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Robust Dispatch with Power Flow Routing and Renewables

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***Abstract*—The uncertainty and variability of renewable energy sources are challenging problems in the operation of power systems. In this paper, we focus on the robust dispatch of generators to overcome the uncertainty of renewable power predictions. The energy management costs of supply, spinning reserve, and power losses, are jointly optimized by the proposed robust optimal power flow (OPF) method with a column-and-constraint generation algorithm. Conic relaxation is applied to the non-convex alternating-current power flow regions with the phase angle constraints for loops retained by linear approximation. The proposed method allows us to solve the robust OPF problem efficiently with good accuracy and to incorporate power flow controllers and routers into the OPF framework. Numerical results on the IEEE Reliability Test System show the efficacy of our robust dispatch strategy in guaranteeing immunity against uncertain renewable generation, as well as in reducing the energy management costs through power flow routing.**

I. Introduction

The intermittent nature of renewable generation poses challenges to the reliability and stability of power systems. Such energy management problems for a power system integrated with renewable energy sources (RESs) have received much attention in recent years [1]–[6]. Existing efforts aim to optimize certain energy-related costs, e.g., costs of generation and reserve, subject to a set of operation constraints. In this paper, we focus on the optimal allocation of supply and spinning reserve of conventional generators given the unit commitment (UC) schedules. Furthermore, we study the effect of power flow routing, an emerging paradigm for the dynamic and responsive control of power flows [7]–[9], on energy management under the uncertainty of RESs.

Stochastic optimization (SO) and robust optimization (RO) are the major mathematical tools to optimize and quantify the impact of uncertainty. The SO approaches have been applied to solve the security-constrained UC (SCUC) [4], [6] and the optimal power flow (OPF) [3] problems with renewables. While SO is a classical method to manage uncertainty, its performance is sensitive to the accuracy of the probability distributions and the selection of representative scenarios of the uncertainty parameters, namely, renewable generation in our context. As a complementary technique of SO, RO deals with the situation when reliable distributions of uncertainty parameters are difficult to obtain. Hence, due to the intermittency of renewables, RO is considered more suitable than SO for handling forecast errors so as to guarantee sufficient energy and ramping capability in SCUC [2], and to determine the energy and reserve requirement in dispatch problems [1], [5].

Power flow routing is enabled by power flow controllers (PFCs), e.g., flexible alternating-current transmission system (FACTS) devices [3], and power flow routers (PFRs) [8]. Due to the development of power electronic over the past two decades, power flow routing is becoming mature and cost-effective for improving asset utilization [7], [9]. Yet, in the literature, while several SO-based proposals have employed FACTS devices [3], we find no RO-based research that studies the potential of power flow routing to facilitate the integration of RESs. This is because all of the current RO methods on energy management adopts linear approximation of power flows, which is usually referred to as direct-current (DC) power flow model, in order to make their RO problems tractable [1], [2], [5]. However, the DC power flow model limits the application of PFCs and PFRs since it does not consider voltage control and reactive power management which are among the key functions of PFCs and PFRs [7]–[9].

Another limitation of the DC power flow model is that the linear approximation is considered acceptable only for the alternating-current (AC) transmission system with small line resistance-to-reactance ratio [10]. Nonetheless, due to the integration of distributed energy resources, e.g., distributed generators and energy storage, dispatching and scheduling of the controllable resources are required at the distribution level, such as microgrids, where the linear model no longer applies.

Although the AC power flow model seems to resolve the issues discussed above, the computational complexity of the resultant RO problem with AC power flows is prohibitive, and therefore convex relaxation on the non-convex flow regions is necessary. Nonetheless, we find only one work [11] that successfully relaxes and solves such a robust AC OPF problem. While [11] only considers minimizing line losses in the radial distribution network, we aim at determining the reserve requirement in a generic meshed network where the relaxation method in [11] fails to guarantee exact results.

This paper fills the research gaps of robust energy management by a robust AC OPF method. Our contributions are summarized as follows:

- To manage the uncertainty of RESs, we are the first to formulate a new robust dispatch problem which introduces power flow routing to the RO-based research in power systems.

- The proposed AC OPF framework is applicable to both meshed and radial networks at different power levels. Inspired by [12], the phase angle constraints for loops are retained by linear approximation with good accuracy.
- We account for the interaction between spinning reserve and line losses on the exactness of convex relaxation. The cost coefficients of line losses for the exact relaxation of the original non-deterministic polynomial-time hard (NP-hard) robust OPF problem is determined.

II. System Model

Consider a power network with N buses modelled as a connected and undirected graph, denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ with the bus set $\mathcal{N} := \{1, 2, \ldots, N\}$ and the set of transmission lines or branches $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$. A branch $(i, k) \in \mathcal{E}$ with its two terminal buses $i, k \in \mathcal{N}$ is modelled by the equivalent π circuit with line admittance $y_{ik} = g_{ik} + \mathrm{j}b_{ik}$, where $g_{ik} \geq 0$ and $b_{ik} \leq 0$ denote the conductance and susceptance of branch (i, k), respectively, and the shunt capacitance $c_{ik} = c_{ki} \geq 0$ as illustrated in Fig. 1. Denote the set of one-hop neighbours of each bus $i \in \mathcal{N}$ as $\Omega_i \subseteq \mathcal{N}$.

