

1 **Breakup dynamics and dripping-to-jetting transition**
2 **in a Newtonian/shear-thinning multiphase**
3 **microsystem**

4 Yong Ren^{*ab}, Zhou Liu^{ac} and Ho Cheung Shum^{*ac}

5 ^a*Department of Mechanical Engineering, The University of Hong Kong, Pokfulam Road,*
6 *Hong Kong.*

7 ^b*Current Address: Department of Mechanical, Materials & Manufacturing Engineering, The*
8 *University of Nottingham, Ningbo, China.*

9 ^c*HKU-Shenzhen Institute of Research and Innovation (HKU-SIRI), Shenzhen, Guangdong, China.*

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11 **Author to whom correspondence should be addressed. E-mail: yong.ren@nottingham.edu.cn;*
12 *ashum@hku.hk; Tel: +852 2859 7904*

13 The breakup dynamics in non-Newtonian multiphase microsystems are associated with a variety
14 of industrial applications such as food production and biomedical engineering. In this study, we
15 numerically and experimentally characterize the dripping-to-jetting transition under various flow
16 conditions in a Newtonian/shear-thinning multiphase microsystem. Our work can help to predict
17 the formation of undesirable satellite droplet, which is one of the challenges in dispensing non-
18 Newtonian fluids. We also demonstrate the variations in breakup dynamics between shear-
19 thinning and Newtonian fluids under the same flow conditions. For shear-thinning fluids, droplet
20 size increases when the Capillary number is smaller than a critical value, while decreases when
21 Capillary number is beyond the critical value. The variations highlight the importance of
22 rheological effects in flows with a non-Newtonian fluid. The viscosity of shear-thinning fluids
23 significantly affects the control over droplet size, therefore necessitating the manipulation of the

24 shear rate through adjusting the flow rate and the dimension of the nozzle. Consequently, the
25 droplet size can be tuned in controlled manner. Our findings can guide the design of novel
26 microdevices for generating droplets of shear-thinning fluids with predetermined droplet
27 size. This enhances the ability to fabricate functional particles using an emulsion-templated
28 approach. Moreover, elastic effects are also investigated experimentally using a model shear-
29 thinning fluid that also exhibits elastic behaviors: droplets are increasingly deformed with
30 increasing elasticity of the continuous phase. The overall understanding in the model multiphase
31 microsystem will facilitate the use of a droplet-based approach for non-Newtonian multiphase
32 applications ranging from energy to biomedical sciences.

33 **1. Introduction**

34 Microfluidic multiphase system can be applied in a wide variety of applications. For example, it
35 can facilitate drug encapsulation and release using emulsion droplets as a template to fabricate
36 core-shell microspheres, capsules and other functional materials.¹⁻³ The system can also be used
37 for the generation of jets, which can work as precursors of microfibers for application in wound
38 dressing and tissue engineering.^{2,4-7} Nevertheless, the practical microfluidic multiphase
39 applications increasingly demand the use of fluids with more complex rheological behaviors,
40 such as non-Newtonian fluids, whose viscosities change with the applied stress. Non-Newtonian
41 fluids are ubiquitous in daily life; examples include blood, lotions, creams, shampoos and
42 toothpaste. They are also widely used in industrial applications such as inkjet printing, spraying
43 and coating.⁸⁻⁹ Microscaled non-Newtonian multiphase systems have been increasingly applied in
44 biomedical engineering, food production, and energy applications. For instance, an electrokinetic
45 microdevice has been developed to convert fluidic mechanical energy to electrical energy by

46 means of electrokinetic phenomena such as streaming currents.⁹By reducing the hydrodynamic
47 conductance while maintaining the same streaming current, addition of an appropriate polymer
48 to the working fluid in a microchannel can enhance the energy conversion efficiency, which is a
49 ratio of electrical output power and hydrodynamic input power.¹⁰⁻¹¹

50 Non-Newtonian multiphase microsystem has become a subject of intense research, and it is of
51 paramount importance to understand the relevant physical phenomena, one of which involves the
52 deformation of liquid threads and subsequently droplet formation. The performance of the
53 droplets is intimately tied to the ability to control droplet size, which has a strong impact on the
54 droplet stability as well as optical and mechanical properties.¹²For example, when the droplets
55 are used as a template to fabricate micro/nano particles for the drug delivery system, the shape
56 and size of emulsion droplets have significant impacts on the drug release kinetics.¹³⁻
57 ¹⁴Monodisperse droplets with precisely controlled sizes can be used to deliver an accurate dosing
58 of contained payload such as drug, flavoring, or chemical reactants.¹⁵Therefore, monodispersity
59 and size tenability are highly desired for ensuring that the droplets exhibit constant, controlled
60 and predictable behaviors.^{4,12}However, the complex rheological properties of non-Newtonian
61 fluids challenge the versatility in droplet size control. For example, the stretching and/or thinning
62 of non-Newtonian liquid filaments will lead to the formation of “bead-on-string” patterns.¹⁶⁻
63 ¹⁷These beads can subsequently become undesirable satellite droplets, increasing the
64 polydispersity of the resultant droplet population. The dynamics of the droplet formation process
65 can be characterized into a dripping and a jetting regime. The dripping-to-jetting transition can
66 be estimated using the Capillary number of the continuous phase, Ca_{out} , a ratio of viscous force to
67 surface tension, and the Weber number of the dispersed phase, We_{in} , a ratio of inertial force to
68 surface tension.¹⁸

$$69 \quad We_{in} = \frac{\rho_{DP} d v_{DP}^2}{\sigma} \quad (1)$$

$$70 \quad Ca_{out} = \frac{\eta_{CP,0} v_{CP}}{\sigma} \quad (2)$$

71 where ρ_{DP} is the density of dispersed phase, d refers to the diameter of the orifice, V_{DP} and V_{CP}
 72 are the velocities of dispersed and continuous phase respectively, σ is the interfacial tension,
 73 and $\eta_{CP,0}$ is the apparent viscosity of the non-Newtonian fluid of continuous phase. The subscript
 74 “0” refers to zero-shear rate when a shear-dependent fluid is used. Droplet formation occurs at
 75 the orifice directly after the two fluids meet in the dripping process, when both Ca_{out} and We_{in} are
 76 small, as surface tension dominates. By contrast, jetting occurs when Ca_{out} or We_{in} is large, as the
 77 viscous stress or the inertial force on the droplet will be large enough to overcome surface
 78 tension. Droplets are generated after breakup of a jet at some distance downstream in this
 79 regime. While the dynamics of droplet breakup has been systematically investigated in
 80 Newtonian fluids,¹⁷⁻¹⁸ the validity of the understanding has not been adequately confirmed in
 81 non-Newtonian fluid systems. In addition, a number of non-Newtonian fluids such as polymeric
 82 solutions, whole blood or protein solutions with large polymeric molecules often exhibit elastic
 83 property due to the stretching and coiling of the polymer chains. The resultant non-Newtonian
 84 rheological behaviors inspire interesting applications.¹⁹ For instance, microparticles driven in
 85 viscoelastic solutions can migrate toward the centerline of a microchannel because of the first
 86 normal stress difference between the centerline and the walls; three-dimensional (3D) particle
 87 focusing can be achieved via a synergistic combination of inertial and elastic forces.²⁰ The
 88 elasticity of the focusing fluid has been shown to facilitate formation of smaller droplets.²¹ These

89 examples attests to the need for a comprehensive understanding of the role of elastic fluids with
90 shear-rate-dependent viscosity in droplet formation using microfluidic systems.

91 In this paper, we focus on a shear-thinning fluid, which is one of the most frequently
92 encountered non-Newtonian fluids,²² and investigate the dripping-to-jetting transition of a
93 Newtonian/shear-thinning two-phase system and characterize the formation of satellite drops in
94 the model system. The breakup time and droplet size are compared with Newtonian/Newtonian
95 two-phase system at the same Weber number and Capillary number. We demonstrate that the
96 degree of control over droplet size can be enhanced by adjusting the flow rate and nozzle size,
97 thus tuning the shear rate experienced by the non-Newtonian fluids. We also present an
98 experimental study of the elasticity effect of a shear-thinning fluid on droplet generation in a
99 microfluidic two-phase system. The shape of the droplets are shown to vary with the
100 Weissenberg number, which represents a ratio of the relaxation time to the hydrodynamic time.
101 Our work helps to elucidate the effects of rheological behaviors on the breakup dynamics in a
102 Newtonian/non-Newtonian two-phase microscaled flow; our understanding inspires new
103 approaches to control sizes and shapes of functional droplets in applications requiring the use of
104 non-Newtonian fluids.

105 **2. Numerical model**

106 The microcapillary co-flow device has been widely used in emulsion generation.²³ A 3D
107 numerical model with the same design is established in the present investigation (see Fig.1a). A
108 Newtonian fluid is injected in a cylindrical capillary as the dispersed phase at a constant mean
109 velocity V_{DP} . This inner fluid is surrounded by a non-Newtonian outer phase, which is injected
110 through the coaxial square capillary as the continuous phase at a constant mean velocity V_{CP} . The

111 cross section (perpendicular to the main flow direction) consists of a circle inside a square (see
 112 inset of Fig.1a). Thus, the continuous phase is flowing through a cross section between an inner
 113 circle and outer square. No-slip condition is applied at the solid boundaries of the walls of
 114 capillaries. Gauge pressure of zero is applied at the outlet of the domain. The maximum entrance
 115 length is 9.4 μm in our numerical computations, and the length of the computational domain is 6
 116 mm, which is long enough to ensure that the fluid flow can be fully developed. The governing
 117 equations for incompressible two-phase fluids are,²⁴

$$118 \quad \frac{\partial \alpha_{CP}}{\partial t} + v \cdot \nabla \alpha_{CP} = 0 \quad (3)$$

$$119 \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (4)$$

$$120 \quad \frac{\partial}{\partial t} (\rho v) + \nabla \cdot (\rho v v) = -\nabla P + \nabla \cdot [\eta (\nabla v + \nabla v^T)] + \frac{\rho \sigma \nabla \alpha_{CP}}{\frac{1}{2}(\rho_{DP} + \rho_{CP})} \nabla \cdot \frac{\nabla \alpha_{CP}}{|\nabla \alpha_{CP}|} \quad (5)$$

$$121 \quad \rho = \alpha_{CP} \rho_{CP} + (1 - \alpha_{CP}) \rho_{DP} \quad (6)$$

