Fourier-cosine Method for Ruin Probabilities

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Abstract

In theory, ruin probabilities in classical insurance risk models can be expressed in terms of an infinite sum of convolutions, but its inherent complexity makes efficient computation almost impossible. In contrast, Fourier transforms of convolutions could be evaluated in a far simpler manner. This feature aligns with the heuristic of the recently popular work by Fang and Oosterlee, in particular, they developed a numerical method based on Fourier transform for option pricing. We here promote their philosophy to ruin theory. In this paper, we not only introduce the Fourier-cosine method to ruin theory, which has $O(n)$ computational complexity, but we also enhance the error bound for our case that are not immediate from Fang and Oosterlee (2009). We also suggest a robust method on estimation of ruin probabilities with respect to perturbation of the moments of both claim size and claim arrival distributions. Rearrangement inequality will also be adopted to amplify the Fourier-cosine method, resulting in an effective global estimation.

Keywords: Fourier-cosine method, Ruin probabilities, Pollaczek-Khinchin formula, Gibbs phenomena, Summation by parts, Rearrangement inequalities

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1. Introduction

In 2008, Fang and Oosterlee [1] was the first to introduce a novel Fourier-cosine method for evaluating European options. The proposed Fourier-cosine

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method not only reduces the computational complexity to linear order for European options, but also provides a concrete error bound for the approximation. The Fourier-cosine method provides an alternative method for solving integration problems such as evaluating $\int_\Gamma f(x)dx$, with a non-explicit integrand f whose only known information is its Fourier transform; before [1], the usual approach for dealing with this problem is to first calculate the inverse Fourier transform, either analytically or numerically, and then substitute the result obtained back into the original integration. Instead, the trick in Fourier-cosine method is to directly incorporate the Fourier-cosine expansion of f under the integration and to derive an approximation with the aid of Fubini's theorem; this method avoids the need of inverting a Fourier transform as to be shown later. Their results have been further applied to other financial derivatives, such as Bermudan options, barrier options and Asian options in the following works [2, 3, 4], all of these illustrate the effectiveness of their method for pricing options with early exercising features and also with stochastic volatility models. In light of the apparent versatile nature of the Fourier-cosine method, we here attempt to promote this elegant approach to ruin theory.

Knowing the contingency of ruin is always the main goal of insurance companies and academia in actuarial science, various researchers devote intense interest in quantitatively studying such risk (See [5, 6, 7, 8, 9]. However, it is usually hard to compute ruin probabilities, even sometimes explicit representation formula exists such as Pollaczek-Khinchin formula, which express ruin probabilities under Lévy subordinator models in terms of infinite sum of convolutions [10]. Nevertheless, this kind of complex formula makes effective computation practically impossible. Instead, researchers turn their focus on approximating ruin probabilities for general surplus process. It has been a long history of effort on trying to approximate ultimate ruin probabilities. Some research dated back to 60's, like [11]. Recently, Grande [12] analysed estimations of ruin probabilities based on claim sizes moments. Shimizu [13] used Edgeworth type expansion for providing a polynomial ruin approximation, when the risk loading is small enough. Also, there are lots of researches on improving the Monte Carlo techniques of finding ruin probabilities, e.g. [14] and [15]. Furthermore, a good number of researchers tried to take advantages of the explicit form of Pollaczek-Khinchin formula; Coulibaly and Lefèvre [16] proposed the estimation for ruin probability by calculating convolutions in the formula with quasi-Monte Carlo method; Albrecher et al. [17] proposed an approximate for ruin probabilities based on higher-order approximation of the tail probability of claim size distribution.

On the other hand, Lévy subordinator's Fourier transform/Laplace transform can be found easily via Lévy -Khinchine's formula as the Fourier transform can be obtained from an algebraic equation; the algebraic equation makes the evaluation of Fourier transforms of ruin probability density a much simpler task. This led to a lot of estimation schemes based on Fourier/Laplace transform. One outstanding example is Albrecher et al. [18], whom developed an approximation for ruin probability based on an improved inverse Laplace transform procedure; they replaced e^{-xu} with a rational function $r_n(-xu)$ in the inverse Laplace transform formula and obtained a simple approximation based on r_n with an explicit error bound. However, their method is limited to models with holomorphic Laplace transform. However, most of the approximations depending on numerical inversion of Laplace transform involves unstable error. Here we propose an alternative method based on the Fourier-cosine expansion. This approach not only fits the rationale of applying Fourier transform via an application of Pollaczek-Khinchin formulas in recent literature, but also provides an explicit error bound. Also note that the complexity of Fourier-cosine method applied to ruin probability approximations should be truly linear as in [1] without any need of prior calculation. As it turns out, our method is similar to the ad-hoc Laplace inversion method in Abate and Whitt [19]; however their derivations based on Poisson summation formula is completely different from our systematic study here.

Furthermore, by sophisticatedly using different Fourier series identities [20] and a number of bounding result with Gibbs phenomenon [21], we significantly improve and enhance the error bound proposed by [1], and these aspect add another dimension of our contribution to the literature.

Lastly, the original Fourier-cosine method does not guarantee the estimation to be monotone, which is a key property of ruin probabilities. We shall further improve the global estimation error for ruin probabilities, by using the rearrangement technique as first proposed in [22].

This paper is organised as follows. Section 2 will introduce the model used in this paper and relevant formulae of ruin probabilities. Section 3 will discuss implementing Fourier-cosine method. The establishment of error bound is the main topic in Section 4. Section 5 will provide a robust approximation based on the moments of claim size and claim arrival distributions and Section 6 will introduce rearrangement technique . Finally, the effectiveness of our method is shown in Section 7 and we conclude the paper in Section 8.

2. Problem Setting

In this section, the model for the underlying surplus process is introduced and relevant formulas of ruin probabilities are also discussed.

Let R_t denote the surplus process of an insurance company

$$R_t = u + ct - L_t, \tag{1}$$

with $u \geq 0$ being the initial reserve of the insurance company. $c > 0$ denotes the premium rate charged by the insurance company. Claim process is modeled by L_t, which is a Lévy subordinator defined as follow. L_t is an infinite divisible stochastic process with $L_0 = 0$ and consists of only a deterministic drift part and a pure positive jump random process. The characteristic function of L_t is given by

$$\phi_{L_t}(\omega) = \mathbb{E}[\exp(i\omega L_t)] = \exp(ib\omega t + t \int_{(0,\infty)} (e^{i\omega x} - 1)\nu(dx)), \tag{2}$$

where $b \in \mathbb{R}$. ν is the Lévy measure on $(0, \infty)$, that is a positive Borel measure with $\int_0^\infty (|x|^2 \wedge 1)\nu(dx) < \infty$. Details can be found in [23] and the reference therein.

Without loss of generality, we assume $b = 0$ throughout this paper. The rationale behind this assumption is that whenever we have a model with non-zero b, we can consider a new model with $c' = c - b$ and L'_t be the pure jump part of L_t and the two models will agree with each other. Here ν is also assumed to satisfy $\mu_1 := \int_{(0,\infty)} x\nu(dx) < \infty$. Moreover, the safety loading condition $c > \mu_1$ is imposed to avoid almost sure ruin.

