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Fluid Coupling in DEM Simulation Using Darcy's Law: Formulation, and Verification

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Abstract. The fluid coupled-DEM has recently become a popular topic in the field of granular material simulation. In most simulations, the averaged Navier–Stokes equations are implemented to consider the fluid flow through particles. In this paper, a simple algorithm based on Darcy's law was discussed to avoid expensive computational effort of solving of the Navier–Stokes equations. The results of this approach were compared quantitatively with the well-known analytical solution of 1D seepage through a soil column as a fully coupled problem in geotechnical engineering. The comparison between the developed pore pressure and induced displacement with analytical values revealed that this algorithm is capable of simulating fluid-particle interaction accurately within the laminar regime.

Keywords: Discrete Element Method, Fluid coupled simulations, Darcy's Law.
PACS: 45.70.-n

INTRODUCTION

In the past decades, the Discrete Element Method (DEM) has been widely used in geotechnical engineering, mineral processing, chemical engineering, and etc, as a powerful method to simulate particulate media. The capability of this method to study such a discontinuous system has been revealed through numerous studies [1, 2]. Due to the fact that fluid-solid interaction is a common condition in some applications of those engineering field, some researchers have been motivated to consider the effect of fluid in DEM simulations.

Fluid coupled-DEM has been introduced to simulate fluidized bed by Tsuji et al. [1]. In their proposed method a background mesh is considered for the fluid phase and continuity and momentum (Navier–Stokes) equations were solved to calculate the averaged velocity and pressure of the fluid on those fixed coarse grids; the fluid equations can take into account the solid part as their porosity. Shimizu [4] added thermal equations to this scheme, and Zeghal and Shamy [5] implemented this approach to look at pore pressure generation in granular deposits under shaking loading. They observed the generated pore pressure and velocity in dynamic conditions in sandy soil and pointed out that fluid velocity in their examples does not exceed laminar regime. Chen et al. [2] coupled the two well-developed open source codes, YADE-Open DEM and Open-FOAM, and verified this method with classical soil mechanics problems. Although this approach is general and can cover wide range of conditions from high flow velocity to laminar fluid flow, the time-consuming iterative solution of Navier–Stokes equations is computationally expensive.

Darcy's Law which is a simplified form of the Navier–Stokes equations for laminar flow has been successfully used in coupled continuum simulations. In most geotechnical engineering applications, particles are usually packed together as opposed to suspensions in a fluidized bed. Therefore, it is appropriate to use the equation of continuity and Darcy's Law on a fixed coarse mesh for taking into account of the fluid flow. Shafipour and Soroush [1] proposed an algorithm for fluid coupled-DEM with fluid phase simplified to Darcy's flow field. They compared the results with constant volume method in a biaxial test.

In this paper, the Biot's poro-elastic theory is combined with Darcy's Law to take into account the coupled fluid-particle interaction. The pore pressure generation is based on poro-elastic theory, and the fluid flow through the particulate media is assumed to conform to Darcy's regime. The flow equations are solved by adopting implicit finite difference method. Finally, this algorithm will be quantitatively evaluated by comparing the results with the well-known analytical solution of 1D seepage flow in a soil column.

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COUPLED FLUID-DEM ALGORITHM

A realistic model to simulate saturated soil should consider both solid and fluid parts and their interactions; the equations of fluid and solid should also be solved simultaneously. In this study a discrete-continuum model is assumed for the saturated soil. The soil grains are idealized by some disks and the fluid flow is considered as a continuum system on some large cells. The velocity and pressure is assumed to be constant inside each cell.

