so far, which are called the personal and global bests, respectively. Since, in the PSO, the exploration ability is important to find a desirable solution, and various kinds of methods have been investigated to improve it. In this paper, we propose a novel PSO method exploiting a steepest descent method with perturbations to a virtual quartic objective function having its global optima at the personal and global best, where elements of each particle's position are updated by the proposed chaotic system or the standard update formula. Thus, the proposed PSO can search for solutions without being trapped at any local minimum due to the chaoticness. Moreover, we show the sufficient condition of parameter values of the proposed system under which the system is chaotic.

Key words: Chaotic system; Particle swarm optimization (PSO); Metaheuristics; Snap-back repeller.

1 INTRODUCTION

The particle swarm optimization (PSO) is a population-based stochastic optimization technique which is inspired by social behavior of bird flocking or fish schooling (J. Kennedy (1995)). In the PSO, a number of candidate solutions called particles are simultaneously updated toward the tentative best solutions called the personal best and global best, respectively, which are found by particles so far. The PSO is a very simple and has a high performance to find desirable solutions, while it is known to suffer from the premature convergence prior to discovering such solutions. Thus, in order to improve the exploration ability, various kinds of improved methods have been investigated (M. Clerc (2006), R. Poli (2007)).

Now, we focus on the PSOs exploiting a chaotic system to improve the exploration ability. Those methods often use chaotic sequences to update positions of particles, in which particles search for solutions extensively because of the chaoticness. It is reported that this kind of PSOs have a wider diversification ability than the original PSO (B. Alatas (2009), B. Liu (2005)). However, since those methods often use a single kind of well-known function such as the logistic function to generate chaotic sequences for any optimization problems, the sequences are not necessarily suitable to solve the optimization problem.

In this paper, we propose a new PSO with a chaotic system which is derived from a steepest descent method with perturbations to a virtual objective function having global optima at the personal and global best. The derived system is theoretically shown to be chaotic an appropriate conditions, and can be used for improvement of search of the PSO.

This paper consists of four sections. In Section 2, we show the standard PSO model and its improved methods. In Section 3, we propose a novel PSO method exploiting a steepest descent method to a virtual quartic objective function with perturbations, and show the sufficient condition of the chaoticness of the proposed model. Finally, we conclude this paper in Section 4.

2 PARTICLE SWARM OPTIMIZATION AND ITS IMPROVED METHODS

In this paper, we focus on the following global optimization problems having many local minima and the rectangular constraint.

$$(P) \quad \min\ f(x) \quad \text{s.t. } x \in X := \prod_{i=1}^{n} [x_i^l,\ x_i^u].$$

In order to solve this problem, in the PSO, a number of candidate solutions called *particle* are simultaneously updated by exchanging the information each other. At each iteration, particle i moves toward a linear combination of two best solutions called *the personal best* $p^i(t)$ and *the global best* $g(t)$, where the former is the best solution obtained by each particle i until iteration t and the latter is the best one obtained by all particles until iteration t. Then, the update formula of the j-th elements of position $x_j^i(t)$ and velocity $v_j^i(t)$, $j \in \{1, \ldots, n\}$ of particle $i \in I^p := \{1, \ldots, l\}$ is given by

$$\text{(SP)} \quad v_j^i(t+1) := wv_j^i(t) + c_1\mu_1(g_j(t) - x_j^i(t)) + c_2\mu_2(p_j^i(t) - x_j^i(t)), \tag{2.1}$$

$$x_j^i(t+1) := x_j^i(t) + v_j^i(t+1), \tag{2.2}$$

where $w,\ c_1,\ c_2 > 0$ are constant weights, while μ_1 and μ_2 are randomized numbers uniformly selected from $(0,\ 1)$. We call (SP) the standard update formula. This extremely simple approach has been surprisingly effective across a variety of problem domains (J. Kennedy (1995)). However if the parameter selection is not appropriate, particles sometimes tend to converge quickly to a local minimum, and it is difficult to find a desirable solution. Hence, in order to improve the ability, various kinds of improved methods have been investigated (M. Clerc (2006)). Recently, PSOs improved by exploiting chaotic systems are proposed, which are called chaotic PSOs (B. Alatas (2009), K. Tatsumi (2009)). Mathematically, the chaos means an aperiodic deterministic behavior which is exceedingly sensitive to its initial conditions. Even though the model of the system is well defined and contains no random parameters, the behavior appears to be random. The chaotic PSOs strengthen the search ability by these properties of the chaos and most of them use well-known functions such as the logistic function which can generate chaotic sequences (B. Alatas (2009), B. Liu (2005)). Since those PSOs use the same function for different optimization problems which is irrelevant to an objective function, the sequences are not necessarily suitable for all problems. On the other hand, it is reported that the system called gradient model with perturbations (GP), which is derived from a steepest descent method with perturbations to an objective function is chaotic, and it can be used to improve diversification of the search in some metaheuristics (K. Tatsumi (2009)).

Therefore, in this paper, we propose a new chaotic PSO which is derived on the basis of the GP model. Then, it is convenience to consider the chaos in the sense of Li-Yorke defined as follows. Now, let us consider a discrete-time system:

$$x(t+1) = F(x(t)), \tag{2.3}$$

where $x(t) \in \Re^n,\ t = 1, 2, \ldots$ and F is a map from $\Re^n$ to itself. A point x satisfying $F(x) = x$ is called a fixed point of F. The ε-neighborhood $N_\varepsilon(x)$ of a point x is defined by $N_\varepsilon(x) := \{y \in \Re^n \,|\|x - y\| \le \varepsilon\}$, where $\|\cdot\|$ denotes the Euclidean norm in $\Re^n$. It is well known that the existence of a fixed point called a snap-back repeller in a system implies that the system is chaotic in the sense of Li-Yorke (C. Li (2003)).

Theorem 2.1 *Suppose that $F(x)$ is continuously differential on a set $X_o \subset \Re^n$ and z is a fixed point of F, and also that*

1. *all eigenvalues of $\nabla F(x)$ exceed 1 in norm for all $x \in N_{\bar{r}}(z) \subset X_0$ for some $\bar{r} > 0$.*

2. *$\nabla F(x)$ is symmetric for all $x \in X_0$, and there exist a point $x^0 \in N_{\bar{r}}(z)$ with $x^0 \neq z$, $F^{(m)}(x^0) = z$ and $\det(\nabla F^{(m)}(x^0)) \neq 0$ for some positive integer m.*

Then, system (2.3) is chaotic in the sense of Li-Yorke. Moreover, the points z and x^0 are called the snap-back repeller and the homoclinic point of F, respectively.

Although a snap-back repeller of F is unstable point, there exist many orbits called homoclinic orbits, which approaches the snap-back repeller, and at the same time are repelled from the point. Then, since the GP model uses a system for updating a tentative solution which has a snap-back repeller closed to interior any local minimum, this model can search for a solution around the local minima extensively without being trapped undesirable solutions.

In this paper, we propose a chaotic system for the PSO based on the same concept as GP model.

3 PSO WITH PROPOSED CHAOTIC SYSTEM

In this section, we propose a new PSO using the system exploiting a steepest descent method to a virtual quartic objective function which have only two optimal solutions at $p^i(t)$ and $g(t)$. In the proposed model, we focus on the distance $r^i(t)$ between the personal and global best of particle i which is defined as

$$r^i(t) := \left| \frac{g(t) - p^i(t)}{2} \right|, \tag{3.1}$$

and update all elements $x^i_j(t)$ of the position such that the elements $r^i(t)$ are not less than a sufficiently small positive constant $r_{\min}$ by the proposed chaotic system, while other elements are updated by (SP) for the detail search. For the sake of simplicity, we suppose that the following inequalities are satisfied for $j = 1, 2, \ldots, n$,

$$r^i_j(t) \geq r_{\min}. \tag{3.2}$$

Let us consider the following minimization problem of a virtual quartic function which has global minima at $p^i(t)$ and $g(t)$ for each particle i at iteration t:

$$\text{(VP)} \ \min \ f^{(i,t)}_v(x) := \frac{1}{\|g(t) - p^i(t)\|^2} \|x - p^i(t)\|^2 \|x - g(t)\|^2. \tag{3.3}$$

Here, note that function values of $f^{(i,t)}_v$ at $p^i(t)$ and $g(t)$ are equal, $f^{(i,t)}_v(p^i(t)) = f^{(i,t)}_v(g(t)) = 0$. If a current solution $x(t)$ is updated by the steepest descent method to $f^{(i,t)}_v$, it may be easily trapped at either of the personal best or global best. Therefore, we add perturbation terms to the virtual objective function as follows:

$$\text{(VP2)} \ \min \ f^{(i,t)}_p(x) := f^{(i,t)}_v(x) - \sum_{j=1}^{n} ar^i_j(t) \cos\left(\omega(x_j - p^i_j(t))\right), \tag{3.4}$$

where $ar^i_j(t)$ and positive constant ω denote the amplitudes and the angular frequency of the perturbations, respectively, and ω is selected as follows:

$$\omega := \frac{2m\pi}{r_{\min}}, \tag{3.5}$$

where m is a positive integer. The problem (VP2) has at least one global minimum at $p^i(t)$. Then, by applying the steepest descent method with step-size α for (VP2), we obtain the system,

$$\text{(D)} \ x^i(t+1) := h(x(t)) = x^i(t) - \alpha \nabla f^{(i,t)}_v(x) - \beta\omega \begin{pmatrix} r^i_1(t) \sin\left(\omega(x^i_1(t) - p^i_1(t))\right) \\ \vdots \\ r^i_n(t) \sin\left(\omega(x^i_n(t) - p^i_n(t))\right) \end{pmatrix}, \tag{3.6}$$

where β are defined as $\beta := a\alpha$. We use (D) as an update formula of particles.

In the following, we show the sufficient conditions of parameter values in which system (D) is chaotic. We show the following relations between α, ω, β and $r_{\min}$ in (D) that the system (D) is chaotic.

Theorem 3.1 *Suppose that ω is sufficiently large. If for particle i, positive parameters α, ω, β and $r_{\min}$ satisfy the following inequalities:*

$$\alpha\lambda^i_\infty \leq \frac{1}{3\pi^2}\beta\omega^2 r_{\min}, \tag{3.7}$$

$$2\pi \leq \beta\omega^2 r_{\min}, \tag{3.8}$$

where λ_∞^i *is defined by*

$$\lambda_\infty^i := \max\left\{ \|\nabla^2 f_v(x)\|_\infty \,\middle|\, \|x - g(t)\|_\infty \le \frac{2\pi}{\omega} \text{ or } \|x - p^i(t)\|_\infty \le \frac{2\pi}{\omega} \right\}, \tag{3.9}$$

then $p^i(t)$ *is a snap-back repeller of* h*, and there exist a snap-back repeller* $\hat{g}^i(t)$ *of* h *such that* $\|\hat{g}(t) - g(t)\|_\infty < (\pi + 0.16)/2\omega$.

The result of Theorem 3.1 yields that $p^i(t)$ and $\hat{g}(t)$ are snap-back repellers of (D) during the period where neither of the global best or personal best is updated. Even if the global best or personal best is updated, (D) is always chaotic as long as the assumptions of Theorem 3.1 are satisfied. Since we suppose that ω is sufficiently large, we can consider that the distance between $g(t)$ and $\bar{g}(t)$ is sufficiently small. Thus, the proposed particle can be expected to search for solutions intensively around $p^i(t)$ and $g(t)$ on the basis of (D), while it does not trapped at any local minima. In addition, the sufficient conditions (3.7) and (3.8) in Theorem 3.1 give us the relations between parameters, α, β and ω for the chaoticness of (D), which also provides a criterion of selecting parameter values in the system (D) in the proposed model.

Although in this section, we assume that (3.2) holds for any j, in general, it is not necessarily satisfied. Therefore, in the proposed model, as mentioned above, the element $x_j^i(t)$ such that does not satisfy (3.2) is updated by (SP), and other elements are updated by (D). Then, by considering modified (VP) and (VP2) with respect to x_j such that $r_j^i(t)$ satisfies (3.2), we can derive the modified system of (D), and show that the system is also chaotic under the assumptions of Theorem 3.1. Hence, if there exist at least one $r_j^i(t)$ satisfies (3.2), particle i can search extensively in constrained region when the distance between $p^i(t)$ and $g(t)$ is so large, else particles can search in detail around the personal and global bests. It is expected that the proposed model can search appropriately for any optimization problem.

