Flippers in Housing Market Search*

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Abstract

We add flippers—specialist investors who attempt to profit from buying low and selling high—to a canonical housing market search model. These agents facilitate the turnover of mismatched houses on behalf of end-users and they may survive even if they face an arbitrarily large cost of financing vis-a-vis ordinary households. Multiple equilibrium may exist. In one equilibrium, most, if not all, transactions are intermediated by flippers, resulting in rapid turnover, a high vacancy rate, and high housing prices. In another equilibrium, few houses are bought and sold by these agents. Turnover is sluggish, few houses are vacant, and prices are moderate. When flippers face a lower cost of financing, their presence can, rather unexpectedly, decline. There may then be lower, not higher, housing prices to follow an interest rate decline.

Key words: Search and matching, housing market, liquidity, flippers

JEL classifications: D83, R30, G12

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1 Introduction

In many housing markets, the purchases of owner-occupied houses by individuals who attempt to profit from buying low and selling high rather than for occupation are commonplace. For a long time, anecdotal evidence abounds as to how the participation of these specialist investors, who are popularly known as flippers in the U.S., in the housing market can be widespread.[1] More recently, empirical studies on the housing market have began to more systematically document the extent to which transactions in the housing market are motivated by a pure investment motive. Notable contributions include Depken et al. (2009) and Bayer et al. (2011). A related strand of investigation, which includes Rosen (2007), Shiller (2008), Wheaton and Nechayev (2008), among others, focuses on the role played by the investment motive in the housing market "bubble" in the U.S. in the late-1990s to the mid-2000s. A common theme in the discussion is that housing market flipping is destabilizing. At times, such sentiments are not uncommon among the public and policy makers in many places.[2]

In contrast, dealers in the financial markets, who buy and sell securities for short-term gains and profit the bid-ask spreads in the interim, are often hailed for their role in providing liquidity to the market.[3] If dealers in the financial market can help improve market liquidity, there should certainly be ample room for the flippers in the housing market to do just the same if not more, given that the housing market appears far less liquid than the financial market.

The latter tendency, of course, is due to the fact that houses differ from one another along a large number of dimensions, and unlike many financial assets, are traded without the benefit of an organized exchange. Beginning with Arnold (1989) and Wheaton (1990), this recognition has prompted a sizeable literature on the appli-

[1] Out of the five transactions in a large developement in Hong Kong in August 2010, three were reported to involve investors who buy in anticipation of short-term gains (September 10, 2010, *Hong Kong Economic Times*). According to one industry insider, among all buyers of a new development in Hong Kong recently, only about 60% are buying for own occupation (November 20, 2010, *Wenweipo*).

[2] In the U.S. in particular, interest deductibility applies to the first and second homes only and capital gains tax may only be exempted for properties sold after the first two years of purchase. In recent times, the Chinese government has put in place severe restrictions on the ownership of a second home in a number of cities. Moreover, in Beijing, individuals who have not lived in the city for up to five years are barred from the purchase of an owner-occupied home altogether. See "Beijing's housing market bubbles," September 27, 2007, *The New York Times*; "Wenzhou investors leave Beijing housing market," February 28, 2011, *CNTV*. Similar measures were also in effect in Hong Kong, where since November 2010, housing units that are sold within two years of purchase are subject to an extra 5-15% ad valorem transaction tax. See "Midland adds to jitters - over extra stamp duty," December 3, 2010, *The Hong Kong Standard.*

[3] Duffe et al (2005), Lagos and Rocheteau (2007), and Lagos et al. (2011) study the liquidity-provision role of dealers in over-the-counter markets. Coughenour and Deli (2002) and Coughenour and Saad (2004) presents empirical evidence on how market makers help provide liquidity in the stock market.

cation of the search and matching framework in Mortensen and Pissarides (1994) first developed for the labor market to study the determination of vacancy, turnover, sales volume, price, among other variables, in the housing market.[4] A common feature of the models in this literature is that the agents are exclusively end-users. A buyer is a household looking for a good match for occupation. A seller is a household which no longer finds its old house a desirable place to live and is trying to sell it to someone who finds it as such. There are no specialist investors or flippers around.

In this paper, we add specialist investors to a canonical housing market search model to study how the presence of these flippers affects price, turnover, vacancy, transaction volume, and welfare. In our model, the central role played by these agents is to facilitate the turnover of mismatched houses on behalf of end-user households. A crucial assumption is that ordinary households cannot hold more than one house at a time. The assumption, of course, can be justified by the usual liquidity constraint argument. In this case, a household which desires to move because the old house is no longer a good match must first sell it before the household can buy up a new house. In the usual housing market search model, the household must wait out a buyer who finds the old house a good match to arrive, which can be a lengthy process, especially in a buyer's market—one in which sellers outnumber buyers by a significant margin. This opens up profitable opportunities for specialist investors to just buy up the mismatched house at a discount in return for the time spent waiting for the eventual end-user buyer to arrive on behalf of the original owner. Less obvious is that mismatched homeowners could similarly prefer to sell quickly to flippers in a seller's market to capitalize on the high prices in such a market sooner, as we shall show in the following. In either case, transaction volume, vacancy, and housing price all increase with the extent of flippers' presence in the market, whereas average Time-On-the-Market (TOM) declines in the interim.

According to Bayer et al. (2011), flippers in the housing market can be novice investors who buy en masse in an up market in the belief that the market may continue to go up and sell in panicky in a down market or can be sophisticated middlemen whose activities help provide liquidity in both up and down markets. Implicit in both popular discussions and previous studies on the investment motive in the housing market is that in reality the majority of flippers are more like novice investors than sophisticated middlemen, and any liquidity the flippers may provide is at best coincidental. If flippers as novice investors destabilize the market and flippers as sophisticated middlemen provide liquidity and help stabilize the market, on the whole, the pure investment motive in the housing market may only end up serving to facilitate the formation of speculative bubbles. Our analysis, however, suggests that the dichotomy may not be clear-cut. For tractability, we restrict attention to analyzing the steady state of a housing market and any kinds of timing market movements and speculative bubbles are ruled out by construction. Even so, wide swings in prices

[4] Notable examples include Williams (1995), Krainer (2001), Albrecht et. al. (2007), Diaz and Jerez (2009), Piazzesi and Schneider (2009), Ngai and Tenreyro (2010), to name just a few.

and transactions are distinct possibilities because multiple equilibrium is a natural outcome in a frictional housing market with specialist investors. In one equilibrium, flippers are numerous, prices are high, and turnover is rapid. In another equilibrium, few transactions are intermediated, prices are moderate, and turnover is sluggish. Thus, the participation of specialist investors in the housing market can be a double-edged sword—on the one hand, the flippers help improve liquidity; on the other hand, when the extent of their participation can be fickle, the housing market can become more volatile as a result.

Our model has a number of readily testable implications. First, it trivially predicts a positive cross-section relation between housing price and TOM—households can either sell to specialist investors at a discount or to wait for a better offer from an end-user buyer to arrive—which agrees with the evidence reported in Merlo and Ortalo-Magne (2004), Leung et al. (2002) and Genesove and Mayer (1997), among others.[5]

An important goal of the recent housing market search and matching literature is to understand the positive time-series correlation between housing price and transaction volume and the negative correlation between the two variables and average TOM.[6] In Kranier (2001), for instance, a positive but temporary preference shock can give rise to higher prices and a greater volume of transaction, whereas Diaz and Jerez's (2009) analysis implies that an adverse shock to construction will shorten TOM, and may possibly lead to higher prices and a greater volume of transaction. The paper by Ngai and Tenreyro (2010) focuses on seasonal cycles in price and sales and they argue that increasing returns in the matching technology play a key role in generating such cycles. In these papers, the increase in sales should be accompanied by a decline in vacancy—given that when a house is sold, it is sold to an end-user, who will immediately occupy it, vacancy must decline, or at least remains unchanged. Across steady-state equilibria, the same positive relation between price and transaction volume and negative relation between the two variables and average TOM also hold in our model. Specifically, with more houses sold to flippers, prices and sales both increase, whereas houses on average stay on the market for a shorter period of time. Unique to our model, however, is that vacancy tends to increase together with prices and transaction volume if the increase in transaction volume is due to more houses sold to flippers, who may then just leave them vacant for the time it takes for the end-user buyers to arrive.

Figure 1 depicts the familiar positive housing price-transaction volume correla-

[5] Albrecht et al. (2007) emphasis another aspect of the results reported in Merlo and Ortalo-Magne (2004), which is that downward price revisions are increasingly likely when a house spends more and more time on the market.

[6] Stein (1995), who explains how the down-payment requirement plays a crucial role in amplifying shocks, is an early non-search-theoretic explanation for the positive relation between price and sales. Hort (2000), Leung et al. (2003), among others, provides recent evidence. Ho and Tse (2006) show that the same relation holds in the cross section.

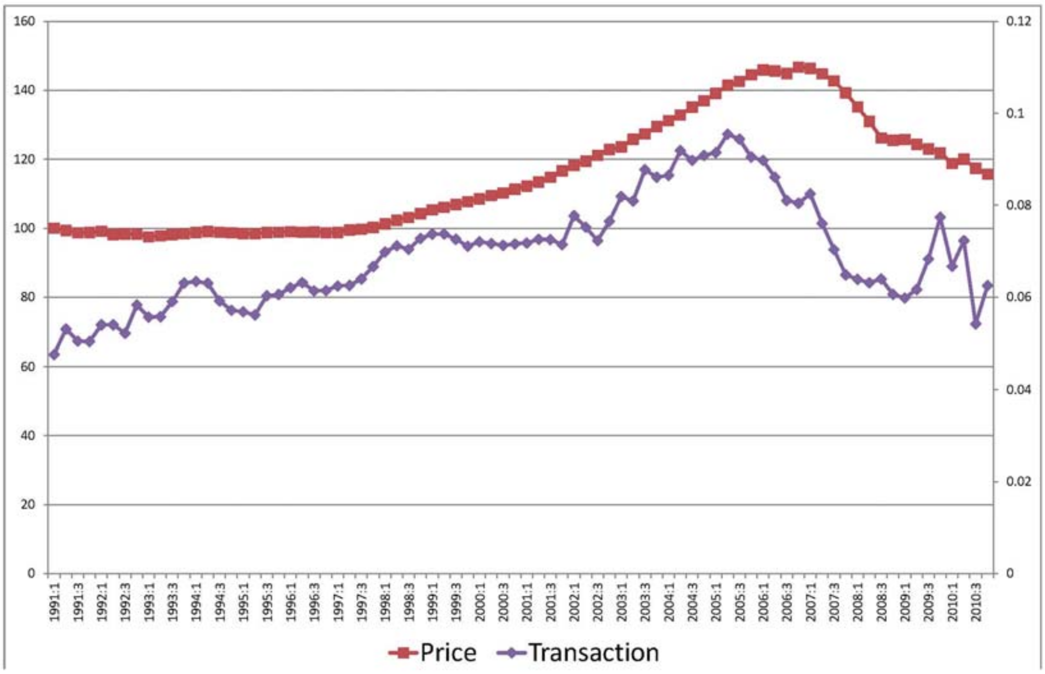

Figure 1: Price and Transaction

tion for the U.S. for the 1991-I to 2010-IV time period.[7] The usual housing market search model predicts that vacancy should decline in the housing market boom in the late-1990s to the mid-2000s and rise thereafter when the market collapses around 2007. Figures 2 and 3, however, show that any decline in vacancy is not apparent in the boom.[8] In fact, if there is any co-movement between vacancy on the one hand and price and transaction on the other hand in the run-up to the peak of the housing market boom in 2006, vacancy appears to have risen along with price and transaction. True, vacancy does not appear to have fallen to follow the market collapse, as predicted by our analysis. But this probably is a combined result of the massive amounts of bank foreclosures and unsold new constructions in the market bust—two forces absent in our analysis.

Insofar as the specialist investors in our model act as middlemen between the original homeowners and the eventual end-user buyers, this paper contributes to the

[7]Housing Price is defined as the nominal house price, which is the transaction-based, seasonally-adjusted house price index from OFHEO (http://www.fhfa.gov), divided by the CPI, from the Federal Reserve Bank at St. Louis, seasonally-adjusted. We set Housing Price equal to 100 at 1991-I. Transaction is measured by the total sales in single-family homes, apartment condos, and co-ops, normed by the stock of such units. The sales data are from Moody's Analytics, whereas the stock data are from the Bureau of Census's CPS/HVS.

[8]Vacancy rate is obtained by dividing the number of vacant and for-sale-only housing units by the stock of such units. The data are from the Bureau of Census's CPS/HVS.