2.1. Ruin Probabilities

The probability of ruin is defined by

$$\psi(u) := \mathbb{P}\left\{\inf_{t\geq 0}\{t : R_t < 0\} < \infty | R_0 = u\right\}. \tag{3}$$

By applying the Pollaczek-Khinchin formula, see Equation (1.3) in [10], it is possible to obtain an explicit infinite sum representation for the ruin probability as below. Define $h(x)$ as $h(x) = \nu(x,\infty)/\mu_1$ and denote $\rho := \mu_1/c$, ruin probability can then be written as:

$$\psi(u) \quad = \quad \rho - (1-\rho)\sum_{j=1}^{\infty}\rho^j \int_0^u h^{*j}(x)dx = \rho - (1-\rho)\int_0^u f(x)dx, \tag{4}$$

where $f := \sum_{j=1}^{\infty} \rho^j h^{j*}$ and h^{*j} denotes the jth order convolution for a function h:

$$h^{*j}(x) = \int_0^x h^{*(j-1)}(x-y)h(y)dy, \tag{5}$$

with $h^{*1} = h$.

Since $h(x) \geq 0$ for all $x \in [0,\infty)$ and $\int_0^\infty h(x)dx = 1$, $((1-\rho)/\rho)f$ can be seen as the probability density of a compound geometric random variable $V := Y_1 + Y_2 + \cdots + Y_M$, where Y is i.i.d. with distribution $h(x)$ and $\mathbb{P}(M = k) = (1-\rho)\rho^{k-1}$. Henceforth, $\int_0^\infty ((1-\rho)/\rho)f(x)dx$ should be equal to 1, so it is natural to assume that $((1-\rho)/\rho)f$ and f are $L^1(\mathbb{R})$ functions.

3. Fourier-cosine Method

In this section, we derive an approximation for ruin probabilities based on Fourier-cosine expansion, this method is inspired by the recent breakthrough of [1]. We shall rewrite the integral in Formula (4) by replacing f with its Fourier-cosine series.

For any function $g : [0, \pi] \to \mathbb{R}$, there is a natural extension for extending this function into an even function on $\mathbb{R}$. This even function is define $\breve{g}$ by

$$\breve{g}(x) = \begin{cases} g(x), & x \geq 0 \\ g(-x), & x < 0 \end{cases}. \tag{6}$$

All even functions can be expressed as Fourier-cosine series [24],

$$\breve{g}(x) = {\sum_{k=0}^{\infty}}' A_k \cos(kx), \tag{7}$$

with

$$A_k = \frac{1}{\pi} \int_{-\pi}^{\pi} \breve{g}(x) \cos(kx) dx = \frac{2}{\pi} \int_0^{\pi} g(x) \cos(kx) dx. \tag{8}$$

The notation $\sum'$ denotes a summation with its first term is weighted by half. Since g is part of $\breve{g}$, the expansion is also valid for g itself. Fourier-cosine series expansion for any function supported on $[0, a]$ can also be obtained through a change-of-variable $y = x\pi/a$.

We now return to ruin probabilities, the first step is to rewrite Formula (4) as the following form:

$$\varphi(u) = \rho - (1 - \rho) \int_0^a \mathbb{1}_{\{x \leq u\}} f(x) dx, \text{ for } a \geq u. \tag{9}$$

Despite f being defined on $[0, \infty)$ in Section 2, here we restrict ourselves to consider f as a function defined on $[0, a]$, a is a fixed number that is greater than the initial reserve u. We shall treat a as a given constant in this section. The method to determine a will be left in Section 4. Since $\mathbb{1}_{x \leq u} f(x) = 0$ in $(u, a]$, the value of the integration in (9) is exactly the same as in (4). We will later show that the domain we pick for f will have an effect on the overall error, therefore, we introduce a new factor a here instead of simply defining f on $[0, u]$.

We can apply Fourier-cosine expansion for function f,

$$f(x) = {\sum_{k=0}^{\infty}}' A_k \cos(k\pi \frac{x}{a}), \tag{10}$$

with

$$A_k = \frac{2}{a}\int_0^a f(s)\cos(k\pi\frac{s}{a})ds, \tag{11}$$

and then substitute f back to Formula (9) by this Fourier-cosine expansion (10),

$$\varphi(u) = \rho - (1-\rho)\int_0^a \mathbb{1}_{\{x\leq u\}} {\sum_{k=0}^{\infty}}' A_k \cos(k\pi\frac{x}{a})dx. \tag{12}$$

Simple application of Fubini's theorem, we obtain

$$\varphi(u) = \rho - (1-\rho){\sum_{k=0}^{\infty}}' A_k \int_0^u \cos(k\pi\frac{x}{a})dx = \rho - (1-\rho){\sum_{k=0}^{\infty}}' A_k\chi_k(0,u), \tag{13}$$

where

$$\chi_k(c,d) := \begin{cases} [\sin(k\pi\frac{d}{a}) - \sin(k\pi\frac{c}{a})]\frac{a}{k\pi}, & k\neq 0 \\ d-c, & k=0 \end{cases}. \tag{14}$$

Next, it is clear that A_k can be rewritten as:

$$A_k \equiv \frac{2}{a}\Re\left(\int_0^a f(x)e^{i\frac{k\pi x}{a}}dx\right), \tag{15}$$

where $\Re(\cdot)$ denotes the real part of a complex function. We can compare the integral in this formula with the characteristic function of f itself.

$$\int_0^a f(x)e^{i\frac{k\pi x}{a}}dx \approx \int_0^\infty f(x)e^{i\frac{k\pi x}{a}}dx = \phi_f(\frac{k\pi}{a}), \tag{16}$$

where ϕ_f is the Fourier transform of f. Due to their similarity, we would use ϕ_f in place of $\int_0^a f(x)e^{i\frac{k\pi x}{a}}dx$ in the original integral, and obtain an approximate value. We define

$$F_k := \frac{2}{a}\Re\{\phi_f(\frac{k\pi}{a})\}, \tag{17}$$

replace all A_K by F_K and obtain the approximation:

$$\varphi(u) \approx \rho - (1-\rho){\sum_{k=0}^{\infty}}' F_k\chi_k(0,u). \tag{18}$$

We then truncate the series summation and only include the first N terms, so we arrive with our approximate $\varphi_e(u,N)$:

$$\varphi_e(u,N) = \rho - (1-\rho){\sum_{k=0}^{N-1}}' F_k\chi_k(0,u). \tag{19}$$

There are two major advantages of implementing the Fourier-cosine method for approximation of ruin probabilities. The first one is that instead of calculating convolution of h directly, we only need to acquire the value of the Fourier transform of f. In fact, ϕ_f can be calculated explicitly through the following Formulae (20) and (21). To begin with, the characteristic function of h is:

$$\phi_h(\omega) = \frac{1}{\mu_1}\int_0^\infty e^{i\omega x}\nu(x,\infty)dx = \frac{1}{\mu_1}\int_0^\infty \frac{e^{i\omega x}-1}{i\omega}\nu(dx), \tag{20}$$

for $\omega \neq 0$; otherwise $\phi_h(0) = 1$. Then one can calculate ϕ_f as:

$$\phi_f(\omega) = \sum_{j=1}^\infty \rho^j \phi_h^j(\omega) = \frac{\frac{\rho}{\mu_1}\int_0^\infty \frac{e^{i\omega x}-1}{i\omega}\nu(dx)}{1-\frac{\rho}{\mu_1}\int_0^\infty \frac{e^{i\omega x}-1}{i\omega}\nu(dx)} = \frac{\int_0^\infty \frac{e^{i\omega x}-1}{i\omega}\nu(dx)}{c-\int_0^\infty \frac{e^{i\omega x}-1}{i\omega}\nu(dx)}, \tag{21}$$

when $\omega \neq 0$; while $\phi_f(0) = \rho/(1-\rho)$.