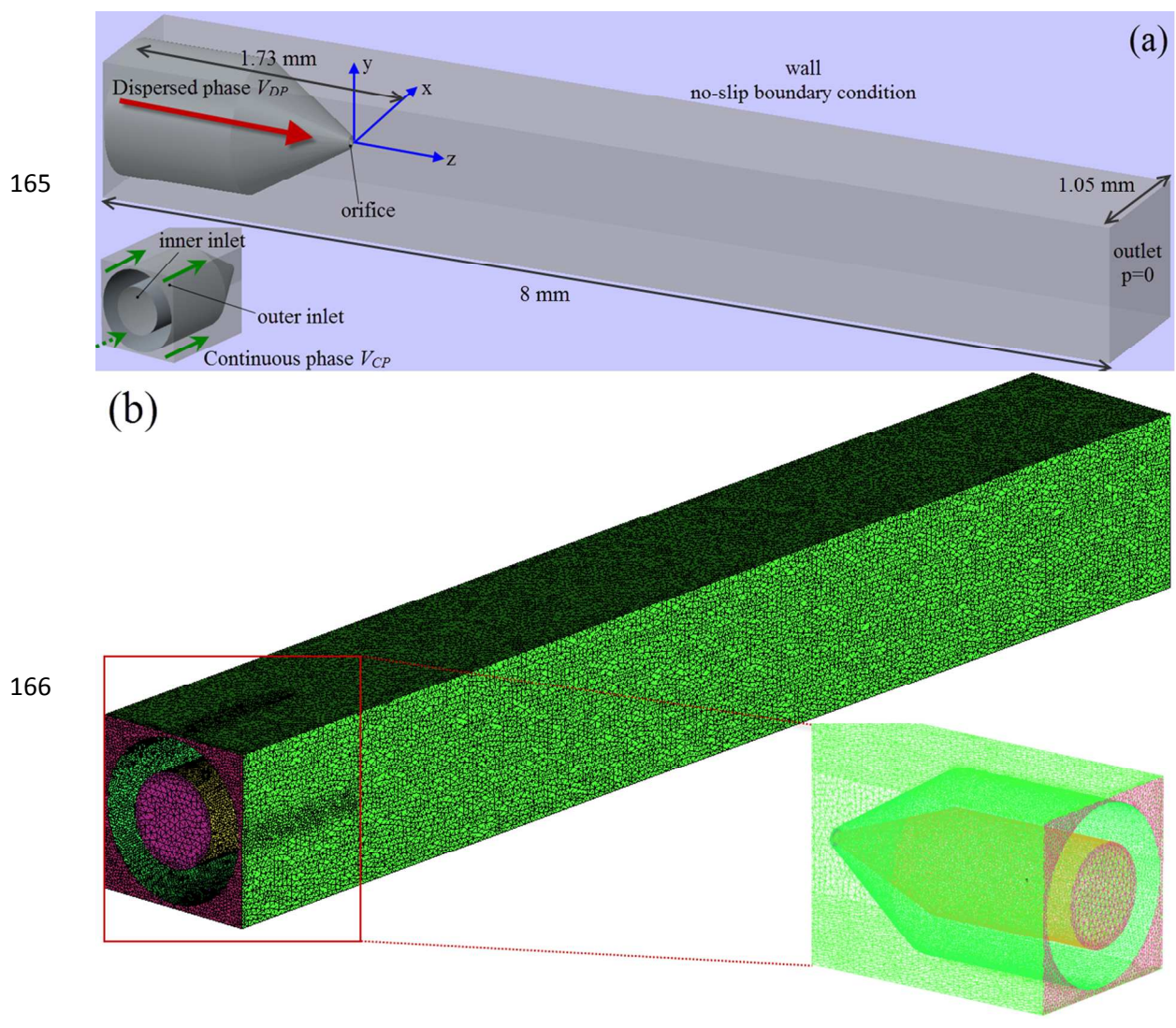
$$122 \quad \eta = \alpha_{CP} \eta_{CP} + (1 - \alpha_{CP}) \eta_{DP} \quad (7)$$

123 where v is the flow velocity, P is the pressure, ρ is the volume averaged density, α_{CP} and ρ_{CP} are
 124 the volume fraction and density of the continuous phase respectively, η_{DP} and η_{CP} are dynamic
 125 viscosities of the dispersed and continuous phases, respectively. The 3D numerical volume of
 126 fluid (VOF) model has been implemented to investigate the droplet breakup in pair of non-
 127 Newtonian/Newtonian fluids in T-shaped microchannel.²⁴The rheological property of the non-
 128 Newtonian continuous phase is modeled using Cross model, which has been widely applied in
 129 the modeling of industrial shear-thinning fluids.²⁵A shear-thinning fluid is used in our present
 130 study and its apparent viscosity is defined by,

$$\eta_{CP} = \eta_{\infty} + \frac{\eta_0 - \eta_{\infty}}{1 + (\lambda_c \dot{\gamma})^n} \quad (8)$$

131 where η_{∞} refers to lower limiting values of the fluid viscosity or infinite shear viscosity; η_0 refers
 132 to upper limiting values of the fluid viscosity or zero shear viscosity; λ_c is the time constant, the
 133 reciprocal of which corresponds to a critical shear rate that provides a useful indicator of the
 134 onset shear rate for shear thinning; $\dot{\gamma}$ is the shear rate and n represents the power law index of
 135 fluid. The solutions with different values of these parameters will have different rheological
 136 behaviors. Any effect due to gravity is neglected for simplicity, because the length scale of
 137 interest is much smaller than the capillary length of 1.33 mm in our model. The two-phase flow is
 138 solved using the VOF method by CFD software Ansys Fluent 14.0. The approach has been
 139 successfully demonstrated to investigate multiphase flow in microsystem.²⁶ The governing
 140 equations are discretized to algebraic equations by using a control-volume-based technique. An
 141 iterative solver was deployed to solve the control-volume discretized equations. The iterative
 142 timestep is 10^{-7} s and the solution converges when the residual is below a tolerance set as 1.0
 143 $\times 10^{-6}$. The simulations were performed using numerical grids composed of triangular elements, as
 144 shown in Fig. 1b. The numerical data was subsequently analyzed by Ansys CFX-Post 14.0 after
 145 simulation was completed. Computational sensitivity study has been carried out to evaluate the
 146 effect of different triangular grid sizes. The generated morphology of interface as indicated by
 147 the green color between dispersed and continuous phases is compared using different grid sizes.
 148 Similar morphology is found at flow rates of the dispersed phase $Q_{DP}=50$ ml/h, and of the
 149 continuous phase $Q_{CP}=70$ ml/h, as shown in Fig. 2. The size of interface gradually increases
 150 along the axial direction, and subsequently adopts cylindrical shape downstream with a uniform
 151 size for grid sizes of 4 μm and 8 μm . The variation in the generated size of interface is less than 5%
 152 after convergence between grid sizes of 4 μm and 8 μm . Furthermore, we investigated the grid
 153

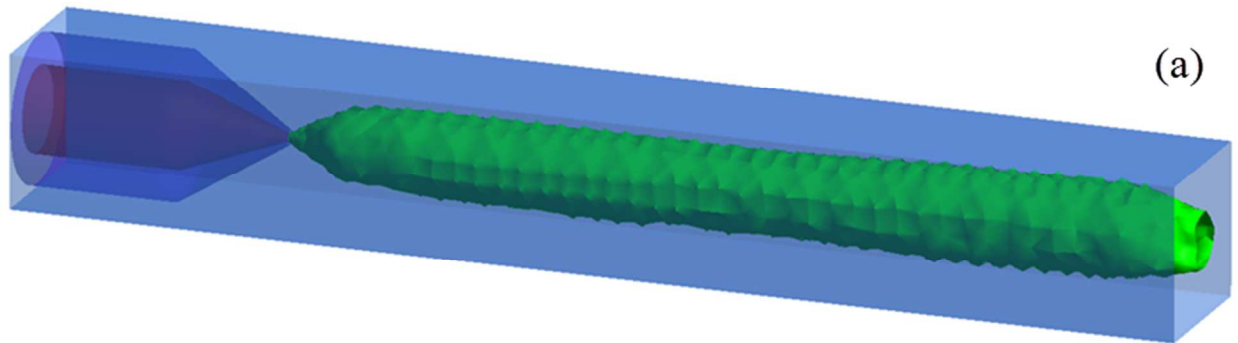
154 size effect by comparing numerical results of axial velocity profile along transversal direction (x
155 direction) for different grid sizes, at two downstream locations, $z=0.02$ mm, where the two
156 phases meet in vicinity of orifice, and $z=5.27$ mm, where the flow has become fully developed,
157 as shown in Fig.3a-b, respectively. At $z=0.02$ mm, the sharp peak in the middle represents the
158 velocity profile of dispersed phase, which is purged out of orifice with very small size, thus the
159 velocity will be increased dramatically due to mass conservation. The magnitude of velocity for
160 the continuous phase is much smaller, because of the much larger cross section. At $z=5.27$ mm,
161 the velocity profiles of dispersed and continuous phases adjoin with a smooth transition at the
162 interface. Compared with coarse grids with a size of $12\ \mu\text{m}$, a good agreement is found between
163 refined grids with a size of $4\ \mu\text{m}$ and $8\ \mu\text{m}$ respectively. 4300956 triangular elements with a size
164 of $8\ \mu\text{m}$ are used in all the following numerical simulations to reduce computation cost.



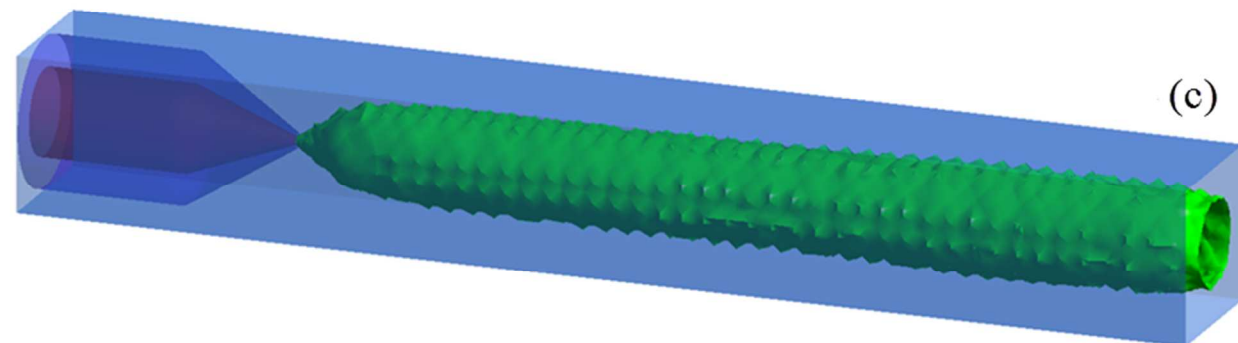
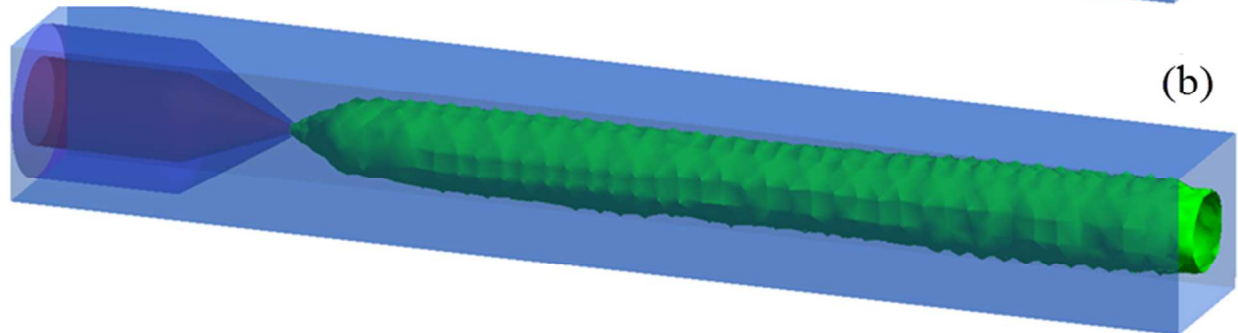
167 **Fig.1**(a) Schematic of the computational domain of multiphase microfluidic system. (b) Meshing
168 grids. The close-up view of meshing of nozzle and inlets is shown in inset.