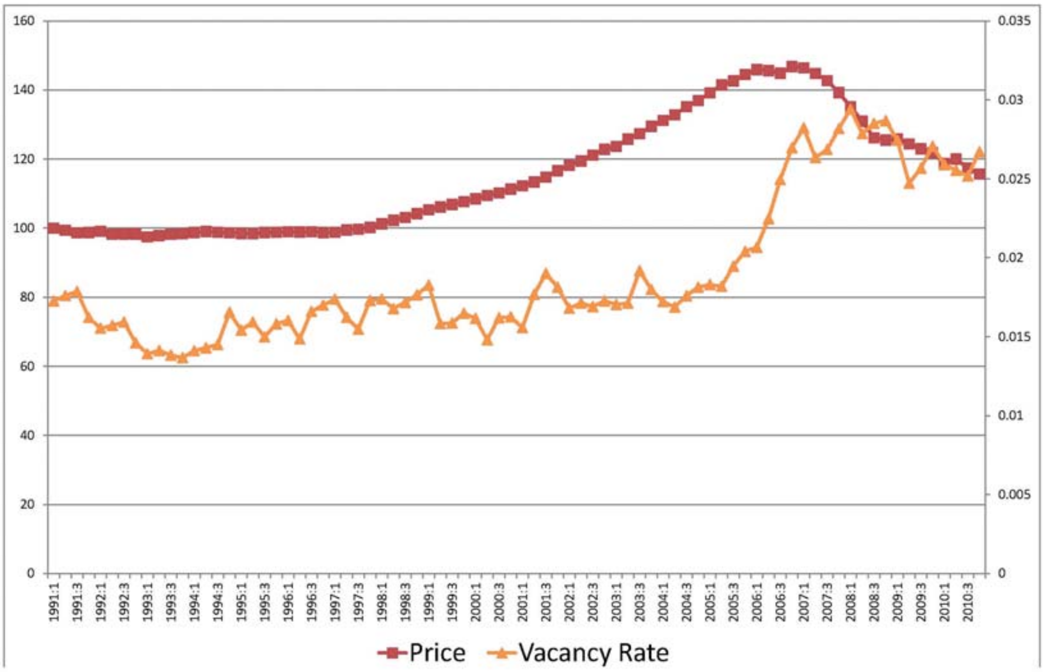

Figure 2: Price and Vacancy

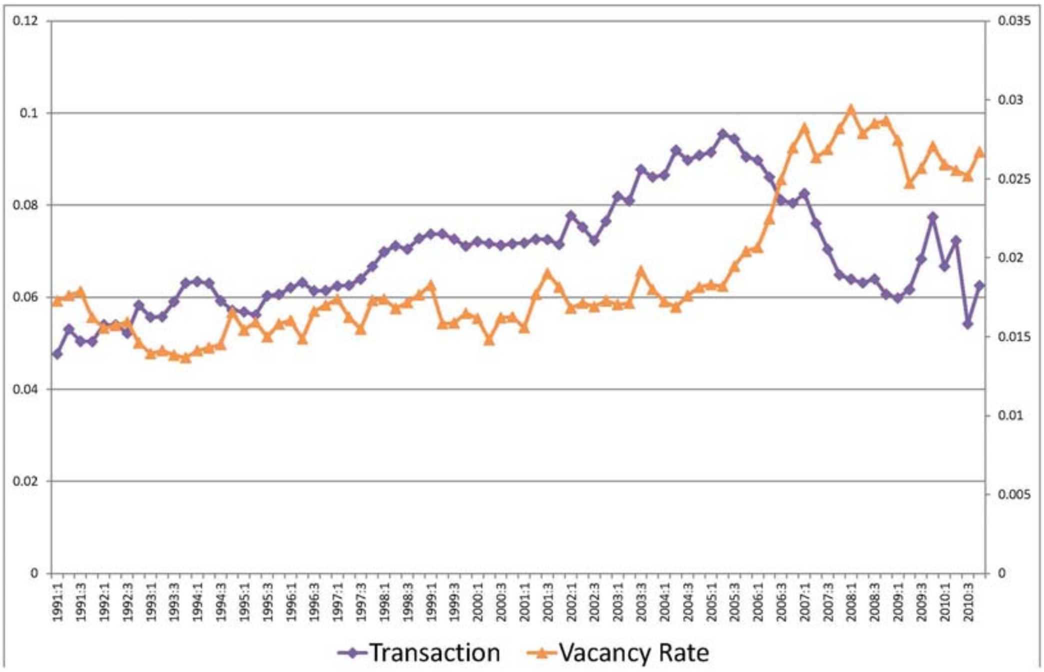

Figure 3: Transaction and Vacancy

literature on middlemen in search and matching pioneered by Rubinstein and Wolinsky (1987). Previously, it was argued that middlemen could survive by developing reputations as sellers of high quality goods (Li, 1998), by holding a large inventory of differentiated products to make shopping less costly for others (Johri and Leach, 2001; Shevchenko, 2004; Smith, 2004), by raising the matching rate in case matching is subject to increasing returns (Masters, 2007), and by lowering distance-related trade costs for others (Tse, 2011). This paper studies the role of middlemen in the provision of market liquidity.

A simple model of housing market flippers as middlemen is also in Bayer et al. (2011). The model though is partial equilibrium in nature and cannot be used to answer many of the questions we ask in this paper. Flippers are also present in the model of the interaction of the frictional housing and labor markets of Head and Lloyd-Ellis (2011), which is fully general equilibrium in nature. Analyzes of how middlemen may serve to improve liquidity in a search market also include Gazza (2011) and Lagos et al. (2011). However, none of these studies allows end-user households a choice of whether to deal with the middlemen and for the multiplicity of equilibrium and how the extent of the presence of these middlemen may vary across the equilibria. A recent paper by Wright and Wong (2011) shows that how, like our model, equilibrium in a search model with middlemen may exhibit bubble-like characters.

The next section presents the model. Section 3 contains the detailed analysis. Section 4 takes a more systematic look at the patterns shown in Figures 1-3 with reference to the model's implications. Section 5 concludes. All proofs are relegated to the Appendix. For brevity, we restrict attention to analyzing steady-state equilibrium in this paper. A companion technical note (Leung and Tse, 2011) covers the analysis of the dynamics.[9]

2 Model

2.1 Basics

The city is populated by a continuum of measure one risk-neutral households, each of whom discounts the future at the same rate r_H. There are two types of housing in the city: owner-occupied, the supply of which is perfectly inelastic at $H < 1$ and rental, which is supplied perfectly elastically for a fixed rental of q. A household staying in a matched owner-occupied house derives a flow utility of $\upsilon > 0$, whereas a household either in a mismatched house or in rental housing none. A household-house match breaks up exogenously at a Poisson arrival rate δ, after which the household may continue to stay in the house but it no longer enjoys the flow utility υ. In the mean time, the household may choose to sell the old house and search out a new match. An important assumption is that a household cannot hold more than one house at

[9]Not for publication, available for download in http://www.sef.hku.hk/~tsechung/index.htm

a time. Then a mismatched homeowner must first sell the old house before she can buy a new one. The qualitative nature of our results should hold as long as there is a limit, not necessarily one, on the number of houses a household can own at a time. The one-house-limit assumption simplifies considerably.

The search market A seller meets a buyer in a search market at a Poisson arrival rate η, while a buyer finds a match in the same market at another Poisson arrival rate μ. The flow of matches in the search market is governed by a CRS matching function $M(B, S)$, where B and S denote, respectively, the measures of buyers and sellers in the market. Hence,

$$\mu = M(1, \theta), \tag{1}$$

$$\eta = \frac{\mu}{\theta}, \tag{2}$$

where $\theta = S/B$ denotes market tightness,[10] and that

$$\frac{\partial \mu}{\partial \theta} > 0, \quad \frac{\partial \eta}{\partial \theta} < 0.$$

Prices in the search market fall out of Nash bargaining.

The Walrasian investment market Instead of waiting out a buyer to arrive in the search market, a mismatched homeowner may sell her old house right away in a Walrasian market populated by specialist investors—agents who do not live in the houses they have bought but rather attempt to profit from buying low and selling high. Because homogeneous flippers do not gain by selling and buying houses to and from one another, the risk-neutral flippers may only sell in the end-user search market and will succeed in doing so at the same rate η that any household-seller does in the market. We allow for flippers to discount the future at a possibly different rate r_F than the households in the city.[11] In the competitive investment market, prices adjust to eliminate any excess returns on real estate investment.

We recognize that the assumption of a Walrasian investment market seemingly completely contradicts the motivations for applying the search and matching framework to the study of the housing market. We believe that what is needed in the analysis is not an investment market altogether free of search frictions of any kind, but one in which the frictions are less severe than in the end-user market. If flippers are entirely motivated by arbitrage considerations and do not care if the houses to

[10] This definition of market tightness measures the intensity of competition among sellers. Equivalently, one can define market tightness as B/S to measure the intensity of competition among buyers. We find the first definition more convenient for this analysis.

[11] While institutional investors may be able to finance investment at a lower interest rate, banks in many places charge higher mortgage interest rates for those who are buying a second home and for those who are not buying a house for occupation. We leave it as an open question as to whether $r_F \lesseqgtr r_H$ is the more empirically relevant case.

be purchased are good matches for their own occupation, search should not a particularly serious problem. In reality, we imagine that households who intend to sell quickly and are willing to accept a lower price will convey their intentions to real estate agents, who in turn will alert any specialist investors the availability of such deals. The competition among flippers should then drive prices up to just enough to eliminate any excess returns on investment. A Walrasian market assumption captures the favor of such arrangements in the simplest possible manner.

2.2 Accounting identities and housing market flows

Accounting identities At any one time, a household can either be staying in a matched house, in a mismatched house, or in rental housing. Let n_M, n_U, and n_R denote the measures of households in the respective states. The population constraint reads

$$n_M + n_U + n_R = 1. \tag{3}$$

Each owner-occupied house in the city must be held either by a household in the city or by a flipper. Hence,

$$n_M + n_U + n_F = H, \tag{4}$$

where n_F denotes both the measures of active flippers and houses held by these individuals.

If each household can hold no more than one house at any moment, the only buyers in the search market are households in rental housing; i.e.,

$$B = n_R. \tag{5}$$

On the other hand, sellers in the search market include mismatched homeowners and flippers, so that

$$S = n_U + n_F. \tag{6}$$

Housing market flows The flows into matched owner-occupied housing are households in rental housing who just manage to buy up a house, whereas a fraction δ of matched households become mismatched in each time unit. In the steady state, with n_M stationary,

$$\mu n_R = \delta n_M. \tag{7}$$

Households' whose matches just break up may choose to sell their houses right away to flippers in the investment market or to wait out a buyer to arrive in the search market. Let α denote the fraction of mismatched households who choose to sell in the investment market and $1 - \alpha$ the fraction of households who choose to sell in the search market. Hence, if the measure of mismatched homeowners (n_U) is to stay unchanging through time,

$$(1 - \alpha)\, \delta n_M = \eta n_U, \tag{8}$$

where ηn_U gives the measure of mismatched homeowners who just manage to dispose of their properties in the search market. Next, those who enter rental housing include households who just sell their properties to flippers and to end users, respectively. The exits are comprised of households who just find a match in owner-occupied housing, so that

$$\alpha\delta n_M + \eta n_U = \mu n_R, \tag{9}$$

for n_R to stay stationary. Finally, the measure of houses held by flippers increase by the measure of houses recently mismatched households decide to dispose right away in the investment market and decline by the measure of houses flippers manage to sell to end-users. Hence, in the steady state,[12]

$$\alpha\delta n_M = \eta n_F. \tag{10}$$

Lemma 1 *In the steady state,*

$$\alpha = \frac{n_F}{n_U + n_F};$$

i.e., the fraction of mismatched households who choose to sell in the investment market (α) *is the same as the market share of flippers in the search market—the fraction of houses for sale in the market by flippers.*

2.3 Flippers' market share, market tightness, and turnovers

Equations (1)-(10) can be used to derive a function for market tightness (θ) of flippers' market share (α) in the search market.

Lemma 2 *From the accounting identities and the housing market flow equations is an implicit function for* θ *of* α*:* $\widetilde{\theta} : [0,1] \to \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$*, where*

$$\frac{\widetilde{\theta}\,(1-H)}{1-\widetilde{\theta}\alpha} - \frac{\delta H - \mu\left(\widetilde{\theta}\right)(1-H)}{\delta + \mu\left(\widetilde{\theta}\right)\alpha} = 0, \tag{11}$$

and that $\partial\widetilde{\theta}/\partial\alpha < 0$*. The lower and upper bounds* $\widetilde{\theta}_L$ *and* $\widetilde{\theta}_U$*, both of which are strictly positive and finite, are given by, respectively, the solutions of* (11) *at* $\alpha = 1$ *and* $\alpha = 0$*. Specifically,* $\widetilde{\theta}_L < 1$*.*

Given α, once θ is determined by (11), the steady-state measures of the various types of agents are uniquely determined. The following summarizes the results.

[12] Where (3) and (4) are two equations in four unknowns, once any two of the four variables are given, the other two are uniquely determined. In this connection, it is straightforward to verify that only two of the four steady-state flow equations (7)-(10) constitute independent restrictions.

Lemma 3

a. *At* $\alpha = 0$,

$$n_F = 0, \quad n_R = 1 - H, \quad n_M = H - n_U,$$

whereas n_U *is given by the solution to*

$$\frac{n_U}{1-H} - \mu^{-1}\left(\delta \frac{H - n_U}{1 - H + n_U}\right) = 0.$$

b. *As* α *increases from 0,*

$$\frac{\partial n_F}{\partial \alpha} > 0, \; \frac{\partial n_R}{\partial \alpha} > 0, \; \frac{\partial n_M}{\partial \alpha} > 0, \text{ whereas } \frac{\partial n_U}{\partial \alpha} < 0.$$

c. *At* $\alpha = 1$,

$$n_U = 0, \quad n_R = 1 - H + n_F, \quad n_M = H - n_F,$$

whereas n_F *is given by the solution to*

$$\frac{n_F}{1-H+n_F} - \mu^{-1}\left(\delta \frac{H - n_F}{1 - H + n_F}\right) = 0.$$

When no houses are sold to flippers ($\alpha = 0$), trivially $n_F = 0$ in the steady state, whereas when all mismatched houses are sold to flippers in the first instance, no households should remain in a mismatched house in the steady state, so that $n_U = 0$. And then as α increases from 0 towards 1, n_F should only increase and n_U decline. Lemma 3 confirms these intuitions. What is less obvious in the Lemma is that both n_R and n_M increase along with the increase in α. The first tendency follows from the fact that if both the city's population and the housing stock are given, a unit increase in the measure of houses held by flippers must be matched by a unit decline in the measure of houses occupied by the households in the city. To follow then is the same unit increase in the city's households in rental housing. For the second tendency, with an increase in α, fewer households spend any time at all selling their old houses in the search market before initiating search for a new match. In the mean time, the decrease in θ (Lemma 2), through lowering μ, lengthens the time a household spends on average in rental housing before a new match can be found. By Lemma 3, the first effect dominates, so that more households are in matched owner-occupied housing in the steady state.

Lemma 2 states that an increase in α lowers market tightness, given by

$$\theta = \frac{S}{B} = \frac{n_U + n_F}{n_R}.$$

Indeed, the relation also follows from the comparative statics results in Lemma 3. First, where $\partial n_R/\partial\alpha > 0$, there will be more buyers to follow an increase in α. Second, given that by (4),

$$n_U + n_F = H - n_M,$$

$$\frac{\partial S}{\partial \alpha} = \frac{\partial\left[n_U + n_F\right]}{\partial \alpha} = -\frac{\partial n_M}{\partial \alpha} < 0.$$

That is, when more households are matched, there must only be fewer houses on the market. The two tendencies—more buyers (n_R) and fewer sellers ($n_U + n_F$)—reinforce each other to result in a smaller θ.

In the model housing market, the entire stock of vacant house comprises of houses held by flippers. With a given housing stock, the vacancy rate is simply equal to n_F/H. A direct corollary of Lemma 3b is that:

Lemma 4 *In the steady state, the vacancy rate for owner-occupied houses is increasing in α.*

Housing market transactions per time unit in the model are comprised of (i) $\alpha\delta n_M$ houses sold from households to flippers, (ii) ηn_F houses flippers sell to households, and (iii) ηn_U houses sold by one household to another, adding up to an aggregate transaction volume,

$$T = \alpha\delta n_M + \eta n_F + \eta n_U.$$

Lemma 5 *In the steady state, transaction volume is increasing in α.*

The usual measure of turnover in the housing market is the time it takes for a house to be sold, what is known as Time-On-the-Market (TOM). Given that houses sold in the investment market are on the market for a vanishingly small time interval and houses sold in the search market for a length of time equal to $1/\eta$ on average, we may define the model's average TOM as

$$\frac{\alpha\delta n_M}{T} \times 0 + \frac{\eta n_F + \eta n_U}{T} \times \frac{1}{\eta}. \tag{12}$$

Lemma 6 *In the steady state, on average, TOM is decreasing in α.*

There is a composition effect (more houses sold to flippers for which TOM is equal to 0) and a "structural" effect (a larger η for houses sold in the search market given that $\partial\eta/\partial\theta < 0$ and $\partial\theta/\partial\alpha < 0$) reinforcing one another. In sum, Lemmas 5 and 6 confirm that when there is a greater market share for flippers, there is also a greater aggregate transaction volume and houses are sold more quickly on average.