4 CONCLUSION

In this paper, we have proposed the new PSO which uses the chaotic system exploiting a steepest descent method with perturbations to a virtual quartic objective function based on the personal and global best solutions. In the proposed model, elements of particle's position are updated by the proposed system for diversification of searching if the corresponding elements of distance between the personal and global bests are not less than a sufficiently small positive constant, while other elements are updated by the standard update formula used in the original PSO for the detailed search. Moreover, we have shown the sufficient conditions under which the proposed system is chaotic.

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119 OPTIMAL CONTROL OF A TIME-DELAYED SWITCHED AUTONOMOUS SYSTEM IN FED-BATCH PROCESS

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Abstract: Considering the existence of time delay and hybrid nature in the constantly fed-batch process, a time-delayed switched autonomous system is proposed to formulate the 1,3-propanediol (1,3-PD) production process. Taking the switching instants and the terminal time as the control variables, a constrained time-delayed optimal control problem is then presented. An equivalently constrained time-delayed optimal control problem is also investigated. Finally, a numerical solution method is developed to seek the optimal control strategy. Numerical simulation results show that the mass of 1,3-PD per unit time at the terminal time is increased considerably.

Key words: Time-delayed switched autonomous system; Optimal control; Numerical method; Constantly fed-batch process.

1 INTRODUCTION

Fed-batch is a technique in microbial processes where one or more nutrients are supplied to the bioreactor during the cultivation and products remain in the containment until the end of the run (Yamanè T. (1984)). In 1,3-Propanediol (1,3-PD) production, one efficient way is the fed-batch process. Unlike the previous researches, we focalize an optimal control problem in constantly fed-batch process, a simple feeding mode has been widely applied for the production of many bioproducts.

2 TIME-DELAYED SWITCHED AUTONOMOUS SYSTEM

Taking the delay effect on the production of new biomass into account, the switched autonomous system with time delay to describe the fed-batch process can be formulated as

$$\begin{cases} \dot{x}(t) = f^i(x(t), x(t-h)),\ t \in (\tau_{i-1}, \tau_i],\ i = 1, 2, \cdots, 2n+1, \\ x(0) = x_0, \\ x(t) = \phi(t), t \in [-\bar{h}, 0], \end{cases} \tag{2.1}$$

where $x(t) := (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t), x_6(t))^T \in R^6_+$ be the continuous state whose components are, respectively, biomass, glycerol, 1,3-PD, acetate and ethanol concentrations and the volume of culture fluid at t in reactor. $x_0 \in R^6$ is a given initial state, h is a delay argument bounded above by a given constant $\bar{h}$ and $\phi(t)$ is a given continuous function on $[-\bar{h}, 0]$. According to the fed-batch process, the switching sequence is preassigned, such that

$$0 = \tau_0 \le \tau_1 \le \cdots \le \tau_{2n} \le \tau_{2n+1} = T \tag{2.2}$$

where the switching instants τ_i, $i = 1, \cdots, 2n$, and the terminal time T are decision variables. In particular, for $t \in (\tau_{2j}, \tau_{2j+1}], j \in \Lambda_1 := \{0, 1, \cdots, n\}$,

$$\begin{aligned} &f^{2j+1}(x(t), x(t-h)) \\ &:= (\mu x_1(t-h), -q_2 x_1(t-h), q_3 x_1(t-h), q_4 x_1(t-h), q_5 x_1(t-h), 0)^T, \end{aligned} \tag{2.3}$$

and for $t \in (\tau_{2j+1}, \tau_{2j+2}], j \in \Lambda_2 = \{0, 1, \cdots, n-1\}$,

$$f^{2j+2}(x(t), x(t-h)) := \left(\begin{array}{l} \mu x_1(t-h) - D(x(t))x_1(t) \\ D(x(t))(\frac{c_{s0}}{1+r} - x_2(t)) - q_2 x_1(t-h) \\ q_3 x_1(t-h) - D(x(t))x_3(t) \\ q_4 x_1(t-h) - D(x(t))x_4(t) \\ q_5 x_1(t-h) - D(x(t))x_5(t) \\ (1+r)v \end{array} \right). \tag{2.4}$$

In (2.2) and (2.4), $c_{s0} > 0$ denotes the concentration of initial feed of glycerol in the medium. $r > 0$ is the velocity ratio of adding alkali to glycerol. $v > 0$ is the velocity of feeding glycerol and is a constant. $D(x(t))$ is the dilution rate defined by

$$D(x(t)) = \frac{(1+r)v}{x_6(t)}. \tag{2.5}$$

The specific growth rate of cells μ, the specific consumption rate of substrate q_2 and the specific formation rates of products $q_\ell, \ell = 3, 4, 5$, are expressed as the ones in Liu C. (2011).

Since biological considerations limit the rate of switching, there are maximal and minimal time durations that are spent on each of the batch and feed processes. On this basis, define the set of admissible switching instants and terminal time as

$$\Gamma := \{(\tau_1, \tau_2, \cdots, \tau_{2n+1})^T \in R^{2n+1} : \rho_i \le \tau_i - \tau_{i-1} \le \varrho_i, i = 1, 2, \cdots, 2n+1\}, \tag{2.6}$$

where ρ_j and ϱ_j are the minimal and the maximal time durations, respectively. Accordingly, any $\tau \in \Gamma$ is regarded as an admissible vector of switching instants and terminal time.

There exist critical concentrations, outside which cells cease to grow, of biomass, glycerol, 1,3-PD, acetate and ethanol. Hence, it is biologically meaningful to restrict the concentrations of biomass, glycerol and products in a set W defined as

$$x^T(t) \in W := \prod_{\ell=1}^{6} [x_{*\ell}, x_\ell^*], \ \forall\, t \in [0, T]. \tag{2.7}$$

3 TIME-DELAYED OPTIMAL CONTROL PROBLEMS

The mass of 1,3-PD per unit time at the terminal time is taken as the cost functional and the time-delayed optimal control problem (TOC) in fed-batch fermentation can be formulated as

$$\begin{array}{lll} \text{(TOC)} & \min & J(\tau) := -\dfrac{x_3(T|\tau)x_6(T|\tau)}{T} \\ & \text{s.t.} & \dot{x}(t) = f^i(x(t), x(t-h)), \ t \in (\tau_{i-1}, \tau_i], \ i = 1, 2, \cdots, 2n+1, \\ & & x(0) = x_0, \\ & & x(t) = \phi(t), t \in [-\bar{h}, 0], \\ & & x^T(t) \in W, \end{array}$$

where $x_3(T|\tau)$ and $x_6(T|\tau)$ are, respectively, the third and the sixth components of the solution to the system (2.1) at the terminal time T.

Note that the (TOC) is of non-standard feature because the terminal time as well as the switching instants is the variable to be determined. Thus, the (TOC) is actually time-delayed optimal control problem with free terminal time. We now employ a time-scaling transformation from $t \in [0, T]$ to $s \in [0, 1]$ as follows:

$$t = Ts. \tag{3.1}$$

Then, let $\tilde{x}(s) := x(t(s)), \tilde{h} := \frac{h}{T}, \bar{\tilde{h}} := \frac{\bar{h}}{T}, g^i(\tilde{x}(s), \tilde{x}(s-\tilde{h}), T) := Tf^i(\tilde{x}(s), \tilde{x}(s-\tilde{h})), \tilde{\phi}(s) := \phi(t(s))$ and $s_i := \frac{\tau_i}{T}, i = 1, 2, \cdots, 2n+1$. As a result, the original system (2.1) takes the form:

$$\begin{cases} \dot{\tilde{x}}(s) = g^i(\tilde{x}(s), \tilde{x}(s-\tilde{h}), T), \ s \in (s_{i-1}, s_i], \ i = 1, 2, \cdots, 2n+1, \\ \tilde{x}(0) = x_0, \\ \tilde{x}(s) = \tilde{\phi}(s), s \in [-\bar{\tilde{h}}, 0]. \end{cases} \tag{3.2}$$

Furthermore, let

$$\xi_i := s_i - s_{i-1}, \ i = 1, 2, \cdots, 2n+1, \tag{3.3}$$

be the duration between s_{i-1} and s_i. Clearly,

$$s_i = \sum_{k=1}^{i} \xi_k, \ i = 1, 2, \cdots, 2n+1. \tag{3.4}$$

Let $\xi := (\xi_1, \xi_2, \cdots, \xi_{2n+1}) \in R^{2n+1}$ be the duration vector. It is obvious that

$$\xi_i \geq 0, \ i = 1, 2, \cdots, 2n+1, \tag{3.5}$$

and

$$\sum_{i=1}^{2n+1} \xi_i = 1. \tag{3.6}$$

With this notation, we note that the determination of the switching vector is equivalent to the determination of the duration vector. Consequently, $\tilde{x}$ can be view as being dependent on the terminal time and the duration vector , i.e.,

$$\tilde{x}(s) = \tilde{x}(s|T, \xi_{i-1}, \xi_{i-2}, \cdots, \xi_1), \tag{3.7}$$

for $s \in (s_{i-1}, s_i], \ i = 1, 2, \cdots, 2n+1$. With this transformation, define the feasible set of the terminal time and the switching instants as

$$\tilde{F} = \{(T, \xi) \in \Xi : \tilde{x}(s|T, \xi) \in W, \forall \ s \in [0, 1]\}. \tag{3.8}$$

As a result, the (TOC) turns into the following equivalently time-delayed optimal control problem (ETOC):

$$\text{(ETOC)} \qquad \min \quad \tilde{J}(T, \xi) := -\frac{\tilde{x}_3(1|T, \xi)\tilde{x}_6(1|T, \xi)}{T}$$
$$\text{s.t.} \quad (T, \xi) \in \tilde{F}.$$

4 NUMERICAL SOLUTION METHODS

The (ETOC) is essentially an optimization problem with continuous state inequality constraint (2.7). By $\varepsilon - \gamma$ approximation method (Teo K. (1991)), (ETOC) can be approximated by the approximately constrained time-delayed optimal control problem as follows:

$$\text{(ETOC}_{\varepsilon,\gamma}\text{)} \qquad \min \quad \tilde{J}_{\varepsilon,\gamma}(T, \xi) := -\frac{\tilde{x}_3(1|T, \xi)\tilde{x}_6(1|T, \xi)}{T} \tag{4.1}$$
$$\text{s.t.} \quad (T, \xi) \in \tilde{F}_{\varepsilon,\gamma},$$

where

$$\tilde{F}_{\epsilon,\gamma} := \{(T, \xi) \in \Xi : \tilde{H}_{\varepsilon,\gamma}(T, \xi) := \gamma + \sum_{l=1}^{12} \int_0^1 \varphi_\varepsilon(h_l(\tilde{x}(s|T, \xi)))ds \geq 0\}, \tag{4.2}$$

$\varepsilon > 0$, $\gamma > 0$ and

$$\varphi_\varepsilon(\eta) = \begin{cases} \eta, & \text{if } \eta < -\varepsilon, \\ -\dfrac{(\eta - \varepsilon)^2}{4\varepsilon}, & \text{if } -\varepsilon \leq \eta \leq \varepsilon, \\ 0, & \text{if } \eta > \varepsilon. \end{cases} \tag{4.3}$$

and

$$h_\ell(\tilde{x}(s|\sigma^p, \delta^p)) := x_\ell^* - \tilde{x}_\ell(s|T, \xi),$$
$$h_{6+\ell}(\tilde{x}(s|\sigma^p, \delta^p)) := \tilde{x}_\ell(s|T, \xi) - x_{*\ell}, \ \ell = 1, 2, \cdots, 6.$$

Now, to solve the (TOC), we need to solve a sequence of problems $\{(\mathrm{ETOC}_{\varepsilon,\gamma})\}$. These problems can be solved using any efficient optimization technique, such as the sequential quadratic programming routine (SQP) (Nocedal J. (1999)). For this, we need the gradients for the cost functional and the constraint with respect to the terminal time and the switching instants as shown in the following theorems.