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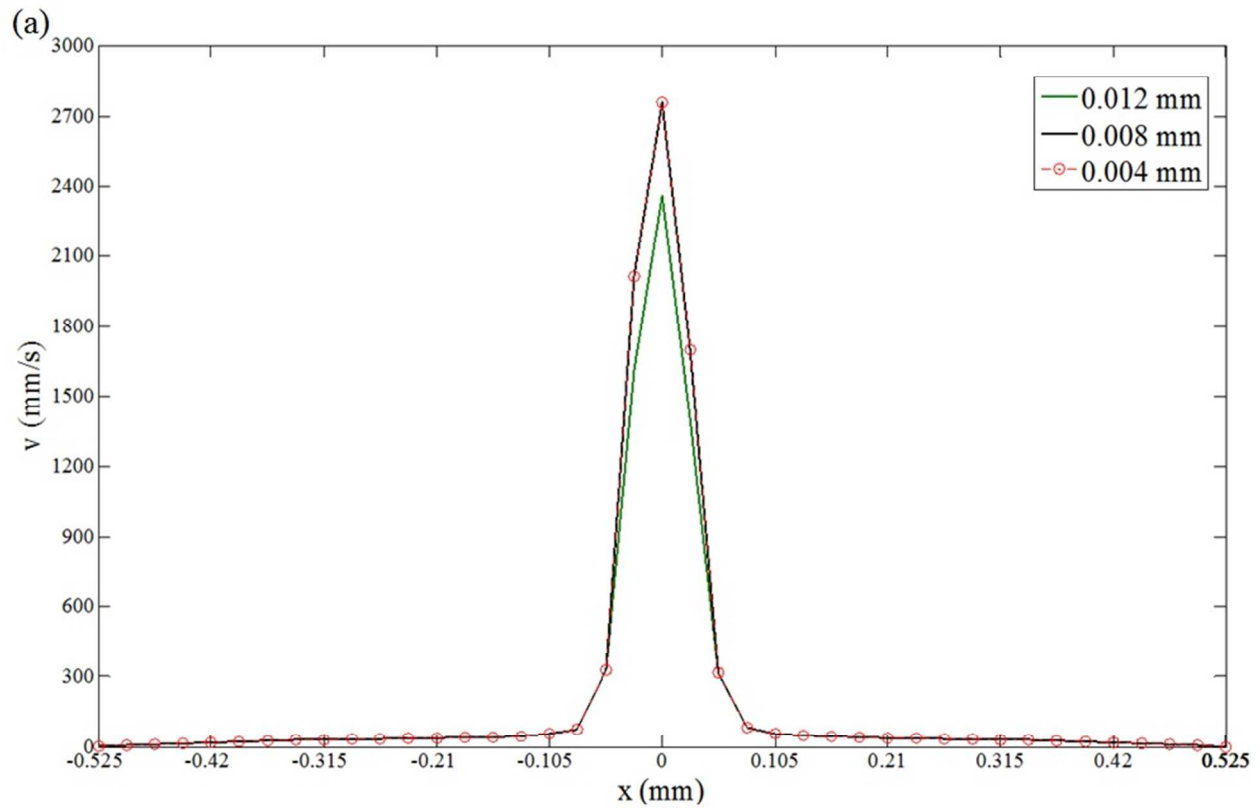
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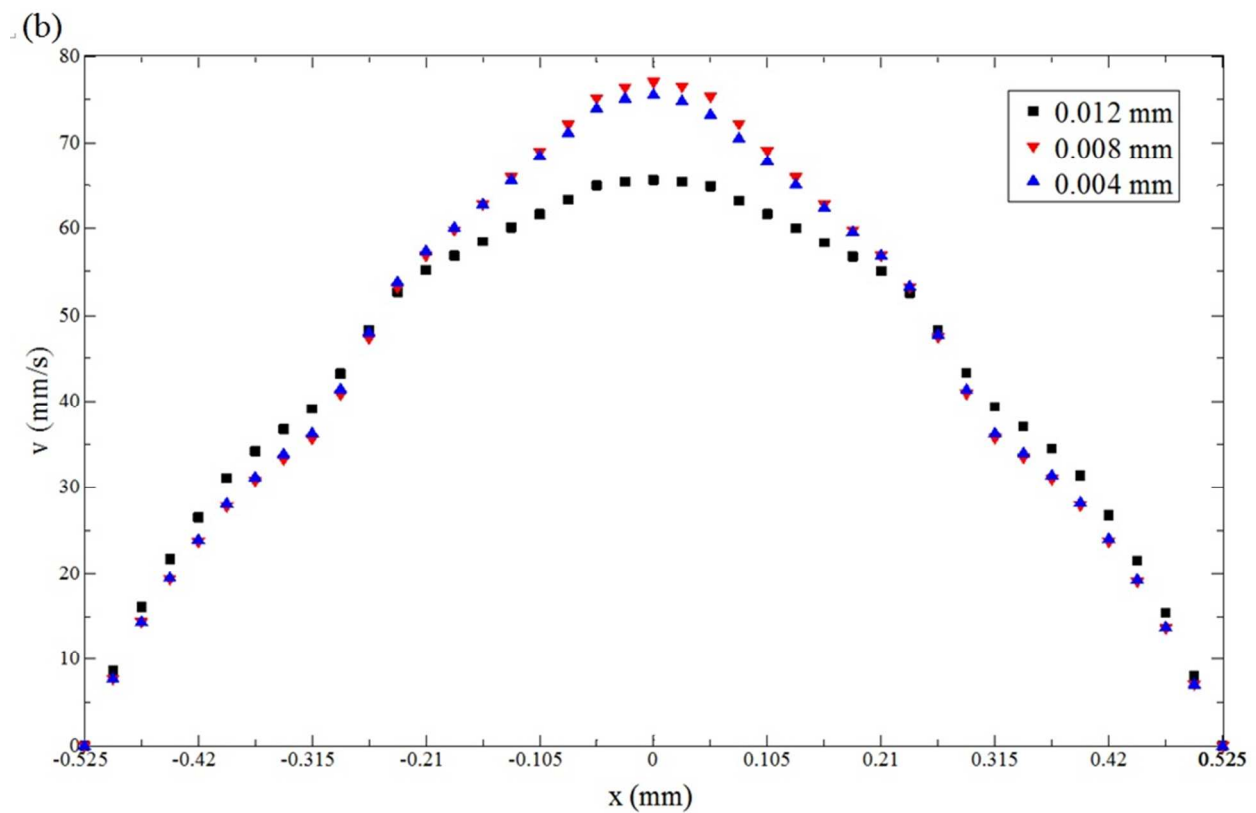
172 **Fig.2** Comparison of morphology of jet using different meshing grid sizes (a) 0.012 mm (b)

173 0.008 mm and (c) 0.004mm. $Q_{DP}=50$ ml/h, $Q_{CP}=70$ ml/h.

174



175



176 **Fig.3** Validation of the numerical model of Newtonian/shear-thinning multiphase microfluidic
177 system by comparison of axial velocity profile along transversal direction (x direction) at (a)
178 $z=0.02$ mm and (b) $z=5.27$ mm using different meshing grid sizes.

179 **3. Experimental section**

180 The Newtonian/shear-thinning two-phase co-flow is generated using a glass microcapillary
181 device reported earlier.²³ A glass slide was used as a substrate to support the capillary device,
182 which is composed of an inner round capillary (World Precision Instruments, Inc) and an outer
183 square capillary (Atlantic International Technology, Inc.). The round capillary has inner and outer
184 diameters of $630\ \mu\text{m}$ and $1.0\ \text{mm}$, and its tip was tapered to the desired diameter by a
185 micropipette puller (Sutter Instrument, Inc.) to obtain an orifice with an inner diameter of about
186 $49\ \mu\text{m}$. The tapered round capillary was fitted into the square capillary, which has an inner
187 dimension of $1.05\ \text{mm}$. Since the outer diameter of the round capillary matches with the inner
188 dimension of the square capillary, coaxial alignment of the two capillaries is ensured. The
189 dispersed phase was injected into the device through the circular capillary, while the continuous
190 phase was injected in the same direction through the square capillary. The fluids were injected
191 into the device through a flexible plastic tubing (Scientific Commodities Inc.), which was
192 connected to syringe pumps (Longerpump, LSP01-2A) at controlled flow rates. Unless otherwise
193 specified, the chemicals used in the study were supplied by Aladdin Reagents (Shanghai) Co.,
194 Ltd. In the present study, the dispersed phase was silicon oil, while the continuous phase was 2%
195 (w/v) aqueous solution of sodium carboxymethyl cellulose (CMC) ($M_w=250,000\ \text{Da}$, $DS=1.2$),
196 which is a pseudo-plastic fluid and demonstrates shear-thinning behaviors. The rheological
197 behavior of CMC solution can be characterized by two critical concentrations: ²⁷a first critical

198 CMC concentration indicates the transition to the semidilute network solution, while a second
199 critical CMC concentration indicates the transition to the concentrated solution. Below the
200 second critical concentration, the CMC solution becomes highly viscous and the dynamics is
201 dominated by viscous effects; above the second critical concentration, the dynamics become
202 dominated by elastic effects. For CMC solution with a nominal molecular weight of 700,000 Da
203 and a degree of substitution of 0.65-0.85, the first critical concentration is $\sim 1\%$ while its second
204 critical concentration is 2.5% .²⁷The effect of concentration and molecular weight on the
205 rheological behavior of aqueous CMC solutions has been investigated experimentally.²⁸Based on
206 their measurements of the rheological properties, solutions of CMC with molecular weights of
207 90,000 Da, 250,000 Da, and 700,000 Da all exhibit predominantly shear-thinning behaviors over
208 a shear rate of 0-1000 /s in absence of yield stress as their concentration changes from 0.1% to
209 3.0%. The average shear rate of CMC solution is below 1000 /s by controlling flow rate in our
210 work. Therefore, it is reasonable to assume that our CMC solution (Mw=250,000 Da, DS=1.2)
211 still has a predominantly viscous behavior; and the elastic effect can be ignored.²⁸

212 We also prepared 5% (w/v) aqueous solution of polyacrylamide (PAA, supplied by Wing Hing
213 Chemical Co., Ltd) as the continuous phase to investigate the elastic effect on the breakup
214 dynamics and droplet formation. The flow behavior inside the microcapillary device was
215 monitored with an inverted microscope (Motic, ocular: WF10 \times 18mm, object lens: EA4). A high-
216 speed camera (Phantomv9.1 high speed camera) was connected to the microscope and the flow
217 through the capillary was captured. The dripping-to-jetting transition using non-Newtonian
218 systems was characterized experimentally, and the breakup dynamics in a non-Newtonian fluid
219 system was compared with a Newtonian fluid system at the same Weber number and Capillary
220 number using the same microdevice. For the Newtonian two-phase flow, silicone oil was also

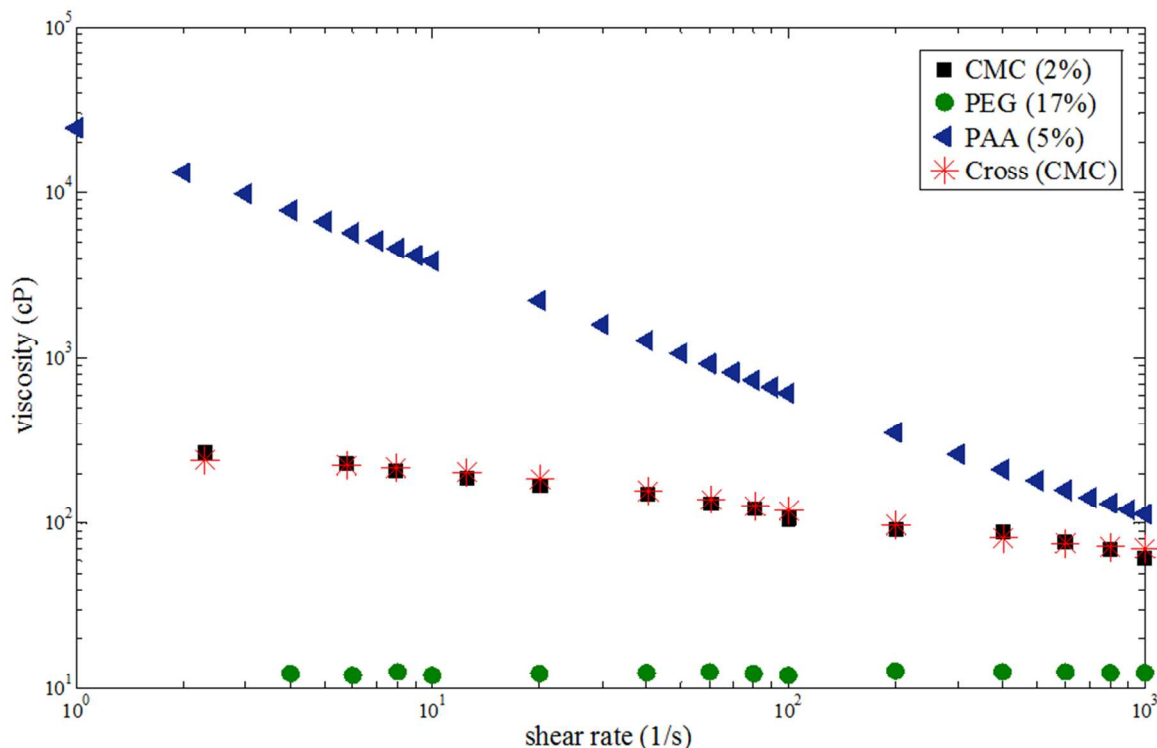
221 used as the dispersed phase, while 17% w/v aqueous solution of polyethylene glycol (PEG)
 222 (Mw=8,000 Da) was used as the continuous phase. The interfacial tension was measured by a
 223 Kruss spinning drop tensiometer-SITE100. The respective viscosity of each solution was
 224 measured at different shear rates by a Brookfield DV-II+Pro programmable viscometer. Shear-
 225 thinning behavior of the CMC solution was observed experimentally. The rheology data of CMC
 226 solution at all concentrations can be well represented by the well-known Cross model;²⁷ therefore
 227 Cross model was used in curve-fitting of the experimental data, and the measured Cross model
 228 parameters (the zero shear viscosity $\eta_0=253.5$ cP, infinite shear viscosity $\eta_\infty=60$ cP, and the
 229 flow index $n=0.9$, as shown in Fig.4) were used in the simulation for computing the simulation
 230 results, which were subsequently compared with the experimental observations. Very small
 231 variations of viscosity were observed at different shear rates (see Fig.4), indicating that 17% w/v
 232 PEG solution exhibits largely Newtonian properties. The concentration of PEG solution can
 233 affect the rheological property. For example, 7% w/w PEG solution shows significantly different
 234 viscosity values at different shear rates, demonstrating a non-Newtonian behavior.²⁹ 5% w/v PAA
 235 solution possesses shear-thinning behavior, as shown in Fig.4. As a typical viscoelastic liquid,
 236 the rheology property of PAA has been well investigated, and the storage modulus G' and loss
 237 modulus G'' have been measured using the oscillatory shear rheometer.³⁰ Based on the reported
 238 data, we find the relaxation time $\lambda=0.017$ s, according to Maxwell model of viscoelastic
 239 liquid.³¹ The relevant dimensionless numbers of viscoelastic liquid include Weissenberg number
 240 Wi and Elasticity number El . The Weissenberg number is defined as the product of the relaxation
 241 time and a characteristic rate of deformation of the flow, and quantifies the nonlinear response of
 242 the liquid.