TOM is a measure of the turnover of houses for sale, and as such it does not carry any direct welfare implications. A more household-centric measure of turnover is the length of time a household (rather than a house) has to stay unmatched. We define what we call Time-Between-Matches (TBM) as the sum of two spells: (1) the time it takes for a household to sell the old house, and (2) the time it takes to find a new match. While the first spell (TOM) on average is shorter with an increase in α, the second is longer where the decline in θ to accompany the increase in α causes μ to fall. A priori then it is not clear what happens to the average length of the whole spell. The old house is sold more quickly. But it also takes longer on average to find a new match in a market with fewer sellers and more buyers. To examine which effect dominates, write the model's average TBM as

$$\alpha\frac{1}{\mu}+(1-\alpha)\left(\frac{1}{\eta}+\frac{1}{\mu}\right)=\frac{1}{\mu}+\frac{1-\alpha}{\eta}.$$

where $1/\mu$ is the average TBM for households who sell in the investment market[13] and $1/\eta+1/\mu$ for households who sell in the search market.[14]

Lemma 7 *In the steady state, on average, TBM is decreasing in α.*

Lemma 7 perhaps may be taken as the dual of Lemma 3a ($\partial n_M/\partial\alpha>0$). When matched households are more numerous in the steady state, on average, people must be spending less time between matches

Up to this point, the model is purely mechanical. Given α, market tightness θ is completely isomorphic of the determination of housing prices in equilibrium. The same conclusion carries over to the determination of vacancy, turnover, and transaction volume. If not for the inclusion of flippers in the model housing market, α is identically equal to 0 and Lemma 2 would have completed the analysis of everything that seems to be of any interest. With the inclusion of flippers and their market share measured by α, Lemmas 6 and 7 show how changes in the latter affect the turnovers of houses and households, which can have important consequences on welfare—a question we shall address in the following. But first α obviously should be made endogenous to which we next turn.

2.4 Asset values and housing prices

Asset values for flippers Let V_F be the value of a vacant house to a flipper and p_{FS} the price she expects to receive for selling it in the search market. In the steady state,

$$r_F V_F=\eta\left(p_{FS}-V_F\right), \tag{13}$$

[13] The household sells the old house instantaneously. Given a house-finding rate μ, the average TBM is then $1/\mu$.

[14] Let t_1 denote the time it takes the household to sell the old house in the search market and t_2-t_1 the time it takes the household to find a new match after the old house is sold. Then the household's TBM is just t_2. On average, $E\left[t_2\right]=\int_0^\infty \eta e^{-\eta t_1}\left(\int_{t_1}^\infty t_2\mu e^{-\mu(t_2-t_1)}dt_2\right)dt_1=1/\eta+1/\mu$.

where η denotes the matching rate for sellers in the search market. Let p_{FB} be the price the flipper has paid for the house in the competitive investment market in the first place. In equilibrium, where any excess returns on real estate investment are eliminated,

$$p_{FB} = V_F. \tag{14}$$

Asset values for households There are three (mutually exclusive) states to which a household can belong:

1. in a matched house; value V_M,
2. in a mismatched house; value V_U,
3. in rental housing; value V_R.

The flow payoff for a matched owner-occupier first of all includes the utility she derives from staying in a matched house υ. The match will be broken, however, with probability δ, after which the household may sell the house right away in the investment market at price p_{FB} and switch to rental housing immediately thereafter. Alternatively, the household can continue to stay in the house while trying to sell it in the search market. In all then,

$$r_H V_M = \upsilon + \delta \left(\max \left\{ V_R + p_{FB}, V_U \right\} - V_M \right). \tag{15}$$

Let p_H denote the price a household-seller expects to receive in the search market. Then the flow payoff of a mismatched owner-occupier is equal to

$$r_H V_U = \eta \left(V_R + p_H - V_U \right). \tag{16}$$

Two comments are in order. First, in (16), the mismatched owner-occupier is entirely preoccupied with disposing the old house while she makes no attempt to search for a new match. This is due to the assumption that a household cannot hold more than one house at a time and the search process is memoryless. Second, under (15) and (16), the household has only one chance to sell the house in the investment market—at the moment the match is broken. Those who forfeit this one-time opportunity must wait out a buyer in the search market to arrive. This restriction is without loss of generality in a steady-state equilibrium, in which the asset values and housing prices stay unchanging over time. No matter, after the old house is disposed of, the household moves to rental housing to start searching for a new match. Hence, with α equal to the fraction of houses offered for sale in the search market held by flippers and $1 - \alpha$ the fraction held by ordinary households,

$$r_H V_R = -q + \mu \left(V_M - \left(\alpha p_{FS} + (1 - \alpha) p_H \right) - V_R \right), \tag{17}$$

where μ denotes the matching rate for buyers in the search market.

Bargaining Prices in the search market fall out of Nash bargaining between a matched buyer-seller pair. There is only one buyer type in the search market—households in rental housing. The sellers, however, are either flippers or mismatched homeowners. When a household-buyer is matched with a flipper, the division of surplus in Nash Bargaining satisfies

$$V_M - p_{FS} - V_R = p_{FS} - V_F, \tag{18}$$

whereas when the household-buyer is matched with a household-seller, the division of surplus in Nash Bargaining satisfies[15]

$$V_M - p_H - V_R = V_R + p_H - V_U. \tag{19}$$

2.5 Which market to sell?

Write

$$\Delta = V_R + p_{FB} - V_U \tag{20}$$

as the difference in payoff for a mismatched homeowner between selling in the investment market ($V_R + p_{FB}$) and in the search market (V_U). By (13), (14), (18), and (19),

$$\Delta = 2\left(\frac{\eta + r_F}{\eta} p_{FB} - p_H\right) = 2\left(p_{FS} - p_H\right).$$

That is, mismatched homeowners prefer to sell right away in the investment market if the given instantaneous reward (p_{FB}) dominates an appropriately-discounted reward of waiting out in the search market $\left(\frac{\eta}{\eta + r_F} p_H\right)$. In turn, the condition is equivalent to whether the price at which flippers sell in the search market (p_{FS}) is greater than or less than the price household-sellers receive in the same market (p_H). Lemma 9 in the Appendix presents the solutions of p_{FS} and p_H, together with the various asset values from (13)-(19). It is then straightforward to show that $p_{FS} - p_H$ has just the same sign as

$$\mathcal{S}_\Delta \equiv \left(\frac{r_H}{r_F} - 1 - z\right)\eta + \mu - 2\left(\delta + r_H\right)z, \tag{21}$$

where $z = q/v$.[16] If $\mathcal{S}_\Delta$ is thought of as measuring the incentives for mismatched homeowners to sell in the investment market, such incentives are weakened at larger r_F and q. Intuitively, flippers may only offer a lower price when they face a higher

[15] With multiple types, the assumption of perfect information in bargaining is perhaps stretching a bit. We could have specified a bargaining game with imperfect information as in Harsanyi and Selten (1972), Chatterjee and Samuelson (1983), or Riddell (1981), for instance. It is not clear what may be the payoffs for the added complications.

[16] Lemma 9 in the Appendix presents two sets of prices and asset values, one derived under the assumption that $\Delta \leq 0$ and the other $\Delta > 0$. In either case, $p_{FS} - p_H$ is seen to have the same sign as $\mathcal{S}_\Delta$ in (21).

cost of financing and it becomes less attractive for mismatched homeowners to switch to rental housing sooner by quickly selling in the investment market if rental housing is more costly. On the other hand, the incentives are strengthened at a larger υ. One interpretation is that it becomes more attractive to shorten Time-Between-Matches by quickly selling in the investment market if there is a higher reward for staying in a matched owner-occupied house. Most of all, however, $\mathcal{S}_\Delta$ can be thought of as a function of θ. In this case, we can define a correspondence $\widehat{\alpha} : \mathcal{R}_+ \rightrightarrows [0,1]$, whereby

$$\widehat{\alpha}(\theta) = \begin{cases} 0 & \mathcal{S}_\Delta(\theta) < 0 \\ [0,1] & \mathcal{S}_\Delta(\theta) = 0 \\ 1 & \mathcal{S}_\Delta(\theta) > 0 \end{cases} \tag{22}$$

that gives the fraction of mismatched homeowners who choose to sell in the investment market.

2.6 Equilibrium

We now have two steady-state relations between α and θ: the $\widetilde{\theta}(\alpha)$ function in (11) and the $\widehat{\alpha}(\theta)$ correspondence in (22). A steady-state equilibrium is any $\{\alpha, \theta\}$ pair that simultaneously satisfies the two relations.

3 Analysis

3.1 Existence of equilibrium

To show the existence of equilibrium, it is useful to define $F(\alpha) \equiv \widehat{\alpha}\left(\widetilde{\theta}(\alpha)\right)$, a correspondence mapping $[0,1]$ into itself. Equilibrium then is any fixed point of F.

Proposition 1 *Equilibrium exists for all positive $\{r_H, r_F, \upsilon, q, \delta, H\}$ tuple.*

3.2 Multiplicity

To check for uniqueness and multiplicity, we begin with inverting the $\widetilde{\theta}(\alpha)$ function in (11) to define $\widetilde{\alpha} \equiv \widetilde{\theta}^{-1}$, whereby $\widetilde{\alpha} : \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right] \to [0,1]$. Given that $\partial\widetilde{\theta}/\partial\alpha < 0$, likewise, $\partial\widetilde{\alpha}/\partial\theta < 0$. That is, $\widetilde{\alpha}(\theta)$ decreases continuously from 1 at $\theta = \widetilde{\theta}_L$ to 0 at $\theta = \widetilde{\theta}_U$. Figure 4 depicts an example of the $\widetilde{\alpha}(\theta)$ schedule. Now, for the $\widehat{\alpha}(\theta)$ correspondence:

Lemma 8 *As a function of θ,*

a. *if $r_F < \widehat{r}_F$, where*

$$\widehat{r}_F \equiv \frac{r_H}{1+z}, \tag{23}$$

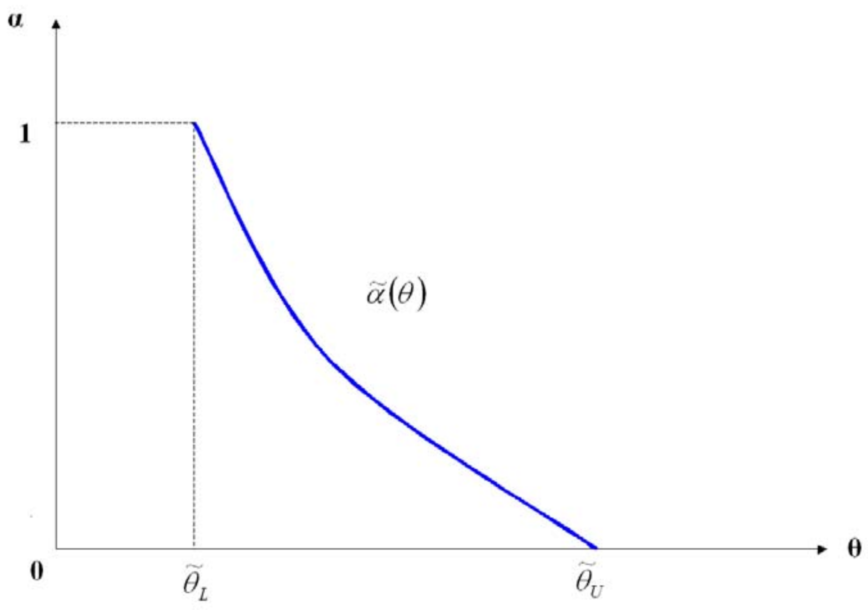

Figure 4: The $\widetilde{\alpha}$ function

and assuming that

$$-\theta\frac{\partial^2\mu}{\partial\theta^2}\mu+2\frac{\partial\mu}{\partial\theta}\left(\theta\frac{\partial\mu}{\partial\theta}-\mu\right)<0,^{17}$$

then S_Δ *is U-shaped, and that* $\lim_{\theta\to0}S_\Delta=\lim_{\theta\to\infty}S_\Delta=\infty$.

b. *if* $r_F\geq\widehat{r}_F$*, then* $\partial S_\Delta/\partial\theta>0$ *throughout, and that* $\lim_{\theta\to0}S_\Delta<0$ *but* $\lim_{\theta\to\infty}S_\Delta>0$.

By Lemma 8a, where $r_F<\widehat{r}_F$, $\mathcal{S}_\Delta$ can stay positive throughout (the upper curve in the left panel of Figure 5). In this case then, $\widehat{\alpha}\,(\theta)=1$ for all $\theta\geq0$, as depicted in Panel A of Figure 6. Alternatively, there can also be two roots to $\mathcal{S}_\Delta=0$ (the lower curve in the left panel of Figure 5), say $\widehat{\theta}_1$ and $\widehat{\theta}_2$, where $\widehat{\theta}_1<\widehat{\theta}_2$. In this case,

$$\widehat{\alpha}\,(\theta)=\begin{cases}0 & \theta\in\left(\widehat{\theta}_1,\widehat{\theta}_2\right)\\ [0,1] & \theta=\widehat{\theta}_1\text{ and }\widehat{\theta}_2\\ 1 & \theta\in\left[0,\widehat{\theta}_1\right)\cup\left(\widehat{\theta}_2,\infty\right)\end{cases}.$$

See Panels C and D of Figure 6 for two examples of such $\widehat{\alpha}\,(\theta)$. By Lemma 8b, where $r_F\geq\widehat{r}_F$, there is a unique root to $\mathcal{S}_\Delta=0$, say $\widehat{\theta}$, as shown in the right panel of 5.

[17] The condition is guaranteed to hold if μ is isoelastic.

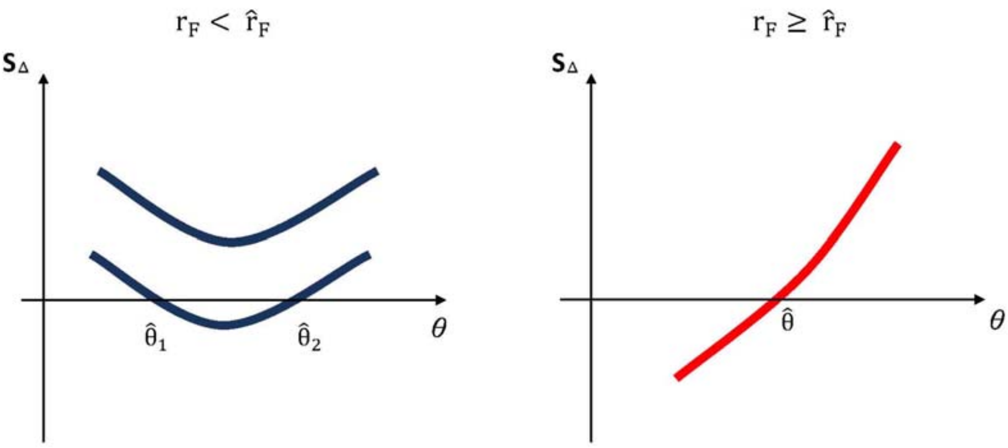

Figure 5: $\mathcal{S}_\Delta$ as a function of θ

The $\widehat{\alpha}(\theta)$ correspondence that follows is thus given by

$$\widehat{\alpha}(\theta) = \begin{cases} 0 & \theta < \widehat{\theta} \\ [0,1] & \theta = \widehat{\theta} \\ 1 & \theta > \widehat{\theta} \end{cases}.$$

See Panels E and F of Figure 6 for two examples of such $\widehat{\alpha}(\theta)$.

