A New Efficient Method for Analysis of Finite Periodic Structures

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Abstract: The electromagnetic modeling of practical finite periodic structures is a topic of growing interest. Due to the truncation of the infinite periodic structures, surface waves will be excited and localized near the discontinuous interfaces leading to the edge effect of finite structures. In this work, surface waves are numerically disentangled from the propagating Bloch waves contributions. Based on the universally exponential decay feature of the surface waves, a novel method is developed by connecting the solution to the large finite periodic structure with that to a relatively small one resulting in low complexity and memory consumption. The method numerically reconstructs propagating Bloch waves and surface waves according to the Bloch-Floquet theorem of periodic structures and translation invariant properties of semi-infinite periodic structures, respectively. Numerical examples are privided to validate the efficiency and accuracy of the newly developed method.

Keywords: Finite Periodic Structures, Surface Waves, Edge Effect, Leaky Wave Antenna

1. Introduction

Periodic structures (PSs) have attracted much attention in the past decades due to numerous applications such as metamaterials, antenna arrays, frequency selective surface (FSS), photonic band gap (PBG) or electromagnetic band gap (EBG) structures, etc [1, 2]. Infinite PSs are impossible in a realistic fabrication. When truncating infinite PSs, discontinuities occur at the interface and surface waves (SWs) will be excited according to mode conversion [3]. In finite PSs, only allowed Bloch waves (BWs) can propagate, and SWs become evanescent and localized around the discontinuities, which leads to the edge effects of finite PSs. The prediction of wave natures in the edge cells of finite PSs is an indispensable procedure during the design process [4, 5].The conventional element-by-element method is rigorous and suitable for small finite PSs. However, it requires heavy computational resources for very large ones. The well-established infinite periodic approximation approach neglecting the edge effect, although reducing the computational domain to a unit cell, is not sufficiently accurate for engineering applications.

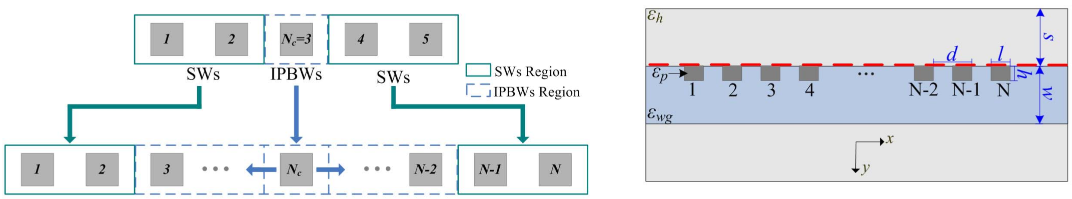
To overcome the difficulties in simulating large finite PSs, two different categories of methods have be proposed. The first category bypassed the difficulties by improving the computational capabilities of the existing methods. Among them, the fast multipole method [6] and the domain decomposition method [7] are very powerful for modeling large finite PSs. The other category of methods consider the edge effect with the help of the information in an infinite array, i.e. the convolution of the infinite array solution with an appropriate window function [8].

This paper aims to develop an efficient method for predicting the electromagnetic (EM) response of large finite PSs accurately from that of relatively small ones. A leaky wave antenna is investigated by the newly developed method. The results are compared to those calculated by the element-by-element method. Excellent agreements are obtained and particularly the proposed method saves considerable computer resources. The presence of SWs is critical for the low side-lobe antenna design.

2. Surface Waves in Finite Periodic Structures

It is known that BWs are normal modes of an infinite PS. However, in finite structures, the fields are given by the superposition of BWs and additional SWs which are excited due to the truncation and localized near the discontinuous interfaces. The BWs in infinite PSs are referred as infinite periodic Bloch waves (IPBWs) to distinguish them from BWs in finite structures. To accurately model finite PSs, we need to take the SWs into account. When the magnitude of SW is large and its decay rate is slow, the edge effect is important and can't be ignored. In our previous work, we have demonstrated that the SWs decay exponentially and are invariant with respect to the increasing array size when the elements N of the structures is sufficiently large for capturing the information of SWs [9, 10].

3. Field Reconstruction in Large Finite Periodic Structures



(a) (b)

Fig. 1. (a) Mechanism for the field reconstruction in large finite periodic structures from that of relative small ones. (b) LWA geometry. The dielectric waveguide is perturbed by the grating inside the waveguide. where n_h=1.45, $n_{wg} = 1.67$, $n_p = 3.48$, $d = 2\ell = 0.97$ um, $w = 1$ um, $h = 0.3$ um, $s = 20$ um and $N = 60$.