$$243 \quad Wi = \frac{\lambda U}{D_t} = \frac{\lambda Q_{CP}}{D_t^3} \quad (9)$$

244 where U is the average axial flow velocity in the capillary device and D_t refers to the dimension
 245 of the capillary tube. Wi is dependent on flow rate and varies in the range between 0.0041 and
 246 0.041 in our experiments. The ratio between elastic and inertial effects is represented by the
 247 Elasticity number,

$$248 \quad El = \frac{Wi}{Re} = \frac{\lambda \eta_{CP,0}}{\rho_{CP} D_t^2} \quad (10)$$

249 where Re refers to Reynolds number. El is thus independent on the flow rate based on the
 250 definition. In our experiments $El=382$, indicating the dominating role of elastic effect relative to
 251 inertia effect. The physical properties of the fluids used in the investigation are shown in Table 1.



252
 253 **Fig.4** Rheological properties of the three aqueous polymer solutions: 2% w/v sodium
 254 carboxymethyl cellulose (CMC) in water (black squares), 17% w/v polyethylene glycol (PEG) in
 255 water (green circles), 5% w/v polyacrylamide (PAA) in water (blue triangle) and Cross model of
 256 CMC solution (red cross).

257
258 **Table 1**

259 Physical property of the test fluids

Test fluids	η (cP)	ρ (g/ml)	σ (mN/m) (silicon oil and CMC)	σ (mN/m) (silicon oil and PEG)	σ (mN/m) (silicon oil and PAA)
Silicon oil	10	0.963			
CMC (2%)	253.5 (zero- shear rate)	0.990			
PEG (17%)	13.6	1.053	7.12	9.624	10.176
PAA (5%)	24569(zero- shear rate)	0.992			

260

261 4. Results and discussion

262 The breakup dynamics and dripping-to-jetting transition in Newtonian/shear-thinning multiphase
 263 microsystem are presented and discussed first, followed by discussion of elastic effect in droplet
 264 shape at the end of this section. Simulation and experiments of Newtonian/shear-thinning
 265 multiphase flow in the capillary microdevice have been conducted under the same flow
 266 conditions to observe the process of jet deformation and droplet formation. Silicon oil and CMC
 267 solution are used as dispersed phase and continuous phase, respectively. The flow pattern at
 268 successive time points from simulation is shown in Fig.5. The flow rates of the dispersed and
 269 continuous phases are 5 and 7 ml/h, respectively, with $Ca_{out} = 0.0631$ and $We_{in} = 5.147$. The
 270 transition regime is observed and the jet size changes along the length of jet. The dispersed phase
 271 is purged out of the orifice, as shown by the inset in Fig.5a. The droplet grows in size and moves
 272 downstream while it is still connected to the fluid neck through the orifice via a filament, as
 273 shown in Fig.5b. The filament gradually becomes thinner (see Fig.5c) and finally breaks up into a

274 droplet (see Fig.5d). The process is repeated afterwards. The interface between the two phases in
275 Fig.5d is tracked by plotting the radius of interface along transversal direction (y direction)
276 versus the streamwise location along z direction. The simulation result is represented by the solid
277 line, and is compared with experimental measurements indicated by the symbols. A reasonable
278 agreement, as shown in Fig.5e, validates the accuracy of the numerical model.

279 A satellite droplet is formed during the breakup of an elongated filament between two adjacent
280 droplets. The profile near the breakup point is highly asymmetric, with the droplet interface
281 being very steep towards the droplet while lying flat towards the neck of the jet. The existence of
282 satellite drops is intimately related to the non-linear properties of the fluid motion close to the
283 breakup point.³²When the filament breaks at both ends before merging with one of the parent
284 droplets, the filament separates from both neighboring parent droplets and recoils into a satellite
285 droplet.³³The number of satellite droplets and their relative sizes strongly depend on the viscosity
286 ratio of the dispersed phase to that of the continuous phase, and are also influenced by the initial
287 disturbance wavenumber, which is defined as $2\pi a/\varepsilon$, where a is the radius of filament and ε is the
288 wavelength.³⁴At a small viscosity ratio, the slender center droplet undulates and pinches off at a
289 number of locations, generating a string of small satellite droplets. By contrast, when the
290 viscosity ratio is large, the internal flow leading to breakup is attenuated, resulting in the
291 formation of fewer satellite droplets. As the wave number increases, the ratio of radius of
292 satellite droplets in different birth regions relative to radius of parent droplet also increases.³⁴

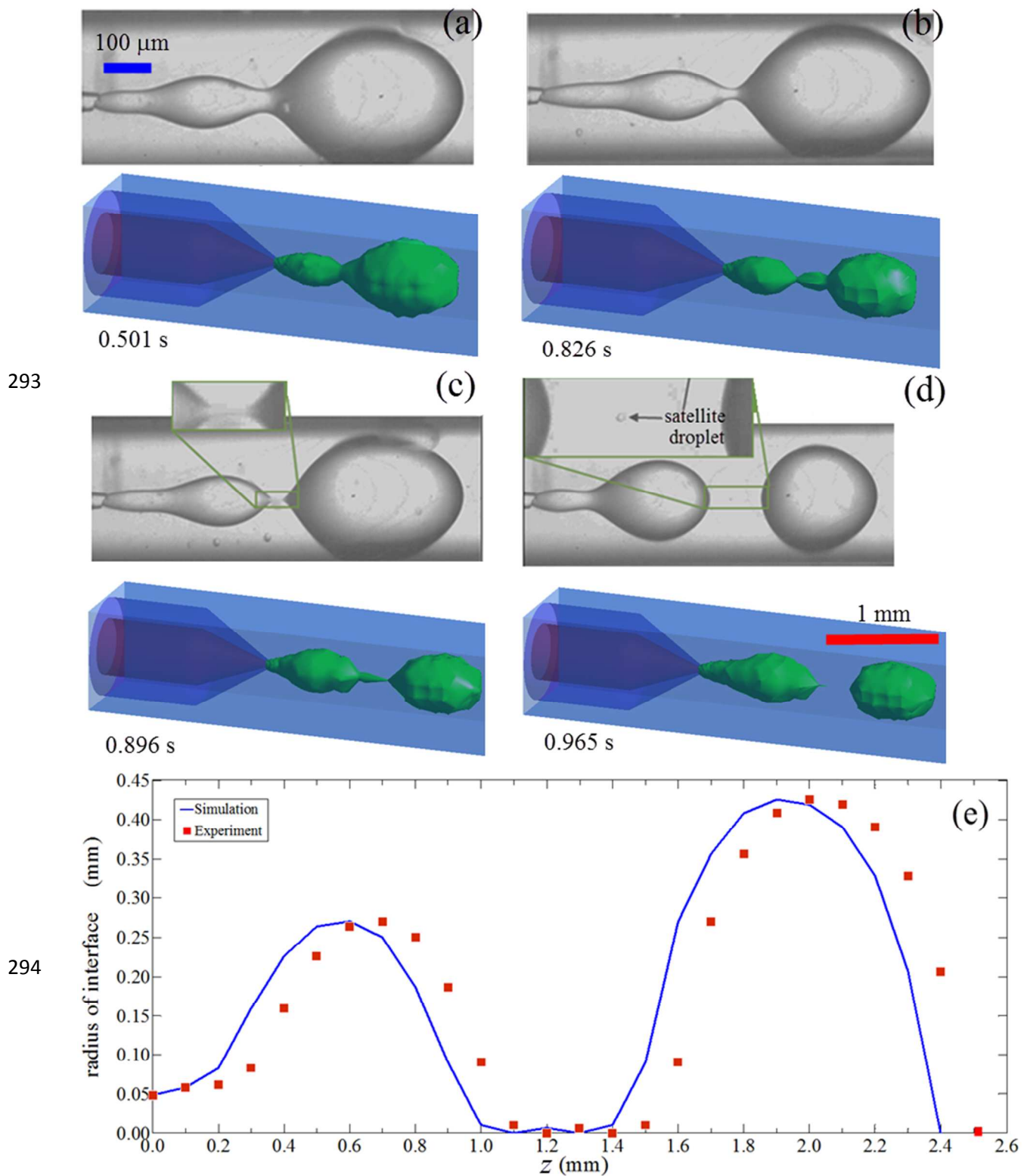


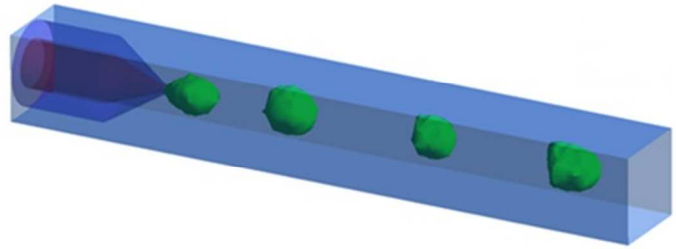
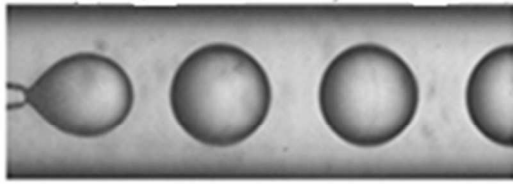
Fig.5 Time-lapse images of jet deformation and droplet formation from simulation (upper one in each sub-figure) and experiments (lower one in each sub-figure) using a Newtonian/shear-

297 thinning two-phase co-flow system (silicon oil as dispersed phase and CMC solution as
298 continuous phase). The blue and red scale bars are applicable for the experimental and
299 simulation results respectively. (a) The dispersed phase is purged out of the orifice; (b) The main
300 droplet is connected with the fluid neck via a thin filament; (c) The filament becomes even much
301 thinner; (d) Satellite droplet occurs after breakup of the jet; (e) Tracking of interface profile by
302 both simulation and experimental observation.