Theorem 4.1 *For each $\varepsilon > 0$ and $\gamma > 0$, the gradients of the cost functional (4.1) and the constraint (4.2) with respect to the terminal time are, respectively, given by*

$$\frac{\partial \tilde{J}_{\varepsilon,\gamma}(T,\xi)}{\partial T} = -\frac{\zeta_3(1|T,\xi)\tilde{x}_6(1|T,\xi)T + \tilde{x}_3(1|T,\xi)\zeta_6(1|T,\xi)T - \tilde{x}_3(1|T,\xi)\tilde{x}_6(1|T,\xi)}{T^2}, \tag{4.4}$$

and

$$\frac{\partial \tilde{H}_{\varepsilon,\gamma}(T,\xi)}{\partial T} = \sum_{l=1}^{12}\int_0^1 \frac{\partial \varphi_\varepsilon(h_l(\tilde{x}(s|T,\xi)))}{\partial h_l}\frac{\partial h_l(\tilde{x}(s|T,\xi))}{\partial \tilde{x}}\zeta(s)ds, \tag{4.5}$$

where $\zeta(s)$ is the solution of the following time-delay system:

$$\begin{aligned}\dot{\zeta}(s) = {} & \frac{\partial g^i(\tilde{x}(s|T,\xi_{i-1},\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_{i-1},\cdots,\xi_1),T)}{\partial \tilde{x}(s)}\zeta(s) + \\ & \frac{\partial g^i(\tilde{x}(s|T,\xi_{i-1},\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_{i-1},\cdots,\xi_1),T)}{\partial \tilde{x}(s-\tilde{h})}\zeta(s-\tilde{h}) + \\ & \frac{\tilde{h}}{T}\frac{\partial g^i(\tilde{x}(s|T,\xi_{i-1},\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_{i-1},\cdots,\xi_1),T)}{\partial \tilde{x}(s-\tilde{h})} \times \\ & g^i(\tilde{x}(s-\tilde{h}|T,\xi_{i-1},\cdots,\xi_1),\tilde{x}(s-2\tilde{h}|T,\xi_{i-1},\cdots,\xi_1),T) + \\ & f^i(\tilde{x}(s|T,\xi_{i-1},\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_{i-1},\cdots,\xi_1)), \forall s \in (s_{i-1},s_i], \\ & i = 1,2,\cdots,2n+1,\end{aligned} \tag{4.6}$$

with

$$\zeta(0) = 0, \tag{4.7}$$

$$\zeta(s) = \frac{\partial \tilde{\phi}(s)}{\partial T}, \ \forall \ s \in [-\bar{\bar{h}}, 0]. \tag{4.8}$$

Theorem 4.2 *For each $\varepsilon > 0$ and $\gamma > 0$, the gradients of the cost functional (4.1) and the constraint (4.2) with respect to the switching instants are, respectively, given by*

$$\frac{\partial \tilde{J}_{\varepsilon,\gamma}(T,\xi)}{\partial \xi_i} = -\frac{\chi_3^i(1|T,\xi)\tilde{x}_6(1|T,\xi) + \tilde{x}_3(1|T,\xi)\chi_6^i(1|T,\xi)}{T}, \tag{4.9}$$

and

$$\begin{aligned}\frac{\partial \tilde{H}_{\varepsilon,\gamma}(T,\xi)}{\partial \xi_i} = {} & \sum_{l=1}^{12}\int_0^1 \frac{\partial \varphi_\varepsilon(h_l(\tilde{x}(s|T,\xi)))}{\partial h_l}\frac{\partial h_l(\tilde{x}(s|T,\xi))}{\partial \tilde{x}}\chi^i(s)ds, \\ & i = 1,2,\cdots,2n,\end{aligned} \tag{4.10}$$

where $\chi^i(s)$ are the solution of the following time-delay systems:

$$\begin{aligned}\dot{\chi}^i(s) = {} & \frac{\partial g^{i+1}(\tilde{x}(s|T,\xi_i,\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_i,\cdots,\xi_1),T)}{\partial \tilde{x}(s)}\chi^i(s) + \\ & \frac{\partial g^{i+1}(\tilde{x}(s|T,\xi_i,\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_i,\cdots,\xi_1),T)}{\partial \tilde{x}(s-\tilde{h})}\chi^i(s-\tilde{h}), s \in (s_i, s_{i+1}], \\ & \cdots \\ \dot{\chi}^i(s) = {} & \frac{\partial g^{2n+1}(\tilde{x}(s|T,\xi_{2n},\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_{2n},\cdots,\xi_1),T)}{\partial \tilde{x}(s)}\chi^i(s) + \\ & \frac{\partial g^{2n+1}(\tilde{x}(s|T,\xi_{2n},\cdots,\xi_1),\tilde{x}(s-\tilde{h}|T,\xi_{2n},\cdots,\xi_1),T)}{\partial \tilde{x}(s-\tilde{h})}\chi^i(s-\tilde{h}), \\ & s \in (s_{2n}, 1],\end{aligned} \tag{4.11}$$

with

$$\chi^i(s_i) = g^i(\tilde{x}(s_i|T, \xi_{i-1}, \cdots, \xi_1), \tilde{x}(s_i - \tilde{h}|T, \xi_{i-1}, \cdots, \xi_1), T), \tag{4.12}$$

$$\chi^i(s) = 0, \forall\ s \in [-\bar{\bar{h}}, s_i]. \tag{4.13}$$

Furthermore,

$$\frac{\partial \tilde{J}_{\varepsilon,\gamma}(T,\xi)}{\partial \xi_{2n+1}} = \frac{\chi_3^{2n+1}(1|T,\xi)\tilde{x}_6(1|T,\xi) + \tilde{x}_3(1|T,\xi)\chi_6^{2n+1}(1|T,\xi)}{T}, \tag{4.14}$$

and

$$\frac{\partial \tilde{H}_{\varepsilon,\gamma}(T,\xi)}{\partial \xi_{2n+1}} = \sum_{l=1}^{12} \int_0^1 \frac{\partial \varphi_\varepsilon(h_l(\tilde{x}(s|T,\xi)))}{\partial h_l} \frac{\partial h_l(\tilde{x}(s|T,\xi))}{\partial \tilde{x}} \chi^{2n+1}(s) ds, \tag{4.15}$$

where

$$\chi^{2n+1}(s) = g^{2n+1}(\tilde{x}(s|T, \xi_{2n}, \cdots, \xi_1), \tilde{x}(s - \tilde{h}|T, \xi_{2n}, \cdots, \xi_1), T), \quad s \in (s_{2n}, 1], \tag{4.16}$$

with

$$\chi^{2n+1}(s) = 0, s \in [-\bar{\bar{h}}, s_{2n}]. \tag{4.17}$$

In view of Theorem 4.1 and 4.2, the following algorithm can now be used to generate an approximately optimal solution of (TOC).

Algorithm 4.1.

Step 1. Choose initial values of ε, γ and (T, ξ), set parameters $\alpha < 1, \beta < 1, \bar{\varepsilon}$ and $\bar{\gamma}$.

Step 2. Compute $(T^*_{\varepsilon,\gamma}, \xi^*_{\varepsilon,\gamma})$.

- Step 2.1 Solve the switched autonomous system with time delay to obtain $\tilde{x}(s|T, \xi_{i-1}, \xi_{i-2}, \cdots, \xi_1), s \in (s_{i-1}, s_i], i = 1, 2, \cdots, 2n+1$.
- Step 2.2 Solve the time-delay systems (4.5) and (4.10) to obtain (4.4),(4.5),(4.9) and (4.9). Furthermore, by (4.15) and Step 2.1, we compute (4.14) and (4.15).
- Step 2.3 Solve $(\mathrm{ETOC}_{\varepsilon,\gamma})$ using SQP to give $(T^*_{\varepsilon,\gamma}, \xi^*_{\varepsilon,\gamma})$.

Step 3. Check feasibility of $H(T^*_{\varepsilon,\gamma}, \xi^*_{\varepsilon,\gamma}) = 0$. If $H(T^*_{\varepsilon,\gamma}, \xi^*_{\varepsilon,\gamma})$ is feasible, then go to Step 4. Else set $\gamma = \alpha\gamma$. If $\gamma \leq \bar{\gamma}$, we have an abnormal exit. Else set go to Step 2.

Step 4. Set $\varepsilon = \beta\varepsilon$. If $\varepsilon > \bar{\varepsilon}$, go to Step 2. Else output $\tau^*_{\varepsilon,\gamma}$ from $(T^*_{\varepsilon,\gamma}, \xi^*_{\varepsilon,\gamma})$ by (3.1) and (3.4) and stop.

Then, $\tau^*_{\varepsilon,\gamma}$ is an approximately optimal solution of (TOC).

5 NUMERICAL SIMULATION RESULTS

Applying Algorithm 4.1 to the (TOC), we obtain the optimal terminal time $T^* = 13.7377$h, in which the corresponding $n^* = 309$, and the optimal switching instants in Bat. Ph. and Phs. I-IX as listed in Table 5.1. It should be noted that the obtained optimal terminal time is much shorter than the original terminal time 24.16h, which is key to reduce the operation costs. Moreover, under the obtained optimal switching instants and the optimal terminal time, the mass of 1,3-PD per unit time is 290.541mmolh^{-1} which is increased by 16.263% in comparison with experimental result 249.9mmolh^{-1} at original terminal time.

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Table 5.1 The optimal switching instants in fed-batch process.

Phases	Switching instants	Optimal values (s)
Bat. Ph.	τ_1	18360.0366
Ph. I ($j = 1, \cdots, 28$)	τ_{2j}	18368.0366+100($j-1$)
	τ_{2j+1}	18360.0366 + 100j
Ph. II ($j = 29, \cdots, 65$)	τ_{2j}	21168.0366+101.662(j-29)
	τ_{2j+1}	21160.0366+101.662(j-28)
Ph. III ($j = 66, \cdots, 126$)	τ_{2j}	24929.5306+100.0328(j-66)
	τ_{2j+1}	24921.5306+100.0328(j-65)
Ph. IV ($j = 127, \cdots, 245$)	$\tau_{2\iota}$	31031.0731+100.98(j-127)
	τ_{2j+1}	31023.5314+100.98(j-126)
Ph. V ($j = 246, \cdots, 309$)	τ_{2j}	43047.843+100.2407(j-246)
	τ_{2j+1}	43040.1514+100.2407(j-245)

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120 DYNAMIC MODELING AND OPTIMAL CONTROL OF FED-BATCH CULTURE INVOLVING MULTIPLE FEEDS

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Abstract: In this paper, we consider modeling and optimal control problem in the fed-batch fermentation of glycerol by Klebsiella pneumoniae with open loop glycerol input and pH logic control. Since it is decisive for increasing the productivity of 1,3- propanediol (1,3-PD) to optimize the feeding volume of glycerol in the fermentation process, we propose a new nonlinear hybrid dynamical system to formulate the process based on the hybrid characteristic of the fed-batch operation. In the system, the feeding volume of glycerol is regarded as the control variable. Some important properties of the proposed system are then discussed. To maximize the concentration of 1,3-PD at the terminal time, an optimal control model is established, and a computational approach is constructed to solve the control model. Finally, the numerical simulations show that the terminal intensity of producing 1,3-PD has been increased obviously by employing the optimal feeding strategy.

Key words: Nonlinear dynamical system; Optimal control; HPSO; Fed-batch fermentation.

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121 A CLASS OF HEMIVARIATIONAL INEQUALITY PROBLEMS INVOLVING NONMONOTONE MAPPINGS

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Abstract: In this paper, we are concerned with the existence of the solutions of hemivariational inequalities for a class of $(S)_+$ mapping and a generalized pseudomonotone mapping.

Key words: Hemivariational inequality; Generalized gradient; Generalized pseudomonotone mapping; $(S)_+$ mapping.

1 INTRODUCTION

Let Ω be a bounded domain of $\mathbb{R}^N$ with smooth boundary, $\mathscr{K}$ be a subset of a real reflexive Banach space $\mathscr{X}$, $\mathscr{X}^*$ the dual space of $\mathscr{X}$, and let $A : \mathscr{K} \to \mathscr{X}^*$ be an operator and $T : \mathscr{X} \to L^p(\Omega; \mathbb{R}^m)$ be a linear continuous operator, where $1 \le p < \infty$. For each $u \in \mathscr{X}$, we denote $\hat{u}$, an element of $L^p(\Omega; \mathbb{R}^m)$, by $\hat{u} := Tu$. Suppose that $j : \Omega \times \mathbb{R}^m \to \mathbb{R}$ is a Carathédory function which is locally Lipschitz with respect to the second variable $y \in \mathbb{R}^m$ and satisfies the following assumption:

(J) there exists $h_1 \in L^{\frac{p}{p-1}}(\Omega, \mathbb{R})$ and $h_2 \in L^\infty(\Omega, \mathbb{R})$ such that

$$|z| \le h_1(x) + h_2(x)|y|^{p-1}$$

for a.e. $x \in \Omega$, every $y \in \mathbb{R}^m$ and $z \in \partial j(x, y)$, where $\partial j(x, y)$ is the Clarke generalized gradient of a locally Lipschitz mapping $j(x, \cdot)$ at $y \in \mathbb{R}^m$.