The basic idea for efficiently calculating large finite periodic structures is inspired by the universal decay feature of SWs. For a sufficiently large layer number N, it is reasonable to assume that the center unit cell only contains IPBWs. As shown in Fig. 1 (a), the center unit cell of the large finite periodic structure is aligned with that of the small one. We assume that the small finite periodic structure is long enough at the periodicity direction to guarantee invariant (converged) SWs. We only simulate the small structure by using rigorous full-wave solvers and obtain its field distributions (or equivalent sources). The procedures to reconstruct the field for the large structure are as follows:

First, the field distribution of the center unit cell of the large structure will exactly copy from that of the small one. Second, the IPBW numbers k_B^m can be extracted by the eigenmode expansion method

$$\mathbf{E}(\mathbf{r}) = \sum_{m=-\infty}^{m=\infty} \mathbf{E}_m(\mathbf{r}) \exp(-jk_B^m x) \tag{1}$$

where $\mathbf{E}$ can be electric or magnetic fields and $\mathbf{E}_m$ are the periodic BW envelope functions: $\mathbf{E}_m(x) = \mathbf{E}_m(x+P)$. The BW vectors k_B^m can be retrieved by using a standard minimization algorithm

$$\min_{k_B} \left\| \mathbf{E}(\mathbf{r}) - \sum_{m=-\infty}^{m=\infty} \mathbf{E}_m(\mathbf{r}) \exp(-jk_B^m x) \right\|_2 \tag{2}$$

The IPBW vectors in both structures are the same and can be complex due to the radiation or intrinsic (Ohmic) loss. Third, numerically propagate EM fields forward and backward at the IPBW region of the large structure by using the extracted BW vectors based on Bloch-Floquet theorem.

$$\mathbf{E}(x+P)=\mathbf{E}(x)\exp(-jk_B^m P) \tag{3}$$

Finally, reconstructing EM fields of the large structure at the surface wave regions by using the translation invariant properties of semi-infinite PSs. With respect to the left (right) SW region, the right (left) IPBW regions can be approximately regarded as semi-infinite PSs due to fast decayed SWs. So we have

$$\frac{\mathbf{E}_{sw}^L(x)}{\mathbf{E}_{IPBW}^L(x\pm lP)}=\frac{\mathbf{E}_{sw}^S(x)}{\mathbf{E}_{IPBW}^S(x\pm lP)} \tag{4}$$

where $\mathbf{E}_{sw}^L$ and $\mathbf{E}_{sw}^S$ are fields at the SW regions for large and small structures, respectively. $\mathbf{E}_{IPBW}^L$ and $\mathbf{E}_{IPBW}^S$ are fields at the IPBW regions for large and small structures, respectively. lP is the distance between the cell in the SW region and its most adjacent cell in the IPBW region (such as the distance between cell 1 and 3 in Fig. 1 (a)).

4. Numerical Examples

An optical leaky wave antenna (LWA) (See Fig. 1 (b)) is efficiently calculated by the developed method [11]. Instead of calculating the original leaky wave antenna with $N=60$, we calculate a relatively smaller one with $N=30$ by using the FDTD method. The middle 10 elements (n=11 to n=20) are considered as the IPBW region. Two Bloch waves located on the third band propagating forward and backward are considered in the IPBW region. Thus the field has the form of $E(n)=a^+E(11)e^{-jk^+d(n-1)}+a^-E(11)e^{-jk^-d(n-1)}$. By employ the least mean square interpolation, we get $k^+=\beta-j\alpha=-2.48e5-j4.40e4$, $a^+=1.097$ and $a^-=-0.097$. After getting the IPBWs of the small structure with $N=30$, the field of the original large structure with $N=60$ can be reconstructed from the information of the $N=30$ structure according to Sec. 3.

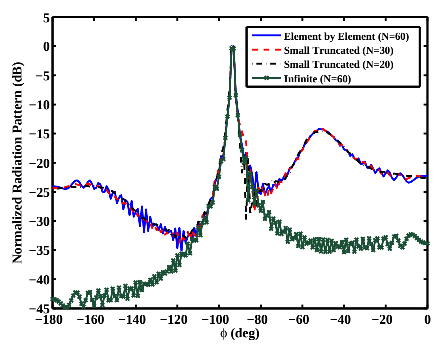


Fig. 2. Comparison of the far-field radiation patterns calculated by different methods.

The magnitude and phase of the field along the radiation aperture of the large finite PSs agree very well with the element-by-element simulation results. Fig. 2 shows that the far-field radiation patterns obtained by the proposed method are also in good agreement with the element-by-element results. The side-lobe around -52° due to the presence of surface waves is not predicted by the infinite array method. Hence, the surface wave effects are critical for predicting the side-lobes of LWA. The radiation pattern from the small structure with $N=20$ is also compared. Only small discrepancies exist which means the $N=30$ structure is large enough to capture all the SWs information. Table 1 compares the computational information of the LWA with different perturbation elements. The proposed method can save lots of CPU time and memory since it only needs to simulate a relatively small structure. Other examples will be given during the conference.

Table 1: The computational information of a LWA with different perturbation elements.

Elements	N=20	N=30	N=60
Domain (μm^2)	32.37×43.97	42.07×43.97	71.17×43.97
Memory (MB)	47	62	104
Time steps (δt)	21440	27840	34240
CPU time (s)	381.65	782.47	1548.84

5. Conclusion

In sum, based on the universally rapid decay character of SWs, an efficient method for modeling large finite PSs is developed. From a small finite PSs which contains all the information of SWs and IPBWs, we can accurately predict the performance of large finite ones without ignoring the edge effect. The proposed method is even more efficient for larger structures.

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