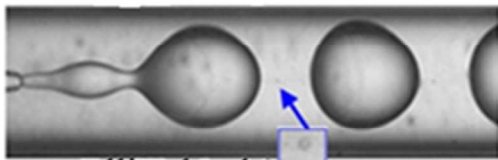
303 Since the satellite droplets have a much smaller size than the main droplet, the global
304 polydispersity of the resultant emulsion increases. To predict the occurrence of satellite droplets,
305 the flow conditions that result in the formation of satellite droplets have been delineated using a
306 state diagram, which shows the two-phase flow pattern as a function of Weber number and
307 Capillary number. Three representative flow regimes are observed when the flow rate ratio is
308 varied, as shown in Fig. 6a: dripping, where droplets are formed in the vicinity of the capillary tip;
309 intermediate, where the growing droplets move downstream while remaining connected to the
310 fluid in the tip through a fluid neck,¹⁸ and jetting, where droplets break up from an extended jet
311 with an incomplete retraction of the fluid neck. For the intermediate regime, the fluid neck
312 retracts completely back to the tip after the droplet pinches off at the detachment point of the
313 fluid neck. This regime is thus still regarded as dripping.¹⁸ Upon further increase of viscous or
314 inertia force, for instance, by changing the flow rate of continuous or dispersed phase, the
315 intermediate regime will transition to the jetting regime. Comparison of results from simulation
316 and experiments shows a good agreement, confirming the validity of our model in capturing the
317 physical behavior of the two-phase flow when a non-Newtonian fluid is used. As We_{in}
318 increases, the viscosity of the shear-thinning continuous phase decreases, when subjected to high
319 shear rate. As a result, a lower viscous stress is exerted on the dispersed phase; the liquid thread is

320 therefore not pulled further downstream. Consequently, the flow remains in the intermediate
321 regime rather than in the jetting regime. However, when We_{in} is larger than 10, jetting occurs
322 because of the larger inertial force. Based on experimental observation, satellite droplets appear
323 only in the intermediate regime, which is characterized as a dripping-to-jetting transition,¹⁸ as
324 shown in the state diagram in Fig.6b. Inertia is necessary to induce the formation of a satellite
325 droplet, because the droplet formation will be suppressed when the viscous force is dominant
326 over inertia. However, a very large inertial force will lead to formation of jetting and the liquid
327 thread will breakup into droplets further downstream. Consequently, the pinch-off time increases,
328 delaying the formation of the parent droplet and satellite droplets. This state diagram can help
329 predict the formation of undesirable satellite droplets in a Newtonian/shear-thinning two-phase
330 system, and can suggest operating conditions for eliminating satellite droplets. This ultimately
331 inspires production of monodisperse droplets by manipulating the breakup profile and
332 controlling the merging of satellite droplets with main droplets; yet these investigations are
333 beyond the scope of the present study.

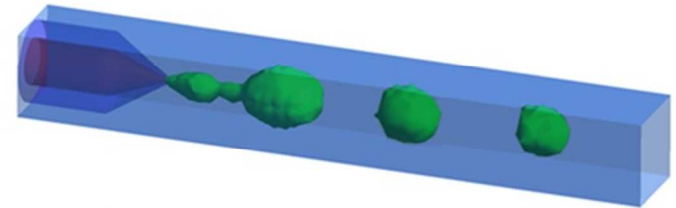
(a) Dripping ($Ca_{out}=0.045$, $We_{in}=0.052$)



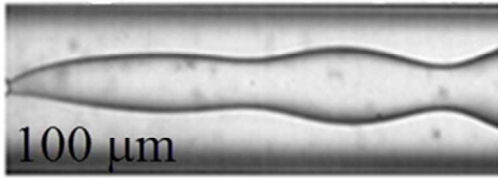
Intermediate ($Ca_{out}=0.091$, $We_{in}=5.15$)



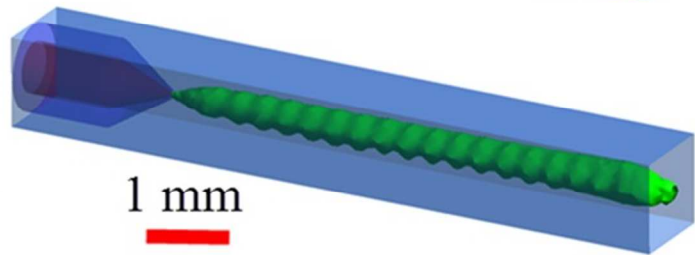
satellite droplet



Jetting ($Ca_{out}=0.18$, $We_{in}=82.29$)

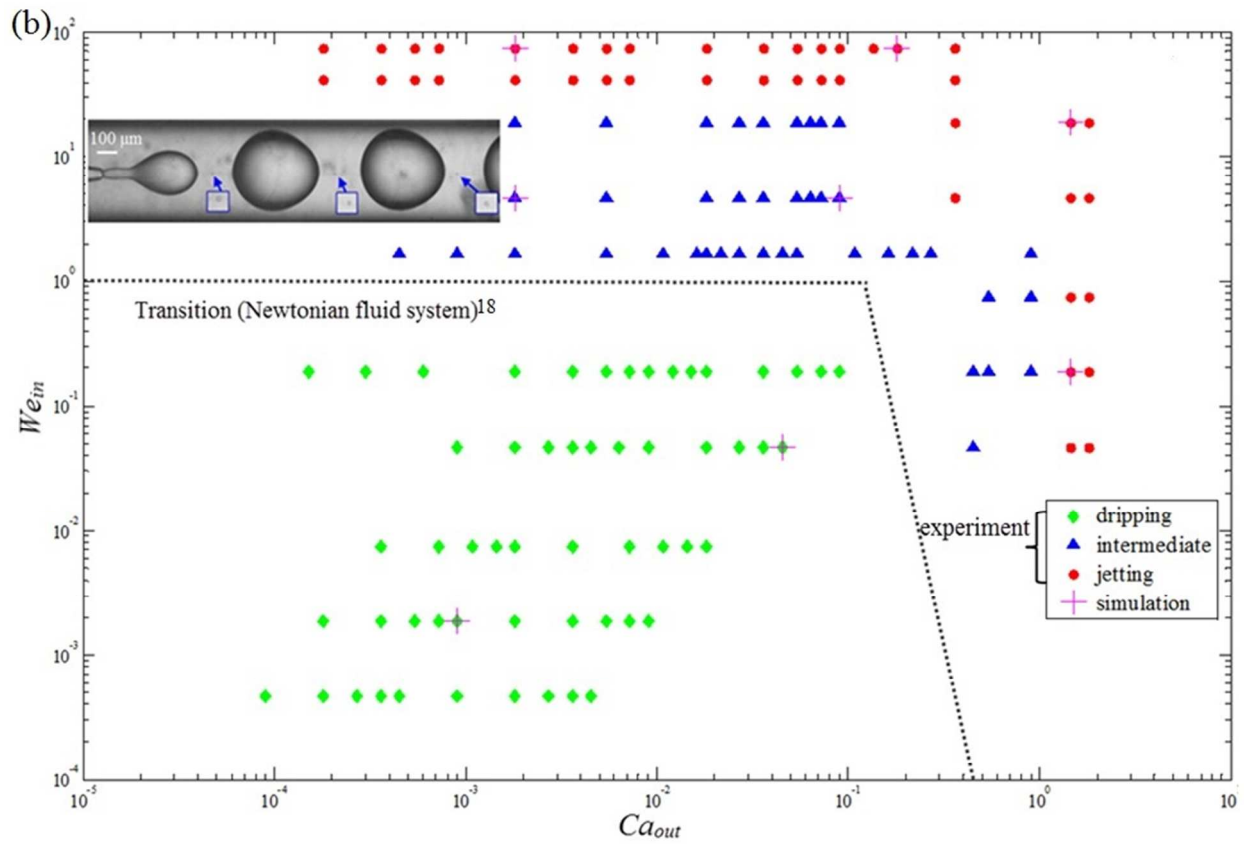


100 μm



1 mm

334



335

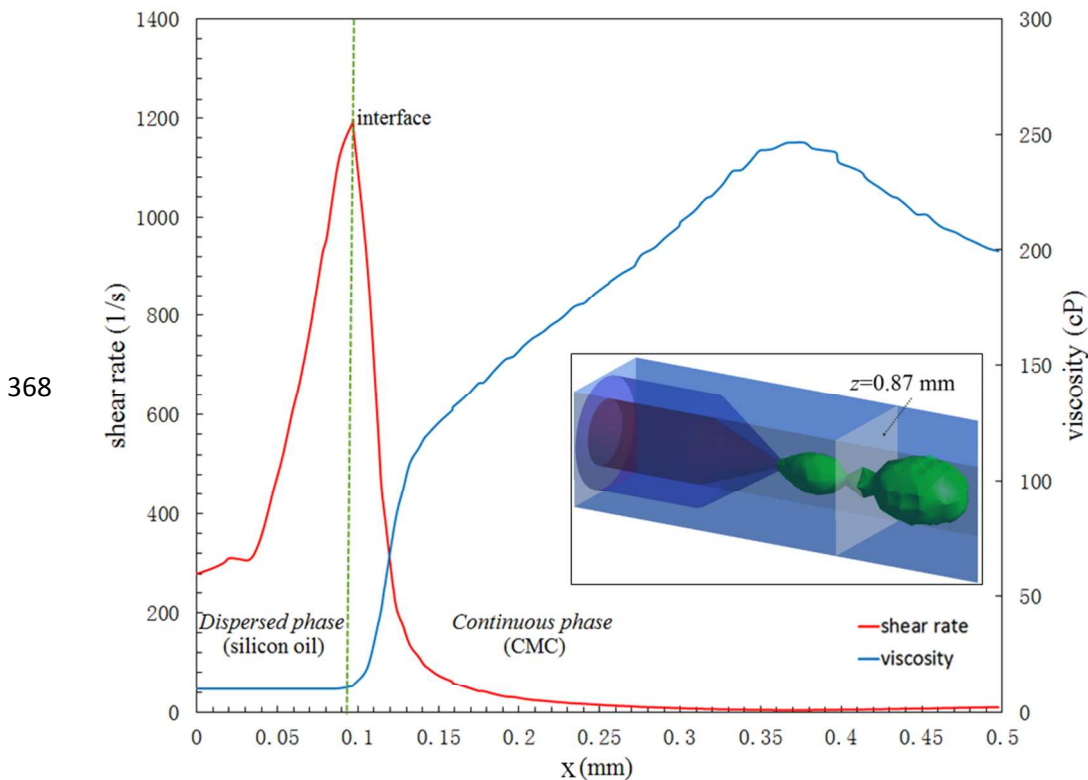
336 **Fig.6(a)** Comparison of results from simulation and experiments at different regimes using a
337 Newtonian/shear-thinning two-phase coflow system (silicon oil as dispersed phase and CMC
338 solution as continuous phase). The blue and red scale bars are applicable for the experimental and
339 simulation results respectively. (b) State diagram showing the dripping-jetting transition as a
340 function of Ca_{out} and We_{in} . The inset shows the satellite droplets appearing periodically between
341 adjacent parent droplets. The dash line (reproduced from reference.¹⁸) shows the dripping-to-
342 jetting transition in a purely Newtonian two-phase coflow system (deionized water and
343 polydimethylsiloxane (PDMS) oils).