To avoid ambiguity, we use PFR to refer to a device that is able to manage multiple power flows, and PFC to one that can only adjust the power flow through one branch or appliance. In our previous work [8], a generic functional model of a PFR that represents the conventional bus has been proposed. In this paper, we use a PFR model [8] shown in Fig. 2, where the local power injections are classified into the powers of conventional generators, RESs, and critical loads. Within the PFR, the line PFC connects the external transmission line to the common bus, which is analogous to the conventional bus, and controls the corresponding branch power flow.

Denote the PFR installed on Bus $i \in \mathcal{N}$ as PFR i. The common bus of PFR i is characterized by the voltage $V_i \in \mathbb{C}$ with the operation range as:

$$U_{i,min} \leq |V_i| \leq U_{i,max} \tag{1}$$

where the operator $|\cdot|$ takes the magnitude.

As illustrated in Fig. 1, for each one-hop connected Bus/PFR $k \in \Omega_i$ of PFR i, denote the output voltage of PFR i for Branch (i, k) as $V_{i_k} \in \mathbb{C}$. According to [8], the operation range of the "branch terminal voltage" $V_{i_k} \in \mathbb{C}$ is:

$$|V_{i_k} - V_i| \leq \gamma_{i_k,max} |V_i| \tag{2}$$

From (2), the line PFC is modelled as a series voltage injection [8], [9], and $\gamma_{i_k,max} \in [0, 1]$ characterizes its capability of voltage control. In addition, the line PFC, as a power electronic device, of PFR i in Branch (i, k) may possess certain extra capability of reactive power compensation which is modelled by the injection $Q_{Ci_k} \in \mathbb{R}$ with the operation range as:

$$Q_{Ci_k,min} \leq Q_{Ci_k} \leq Q_{Ci_k,max} \tag{3}$$

Denote the complex power flow from PFR i to Bus/PFR $k \in \Omega_i$ as $S_{ik} = P_{ik} + \mathrm{j}Q_{ik}$ which satisfies Ohm's law as:

$$S_{ik} = V_{i_k}(V_{i_k} - V_{k_i})^* y_{ik}^* \tag{4}$$

where * denotes the conjugate operator of a complex number.

Let $S_i = P_i + \mathrm{j}Q_i$ denote the complex power of the aggregate local power injection of PFR i. If the conversion losses of line PFCs are neglected, the power balance equation for PFR i is formulated as:

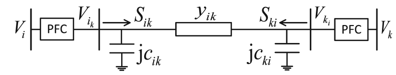

Fig. 1. Notations for a branch (i, k).

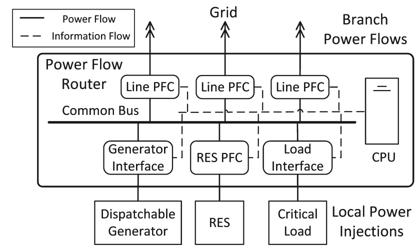

Fig. 2. Schematic of a power flow router that represents a conventional bus.

$$S_i = \sum_{k \in \Omega_i} \left(S_{ik} - \mathrm{j} |V_{i_k}|^2 c_{ik} + \mathrm{j}Q_{Ci_k} \right) \tag{5}$$

III. Robust Optimal Power Flow

Denote the set of buses that are installed with renewable generators as $\mathcal{N}_W \subseteq \mathcal{N}$. Let $S_{Wi} := P_{Wi} + \mathrm{j}Q_{Wi}$ denote the complex power output of the renewable generator at Bus $i \in \mathcal{N}_W$. We focus on the impact of the uncertain real power output of RESs, and assume that Q_{Wi} is kept proportional to $P_{Wi} \geq 0$ with a fixed ratio η_{Wi} for every renewable generator $i \in \mathcal{N}_W$ by its own reactive support [13], which means:

$$Q_{Wi} = \eta_{Wi} P_{Wi}, \forall i \in \mathcal{N}_W \tag{6}$$

As a common practice of the RO-based research [2], [5], [11], we model the uncertainty of renewable generation by a polyhedral uncertainty set $\mathcal{P}_W$ as follows:

$$\begin{aligned} \mathcal{P}_W := \Bigg\{ & P_{Wi} = P_{Wi,f} + P_{Wi,e}, \forall i \in \mathcal{N}_W \Bigg| \\ & \sum_{i \in \mathcal{N}_W} \left| \frac{P_{Wi,e}}{P_{Wi,emin}} \right| \leq \Gamma_W, \sum_{i \in \mathcal{N}_W} \left| \frac{P_{Wi,e}}{P_{Wi,emax}} \right| \leq \Gamma_W, \\ & P_{Wi,e} \in [P_{Wi,emin}, P_{Wi,emax}], \forall i \in \mathcal{N}_W \Bigg\} \end{aligned} \tag{7}$$

From (7), $P_{Wi,f} \geq 0$ and $P_{Wi,e} \in \mathbb{R}$ are the forecasted real power output and prediction error, respectively, of the renewable generator at Bus $i \in \mathcal{N}_W$. The uncertainty of P_{W_i} lies in $P_{Wi,e}$ which is bounded between $P_{Wi,emin} \leq 0$ and $P_{Wi,emax} \geq 0$. The two inequalities upper-bounded by $\Gamma_W \geq 0$ represent the spatial correlation of the renewable generators and define the polyhedral property of $\mathcal{P}_W$. The parameter Γ_W is called the "budget of uncertainty" [2], [5], [11]. The larger the value of Γ_W, the higher the system reliability achieved, and the more energy management costs incurred.