Remark 3.1. Since the safety loading assumption ensures that $c > \mu_1$, together with $|e^{i\omega x} - 1| \leq |\omega x|$, we clearly have

$$\left| c - \int_0^\infty \frac{e^{i\omega x}-1}{i\omega}\nu(dx)\right| \geq c - \int_0^\infty \left|\frac{e^{i\omega x}-1}{i\omega}\right| \nu(dx) \geq c-\mu_1 > 0,$$

and hence, (21) is well-defined.

The second advantage of using the Fourier-cosine approach is that we can derive an explicit error bound, which will be shown in the next section; while for other approaches in the literature, the derivation of error bound is normally hard, if not impossible. Also, our proposed approximate only involves elementary arithmetic operations and has a linear computational complexity.

4. Error Estimate

We now aim to show that there is a reasonable error bound for our approximation. Following the derivation in Section 3, the total error of the proposed estimation consists of two parts:

1. The error related to approximating A_k by F_k in (19):

$$\epsilon_1 = \left| \frac{2(1-\rho)}{a} {\sum_{k=0}^\infty}' \mathrm{Re}\left\{ \int_a^\infty e^{ik\pi\frac{x}{a}} f(x)dx \right\} \chi_k(0,u) \right|; \tag{22}$$

2. The series truncation error on [0,a]:

$$\epsilon_2 = \left| -(1-\rho) \sum_{k=N}^{\infty} F_k \chi_k(0,u) \right| = \left| (\rho - 1) \sum_{k=N}^{\infty} \frac{aF_k}{k\pi} \sin(k\pi \frac{u}{a}) \right|. \qquad (23)$$

The total error $\epsilon \leq \epsilon_1 + \epsilon_2$ is obviously bounded by these two parts. In this section, we shall consider the error bound for these parts one by one. Although we adopted the same philosophy as in [1] to derive our approximation, the establishment of error bound has been fundamentally enhanced here to cater for our specific situation.

4.1. Approximation Error for A_k by F_k

Firstly, we shall show that ϵ_1 is bounded by an integration range error. We start with a finite sum instead of the infinite sum in ϵ_1. Since f is a positive function in our setting, we have

$$\begin{aligned} & (1-\rho) \left| \frac{2}{a} \sum_{k=0}^{n}{}' \chi_k(0,u) \int_a^{\infty} \cos(k\pi \frac{x}{a}) f(x) dx \right| \\ \leq \; & (1-\rho) \int_a^{\infty} \left| \frac{2}{a} \sum_{k=0}^{n}{}' \chi_k(0,u) \cos(k\pi \frac{x}{a}) \right| f(x) dx. \end{aligned} \qquad (24)$$

We claim the following at the moment.

Claim 4.1. $\left| \frac{2}{a} \sum_{k=0}^{n}{}' \chi_k(0,u) \cos(k\pi \frac{x}{a}) \right| \leq 1 + \frac{2}{\pi} \int_0^{\pi} \frac{\sin t}{t} dt$, *which holds independent of a and n.*

Its proof shall be given after establishing bound for ϵ_1. Applying Claim 4.1,

$$\begin{aligned} & (1-\rho) \int_a^{\infty} \left| \frac{2}{a} \sum_{k=0}^{n}{}' \chi_k(0,u) \cos(k\pi \frac{x}{a}) \right| f(x) dx \\ \leq \; & (1-\rho) \left(1 + \frac{2}{\pi} \int_0^{\pi} \frac{\sin t}{t} dt \right) \int_a^{\infty} f(x) dx. \end{aligned} \qquad (25)$$

Taking the limit on both sides as n tends to infinity, we have

$$\epsilon_1 \leq (1-\rho) \left(1 + \frac{2}{\pi} \int_0^{\pi} \frac{\sin(t)}{t} dt \right) \int_a^{\infty} f(x) dx. \qquad (26)$$

As $\int_a^{\infty} f(x) dx \to 0$ when $a \to +\infty$ and $a > u$ is arbitrary, we can reduce this part of error by choosing a suitably large value of a.

Proof of Claim 4.1. For the sum $\frac{2}{a}\sum_{k=0}^{n}{}'\chi_k(0,u)\cos(k\pi\frac{x}{a})$, where $x \in [a,\infty)$, we have

$$\begin{aligned}\frac{2}{a}\sum_{k=0}^{n}{}'\chi_k(0,u)\cos(k\pi\frac{x}{a}) &= \frac{u}{a}+\frac{2}{\pi}\sum_{k=1}^{n}\frac{\sin(k\pi\frac{u}{a})\cos(k\pi\frac{x}{a})}{k}\\ &= \frac{u}{a}+\frac{1}{\pi}\sum_{k=1}^{n}\frac{\sin(k\pi\frac{u+x}{a})}{k}+\frac{1}{\pi}\sum_{k=1}^{n}\frac{\sin(k\pi\frac{u-x}{a})}{k}.\\ &= \frac{u}{a}+S_n\left(\frac{u+x}{a}\right)+S_n\left(\frac{u-x}{a}\right) \qquad (27)\end{aligned}$$

where $S_n(y) := \sum_{k=1}^{n}\sin(ky)/k$. Since $S_N(y)$ is periodic with a period 2π, and $S_n(0) = S_n(\pi) = 0$ and $S_n(2\pi - y) = -S_n(y)$ for $y \in (0,\pi)$, we only consider $S_n(y) = \sum_{k=1}^{n}\sin(ky)/k$ for $y \in (0,\pi)$. This $S_n(y)$ is a well-known series with an important role on the study of Gibbs phenomena. For the proof and further information, we refer to check [21] and references therein.

Lemma 4.2. *For all positive integer n, $l = 0, 1, \ldots, \lfloor(n-1)/2\rfloor$ and $y \in (0,\pi)$, the followings are true:*

1. $S_n(\pi/(n+1)) \geq S_n(y)$;
2. $S_{n+1}((2l+1)\pi/(n+2)) > S_n(((2l+1)\pi)/(n+1))$.