344 The breakup time is one of the determining factors on the droplet production rate. It is thus of
345 fundamental interest and industrial relevance to investigate the breakup time when non-
346 Newtonian fluids, for example, shear-thinning fluids, are used, in comparison to the counterpart
347 of Newtonian multiphase system at the same We_{in} and Ca_{out} . Droplet breakup occurs due to the
348 interplay between the viscous stress and the interfacial tension between the two fluids. The
349 viscous stress is closely associated with the shear rate, which varies greatly depending on the
350 position and the shear rate, which affects the viscosity in all directions in the 3D flow. The
351 highly deformed filament connecting the fluid neck with the droplet (see Fig.5b) is of particular
352 interest, because it is the region where breakup occurs. We will therefore use the filament region,
353 for instance, $z=0.87$ mm, as the focal point to characterize the shear-rate-dependent viscosity
354 profile. Given the continuity of velocity at the interface and the small diameter of filament, the
355 shear rate reaches a sharp peak at the interface between the two phases in the filament region.
356 Due to shear-thinning characteristics of CMC solution, the viscosity will be dramatically reduced
357 at the interface, as shown by the numerically calculated results of viscosity and shear rate as a
358 function of the radial location along the transverse direction of the capillary device in Fig.7. The

359 shear rate of the continuous phase in the co-flowing microfluidic device with a dimension of D_t is
 360 related to the droplet diameter D_d through,²⁶

$$361 \quad \dot{\gamma} \sim \frac{Q_{CP}}{\pi D_t^3 \left[1 - \left(\frac{D_d}{D_t} \right)^2 \right] \left(1 - \frac{D_d}{D_t} \right)} \quad (11)$$

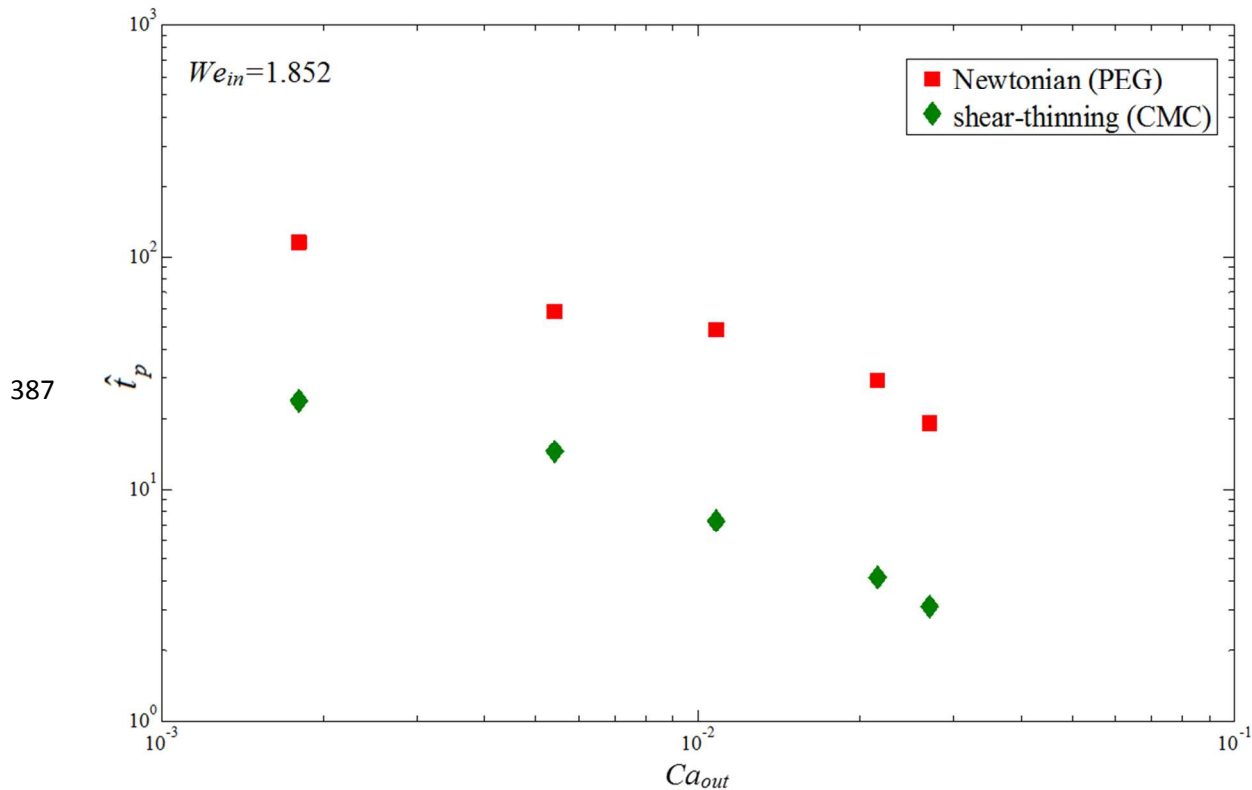
362 Given the flow condition in Fig.5, where $Q_{CP}=7$ mL/h and $\frac{D_d}{D_t} = 0.5$, $\dot{\gamma} \sim 1.65/s$. The calculated
 363 shear rate of the continuous phase is of the same order of magnitude as the simulation result. A
 364 local maximum shear rate of 9.68/s for continuous phase is obtained at the wall of collection tube,
 365 because of no-slip at the wall and large velocity gradient in boundary layer adjacent to the wall.
 366 The shear rate is reduced in the region far from the wall, and the minimum value of 3.87/s is
 367 obtained at $r=0.375$ mm.



369 **Fig. 7** The numerically calculated viscosity and shear rate as a function of the radial position
 370 along the transverse direction of the capillary device of the Newtonian/shear-thinning two-phase

371 coflow system (silicon oil as dispersed phase and CMC solution as continuous phase) at $z=0.87$
372 mm, which is illustrated in inset.

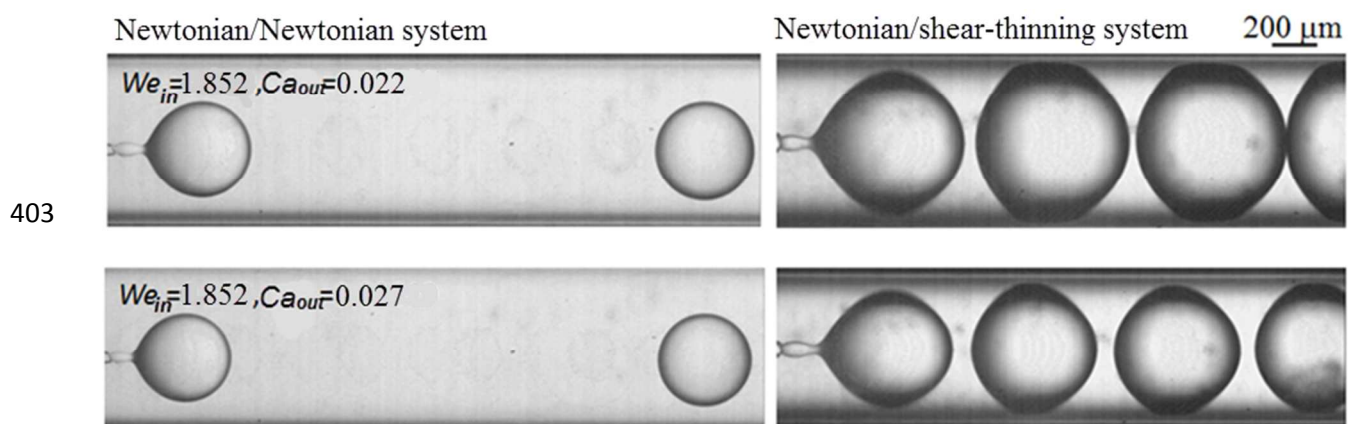
373 For shear thinning fluids, the viscosity is inversely proportional to the shear rate; thus the viscous
374 effect will be attenuated near regions with high shear rate, facilitating the pinchoff of the jet.
375 Consequently, the breakup process will be sped up so that a higher production rate of droplets
376 can be expected. To verify the hypothesis that shorter breakup time can be achieved when a
377 shear thinning fluid is involved, control tests have been conducted to monitor the breakup time
378 for the purely Newtonian two-phase system. The dimensionless pinching time,
379 $\hat{t}_p = t_p / t_v = t_p / (\eta_0 d / \sigma)$, was measured experimentally at the same We_{in} and Ca_{out} , where t_p and
380 t_v refer to the measured time interval between two successive pinching, and the viscous time
381 scale, respectively. When compared with the Newtonian system with constant viscosity, the
382 shear-thinning fluid near the liquid-liquid interface has a much reduced viscosity due to high
383 shear rate; as a result, at the same Capillary number, surface force dominates and the pinching is
384 faster when a shear-thinning fluid is used as the continuous phase. The dimensionless breakup
385 time is also reduced when Ca_{out} increases due to the enhanced viscous shear stress exerted on the
386 droplets, as shown in Fig. 8 where $We_{in}=1.852$.



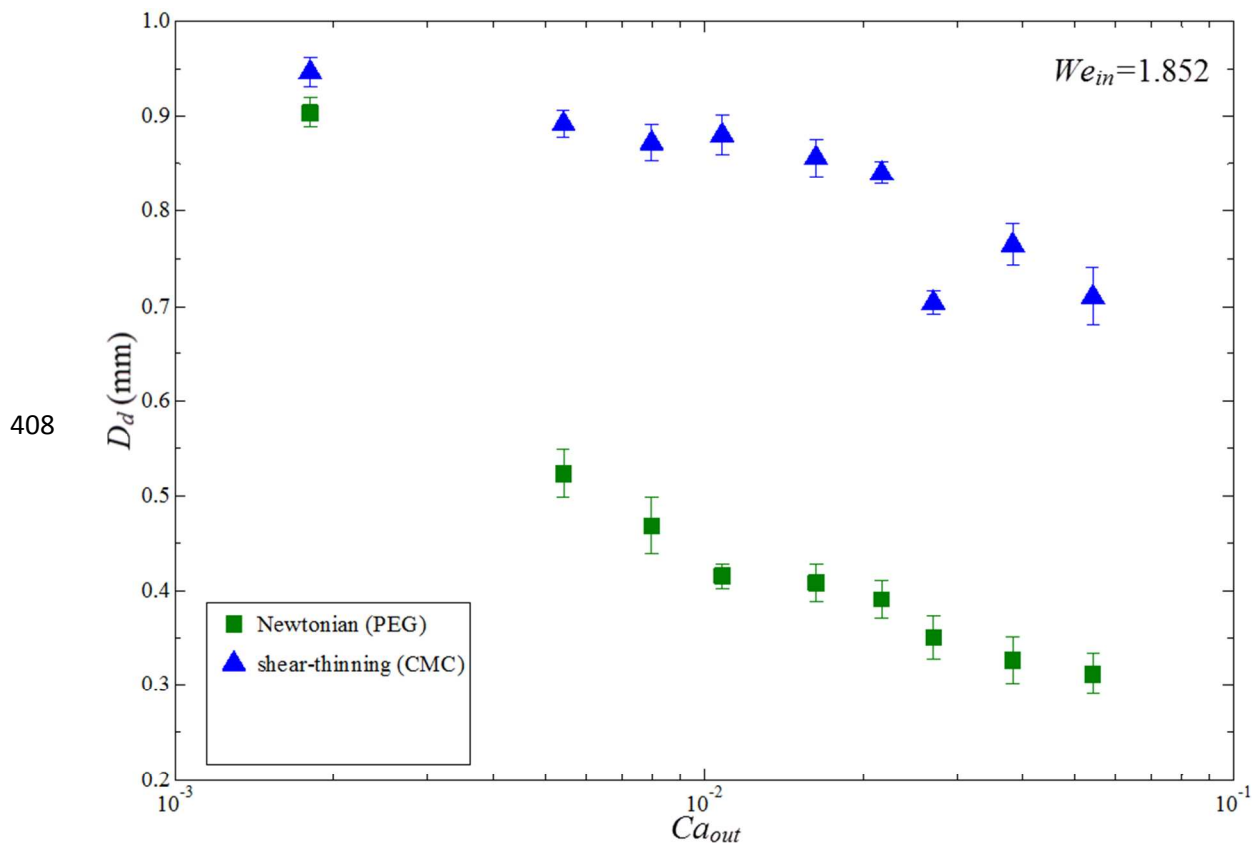
388 **Fig.8** Comparison of the experimentally observed breakup time of jet into droplets using the
 389 Newtonian/shear-thinning two-phase coflow system (silicon oil as dispersed phase and CMC
 390 solution as continuous phase) versus Newtonian/Newtonian two-phase coflowsystem (silicon oil
 391 as dispersed phase and PEG solution as continuous phase) when $We_{in}=1.852$.

392 In the breakup process of liquid jet of dispersed phase, the droplets are formed and surrounded
 393 by the continuous phase. The droplet size is influenced by the viscous force and the surface
 394 tension force. Due to the reduced viscosity of the continuous phase, the droplet size is larger
 395 when a shear-thinning fluid is used at the same We_{in} and Ca_{out} .³⁵This is confirmed by the
 396 observed snapshots of droplets after breakup at the same Weber number but at different Capillary
 397 numbers in two different multiphase systems, as shown in Fig.9. In both cases, silicone oil is
 398 used as the dispersed phase, while solutions of PEG and CMC are used as the Newtonian and

399 non-Newtonian continuous phases respectively. At the same Weber number of 1.852, both
 400 systems show a decrease in the droplet size when the Capillary number is increased from 0.0018
 401 to 0.054 (see Fig.10), due to the higher shear stress and the enhanced hydrodynamic focusing
 402 effect by the continuous phase.



