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Mathematics education in East Asia from antiquity to modern times

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Abstract

Since the early 1990s the learning process of Asian students brought up in the tradition of the Confucian heritage culture (CHC) has become a much discussed issue. As a consequence the teaching process of Asian teachers in CHC classrooms has attracted the same attention. These two related issues are brought into focus in the so-called "CHC Learner / Teacher Paradox". It is therefore natural to look at the history of mathematics education in some Asian countries such as China, Japan and Korea. This paper attempts to give an account of this long episode from ancient to medieval to modern times with illustrative examples.

Introduction

It is impossible to do justice to the subject indicated by the title in such a short paper, which is the text of a talk that attempts to condense what happened throughout four thousand years into forty minutes! (The talk was given on Summer Solstice Day of 2009 in Iceland.) The author can only hope to share briefly with the readers some of his views on aspects of mathematics education in East Asia from antiquity to modern times by addressing the following questions.

(i) What were the main features of mathematics education in (ancient) China / East Asia?
(ii) What were some factors that led to such features?
(iii) What influence did such features exert upon the development of mathematics in (ancient) China / East Asia?
(iv) What lesson in mathematics education do we learn from this study?

The last question is of particular interest in view of the upsurge in the recent decade of the attention paid to the process of learning and teaching in a classroom environment dominated by the so-called Confucian heritage culture[1] (CHC) [see

[1] The nebulous term CHC is used here in a general sense to cover the cultural background of communities in mainland China, Hong Kong, Japan, Korea, Singapore, Taiwan and Vietnam. It is

(Stevenson & Stigler, 1992; Stigler & Hiebert, 1999; Watkins et al, 1996; 2001)]. These two issues are brought into focus in the form of two paradoxes, namely

(1) The CHC Learner Paradox: CHC students are perceived as using low-level, rote-based strategies in a classroom environment which should not be conducive to high achievement, yet CHC students report a preference for high-level, meaning-based learning strategies and they achieve significantly better in international assessments!

(2) The CHC Teacher Paradox: Teachers in CHC classrooms produce a positive learning outcome under substandard conditions that western educators would regard as most unpromising!

A usual explanation of these paradoxes lies in a careful differentiation between repetitive learning and rote learning. Such a differentiation is succinctly captured in the writings of the leading neo-Confucian scholar Zhu Xi (1130-1200) who said (translation taken from (Gardner, 1990)):

> "Generally speaking, in reading, we must first become intimately familiar with the text so that its words seem to come from our own mouths. We should then continue to reflect on it so that its ideas seem to come from our own minds. Only then can there be real understanding. Still, once our intimate reading of it and careful reflection on it have led to a clear understanding of it, we must continue to question. Then there might be additional progress. If we cease questioning, in the end there'll be no additional progress."
>
> "Learning is reciting. If we recite it then think it over, think it over then recite it, naturally it'll become meaningful to us. If we recite it but don't think it over, we still won't appreciate its meaning. If we think it over but don't recite it, even though we might understand it, our understanding will be precarious. (…) Should we recite it to the point of intimate familiarity, and moreover think about it in detail, naturally our mind and principle will become one and never shall we forget what we have read."

Still, it remains a fact that in CHC a strong tradition of examination prevails and that it is commonly believed that an examination-oriented culture will hinder the learning process. Is examination really that bad? Or is it a necessary evil? Or is it even beneficial to the learning process in some sense? It may be helpful to look at the issue from a historical perspective.

Traditional mathematics education in China

beyond the scope of this short paper as well as beyond the capacity of the author to elaborate on this term in depth.

In China the school system in its formal setting began during the latter part of the Xia Dynasty (c.21st century B.C. to 16th century B.C.), run by the state and intended as a training ground for youths and children of the aristocracy. The system became more institutionalized in subsequent dynasties, persisted and evolved up to the last imperial dynasty of Qing (1616-1911), with rise and decline in its strength along with events and happenings in different epochs of history. A long period spanning half a millennium (722 B.C. to 221 B.C.) beset with conflicts and unrest caused decline in state-run institutions of learning, but the decline was more than compensated for by the formation of private academies around some eminent scholars in the community. This dual system of learning, which comprised state-run institutions and private academies side by side, persisted in China for the next two millennia. Embedded within the general education system was that of mathematics. For details see (Siu, 1995; 2001; 2004; Siu & Volkov, 1999) plus its extensive bibliography. A summary is depicted in a time-line with certain important events added alongside (see Figure 1).