Applying Lemma 4.2, in particular with $l = 0$, for any $y \in (0,\pi)$

$$\begin{aligned}S_n(y) &\leq \lim_{n\to\infty} S_n\left(\frac{1}{n+1}\pi\right)\\ &= \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{n+1}{\pi}\sin\left(\frac{k}{n+1}\pi\right)\right)\frac{\pi}{n+1}\\ &= \int_0^{\pi}\frac{\sin(t)}{t}dt. \qquad (28)\end{aligned}$$

Together with the periodicity of $S_n(y)$, we have $|S_n(y)| \leq \int_0^{\pi}(\sin(t)/t)dt$ for all positive integer n and all real number y. Therefore,

$$\left|\frac{u}{a}+\frac{1}{\pi}\sum_{k=1}^{n}\frac{\sin(k\pi\frac{u+x}{a})}{k}+\frac{1}{\pi}\sum_{k=1}^{n}\frac{\sin(k\pi\frac{u-x}{a})}{k}\right| \leq 1+\frac{2}{\pi}\int_0^{\pi}\frac{\sin(t)}{t}dt. \qquad (29)$$

We complete our proof for the Claim 4.1. □

Remark 4.3. Classical Lundberg inequality can give us hints on finding a suitable value of a so that the error incurred could be within a given tolerance level. In particular, note that $\int_a^\infty f(x)dx = (\varphi(a) - \varphi(\infty)/(1-\rho) = \varphi(a)/(1-\rho)$ and let $S_t = u - R_t = L_t - ct$ for general surplus process R_t. Citing the result from [5], if $\{e^{\kappa S_t}\}_{t\geq 0}$ is a martingale for some constant $\kappa > 0$ and S_t tends to $-\infty$ almost surely, then $\varphi(a) \leq e^{-\kappa a}$ for all $a \geq 0$, so one can use the Lundberg inequality to find an upper bound for $\int_a^\infty f(x)dx$ and hence to pick an appropriate value of a. Of course, other versions of Lundberg inequality are required for other types of surplus process, but the rationale behind choosing a remains unchanged.

4.2. Series Truncation Error

For ϵ_2, we consider the convergent properties of ϕ_f. The algebraic index of convergence is defined as follows.

Definition 4.4. ([24] Definition 2 in Section 2.3) A sequence A has algebraic index of convergence of s if s is the greatest number such that

$$\limsup_{k\to\infty} |A_k| k^s < \infty. \tag{30}$$

It is natural to consider this property since for any $f \in L^1$ which is differentiable with non-zero derivative and $f' \in L^1$,

$$\frac{aF_k}{2} = \int_0^\infty f(x)\cos\left(k\pi\frac{x}{a}\right)dx = -\frac{a}{k\pi}\int_0^\infty f'(x)\sin\left(k\pi\frac{x}{a}\right)dx, \tag{31}$$

in accordance with integration by parts. It shows that $\frac{a}{2}|F_k| \leq (a/k\pi)\int_0^\infty |f'(x)|dx \leq C/k$ and suggests that F_k commonly encountered under our model for surplus process should have an algebraic index of convergence of at least one.

Assuming that $(a/2)F_k$ has an algebraic index of convergence of β, which also means that

$$\begin{aligned} \epsilon_2 &= (1-\rho)\left|\sum_{k=N}^\infty \frac{aF_k}{k\pi}\sin(k\pi\frac{u}{a})\right| \leq (1-\rho)\sum_{k=N}^\infty \left|\frac{aF_k}{k\pi}\sin(k\pi\frac{u}{a})\right| \\ &\leq (1-\rho)\frac{2}{\pi}\sum_{k=N}^\infty \frac{C}{k^{\beta+1}} \leq (1-\rho)\frac{\bar{C}}{(N-1)^\beta}. \end{aligned} \tag{32}$$

The last inequality comes from calculating the summation asymptotically by carrying out an integration. Note that $\bar{C}$ is a constant depends on and increases with a.

Therefore, the total error for applying Fourier-cosine method in approximating ruin probabilities which has a characteristic function with algebraic index of convergence β is

$$\epsilon = \epsilon_1 + \epsilon_2 \leq (1-\rho)\left[\left(1 + \frac{2}{\pi}\int_0^{\pi}\frac{\sin(t)}{t}dt\right)\int_a^{\infty} f(x)dx + \frac{\bar{C}}{(N-1)^{\beta}}\right]. \quad (33)$$

The first part in the square-bracket can be made as small as possible by increasing the value of a and is independent of N. The second term depends on both a and N. It increases when a increases while decreases when N increases . When applying our approximation, one should choose a first through the control of ϵ_1 and with such a fixed value of a, we can then pick an N such that it can reduce the magnitude of ϵ_2.

4.2.1. Enhancing Error Bound

The error bound can be further improved if we assume more properties on $\Re\{\phi_f\}$ such as its monotonicity. For any sequence $\{a_n\}$, define $\Delta a_n := a_{n+1} - a_n$ and we have the following theorem.

Theorem 4.5. *For $u \in [\theta, a-\theta]$ where $\theta > 0$ and $F_k = (2/a)\Re\{\phi_f(k\pi/a)\}$ satisfying:*

1. *$(a/2)F_k$ has algebraic index of convergence $\beta > 0$, and so $F_k \to 0$ as $k \to \infty$.*
2. *There exists a large enough N' such that ΔF_k are of the same sign for all $k \geq N'$.*

Then $\epsilon_2 = (1-\rho)\left|\sum_{k=N}^{\infty} aF_k \sin(k\pi u/a)/(k\pi)\right| \leq C_\theta/N^{\beta+1}$ for some constant C_θ (depending on θ and a) and $N \geq N'$

Remark 4.6. These two conditions also imply that all F_k are of the same sign for large enough k. Indeed, for if $\Delta F_k \leq 0$, it implies that F_k is decreasing for large k and F_k tends to 0 when k goes to infinity. This means that F_k has to be positive when $k > N'$. The argument for positive ΔF_k is similar.

Remark 4.7. As in most common models, F_k satisfies Condition 1 in Theorem 4.5. We only need to check Condition 2 in order to strengthen our error bound. In particular, both conditions are clearly satisfied when $\Re(\phi(\omega))$ is a positive differentiable decreasing function or negative differentiable increasing function for large enough ω.