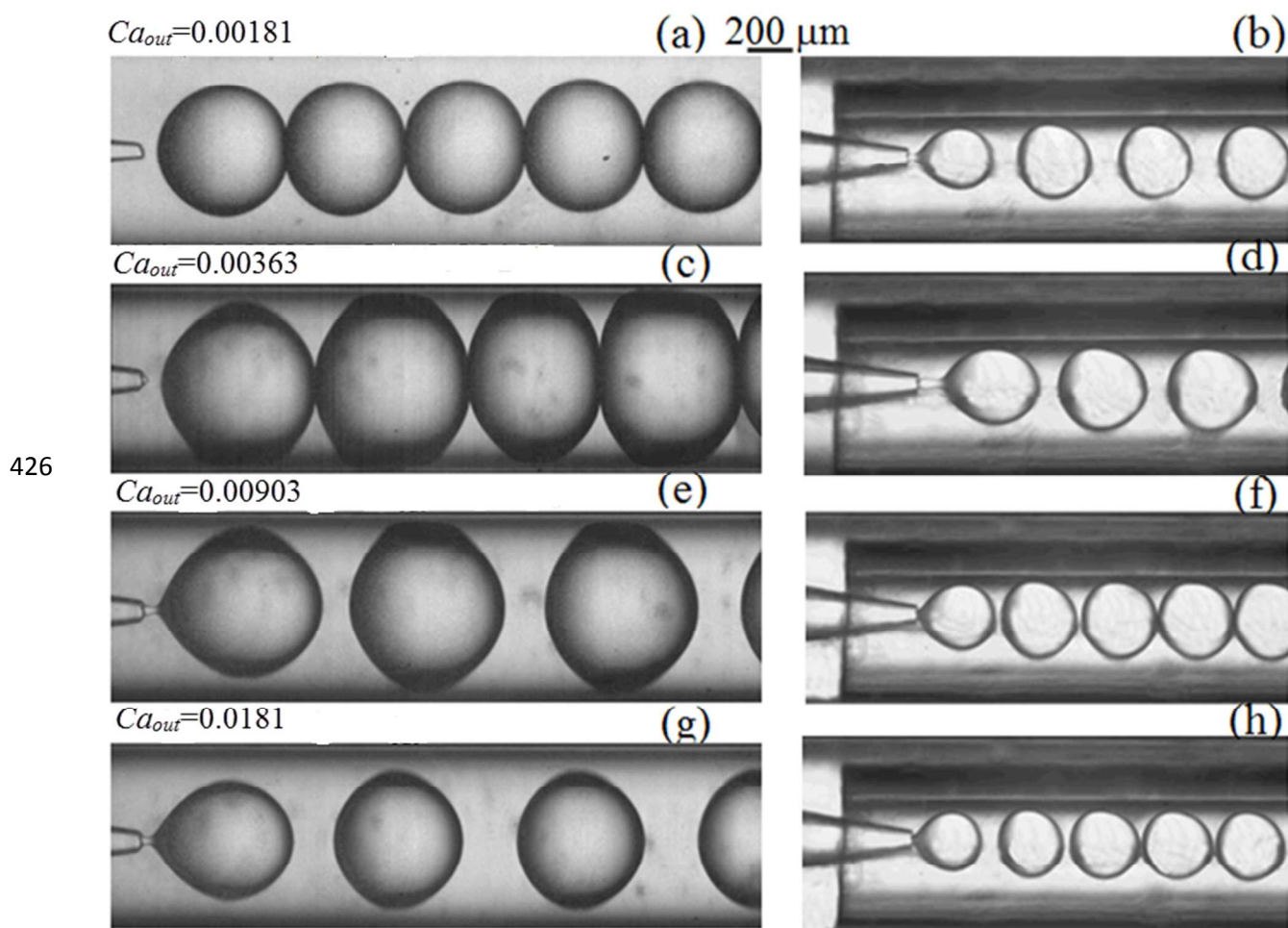
404 **Fig.9** Comparison of droplet size between Newtonian/shear-thinning two-phase coflow system
 405 (silicon oil as dispersed phase and CMC solution as continuous phase) and
 406 Newtonian/Newtonian multiphase coflow system (silicon oil as dispersed phase and PEG solution
 407 as continuous phase).



409 **Fig.10** Comparison of the experimentally observed droplet diameter between the
 410 Newtonian/shear-thinning two-phase system (silicon oil as dispersed phase and CMC solution as
 411 continuous phase) and the Newtonian/Newtonian two-phase system (silicon oil as dispersed
 412 phase and PEG solution as continuous phase). $We_{in}=1.852$ in all cases.

413 Control of the droplet size is also an important aspect of droplet-based microfluidic applications.
 414 For example, the volume of the droplet which contains reagents or analytes is one of the key
 415 parameters that determinethe efficiency and the overall throughput of the system.³⁶Since non-
 416 Newtonian fluids are ubiquitous in biochemical applications of microfluidics, it is also crucial to
 417 control droplet size. To achieve smaller droplets, we vary the dimensions of the nozzles by
 418 fitting a round capillary with radius R into the square capillary (see Fig.11). At a constant flow
 419 rate of outer-phase and constant Capillary number, evaluated based on the apparent viscosity, the

420 shear rate increases with decreasing inner diameter of the collection capillary, leading to an
 421 increase in the shear stress and thus reduced droplet sizes, as shown by the observed droplets in
 422 the collection tube with two different sizes in Fig.11. The droplet size is significantly reduced
 423 when collection tube with a smaller radius of 0.3mm is used (see the sub-figures on the right) as
 424 compared to that with a radius of 0.5 mm (see the sub-figures on the left), at the same Weber
 425 number and Capillary number.



427 **Fig.11** Observed droplets in collection tubes with different sizes ($R=0.5\text{mm}$ in a, c, e and g;
 428 $R=0.3\text{ mm}$ in b, d, f and h) using Newtonian/shear-thinning two-phase system (silicon oil as

429 dispersed phase and CMC solution as continuous phase). $Ca_{out}=0.00181$ in a-b, $Ca_{out}=0.00363$ in
 430 c-d, $Ca_{out}=0.00903$ in e-f, and $Ca_{out}=0.0181$ in g-h. $We_{in}=0.206$ in all cases.

431 As Ca_{out} and the shear rate of continuous phase increase, the viscosity of continuous phase is
 432 reduced for shear-thinning fluids such as CMC; this results in larger droplets.³⁵ However, when
 433 Ca_{out} increases beyond a critical value, the viscous drag can overcome the surface tension effects
 434 that would otherwise minimize the stretching of the fluid neck by drawing the fluid interfaces
 435 closer to the orifice. As a result, the fluid neck becomes stretched and elongated at high Ca_{out} .
 436 The radius of fluid neck r_j can be characterized by a scaling law that correlates the radius of fluid
 437 neck as a function of parameters including flow rate ratio, viscosity ratio and radius of collection
 438 tube, when the dispersed phase is not perturbed,³⁷

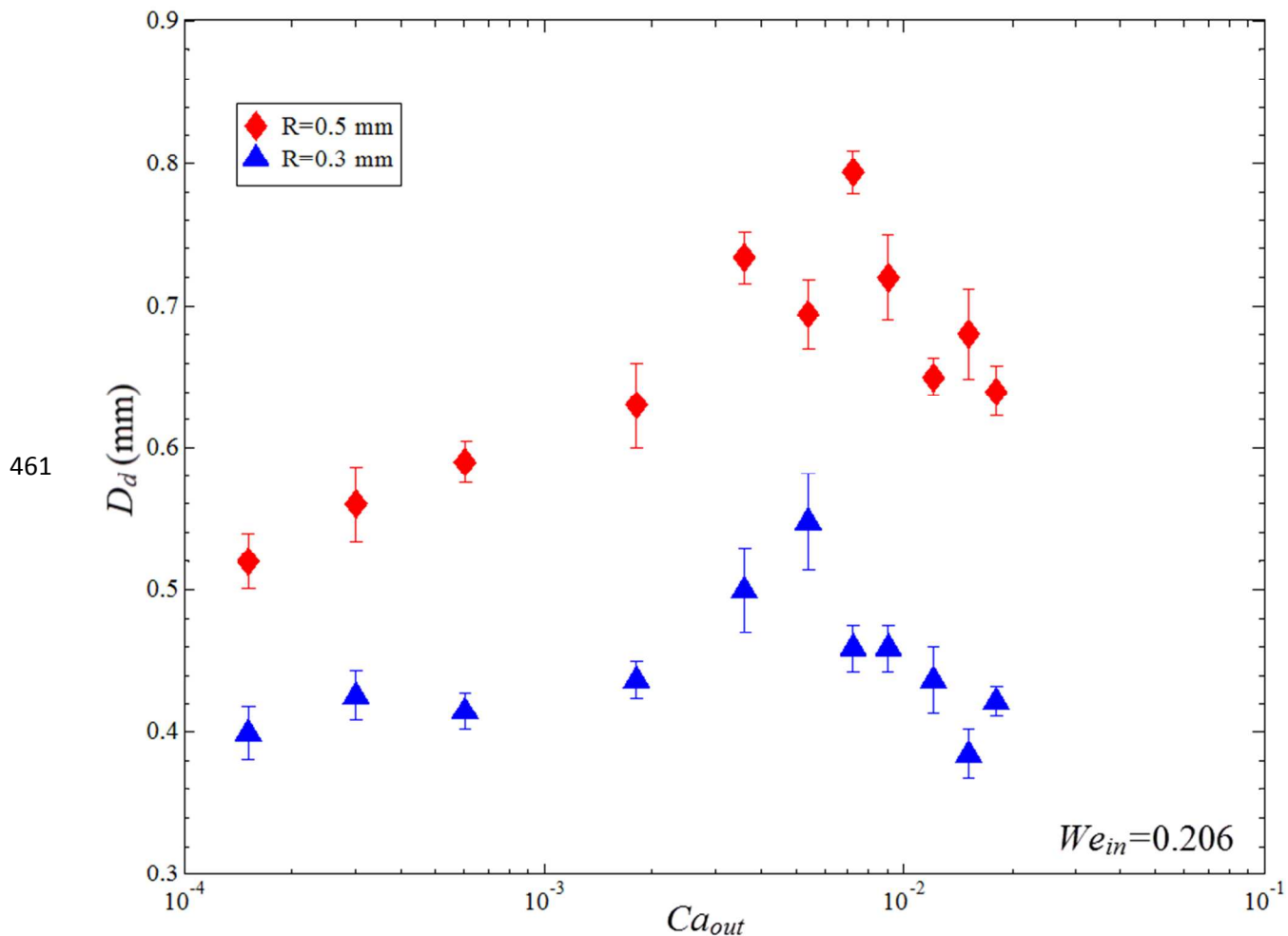
$$439 \quad r_j = R \left[\frac{\left(1 + \frac{Q_{DP}\eta_{DP}}{Q_{CP}\eta_{CP}}\right)^{0.5} - \left(1 + \frac{Q_{DP}}{Q_{CP}}\right)}{\frac{\eta_{DP}}{\eta_{CP}} - 2 - \frac{Q_{DP}}{Q_{CP}}} \right]^{0.5} \quad (12)$$

440 The shear rate is defined by,

$$441 \quad \dot{\gamma} = \frac{Q_{CP}}{\pi(R^2 - r_j^2)(R - r_j)} \quad (13)$$

442 The thinning of fluid neck occurs when it is stretched and elongated at high Ca_{out} . Thus, the
 443 shear rate of the continuous phase (i.e., CMC in our case) starts to decrease, and the viscosity of
 444 continuous phase will increase for a shear-thinning fluid. As a result, the droplet size will start to
 445 be reduced beyond a critical Ca_{out} , as shown in Fig.12 when $We_{in}=0.206$. When the inner radius
 446 of the collection tube is 0.3 mm, the droplet size increases from 0.40 mm to 0.55 mm as Ca_{out} is
 447 increased from 1.5×10^{-4} to 0.0053, while the droplet size is reduced to 0.38 mm when Ca_{out} is
 448 further increased to 0.015. At a given Weber number, the droplet size first increases below a

449 certain critical Capillary number due to the reduced viscosity, and then decreases above the
450 critical Capillary number, when the viscosity starts to increase again. The correlation between
451 the droplet size and the Capillary number in the Newtonian/shear-thinning two-phase system is
452 different from that in the Newtonian/Newtonian two-phase system, which normally shows that
453 droplet size scales inversely with the Capillary number of the continuous phase in a monotonous
454 fashion.³⁸The different correlation highlights the complex viscosity effect towards the control
455 over emulsion droplets generated with fluid-fluid systems involving shear-thinning non-
456 Newtonian fluids. This understanding can provide information needed for designing
457 microdevices for generating droplets with well-defined volumes when shear-thinning non-
458 Newtonian multiphase systems are involved. For example, given certain flow conditions and
459 fluid properties (thus We_{in} and Ca_{out} are known), an analysis can be made to determine the size of
460 collection tube of the microdevice, according to the expected size of droplets in demand.