Denote the vector of real power outputs of renewables as $\mathbf{P_W} := [P_{Wi}]_{i \in \mathcal{N}_W}$. Before the uncertain renewable generation is realized, the system operator will "pre-dispatch" the conventional generators based on the uncertainty set $\mathcal{P}_W$, so that for any realization of $\mathbf{P_W} \in \mathcal{P}_W$, the operator can find a "re-dispatch" decision to offset the prediction errors of $\mathbf{P_W}$.

Define the set of buses with conventional generators as $\mathcal{N}_G \subseteq \mathcal{N}$. The set of the first-stage pre-dispatch decision variables $\mathbf{X}$ includes the real power output $P_{Gi} \geq 0$, up-spinning reserve $R_{Ui} \geq 0$, and down-spinning reserve $R_{Di} \geq 0$ of every conventional generator $i \in \mathcal{N}_G$, and is defined as:

$$\mathbf{X} := \{(P_{Gi}, R_{Ui}, R_{Di}) \mid i \in \mathcal{N}_G\} \tag{8}$$

Define the sets of the second-stage re-dispatch decision variables $\mathbf{Z}$ as:

$$\begin{aligned}\mathbf{Z} := &\{(P_{Ui}, P_{Di}, Q_{Gi}) \mid i \in \mathcal{N}_G\} \cup \{V_i \mid i \in \mathcal{N}\}\\ &\cup \{(V_{i_k}, V_{k_i}, Q_{Ci_k}, Q_{Ck_i}) \mid (i,k) \in \mathcal{E}\}\end{aligned} \tag{9}$$

where $P_{Ui}, P_{Di} \geq 0$ denote the up-regulation and down-regulation powers, respectively, and $Q_{Gi} \in \mathbb{R}$ denotes the reactive power output of the conventional generator $i \in \mathcal{N}_G$.

We propose a two-stage robust dispatch problem in an AC-OPF formulation incorporating power flow routing and the uncertainty of RESs as follows:

$$\min_{\mathbf{X}} \sum_{i \in \mathcal{N}_G} \left(C_{Gi}\left(P_{Gi}\right) + c_{Ui} R_{Ui} + c_{Di} R_{Di}\right) + C_{ll}\left(P_{ll}\right) \tag{10a}$$

subject to

$$P_{Gi} + R_{Ui} \leq \min\left\{P_{Gi,max}, P_{Gi,c} + r_{Ui}\Delta t\right\}, \forall i \in \mathcal{N}_G \tag{10b}$$

$$P_{Gi} - R_{Di} \geq \max\left\{P_{Gi,min}, P_{Gi,c} - r_{Di}\Delta t\right\}, \forall i \in \mathcal{N}_G \tag{10c}$$

$$0 \leq R_{Ui} \leq r_{Ui}\Delta t, \forall i \in \mathcal{N}_G \tag{10d}$$

$$0 \leq R_{Di} \leq r_{Di}\Delta t, \forall i \in \mathcal{N}_G \tag{10e}$$

$\forall \mathbf{P_W} \in \mathcal{P}_\mathcal{W}$, we can find a re-dispatch decision $\mathbf{Z}$ such that

$$P_i = P_{Gi} + P_{Ui} - P_{Di} + P_{Wi} - P_{Li}, \forall i \in \mathcal{N} \tag{10f}$$

$$Q_i = Q_{Gi} + \eta_{Wi} P_{Wi} - Q_{Li}, \forall i \in \mathcal{N} \tag{10g}$$

$$\begin{cases} P_{Ui} = P_{Di} = 0, \forall i \in \mathcal{N}_G, & \text{if } P_{Wi,e} = 0, \forall i \in \mathcal{N}_W \\ 0 \leq P_{Ui} \leq R_{Ui}, \text{and} & \text{otherwise} \\ 0 \leq P_{Di} \leq R_{Di}, \forall i \in \mathcal{N}_G, & \end{cases} \tag{10h}$$

$$\left|S_{ik} - \mathrm{j}c_{ik}\left|V_{i_k}\right|^2\right| \leq S_{ij,max}, \forall i \in \mathcal{N}, k \in \Omega_i \tag{10i}$$