We will be concerned with the existence of solutions of the following hemivariational inequality problem: Find $u \in \mathscr{K}$ such that, for every $v \in \mathscr{K}$,

$$\langle Au, v - u \rangle + \int_\Omega j^0(x, \hat{u}(x); \hat{v}(x) - \hat{u}(x))\, dx \ge 0, \quad \forall v \in \mathscr{K}, \tag{1.1}$$

where $\langle \cdot, \cdot \rangle$ means the duality pairing between $\mathscr{X}$ and $\mathscr{X}^*$, for the point $x \in \Omega$, $j^0(x, y; h)$ denotes the Clarke generalized directional derivative of the locally Lipschitz mapping $j(x, \cdot)$ at the point $y \in \mathbb{R}^m$ with respect to the direction $h \in \mathbb{R}^m$.

The concept of hemivariational inequality was introduced by Panagiotopoulos as the mathematical models of many problems coming from mechanics, engineering and economics (cf. Panagiotopoulos P.D. (1993)). Because of their wide applicability, existence of solutions for hemivariational inequalities like

(1.1) has been investigated by many authors in recent years (cf. Liu Z.H. (2003); Dályay Z. (2004); Noor M.A. (2005); Costea N. (2009); Costea N. (2010); Costea N. (2011); Denkowski Z. (2007); Filippakis M. (2005); Filippakis M. (2006); Gasinski L., Papageorgiou N.S. (2005); Goeleven D. (2003); Motreanu D. (1999); Motreanu D. (2003); Papageorgiou N.S. (2009); Huang Y.S. (2009)), and the references therein, where the treatment relies on monotonicity principles, projection arguments, topological method and nonsmooth critical point theory. For example, by using a surjectivity result for multivalued (S_+) type mapping, Liu Z.H. (2003) obtained some existence results for a class of evolution hemivariational inequalities. By employing nonsmooth version of the Mountain Pass Theorem, Dályay Z. (2004) gave some sufficient conditions to ensure the hemivariational inequality with strongly monotone and homogeneous of degree $p-1$ operator has a nontrivial solution. In Noor M.A. (2005), by means of the auxiliary principle technique, Noor studied the hemivariational inequalities with pseudomonotone operator and partially relaxed strongly monotone operator in a Hilbert space. By using the Galerkin approximation, Huang Y.S. (2009) obtained an existence result for the hemivariational inequality problem with a generalized pseudomonotone mapping satisfying the Karamandian condition, and then established a new existence result for an elliptic hemivariational inequality problem which was considered in Motreanu D. (2003), Motreanu D. (2004) and Denkowski Z. (2007). By employing a fixed point theorem for set valued mappings, Costea N. (2009) studied the hemivariational inequalities with relaxed $\eta-\alpha$ monotone mappings in a reflexive Banach space, and in Costea N. (2010), they obtained some existence theorems of solutions for hemivariational inequalities of Hartman-Stampacchia type involving stably pseudomonotone operators.

In this paper, we are concerned with the existence results for the nonlinear hemivariational inequality (1.1) in a reflexive Banach space for general mapping, without monotonicity assumption. The purpose of this paper is threefold. First, we are interested in studying the existence of solutions of (1.1) for the mapping A being of $(S)_+$, and we establish a existence result of (1.1) on a bounded, closed and convex subset in a reflexive Banach space under mild conditions (see Theorem 4). Next, we give some sufficient conditions to guarantee the existence of solutions of (1.1) in a unbounded subset (see Theorem 5). Finally, we study the existence of solutions of (1.1) for a generalized pseudomonotone mapping A (see Theorem 6). Note that the assumptions and the methods given in this paper are different from those in the references mentioned above.

The following definitions can be found in Clarke F.H. (1983) and Pacali D. (1978).

Definition 1.1 *The generalized directional derivative of a locally Lipschitz functional $f:\mathscr{X}\to\mathbb{R}$ at a point $u\in\mathscr{X}$ in the direction $v\in\mathscr{X}$, denoted $f^0(u;v)$, is defined by*

$$f^0(u;v)=\limsup_{\substack{u'\to u\\ t\downarrow 0}}\frac{f(u'+tv)-f(u')}{t}.$$

Definition 1.2 *The generalized gradient of a locally Lipschitz functional $f:\mathscr{X}\to\mathbb{R}$ at a point $u\in\mathscr{X}$, denoted $\partial f(u)$, is defined by*

$$\partial f(u)=\{\zeta\in\mathscr{X}^*: f^0(u;v)\geq\langle\zeta,v\rangle,\ \forall\ v\in\mathscr{X}\}.$$

Definition 1.3 *The operator $A:\mathscr{K}\to\mathscr{X}^*$ is w^* -demicontinuous if for any sequence $\{u_n\}\in\mathscr{K}$ converging to u, the sequence $\{Au_n\}$ converges to Au for the w^* -topology.*

Definition 1.4 *A mapping $f:\mathscr{X}\to\mathscr{X}^*$ is called to satisfy the condition $(S)_+$ if, $\{u_k\}_{k\in\mathbb{N}}\subset\mathscr{X}$ satisfies $u_k\rightharpoonup u$ and $\limsup\langle f(u_k),\, u_k-u\rangle\leq 0$, then $u_k\to u$.*

Definition 1.5 *A mapping $A:\mathscr{K}\to\mathscr{X}^*$ is said to be generalized pseudomonotone if, $\{u_n\}_{n\in\mathbb{N}}\subset\mathscr{K}$ satisfies $u_n\rightharpoonup u$, $Au_n\rightharpoonup w_0$ and $\limsup\limits_{n\to\infty}\langle Au_n,\, u_n-u\rangle\leq 0$, we have $w_0=Au_0$ and $\langle Au_n,u_n\rangle\to\langle w_0,u_0\rangle$ as $n\to\infty$.*

Let $J:L^p(\Omega,\mathbb{R}^m)\to\mathbb{R}$ be the function defined by

$$J(u)=\int_\Omega j(x,u(x))dx.$$

We have

$$\int_\Omega j^0(x,u(x);v(x))dx\geq J^0(u,v),\quad \forall u,v\in L^p(\Omega,\mathbb{R}^m).$$

Then

$$\int_{\Omega} j^0(x, \hat{u}(x); \hat{v}(x))dx \geq J^0(\hat{u}, \hat{v}), \quad \forall u, v \in \mathscr{K}.$$

By Lemma 6.1 and Corollary 6.1 in Motreanu D. (2003), we have

Lemma 1.1 (Lemma 6.1 in Motreanu D. (2003)) Assume that j satisfies the assumption (J) and X_1, X_2 are nonempty subsets of $\mathscr{X}$, then the mapping $(u, v) \mapsto \int_{\Omega} j^0(x, \hat{u}(x); \hat{v}(x))dx$ from $X_1 \times X_2$ to $\mathbb{R}^N$ is upper semicontinuous. Moreover, if $T : \mathscr{X} \to L^p(\Omega, \mathbb{R}^m)$ is a linear compact operator, then the above mapping is weakly upper semicontinuous.

Lemma 1.2 (Corollary 6.1 in Motreanu D. (2003)) Let $\mathscr{V}$ be a finite dimensional Banach space and let $\mathscr{K}$ be a compact and convex subset of $\mathscr{V}$. If assumption (J) is fulfilled and if $A : \mathscr{K} \to \mathscr{V}*$ is a continuous operator, then problem *(P)* has at least a solution.

Denote Λ by the family of all finite dimensional subspaces $\mathscr{F}$ of $\mathscr{X}$, ordered by inclusion, $\mathscr{K}_{\mathscr{F}} = \mathscr{K} \cap \mathscr{F}$. Let $i_{\mathscr{K}_{\mathscr{F}}}$ be the canonical injection of $\mathscr{K}_{\mathscr{F}}$ into $\mathscr{K}$ and $i^*_{\mathscr{F}}$ be the adjoint of the canonical injection $i_{\mathscr{F}}$ of $\mathscr{F}$ into $\mathscr{X}$. We have

Lemma 1.3 (Lemma 6.2 in Motreanu D. (2003)) Let $A : \mathscr{K} \to \mathscr{X}^*$ is w^*-demicontinuous. Then the operator $B : \mathscr{K}_{\mathscr{F}} \to \mathscr{F}^*$, $B = i^*_{\mathscr{F}} A i_{\mathscr{K} \bigcap \mathscr{F}}$ is continuous.

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122 STABILITY ANALYSIS OF A TYPE OF STOCHASTIC INTEGRO-DIFFERENTIAL INTERVAL SYSTEM

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Abstract: We discuss a type of stochastic integro-differential interval system in this paper. We firstly prove that the solution of this system exists and is unique, and then we give a sufficient criterion to show the exponential stability property for such system. Finally, we generalize these results to multiple time delays cases.

Key words: exponential stability; stochastic integro-differential interval system; Brownian motion; Ito's formula

1 INTRODUCTION

Recently, lots of advances on stochastic differential interval delay systems could be found in Mao's (Mao X. (2001)) and other authors' papers (Liao X.X(2000)). In last decade, Mao generalizes the results in the case of such system with Markov switching (Mao X. (2002), Mao X (2006)). Motivate by these results, in this paper, we discuss a stochastic integro-differential interval system, which is a generalization of Mao's studies. Here we refer to such system as following

$$\begin{aligned} dx(t) &= [A_0x(t) + A_1x(t-\tau) + A_2\textstyle\int_{-\tau}^{0} x(t+\theta)d\mu(\vartheta)]dt \\ &\quad +[B_0x(t) + B_1x(t-\tau) + B_2\textstyle\int_{-\tau}^{0} x(t+\theta)d\nu(\vartheta)]dB_t \end{aligned} \tag{1.1}$$

where $A_0, A_1, A_2, B_0, B_1, B_2$ are constant matrices and μ, ν denote probability measures, τ is a positive constant. Consider an interval system of the form

$$\begin{aligned} dx(t) &= [(A_0+\Delta A_0)x(t) + (A_1+\Delta A_1)x(t-\tau) + (A_2+\Delta A_2)\textstyle\int_{-\tau}^{0} x(t+\theta)d\mu(\vartheta)]dt \\ &\quad +[(B_0+\Delta B_0)x(t) + (B_1+\Delta B_1)x(t-\tau) + (B_2+\Delta B_2)\textstyle\int_{-\tau}^{0} x(t+\theta)d\nu(\vartheta)]dB_t \end{aligned} \tag{1.2}$$

where $A_{0m}, A_{1m}, A_{2m}, B_{0m}, B_{1m}, B_{2m}$ are constant matrices and $\Delta A_0 \in [-A_{0m}, A_{0m}]$, $\Delta A_1 \in [-A_{1m}, A_{1m}]$, $\Delta A_2 \in [-A_{2m}, A_{2m}]$, $\Delta B_0 \in [-B_{0m}, B_{0m}]$, $\Delta B_1 \in [-B_{1m}, B_{1m}]$, $\Delta B_2 \in [-B_{2m}, B_{2m}]$ are constant matrices, and μ, ν denote probability measures, τ is a positive constant.

In the past few years, a great dedication on stability of deterministic interval system has been studied, for example,

$$\dot{x}(t) = (A + \Delta A)x(t),$$

here we refer to Han H.S. (1993), Sun Y (1997) and Wang K (1994). This paper utilizes methods which are used in those papers to cope with the case of system (2.2).

2 PRELIMINARIES

We let $|\bullet|$ denote the Euclidean norm in the Euclidean space R^n . A is a matrix and its transpose is denoted by A^T . If A is a symmetric matrix, let $\lambda_{max}(A)$ and $\lambda_{min}(A)$ represent its largest and smallest eigenvalue respectively. Define a norm of A as $\| A \| = sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{max}(AA^T)}$. Obviously, if A is a symmetric matrix, then $\lambda_{max}(A) \leq \|A\|$.

For $A^m = [a^m_{ij}]_{n\times n}$ and $A^M = [a^M_{ij}]_{n\times n}$ satisfying $a^m_{ij} \leq a^M_{ij}$, $\forall 1 \leq i,j \leq n$, the interval matrix $[A^m, A^M]$ is defined by

$$[A^m, A^M] = \{A = [a_{ij}]_{n\times n} : a^m_{ij} \leq a_{ij} \leq a^M_{ij}, \forall 1 \leq i,j \leq n\}.$$

For $A, A_m \in R^{n\times n}$, where A_m is a nonnegative matrix, we note that any interval matrix $[A^m, A^M]$ has a unique representation of the form $[A - A_m, A + A_m]$, where $A = \frac{1}{2}(A^m + A^M)$, and $A_m = \frac{1}{2}(A^M - A^m)$.