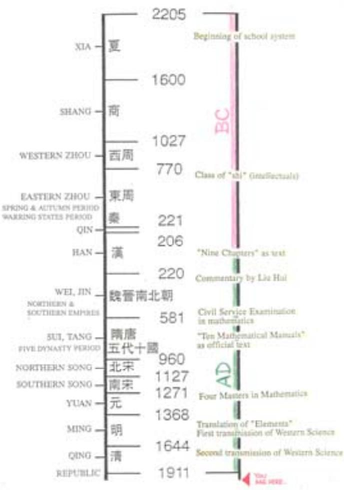

Figure 1.

To address the question on examination-oriented culture, Siu and Volkov (1999) report their study of the state examination system in the Tang Dynasty (618-907), based on the detailed accounts recorded in official chronicles including (a) *Jiu Tang Shu* (Old History of the Tang Dynasty, c.941-945), (b) *Xin Tang Shu* (New History of the Tang Dynasty, c.1044-1058), (c) *Tang Liu Dian* (Six Codes of the Tang Dynasty, 738), (d) *Tong Dian* (Complete Structure of Government, c.770-801), (e) *Tang Hui Yao* (Collection of Important Documents of the Tang Dynasty, 961). Furthermore, Siu even ventures to offer a (perhaps fictitious but with some partial evidence) "re-constructed" examination question to support his belief that candidates in ancient China did not just recite by heart mathematical texts in learning the subject at state-run institutions [see (Siu, 2001; 2004) for more details].

The “re-constructed” question is: Compute the volume of an ‘oblong pavilion’ of height h with top and bottom being rectangles of sides a_1, a_2 and b_1, b_2 respectively $(a_1 \neq a_2, b_1 \neq b_2)$ (see Figure 2).

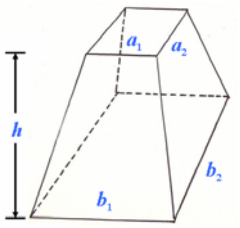

Figure 2.

A special case of the problem (when $a_1 = a_2, b_1 = b_2$) is indeed Problem 10 of Chapter 5 of the most famous ancient Chinese mathematical classics *Jiu Zhang Suan Shu* (Nine Chapters on the Mathematical Art) compiled between 100 B.C. and A.D. 100 (See Figure 3).

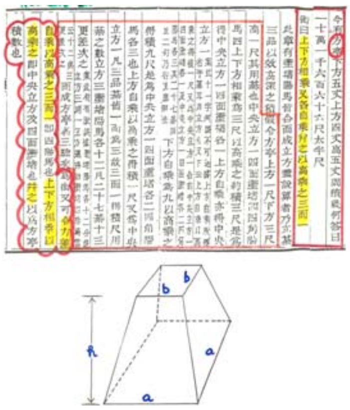

Figure 3.

It is not easy to hit upon the correct formula

$$V = \frac{1}{3}[a_1a_2 + b_1b_2 + \frac{1}{2}(a_1b_2 + a_2b_1)]h$$

of the volume of an ‘oblong pavilion’ if one merely learns the formula of the pavilion (with square top and bottom) given in *Jiu Zhang Suan Shu* by rote. Besides, the problem is of practical interest, because the candidates might well be facing in their subsequent career problems which were variations of those they learnt in the textbooks.