$$\text{(1), (5)}, \forall i \in \mathcal{N} \tag{10j}$$

$$\text{(2), (3), (4)}, \forall i \in \mathcal{N}, k \in \Omega_i \tag{10k}$$

From (10a), $C_{Gi}(P_{Gi})$ is the non-negative generation cost function of the conventional generator, Generator $i \in \mathcal{N}_G$. $c_{Ui}, c_{Di} \geq 0$ are the cost coefficients of up- and down-spinning reserve, respectively, of Generator i. In addition, the non-negative cost function of real power losses, $C_{ll}(P_{ll})$, is introduced to the objective function to account for the effect of the total line loss P_{ll} on the power balance and thus on the requirement of spinning reserve, where P_{ll} is defined as:

$$P_{ll} := \sum_{(i,k) \in \mathcal{E}} \left(P_{ik} + P_{ki}\right) \tag{11}$$

Since P_{ll} is obtained from the re-dispatch decision $\mathbf{Z}$ which corresponds to the uncertain parameter $\mathbf{P_W} \in \mathcal{P}_W$ with infinitely many possible realizations, the implementation of $C_{ll}(P_{ll})$ depends on the solution method applied.

From (10b)–(10e), $P_{Gi,c} \geq 0$ is the current real power output, and $r_{Ui}, r_{Di} \geq 0$ are the maximum up and down ramping limits of Generator $i \in \mathcal{N}_G$. $\Delta t > 0$ is the duration of the dispatch interval. From (10f) and (10g), $P_{Li} \geq 0$ and $Q_{Li} \in \mathbb{R}$ denote the active and reactive power demand, respectively, of the critical load at Bus $i \in \mathcal{N}$. From (10i), $S_{ij,max} \geq 0$ is the flow limit of Branch $(i,k) \in \mathcal{E}$. The constraint in (10h) indicates that the generator outputs of the pre-dispatch decision should meet the power balance without re-dispatching the generators when the output of each RES is equal to its forecasted value. The PFR model with AC power flows is incorporated into the problem by (10j) and (10k).

IV. Solution Approach

A. Second-Order Cone Programming Relaxation

We apply second-order cone programming (SOCP) relaxation to the non-convex AC power flow regions constrained by (4) and (5) to make the problem in (10) tractable. Introduce the auxiliary variables as:

$$M_i := |V_i|^2, \forall i \in \mathcal{N} \tag{12}$$

$$M_{i_k} := |V_{i_k}|^2, \forall i \in \mathcal{N}, k \in \Omega_i \tag{13}$$

$$W_{i_k} := V_{i_k} V_{k_i}^*, \forall i \in \mathcal{N}, k \in \Omega_i \tag{14}$$

Then, the set of the re-dispatch decision variables becomes:

$$\begin{aligned}\widehat{\mathbf{Z}} := &\{(P_{Ui}, P_{Di}, Q_{Gi}) \mid i \in \mathcal{N}_G\} \cup \{M_i \mid i \in \mathcal{N}\}\\ \cup &\{(M_{i_k}, M_{k_i}, W_{i_k}, W_{k_i}, Q_{Ci_k}, Q_{Ck_i}) \mid (i,k) \in \mathcal{E}\}\end{aligned} \tag{15}$$

We apply the conic relaxation to (13) and (14) as follows:

$$M_{i_k} M_{k_i} \geq W_{i_k} W_{k_i}, \forall (i,k) \in \mathcal{E} \tag{16}$$

Derived from (4), the angle difference θ_{ik} between the phase angles of the terminal voltages V_{i_k} and V_{k_i} is given as:

$$\left|V_{i_k} V_{k_i}^*\right| \sin\theta_{ik} = \frac{-b_{ik} P_{ik} - g_{ik} Q_{ik}}{g_{ik}^2 + b_{ik}^2}, \forall (i,k) \in \mathcal{E} \tag{17}$$

According to Kirchhoff's voltage law (KVL), the angle differences around a loop sum to zero. This non-convex angle constraint can be relaxed by installing a phase shifting device in every independent loop of the network [8]. Alternatively, in this work, the approximation method proposed in [12] is adopted to preserve the angle constraints without requiring the use of phase shifting devices. We use $l \in \mathbb{N}^+$ to label a closed path where no phase shifting device is available. Loop l with $n_l \geq 2$ buses is described as Buses $i_0 \to i_2 \to \cdots \to i_{n_l - 1} \to i_0$. According to KVL, we have:

$$\sum_{m=1}^{n_l} \theta_{i_{m-1} i_{m \bmod n_l}} = 0, \forall l \in \mathcal{L} \tag{18}$$

where "mod" represents the modulo operator, and $\mathcal{L}$ is the set of the independent loops without any phase shifting device.