Particles are displaced by the forces originated from external loading or fluid phase. This movement leads to pore volume change in fluid cells which will be followed by excess pore pressure generation. The fluid flows because of the differential pore pressure between cells, and the pore pressure will be redistributed in the domain. In reality, these steps are a simultaneous procedure which can be summarized in a simple equation based on the Biot's poroelastic theory as following [6]:

$$\frac{\Delta V_f}{V} = n\left(\frac{\Delta V_p}{V_p} + \frac{\Delta u}{K_f}\right) \quad (1)$$

where n is the porosity, ΔV_f is the change of the volume of fluid due to flow in or out of the cell, ΔV_p is the change in void volume, V_p is the void volume, V is the total volume of the solid-fluid system, and K_f is the bulk modulus of the fluid. It should be noted that for incompressible solid particles, the volume change of the system is equal to the volume change of the pores, so the volumetric strain (ε_v) is:

$$\varepsilon_v = n\,\Delta V_p / V_p \quad (2)$$

In this continuum-discrete model, the fully coupled procedure in porous media will be divided into two steps. One step is the pore pressure generation due to volumetric strain in undrained condition which implies that $\Delta V_f = 0$, so Eq. (1) will become:

$$\Delta u^{i,j} = \frac{K_f}{n}\varepsilon_v^{i,j} \quad (3)$$

In the second step, the fluid is exchanged between cells due to differential pressure under constant volumetric strain ($\varepsilon_v = n\Delta V_p / V_p = 0$), and it causes redistribution of pore pressure in whole system. Therefore, Eq. (1) will become Eq. (4). Considering continuity in each cell, the change of the pore fluid in each time-step is equal to the sum of all flow discharges from neighboring cells (Eq. 5).

$$u_{t+\Delta t}^{i,j} - u_t^{i,j} = \frac{K_f}{n}\frac{\Delta V_f}{V^{i,j}} \quad (4)$$

$$\Delta V_f = \Delta t \sum_{k=1}^{4} Q_k \quad (5)$$

where (i, j) shows the cell position (Fig. 1), Δt and t are denoted as the time-step and the current time, Q_k is the discharge from neighboring cells.

The equations of fluid flow between neighboring cells can be derived from Darcy's Law. For instance, the flow between two cells like (i, j) and $(i+1, j)$ is:

$$Q_1 = k_x \frac{u_{t^*}^{i-1,j} - u_{t^*}^{i,j}}{\gamma_w dx} dy \quad (6)$$

where k_x is the hydraulic conductivity in x direction, γ_w is the unite weight of water, dx and dy are the cell dimensions. The t^* is a time between t to $t+\Delta t$. By adopting finite difference solution and choosing $t^* = t + \Delta t$, the flow equations can be solved with implicit finite difference which does not have instability problems. Calculating the four discharges for each cell based on Eqs. (5) and (6) and then substituting it into Eq. (4), a system of linear equations will be obtained for the whole domain. By inverting the coefficient matrix, the unknowns which are pore pressure of the cells can be obtained in one step. Therefore, the assumption of validity of Darcy's Law avoids the enormous computational effort of the iterative solution of the Navier–Stokes momentum equation.

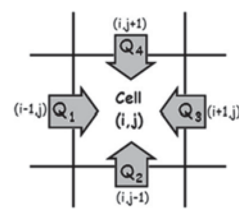

FIGURE 1. Fluid exchange between cells [1].

The total pore pressure at the end of each step is the sum of the induced excess pressure due to particle movement and the residual pore pressure after dissipation (Eq. 7):

$$u_{t+\Delta t}^{i,j} = u_{t+\Delta}^{i,j} + \Delta u^{i,j} \quad (7)$$

This equation is essential to generate coupled behavior for the algorithm which means solid movement will induce pore pressure. To complete the coupling procedure, the solid particles should also receive the effects of the fluid phase. This can be done through applying buoyancy forces and drag forces to the solid media. The first force is due to the pressure gradient around particles, and it can be directly determined by calculating the pressure gradient around each fluid cell. The second force is due to the difference between soil particle velocity and the fluid velocity. The amount of this force can be estimated by using semi–empirical equations. The Ergun's equation is one of the most well-known equations in this field [2, 5] which can be used to estimate the drag force on particles from laminar to turbulent flow. Thus, the total force acting on each particle in a fluid cell is:

$$f_i = \left[-\frac{\beta}{1-n}(v - v_p) - \nabla p \right] V^p \tag{8}$$

where ∇P is the hydrostatic pressure gradient around the cell, v is the fluid velocity inside the cell, v_p is the translational velocity of the particle, V^p is the particles volume, and β is the empirical coefficient that can be determined through the Ergun's equation. For a porosity less than 0.8, we have:

$$\beta = \frac{\mu(1-n)}{d^2 n}[150(1-n) + 1.75\frac{(v_p - v)\rho_f n d}{\mu}] \tag{9}$$

where d is the mean particles diameter, μ is the fluid viscosity, and ρ_f is the fluid density. It is noted that the velocity in this equation is the real velocity of the fluid, and it is different form superficial velocity (v^*) that is obtained from Darcy's equation. The relationship between the superficial velocity and the pore fluid velocity is:

$$v^* = n.v \tag{10}$$

SIMULATION OF 1D SEEPAGE FLOW THROUGH A SOIL COLUMN

A simple example was designed by Suzuki et al. [7] and followed and developed by Chen et al. [2] to check their coupled codes using Navier–Stokes equations. They compared their results with a classical analytical solution in soil mechanics. Here, the 1D upward seepage flow was chosen. In this problem, a column of soil with fixed lateral boundaries and a drained boundary on top was created; then, a constant fluid velocity was applied from the bottom of the soil column. The analytical solutions for the pore pressure and uplift displacement of the soil column are presented through Eqs. (11), (12) and (13).

$$C_v = k/(m_v \gamma_w) \Longleftrightarrow T_v = C_v t / H^2 \tag{11}$$

$$P = \frac{H\gamma_w u^*}{k}\left(\frac{z}{H} - \sum_{n=0}^{\infty} \frac{8(-1)^n}{\pi^2 (2n+1)^n} \sin\frac{\pi(2n+1)z}{2H} e^{\frac{-T_v(2n+1)^2\pi^2}{4}} \right) \tag{12}$$

$$S_t = \frac{u^* H^2}{2C_v}\left(1 - \sum_{n=0}^{\infty} \frac{32(-1)^n}{\pi^3 (2n+1)^3} e^{\frac{-T_v(2n+1)^2\pi^2}{4}} \right) \tag{13}$$

where S_t is the upward displacement of the top of the soil column, P is the pore pressure along the soil column at any points with the distance of z from the drained boundary, m_v is the coefficient of volume compressibility, k is the permeability, H is the length of the soil column, u^* is the input flow velocity, and t is the real time.

To simplify the determination of soil skeleton parameters, one column of particles with 50 particles which are placed on top of each other without any overlapping is considered. Each fluid cell contains two particles. Moreover, for this special example, cell boundaries are attached to some particles, so the ε_v of the cells can be obtained precisely based on particles movement. The initial configuration of the model is plotted in Fig. 2, and its details are presented in Table 1.

TABLE 1. Parameters of the seepage example

Parameters	Values
Number of Particles	50 Disc Shaped
Particles Diameter (d)	0.001 mm
Particles Density (ρ_s)	2650 kg/m^3
Contact Stiffness (k_n)	1e5 N/m
Fluid Density (ρ_f)	1000 kg/m^3
Initial Porosity	0.2146
Submerged Density of the soil skeleton	1.2959e3 kg/m^3
Fluid Viscosity (μ)	1.004e-3 $N.s.m^2$
Fluid Bulk Modulus (K_f)	1e7, 1e5 and 1e4 Pa
Gravity (g)	9.80665 m/s^2

First, we let the saturated sample settle under gravity and buoyancy force. The amount of settlement is 0.16 mm, and the height of the column is 50 mm, so the strain in the column due to its submerged weight is 0.324%. Because of the simple geometry of the sample the coefficient of compressibility can be calculated as:

$$\varepsilon = m_v \int_0^H \sigma_v^{'} dz \Longleftrightarrow \sigma_v^{'} = zg\rho_{Submerged} \tag{14}$$