To summarize, for large $\theta = S/B$, $\widehat{\alpha}(\theta) = 1$ must hold, whether or not $r_F \lesseqgtr \widehat{r}_F$—whereas it can take a long time to sell in a tight search market in which sellers outnumber buyers by a significant margin, mismatched homeowners are better off to just sell in the investment market at a discount. On the other hand, where the bargaining power sellers acquire under a small θ should allow them to sell at a high price, they can similarly prefer to just sell right away in the investment market to capitalize on the high housing price sooner. This effect is strongest for smaller r_F ($r_F < \widehat{r}_F$) under which flippers can afford to pay higher prices. For intermediate θ, neither of the two incentives to sell in the investment market is strong enough to cause $\mathcal{S}_\Delta \geq 0$, so that mismatched homeowners would prefer to wait it out in the search market.

Given the $\widetilde{\alpha}(\theta)$ schedule in Figure 4 and the $\widehat{\alpha}(\theta)$ graphs in Figure 6, equilibrium is any θ at which $\widetilde{\alpha}(\theta) \subset \widehat{\alpha}(\theta)$. Now, if $1 \subset \widehat{\alpha}\left(\widetilde{\theta}_L\right)$, then $\theta = \widetilde{\theta}_L$ and $\alpha = 1$ is a steady-state equilibrium since by construction, $\widetilde{\alpha}\left(\widetilde{\theta}_L\right) = 1$. In this equilibrium, all transactions are intermediated and turnover is fastest. On the other hand, if $0 \subset \widehat{\alpha}\left(\widetilde{\theta}_U\right)$, then $\theta = \widetilde{\theta}_U$ and $\alpha = 0$ is a steady-state equilibrium since by construction, $\widetilde{\alpha}\left(\widetilde{\theta}_U\right) = 0$. In this equilibrium, all sales and purchases are between two end-users while turnover is slowest. In between, there can be equilibrium in which $\theta = \widehat{\theta}$,

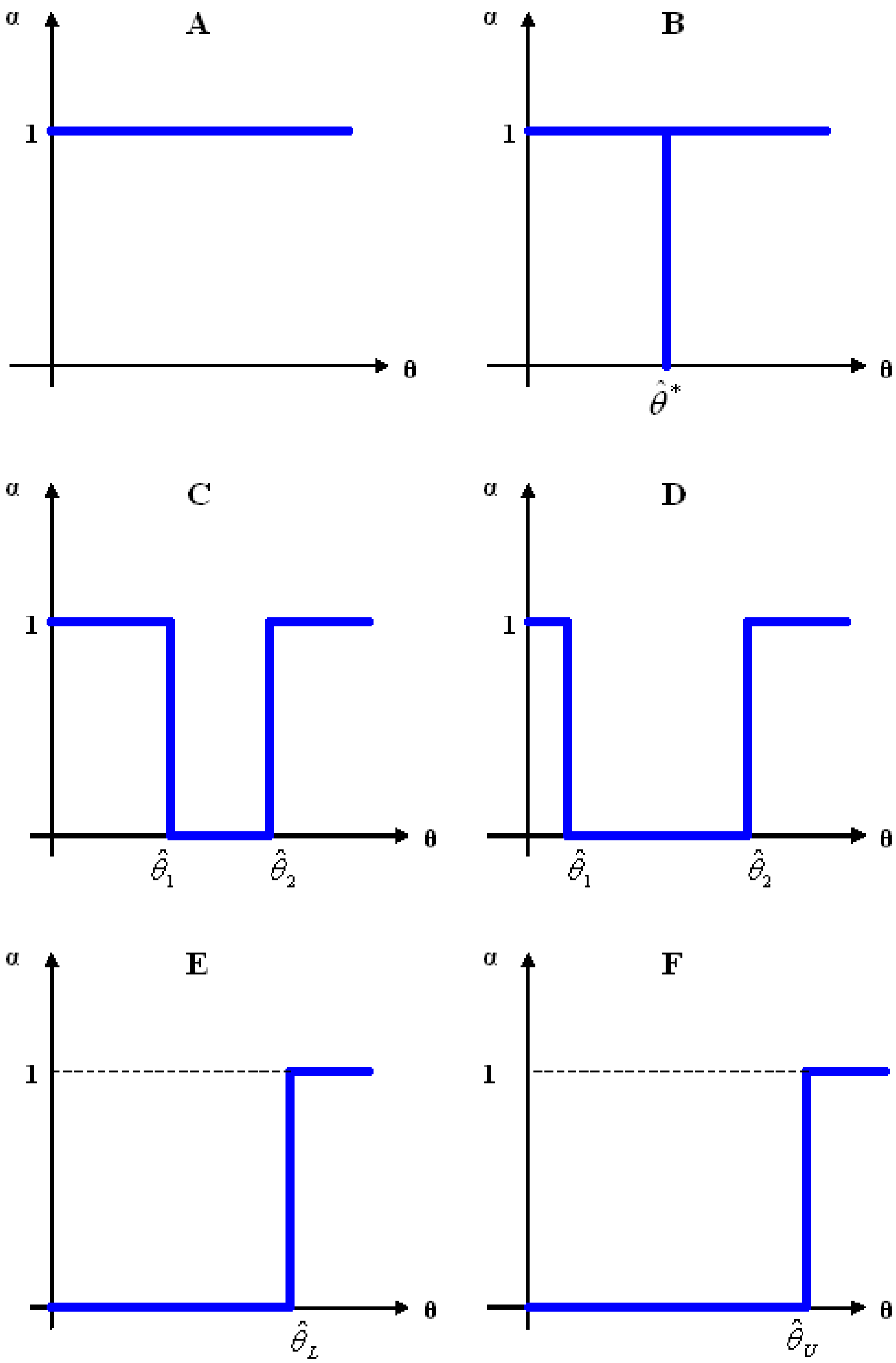

Figure 6: The $\widehat{\alpha}(\theta)$ correspondence

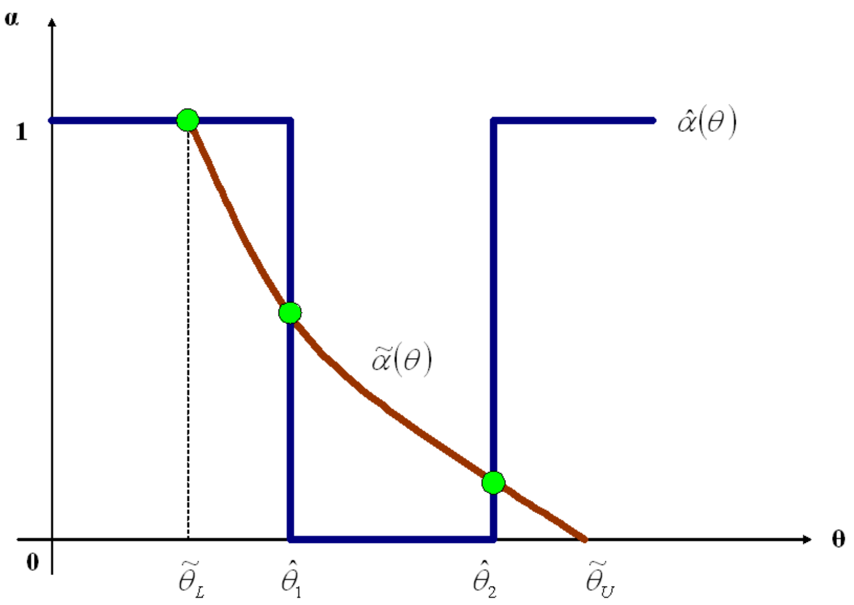

Figure 7: Multiple equilibrium – $\widehat{\theta}_2 \leq \widetilde{\theta}_U$

$\widehat{\theta}_1$, or $\widehat{\theta}_2$ and $\alpha = \widetilde{\alpha}(\theta)$ if the given $\theta \in \left(\widetilde{\theta}_L, \widetilde{\theta}_U\right)$. In such an equilibrium, with mismatched homeowners indifferent between selling in the investment and search markets, a fraction, but only a fraction, of all transactions are intermediated.

Proposition 2 *For arbitrarily small r_F, the unique equilibrium is $\theta = \widetilde{\theta}_L$; for $r_F \geq \widehat{r}_F$, there is a unique equilibrium at either $\theta = \widetilde{\theta}_L$, $\widehat{\theta}$, or $\widetilde{\theta}_U$. For $r_F < \widehat{r}_F$ and where there are two θ that solves $S_\Delta = 0$ (Panels C and D of Figure 6), there exist at least two equilibria at $\theta = \widetilde{\theta}_L$ and $\theta = \widehat{\theta}_2$ or $\widetilde{\theta}_U$ if and only if $\widehat{\theta}_1 \in \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$. A third equilibrium at a distinct $\widehat{\theta}_1$ exists if $\widehat{\theta}_1 > \widetilde{\theta}_L$.*

Figures 7 and 8 illustrate the situations covered by the second part of the Proposition. In both figures, there are three equilibria where $\widehat{\theta}_1 > \widetilde{\theta}_L$. Intuitively, for small $\theta = S/B$, under which houses are sold at relatively high prices, households find it advantageous to sell their old houses quickly in the investment market. And then if all mismatched houses are sold in the investment market in the first instance, there will be rapid turnover and few houses are for sale in the search market to cause a small θ. In this way, $\theta = \widetilde{\theta}_L$ and $\alpha = 1$ is equilibrium in Figures 7 and 8. With larger θ, households' incentives to sell in the investment market are weakened. Meanwhile, if fewer or none at all mismatched houses are sold in the investment market, turnover slows down and more houses are for sale in the search market to cause an increase in θ. As a result, a smaller α and a larger θ is also equilibrium in Figures 7 and 8.

With the existence of multiple steady-state equilibrium, the presence of flippers'

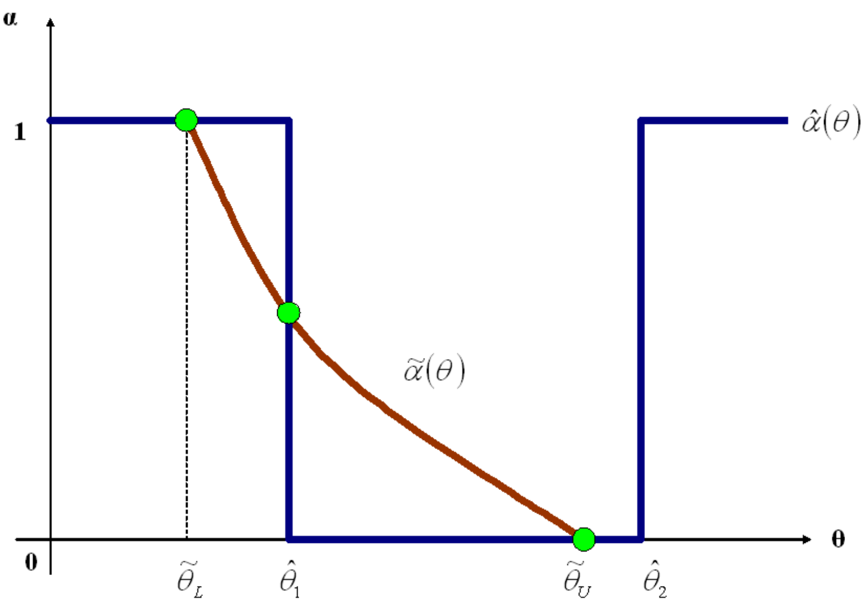

Figure 8: Multiple equilibrium – $\widehat{\theta}_2 > \widetilde{\theta}_U$

in the market can be fickle, especially when the equilibrium the market happens to be in is unstable, a subject we should turn to shortly. In any case, where there are multiple equilibrium, any seemingly unimportant shock can completely dislocate the market from one equilibrium and move it to another, causing catastrophic declines in flippers' market share, turnover, and transaction volume. To accompany such discrete declines in the activities of flippers can be discrete declines in housing price, a subject we should follow up in Section 3.5. In sum, the model housing market can be prone to apparent boom-bust cycles, just its real-world counterpart often does.

3.3 Dynamics and stability

In Leung and Tse (2011), we show that, of the five types of steady-state equilibrium: (1) $\theta = \widetilde{\theta}_L$, (2) $\theta = \widetilde{\theta}_U$, (3) $\theta = \widehat{\theta}_1$, (4) $\theta = \widehat{\theta}_2$, and (5) $\theta = \widehat{\theta}$, all except the $\widehat{\theta}_1$ equilibrium are guaranteed to be locally stable, whereas the $\widehat{\theta}_1$ equilibrium is almost always unstable.[18] Intuitively, the $\widehat{\theta}_1$ equilibrium is unstable because it lies at where the $\widehat{\alpha}(\theta)$ correspondence is "decreasing". Consider for instance, a small positive perturbation from the $\widehat{\theta}_1$ equilibrium in Figure 7. Then, Δ should turn negative, after which no mismatched homeowners should still sell in the investment market. When turnover slows down as a result, market tightness rises. To follow is a further increase

[18] We cannot completely rule out local stability for the $\widehat{\theta}_1$ equilibrium but find the conditions for it to be the case highly improbable.

in θ. Eventually, the market should just settle on the $\widehat{\theta}_2$ steady-state equilibrium in the Figure. Likewise, a negative perturbation from the the $\widehat{\theta}_1$ equilibrium should send the market to the $\widetilde{\theta}_L$ steady-state equilibrium.[19]

We also show in Leung and Tse (2011) that local stability notwithstanding, convergence to the $\widetilde{\theta}_U$, $\widehat{\theta}_2$, and $\widehat{\theta}$ steady-state equilibria is almost always oscillatory, with θ under- and overshooting the given steady state and $\dot{\theta}$ and Δ changing signs in the transition dynamics.

Right at the $\widetilde{\theta}_U$ steady state, recall that $n_F = 0$ where $\alpha = 0$. Off steady state, n_F need not be equal to zero despite $\alpha = 0$ as it may take time for the flippers to dispose of any inventories they may have previously acquired. The oscillation in the convergence to the $\widetilde{\theta}_U$ steady state is due to how this running down of the inventory affects the dynamics of the other state variables. The $\widetilde{\theta}_U$ steady state is a low-level steady state, with the largest $\theta = S/B$ and smallest n_M. In the approach to this steady state then, θ must eventually increase and n_M decline. But if there was a large n_F to begin with, its gradual disposal may temporarily help lower θ and raise n_M.