In this paper, we let $(\Omega, \mathfrak{F}, \{\mathfrak{F}\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathfrak{F}\}_{t\geq 0}$ satisfying the usual conditions. Let B_t denotes a Brownian motion defined on the probability space. Let τ be a positive number and $C([-\tau, 0]; R^n)$ be the family of all continuous R^n-valued functions on $[-\tau, 0]$ with the values in R^n. We define a norm as $\|y\|_\tau = \sup_{-\tau\leq s\leq 0} |y(t)|$ for any $y \in C([-\tau, 0]; R^n)$. Let $L^2(\Omega, \mathfrak{F}_{t_0}, C([-\tau, 0]; R^n))$ represents all F_{t_0}-measurable $C([-\tau, 0]; R^n)$-valued random variables ξ with $E\|\xi\|^2_\tau < \infty$ and here we write L^2 for short unless otherwise specified. If $x(t), t \geq t_0 - \tau$ is an n-dimensional continuous stochastic process, we denote $\hat{x}(t) = x(t + s) : -\tau \leq s \leq 0$ as a $C([-\tau, 0]; R^n)$-valued process on $t \geq 0$. For any initial data $\hat{x}(t_0) = \xi \in L^2(\Omega, \mathfrak{F}_{t_0}, C([-\tau, 0]; R^n))$, there exists a unique global solution to 2.1 which is denoted by $x(t, t_0, \xi)$.

Definition 2.1 *The system (2.1) is said to be*

(a) *exponentially stable, if there exists constants M and γ such that for all $t_0 \geq 0$ and $\xi \in L^2(\Omega, \mathfrak{F}_{t_0}, C([-\tau, 0]; R^n))$,*

$$E\|\hat{x}(t, t_0, \xi)\|^2_\tau \leq Me^{-\gamma(t-t_0)}E\|\xi\|^2_\tau.$$

(b) *almost surely exponentially stable if*

$$\lim_{t\to\infty} sup\frac{1}{t}log|x(t, t_0, \xi| < 0. \qquad a.s.$$

3 MAIN RESULT

In this section, we will study the stability properties of system (2.2).

Let

$$\aleph = \{\xi|\xi(\theta) : -\tau \leq \theta \leq 0 \ \ is\ \ random\ \ variable;\ \ \xi \in \mathfrak{F}_{t_0} \cap C([-\tau, 0]; R^d)\ \ and\ \ E\|\xi\|^2 < \infty\}.$$

Theorem 3.1 *For any $\xi \in \aleph$, there exists a unique solution $x(t)$ of system (2.1) satisfies $x_{t_0} = \xi$.*

Lemma 3.1 *If $x(t)$ is a solution to equation (2.1), then for any $T > t_0$, $\exists C > 0$, such that*

$$E(\sup_{t_0-\tau\leq t\leq T} |x(t)|^2) < C.$$

In particular, $x(t)$ belongs to $L^2([t_0 - \tau, T]; R^d))$.

In order to study the stability properties of system (2.2), firstly, we should consider system (2.1).

Theorem 3.2 *Assume there exists a symmetric positive-definite matrix Q such that*

$$\begin{aligned}
& 2\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}A_1^TQA_1Q^{-\frac{1}{2}})} + 2\sqrt{\varpi_A(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}A_2^TQA_2Q^{-\frac{1}{2}})} \\
+ \quad & (\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}B_0^TQB_0Q^{-\frac{1}{2}})} + \sqrt{\lambda_{max}(Q^{-\frac{1}{2}}B_1^TQB_1Q^{-\frac{1}{2}})} + \sqrt{\varpi_B(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}B_2^TQB_2Q^{-\frac{1}{2}}))^2} \\
< \quad & -\lambda_{max}(Q^{-\frac{1}{2}}(QA_0 + A_0^TQ)Q^{-\frac{1}{2}}),
\end{aligned}$$

$$\varpi_A(\lambda) = \int_{-\tau}^{0} e^{-\lambda\theta} d\mu(\theta), \quad \varpi_B(\lambda) = \int_{-\tau}^{0} e^{-\lambda\theta} d\nu(\theta). \tag{3.1}$$

Then system (2.1) is exponentially stable in $L^2(\Omega, C([-\tau, 0]; R^n))$ and moreover, it is almost surely exponentially stable.

Proof. Firstly, we note that $Q^{-\frac{1}{2}}(QA_0 + A_0^T Q)Q^{-\frac{1}{2}}$ must be negative definite.

Set

$$\lambda = -\lambda_{max}(Q^{-\frac{1}{2}}(QA_0 + A_0^T Q)Q^{-\frac{1}{2}}) > 0. \tag{3.2}$$

By the condition of Theorem 3.2, we can find a constant $\gamma \in (0, \lambda)$ such that

$$\begin{aligned}
& (1+e^{\gamma\tau})\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}A_1^T QA_1 Q^{-\frac{1}{2}})} + (1+e^{\gamma\tau})\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}B_0^T QB_0 Q^{-\frac{1}{2}})\lambda_{max}(Q^{-\frac{1}{2}}B_1^T QB_1 Q^{-\frac{1}{2}})} \\
+ \;& 2\sqrt{\varpi_A(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}A_2^T QA_2 Q^{-\frac{1}{2}})} + 2\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}B_0^T QB_0 Q^{-\frac{1}{2}})\varpi_B(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}B_2^T QB_2 Q^{-\frac{1}{2}})} \\
+ \;& (1+e^{\gamma\tau})\sqrt{\varpi_B(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}B_2^T QB_2 Q^{-\frac{1}{2}})\lambda_{max}(Q^{-\frac{1}{2}}B_1^T QB_1 Q^{-\frac{1}{2}})} + \lambda_{max}(Q^{-\frac{1}{2}}B_0^T QB_0 Q^{-\frac{1}{2}}) \\
+ \;& \varpi_B(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}B_2^T QB_2 Q^{-\frac{1}{2}}) + e^{\gamma\tau}\lambda_{max}(Q^{-\frac{1}{2}}B_1^T QB_1 Q^{-\frac{1}{2}}) < \lambda - \gamma.
\end{aligned} \tag{3.3}$$

We claim that there exists a such that

$$\int_{t_0}^{\infty} e^{\gamma t} E(x(t)^T Qx(t))dt \le Ce^{\gamma t_0} E\|\xi^T Q\xi\|, \tag{3.4}$$

for all $t_0 \ge 0$ and $\xi \in L^2(\Omega, F_{t_0}, C([-\tau, 0]; R^n))$.

In addition, we also affirm that there exists another constant $C' > 0$ such that

$$E\|\hat{x}(t)^T Q\hat{x}(t)\| \le C' e^{-\gamma(t-t_0)} E\|\xi^T Q\xi\|, \tag{3.5}$$

which is hold in $L^2(\Omega, F_{t_0}, C([-\tau, 0]; R^n))$. Then apply (3.4), we imply that Equation (2.2) is almost surely exponentially stable.

Apply Theorem 3.2, now it is capable to cope with such system with interval matrix coefficient, exactly for all matrices belong to the interval satisfy the sufficient condition of Theorem 3.2, and here we can prove that

Theorem 3.3 *If there exists a symmetric positive-definite matrix Q such that*

$$\begin{aligned}
& 2[\lambda_{max}(Q^{-\frac{1}{2}}A_1^T QA_1 Q^{-\frac{1}{2}}) + \tfrac{\|Q\|}{\lambda_{min}(Q)}(2\|A_1\|\|A_{1m}\| + \|A_{1m}\|^2)]^{1/2} \\
+ \;& 2[(\lambda_{max}(Q^{-\frac{1}{2}}A_2^T QA_2 Q^{-\frac{1}{2}})) + \tfrac{\|Q\|}{\lambda_{min}(Q)}(2\|A_2\|\|A_{2m}\| + \|A_{2m}\|^2)\varpi_A(\lambda)]^{1/2} \\
+ \;& \{[\lambda_{max}(Q^{-\frac{1}{2}}B_0^T QB_0 Q^{-\frac{1}{2}}) + \tfrac{\|Q\|}{\lambda_{min}(Q)}(2\|B_0\|\|B_{0m}\| + \|B_{0m}\|^2)]^{1/2} \\
+ \;& [\lambda_{max}(Q^{-\frac{1}{2}}B_1^T QB_1 Q^{-\frac{1}{2}}) + \tfrac{\|Q\|}{\lambda_{min}(Q)}(2\|B_1\|\|B_{1m}\| + \|B_{1m}\|^2)]^{1/2} \\
+ \;& [(\lambda_{max}(Q^{-\frac{1}{2}}B_2^T QB_2 Q^{-\frac{1}{2}}) + \tfrac{\|Q\|}{\lambda_{min}(Q)}(2\|B_2\|\|B_{2m}\| + \|B_{2m}\|^2))\varpi_B(\lambda)]^{1/2}\}^2 \\
\le \;& -\lambda_{max}(Q^{-\frac{1}{2}}(QA_0 + A_0^T Q)Q^{-\frac{1}{2}}) - \tfrac{2\|A_{0m}\|\|Q\|}{\lambda_{min}(Q)}
\end{aligned} \tag{3.6}$$

Then Equation (2.2) is exponentially stable in $L^2(\Omega, C([-\tau, 0]; R^n))$ and moreover, it is almost surely exponentially stable.

Before prove the theorem, we first give some lemmas.

Lemma 3.2 *For a positive-definite symmetric matrix Q,*

$$\|Q^{-\frac{1}{2}}\|\|Q^{\frac{1}{2}}\| \le \frac{\|Q\|}{\lambda_{min}(Q)}.$$

Lemma 3.3 *For a positive-definitive, symmetric matrix Q and an $n \times n$ matrix A,*

$$\lambda_{max}(Q^{-\frac{1}{2}}(QA + A^T Q)Q^{-\frac{1}{2}}) \le \frac{2\|A\|\|Q\|}{\lambda_{min}(Q)}.$$

Lemma 3.4 *If $\Delta A \in [-A_m, A_m]$, then $\|\Delta A\| \le \|A_m\|$.*

Lemma 3.5 *For a positive-definitive, symmetric matrix Q, and B and $\Delta B \in [-B_m, B_m]$,*

$$\lambda_{max}(Q^{-\frac{1}{2}}(B^TQ\Delta B + (\Delta B)^TQB) + (\Delta B)^TQ(\Delta B))Q^{-\frac{1}{2}}) \leq \frac{2\|B\|\|Q\|\|B_m\|}{\lambda_{min}(Q)} + \frac{\|Q\|\|B_m\|^2}{\lambda_{min}(Q)}.$$

Proof of Theorem 3.3: In order to prove the exponential stability property for the interval system (2.2), we must guarantee that the condition (3.1) of Theorem 3.2 is hold for all the matrix coefficients $\Delta A_1 \in [-A_{1m}, A_{1m}]$, $\Delta A_2 \in [-A_{2m}, A_{2m}]$, $\Delta B_0 \in [-B_{0m}, B_{0m}]$, $\Delta B_1 \in [-B_{1m}, B_{1m}]$, $\Delta B_2 \in [-B_{2m}, B_{2m}]$, i.e.

$$\begin{array}{rl} & 2\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}(A_1+\Delta A_1)^TQ(A_1+\Delta A_1)Q^{-\frac{1}{2}})} + 2\sqrt{\varpi_A(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}(A_2+\Delta A_2)^TQ(A_2+\Delta A_2 Q^{-\frac{1}{2}})} \\ + & (\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}(B_0+\Delta B_0)^TQ(B_0+\Delta B_0)Q^{-\frac{1}{2}})} + \sqrt{\lambda_{max}(Q^{-\frac{1}{2}}(B_1+\Delta B_1)^TQ(B_1+\Delta B_1)Q^{-\frac{1}{2}})} \\ + & \sqrt{\varpi_B(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}(B_2+\Delta B_2)^TQ(B_2+\Delta B_2)Q^{-\frac{1}{2}})})^2 \\ < & -\lambda_{max}(Q^{-\frac{1}{2}}(Q(A_0+\Delta A_0) + (A+\Delta A_0)^TQ)Q^{-\frac{1}{2}}) \end{array}$$

According to lemma 3.3 and lemma 3.4, we note that

$$\begin{array}{rl} & -\lambda_{max}(Q^{-\frac{1}{2}}(Q(A_0+\Delta A_0) + (A+\Delta A_0)^TQ)Q^{-\frac{1}{2}}) \\ \geq & -\lambda_{max}(Q^{-\frac{1}{2}}(QA_0 + A_0^TQ)Q^{-\frac{1}{2}}) - \frac{2\|A_{0m}\|\|Q\|}{\lambda_{min}(Q)} \\ \triangleq & I_1. \end{array}$$