By substituting the parameters from Table 1, m_v can be obtained around 1.012*10^{-5} Pa^{-1}. Because of the settlement, the porosity will reduce and reach to 0.2121. The other parameter which should be determined for both analytical and numerical formulations is the coefficient of permeability. The well-known Kozeny-Carman equation (Eq. 15) was implemented to estimate the coefficient of permeability. Based on the values presented in Table 1 and the modified porosity, k is found to be 0.001 m/s.

$$k = d^2 n^3 \rho_f g / (150\mu(1-n)^2) \tag{15}$$

After this stage a constant fluid velocity equals to 0.001 m/s was applied at the bottom of the soil column. This velocity should not cause liquefaction in the soil because the analytical solution is not valid for that condition. The numerical and analytical results of the uplift of the top of the soil and the pore pressure along the column for the mentioned input velocity are plotted in Figs. 3 and 4, respectively.

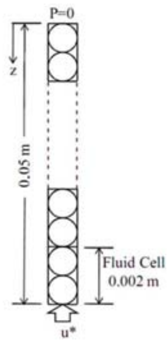

FIGURE 2. Initial geometry of the model.

From the results, it can be observed that the proposed method is capable of replicating the fully coupled procedure of soil deformation under seepage condition. The early stage behavior of the sample is related to the amount of damping. In fact, as static condition is assumed in the analytical solution, the discrete system should be damped enough to show a quasi-static behavior. Another important factor in this study is the fluid bulk modulus because incompressible fluid is also an assumption in the analytical solution. In addition, incompressibility is a relative concept and for geotechnical applications, incompressibility of solid particles or fluid means incompressible in comparison with the soil skeleton deformability. The one dimensional bulk modulus of the soil body ($1/m_v$) is 0.98*10^{-5} Pa, so for the values of bulk modulus which are not significantly greater than soil skeleton stiffness, the error is noticeable (Fig. 3).

CONCLUSION

In this paper, a simple model based on Biot's poro-elastic theory and Darcy's Law is formulated and coupled with DEM. Although implementing Darcy's Law limits the application of this algorithm for fluid flow in laminar regime, the simplicity of its numerical solution can help to significantly reduce the computational time of these CFD-DEM simulations to solve geotechnical engineering problems. The comparison of the numerical results with analytical solution revealed the capability of this algorithm.

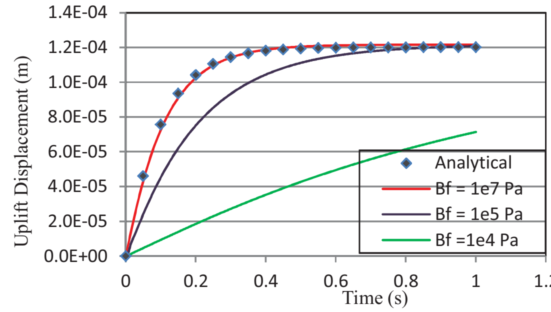

FIGURE 3. Analytical and Numerical Results of the Uplift.

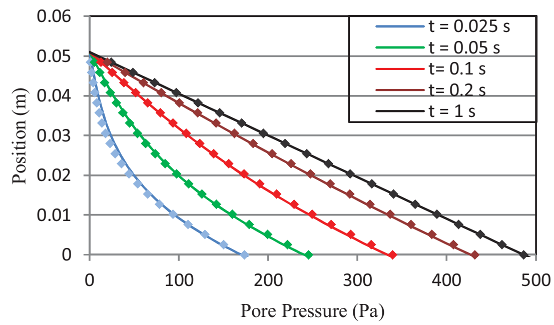

FIGURE 4. Analytical (points) and numerical (lines) results of pore pressure development along the soil column (Points and lines with a same color are corresponding to a same time).

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