In a $\widehat{\theta}_2$ steady-state equilibrium, $\Delta = 0$ and mismatched homeowners are indifferent between selling in the two markets. It turns out that almost any dynamic equilibrium in which $\Delta = 0$ holds off steady state can only be divergent. In the approach to a locally stable $\widehat{\theta}_2$ steady state then, either $\Delta < 0$ or $\Delta > 0$. Now, let's say suppose to begin with, $\Delta < 0$. Where $\alpha = 0$, θ should generally increase towards the $\widetilde{\theta}_U$ steady state, with $\dot{n}_F < 0$ in the transition. Sooner or later, as θ is moving away from $\widehat{\theta}_2$ and nearing $\widetilde{\theta}_U$, in Figure 7, Δ must turn positive. At the moment this happens, all mismatched homeowners will find it optimal to just sell in the investment market. The convergence now is to the $\widetilde{\theta}_L$ steady state, with n_U remaining equal to zero throughout. This means that when θ hits $\widehat{\theta}_2$ in the first instance, it will not come to rest as $\widehat{\theta}_2$ is steady state only for n_U equal to some particular positive value. Eventually, when θ is sufficiently far away from $\widehat{\theta}_2$ in its approach to $\widetilde{\theta}_L$, in Figure 7, Δ must turn negative again. Given that, like a $\widehat{\theta}_2$ steady-state equilibrium, a $\widehat{\theta}$ steady-state equilibrium is also at where $\widehat{\alpha}(\theta)$ is "increasing", the same analysis applies. The analysis in Leung and Tse (2011) implies that this kind of oscillation should dampen over time to come to rest at the given steady state.

The oscillation and limited cycles in the dynamics are, not surprisingly, entirely due to the presence of flippers. If both α and n_F are forced to zero a priori, the only equilibrium is a $\widetilde{\theta}_U$ equilibrium, the convergence of which must be direct if $n_F = 0$.

[19] Granted that the $\mathcal{S}_\Delta$ and $\widetilde{\theta}$ functions are merely steady-state relations, the arguments above no doubt involve a good dose of hand waving. The full-fetched dynamic analysis is in Leung and Tse (2011).

3.4 Cost of financing and flippers' market share

By Proposition 2, $\alpha = 1$ for small r_F and then at larger r_F, α can fall below unity. This is intuitive. Flippers can afford to pay the highest price when they can finance investment at the least cost. As r_F increases, their dominance can only fall. More precisely, by Lemma 10 in the Appendix:

(A) For the smallest r_F, $\mathcal{S}_\Delta > 0$ for all θ, so that the $\widehat{\alpha}(\theta) = 1$ throughout as shown in Panel A of Figure 6.

(B) As r_F increases to some given level below $\widehat{r}_F$, the $\mathcal{S}_\Delta$ curve in the left panel in Figure 5 would just be tangent to the horizontal axis at which $\widehat{\alpha}(\theta) = [0, 1]$, whereas $\widehat{\alpha}(\theta)$ remains equal to unity at all other θ; the $\widehat{\alpha}(\theta)$ correspondence turns into the one in Panel B of Figure 6.

(C) Thereafter, the minimum of the $\mathcal{S}_\Delta$ curve in the left panel in Figure 5 dips below zero. With two roots to $\mathcal{S}_\Delta = 0$, the $\widehat{\alpha}(\theta)$ correspondence becomes like the one in Panel C of Figure 6.

(D) While r_F remains below $\widehat{r}_F$, $\partial\widehat{\theta}_1/\partial r_F < 0$ and $\partial\widehat{\theta}_2/\partial r_F > 0$. Panel C turns into D.

(E) In the limit as $r_F \to \widehat{r}_F$, $\widehat{\theta}_1 = 0$ and $\widehat{\theta}_2 = \widehat{\theta}_L$—the unique root of $\mathcal{S}_\Delta = 0$ at which $r_F = \widehat{r}_F$ just holds. Panel D turns into E

(F) Given that $r_F \geq \widehat{r}_F$, $\partial\widehat{\theta}/\partial r_F > 0$, while the limiting value of $\widehat{\theta}$ as r_F becomes arbitrarily large, $\widehat{\theta}_U$, is finite. Panel E gradually evolves towards F.

Granted that $\widetilde{\alpha}(\theta)$ is independent of r_F, the effects of an increase in r_F on equilibrium $\{\alpha, \theta\}$ can then be read off by superimposing the same given $\widetilde{\alpha}(\theta)$ successively into Panels A to F of Figure 6. We can conclude from this exercise the following.

Proposition 3

a. *For small r_F, equilibrium is $\theta = \widetilde{\theta}_L$ and $\alpha = 1$.*

b. *For large r_F, α must fall below unity in equilibrium.*

    i. *If $\widehat{\theta}_U \geq \widetilde{\theta}_U$, as r_F becomes large, $\theta = \widetilde{\theta}_U$ and $\alpha = 0$.*

    ii. *Otherwise, α stays positive for arbitrarily large r_F.*

c. *If $\theta = \widetilde{\theta}_L$ and $\alpha = 1$ is not equilibrium at r_F, the pair is not equilibrium at $r'_F > r_F$. If $\theta = \widetilde{\theta}_U$ and $\alpha = 0$ is equilibrium at r_F, the pair is equilibrium at $r'_F > r_F$.*

d. *Any $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium is increasing in r_F, and therefore the accompanying α is decreasing in r_F.*

e. *Any $\widehat{\theta}_1$ equilibrium is decreasing in r_F, and therefore the accompanying α is increasing in r_F.*

Parts (a)-(d) of the Proposition conform to the intuitive notion that an increase (decrease) in r_F should have a negative (positive) impact on flippers's market share. Still, it is surprising that α can remain strictly positive even for arbitrarily large r_F, for which flippers can only finance investment at a huge disadvantage vis-a-vis ordinary households. The condition for this to be the case, $\widehat{\theta}_U < \widetilde{\theta}_U$, holds for large $\widetilde{\theta}_U$. By (51) in the proof of Lemma 2, $\partial\widetilde{\theta}_U/\partial H > 0$, and that $\lim_{H\to 1}\widetilde{\theta}_U = \infty$. In general, we can show that $\partial\widetilde{\theta}/\partial H > 0$ for each $\alpha \in [0, 1]$. With a larger housing stock, there can only be more units for sale, other things being equal. In the mean time, where there are fewer households in rental housing, there can only be fewer buyers in the search market. For a large enough $\widetilde{\theta}_U$, a role for flippers can remain no matter what, if mismatched homeowners find it too difficult to sell in the end-user search market.

Part (e) is rather counterintuitive too for it says that an increase (decrease) in r_F can have a positive (negative) impact on flippers' market share in equilibrium. In Figures 7 and 8, $\widehat{\theta}_1$ is the endpoint of an interval of θ over which mismatched homeowners strictly prefer to sell in the investment market to quickly capitalize on the high housing price due to a small θ. Now, flippers would be able to pay even higher prices at a smaller r_F and this should only cause the interval $\left[0, \widehat{\theta}_1\right]$ to expand. If equilibrium is at $\widehat{\theta}_1$, the increase in $\widehat{\theta}_1$ is a movement down $\widetilde{\alpha}(\theta)$, giving rise to a smaller α.

On the contrary, by Part (d), an increase (decrease) in r_F will have the expected negative (positive) impact on flippers' market share in a $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium. In Figures 7, $\widehat{\theta}_2$ is the smallest θ within the interval $\left[\widehat{\theta}_2, \infty\right)$, over which mismatched homeowners find it advantageous to sell in the investment market because it takes too long to sell in the search market. A decline in r_F, by raising the price flippers are able to offer to mismatched homeowners, enlarges the interval. Where the decline in $\widehat{\theta}_2$ is a movement up the downward-sloping $\widetilde{\alpha}(\theta)$, in equilibrium α increases. A similar analysis applies to a $\widehat{\theta}$ equilibrium.

On the whole, one can conclude that at a smaller r_F, flippers are more numerous and more transactions are intermediated if one is willing to dismiss any $\widehat{\theta}_1$ equilibrium on stability grounds and the possibility that agents may coordinate to a smaller α equilibrium in case there exist multiple equilibrium. However, we do not think that the analysis, strictly speaking, allows us to reach any such unambiguous conclusions. And we cannot rule out occasions, admittedly rare, in which a decline in r_F can, rather perversely, be followed by a decline in the presence of flippers in the model housing market.

3.5 Housing prices

Housing prices in no-intermediation equilibrium Absent flippers, all housing market transactions are between end-user households at price p_H, given by (34) evaluated at $\alpha = 0$,[20]

$$p_H = \frac{(\eta + r_H - \mu)\, v + (2\delta + \eta + 2r_H)\, q}{(2\delta + \eta + 2r_H)\, r_H}. \tag{24}$$

In Leung and Tse (2011), we find that, nearing the given steady state whereby $n_F \simeq 0$, housing price and transaction volume move in opposite directions in the transition dynamics. Vacancy either stays equal to zero (when $n_F = 0$ in the transition) or declines monotonically (when $n_F > 0$ in the transition).

Housing prices in fully-intermediated equilibrium In a fully-intermediated equilibrium, all houses are first sold from mismatched homeowners to flippers at p_{FB}, given by (42), in the investment market and then at p_{FS}, given by (41), from flippers to end-user households in the search market, where $p_{FB} < p_{FS}$. Now, houses sold from households to flippers stay on the market for a vanishingly small time interval, whereas houses sold from flippers to households in the search market stay on the market for, on average, $1/\eta > 0$ units of time. There should then be a positive cross-section relation between price and TOM in the model housing market, as in the real-world housing market. Besides, with $p_{FB} < p_{FS}$, the model trivially predicts that houses bought by flippers are at lower prices than are houses bought by non-flippers. Depken et al. (2009) find the tendency to hold in their hedonic price regressions. In Leung and Tse (2011), we find that around the $\widetilde{\theta}_L$ steady state, price and vacancy move in opposite directions, whereas there exists no definite relationship between the two variables and transaction volume in the transition dynamics.

Housing prices in partially-intermediated equilibrium In a steady-state equilibrium in which mismatched homeowners sell in both the investment and search markets, in addition to the two prices p_{FB} and p_{FS} for transactions between a flipper and an end-user household, there will also be transactions between two end-user households, carried out at price p_H. In any partially-intermediated equilibrium, $p_H = p_{FS}$, however, so that all transactions in the search market are at the same price after all. In this case, $p_H = p_{FS}$ is given by either (34) or (35) evaluated at $\mathcal{S}_\Delta = 0$,

$$p_H = p_{IS} = \frac{(\eta + r_I)\, v}{r_F\, (2\delta + \eta + 2r_H)}, \tag{25}$$

whereas in the investment market, p_{FB} is given by (36), similarly evaluated at $\mathcal{S}_\Delta = 0$,

$$p_{IB} = \frac{\eta v}{r_F\, (2\delta + \eta + 2r_H)}. \tag{26}$$

Just as in the fully-intermediated equilibrium, here there exists a positive relation between TOM and price in the cross section and houses bought by flippers are at lower prices.

[20] The equations for the housing prices and asset values referred to hereinafter can be found in Lemma 9 in the Appendix.

Prices across equilibria Recall that across steady-state equilibria, $\theta = S/B$ is smallest in the equilibrium where flippers are most numerous. A priori, one would expect that prices are also highest in such an equilibrium where there is least competition among sellers.

Proposition 4 *If $\theta = \widetilde{\theta}_L$, $\widehat{\theta}_1$, and $\widehat{\theta}_2$ are all steady-state equilibria, prices in the search and investment markets are highest in the $\widetilde{\theta}_L$ equilibrium and lowest in the $\widehat{\theta}_2$ equilibrium. If $\theta = \widetilde{\theta}_L$, $\widehat{\theta}_1$, and $\widetilde{\theta}_U$ are all steady-state equilibria, prices in the search and investment markets are highest in the $\widetilde{\theta}_L$ equilibrium and lowest in the $\widetilde{\theta}_U$ equilibrium if μ is isoelastic and that $\frac{(\delta + r_H)^2}{2\delta + r_H} \leq \frac{3}{4}$.*

We should emphasis that the condition in the latter part of the Proposition is merely a sufficient condition. Indeed, we fail to find a single instance in which the conclusion fails across a large number of numerical experiments notwithstanding the condition holding in reverse. In any case, the condition is easily met for conventional values for δ and r_H.[21] Now, a direct corollary of the Proposition and Lemmas 4-7 is that:

Proposition 5 *Across steady-state equilibria in case multiple equilibrium exist, price, vacancy, and transaction volume increase or decrease together from one to another equilibrium, whereas average TOM and TBM move with the former set of variables in the opposite direction.*

Interest rate shocks As usual, in the present model, interest rates can play an important role in determining housing prices. But first, in the entire absence of flippers, housing price p_H in a no-intermediation equilibrium only depend on r_H but not on the rate r_F at which flippers may be able to finance investment. Specifically, a decline in r_H will lead to higher prices, as can be verified by differentiating (24), but market tightness, vacancy, turnover, and transaction volume will just stay at the given levels entirely determined by the housing stock, the rate matched households becomes mismatched, and the matching technology in the search market, as described in Lemmas 2-7 with $\alpha = 0$.

On the other hand, in a fully-intermediated equilibrium, prices in both the investment and search market, given by p_{FB} and p_{FS}, respectively, are decreasing in r_F, as can be verified by differentiating (42) and (41). Just as in the no-intermedation equilibrium, such interest rate shocks will leave no impact on market tightness, transaction volume, turnover, and vacancy, if all transactions were already intermediated in the first place.

In a partially-intermediated equilibrium where $\mathcal{S}_\Delta = 0$, prices in the search market p_{FS} $(= p_H)$, as well as in the investment market, p_{FB}, are decreasing in r_F, just as they are in a fully-intermediated equilibrium. But where θ was not already fixed at

[21] If $r_H = 0$, the condition is satisfied for $\delta \leq 3/2$. If $\delta = 0$, the condition is satisifed for $r_H \leq 3/4$.

the boundary of $\widetilde{\theta}_U$, housing prices can also change to follow any movements in θ triggered by the given interest rate shock. Differentiating (25) and (26) with respect to θ confirms that prices in both markets are decreasing in θ, so that prices are higher when the competition among sellers is less intense. Hence, if a given positive (negative) interest rate shock should cause θ to increase (decrease), there will be lower (higher) housing prices to follow because of its direct negative (positive) impact and of an indirect effect through lowering (raising) flippers' market share and then raising (lowering) market tightness. However, a positive interest rate shock need not cause θ to increase and α to fall. By Proposition 3e, along any $\widehat{\theta}_1$ equilibrium, the given interest rate shock will be followed by a decline in θ and an increase in α. Furthermore, in case there exist multiple equilibrium, θ can fall and α can increase just by itself when the housing market, for whatever reason, moves to a high α equilibrium from an initial low α equilibrium, without any change in parameter values. In what direction housing prices will move then cannot be unambiguously read off from (25) and (26) as the direct effect of any interest rate shock and the indirect effect through the changes in θ can affect housing prices differently. To proceed, we solve $\mathcal{S}_\Delta = 0$ for r_F and substitute the result into (25) and (26), respectively,

$$p_{FS} = p_H = \frac{(\eta + r_H - \mu)\,\upsilon + (2\delta + \eta + 2r_H)\,q}{r_H\,(2\delta + \eta + 2r_H)}, \tag{27}$$

$$p_{FB} = \frac{(\eta - \mu)\,\upsilon + (2\delta + \eta + 2r_H)\,q}{r_H\,(2\delta + \eta + 2r_H)}. \tag{28}$$

The two equations are independent of r_F; whatever effects a given change in r_F will have on housing prices are subsumed through the effects of changes in θ that follow the change in r_F obtained from holding $\mathcal{S}_\Delta = 0$. To evaluate the the effects of r_F on housing prices then is to differentiate these two expressions just with respect to θ.