By examining the content and style of mathematical classics in China we discern two main features of traditional Chinese mathematics: (1) It responded to demands on solving real world problems more than to demands on explicating problems created within the theory itself. (2) It was more inductive than deductive in nature. We can perhaps trace the root of these features to a basic tenet of traditional Chinese philosophy of life shared by the class of *shi*[2] (intellectuals), namely, self-improvement and social interaction, leading to an aspiration for public service and inclination to pragmatism. Such an attitude is basically good and positive, but it also invites the possibility of exerting a negative influence on the study of certain disciplines, in this case mathematics. Mathematics is regarded primarily as a tool in dealing with practical matters, and that the worth of mathematics can only be so justified. As a result, mathematics did not play a role in traditional Chinese culture and thought as the role it played in western culture, for instance, as described in (Grabiner, 1988). These same features were reflected in mathematics education in China.

Let us illustrate with one example. In the famous mathematical treatise *Shu Shu Jiu Zhang* (Mathematical Treatise in Nine Sections) written by QIN Jiushao (c.1202-1261) in 1247, there appeared a problem (Problem 5 of Chapter 8) that says, "A circular castle has four gates to each direction. A tall tree stands 3 miles to the north. If one goes out by the South Gate and walks towards the east for 9 miles, one shall just see that tree. What is the circumference and diameter of the castle?" (See Figure 4)

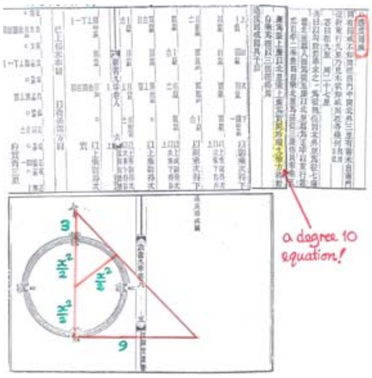

Figure 4.

Phrased in modern notation, what Qin did is to put the diameter as x^2 and set up an equation of tenth degree, namely,

$$x^{10} + 15x^8 + 72x^6 - 864x^4 - 11664x^2 - 34992 = 0.$$

He then solved the equation and obtained $x = 3$ as an answer, so the diameter is 9 miles, which is correct. One may query, "Why x^2; why not simply y?" Indeed,

[2] The class of *shi* is a rather peculiar but extremely important social class throughout the whole cultural history of China. It is sometimes rendered in translation as 'literati' or 'scholar' or 'scholar-official' or 'intellectual', but none of these terms individually can capture a holistic meaning of the word.

Qin's contemporary, LI Ye (1192-1279), posed a similar problem (Problem 4 of Chapter 4) in his *Ce Yuan Hai Jing* (Sea Mirror of Circle Measurement) of 1248, and answered it by solving an equation of third degree, namely,

$$y^3 + ky^2 - 4k\ell^2 = 0,$$

in which y stands for the diameter, k stands for the distance of the tree from the North Gate and ℓ stands for the distance one walks to the east. (The technical details will provide a good exercise in school mathematics.) In the late 18th century another mathematician, LI Rui (1768-1817), even reprimanded Qin for missing the point in solving the problem in an unnecessarily harder way. (Indeed, it is much more straight-forward to arrive at the third degree equation than the tenth-degree equation! See (Guo, 1982) for the detail.) It seems that LI Rui missed the point himself, as it seems unlikely that a mathematician of the caliber of Qin would miss noticing the easier equation of the third degree. Why then did Qin solve the problem in a harder way? He might have done it on purpose with a purely pedagogical motive. He wanted to offer an example to illustrate his method of solving an equation of high degree, but in the good old Chinese tradition, a problem should not be discussed in a purely theoretical context but should arise in a practical context, or else it would not be accorded its deserved value and attention.

Solving an equation of high degree was a high-point in medieval Chinese mathematics, which accomplished what western mathematicians re-discovered (independently) six centuries later. But ironically, this high-point was also the beginning of its standstill! One reason is that there was no need at the time to solve an equation of such high degree in practice. The pragmatic viewpoint would not encourage nor induce mathematicians to think about questions such as the existence of a root of an equation or solvability by radical. When the technical capability far exceeded the demand imposed by practical matters, motivation otherwise arising from an inner curiosity did not arise, leading to a standstill.