PROOF. Firstly, u lying in $[\theta, a-\theta]$ suggests that $x := \frac{u}{a}\pi$ is bounded away from 0 or π. Secondly, $\Delta\left(F_k/k\right)$ is also of the same sign whenever $k > N'$; indeed,

$$\Delta\left(\frac{F_k}{k}\right) = \frac{F_{k+1}}{k+1} - \frac{F_k}{k} = \frac{k\Delta F_k - F_k}{k(k+1)}, \tag{34}$$

which will always be of the same sign in accordance with Remark 4.6. Next, let $b_k := F_k/k$ and clearly $b_k = -\sum_{l=k}^{\infty} \Delta b_l$, which is well-defined since $F_k/k \to 0$ as $k \to \infty$.

$$\begin{aligned}
\sum_{k=N}^{\infty} \frac{F_k}{k} \sin(kx) &= \sum_{k=N}^{\infty} b_k \sin(kx) \\
&= \sum_{k=N}^{\infty} \left[\left(-\sum_{l=k}^{\infty} \Delta b_l\right) \sin(kx)\right] \\
&= -\sum_{l=N}^{\infty} \Delta b_l \left(\sum_{k=N}^{l} \sin(kx)\right) \\
&= -\frac{1}{2\sin\frac{x}{2}} \sum_{l=N}^{\infty} \Delta b_l \left(\cos\left[(N-\frac{1}{2})x\right] - \cos\left[(l+\frac{1}{2})x\right]\right) \\
&= \frac{\cos\left[(N-\frac{1}{2})x\right]}{2\sin\frac{x}{2}} \left(-\sum_{l=N}^{\infty} \Delta b_l\right) + \frac{1}{2\sin\frac{x}{2}} \sum_{l=N}^{\infty} \Delta b_l \cos\left[(l+\frac{1}{2})x\right] \\
&= \frac{\cos\left[(N-\frac{1}{2})x\right]}{2\sin\frac{x}{2}} b_N + \frac{1}{2\sin\frac{x}{2}} \sum_{l=N}^{\infty} \Delta b_l \cos\left[(l+\frac{1}{2})x\right].
\end{aligned} \tag{35}$$

Taking absolute value of both sides of (35),

$$\begin{aligned}
\left|\sum_{k=N}^{\infty}\frac{F_k}{k}\sin(kx)\right| &\leq \left|\frac{\cos\left[(N-\frac{1}{2})x\right]}{2\sin\frac{x}{2}}b_N\right| + \frac{1}{2\sin\frac{x}{2}}\sum_{l=N}^{\infty}\left|\Delta b_l\cos\left[(l+\frac{1}{2})x\right]\right| \\
&\leq \frac{|b_N|}{2\sin\frac{x}{2}} + \frac{1}{2\sin\frac{x}{2}}\sum_{l=N}^{\infty}|\Delta b_l| \\
&= \frac{|b_N|}{2\sin\frac{x}{2}} + \frac{1}{2\sin\frac{x}{2}}\left|\sum_{k=N}^{\infty}\Delta b_k\right| \\
&= \frac{|b_N|}{\sin\frac{x}{2}}. \qquad (36)
\end{aligned}$$

Note that we use the fact that $\sum|\Delta\left(F_k/k\right)| = |\sum\Delta\left(F_k/k\right)|$ since $\Delta\left(F_k/k\right)$ is of the same sign. Finally we have

$$\epsilon_2 = (1-\rho)\left|\sum_{k=N}^{\infty}\frac{aF_k}{k\pi}\sin\left(k\pi\frac{u}{a}\right)\right| \leq (1-\rho)\frac{a}{\pi}\frac{1}{\sin\frac{x}{2}}\frac{|F_N|}{N} \leq \frac{C_\theta}{N^{\beta+1}}, \qquad (37)$$

where C_θ is a constant depending on a and θ but is independent of N.

Remark 4.8. In this section, the algebraic index of convergence of F_k is assumed to be known and a strong error bound has been derived. However, the algebraic index of convergence may not be explicit in general even if F_k is explicitly given. A more robust (but not as optimal) integrability condition will be proposed to guarantee the convergence of F_k in our forthcoming paper on constructing modern Fourier-cosine method for Gerber-Shiu theory [25].

5. Robust Approximation

The Fourier-cosine method we have developed here relies on the structure of the underlying model of the surplus process. Our approximation may change dramatically if one switches from one model to another. However, the estimation of surplus process models may not often be reliable. Hence, we here supplement the Fourier-cosine method with a more robust approximation based on the moments of the distribution h, which is easier to be estimated statistically from real data. Let Y be a random variable with probability density function h as given in Section 2 and ι_k $(k = 1, 2, \cdots)$ be the k-th moment of h as given by

$$\iota_k = \int_0^\infty x^k h(x)dx = \int_0^\infty x^k \frac{\nu(x,\infty)}{\mu_1}dx = \frac{1}{(k+1)\mu_1}\int_0^\infty x^{k+1}\nu(dx). \quad (38)$$

Assuming that $\lim_{x\to\infty} x^{k+1}\nu(x,\infty)/(k+1) = 0$, so that the last equality holds, and ι_0 set to be 1. Using Taylor series expansion, we can express the characteristic function of h in terms of its moments.

$$\begin{aligned}\phi_h(\omega) = \mathbb{E}[e^{iY\omega}] &= \mathbb{E}\left[\sum_{k=0}^{m}\frac{(iY\omega)^k}{k!} + \int_0^{Y\omega}\cdots\int_0^{s_m} e^{is_{m+1}}ds_{m+1}\cdots ds_1\right] \\ &= \sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k + R, \quad (39)\end{aligned}$$

where $R := \mathbb{E}\left[\int_0^{Y\omega}\int_0^{s_1}\cdots\int_0^{s_m} e^{is_{m+1}}ds_{m+1}\cdots ds_2 ds_1\right]$ with

$$|R| \le \frac{|\omega|^{m+1}}{(m+1)!}\iota_{m+1}. \quad (40)$$

Assuming that the moment of h has no greater than polynomial growth, i.e. there exists $c > 0$ such that $\iota_k \le c^k$ for all k, $|R| \to 0$ as $m \to \infty$.

Next, we can derive an approximation of ϕ_f based on the above results:

$$\phi_f(\omega) = \frac{\rho\phi_h(\omega)}{1-\rho\phi_h(\omega)} = \frac{\rho\phi_h(\omega)}{1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k - \rho R}. \quad (41)$$

Consider $\left|\rho R/(1-\rho\sum_{k=0}^{m}(i\omega)^k\iota_k/k!)\right|$ for fixed ω. Since $\lim\limits_{m\to\infty} 1-\rho\sum\limits_{k=0}^{m}(i\omega)^k\iota_k/k! = 1-\rho\phi_h(\omega) \neq 0$ from Taylor series expansion, $\left|\rho R/(1-\rho\sum_{k=0}^{m}(i\omega)^k\iota_k/k!)\right|$ tends to zero as a whole as m tends to infinity. Therefore, $\left|\rho R/(1-\rho\sum_{k=0}^{m}(i\omega)^k\iota_k/k!)\right| \le 1$ for large enough m and we can write:

$$\begin{aligned}\phi_f(\omega) &= \frac{\rho\phi_h(\omega)}{1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k - \rho R} \\ &= \frac{\rho\phi_h(\omega)}{1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k} + O\left(\frac{\rho R}{1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k}\right) \\ &= \frac{\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k}{1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k} + O\left(\frac{\rho R}{1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k}\right). \quad (42)\end{aligned}$$