According to [12], we use the approximations in (19) and (20) to linearize (17) for every loop $l \in \mathcal{L}$ as:

$$\sin\theta_{i_{m-1} i_{m \bmod n_l}} \approx \theta_{i_{m-1} i_{m \bmod n_l}}, \forall l \in \mathcal{L}, m = 1, \ldots, n_l \tag{19}$$

$$\left|W_{i_{m-1} i_{m \bmod n_l}}\right| \approx 1, \forall l \in \mathcal{L}, m = 1, \ldots, n_l \tag{20}$$

From (18)–(20), we further obtain:

$$\begin{aligned}\sum_{m=1}^{n_l} \big(&b_{i_{m-1} i_{m \bmod n_l}} P_{i_{m-1} i_{m \bmod n_l}}\\ &+ g_{i_{m-1} i_{m \bmod n_l}} Q_{i_{m-1} i_{m \bmod n_l}}\big) = 0, \forall l \in \mathcal{L}\end{aligned} \tag{21}$$

B. Master-Subproblem Framework

We reformulate the RO problem into a master/subproblem structure so as to solve it by an iterative process [14].

1) Master Problem: We take a finite subset of $\mathcal{P}_W$ to obtain a reduced problem of the original problem in (10). We use $h \in \mathbb{N}$ to label a possible realization of renewable generation as $\mathbf{P}_{\mathbf{W}(h)} \in \mathcal{P}_W$. Correspondingly, for such Scenario h, the set of re-dispatch decision variables is indexed by h as $\mathbf{Z}_{(h)}$. Let $\mathcal{H} := \{0, 1, \ldots, H\}$ be the index set of the selected scenarios. Denote the set of scenario-wise re-dispatch variables as $\mathbf{Z}_{(\mathcal{H})} := \{\mathbf{Z}_{(h)} \mid h \in \mathcal{H}\}$.

In particular, define a baseline scenario, namely, Scenario $h = 0$, where the renewable powers are equal to their predictions, i.e.:

$$\mathbf{P_{W}}_{(0)} := [P_{Wi,f}]_{i \in \mathcal{N}_W} \tag{22}$$

The master problem is formulated as follows:

$$\min_{\mathbf{X}, \widehat{\mathbf{Z}}_{(\mathcal{H})}} \sum_{i \in \mathcal{N}_G} \left(C_{Gi}\left(P_{Gi}\right) + c_{Ui} R_{Ui} + c_{Di} R_{Di} \right) + C_{ll}\left(P_{ll(\mathcal{H})}\right) \tag{23a}$$

subject to

$$P_{i(h)} = P_{Gi} + P_{Ui(h)} - P_{Di(h)} + P_{Wi(h)} - P_{Li}, \quad \forall i \in \mathcal{N}, h \in \mathcal{H} \tag{23b}$$

$$Q_{i(h)} = Q_{Gi(h)} + \eta_{Wi} P_{Wi(h)} - Q_{Li}, \forall i \in \mathcal{N}, h \in \mathcal{H} \tag{23c}$$

$$\widehat{\mathbf{Z}}_{(h)} \in \widehat{\mathcal{Z}}\left(\mathbf{X}, \mathbf{P_{W}}_{(h)}\right), \forall h \in \mathcal{H} \tag{23d}$$

$$\text{and (10b)–(10e)} \tag{23e}$$

where

$$\widehat{\mathcal{Z}}\left(\mathbf{X}, \mathbf{P_W}\right) := \left\{ \widehat{\mathbf{Z}} \mid \text{(10h)–(10k),(16),(21)} \right\} \tag{24}$$

is the relaxed feasible region of the re-dispatch decisions given the pre-dispatch decision $\mathbf{X}$ and the renewable generation $\mathbf{P_W}$.

From (23a), $P_{ll(\mathcal{H})} := \{P_{ll(h)} \mid h \in \mathcal{H}\}$ is the set of scenario-wise total line losses. The cost function of total line losses, $C_{ll}(P_{ll(\mathcal{H})})$, is implemented as follows:

$$C_{ll}\left(P_{ll(\mathcal{H})}\right) := c_{ll} \sum_{h \in \mathcal{H}} P_{ll(h)} \tag{25}$$

where $c_{ll} \geq 0$ is the cost coefficient of line losses.

2) Subproblem: Suppose we obtain a temporary pre-dispatch decision $\mathbf{X}_{temp}$ by solving the master problem in (23). The robust feasibility of $\mathbf{X}_{temp}$ over the uncertainty set $\mathcal{P}_W$ is examined by solving the subproblem as follows:

$$\max_{\mathbf{P_W} \in \mathcal{P}_\mathbf{W}} \min_{\mathbf{Z}, \mathbf{P_e}} \sum_{i \in \mathcal{N}} \left(P_{ei+} + P_{ei-}\right) + c_e P_{ll} \tag{26a}$$

subject to

$$P_i = P_{Gi,temp} + P_{Ui} - P_{Di} + P_{Wi} - P_{Li} + P_{ei+} - P_{ei-}, \forall i \in \mathcal{N} \tag{26b}$$

$$Q_i = Q_{Gi} + \eta_{Wi} P_{Wi} - Q_{Li}, \forall i \in \mathcal{N} \tag{26c}$$

$$P_{ei+}, P_{ei-} \geq 0, \forall i \in \mathcal{N} \tag{26d}$$

$$\widehat{\mathbf{Z}} \in \widehat{\mathcal{Z}}\left(\mathbf{X}_{temp}, \mathbf{P_W}\right) \tag{26e}$$