Proposition 6 *Across steady-state equilibria and holding $S_\Delta = 0$, a shock to r_F, whether positive or negative, will cause housing price to increase (decrease), as long as to follow the interest rate shock is a decrease (increase) in θ and an increase (decrease) in α.*

By Proposition 6 then, the indirect effect of an interest rate shock on housing prices through the changes in flippers' presence and then in market tightness always dominates the direct effect shall the two be of opposite tendencies. A surprising implication then is that a given increase in flippers' cost of financing can actually lead to an increase in housing prices, if to follow the higher interest rate is also a heightened presence of flippers' in the market. In any case, a direct corollary of Lemmas 4-7 and Proposition 6 is that:

Proposition 7 *Across steady-state equilibria and holding $S_\Delta = 0$, a shock to r_F will cause housing price, transaction volume, and vacancy to move in the same direction, whereas average TOM and TBM will move in the opposite direction.*

3.6 A general interest rate shock

So far, we have restricted attention to analyzing the effects of an increase in r_F alone on flippers' presence and housing prices. Many of the implications, however, survive for a general change in interest rate that affects both flippers and ordinary households alike. Specifically, write R for r_H/r_F in (21),

$$\mathcal{S}_\Delta = (R - 1 - z)\,\eta + \mu - 2\,(\delta + r_H)\,z. \tag{29}$$

Then equiproportionate increases in r_H and r_F, while leaving R unchanged, lower $\mathcal{S}_\Delta$. A general increase in interest rate thus weakens mismatched homeowners' incentives to sell in the investment market, just as an increase in r_F, holding fixed r_H, does. Analogous to Proposition 3 is that:

Proposition 8 *Holding constant R at some given level,*

a. *for sufficiently large r_H, in equilibrium, $\theta > \widetilde{\theta}_L$ and $\alpha < 1$. Eventually, as r_H rises above a certain level, $\theta = \widetilde{\theta}_U$ and $\alpha = 0$ must obtain.*

b. *if $\theta = \widetilde{\theta}_L$ and $\alpha = 1$ is not equilibrium at r_H, the pair is not equilibrium at $r'_H > r_H$. If $\theta = \widetilde{\theta}_U$ and $\alpha = 0$ is equilibrium at r_H, the pair is equilibrium at $r'_H > r_H$.*

c. *any $\widehat{\theta}_2$ or $\widehat{\theta}$ equilibrium is increasing in r_H, and therefore the accompanying α is decreasing in r_H.*

d. *any $\widehat{\theta}_1$ equilibrium is decreasing in r_H, and therefore the accompanying α is increasing in r_H.*

Intuitively, at a higher cost of financing in general, housing prices fall, and the reward to quickly selling in the investment market diminishes. But just as in the case of an increase in r_F alone, it is not possible to conclude unambiguously that there must be fewer flippers around, where there exist multiple equilibrium, and that any $\widehat{\theta}_1$ equilibrium is actually decreasing in r_H.

The effects of the general increase in the cost of financing on housing prices are similar to those of an increase in r_F by itself. The following proposition summarizes the results.

Proposition 9

a. *In both the no-intermediation and fully-intermediated equilibria, equiproportionate increases in r_H and r_F lower housing prices.*

b. *In a partially-intermediated equilibrium,*

i. *equiproportionate increases in r_H and r_F, holding θ fixed, lower housing prices;*

ii. *across steady-state equilibria and holding $S_\Delta = 0$, equiproportionate changes in r_H and r_F, whether positive or otherwise, cause p_{FB} to increase (decrease) as long as to follow the interest rate shocks is a decline (increase) in θ and an increase (decrease) in α and for $R \in [0, 1+z]$; the same effect is felt on $p_{FS} = p_H$ for R in neighborhoods of $R = 0$, 1, and $1+z$.*

Notice that by (b.i), if to follow the equiproportionate increases in r_H and r_F is a decline in flippers' presence, housing prices must unambiguously decline, just as when an increase in r_F alone causes θ to rise and α to fall will lower housing prices for sure. More generally, (b.ii) is concerned with how prices may change when the interest rate shocks may be followed by either an increase or a decline in θ, as in the situations covered in Proposition 6. Also as in Proposition 6, here prices will increase if θ happens to fall and α rise to follow the interest rate shocks, positive or otherwise, if the values of R are appropriately chosen. The last restrictions are sufficient, but not necessary, conditions, and that the conclusions should hold under weaker conditions.

3.7 Welfare

In a steady-state equilibrium where $\mathcal{S}_\Delta = 0$, asset values for matched and mismatched homeowners, renters, and flippers are given by respectively,[22]

$$V_M = \frac{(\eta + 2r_H)\,\upsilon}{r_H\,(2\delta + \eta + 2r_H)}, \tag{30}$$

$$V_U = \frac{\eta\upsilon}{r_H\,(2\delta + 2r_H + \eta)}, \tag{31}$$

$$V_R = \frac{(r_F - r_H)\,\eta\upsilon}{r_F r_H\,(2\delta + \eta + 2r_H)}, \tag{32}$$

$$V_F = \frac{\eta\upsilon}{r_F\,(2\delta + \eta + 2r_H)}. \tag{33}$$

It is straightforward to verify that V_M, V_U, and V_F are all decreasing in θ. Any homeowners—matched or mismatched, end-users or flippers—benefit from the higher housing prices in a less tight market. The asset value for households in rental housing V_R, however, is decreasing in θ if $r_F < r_H$, which is a necessary condition for multiple equilibrium to exist (Lemma 8a). In this case, would-be buyers are worse off with the higher housing prices in the less tight market. Now, suppose both $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are

[22] The first two equations are from (37) and (38), respectively. The last two are from (39) and (36), respectively, both evaluated at $\mathcal{S}_\Delta = 0$.

steady-state equilibria with the two θ lying within the interval $\left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$. In this case, homeowners are better off in the $\widehat{\theta}_1$ equilibrium than in the $\widehat{\theta}_2$ equilibrium, whereas renters are better off in the second than in the first equilibria. The two steady-state equilibria then cannot be Pareto-ranked. The same holds for comparison between the $\widetilde{\theta}_L$ and $\widehat{\theta}_1$ equilibria and between the $\widehat{\theta}_1$ and the $\widetilde{\theta}_U$ equilibria.

Even though the equilibria cannot be Pareto-ranked, perhaps they can be ranked by aggregate welfare as measured by the sum of the asset values for all agents,

$$W = n_M V_M + n_U V_U + n_R V_R + n_F V_F.$$

At where $\mathcal{S}_\Delta = 0$, the asset values are given by (30)-(33). Again, consider a comparison between the $\widehat{\theta}_1$ and $\widehat{\theta}_2$ steady-state equilibria. Substituting from (54) to (57) in the Appendix for the various steady-state measures of agents and simplifying,

$$W = \frac{\eta v}{2\delta + \eta + 2r_H} \left(\frac{2H}{\eta + \delta} + \frac{H-1}{r_F} + \frac{1}{r_H} \right).$$

This expression is guaranteed to be decreasing in θ for large H. In this case, there is a larger aggregate asset value in the less-tight and higher-priced $\widehat{\theta}_1$ equilibrium where more transactions are intermediated. For smaller H, however, W above is increasing in θ, so that there is only a smaller W in the $\widehat{\theta}_1$ equilibrium than in the $\widehat{\theta}_2$ equilibrium. Thus, it seems that the equilibria cannot in general be ranked by even aggregate asset value. While all agents, except for households in rental housing, benefit from the higher housing prices in the more active $\widehat{\theta}_1$ equilibrium and that more households are matched in the steady state amid a shorter average Time-Between-Matches, W needs not be higher. Would-be buyers in rental housing are more numerous and they suffer a lower asset value with the higher prices. Such a negative impact on W can more than offset the positive effects of a more active market, especially when there is a small housing stock. Intuitively, given a small H, there can only be few house owners to benefit from the higher prices and faster turnover, while there are many would-be buyers to suffer from the same higher prices and the longer wait for owner-occupied housing. Apparently, when owner-occupied houses are scarce to begin with, leaving more houses vacant in the hands of flippers can be disproportionately costly.[23] Conversely, in a market endowed with a large H, it is hardest to sell and flippers' role in speeding up turnover is most valued.

[23] Masters (2007) is also a model in which intermediation in a search and matching environment can be wasteful.

4 Time-series relations among housing price, transaction volume, and vacancy

By Propositions 5, 7, and 9, the model predicts housing price, transaction volume, and vacancy should move together over time from one to another steady-state equilibria.[24] The positive time-series relation between housing price and transaction volume is well-known and numerous models have been constructed to account for it. Unique to our analysis is that vacancy should also move in the same direction with the two variables.

The prediction is not obviously inconsistent with the pictures depicted in Figures 1-3 in the Introduction. In a more systematic analysis, we first verify that in the 1991-I to 2010-IV sample period, the three variables are all $I(1)$ at conventional significance levels. Next, we find that the variables are indeed cointegrated by the Johansen Cointegration Test under all the usual trend assumptions. In particular, assuming a quadratic deterministic trend, the Trace test indicates 2 cointegrating equations:

$$\text{Price} - 13034 \times \text{Transaction} = 0,$$

$$\text{Transaction} - 3.87 \times \text{Vacancy} = 0,$$

which together imply that there exist long-run positive relations among the three variables. With other trend assumptions, the Trace tests only indicate 1 cointegrating equation. In a single cointegrating equation with non-zero coefficients for all three variables, the three cannot move together in the same direction. But if one imposes a priori two cointegrating equations in the estimation, the same qualitative results survive.

5 Concluding remarks

By allowing for the presence of flippers, without any assumed or acquired heterogeneity and endogenous search efforts, our model predicts a positive relation between housing price and TOM in the cross section, a relation found in numerous empirical studies. Our model can also generate the well-known relation between price and transaction volume in the time series. Previous models rely on preference and construction shocks and increasing returns in the matching technology to generate such relations. In our model, such relations are the relations among the different steady-state equilibria, as well as from interest rate shocks. Unique to our analysis is

[24] More precisely, Propositions 7 and 9 are only concerned with partially-intermediated steady-state equilibria. In both the no-intermediation and fully-intermediated equilibria, housing price is uncorrelated with transaction and vacancy. The fully-intermediated equilibrium is perhaps less empirically relevant. In this case, strictly speaking, our analysis predicts that interest rate shocks should only cause price, transaction, and vacancy to move together when the three variables are at modest to high levels. This observation is due to our discussant Enrique Schroth.

that vacancy should move together with price and transaction volume. This relation appears to be borne out in the data.

If the existence of multiple equilibrium is a natural outcome of the investment motive in a frictional housing market, the extent of flippers' presence in the housing market can be fickle, and then prices can change discretely in response to a discrete change in the former. The model housing market can then exhibit bubble-like characteristics with prices fluctuating widely without any apparent changes in "market fundamentals". Undoubtedly, our analysis cannot be the complete analysis of "speculative bubbles" in the housing market. Credit market conditions, market psychology, and the dynamics of price movements must also feature prominently. Nevertheless, we show that even in the absence of such factors, the interaction of the strength of the incentives to sell quickly to flippers and the influence of these agents' activities on market tightness suffices to imply an intrinsically volatile housing market.

Throughout the analysis, we assume a constant population and a given housing stock. A useful extension would be to incorporate a secular growing population and endogenous housing supply. With a varying population, one on the right side of the population constraint in (3) would be replaced by the population at the given point in time, equal to say n. In this case, one can show that the generalization to $\widetilde{\theta}(\alpha)$ in (11) is such that $\partial\widetilde{\theta}(\alpha)/\partial n < 0$, whereas $\partial\widetilde{\theta}(\alpha)/\partial H > 0$, as previously remarked in the discussions following Proposition 3. That is, as owner-occupied housing becomes relatively abundant (scarce), there will be more (fewer) houses for sale for each buyer. Now, suppose the market is at a $\widetilde{\theta}_L$ equilibrium but where the prevailing housing price justifies further development. As H increases relative to n, in Figure 7, for example, the $\widetilde{\alpha}(\theta)$ schedule will gradually shift to the right. To follow the increase in market tightness is a gradual decrease in housing prices. If at some point in time, $\widetilde{\theta}_L$ rises above $\widehat{\theta}_1$, $\theta = \widetilde{\theta}_L$ and $\alpha = 1$ can no longer be steady-state equilibrium. Then, even without further housing development, prices will continue to fall when the market converges to a $\widehat{\theta}_2$ or $\widetilde{\theta}_U$ from an initial $\widetilde{\theta}_L$ equilibrium. Meanwhile, the low prices at the $\widehat{\theta}_2$ or $\widetilde{\theta}_U$ equilibrium should choke off any housing development for a while until the population growth has caught up with the increase in the housing stock to send the $\widetilde{\alpha}(\theta)$ schedule back to where it began. What we usually describe as a cycle of market crash and recovery may then be the endogenous outcome in this analysis. No doubt, this conjecture is a widely speculative conjecture but it is also an intriguing one, which warrants a full-fetched analysis.