So we can replace ϕ_f by $\phi_R(\omega) := \rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k/(1-\rho\sum_{k=0}^{m}\frac{(i\omega)^k}{k!}\iota_k)$ in our approximate (19) and have a more robust estimation. This will give an extra error term:

$$\begin{aligned}&\left|\sum_{k=1}^{N}\frac{2(1-\rho)}{k\pi}\Re\left(\phi_f\left(\frac{k\pi}{a}\right)-\phi_R\left(\frac{k\pi}{a}\right)\right)\sin\left(\frac{k\pi u}{a}\right)\right|\\ \leq\ & C(1-\rho)\log N\max_{1\leq l\leq N}\left\{\frac{\rho|R|}{\left|1-\rho\sum_{k=0}^{m}\frac{(il\pi)^k}{a^k k!}\iota_k\right|}\right\}\end{aligned} \tag{43}$$

for some constant C. Therefore, the overall error bound for this robust approximate is

$$\begin{aligned}\epsilon_R =\ & (1-\rho)\left[\left(1+\frac{2}{\pi}\int_0^{\pi}\frac{\sin t}{t}dt\right)\int_a^{\infty}f(x)dx+\frac{C_1}{(N-1)^{\beta}}\right.\\ &\left.+C_2\log N\max_{1\leq l\leq N}\left\{\frac{\rho|R|}{\left|1-\rho\sum_{k=0}^{m}\frac{(il\pi)^k}{a^k k!}\iota_k\right|}\right\}\right]\end{aligned} \tag{44}$$

for some constants C_1 and C_2 independent of N and m, with C_2 depending on c in light of our assumption of $\iota_k \leq c^k$. It is clear from the expression that with our assumption of $\iota_k \leq c^k$, the right hand side of (43) tends to zero as m goes to infinity. Finally, when the safety loading is large, ρ can be smaller. It is obvious that $\rho|R|/\left|1-\rho\sum_{k=0}^{m}(il\pi)^k\iota_k/a^k k!\right|$ goes to zero when ρ goes to 0 for fixed m. So we conclude that our robust approximate works better in a large loading environment and a smaller value of m can be used. It is suggested that for a robust approximation on ruin probabilities in the case of small safety loading, readers can refer to the interesting work of [13].

6. Rearrangement Inequality

One fundamental property of ruin probability is that it is monotonically decreasing with respect to the initial reserve u. However, our approximate (19) is not necessarily decreasing. This results from the fact that trigonometric functions used in our approximation are periodic instead of monotonic. Nevertheless, a refining procedure proposed by Chernozhukov et al. [22] called rearrangement can be adopted here to further improve our approximation. The resulting modified approximate not only is decreasing, but also reduces the L^p-norm of the global error of the approximation for a suitable chosen $p \geq 1$.

Firstly, consider a measurable function m defined on the compact interval $[0, a]$ and mapping to a bounded set $K \subset \mathbb{R}$. The decreasing rearrangement is defined as follow:

$$m^*(x) := \inf \left\{ y \in \mathbb{R} \middle| \left[\int_0^a \mathbb{1}_{\{m(u) \geq y\}} du \right] \geq x \right\}. \tag{45}$$

The effect of this method is given by the following proposition. The proposition is from the paper [22]. Interested readers can refer to that paper for more information.

Proposition 6.1. *Consider a decreasing and bounded measurable function $m : [0, a] \to K \subset \mathbb{R}$, K is a bounded subset in $\mathbb{R}$ and $\widehat{m}(x)$, the approximation for the given function m. For the rearrangement of $\widehat{m}$, $\widehat{m}^*(x)$, the following statement is true.*

1. *$\widehat{m}^*(x)$ is a weakly improved approximation in terms of L^p norm for $p \in [1, \infty]$,*

$$\left[\int_0^a |\widehat{m}^*(x) - m(x)|^p dx \right]^{\frac{1}{p}} \leq \left[\int_0^a |\widehat{m}(x) - m(x)|^p dx \right]^{\frac{1}{p}}.$$

2. *The improvement of rearranging become strict if the following conditions are true. There exist two sets $\aleph$ and $\aleph'$ such that their measures are greater than $\delta > 0$ and satisfy the following conditions. For any $x \in \aleph$ and $x' \in \aleph'$, $x' > x$, $\widehat{m}(x) < \widehat{m}(x') + \epsilon$ and $m(x') < m(x) + \epsilon$ for some $\epsilon > 0$. For $p \in [1, \infty)$,*

$$\left[\int_0^a |\widehat{m}^*(x) - m(x)|^p dx \right]^{\frac{1}{p}} \leq \left[\int_0^a |\widehat{m}(x) - m(x)|^p dx - \frac{\delta}{a} \eta_p \right]^{\frac{1}{p}}.$$

   $\eta_p := \inf\{|v - t'|^p + |v' - t|^p - |v - t|^p - |v' - t'|^p\}$ with the infimum taken over all v, v', t, t' in the set K such that $v' \geq v + \epsilon$ and $t' \geq t + \epsilon$. It is clear that $\eta_p \geq 0$ for $p \in (0, \infty)$.

Since ruin probabilities must satisfy all the conditions in Proposition 6.1. This rearrangement technique can be directly applied to the approximation of ruin probabilities and helps us obtain better results. In practice, while we cannot derive the rearrangement analytically, it can be computed as follows. Let $\Pi := \{x_i = ai/n | i = 0, 1, \ldots, n\}$ be the equidistant partition of $[0, a]$. Consider the set of $\{m(x_i)|x_i \in \Pi\}$. By sorting the elements in this set in a decreasing order, one can get a rearrangement approximation for the original function m. The number of points n needed for an accurate result depends on the original functions and the computational complexity of this method is the same as the ordering operations of a sample size of n, where the best possible order is $O(n \log n)$. Alternatively, a stochastic method can be used to compute the rearrangement. One can first generate a sample set of independently and uniformly distributed random variables $\{U_i : i = 1, \ldots, n\}$ on $[0, a]$, and then sort the elements in $\{m(U_i) : i = 1, \ldots, n\}$ in decreasing order. More information can be found in [22] and the references therein.

7. Numerical Studies

We provide two studies on applying the Fourier-cosine method to approximate ruin probabilities. The first model is compound Poisson processes with exponential claim size distributions. This model satisfies the the conditions in Theorem 4.5. Next, we conduct another study by assuming L_t a Poisson process. It is used to demonstrate the case when conditions in Theorem 4.5 fail to hold. Also, one more example of using rearrangement inequality will be illustrated for examining its global effect. Note that graphs in this section may be in different scale for the purpose of demonstration. Computational time is generated by a normal laptop computer with *Mathematica*.

Example 7.1. (Compound Poisson-Exponential Claim) Let L_t be a compound Poisson process whose intensity is 20 with exponentially distributed claim sizes of mean 1. The Lévy measure for such process is $\nu(dx) = 20e^{-x}dx$. The premium rate is set as 25. The exact ruin probability in this case is given by $\psi(u) = 0.8e^{-0.2u}$. Here a is chosen to be 90. The result can be seen in Figure 1.