"Westernization" of mathematics education in East Asian countries

In 1607 there appeared the first[3] Chinese translation of a European treatise in mathematics, namely, the fifteen-volume compilation in Latin of Euclid's *Elements* by Christopher Clavius (1537-1612) in the late 16th century. The translation was a landmark collaboration between the Ming Dynasty scholar-minister XU Guangqi (1562-1633) and the Italian Jesuit Matteo Ricci (1552-1610). The remaining nine books were translated (from a compilation other than that of Clavius) in 1857 under the collaboration between the Qing Dynasty mathematician LI Shanlan (1811-1882) and the English missionary Alexander Wylie (1815-1887). However,

[3] Some historians believe that a translated version of Euclid's *Elements* was in existence in China in the 13th century during the Yuan Dynasty (1279-1368). It was translated from a fifteen-volume compilation in Arabic.

this famous mathematical treatise of significance in the western world did not seem to exert an influence of equal magnitude in China at the time. In (Siu, 2007) an account on this aspect is given on the occasion of the 400 anniversary of the translation of *Elements* in China.

Although the transmission of Euclid's *Elements* did not seem to exert as much influence in China as in the western world, unexpectedly its influence blossomed in a more politically oriented arena. Study of western science in general, and western mathematics in particular, attracted the attention of some active liberal intellectuals of the time, among whom three prominent figures KANG Youwei (1858-1927), LIANG Qichao (1873-1929) and TAN Sitong (1865-1898) played an important role in the history of modern China as leading participants in the episode of "Hundred-day Reform" of 1898. Unfortunately the episode ended tragically, with Kang and Liang fleeing to Japan and Tan being executed for trying to overthrow the regime of the Qing Dynasty. As far as mathematics education is concerned it is of interest to note that TAN Sitong founded a private academy, the Liuyang College of Mathematics, in his hometown in 1897. In a document about the establishment he clearly stated (what nowadays would be labeled as "vision and mission") how a mathematical training benefits one's upbringing.

The regime of the Qing Dynasty also paid much attention to the study of western science and mathematics, but for another reason, namely, to "learn form the westerners in order to resist their invasion". Activities towards this goal are called the "Self-strengthening Movement" in the history of modern China [see (Fairbank & Reischauer, 1973; Hsu, 1970/1995; Swetz, 1974)]. Headed by Prince Gong (Yixin) (1833-1898) and supported strongly by some officials of high rank including ZENG Guofan (1811-1872), LI Hongzhang (1823-1901) and ZHANG Zhidong (1837-1909), schools were established to learn western science and mathematics, and offices were set up to translate western texts in science and mathematics. In particular, an establishment known by the name of *Tong Wen Guan* (College of Foreign Languages) was set up in 1862 by decree, with the section on mathematics and astronomy established in 1866. (This same college was extended to form the Peking Imperial University, which was later renamed as Beijing University, in 1902.) The American missionary of the Presbyterian Church, William Alexander Parsons Martin (1827-1916), was appointed the President of *Tong Wen Guan*. LI Shanlan was appointed the head of the mathematics and astronomy section. (By the way, the first complete translation of *Elements* by Li and Wylie was destroyed soon after its publication during the tumultuous period that saw the inner strife of the Taiping Rebellion and the foreign invasion of the Anglo-Franco expeditionary force. It was to the credit of ZENG Guofan, a patron of Li, that the translation got republished in 1895.) Many more translated texts in western science and mathematics resulted from the industrious collaboration of the famed pairs — LI Shanlan and Alexander Wylie, HUA Hengfang (1833-1902) and John Fryer (1839-1928).

Another American missionary, Calvin Wilson Mateer (1836-1908), founded in 1877 the *School and Textbooks Series Committee* (with a Chinese title that means literally "Book Club of Benefit to Wisdom"), which became the *Educational Association of China* in 1890 and finally the *Zhonghua* (*China*) *Association for Education* in 1905. Mateer brought in many school textbooks, some through translation and some through compilation (by himself or other missionaries), in all the basic subjects: *Bi Suan Shu Xue* (Arithmetic) in 1892, *Xing Xue Bei Zhi* (Complete Meaning of the Science of Figures) in 1884, *Dai Shu Bei Zhi* (Complete Meaning of Algebra) in 1891, *Ba Xian Bei Zhi* (Complete Meaning of Trigonometry) in 1894 and *Dai Xing He Can* (Combined Study on Algebra and Geometry) in 1893 [see (Chen & Zhang, 2008; Li, 2005; Tian, 2005)].