6 Appendix

6.1 Lemma

Lemma 9 *For* $\Delta \leq 0$*, so that* $\max\{V_R + p_{FB}, V_U\} = V_U$*,*

$$p_H = \frac{((\eta + r_H)(\eta + 2r_F) - ((1-\alpha)(\eta + r_F) + r_F)\mu)\upsilon + (2\delta + \eta + 2r_H)(\eta + 2r_F)q}{(2\delta + \eta + 2r_H)(\alpha\mu r_F + \eta r_H + 2r_F r_H)}, \tag{34}$$

$$p_{FS} = \frac{(\eta + r_F)((\eta + 2r_H - (1-\alpha)\mu)\upsilon + (2\delta + \eta + 2r_H)q)}{(2\delta + \eta + 2r_H)(\alpha\mu r_F + \eta r_H + 2r_I r_H)}, \tag{35}$$

$$p_{FB} = V_F = \frac{\eta((\eta + 2r_H - (1-\alpha)\mu)\upsilon + (2\delta + \eta + 2r_H)q)}{(2\delta + \eta + 2r_H)(\alpha\mu r_F + \eta r_H + 2r_I r_H)}, \tag{36}$$

$$V_M = \frac{(\eta + 2r_H)\upsilon}{r_H(2\delta + \eta + 2r_H)}, \tag{37}$$

$$V_U = \frac{\eta\upsilon}{r_H(2\delta + 2r_H + \eta)}, \tag{38}$$

$$V_R = \frac{((\eta + 2r_H)r_F + (1-\alpha)\eta(r_H - r_F))\mu\upsilon - r_H(\eta + 2r_F)(2\delta + \eta + 2r_H)q}{r_H(2\delta + \eta + 2r_H)(r_H(\eta + 2r_F) + \alpha\mu r_F)}. \tag{39}$$

For $\Delta > 0$*, so that* $\max\{V_R + p_{FB}, V_U\} = V_R + p_{FB}$ *and* $\alpha = 1$*,*

$$p_H = \frac{(\eta r_H - \mu r_I + 2\eta r_I + 2r_H r_F + \eta^2)\upsilon + (2\eta r_H + 2\delta r_I + 2\eta r_I + 4r_H r_F + \eta^2)q}{(\eta + 2r_H)(\eta r_H + \mu r_F + 2\delta r_F + 2r_H r_F)}, \tag{40}$$

$$p_{FS} = \frac{(r_F + \eta)(\upsilon + q)}{r_H(\eta + 2r_F) + r_F(\mu + 2\delta)}, \tag{41}$$

$$p_{FB} = V_F = \frac{\eta(\upsilon + q)}{r_H(\eta + 2r_F) + r_F(\mu + 2\delta)}, \tag{42}$$

$$V_M = \frac{(r_H(\eta + 2r_F) + \mu r_F)\upsilon - 2\delta r_F q}{r_H(r_H(\eta + 2r_F) + r_F(\mu + 2\delta))}, \tag{43}$$

$$V_U = \frac{\eta}{\eta + 2r_H}\frac{(r_H(\eta + 2r_F) + \mu r_I)\upsilon - 2\delta r_I q}{r_H(r_H(\eta + 2r_F) + r_F(\mu + 2\delta))}, \tag{44}$$

$$V_R = \frac{\mu r_F\upsilon - (r_H(\eta + 2r_F) + 2\delta r_F)q}{r_H(r_H(\eta + 2r_F) + r_F(\mu + 2\delta))}. \tag{45}$$

Lemma 10

a. *For $r_F < \widehat{r}_F$, S_Δ is U-shaped, with a well-defined minimum. Write $S^*_\Delta = \min_\theta S_\Delta$.*

   i. *For small r_F, $S^*_\Delta > 0$.*

   ii. *$\partial S^*_\Delta / \partial r_F < 0$. As r_F increases, before r_F reaches $\widehat{r}_F$, $S^*_\Delta = 0$ at some $\theta = \widehat{\theta}^*$.*

   iii. *Thereafter, as r_F continues to increase, S^*_Δ falls below 0, and that the two roots of $S_\Delta = 0$, $\widehat{\theta}_1$ and $\widehat{\theta}_2$ diverge as $\partial\widehat{\theta}_1/\partial r_F < 0$ but $\partial\widehat{\theta}_2/\partial r_F > 0$.*

   iv. *As $r_F \to \widehat{r}_F$, $\widehat{\theta}_1 \to 0$ and $\widehat{\theta}_2 \to \widehat{\theta}_L$ for some limiting value $\widehat{\theta}_L$.*

b. *For $r_F \geq \widehat{r}_F$, S_Δ becomes upward-sloping throughout.*

   i. *At $r_F = \widehat{r}_F$, the unique root of $S_\Delta = 0$, $\widehat{\theta} = \widehat{\theta}_L$, the limiting value of $\widehat{\theta}_2$.*

   ii. *Thereafter, $\partial\widehat{\theta}/\partial r_F > 0$; while as r_F becomes arbitrarily large, $\widehat{\theta} \to \widehat{\theta}_U$ for some finite $\widehat{\theta}_U > 1$.*

6.2 Proofs

Proof of Lemma 1 Substitute

$$\eta = \frac{\mu}{\theta} = \mu\frac{B}{S} = \mu\frac{n_R}{n_U + n_F},$$

into (8),

$$\mu n_R = (1-\alpha)\,\delta n_M \frac{n_U + n_F}{n_U}.$$

Combine this equation with (7) and simplify,

$$n_U = \frac{1-\alpha}{\alpha} n_F. \tag{46}$$

Substitute this equation for n_U into the fraction $n_F/(n_U + n_F)$ gives just α.

Proof of Lemma 2 The first step is to use (3) and (4) to write

$$n_R = 1 - H + n_F. \tag{47}$$

Then, by (46) and (47),

$$\theta = \frac{S}{B} = \frac{n_U + n_F}{n_R} = \frac{\frac{1-\alpha}{\alpha} n_F + n_F}{1 - H + n_F} = \frac{n_F}{\alpha\,(1 - H + n_F)}. \tag{48}$$

Solve the equation for n_F,

$$n_F = \frac{\theta\alpha (1 - H)}{1 - \theta\alpha}. \tag{49}$$

Next, by (7) and (3),

$$\mu n_R = \delta (1 - n_U - n_R).$$

Rearrange and then substitute from (46) and (47),

$$(\mu + \delta)(1 - H + n_F) = \delta \left(1 - \frac{1-\alpha}{\alpha} n_F\right).$$

Solve the equation for n_F,

$$n_F = \alpha \frac{\delta H - \mu(\theta)(1 - H)}{\delta + \mu(\theta)\alpha}. \tag{50}$$

Setting the LHSs of (49) and (50) equal yields (11). Implicitly differentiating yields a negative partial derivative. From (11), $\widetilde{\theta}_U$ is given by

$$\delta\widetilde{\theta}_U + \mu\left(\widetilde{\theta}_U\right) = \frac{\delta H}{1 - H}, \tag{51}$$

while $\widetilde{\theta}_L$ solves

$$\frac{\delta\widetilde{\theta}_L + \mu\left(\widetilde{\theta}_L\right)}{1 - \widetilde{\theta}_L} = \frac{\delta H}{1 - H}. \tag{52}$$

Given that the LHSs of both conditions are positive and finite for $H < 1$, $\widetilde{\theta}_L$ and $\widetilde{\theta}_U$ are strictly positive and finite, and that $\widetilde{\theta}_L < 1$.

Proof of Lemma 3 *Comparative statics–* Solve (11) for

$$\alpha = \frac{\delta H - (1 - H)(\mu + \delta\theta)}{\theta\delta H}. \tag{53}$$

Substituting (53) into (49) yields

$$n_F = \frac{\delta H - (1 - H)(\mu + \delta\theta)}{\mu + \delta\theta}. \tag{54}$$

Substituting (54) into (46) and (47), respectively, yields

$$n_U = \frac{\delta H(\theta - 1) - (1 - H)(\mu + \delta\theta)}{\mu + \delta\theta}, \tag{55}$$

$$n_R = \frac{\delta H}{\mu + \delta\theta}. \tag{56}$$

Finally, by (3), (55), and (56),

$$n_M = \frac{\mu H}{\mu + \delta\theta}. \tag{57}$$

The comparative statics in the Lemma can be obtained by differentiating (54)-(57), respectively, with respect to θ, and then noting that $\partial\widetilde{\theta}/\partial\alpha < 0$.

Boundary values– At $\alpha = 0$, by (10), $n_F = 0$. Then, by (47), $n_R = 1 - H$ and by (4), $n_M = H - n_U$. To obtain the equation for n_U, substitute (51) into (55) and simplify. For $\alpha = 1$, by (8), $n_U = 0$. And then by (47), $n_R = 1 - H + n_F$. Thus, with (3), $n_M = H - n_F$. The equation for n_F is obtained by substituting (52) into (54) and simplifying.

Proof of Lemma 5 First substitute from (4), and then from (53) and (57),

$$T = \alpha\delta n_M + (H - n_M)\,\eta = \delta\frac{\eta H}{\eta + \delta}\frac{1+\theta}{\theta} - \eta\,(1 - H)\,. \tag{58}$$

Differentiating and simplifying,

$$\frac{\partial T}{\partial \theta} = \frac{\delta H}{\theta}\left[\theta\frac{\partial \eta}{\partial \theta}\left(\left(\frac{1}{\eta+\delta}\right)^2\delta\frac{1+\theta}{\theta} - \frac{1-H}{\delta H}\right) - \frac{\eta}{\eta+\delta}\frac{1}{\theta}\right].$$

From (51),

$$\frac{1-H}{\delta H} \leq \frac{1}{\theta\,(\eta+\delta)},$$

since $\theta \leq \widetilde{\theta}_U$. Thus

$$\begin{aligned}
\frac{\partial T}{\partial \theta} &\leq \frac{\delta H}{\theta}\left[\theta\frac{\partial \eta}{\partial \theta}\left(\left(\frac{1}{\eta+\delta}\right)^2\delta\frac{1+\theta}{\theta} - \frac{1}{\theta\,(\eta+\delta)}\right) - \frac{\eta}{\eta+\delta}\frac{1}{\theta}\right] \\
&= \frac{\delta H}{\theta^2\,(\eta+\delta)}\left[\left(\frac{\delta\theta-\eta}{\eta+\delta}\right)\frac{\partial \eta}{\partial \theta}\theta - \eta\right] \\
&\leq \frac{\delta H}{\theta^2\,(\eta+\delta)}\left[\left(\frac{\delta\theta-\eta}{\eta+\delta}\right)\frac{\partial \eta}{\partial \theta}\theta - \frac{\partial \eta}{\partial \theta}\theta\right] \\
&= \frac{\delta H}{\theta^2\,(\eta+\delta)}\left[\frac{\delta\theta+\delta}{\eta+\delta}\right]\frac{\partial \eta}{\partial \theta}\theta < 0,
\end{aligned}$$

where the second inequality follows from the fact that $\partial\mu/\partial\theta > 0$ and that $\eta = \mu/\theta$. But then $\partial\widetilde{\theta}/\partial\alpha < 0$; hence $\partial T/\partial\alpha > 0$.

Proof of Lemma 6 Substituting from (4) and (58), (12) becomes

$$\frac{H - n_M}{\alpha\delta n_M + (H - n_M)\,\eta}.$$

Differentiating with respect to α yields

$$\frac{\delta\left(n_M - H\right)n_M - \delta H\alpha\frac{\partial n_M}{\partial\alpha} - \left(H - n_M\right)^2\frac{\partial\eta}{\partial\theta}\frac{\partial\theta}{\partial\alpha}}{\left[\alpha\delta n_M + \left(H - n_M\right)\eta\right]^2} < 0,$$

since $\partial n_M/\partial\alpha > 0$, $\partial\eta/\partial\theta < 0$, and $\partial\theta/\partial\alpha < 0$.

Proof of Lemma 7 Substituting from (7), (8), and then (3),

$$\frac{1}{\mu} + \frac{1-\alpha}{\eta} = \frac{1-n_M}{\delta n_M},$$

a decreasing function of n_M. But where $\partial n_M/\partial\alpha > 0$, there must be a smaller average TBM.

Proof of Lemma 8 Starting with $\theta = 0$, $\mu = 0$, while $\eta \to \infty$, so that $\mathcal{S}_\Delta$ becomes

$$\lim_{\theta\to 0}\left(\frac{r_H}{r_F} - 1 - z\right)\eta - 2\left(\delta + r_H\right)z,$$

which is equal to positive infinity/a finite negative number/negative infinity if $r_F \lesseqgtr \widehat{r}_F$. On the other hand, as $\theta \to \infty$, $\eta = 0$ and $\mu \to \infty$, so that $\mathcal{S}_\Delta$ becomes

$$\lim_{\theta\to\infty}\mu - 2\left(\delta + r_H\right)z, \tag{59}$$

an expression that tends to positive infinity. Differentiating,

$$\frac{\partial\mathcal{S}_\Delta}{\partial\theta} = \left(\frac{r_H}{r_F} - 1 - z\right)\frac{\partial\eta}{\partial\theta} + \frac{\partial\mu}{\partial\theta}, \tag{60}$$

which is guaranteed positive if $r_F \geq \widehat{r}_F$. In this case, $\mathcal{S}_\Delta$ starts out at either negative infinity or a finite negative number, is increasing throughout and eventually tends to positive infinity. A unique θ then solves $\mathcal{S}_\Delta = 0$. On the other hand, if $r_F < \widehat{r}_F$, $\mathcal{S}_\Delta$ starts out and ends up equal to positive infinity. It must therefore be initially decreasing but eventually increasing. If the condition in the Lemma holds, (60) changes sign just once. This can be shown by differentiating (60) and evaluating at where it is equal to zero, which leads to an expression which is positive if the condition holds. Then, at where (60) vanishes, $\mathcal{S}_\Delta$ is convex.

Proof of Lemma 10 By (21), $\lim_{r_F\to 0}\mathcal{S}_\Delta = \infty$. Thus, for arbitrarily small r_F, $\mathcal{S}_\Delta^* > 0$. This proves (a.i). Differentiating (21) and by the Envelope Theorem,

$$\frac{\partial\mathcal{S}_\Delta^*}{\partial r_F} = -\frac{r_H}{r_F}\eta < 0.$$

This proves the first part of (a.ii). As to the second part, notice that

$$\lim_{r_F \to \widehat{r}_F} \mathcal{S}_\Delta = \mu - 2(\delta + r_H) z, \tag{61}$$

which is minimized at $\theta = 0$, yielding a negative $\mathcal{S}_\Delta^*$ in the limit. Given that $\mathcal{S}_\Delta^*$ is continuous in r_F, $\mathcal{S}_\Delta^* = 0$ must hold before r_F has reached $\widehat{r}_F$. For (a.iii), differentiating $\mathcal{S}_\Delta = 0$ and for $i = 1, 2$,

$$\frac{\partial \widehat{\theta}_i}{\partial r_F} = \frac{\frac{r_H}{r_F}\eta}{\frac{\partial \mathcal{S}_\Delta}{\partial \theta}}.$$

Where $\mathcal{S}_\Delta$ is decreasing at $\widehat{\theta}_1$ and increasing at $\widehat{\theta}_2$, $\partial\widehat{\theta}_1/\partial r_F < 0$ and $\partial\widehat{\theta}_2/\partial r_F > 0$. For (a.iv), notice that $\partial \mathcal{S}_\Delta/\partial\theta$, as given by (60), can only be negative as $r_F \to \widehat{r}_F$ if $\theta \to 0$ in the interim. This proves $\widehat{\theta}_1 \to 0$ as $r_F \to \widehat{r}_F$. The limiting value for $\widehat{\theta}_2$ is given by the solution to

$$\mu\left(\widehat{\theta}_2\right) = 2(\delta + r_H) z.$$

Denote this as $\widehat{\theta}_L$. The positivity of $\partial\widehat{\theta}/\partial r_F$ is due to the same reason for the positivity of $\partial\widehat{\theta}_2/\partial r_F$. The limiting value of $\widehat{\theta}$ as r_F becomes arbitrarily large is given by the solution to

$$\mu\left(\widehat{\theta}_U\right)\left(1 - \frac{1+z}{\widehat{\theta}_U}\right) = 2(\delta + r_H) z.$$

Given that the RHS is positive and finite, $\widehat{\theta}_U$ is finite and satisfies $\widehat{\theta}_U > 1 + z > 1$. This completes the proof of (b).