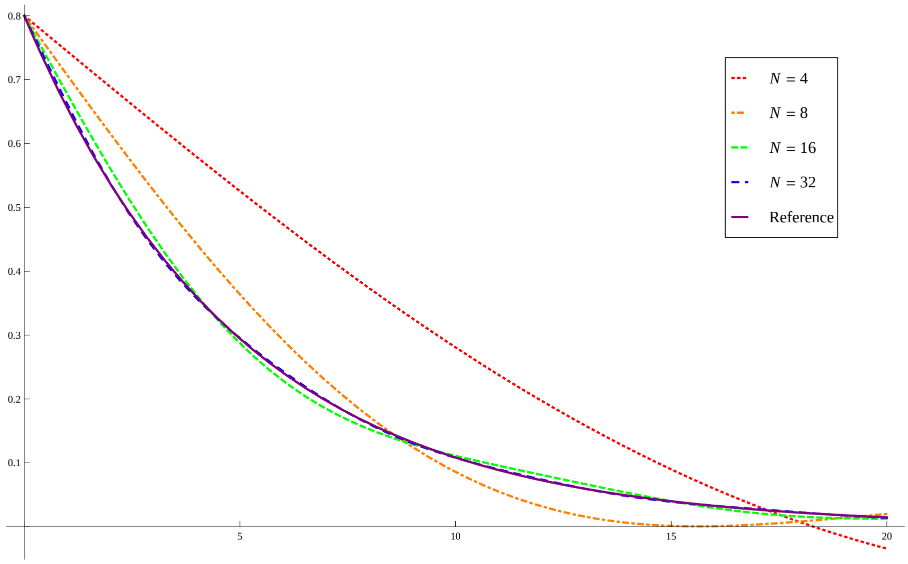

Figure 1: Comparison of Fourier-cosine approximation with reference curve where compound Poisson process with exponential claim size as underlying model. Truncation range N is set to be $4, 8, 16$ and 32

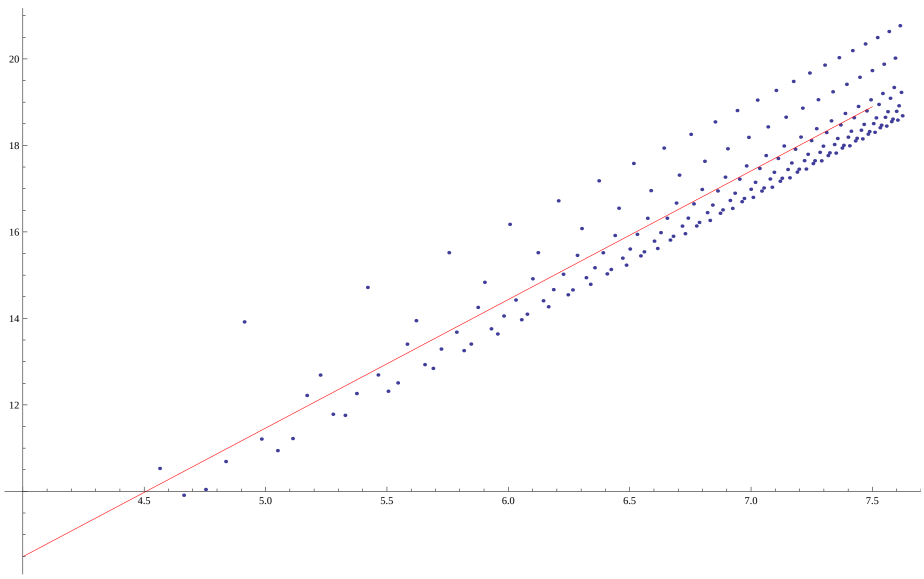

Figure 2: The graph of $-\log(|\varphi_e(7, N) - \varphi(7)|)$ against $\log N$ in Example 7.1.

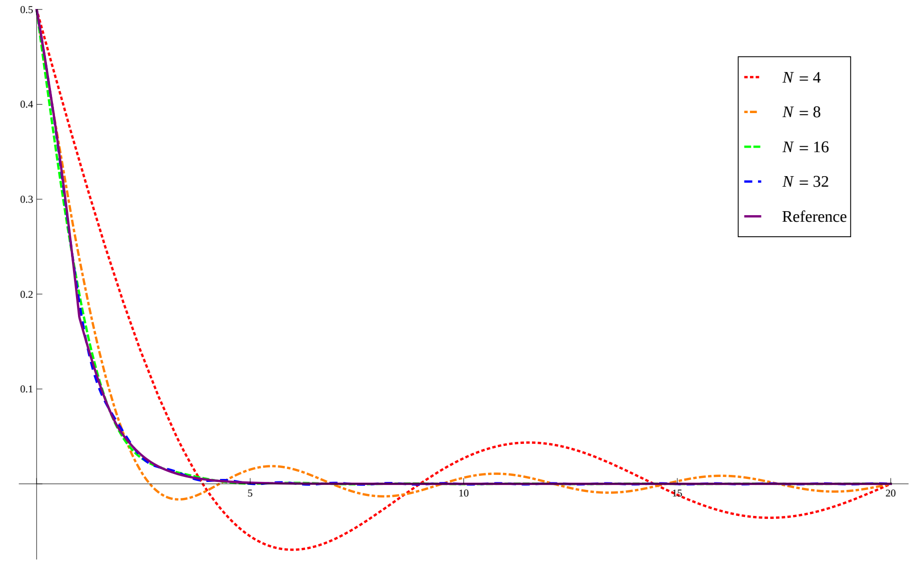

Figure 3: Comparison of Fourier-cosine approximation with reference curve using Poisson process. Truncation range N is set to be $4, 8, 16$ and 32