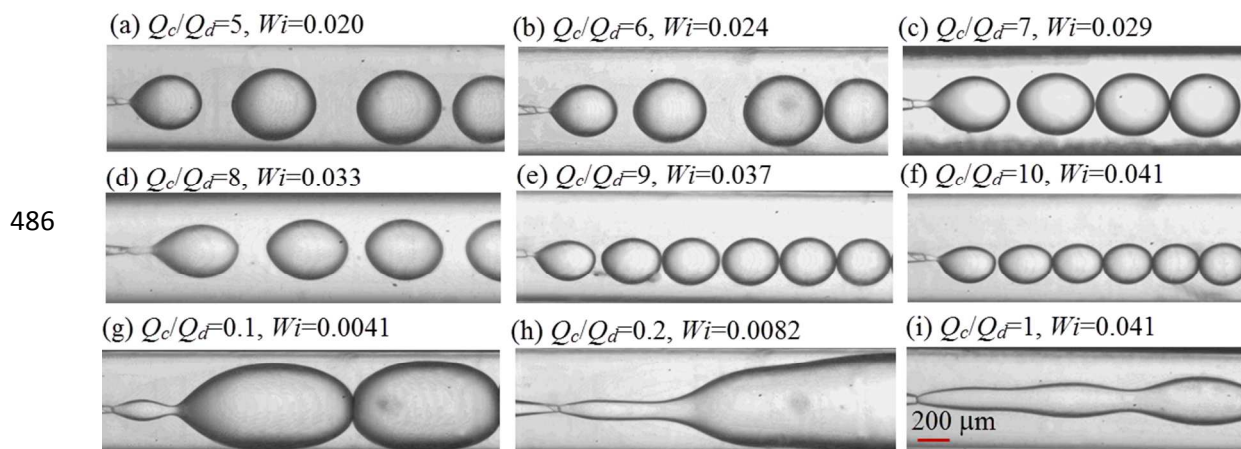
462 **Fig.12** Experimental result of droplet size as a function of the Capillary number for a given
 463 collection tube in a Newtonian/shear-thinning two-phase system (silicon oil as dispersed phase
 464 and CMC solution as continuous phase). $We_{in}=0.206$.

465 When a Newtonian liquid is surrounded by a shear-thinning liquid with pronounced elastic
 466 property, the droplet breakup dynamics gets modified. We observe flow patterns at different
 467 ratios of flow rate of 5% w/v PAA solution, which is the viscoelastic continuous phase, to that of
 468 silicon oil as dispersed phase, as shown in Fig.13. The flow rate of dispersed phase is fixed at 1
 469 ml/h in Fig.13a-f, and 10 ml/h in Fig.13g-i. The elastic forces from viscoelastic continuous phase
 470 can help to overcome interfacial tension, and thus facilitate transition to jetting at smaller

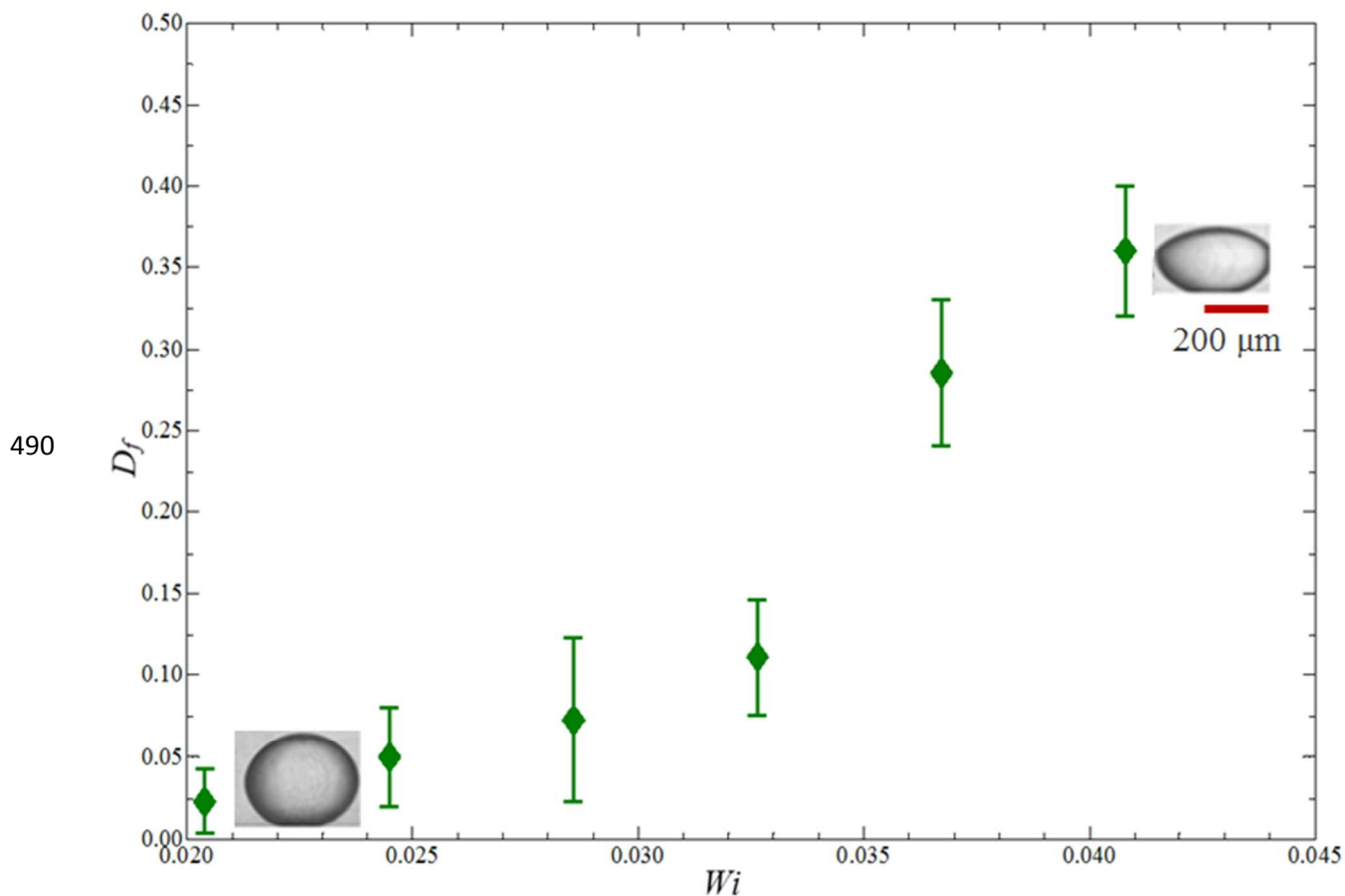
471 magnitudes of the viscous forces. Compared to the aforementioned Newtonian/Newtonian or
 472 Newtonian/shear-thinning multiphase microsystem where droplets normally adopt a spherical or
 473 nearly spherical shape, the droplets experience significant deformation in the viscoelastic non-
 474 Newtonian continuous phase. For instance, the droplets adopt an elliptical shape after breakup
 475 and relax into pointed shapes. Droplets become more pointed as the radius decreases and the
 476 flow rate ratio increases before it transitions to the jetting regime (see Fig.13a-f). The degree of
 477 droplet deformation has been characterized based on Taylor's analysis,³⁹

$$478 \quad D_f = \frac{L-W}{L+W} \quad (14)$$

479 where L is the half-length and W the half-breadth of the droplet. D_f increases with increasing
 480 Weissenberg number (thus more pronounced elastic effect), leading to formation of droplets with
 481 more pointed shape, as shown in Fig.14. This observation is consistent with the previous finding
 482 that elasticity of the suspending liquid can facilitate the deformation of the Newtonian
 483 droplets.⁴⁰ Our work provides a platform which will help to facilitate understanding of the
 484 complex rheology behavior of viscoelastic fluid and how this behavior can affect the breakup
 485 dynamics and droplet formation in multiphase microfluidic system.



487 **Fig.13** Flow patterns at selected flow rate ratios of continuous phase over dispersed phase in a
 488 Newtonian/elastic-shear-thinning two-phase system (silicon oil as dispersed phase and PAA
 489 solution as continuous phase). $Q_d=1$ ml/h for (a)-(f), and $Q_d=10$ ml/h for (g)-(i).



491 **Fig.14** Degree of deformation of droplet as a function of the Weissenberg number in a
 492 Newtonian/elastic-shear-thinning two-phase system (silicon oil as dispersed phase and PAA
 493 solution as continuous phase). The inset images show droplet adopts nearly spherical shape at
 494 low Wi , while pointed shape at high Wi , respectively. The scale bar applies for both inset images.

495

496 5. Conclusions

497 The work reports investigation of emulsion formation using a Newtonian/shear-thinning two-
498 phase microsystem. With a shear-thinning continuous phase, the droplet dynamics is
499 characterized to predict the dripping-to-jetting transitions under different flow conditions (as
500 function of Weber number and Capillary number), where undesirable satellite droplets may be
501 formed. Inertial effects have been shown to be important for inducing the formation of a satellite
502 droplet, while the droplet formation can be suppressed by increasing viscous effect. Due to the
503 shear-thinning characteristics of the continuous phase used in the present study, the viscosity is
504 dramatically reduced at the interface; viscous effect is therefore attenuated, leading to a faster
505 breakup and a larger droplet size. Emulsions generated with non-Newtonian fluids are routinely
506 involved in drug delivery and other biochemical applications. Accurate dosing must be ensured
507 for reliable operation, and this requires excellent control over the droplet size. We also identify
508 the correlation between the droplet size and Capillary number, potentially enabling a higher
509 degree of control over the size of emulsion droplet generated with shear-thinning fluids. Finally, a
510 viscoelastic fluid is used as continuous non-Newtonian phase, and the degree of Newtonian
511 droplet deformation increases with increasing Weissenberg number of suspending non-
512 Newtonian fluid, indicating the important impact of elasticity on the droplet shape. Our work
513 provides a first framework for understanding a range of behaviors in non-Newtonian multiphase
514 microsystems, although we recognize the vast variety of non-Newtonian fluids besides the two
515 types (shear-thinning solution with or without elastic property) we have investigated and the
516 large regions of unexplored operating conditions. For example, it is challenging to generate
517 droplets with non-Newtonian fluids in aqueous two phase systems (ATPS) due to much lower
518 interfacial tension when compared with oil-water systems.⁴¹⁻⁴² Furthermore, it is also of great
519 interest to investigate the cases when non-Newtonian fluid is used as the dispersed phase, rather

520 than just the continuous phase. Improvement in the polydispersity of the resultant droplets can
521 also be achieved potentially by developing and implementing methods to dynamically control the
522 merging of satellite droplets with the main droplets. This approach of elimination of satellite
523 droplets will also lead to droplets with higher size uniformity in applications that inevitably
524 involve non-Newtonian multiphase flows.

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