Proof of Proposition 1 To establish existence, we apply Kakutani's fixed point theorem to show that F has a fixed point. First, the unit interval is clearly a compact, convex, and nonempty subset of the one-dimensional Euclidean space. Second, since $\widetilde{\theta}(\alpha)$ is defined for all $\alpha \in [0,1]$ and is positive-valued, and that $\widehat{\alpha}$ is nonempty for all $\theta > 0$, F must be positive-valued and nonempty for all $\alpha \in [0,1]$. Whenever F is multi-valued, F is the entire unit interval. Then it must be convex. Finally, with $\widetilde{\theta}$ continuous and $\widehat{\alpha}$ possessing a closed graph by virtue of the continuity of $\mathcal{S}_\Delta$, F must have a closed graph as well. Then by Kakutani's fixed point theorem, F has a fixed point.

Proof of Proposition 2 First define the graph of the $\widehat{\alpha}(\theta)$ correspondence as being:

1. *Nondecreasing* over a given interval of θ if for any two θ' and θ'' in the interval, where $\theta' < \theta''$, no element of $\widehat{\alpha}(\theta')$ is strictly greater than any element of $\widehat{\alpha}(\theta'')$.

2. *Decreasing* at some θ' if there exists an nonempty interval $(\theta', \theta'']$ where at least one element of $\widehat{\alpha}(\theta')$ is greater than all elements of $\widehat{\alpha}(\theta)$ for some $\theta \in (\theta', \theta'']$.

Then, if $r_F \geq \widehat{r}_F$, the $\widehat{\alpha}(\theta)$ graph, as depicted in either Panel E or F of Figure 6, is nondecreasing throughout. Then there can be one and only one $\{\alpha, \theta\}$ pair at which the downward-sloping $\widetilde{\alpha}(\theta)$ in Figure 4 can meet a nondecreasing $\widehat{\alpha}(\theta)$. Next, by (21), $\lim_{r_F \to 0} \mathcal{S}_\Delta = \infty$. Hence for arbitrarily small r_F, $\mathcal{S}_\Delta$ stays positive for all θ and the $\widehat{\alpha}(\theta)$ graph is as depicted in Panel A of Figure 6. Clearly, the unique equilibrium is $\alpha = 1$ and $\theta = \widetilde{\theta}_L$.

Otherwise, the $\widehat{\alpha}(\theta)$ graph must be like the two depicted in Panels C and D in Figure 6. In this case, by construction, for all $\theta \leq \widehat{\theta}_1$, $\mathcal{S}_\Delta \geq 0$, so that $1 \subset \widehat{\alpha}(\theta)$ for all $\theta \leq \widehat{\theta}_1$. Thus if $\widetilde{\theta}_L \leq \widehat{\theta}_1$, $\alpha = 1$ and $\theta = \widetilde{\theta}_L$ is equilibrium. Given that $\widetilde{\theta}_U \geq \widehat{\theta}_1$, either that $\widetilde{\theta}_U \in \left[\widehat{\theta}_1, \widehat{\theta}_2\right]$ or that $\widetilde{\theta}_U > \widehat{\theta}_2$. In the first case, since $0 \subset \widehat{\alpha}(\theta)$ for all $\theta \in \left[\widehat{\theta}_1, \widehat{\theta}_2\right]$ where $\mathcal{S}_\Delta \leq 0$, $\alpha = 0$ and $\theta = \widetilde{\theta}_U$ is equilibrium. Next, consider the case of $\widetilde{\theta}_U > \widehat{\theta}_2$. But if $\widetilde{\theta}_L \leq \widehat{\theta}_1$, $\widetilde{\theta}_U > \widehat{\theta}_2 > \widehat{\theta}_1 \geq \widetilde{\theta}_L$; i.e., $\widehat{\theta}_2 \in \left(\widetilde{\theta}_L, \widetilde{\theta}_U\right)$. Then there exists an $\alpha \in (0,1)$ that satisfies $\alpha = \widetilde{\alpha}\left(\widehat{\theta}_2\right)$. At $\theta = \widehat{\theta}_2$, any $\alpha \in [0,1] \subset \widehat{\alpha}\left(\widehat{\theta}_2\right)$. Thus, $\alpha = \widetilde{\alpha}\left(\widehat{\theta}_2\right)$ and $\theta = \widehat{\theta}_2$ is equilibrium. This proves that $\widehat{\theta}_1 \in \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$ is sufficient for multiplicity given an $\widehat{\alpha}(\theta)$ graph as depicted in either Panel C or D in Figure 6. To show that it is also necessary, notice that if $\widehat{\theta}_1 \notin \left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$, the point at which $\widehat{\alpha}(\theta)$ is decreasing, $\widehat{\alpha}(\theta)$ is nondecreasing throughout $\left[\widetilde{\theta}_L, \widetilde{\theta}_U\right]$, and as in where $r_F \geq \widehat{r}_F$, there can be just one point at which $\widehat{\alpha}(\theta)$ and $\widetilde{\alpha}(\theta)$ intersect.

Proof of Proposition 3 Given that for small r_F, the $\widehat{\alpha}(\theta)$ correspondence is given by the one in Panel A of Figure 6, (a) follows immediately. For large r_F, the $\widehat{\alpha}(\theta)$ correspondence tends to the one in Panel F. In this case, θ can remain equal to $\widetilde{\theta}_L$ only if the entire $\widetilde{\alpha}(\theta)$ schedule lies to the right of $\widehat{\theta}_U$; i.e., $\widetilde{\theta}_L \geq \widehat{\theta}_U$. Given that $\widetilde{\theta}_L < 1$ (Lemma 2) and $\widehat{\theta}_U > 1$ (Lemma 9), the condition cannot hold. Hence, for large r_F, in equilibrium, $\alpha < 1$ and $\theta > \widetilde{\theta}_L$. Next, if $\widehat{\theta}_U \geq \widetilde{\theta}_U$, in the limit when r_F becomes arbitrarily large, the downward-sloping $\widetilde{\alpha}$ must meet the horizontal axis at where $\widehat{\alpha}(\theta) = 0$, in which case equilibrium is $\alpha = 0$ and $\theta = \widetilde{\theta}_U$. Otherwise, the downward-sloping $\widetilde{\alpha}$ must meet the vertical segment of $\widehat{\alpha}$, yielding a $\widehat{\theta}$ equilibrium, where $\alpha > 0$. This proves (b). For (c), note that as r_F increases, the set of θ over which $1 \subset \widehat{\alpha}(\theta)$ shrinks and the set of θ over which $0 \subset \widehat{\alpha}(\theta)$ expands. Parts (d) and (e) are direct corollaries of (a.iii) and (b.ii) of Lemma 10, given that $\widetilde{\alpha}(\theta)$ is downward-sloping.

Proof of Proposition 4 We begin with showing that price in the search market in the $\widetilde{\theta}_L$ equilibrium (p_{FS}) is higher than in the $\widehat{\theta}_1$ equilibrium ($p_H = p_{FS}$). First, if $\widetilde{\theta}_L$ is equilibrium, p_{FS} as given by (41) must exceed p_H as given by (40), both evaluated at $\theta = \widetilde{\theta}_L$. Next, it is straightforward but tedious to verify that with $\mathcal{S}_\Delta \geq 0$ and

$r_F < r_H$, p_H as given by (40) is decreasing in θ. Then the value of p_H with $\mathcal{S}_\Delta \geq 0$ and $\theta = \widetilde{\theta}_L$ exceeds the value with $\mathcal{S}_\Delta \geq 0$ and $\theta = \widehat{\theta}_1$. The latter, however, is identical to p_H with $\mathcal{S}_\Delta \leq 0$ as given by (34), when both expressions are evaluated at $\mathcal{S}_\Delta = 0$. Next, with $\partial p_H/\partial\theta < 0$ as given by (25), search market housing price in the $\widehat{\theta}_1$ equilibrium must exceed that in the $\widehat{\theta}_2$ equilibrium. The final comparison is between search market prices in the $\widehat{\theta}_1$ and $\widetilde{\theta}_U$ equilibria. At where $\theta = \widetilde{\theta}_U$, search market housing price p_H is given by (24). Differentiating with respect to θ yields an expression whose sign is the opposite of the sign of

$$\Phi \equiv 2(\delta + r_H)\theta + \theta\eta + 2(\delta + r_H)\theta\frac{\partial\eta}{\partial\theta} - (2\delta + r_H)\frac{\partial\eta}{\partial\theta}.$$

If μ is isoelastic, given by $\mu = \theta^a$, where $a \in (0,1)$, then $\eta = \theta^{a-1}$. The above becomes

$$\Phi = 2(\delta + r_H)\theta + \theta^a - (2\delta + r_H)(a-1)\theta^{a-2} + 2(\delta + r_H)(a-1)\theta^{a-1}, \tag{62}$$

whereby $\lim_{\theta\to 0}\Phi = \lim_{\theta\to\infty}\Phi = \infty$. Differentiating,

$$\frac{\partial\Phi}{\partial\theta} = 2(\delta + r_H) + a\theta^{a-1} - (2\delta + r_H)(a-1)(a-2)\theta^{a-3} + 2(\delta + r_H)(a-1)^2\theta^{a-2}.$$

Set this derivative equal to 0 and substitute the result back into (62),

$$\Phi = (1-a)\theta^{a-2}\left(\theta^2 - 2(\delta + r_H)(2-a)\theta - (a-3)(2\delta + r_H)\right).$$

This is the value for Φ at any local minimum, which is guaranteed positive if the condition in the second part of the Proposition holds. This establishes that p_H given by (34) evaluated at $\alpha = 0$ is decreasing in θ. Then p_H at $\theta = \widetilde{\theta}_U$ must fall below the value of (34) evaluated at $\alpha = 0$ and $\theta = \widehat{\theta}_1$. The latter when also evaluated at $\mathcal{S}_\Delta = 0$ is just the search market housing price at $\theta = \widehat{\theta}_1$. This completes the proof that search market housing prices across steady-state equilibria can be ranked inversely by the value of θ. Given that $p_{FB} = \frac{\eta}{\eta + r_F}p_{FS}$, investment market housing prices are ranked in the same way as in search market housing prices.

Proof of Proposition 8 Hold constant R and allow r_H to increase; by (29),

$$\frac{\partial\mathcal{S}^*_\Delta}{\partial r_H} = 2z < 0.$$

For large r_H then, the $\widehat{\alpha}(\theta)$ graph cannot be like the ones in Panels A and B of Figure 6. For $r_F \geq \widehat{r}_F$ (i.e., $R \leq 1 + z$), so that there exists a unique root $\widehat{\theta}$ to $\mathcal{S}_\Delta = 0$, $\partial\widehat{\theta}/\partial r_H > 0$, given that $\partial\mathcal{S}_\Delta/\partial r_H < 0$. By (29) and with $R - 1 - z < 0$, $\lim_{r_H\to\infty}\widehat{\theta} = \infty$. For $r_F < \widehat{r}_F$ (i.e., $R > 1 + z$), there are two roots $\widehat{\theta}_1$ and $\widehat{\theta}_2$ to $\mathcal{S}_\Delta = 0$. Again, given that $\partial\mathcal{S}_\Delta/\partial r_H < 0$, $\partial\widehat{\theta}_1/\partial r_H < 0$ and $\partial\widehat{\theta}_2/\partial r_H > 0$. By (29) and with $R - 1 - z > 0$, as $r_H \to \infty$, there can just be two θ that solves $\mathcal{S}_\Delta = 0$, equal to zero and infinity; thus $\lim_{r_H\to\infty}\widehat{\theta}_1 = 0$ and $\lim_{r_H\to\infty}\widehat{\theta}_2 = \infty$. Parts (a)-(d) of the Propositions then follow immediately.

Proof of Proposition 9 In a no-intermediate equilibrium, p_H is given by (34), evaluated at $\alpha = 0$, which is independent of r_F but decreasing in r_H. This proves the first part of (a). For the second part, substitute $r_F = r_H R^{-1}$ into (41) and (42) and differentiate. For (b), substituting $r_F = r_H R^{-1}$ into (25) and (26), respectively, yields,

$$p_{FS} = p_H = \frac{(\eta + r_H R^{-1})\,\upsilon}{(2\delta + \eta + 2r_H)\,r_H R^{-1}}, \tag{63}$$

$$p_{FB} = \frac{\eta\upsilon}{(2\delta + \eta + 2r_H)\,r_H R^{-1}}, \tag{64}$$

both of which are decreasing in r_H and θ. Solve $\mathcal{S}^*_\Delta = 0$ from (29) for r_H,

$$r_H = \frac{(R - 1 - z)\,\eta + \mu}{2z} - \delta, \tag{65}$$

and substituting the result into (63) and (64), respectively, gives

$$p_{FS} = p_H = \frac{(2\eta z R + \eta R - \eta - \eta z + \mu - 2\delta z)\,q}{(\eta R - \eta + \mu)\,(\eta R - \eta - \eta z + \mu - 2\delta z)}, \tag{66}$$

$$p_{FB} = \frac{2\upsilon z^2 R\eta}{(\eta R - \eta + \mu)\,(\eta R - \eta - \eta z + \mu - 2\delta z)}. \tag{67}$$

Differentiating (67) with respect to θ and then substituting for η and $\partial\eta/\partial\theta$ using the identity $\eta = \mu/\theta$ yield an expression whose sign is given by that of

$$\left(\mu - \theta\frac{\partial\mu}{\partial\theta}\right)(R-1)\,(R-z-1) - \left((2R - 2 - z + \theta)\,\frac{\partial\mu}{\partial\theta} + \mu - 2z\delta\right)\theta^2.$$

The expression is strictly negative at $R = 0$ and $R = 1 + z$ if the RHS of (65) is positive. And then differentiating twice with respect to R yields

$$2\left(\mu - \theta\frac{\partial\mu}{\partial\theta}\right) > 0,$$

given the concavity of μ. This establishes that p_{FB} in (67) must be decreasing in θ for $R \in [0, 1 + z]$. For $p_{FS} = p_H$, differentiating (66) with respect to θ and evaluating at $R = 0$, 1, and $1 + z$, respectively, all yield a strictly negative expression as long as the RHS of (65) is positive. This establishes that $p_{FS} = p_H$ in (66) must be decreasing in θ for R in neighborhoods of 0, 1, and $1 + z$.

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