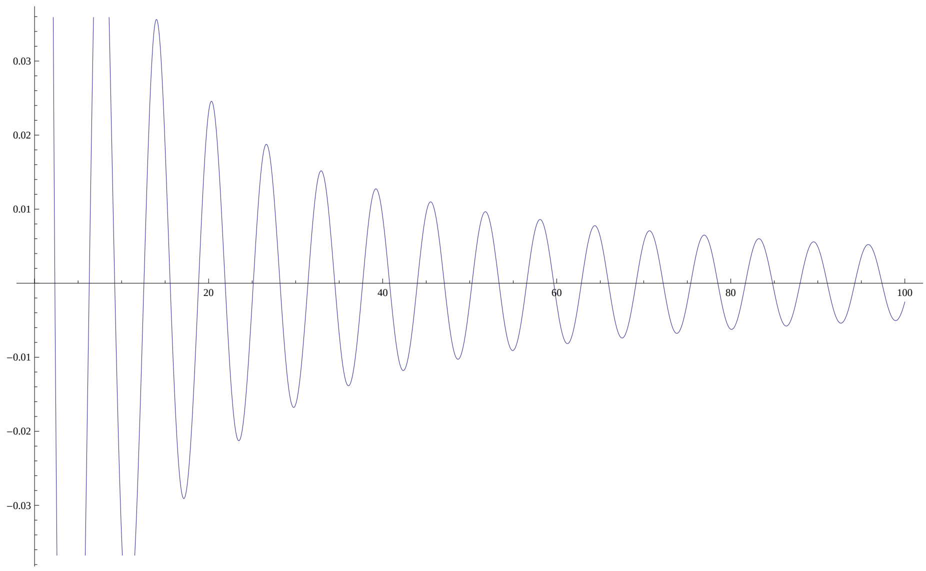

Figure 4: The graph of the real part of Fourier transform used in Example 7.2

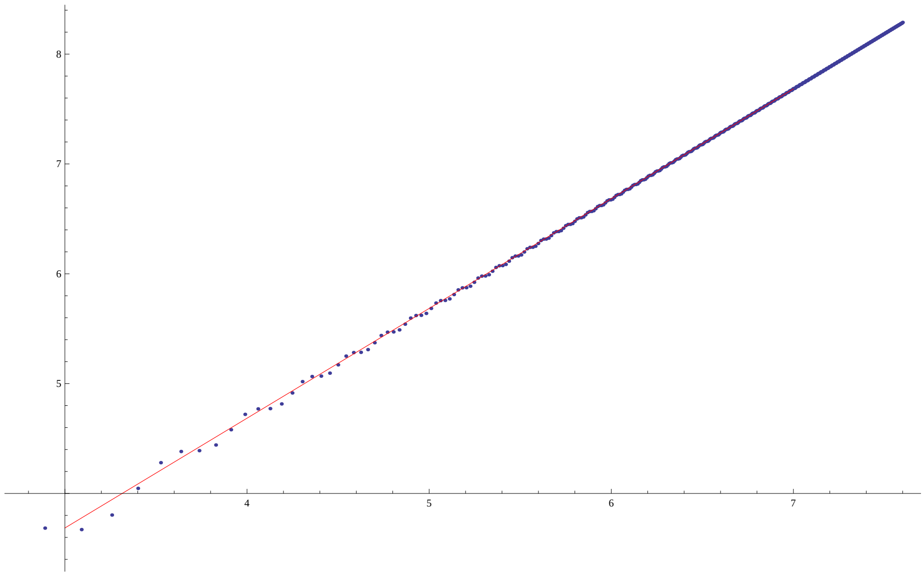

Figure 5: The graph of $-\log(|\varphi_e(1, N) - \varphi(1)|)$ against $\log N$ in Example 7.2

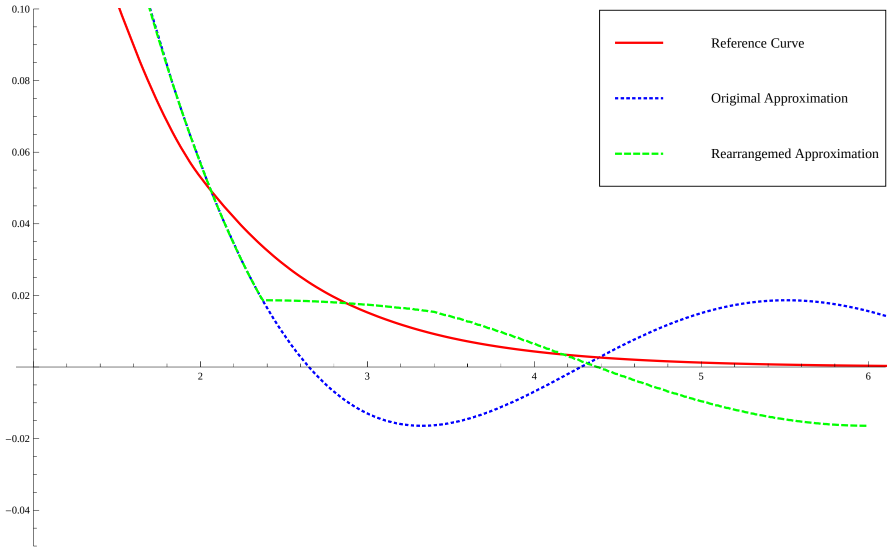

Figure 6: Improved approximation for Poisson process model by rearrangement inequality. Being enlarged for showing the rearrangement result.

Since $\Re(\phi_f(\omega)) = 4/(25\omega^2 + 1)$ in this model, it not only has an algebraic index of convergence of 2, but also tends to $4/25\omega^2$ when ω is large. This gives that $\Re(\phi_f(\omega))$ tends to a positive decreasing function. Consequently, our improved error bound is applicable here and so one would expect that $\epsilon_2 \approx O\left(1/N^3\right)$. Figure 2 demonstrates that it is the case, indeed using simple linear regression, we find that the slope of this graph is 2.97308 and it suggests that $\epsilon \approx O\left(1/N^{2.97308}\right)$. Also note that since ϕ_f is monotone in this example, $\varphi_e(u, N)$ behaves like an alternating series in N for fixed u. As a result, the plot points fluctuate around the regression curve in Figure 2.

Table 1: Fourier-cosine method for compound Poisson process with exponential claim model as in Example 7.1 with $u = 7$.

N	32	64	128	256	512
Error	9.17×10^{-4}	2.40×10^{-4}	3.07×10^{-5}	3.70×10^{-6}	3.90×10^{-7}

Example 7.2. L_t is assumed to be a Poisson process with $\lambda = 1$ and $c = 2$ in this example. Figure 3 shows the result. It is clear that the approximation curves do not converge to the true curve as quickly as the previous examples. Figure 4 shows that $\Re(\phi_f(\omega))$ is not monotone. Since the algebraic rate of convergent is 1 for $\Re(\phi_f(\omega))$, it would suggest that $\epsilon_2 \approx O\left(\frac{1}{N}\right)$. Figure 5 shows the convergence of error with respect to N when $u = 1$. The slope of this graph is approximately 0.999466.

Table 2: Fourier-cosine method for Poisson model as in Example 7.2 with $u = 1$. ; Reference value = 0.17564129722......

N	32	64	128	256	512
Error	1.54×10^{-2}	8.35×10^{-3}	4.04×10^{-3}	1.96×10^{-3}	9.86×10^{-4}

Example 7.2 also demonstrates another important property of the Fourier-cosine method. Most of the numerical inversions of Laplace transform, for example, the GWR and FT algorithms in [26], only provide tentative error bounds without mathematical justification; indeed, the error bounds are not binding. In fact, both GWR and FT algorithms fail to realize their errors bounds in

Example 7.2, yet the error bound based on the Fourier-cosine method can still be applied.

Example 7.3. (Rearrangement) Using the same setting as in Example 7.2 on the range $[0, 6]$ and let $N = 8$. We performed rearrangement to improve our approximation. Obviously, Figure 6 shows that the curve is much closer to the true curve after rearrangement.

8. Conclusion

In this paper, we promote the philosophy of Fourier-cosine method from [1] to ruin theory with Lévy subordinator models and derive an error bound that follows its line of reasoning. We also shown that a stronger bound for error can be obtained assuming monotonicity for $\Re(\phi_f(u))$. Moreover, we modified our method to provide a robust approximation of ruin probabilities. Furthermore, rearrangement technique is introduced for further improvement for the global error. Finally, our numerical studies show the effectiveness of our approximation.

Further research can be done on adopting our method to general situations, for examples, extending the method to models including diffusion. Enhancing the converging rate of the Fourier series through series acceleration method is another possible research direction.

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