Using lemma 3.4 and lemma 3.5, we have

$$\begin{array}{rl} & \lambda_{max}(Q^{-\frac{1}{2}}(A_1+\Delta A_1)^TQ(A_1+\Delta A_1)Q^{-\frac{1}{2}}) \\ < & \lambda_{max}(Q^{-\frac{1}{2}}A_1^TQA_1Q^{-\frac{1}{2}}) + \lambda_{max}(Q^{-\frac{1}{2}}(A_1^TQ\Delta A_1 + (\Delta A_1)^TQA_1 + (\Delta A_1)^TQ\Delta A_1)Q^{-\frac{1}{2}}) \\ \leq & \lambda_{max}(Q^{-\frac{1}{2}}A_1^TQA_1Q^{-\frac{1}{2}}) + \frac{\|Q\|}{\lambda_{min}(Q)}(2\|A_1\|\|A_{1m}\| + \|A_{1m}\|^2) \end{array}$$

Then

$$\begin{array}{rl} & 2\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}(A_1+\Delta A_1)^TQ(A_1+\Delta A_1)Q^{-\frac{1}{2}})} + 2\sqrt{\varpi_A(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}(A_2+\Delta A_2)^TQ(A_2+\Delta A_2)Q^{-\frac{1}{2}})} \\ + & (\sqrt{\lambda_{max}(Q^{-\frac{1}{2}}(B_0+\Delta B_0)^TQ(B_0+\Delta B_0)Q^{-\frac{1}{2}})} + \sqrt{\lambda_{max}(Q^{-\frac{1}{2}}(B_1+\Delta B_1)^TQ(B_1+\Delta B_1)Q^{-\frac{1}{2}})} \\ + & \sqrt{\varpi_B(\lambda)\lambda_{max}(Q^{-\frac{1}{2}}(B_2+\Delta B_2)^TQ(B_2+\Delta B_2)Q^{-\frac{1}{2}})})^2 \\ \leq & 2[\lambda_{max}(Q^{-\frac{1}{2}}A_1^TQA_1Q^{-\frac{1}{2}}) + \frac{\|Q\|}{\lambda_{min}(Q)}(2\|A_1\|\|A_{1m}\| + \|A_{1m}\|^2)]^{1/2} \\ + & 2[\lambda_{max}(Q^{-\frac{1}{2}}A_2^TQA_2Q^{-\frac{1}{2}}) + \frac{\|Q\|}{\lambda_{min}(Q)}(2\|A_2\|\|A_{2m}\| + \|A_{2m}\|^2)\varpi_A(\lambda)]^{1/2} \\ + & \{[\lambda_{max}(Q^{-\frac{1}{2}}B_0^TQB_0Q^{-\frac{1}{2}}) + \frac{\|Q\|}{\lambda_{min}(Q)}(2\|B_0\|\|B_{0m}\| + \|B_{0m}\|^2)]^{1/2} \\ + & \{[\lambda_{max}(Q^{-\frac{1}{2}}B_1^TQB_1Q^{-\frac{1}{2}}) + \frac{\|Q\|}{\lambda_{min}(Q)}(2\|B_1\|\|B_{1m}\| + \|B_{1m}\|^2)]^{1/2} \\ + & \{[\lambda_{max}(Q^{-\frac{1}{2}}B_2^TQB_2Q^{-\frac{1}{2}}) + \frac{\|Q\|}{\lambda_{min}(Q)}(2\|B_2\|\|B_{2m}\| + \|B_{2m}\|^2)\varpi_B(\lambda)]^{1/2} \\ \triangleq & I_2. \end{array}$$

If $I_1 \geq I_2$, we can conclude that the matrix coefficient be of interval type satisfy the condition of Theorem 3.2, which leads to the exponential stability property for the stochastic interval system.

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123 NONLINEAR DYNAMICS OF JEFFCOTT ROTORS UNDER AIR EXCITING VIBRATION FORCE

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Abstract: This paper studies nonlinear dynamic characteristics of the Jeffcott rotor with air exciting vibration force generated by the impeller blade tip clearance of eccentric and seal clearance force. First, a direct integral method is applied to calculate the system parameters. Then, combined with the computational results, phase diagram, axis path and poincare maps, the nonlinear dynamics of the rotor under different rotor speeds and blade tip clearance force is analyzed. Our results show that with the increase of the blade tip clearance force and the rotary speed, the rotor system will experience more and more probable periodic motion and period-doubling motion transformation during the process to the chaotic status.

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124 STABILIZATION OF SWITCHED LINEAR SYSTEMS WITH TIME-DELAY

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Abstract: In this paper, we discuss a class of switched integro-differential control system with time delay in detection of switching signal, sufficient conditions are given to guarantee the system being asymptotically stable or exponentially stable.

Key words: Switched systems; asymptotically stable; exponentially stable; delay; switched signal

1 INTRODUCTION

Switched systems are systems that consist of several subsystems and controlled by switching laws. Such systems are often encountered in reality, such as in computer science, control systems and etc. Now, Switched systems is one of the most important part of hybrid systems. For switched systems, one of the most important and challenging problems is the stability, i.e. what switching laws can guarantee the switched systems stable.

Recently, there has been increasing interest in the stability analysis of switched systems and switching control design of such systems (Sun X.M.(2006), Xie G(2000),Daafouz,J (2002), Liu X.Z.(2006)), Sun X.M.(2006) considered stability of switched systems with time-delay, based on the result of average dwell time and Lyapunov functions method, the result of exponentially stability is obtained under time-delay. Using inequality and multiple Lyapunov functions method, Exponential stability is studied for some special linear switched system in Xie G(2000). LMI method is also used to study the stability of switched systems in (Zhang Y (2007), Liu J(2008)). Similar results are presented in (Sun X.M (2006), Gurvits L (1995)). Liu X.Z.(2006) studied the problems of asymptotical stability and stabilization of a class switched control systems, a delay-dependent stability criterion is formulated in term of linear matrix inequalities (LMIs) by using quadratic Lyapunov functions and inequality analysis technique. Some authors considered discrete systems and give some results (Liberzon D(1999), Zhai G(2002), Zhai G(2001). There are other methods are used in studying switched systems, such as dwell time and average dwell time and so on. The stability of some slow-switched control systems is studied in Solo V(1994). Hu K(2008) investigates the exponentially stability of the switched delay systems with integral item.

In the above results, the switching signal do not include time delay. But in real world, it is well known that we need time to receive the control signal, it means that switching signal should include time delay. Recently, Xie G. M(2003), Xie G. M(2005) investigate the switched linear systems with time-delay in switching signal and some sufficient conditions are given for stability. In this paper, we study linear switched integro-differential system with time delay in switching signal and some sufficient conditions are obtained to guarantee the system is asymptotically stable or exponentially stable.

The paper is organized as follows. Firstly, we proved some notations and assumptions which are useful in this paper in section 2. Then, main results are given in Section 3 and a brief conclusion is given in the last section.

2 PRELIMINARIES

In this paper, we consider following switched linear system:

$$\begin{cases} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + C_{\sigma(t)}\int_{t-\tau}^{t} x(s)ds \\ \gamma(t) &= \sigma(t-\tau) \end{cases} \tag{2.1}$$

where $x(t) \in R^n$, $u(t) \in R$ is the signal input, and $A_{\sigma(t)}$, $B_{\sigma(t)}$, $C_{\sigma(t)} \in R^{n\times n}$ are matrix, the right continuous function $\sigma(t) : R \to \Theta = \{1, 2, \cdots, N\}$ is the switching signal, $\gamma(t)$ is the detection function of $\sigma(t)$, time-delay $\tau > 0$.

In this paper, we always assume that

Assumption $\mathfrak{A}$

i) there exist $\alpha > 0, \beta > 0, M \geq 1$ such that $\|e^{(A_i+B_iK_i)t}\| \leq Me^{-\alpha t}$, $\left\|e^{(A_i+B_iK_{i-1})t}\right\| \leq Me^{-\beta t}$ for all $i \in \Theta$;

ii) (A_i, B_i, C_i) is activated, then it will hold at least for a period of $T_D > \tau$, T_D is dwell time.

For system (2.1), we introduce the piecewise constant state feedback as follows:

$$u(t) = K_{\gamma(t)}x(t)$$

where K_i is to be designed, $i = 1, 2, \cdots, N$, then we get the closed-loop system

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma(t)K_{\gamma(t)}x(t) + C_\sigma(t)\int_{t-\tau}^{t} x(s)ds \tag{2.2}$$

We will use the following definition (see Xie G. M(2005)).

Definition 2.1 *(Asymptotically stabilizability) System (2.1) is said to be asymptotically stabilizable via state feedback, if for any switching signal $\sigma(t)$, the closed-loop systems (2.2) satisfies $\lim_{t\to\infty} x(t) = 0$.*

Definition 2.2 *(Exponential stabilizability) System (2.1) is said to be exponentially stabilizable via state feedback, if for any switching signal $\sigma(t)$, there exist two constant $C > 0$, and $\lambda > 0$, such that the closed-loop system (2.2) satisfies $\|x(t)\| \leq C\|x(0)\|e^{(-\lambda t)}$.*

3 MAIN RESULT

In this section, we will give some sufficient conditions to guarantee the switched system (2.1) is asymptotically stabilizable and exponentially stabilizable.

Following lemma is needed for our results.

Lemma 3.1 *(Gronwall inequality) Suppose that $g, u \in C$, $g, u \geq 0$ and c is a nonnegative real constant. Then $\forall t \in [t_0, t_1,]$*

$$u(t) \leq c + \int_{t_0}^{t} g(s)u(s)ds$$

implies that the inequality

$$u(t) \leq ce^{\int_{t_0}^{t} g(s)ds}$$

is true.

Using this lemma, we can prove the following result.

Theorem 3.1 *For system (2.1), suppose that*

(a) Assumption $\mathfrak{A}$ are satisfied;

(b) $\exists C > 0$ such that $\|C_m\| \leq C$ for all $m \in \Theta$, and $-\gamma = CM\tau e^{\alpha\tau} - \alpha < 0$;

(c) $\tilde{M} = M^2 e^{(2\alpha + CMe^{\beta\tau} - CM\tau e^{\alpha\tau})\tau}$, $\lim_{m\to\infty} \tilde{M}^m e^{-\gamma(t_m - t_0)} = 0$.

Then the systems is asymptotically stabilizable by state feedback mechanism.

Proof. Consider system (2.1), for given switching signal $\sigma(t) = m$, if $t \in [t_{m-1}+\tau, t_m], m = 1, 2, \cdots,$

$$x(t) = e^{(A_m + B_m K_m)(t - t_{m-1} - \tau)} x(t_{m-1}+\tau) + \int_{t_{m-1}+\tau}^{t} e^{(A_m + B_m K_m)(t-s)} \int_{s-\tau}^{s} C_m x(u) du ds$$

$$\begin{aligned} \|x(t)\| &\le Me^{-\alpha(t-t_{m-1}-\tau)}\|x(t_{m-1}+\tau)\| + \textstyle\int_{t_{m-1}+\tau}^{t} e^{-\alpha(t-s)} \int_{s-\tau}^{s} CM\|x(u)\|duds \\ &\le Me^{-\alpha(t-t_{m-1}-\tau)}\|x(t_{m-1}+\tau)\| + \textstyle\int_{t_{m-1}+\tau}^{t} e^{-\alpha(t-s)} \int_{s-\tau}^{s} CM\|x_u\|duds \\ &\le Me^{-\alpha(t-t_{m-1}-\tau)}\|x(t_{m-1}+\tau)\| + \textstyle\int_{t_{m-1}+\tau}^{t} e^{-\alpha(t-s)} CM\|x_u\|\tau ds \end{aligned}$$

$$e^{\alpha(t-t_{m-1}-\tau)}\|x(t)\| \le M\|x(t_{m-1}+\tau)\| + \int_{t_{m-1}+\tau}^{t} CMe^{\alpha(s-t_{m-1}-\tau)}\|x_u\|\tau ds$$

since

$$e^{-\alpha\tau}e^{\alpha(t-t_{m-1}-\tau)}\|x_t\| \le sup_{t-\tau\le u\le t} e^{\alpha(u-t_{m-1}-\tau)}\|x(u)\|$$

thus

$$\begin{aligned} e^{\alpha(t-t_{m-1}-\tau)}\|x_t\| &\le e^{\alpha\tau} sup_{t-\tau\le u\le t} e^{\alpha(u-t_{m-1}-\tau)}\|x(u)\| \\ &\le e^{\alpha\tau} M\|x_{t_{m-1}+\tau}\| + CM\tau e^{\alpha\tau} \textstyle\int_{t_{m-1}+\tau}^{t} e^{\alpha(s-t_{m-1}-\tau)}\|x_u\|ds \\ e^{\alpha(t-t_{m-1}-\tau)}\|x_t\| &\le e^{\alpha\tau} M\|x_{t_{m-1}+\tau}\| e^{CM\tau e^{\alpha\tau}(t-t_{m-1}-\tau)} \end{aligned}$$