The "westernization" of mathematics education went on in other Asian countries besides China in the 19th century. A brief look at what happened in Korea and Japan will convey a general idea of the scenario.

Since very ancient time Korea was basically divided into three kingdoms, Koguryo (37 B.C. to 668), Paekche (18 B.C. to 660) and Silla (57 B.C. to 935), evolving into Koryo (918-1392) succeeded by Choson (1392-1910) [see (Lee, 1961/1984)]. From fairly early time these kingdoms had been under Chinese influence along with its culture and learning. During the last decade of the 16th century the Japanese warlord TOYOTOMI Hideyoshi (1536-1598) invaded Korea, with the real objective of invading the Ming Empire of China. As a byproduct (probably unintended) numerous books in Korea were seized and brought back to Japan, among them Chinese mathematical texts. Two texts became particularly prominent and exerted significant influence in the formation of the subject of *Wasan* in the Edo period (1603-1868): *Suan Xue Qi Meng* (Introduction to the Computational Science) by ZHU Shijie (c.1260-c.1320) of 1299, *Suan Fa Tong Zhong* (Systematic Treatise on Calculating Methods) by CHENG Dawei (1533-1606) of 1592. *Wasan* is the name given to traditional Japanese mathematics which is an elaborate development based on Chinese tradition. [See (Fukagawa & Rothman, 2008). See also (Hirabayashi, 2006) for an interesting comparison between *Wasan* and traditional Chinese mathematics.]

Let us illustrate with one problem in the famous Japanese mathematical treatise *Jinkoki* composed by YOSHIDA Mitsuyoshi (1598-1672) in 1627. The problem reads: "Some thieves stole a long roll of silk cloth from a warehouse. In a bush far from the warehouse, they counted the length of the cloth. If each thief gets 6 *hiki*, then 6 *hiki* is left over. But if each thief takes 7 *hiki* then the last thief gets no cloth at all. Find the number of thieves and the length of the cloth." This problem appears (with exactly the same numerical data) as Problem 28 of Part II of the Chinese classics *Sun Zi Suan Jing* (Master Sun's Mathematical Manual) of the 4th century. Similar problems also appear in Chapter 16 of *Suan Fa Tong Zhong* mentioned above.

Along with mathematical treatises the school system and examination system in China were also adopted in Korea and Japan. In 718 the *Yoro rei* (Decree in the

Reign of Yoro) described in detail the state education system, including the structure of schools, the curriculum and the state examination system, which closely resembled that of the Tang Dynasty (618-907) in China. Like the private academies of learning in China, Japan was also noted for its *juku* during the Edo period when *Wasan* flourished. Many samurais moonlighted as teachers in these *jukus*, about 80,000 existing throughout the country in the late Edo period. One *juku*, the *Yōken Juku* in Tamura City, has its building still preserved to this date. It was run by the Japanese mathematician YŌKEN Sakuma (1819-1896). Its roster indicated that 2144 students attended it over a span of fifty years (Fukagawa & Rothman, 2008).

Western mathematics took its root in Japan for a more or less similar reason as it was in China. In July of 1853, when Commodore Matthew Calbraith Perry (1794-1858) led an American fleet to reach Japan and anchored in Edo Bay (now Bay of Tokyo), the closed door of the country was forced open under military threat. Besides ending the seclusion of Japan this incident also led to the establishment of the Nagasaki Naval Academy and the *Bansho Shirabe-sho* (literally meaning "Office for the Investigation of Barbarian Books"), both of which were important for instituting systematic study of western science and mathematics in Japan. With the Meiji Restoration western learning in Japan was no longer confined to military science for self-defence but was regarded as an integral means for modernization of the country (Sasaki, 1994). Foreigners were brought into Japan to teach western science and mathematics, among them was another famous Perry —— John Perry (1850-1920) —— who was well-known for the reform of school curriculum in mathematics he promoted through an influential address delivered in Glasgow before the *British Association for the Advancement of Science* in 1901. During 1875 to 1879 Perry was a professor of civil and mechanical engineering in the *Kobu Daigakko* (Imperial College of Engineering) in Tokyo, which became in 1886 College of Engineering of University of Tokyo (Kota, 2001).