From (26a), P_{ei+} and P_{ei-} are the slack variables to compensate the positive and negative real power mismatches, respectively, at Bus $i \in \mathcal{N}$. $\mathbf{P_e} := \{(P_{ei+}, P_{ei-}) \mid i \in \mathcal{N}\}$ groups all of the slack variables. The effect of the total line loss on the power balance is considered by adding the term $c_e P_{ll}$ to the objective function where $c_e \geq 0$. Given $\mathbf{X}_{temp}$, suppose we solve the feasibility-check subproblem in (26) to obtain the optimal solutions $\mathbf{P}_{\mathbf{W},cri}$ and $\mathbf{P}_{\mathbf{e},cri}$. Then, the largest total real power mismatch $e(\mathbf{X}_{temp})$ over $\mathcal{P}_W$ is:

$$e\left(\mathbf{X}_{temp}\right) := \sum_{i \in \mathcal{N}} \left(P_{ei+,cri} + P_{ei-,cri}\right) \tag{27}$$

If $e\left(\mathbf{X}_{temp}\right) = 0$, the temporary decision $\mathbf{X}_{temp}$ can already guarantee robustness over $\mathcal{P}_W$. Otherwise, $e\left(\mathbf{X}_{temp}\right) > 0$. There exists at least one realization of $\mathbf{P_W} \in \mathcal{P}_W$, e.g. $\mathbf{P}_{\mathbf{W},cri}$, whereby the system fails to maintain the power balance with the pre-dispatch decision $\mathbf{X}_{temp}$.

C. Exactness of SOCP Relaxation

Assumption 1. *The robust OPF problem in* (10) *is feasible.*

Assumption 2. *The constraint*

$$|\angle V_{i_k} - \angle V_{k_i}| \leq \arctan\left(-\frac{b_{ik}}{g_{ik}}\right), \forall (i,k) \in \mathcal{E} \tag{28}$$

for the phase angle difference over a transmission line holds for the master problem in (23) *and the subproblem in* (26)*.*

Assumptions 1 and 2 are practical. Refer to our technical report [15] for their justifications.

Theorem 1. *Under Assumptions 2 and 1, a sufficient condition for exact SOCP relaxation of the master problem in* (23) *is:*

$$c_{ll} > \max\left\{ \max_{i \in \mathcal{N}_G} c_{Ui}, \max_{i \in \mathcal{N}_G} c_{Di} \right\} \tag{29}$$

Proof: Refer to our technical report [15]. ∎

Theorem 2. *Under Assumptions 2 and 1, a sufficient condition for exact SOCP relaxation of the inner minimization problem of the subproblem in* (26) *with any* $\mathbf{P_W} \in \mathcal{P}_W$ *is:*

$$c_e > 1 \tag{30}$$

Proof: Refer to our technical report [15]. ∎

According to Theorem 2, the subproblem in (26) can be converted into a mixed-integer SOCP problem which can be solved by existing solvers. Due to the convexity of the solution space and the polyhedral property of the uncertainty set $\mathcal{P}_W$, optimality can be guaranteed by searching the extreme points of $\mathcal{P}_W$. The detailed procedure for the conversion of the problem can be found in our technical report [15].

D. Column-and-Constraint Generation Algorithm

Based on the column-and-constraint generation algorithm proposed in [14], we design an iterative algorithm, namely, Algorithm 1, to solve the robust dispatch problem in (10) with the master problem in (23) and the subproblem in (26).

Algorithm 1 Robust Dispatch with Renewable Generation

Input: The uncertainty set $\mathcal{P}_W$, and all specifications of the components in the power network required by the master problem in (23) and the subproblem in (26).

Output: The final pre-dispatch decision $\mathbf{X}_{opt}$, and the set of re-dispatch decisions for the critical scenarios, $\widehat{\mathbf{Z}}_{(\mathcal{H}),opt}$.

Initialize the index set of critical scenarios $\mathcal{H} \leftarrow \{0\}$, the iteration indicator $H \leftarrow 0$, and the power mismatch $e_p \leftarrow 1$.

Repeat Steps 1–3 until $e_p = 0$.

1) Pre-dispatch: Solve the master problem in (23) to obtain the temporary pre-dispatch decision $\mathbf{X}_{temp}$, and the set of re-dispatch decisions $\widehat{\mathbf{Z}}_{(\mathcal{H})}$.
2) Robust feasibility check: Given $\mathbf{X}_{temp}$, solve the subproblem in (26) to obtain the critical scenario of renewable generation $\mathbf{P}_{\mathbf{W},cri}$, and the largest power mismatch $e(\mathbf{X}_{temp})$ over $\mathcal{P}_W$. Set $e_p \leftarrow e(\mathbf{X}_{temp})$.
3) If $e_p > 0$, add $\mathbf{P}_{\mathbf{W},cri}$ to the set of critical scenarios by setting $\mathbf{P_W}_{(H+1)} \leftarrow \mathbf{P}_{\mathbf{W},cri}$ and $\mathcal{H} \leftarrow \mathcal{H} \cup \{H+1\}$. Set $H \leftarrow H+1$, go to Step 1.