So

$$\|x_t\| \le e^{\alpha\tau}M\|x_{t_{m-1}+\tau}\|e^{(CM\tau e^{\alpha\tau}-\alpha)(t-t_{m-1}-\tau)}$$

When $t \in [t_{m-1}, t_{m-1}+\tau]$

$$\|x(t)\| \le Me^{-\beta(t-t_{m-1})}\|x(t_{m-1})\| + \textstyle\int_{t_{m-1}}^{t} e^{-\beta(t-s)} CM\|x_u\|\tau ds$$

$$e^{\beta(t-t_{m-1})}\|x(t)\| \le M\|x(t_{m-1})\| + \textstyle\int_{t_{m-1}}^{t} CMe^{\beta(s-t_{m-1})}\|x_u\|\tau ds$$

$$e^{-\beta\tau}e^{\beta(t-t_{m-1})}\|x_t\| \le \sup_{t-\tau\le u\le t} e^{\beta(u-t_{m-1})}\|x(u)\|$$

since

$$e^{-\beta\tau}e^{\beta(t-t_{m-1})}\|x_t\| \le M\|x(t_{m-1})\| + \int_{t_{m-1}}^{t} CMe^{\beta(s-t_{m-1})}\|x_u\|\tau ds$$

thus

$$e^{\beta(t-t_{m-1})}\|x_t\| \le M\|x_{t_{m-1}}\|e^{\beta\tau} + e^{\beta\tau}\int_{t_{m-1}}^{t} CMe^{\beta(s-t_{m-1})}\|x_u\|\tau ds$$

$$e^{\beta(t-t_{m-1})}\|x_t\| \le M\|x_{t_{m-1}}\|e^{\beta\tau}e^{CMe^{\beta\tau}(t-t_{m-1})}$$

$$\|x_t\| \le M\|x_{t_{m-1}}\|e^{\beta\tau}e^{(CMe^{\beta\tau}-\beta)(t-t_{m-1})}$$

$$\|x_{t_{m-1}+\tau}\| \le M\|x_{t_{m-1}}\|e^{\beta\tau}e^{(CMe^{\beta\tau}-\beta)\tau}$$

So

$$\begin{aligned} \|x_t\| &\le e^{\alpha\tau}MM\|x_{t_{m-1}}\|e^{\beta\tau}e^{(CMe^{\beta\tau}-\beta)\tau}e^{(CM\tau e^{\alpha\tau}-\alpha)(t-t_{m-1}-\tau)} \\ \|x_t\| &\le M^2\|x_{t_{m-1}}\|e^{(2\alpha+CMe^{\beta\tau}-CM\tau e^{\alpha\tau})\tau}e^{(CM\tau e^{\alpha\tau}-\alpha)(t-t_{m-1})} \end{aligned}$$

especially when $t = t_m$,

$$\|x_{t_m}\| \le M^2\|x_{t_{m-1}}\|e^{(2\alpha+CMe^{\beta\tau}-CM\tau e^{\alpha\tau})\tau}e^{(CM\tau e^{\alpha\tau}-\alpha)(t_m-t_{m-1})}$$

Denote $\tilde{M} = M^2 e^{(2\alpha + CMe^{\beta\tau} - CM\tau e^{\alpha\tau})\tau}$, $-\gamma = CM\tau e^{\alpha\tau} - \alpha$, then

$$\begin{aligned}\|x_{t_m}\| &\leq \tilde{M}\|x_{t_{m-1}}\|e^{-\gamma(t_m - t_{m-1})} \leq \tilde{M}^2 e^{-\gamma(t_m - t_{m-2})}\|x_{t_{m-2}}\| \\ &\leq \tilde{M}^m e^{-\gamma(t_m - t_0)}\|x_{t_0}\|\end{aligned}$$

This inequality implies that the systems is asymptotically stabilizable by state feedback mechanism. The proof is finished.

If we prolong the dwell time properly, we can obtain the following result.

Theorem 3.2 *For system (2.1), suppose that*

(a) Assumption $\mathfrak{A}$ *are satisfied;*

(b) $\exists C > 0$ *such that* $\|C_m\| \leq C$ *for all* $m \in \Theta$, *and* $-\gamma = CM\tau e^{\alpha\tau} - \alpha < 0$*;*

(c) $\exists \bar{M} > 0, 0 < \lambda < \gamma$, *such that*

$$m \begin{cases} < \frac{1}{\tilde{M}} \ln \bar{M} e^{\lambda(t_m - t_0)}, & if \quad \tilde{M} > 1, \\ \geq \frac{1}{\tilde{M}} \ln \bar{M} e^{\lambda(t_m - t_0)}, & if \quad \tilde{M} \leq 1, \end{cases}$$

where $\tilde{M} = M^2 e^{(2\alpha + CMe^{\beta\tau} - CM\tau e^{\alpha\tau})\tau}$.

Then the systems is exponentially stabilizable by state feedback mechanism.

Proof. Based on the proof of Theorem 3.1, we can obtain

$$\|x_{t_m}\| \leq \tilde{M}^m e^{-\gamma(t_m - t_0)}\|x_{t_0}\|$$

which implies that

$$\|x_{t_m}\| \leq \bar{M} e^{-(\gamma - \lambda)(t_m - t_0)}\|x_{t_0}\|$$

Thus, the systems is exponentially stabilizable by state feedback mechanism. The proof is completed.

4 CONCLUSION

In this paper, asymptotically stabilizable and exponentially stabilizable of a class of switched linear systems with time-delay in switching signal is studied. Switching law and controllers designing to guarantee the system is asymptotically stabilizable or exponentially stabilizable are obtained by inequality analysis technique. This method can be extended to deal with nonlinear switched system.

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125 OPTIMAL BANDWIDTH DESIGN WITH UNCERTAIN TRAFFIC DEMAND IN COMMUNICATION NETWORKS

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Abstract: In this paper, a fuzzy bandwidth design methodology with uncertain traffic demand is presented for bandwidth capacity optimization in communication networks. In the proposed methodology, uncertain traffic demand is first handled by a triangular fuzzification method. Then a fuzzy optimization analysis of the bandwidth capacity optimization problems considering the trade-off between resource utilization and network performance is presented and accordingly the optimal bandwidth to obtain maximum revenue are derived. Finally, the relationships between fuzzy bandwidth design and stochastic bandwidth design are explored when the traffic demand follows an exponential distribution process.

Key words: Communication Networks; Fuzzy Traffic Engineering; Bandwidth Optimization Design; Uncertain Traffic Demand.

1 INTRODUCTION

In the field of communication networks (CNs), traffic engineering is of utmost importance for optimizing resource utilization and improving network performance. Usually, online planning and offline planning are two common forms of traffic engineering.

Online traffic engineering focuses on instantaneous network states and individual connections. While offline traffic engineering simultaneously examines each channel's resource constraints as well as examining what is needed of each Local Service Provider (LSP) in order to provide global calculations and solutions for the CNs from a centralized view (Wu J. (2006)).

The online planning is very complexity because of requirement of instantaneous network performance optimization and thus offline optimization based on the global view has attracted much attention in the past decades.

Previously, the offline traffic engineering optimizations were realized by a deterministic multi-commodity flow (MCF) model with the objective to optimize the network total revenue from serving traffic demand.

In the deterministic MCF model, the demand of each channel was assumed to be a fixed quantity and the network revenue was also assumed to be a linearly increasing function of the amount of bandwidth provisioned up to the capacity, where all traffic demand was satisfied (Wu J. (2006)), (Mitra D. (1999)), (Suri S. (2003)).

However, the deterministic MCF approach may be infeasible in the case of traffic demand uncertainty. When the traffic demand is uncertain, we cannot know the traffic load exactly and thus it is difficult to design a suitable bandwidth capacity.

If the network bandwidth capacity is initially designed to be large enough, the network performance may be good, but it may also lead to inefficient utilization of network resources and thus generating some unnecessary maintenance costs for the over-provisioned bandwidth capacity.

To ensure effective resource utilization, the network bandwidth capacity should be small, but with a small bandwidth capacity the network may not satisfy the possible traffic demand and thus increasing a risk of reduction of network total revenue. Also, less-provisioned bandwidth capacity may depress service performance in CNs. For the reasons, it is extremely important for CNs to design a suitable bandwidth capacity within the environment of traffic demand uncertainty.

In the past studies, the uncertain demand is treated as a random variable to design an appropriate bandwidth capacity in CNs. Typical examples include Mitra and Wang (Mitra D. (2001)), (Mitra D. (2003)), (Mitra D. (2005)) and Wu et al. (Wu J. (2006)), (Wu J. (2005-1)), (Wu J. (2005-2)). Although demand randomization is an effective method to handle uncertainty, random traffic demand is only an estimation of traffic demand data in a statistical sense during a specified period.

However, the traffic demand usually varies within the confidence interval due to uncertain environment; random traffic demand can be reasonably treated as a triangular fuzzy number corresponding to the confidence interval. Based on the fuzzification treatment for traffic demand, this paper proposes a fuzzy bandwidth design scheme (i.e., fuzzy traffic engineering) to optimize bandwidth capacity in CNs. In the proposed scheme, uncertain traffic demand is first handled by a triangular fuzzification method. Then a basic analysis of the bandwidth allocation problems based on the fuzzy traffic demand is given. Finally, some important results about the optimal bandwidth design are derived based on the fuzzy optimization framework.

2 SYSTEM MODEL FOR THE CN SYSTEM

A communication network (CN) system can usually be regarded as a collection of nodes and links. Similar to the previous descriptions in (Wu J. (2006)), (Mitra D. (2001)), (Wu J. (2005-1)), let (N, L) be a CN system decomposed of nodes n_i $(n_i \in N, 1 \leq i \leq N)$ and links l $(l \in L)$, where N is the total number of nodes and L is the total number of links in the CN system. For any link l, it has maximal bandwidth capacity C_{lmax} for serving user demand including voice, packet data, image and full-motion video. Let V be the set of all node pairs and $n \in V$ denote an arbitrary node pair, where $n = (n_i, n_j)$ and $n_i, n_j \in N$.

Usually, a link l between two nodes n_i and n_j can formulate a route r, but there may be more than one route to be routed for a node pair, we use $R(n)$ to represent an admissible route set for $n \in V$. Denote the traffic load or traffic demand on an arbitrary node pair $n = (n_i, n_j)$ by d_n $(n \in V)$ and let b_n $(n \in V)$ be the amount of bandwidth capacity provisioned to an arbitrary node pair $n = (n_i, n_j)$ and ξ_r $(r \in R(n))$ denote the amount of capacity provisioned on route r, then we have $b_n = \Sigma_{r \in R(n)} \xi_r$.

In this paper, we consider the CN to be a whole system. We let B denote the amount of bandwidth capacity provisioned to the CN system, we have $B = \Sigma_{n \in V} b_n$. Similarly we let D be the total traffic demand in the CN system, we also have $D = \Sigma_{n \in V} d_n$, which is characterized by a random distribution with its probability density function $f(D)$.

Usually, a CN system can gain its revenue by serving traffic demand to and from its users. Let a be the unit revenue by transmitting the traffic load or serving the traffic demand and denote the unit cost for unit bandwidth capacity allocated to the network.

If a network bandwidth is designed to be too small to satisfy traffic demand, a unit penalty cost p should be charged. If a network bandwidth is initially designed to be too large, a unit otiose or maintenance cost h for redundant bandwidth capacity should be considered in the whole CN system.

The objective of the CN system is to maximize the revenue of the network with a good trade-off between resource utilization and network performance. To avoid unrealistic and trivial cases, we assume $a > p > 0$, $a > c > 0$, $a > h > 0$.

Now our main task is how to design a reasonable bandwidth capacity B to maximize the network revenue under demand uncertainty with the consideration of the trade-off between resource utilization and network performance. In the next section, we come to solve this task.

3 OPTIMAL BANDWIDTH DESIGN WITH FUZZY DEMAND

In this section, we formulate a model for the network bandwidth design problem and derive the optimal bandwidth capacity with an uncertain demand.

Let $\pi(B, D)$ denote the expected profit function by transmitting messages in the network. According to the previous notations presented in Section 2, the expected profit function of the network can be formulated as

$$\pi(B, D) = \int_0^B [aD - cB - h(B - D)]f(D)dD + \int_B^\infty [(a - c)B - p(D - B)]f(D)dD. \tag{3.1}$$

In the above equation, D is random traffic demand with its probability density function $f(D)$, which can be estimated by some actual traffic demand data within a specified service period. Usually, at the beginning of a specified service period, random traffic demand is equal to actual traffic demand.