However, the route to "westernization" of mathematics education in Japan took a much faster and more drastic turn. The *Gakurei* (Fundamental Code of Education) of Japan in 1872 decreed that *Wasan* was not to be taught at school; only western mathematics was taught. [See (Hirabayashi, 2006; Sasaki, 1994; Ueno, 2006).] As pointed out in (Siu, 1995/96), "It will be a meaningful task to try to trace the "mental struggle" of China in the long process of learning Western science, from the endeavour of XU Guangqi, to the resistance best portrayed by the vehement opposition of YANG Guangxin, to the promulgation of the theory that "Western science had roots in ancient China", to the self-strengthening movement, and finally to the "naturalization" of Western science in China. It is a complicated story embedded in a complicated cultural-socio-political context". It is the author's plan of continued research (in collaboration with CHAN Yip Cheung of the Hong Kong Institute of Education) to study:

(1) the role in mathematics education played by the private *shu yuan* (academy of classical learning) that lay outside the state education system in China, particularly in the Qing Dynasty during the 19th century,

(2) mathematics education in China-Korea-Japan in the 16th to 19th centuries, in the context of *shi xue* (concrete and useful learning).

Dialectic and algorithmic mathematics

To some extent the term "modernization" becomes synonymous with "westernization". Historical happenings brought about a dominance of western civilization since the 17th century. In the long river of history, a few centuries form but a fraction of the long span of time. Indeed, as pointed out by Oswald Spengler (1880-1936), who carried out a study of comparative cultures in his *Der Untergang des Abendlandes* (The Decline of the West) (Spengler, 1918/1922), the histories of various cultures resemble the regular course of birth, growth, maturity and decay of a living organism, or metaphorically analogous to the change of seasons. Within each culture, certain basic attitudes, which are exemplified in different expression-forms, give the key or clue to the history of the whole culture. In particular, mathematics is one such expression-form, expounded in Chapter 2 of Volume I of (Spengler, 1918/1922).

Thus, it would be instructive to look at different mathematical cultures and learn from each other. By studying the history of mathematics education in East Asia and the western world we can compare two styles in doing mathematics, which this author, borrowing the terms from (Henrici, 1974), labels as "dialectic" and "algorithmic". Broadly speaking, dialectic mathematics is a rigorously logical science, in which statements are either true or false, and in which "objects with specified properties either do or do not exist". On the other hand, algorithmic mathematics is a tool for solving problems, in which "we are concerned not only with the existence of a mathematical object but also with the credentials of its existence" (Henrici, 1974). They complement each other in that procedural approach helps us to prepare more solid ground on which to build up conceptual understanding, and conversely, better conceptual understanding enables us to handle the algorithm with more facility. Indeed, several main issues in mathematics education are, in some sense, rooted in an understanding of these two complementary aspects —— "dialectic mathematics" and "algorithmic mathematics". These include:

(1) procedural versus conceptual knowledge,

(2) process versus object in learning theory,

(3) computer versus no-computer in learning environment,

(4) "symbolic" versus "geometric" emphasis in learning/teaching,

(5) "Eastern" versus "Western" learners/teachers

[In (Siu, 2009) one can find a more detailed exposition on this theme with illustrative examples. In a seminal paper (Sfard, 1991), Sfard explicates this duality and develops it into a deeper model of concept formation through an interplay of the "operational" and "structural" phases.]

To conclude, the author will reiterate a passage from his paper (Siu, 2008):
"A broader message that I would like to convey is that mathematics constitutes a part of human endeavour rather than standing on its own as a technical subject, as it is commonly taught in the classroom. (…) It may not yield specific tactics or a comprehensive theory. But it serves to remind us that to make the subject more "humanistic" so that students feel that it makes good sense to spend time on it, mathematics is best studied along with its influence to and from other human endeavours."

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