Return $\mathbf{X}_{opt} \leftarrow \mathbf{X}_{temp}, \widehat{\mathbf{Z}}_{(\mathcal{H}),opt} \leftarrow \widehat{\mathbf{Z}}_{(\mathcal{H})}$.

Let D be the number of extreme points of the polyhedral uncertainty set $\mathcal{P}_W$. Since we only need to search the extreme points of $\mathcal{P}_W$ to achieve robustness, Algorithm 1 will converge to $e(\mathbf{X}_{temp}) = 0$ in no greater than $D+1$ iterations.

V. Case Study

A. Test Systems

The 24-bus IEEE Reliability Test System (RTS) [16] is chosen as the baseline test system. Without loss of generality, we use wind generators as the RESs in this study. The RTS is modified by installing six wind generators on six buses, namely, Buses 1, 2, 6, 8, 17, and 22. Four test systems with different configurations of PFCs and PFRs are analyzed:

1) Baseline system: RTS without any PFC or PFR.
2) System with 11 PFCs: We install one PFC at each of the 11 independent loops that contain at least three buses so that the phase angle constraints in (18) are relaxed by the phase shifting devices.
3) System with 5 PFRs: Each of Buses 1, 10, 13, 16, and 21 is represented by a PFR so that there is at least one PFR in each of the loops with at least three buses.
4) System with full coverage of PFRs: Each of the 24 buses of the RTS is represented by a PFR.

B. Performance Metrics

Given the pre-dispatch decision $\mathbf{X}$ and the set of re-dispatch decisions for the selected scenarios, $\widehat{\mathbf{Z}}_{(\mathcal{H})}$, define a composite energy management cost function $C_{comp}(\mathbf{X}, \widehat{\mathbf{Z}}_{(\mathcal{H})})$ as follows:

$$C_{comp}\left(\mathbf{X}, \widehat{\mathbf{Z}}_{(\mathcal{H})}\right) := \sum_{i\in\mathcal{N}_G} \left(C_{Gi}(P_{Gi}) + c_{Ui}P_{Ui} + c_{Di}P_{Di}\right) + c_{ll} \max_{h\in\mathcal{H}} P_{ll(h)} \tag{31}$$

The performance metric of this study is the composite energy management cost $C_{comp}(\mathbf{X}_{opt}, \widehat{\mathbf{Z}}_{(\mathcal{H}),opt})$ where $\mathbf{X}_{opt}$ and $\widehat{\mathbf{Z}}_{(\mathcal{H}),opt}$ are obtained by performing Algorithm 1.

C. General Setup

The specifications of the conventional generators are obtained from [16] and the MATPOWER [17] power flow data file for the IEEE RTS. The generation cost function $C_{Gi}(P_{Gi})$ at Bus i is set to be quadratic. The coefficient for total power losses c_{ll} is set as 1.01 times of the maximum value among all c_{Ui}'s and c_{Di}'s. The parameter c_e in (26a) is set as $c_e = 1.1$.

We investigate the energy management costs when the wind energy penetrations, i.e., the ratios between wind generation and consumption, are 25% and 40%, respectively. Accounting for the increase in loadability due to the wind energy, the total demand is scaled up to around 4000 MW from 2850 MW which is the total load of the RTS nominal load scenario.

We use the data of total wind power and load published online in [18] to simulate the temporal variations. The dispatch interval is set as $\Delta t = 10$ minutes, which is the same as the data sampling period of the data in [18]. Similar to [2], we assume that the load forecasts are accurate while the maximum prediction error of the wind power output is 15% of its forecast. We perform the simulations over a two-hour horizon with twelve dispatch intervals. The budget of uncertainty Γ_W is set as 0, 2, 4, and 6, respectively.

For ease of analysis, we assume that the voltage control capability of the line PFC $\gamma_{i_k,max} = 0.05$, and the limits of the branch reactive power compensation $Q_{Cik,max} = -Q_{Cik,min} = 5$ Mvar for every PFC and PFR in this study.

D. Numerical Results

Table I summarizes the robust dispatch results in terms of the total composite energy management costs defined in (31), and the cost reductions of the three test systems with PFCs or PFRs as percentage savings compared to the baseline system. Consistent with our intuition, the total cost increases as Γ_W increases while it decreases as the wind penetration increases. The results show that, by introducing power flow routing to the grid, remarkable cost reductions are achieved. Given the same value of Γ_W, the composite energy management costs of the systems with 11 PFCs and 5 PFRs, respectively, are close to that of the system with full coverage of PFRs. This suggests that a placement scheme that ensures all the phase angle constraints for loops in (18) are relaxed by PFCs or PFRs can be very effective in improving the performance.