However, in the whole specified service period, the actual traffic demand usually varies because of the demand uncertainty. The random traffic demand is not necessarily equal to the actual traffic demand at the end of specified service period but rather may vary within the confidence interval $[D - \Delta_1, D + \Delta_2]$, where $0 < \Delta_1 < D$, $\Delta_2 > 0$ and Δ_1, Δ_2 can be appropriately determined by the decision maker. Thus differing in the random demand D, uncertain demand or fuzzy demand $\tilde{D}$ can be reasonably regarded as the following triangular fuzzy number corresponding to the confidence interval $[D - \Delta_1, D + \Delta_2]$, namely,

$$\tilde{D} = (D - \Delta_1, D, D + \Delta_2), \tag{3.2}$$

which has the membership function:

$$\mu_{\tilde{D}}(x, D) = \begin{cases} (x - D + \Delta_1)/\Delta_1, & D - \Delta_1 \le x \le D \\ (D + \Delta_2 - x)/\Delta_2, & D \le x \le D + \Delta_2 \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

In order to derive the relationship between random demand D and fuzzy demand $\tilde{D}$, we can calculate the centroid of fuzzy demand $\tilde{D}$, that is,

$$\begin{aligned} Z &= C(\tilde{D}) = \int_{-\infty}^{+\infty} x\mu_{\tilde{D}}(x, D)dx \Big/ \int_{-\infty}^{+\infty} \mu_{\tilde{D}}(x, D)dx \\ &= \left(\int_{D-\Delta_1}^{D} \frac{x^2 - Dx + \Delta_1 x}{\Delta_1}dx + \int_{D}^{D+\Delta_2} \frac{Dx + \Delta_2 x - x^2}{\Delta_2}dx\right) \\ &\quad \Big/ \left(\int_{D-\Delta_1}^{D} \frac{x - D + \Delta_1}{\Delta_1}dx + \int_{D}^{D+\Delta_2} \frac{D + \Delta_2 - x}{\Delta_2}dx\right) \\ &= D + (\Delta_2 - \Delta_1)/3 \end{aligned} \tag{3.4}$$

where Z can be regarded as the estimated value of the traffic demand in the fuzzy sense, i.e., the point estimate of random demand D in the probabilistic sense plus a fuzzy term $(\Delta_2 - \Delta_1)/3$.

If $\Delta_2 > \Delta_1$, then $Z > D$; if $\Delta_1 > \Delta_2$, then $Z < D$, and if $\Delta_1 = \Delta_2$, then $Z = D$.

Theorem 1 If the probability density function of random demand D is $f(D)$ and uncertain demand is fuzzified to a triangular fuzzy number $\tilde{D}$ with the centroid Z, then

(1) The fuzzy demand Z can be seen as a new random variable, the probability density function of which can be expressed as $\varphi(Z) = f(Z - (\Delta_2 - \Delta_1)/3)$ for $\Delta_1 \le \Delta_2$.

(2) The expected total profit in the fuzzy sense is $\pi(B, Z) = \int_0^B [aZ - cB - h(B - Z)]\varphi(Z)dZ + \int_B^\infty [(a - c)B - p(Z - B)]\varphi(Z)dZ$ for $\Delta_1 \le \Delta_2$.

(3) The optimal bandwidth capacity in the fuzzy sense is the B^*, which satisfied $\int_0^{B^*} \varphi(Z)dZ = \frac{a+p-c}{a+p+h}$ where $\Delta_1 \le \Delta_2$. And the maximum profit is $\pi(B^*, Z)$.

Proof:

(1) From Eq. (3.4) we have $Z = D + (\Delta_2 - \Delta_1)/3$. By reformulation, the fuzzy demand Z can be represented as $Z = D + \frac{\Delta_2 - \Delta_1}{3} = \frac{2}{3}D + \frac{\Delta_2}{3} + \frac{D - \Delta_1}{3} > 0$. Thus Z is a measurable random variable with respect to D.

From $Z = D + (\Delta_2 - \Delta_1)/3$ we have $D = Z - (\Delta_2 - \Delta_1)/3$. Because the probability density function of random traffic demand D is $f(D)$, we have $f(D) = f(Z - (\Delta_2 - \Delta_1)/3)$ and thus

$f'(D)dD = f'(Z - (\Delta_2 - \Delta_1)/3)dZ$. According to the differential invariance principle, we have $\varphi(Z) = f(Z - (\Delta_2 - \Delta_1)/3)$.

According to the property of probability density function, we have $f(D) > 0$, $\forall D \geq 0$ and $f(D) = 0$, $\forall D < 0$. Similarly, we also have $\varphi(Z) > 0$, $\forall Z \geq 0$ and $\varphi(Z) = 0$, $\forall Z < 0$. If $Z : 0 \to \infty$, then $D : (\Delta_2 - \Delta_1)/3 \to \infty$, due to $D = Z - (\Delta_2 - \Delta_1)/3$, thus,

$$\begin{aligned}
\int_0^\infty \varphi(Z)dZ &= \int_0^\infty f(Z - (\Delta_2 - \Delta_1)/3)dZ = \int_{\frac{\Delta_1 - \Delta_2}{3}}^\infty f(D)dD \\
&= \begin{cases} \int_{\frac{\Delta_1 - \Delta_2}{3}}^\infty f(D)dD = \int_0^\infty f(D)dD = 1, \ if \ \Delta_1 \leq \Delta_2 \\ \int_{\frac{\Delta_1 - \Delta_2}{3}}^\infty f(D)dD < 1, \ if \ \Delta_1 > \Delta_2. \end{cases}
\end{aligned}$$

According to the property of probability density function, we only consider the case of $\Delta_1 \leq \Delta_2$ and $\Delta_1 > \Delta_2$ should not be considered in this case.

(2) Similar to the Eq. (3.1), when $\Delta_1 \leq \Delta_2$ the expected total profit in the fuzzy sense can be represented as

$$\pi(B, Z) = \int_0^B [aZ - cB - h(B - Z)]\varphi(Z)dZ + \int_B^\infty [(a - c)B - p(Z - B)]\varphi(Z)dZ.$$

(3) In order to maximize the expected total profit of network in the fuzzy sense, an optimal bandwidth B^*, which is a solution to $\partial\pi(B, Z)/\partial B = 0$ (first order condition), should be designed. When $\Delta_1 \leq \Delta_2$, from the first order condition we have $\int_0^{B^*} \varphi(Z)dZ = \frac{a+p-c}{a+p+h}$ where B^* is defined only if $\Delta_1 \leq \Delta_2$.

And $\partial^2\pi(B, Z)/\partial B^2 = -(a + p + h)\varphi(Z) < 0$ (second order condition), since $\partial^2\pi(B, Z)/\partial B^2 < 0$, the optimal second order condition is satisfied and B^* is the optimal solution. Thus the maximum expected profit is $\pi(B^*, Z)$.

From the results of Eq. (3.4), if $\Delta_1 = \Delta_2$, then $Z = D$, thus we have the following corollary.

Corollary 1 If $\Delta_1 = \Delta_2$, then $Z = D$ and $\varphi(Z) = f(D)$. Thus the fuzzy traffic engineering optimization case becomes the pure stochastic traffic engineering optimization case. That is, the fuzzy traffic engineering optimization problem is an extension of stochastic traffic engineering optimization problem.

In order to distinguish the difference between the two types of traffic engineering problem, we use a special distribution case to explore their relationships in next section.

4 OPTIMAL BANDWIDTH DESIGN WITH EXPONENTIAL DISTRIBUTION DEMAND

In this section, we assume the demand follows the exponential distribution process to derive the optimal bandwidth capacity in CN system.

To compare the difference between stochastic traffic engineering and fuzzy traffic engineering, we have the following two theorems. It is worth noting that the selection of exponential distribution is just one illustrative example for the proposed fuzzy traffic engineering optimization model. As for the use of other distribution (e.g., Gaussian distribution process), it would be able to obtain the similar results by employing the following theorems.

Theorem 2 (Stochastic traffic engineering case) If the traffic demand follows exponential distribution process,

$$f(D) = \begin{cases} \frac{1}{\lambda} e^{-\frac{D}{\lambda}}, \ D \geq 0 \\ 0, \ D < 0 \end{cases}$$

where λ is known and $0 < \lambda \leq 1$, then

(1) The optimal bandwidth is given as follows:

$$B^*(\lambda) = \lambda \ln\left(\frac{a + p + h}{h + c}\right).$$

(2) The maximum expected profit is given as follows:

$$\pi(B^*(\lambda), D) = \lambda(a-c) - (h+c)B^*(\lambda).$$

Proof:

(1) If the demand follows the exponential distribution process, then optimal bandwidth $B^*(\lambda)$ can be calculated from the Theorem 1,

$$\int_0^{B^*} \varphi(Z)dZ = \int_0^{B^*} \frac{1}{\lambda}e^{-\frac{D}{\lambda}}dD = 1 - e^{-\frac{B^*}{\lambda}} = \frac{a+p-c}{a+p+h}$$
$$\Rightarrow e^{-\frac{B^*}{\lambda}} = (h+c)/(a+p+h) \Rightarrow B^*(\lambda) = \lambda \ln\left(\frac{a+p+h}{h+c}\right).$$

(2) Substituting $e^{-\frac{B^*}{\lambda}} = (h+c)/(a+p+h)$ into $\pi(B, D)$, we have

$$\begin{aligned}\pi(B^*(\lambda), D) &= \int_0^{B^*} [aD - cB - h(B^* - D)]\frac{1}{\lambda}e^{-\frac{D}{\lambda}}dD + \int_{B^*}^{\infty} [(a-c)B^* - p(D-B^*)]\frac{1}{\lambda}e^{-\frac{D}{\lambda}}dD \\ &= -(h+c)B^* - \lambda(a+p+h)e^{-\frac{B^*}{\lambda}} + \lambda(a+h) \\ &= \lambda(a-c) - (h+c)B^*(\lambda)\end{aligned}$$

where $B^*(\lambda) = \lambda \ln\left(\frac{a+p+h}{h+c}\right)$.

Theorem 3 (Fuzzy traffic engineering case) If the traffic demand follows the exponential distribution process,

$$f(D) = \begin{cases} \frac{1}{\lambda}e^{-\frac{D}{\lambda}}, & D \geq 0 \\ 0, & D < 0 \end{cases}$$

where λ is known and $0 < \lambda \leq 1$, and demand D is fuzzified into a triangular fuzzy number $\tilde{D}$, i.e., $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$, D, $0 < \Delta_1 < D$, $\Delta_2 > 0$, then

(1) The optimal bandwidth is given as follows:

$$B^*(\lambda; \Delta_1, \Delta_2) = -\lambda \ln\left[1 - \left(\frac{a+p-c}{a+p+h}\right)e^{\frac{\Delta_1 - \Delta_2}{3\lambda}}\right].$$

(2) With the above (1), the maximum expected profit is given as follows:

$$\pi(B^*(\lambda; \Delta_1, \Delta_2), D) = \lambda(a+p-c)e^{\frac{\Delta_1-\Delta_2}{3\lambda}} - \lambda p - (h+c)B^*(\lambda; \Delta_1, \Delta_2).$$

Proof of Theorem 3 is omitted for restriction of the length of the paper.

Corollary 2 In Theorem 3, if $\Delta_1 = \Delta_2 \equiv \Delta \to 0$ for each $\lambda \in [0, 1]$, then we have

(1) $\lim_{\Delta \to 0} B^*(\lambda; \Delta_1, \Delta_2) = B^*(\lambda)$.

(2) $\lim_{\Delta \to 0} \pi(B^*(\lambda; \Delta_1, \Delta_2), D) = \pi(B^*(\lambda), D)$.

Proof of Corollary 2 is omitted for restriction of the length of the paper.

From the above Corollary 2, we once confirm that fuzzy traffic engineering case is an extension of stochastic traffic engineering case.

5 CONCLUSIONS

In this paper, we employed the fuzzy method to derive an optimal bandwidth capacity for the CN system under demand uncertainty in order to obtain the maximum expected revenue. First of all, the general expression for optimal network bandwidth design was derived. Based on the general expression, we used the exponential distribution to illustrate the relationships between the fuzzy traffic engineering optimization and stochastic traffic engineering optimization. The results obtained reveal that the proposed fuzzy traffic engineering optimization problem is an extension of stochastic traffic engineering optimization problem.

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