Figs. 3 and 4 present the breakdowns of the composite energy management costs when the wind energy penetrations are 25% and 40%, respectively. As we compare the cost breakdowns across the four test systems at a given Γ_W, it can be observed that the savings of the composite costs achieved by power flow routing mainly come from the reductions of the generation costs, although the introduction of PFCs and PFRs also contributes to reducing the costs of spinning reserve. Furthermore, while the generation costs of the baseline system show obvious growth as Γ_W increases from two to six, those of the test systems with PFCs and PFRs remain almost unchanged. This exhibits another merit of power flow routing in that the system with PFCs and PFRs is able to maintain a more stable dispatch schedule against uncertainty than the system without PFC nor PFR.

The average numbers of iterations performed in the robust dispatch algorithm, namely, Algorithm 1, in various test cases are presented in Table II. The result for each test case is the number of iterations averaged over the twelve dispatch intervals. Except for the cases with $\Gamma_W = 0$ where the system

TABLE I
TOTAL COMPOSITE ENERGY MANAGEMENT COSTS AND SAVINGS COMPARED TO BASELINE TEST CASES OVER TWO-HOUR HORIZON

Wind	Γ_W	Baseline	With 11 PFCs		With 5 PFRs		With Full PFRs	
		Cost / $\times 10^3$\$	Cost / $\times 10^3$\$	Saving	Cost / $\times 10^3$\$	Saving	Cost / $\times 10^3$\$	Saving
25%	0	134.2	124.5	7.18%	122.5	8.72%	**120.4**	**10.29%**
	2	140.7	133.4	5.21%	131.4	6.58%	**129.1**	**8.27%**
	4	146.5	135.1	7.81%	132.7	9.44%	**130.5**	**10.93%**
	6	147.9	136.8	7.52%	134.7	8.93%	**132.5**	**10.45%**
40%	0	110.5	106.5	3.60%	103.9	5.98%	**102.0**	**7.65%**
	2	122.0	114.9	5.82%	113.1	7.25%	**110.8**	**9.17%**
	4	129.5	119.3	7.86%	116.3	10.20%	**113.4**	**11.96%**
	6	134.0	122.3	8.73%	118.9	11.23%	**116.6**	**12.97%**

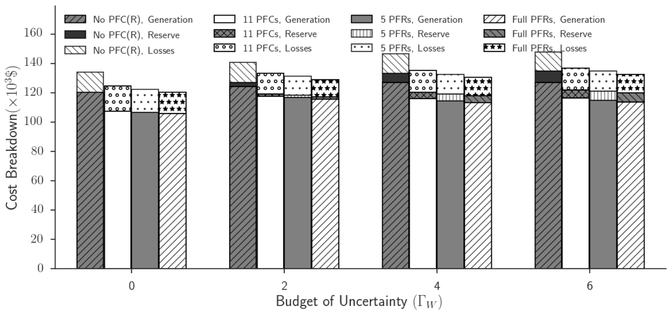

Fig. 3. Breakdowns of the composite energy management costs when the wind energy penetration is 25%.

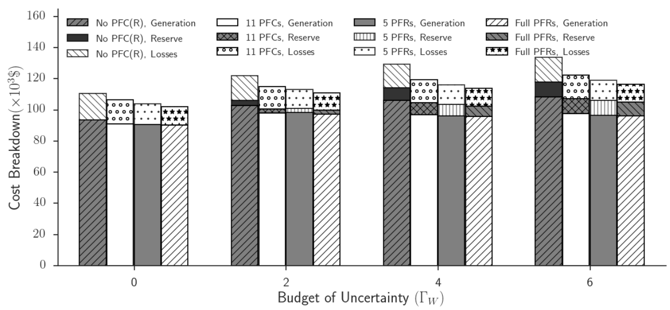

Fig. 4. Breakdowns of the composite energy management costs when the wind energy penetration is 40%.

operator does not consider the prediction errors, most of the test cases that can account for the uncertainty converge in three or four iterations. When $\Gamma_W > 0$, since we consider both over and under generation of RESs in the simulations, Algorithm 1 has to find at least one critical scenario for down-spinning reserve and another for up-spinning reserve at each dispatch interval. This means that it would run for at least three iterations before converging. In other words, the results suggest that the proposed method is efficient in obtaining the optimal dispatch decisions.

VI. Conclusions

The robust dispatch problem of conventional generators under the uncertainty of RESs is studied. A novel robust OPF formulation with the AC power flow model is proposed to incorporate power flow routing into the network and improve the accuracy of power flow calculation. Our method optimizes the allocation of energy supply and spinning reserve given the requirement of robustness, as well as accounts for the effect of real power losses on determining the requirement of spinning reserve and attaining exact SOCP relaxation. The numerical study shows that, with just a few PFRs installed, the system can already achieve remarkable reduction of energy management costs. Future work will study the optimal placement scheme of PFCs and PFRs while considering the infrastructure costs of power flow routing.

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TABLE II
Average Number of Iterations Performed in Algorithm 1

Wind	25%				40%			
Γ_W	0	2	4	6	0	2	4	6
Baseline	1	3.17	3.33	3.17	1	3	3.33	3.17
With 11 PFCs	1	3.92	3	3	1	4	3	3
With 5 PFRs	1	4	3	3	1	4.08	3	3
With Full PFRs	1	4	3	3	1	4.08	3	3

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